Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.4-u-a+b-arctanh-c-x-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [538]. This is test number [194].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (538)	% 0.00 (0)
Mathematica	% 99.63 (536)	% 0.37 (2)
Maple	% 94.42 (508)	% 5.58 (30)
Maxima	% 50.19 (270)	% 49.81 (268)
Fricas	% 47.58 (256)	% 52.42 (282)
Sympy	% 26.77 (144)	% 73.23 (394)
Giac	% 32.53 (175)	% 67.47 (363)
Mupad	% 32.53 (175)	% 67.47 (363)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

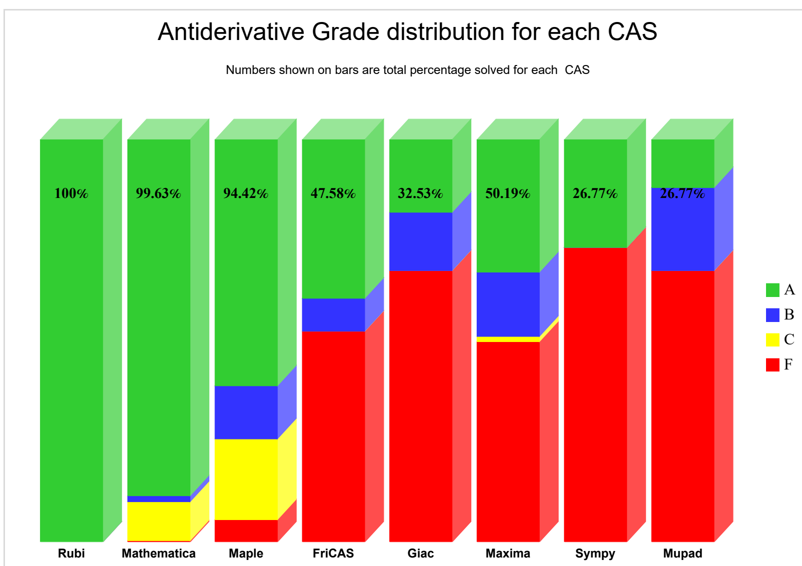
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

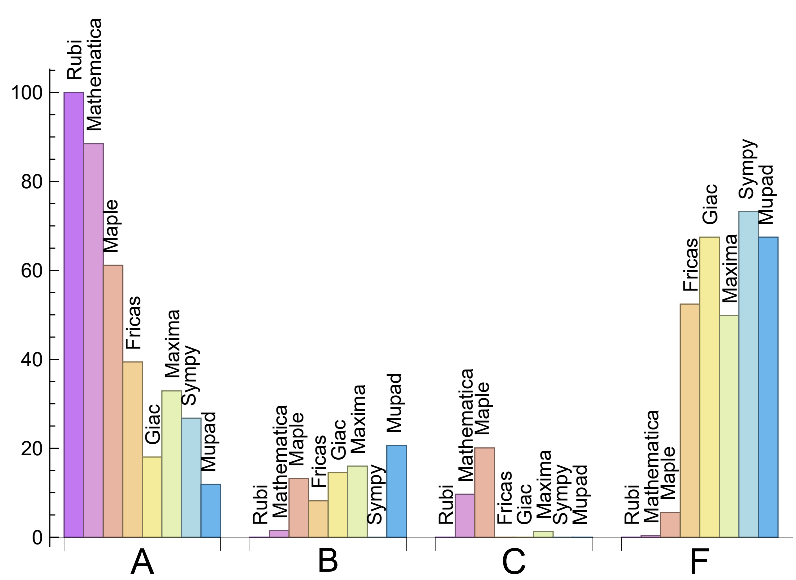
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	88.48	1.49	9.67	0.37
Maple	61.15	13.20	20.07	5.58
Maxima	32.90	15.99	1.30	49.81
Fricas	39.41	8.18	0.00	52.42
Sympy	26.77	0.00	0.00	73.23
Giac	18.03	14.50	0.00	67.47
Mupad	11.90	20.63	0.00	67.47

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	30	80.00 %	0.00 %	20.00 %
Maxima	268	98.88 %	0.00 %	1.12 %
Fricas	282	98.94 %	0.00 %	1.06 %
Sympy	394	97.97 %	2.03 %	0.00 %
Giac	363	86.50 %	0.28 %	13.22 %
Mupad	363	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

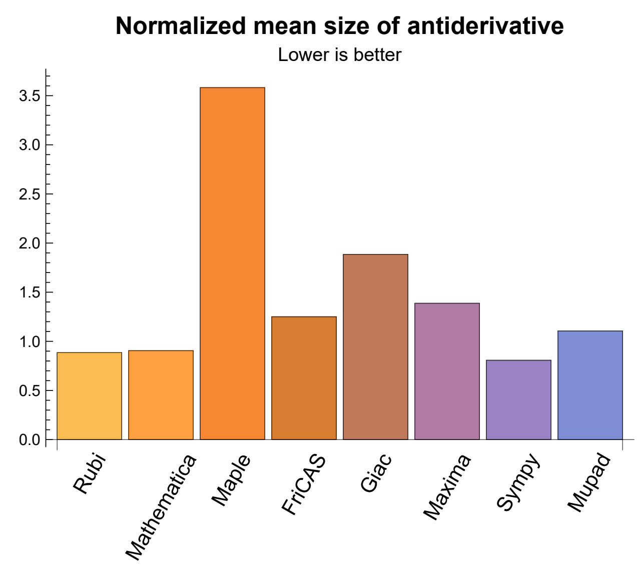
1.3 Performance

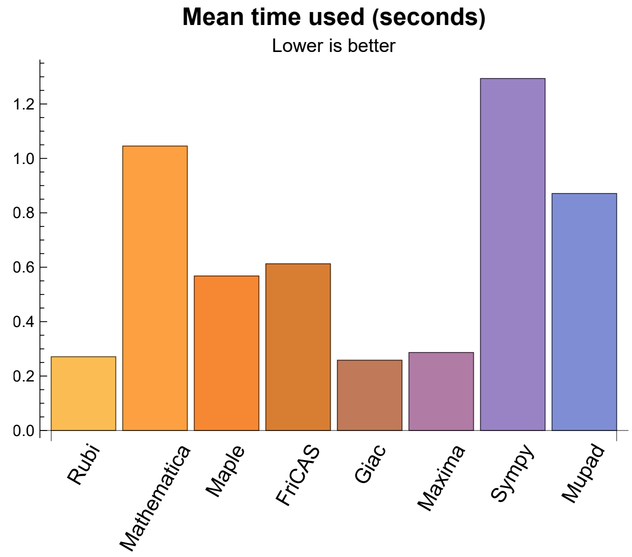
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	140.45	0.89	125.00	1.00
Mathematica	1.05	153.47	0.91	102.00	0.89
Maple	0.57	656.32	3.58	180.50	1.40
Maxima	0.29	184.50	1.39	126.00	1.22
Fricas	0.61	114.36	1.25	91.00	0.99
Sympy	1.29	83.40	0.81	39.00	0.87
Giac	0.26	208.11	1.88	135.00	1.48
Mupad	0.87	136.34	1.11	65.00	0.93

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 249, 251, 252, 254, 255, 257, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 421, 422, 425, 479, 480, 485, 486, 491, 492, 511, 512, 535}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 178, 180, 182, 183, 184, 197, 201, 203, 205, 207, 209, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 306, 312, 318, 319, 320, 348, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 497, 503, 504, 508, 510, 517, 518, 529, 531, 533, 536, 537}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
```

```
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

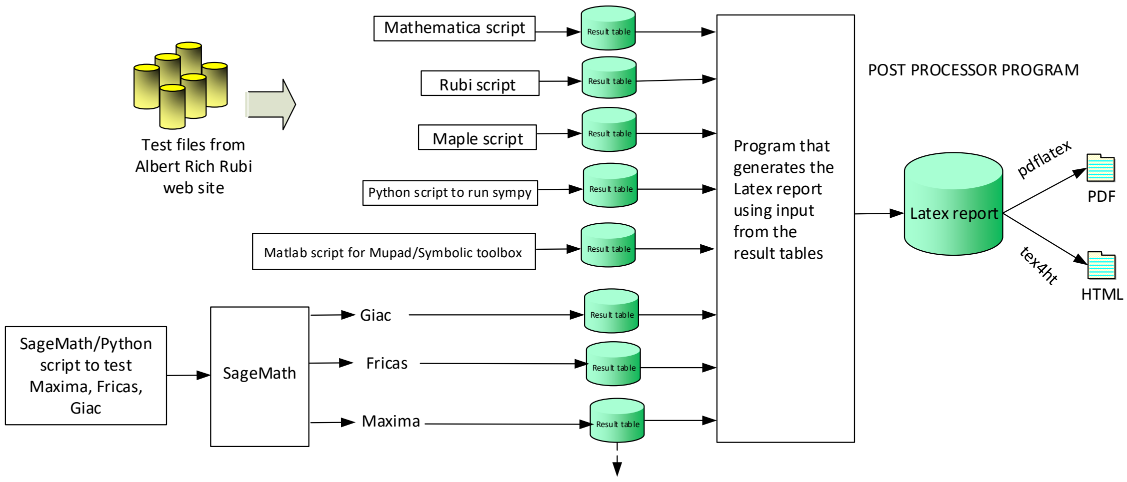
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form “{n,n,..}” which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 111, 112, 113, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 505, 507, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 529, 531, 535, 538 }

B grade: { 4, 120, 121, 383, 405, 504, 528, 530 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 99, 100, 101, 102, 108, 109, 110, 116, 117, 132, 133, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 212, 240, 246, 270, 272, 278, 312, 319, 502, 506, 508, 509, 510, 533, 534, 536, 537 }

F grade: { 527, 532 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 76, 77, 78, 84, 85, 94, 127, 134, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 176, 178, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 230, 237, 241, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 367, 369, 370, 371, 372, 373, 375, 377, 378, 379, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 419, 420, 421, 422, 424, 425, 426, 428, 430, 431, 432, 433, 434, 435, 436, 437, 439, 441, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 491, 492, 493, 498, 499, 500, 501, 505, 506, 507, 508, 511, 512, 517, 518, 519, 520, 521, 535 }

B grade: { 23, 28, 34, 40, 46, 47, 48, 49, 67, 71, 74, 75, 79, 83, 86, 87, 92, 93, 107, 114, 115, 118, 133, 139, 140, 173, 175, 180, 181, 182, 214, 227, 228, 229, 231, 232, 233, 247, 259, 260, 262, 263, 264, 265, 268, 278, 279, 303, 305, 306, 307, 308, 310, 319, 345, 413, 418, 423, 488, 489, 490, 494, 495, 496, 502, 503, 504, 509, 510, 537, 538 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 135, 136, 137, 138, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 235, 236, 238, 239, 240, 242, 243, 245, 246, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 309, 311, 312, 313, 314, 315, 316, 317, 318, 346, 347, 364, 366, 368, 388, 427, 429, 448, 450, 497, 522, 523, 524, 525, 526, 527, 533 }

F grade: { 280, 320, 348, 374, 376, 380, 381, 382, 383, 397, 404, 405, 438, 440, 442, 443, 444, 471, 475, 513, 514, 515, 516, 528, 529, 530, 531, 532, 534, 536 }

2.1.4 Maxima

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 14, 18, 19, 20, 21, 24, 25, 30, 31, 35, 36, 42, 54, 62, 66, 69, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 227, 249, 251, 252, 254, 255, 257, 259, 261, 281, 282, 286, 287, 291, 292, 296, 302, 304, 307, 310, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 364, 366, 368, 371, 387, 388, 390, 391, 393, 395, 398, 399, 403, 406, 407, 411, 412, 415, 416, 417, 420, 421, 422, 425, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 469, 470, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 503, 505, 506, 507, 508, 511, 512, 519, 520, 521, 525, 527, 535 }

B grade: { 4, 13, 17, 22, 23, 28, 29, 32, 33, 34, 37, 40, 41, 61, 67, 71, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 107, 114, 115, 118, 124, 125, 126, 166, 181, 228, 229, 230, 231, 232, 233, 235, 237, 239, 244, 250, 253, 256, 260, 262, 263, 264, 265, 267, 268, 269, 271, 274, 275, 276, 303, 305, 306, 308, 309, 311, 313, 314, 315, 316, 317, 345, 346, 347, 468, 472, 473, 474, 504, 513, 514, 515, 516 }

C grade: { 502, 509, 522, 523, 524, 526, 538 }

F grade: { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 68, 70, 72, 73, 74, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 236, 238, 240, 241, 242, 243, 245, 246, 247, 248, 258, 266, 270, 272, 273, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 312, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 510, 517, 518, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 126, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 218, 219, 220, 221, 222, 223, 224, 228, 230, 232, 237, 244, 249, 251, 252, 253, 254, 255, 256, 257, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 281, 282, 286, 287, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 321, 322, 328, 329, 335, 341, 344, 345, 346, 347, 349, 350, 358, 359, 364, 366, 368, 371, 387, 388, 390, 391, 393, 395, 398, 399, 403, 406, 407, 411, 412, 415, 416, 417, 420, 421, 422, 425, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 468, 469, 470, 472, 473, 474, 476, 477, 478, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 505, 511, 512, 519, 520, 521, 522, 523, 524, 525, 526, 535, 538 }

B grade: { 248, 250, 258, 283, 284, 285, 288, 289, 290, 293, 294, 295, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 513, 514, 515, 516 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 280, 306, 312, 313, 318, 319, 320, 348, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 517, 518, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 218, 219, 220, 221, 222, 223, 224, 228, 230, 232, 237, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 264, 281, 282, 286, 287, 291, 292, 296, 302, 304, 307, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 422, 425, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 505, 511, 512, 522, 523, 524, 525, 526 }

B grade: { }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 260, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 421, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.7 Giac

A grade: { 54, 61, 62, 107, 114, 115, 124, 125, 141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 230, 237, 244, 249, 251, 252, 253, 254, 255, 256, 257, 268, 269, 275, 276, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 368, 387, 390, 391, 395, 403, 411, 412, 416, 417, 421, 422, 462, 463, 464, 468, 469, 470, 479, 480, 485, 486, 491, 492, 511, 512, 513, 514, 515, 516, 519, 520, 521, 522, 524, 535, 538 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 66, 76, 78, 118, 126, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 250, 261, 262, 302, 304, 308, 310, 314, 316, 371, 393, 498, 499, 500, 501, 505, 525, 526 }

C grade: { }

F grade: { 5, 6, 14, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 258, 259, 260, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 307, 309, 311, 312, 313, 315, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 392, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 415, 418, 419, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 465, 466, 467, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 517, 518, 523, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.8 Mupad

A grade: { 141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 249, 251, 252, 254, 255, 257, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 421, 422, 425, 479, 480, 485, 486, 491, 492, 511, 512, 535 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 126, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 228, 230, 232, 237, 244, 248, 250, 253, 256, 258, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 345, 346, 347, 498, 499, 500, 501, 505, 522, 523, 524, 525, 526, 538 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 306, 312, 313, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	101	121	114	124	491	103
normalized size	1	1.00	0.90	0.94	1.12	1.06	1.15	4.55	0.95
time (sec)	N/A	0.097	0.073	0.030	0.311	0.482	1.533	0.246	0.989

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	87	91	110	102	112	394	92
normalized size	1	1.00	0.91	0.95	1.15	1.06	1.17	4.10	0.96
time (sec)	N/A	0.097	0.064	0.028	0.320	0.656	1.201	0.199	0.944

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	81	99	93	100	305	83
normalized size	1	1.00	0.94	0.96	1.18	1.11	1.19	3.63	0.99
time (sec)	N/A	0.075	0.055	0.031	0.307	0.543	0.924	0.162	0.901

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	95	65	85	77	75	211	65
normalized size	1	1.00	2.16	1.48	1.93	1.75	1.70	4.80	1.48
time (sec)	N/A	0.030	0.010	0.029	0.314	0.588	0.577	0.177	0.864

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	86	0	0	0	0	-1
normalized size	1	1.00	0.90	1.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.075	0.047	0.000	0.419	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	105	0	0	0	0	-1
normalized size	1	1.00	1.01	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.066	0.054	0.000	0.421	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	76	84	89	89	95	192	75
normalized size	1	1.00	1.36	1.50	1.59	1.59	1.70	3.43	1.34
time (sec)	N/A	0.054	0.061	0.040	0.315	0.481	1.076	0.196	0.902

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	95	99	101	117	306	110
normalized size	1	1.00	0.88	0.97	1.01	1.03	1.19	3.12	1.12
time (sec)	N/A	0.087	0.067	0.041	0.314	0.451	1.422	0.286	0.900

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	94	105	114	110	129	401	120
normalized size	1	1.00	0.85	0.95	1.04	1.00	1.17	3.65	1.09
time (sec)	N/A	0.092	0.066	0.040	0.315	0.457	1.881	0.167	0.846

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	125	159	210	162	196	620	146
normalized size	1	1.00	0.80	1.01	1.34	1.03	1.25	3.95	0.93
time (sec)	N/A	0.168	0.108	0.030	0.330	0.452	2.155	0.276	1.043

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	115	147	184	146	177	525	134
normalized size	1	1.00	0.80	1.03	1.29	1.02	1.24	3.67	0.94
time (sec)	N/A	0.154	0.092	0.030	0.312	0.504	1.713	0.233	1.032

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	107	135	179	137	167	425	122
normalized size	1	1.00	0.83	1.05	1.39	1.06	1.29	3.29	0.95
time (sec)	N/A	0.130	0.086	0.028	0.313	0.691	1.327	0.408	0.966

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	92	121	147	114	131	330	105
normalized size	1	1.00	1.30	1.70	2.07	1.61	1.85	4.65	1.48
time (sec)	N/A	0.043	0.106	0.028	0.315	0.453	0.958	0.203	0.962

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	103	142	173	0	0	0	-1
normalized size	1	1.00	0.90	1.25	1.52	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.103	0.049	0.472	0.411	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	80	73	123	0	0	0	410	-1
normalized size	1	1.31	1.20	2.02	0.00	0.00	0.00	6.72	-0.02
time (sec)	N/A	0.126	0.106	0.047	0.000	0.462	0.000	1.802	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	143	176	0	0	0	0	-1
normalized size	1	1.00	1.04	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.099	0.056	0.000	0.615	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	103	141	157	128	158	330	116
normalized size	1	1.00	1.27	1.74	1.94	1.58	1.95	4.07	1.43
time (sec)	N/A	0.085	0.094	0.043	0.320	0.633	1.578	0.153	0.896

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	114	153	178	147	189	431	168
normalized size	1	1.00	0.78	1.04	1.21	1.00	1.29	2.93	1.14
time (sec)	N/A	0.149	0.095	0.040	0.319	0.696	2.021	0.282	1.008

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	122	165	194	156	199	532	182
normalized size	1	1.00	0.76	1.02	1.20	0.97	1.24	3.30	1.13
time (sec)	N/A	0.160	0.097	0.042	0.319	0.535	2.564	0.271	0.952

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	151	199	285	190	243	722	177
normalized size	1	1.00	0.79	1.04	1.48	0.99	1.27	3.76	0.92
time (sec)	N/A	0.181	0.136	0.030	0.320	0.441	2.835	0.274	1.054

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	142	187	265	178	235	621	165
normalized size	1	1.00	0.80	1.05	1.49	1.00	1.32	3.49	0.93
time (sec)	N/A	0.176	0.126	0.029	0.321	0.439	2.291	0.231	1.032

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	173	244	165	211	527	153
normalized size	1	1.00	0.99	1.28	1.81	1.22	1.56	3.90	1.13
time (sec)	N/A	0.101	0.109	0.030	0.318	0.489	1.793	0.518	0.978

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	115	162	219	149	182	425	136
normalized size	1	1.00	1.37	1.93	2.61	1.77	2.17	5.06	1.62
time (sec)	N/A	0.051	0.145	0.028	0.320	0.486	1.314	0.217	0.956

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	148	182	228	0	0	0	-1
normalized size	1	1.00	0.97	1.20	1.50	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.133	0.049	0.460	0.407	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	189	229	0	0	0	-1
normalized size	1	1.00	0.99	1.26	1.53	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.144	0.054	0.488	0.459	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	165	200	0	0	0	0	-1
normalized size	1	1.00	1.03	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.145	0.057	0.000	0.577	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	175	216	0	0	0	0	-1
normalized size	1	1.00	0.99	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.135	0.057	0.000	0.445	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	131	181	228	163	207	431	147
normalized size	1	1.00	1.41	1.95	2.45	1.75	2.23	4.63	1.58
time (sec)	N/A	0.099	0.118	0.041	0.328	0.423	2.021	0.244	0.949

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	140	193	250	175	233	533	233
normalized size	1	1.00	1.02	1.41	1.82	1.28	1.70	3.89	1.70
time (sec)	N/A	0.119	0.127	0.041	0.326	0.452	2.624	0.200	0.980

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	149	205	273	188	257	634	220
normalized size	1	1.00	0.76	1.05	1.39	0.96	1.31	3.23	1.12
time (sec)	N/A	0.177	0.130	0.040	0.328	0.480	3.306	0.164	1.067

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	177	237	373	222	294	817	337
normalized size	1	1.00	0.79	1.06	1.67	0.99	1.31	3.65	1.50
time (sec)	N/A	0.213	0.159	0.033	0.333	0.548	3.756	0.343	1.839

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	168	225	339	208	279	723	196
normalized size	1	1.00	0.98	1.32	1.98	1.22	1.63	4.23	1.15
time (sec)	N/A	0.181	0.150	0.030	0.333	0.520	2.990	0.188	1.077

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	159	215	326	198	269	621	185
normalized size	1	1.00	1.04	1.41	2.13	1.29	1.76	4.06	1.21
time (sec)	N/A	0.116	0.141	0.030	0.336	0.505	2.359	0.276	1.048

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	146	202	283	177	226	526	168
normalized size	1	1.00	1.36	1.89	2.64	1.65	2.11	4.92	1.57
time (sec)	N/A	0.055	0.176	0.028	0.334	0.428	1.849	0.239	1.048

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	179	222	276	0	0	0	-1
normalized size	1	1.00	0.97	1.20	1.49	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.171	0.050	0.472	0.486	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	194	229	281	0	0	0	-1
normalized size	1	1.00	1.09	1.29	1.58	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.178	0.054	0.474	0.488	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	143	210	293	0	0	0	-1
normalized size	1	1.00	0.92	1.35	1.88	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.165	0.052	0.477	0.421	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	197	240	0	0	0	0	-1
normalized size	1	1.00	1.04	1.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.175	0.056	0.000	0.834	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	256	0	0	0	0	-1
normalized size	1	1.00	0.99	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.160	0.057	0.000	0.413	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	157	221	299	191	253	532	179
normalized size	1	1.00	1.44	2.03	2.74	1.75	2.32	4.88	1.64
time (sec)	N/A	0.105	0.146	0.040	0.329	0.478	2.701	0.233	0.955

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	166	233	329	208	291	634	248
normalized size	1	1.00	1.10	1.54	2.18	1.38	1.93	4.20	1.64
time (sec)	N/A	0.125	0.155	0.040	0.327	0.470	3.365	0.557	1.235

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	175	245	353	218	301	735	260
normalized size	1	1.00	0.76	1.07	1.54	0.95	1.31	3.21	1.14
time (sec)	N/A	0.197	0.178	0.043	0.327	0.605	4.188	0.328	1.209

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	129	253	0	0	0	0	-1
normalized size	1	1.00	0.73	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.391	0.053	0.000	0.596	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	97	213	0	0	0	0	-1
normalized size	1	1.00	0.67	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.247	0.050	0.000	0.434	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	157	0	0	0	0	-1
normalized size	1	1.00	0.80	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.156	0.047	0.000	0.450	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	112	0	0	0	0	-1
normalized size	1	1.00	1.02	2.20	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.101	0.045	0.000	0.419	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	156	0	0	0	0	-1
normalized size	1	1.00	1.20	3.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.113	0.052	0.000	0.521	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	225	0	0	0	0	-1
normalized size	1	1.00	1.00	2.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.199	0.062	0.000	0.605	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	133	286	0	0	0	0	-1
normalized size	1	1.00	0.91	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.341	0.063	0.000	0.634	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	172	328	0	0	0	0	-1
normalized size	1	1.00	0.93	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.346	0.426	0.062	0.000	0.610	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	142	265	0	0	0	0	-1
normalized size	1	1.00	0.78	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.703	0.059	0.000	0.522	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	216	0	0	0	0	-1
normalized size	1	1.00	0.81	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.579	0.054	0.000	0.630	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	99	192	0	0	0	0	-1
normalized size	1	1.00	0.93	1.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.373	0.056	0.000	0.646	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	64	84	96	49	121	63	45
normalized size	1	1.00	1.12	1.47	1.68	0.86	2.12	1.11	0.79
time (sec)	N/A	0.045	0.069	0.035	0.304	1.292	1.385	0.243	1.075

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	101	221	0	0	0	0	-1
normalized size	1	1.00	0.81	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.418	0.062	0.000	0.405	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	269	0	0	0	0	-1
normalized size	1	1.00	0.82	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.827	0.063	0.000	0.649	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	189	338	0	0	0	0	-1
normalized size	1	1.00	0.89	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	1.107	0.074	0.000	1.532	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	189	319	0	0	0	0	-1
normalized size	1	1.00	0.83	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.817	0.059	0.000	0.760	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	167	270	0	0	0	0	-1
normalized size	1	1.00	0.86	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.696	0.056	0.000	0.517	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	145	246	0	0	0	0	-1
normalized size	1	1.00	0.97	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.489	0.056	0.000	0.562	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	99	136	152	84	306	114	81
normalized size	1	1.00	1.29	1.77	1.97	1.09	3.97	1.48	1.05
time (sec)	N/A	0.082	0.111	0.039	0.328	0.417	2.259	0.201	1.266

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	86	100	134	75	250	118	123
normalized size	1	1.00	1.12	1.30	1.74	0.97	3.25	1.53	1.60
time (sec)	N/A	0.055	0.073	0.036	0.318	0.413	2.154	0.218	1.089

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	147	264	0	0	0	0	-1
normalized size	1	1.00	0.91	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.531	0.063	0.000	0.542	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	186	319	0	0	0	0	-1
normalized size	1	1.00	0.85	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	1.238	0.060	0.000	0.446	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	220	394	0	0	0	0	-1
normalized size	1	1.00	0.82	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	1.474	0.068	0.000	0.433	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	95	132	91	294	161	139
normalized size	1	1.00	0.94	1.19	1.65	1.14	3.68	2.01	1.74
time (sec)	N/A	0.054	0.114	0.036	0.311	0.432	2.756	0.199	1.096

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	126	120	0	0	0	-1
normalized size	1	1.00	0.95	3.07	2.93	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.077	0.054	0.315	0.402	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	271	422	0	0	0	0	-1
normalized size	1	1.00	1.00	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	0.760	0.059	0.000	0.437	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	234	383	402	0	0	0	-1
normalized size	1	1.00	0.99	1.62	1.70	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	0.570	0.063	0.659	0.506	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	201	341	0	0	0	0	-1
normalized size	1	1.00	1.03	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.394	0.506	0.060	0.000	0.606	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	156	296	290	0	0	0	-1
normalized size	1	1.00	1.39	2.64	2.59	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.322	0.058	0.564	0.454	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	228	3644	0	0	0	0	-1
normalized size	1	1.00	1.19	19.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.451	0.514	0.457	0.000	0.472	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	249	3104	0	0	0	0	-1
normalized size	1	1.00	1.24	15.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	0.518	0.983	0.000	0.472	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	206	400	0	0	0	0	-1
normalized size	1	1.00	1.36	2.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.365	0.283	0.078	0.000	0.509	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	246	440	417	0	0	0	-1
normalized size	1	1.00	1.19	2.14	2.02	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.482	0.081	0.924	0.459	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	329	569	766	0	0	1135	-1
normalized size	1	1.00	0.92	1.60	2.15	0.00	0.00	3.19	-0.00
time (sec)	N/A	1.024	1.096	0.059	0.688	0.745	0.000	4.222	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	297	521	604	0	0	0	-1
normalized size	1	1.00	0.95	1.67	1.94	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.884	1.029	0.059	0.658	0.455	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	263	478	610	0	0	761	-1
normalized size	1	1.00	0.94	1.71	2.18	0.00	0.00	2.72	-0.00
time (sec)	N/A	0.652	0.777	0.054	0.673	0.641	0.000	2.364	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	227	372	464	0	0	0	-1
normalized size	1	1.00	1.30	2.13	2.65	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.663	0.057	0.553	0.766	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	324	1082	0	0	0	0	-1
normalized size	1	1.00	1.17	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.586	0.707	1.099	0.000	0.710	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	341	6039	0	0	0	0	-1
normalized size	1	1.00	1.20	21.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	0.525	1.010	0.000	0.651	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	370	1167	0	0	0	0	-1
normalized size	1	1.00	1.18	3.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.673	0.754	1.735	0.000	0.463	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	270	550	555	0	0	0	-1
normalized size	1	1.00	1.11	2.25	2.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.635	0.079	0.936	0.646	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	385	662	928	0	0	0	-1
normalized size	1	1.00	0.93	1.60	2.24	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.459	1.664	0.060	0.703	0.541	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	356	618	775	0	0	0	-1
normalized size	1	1.00	0.94	1.64	2.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.230	1.330	0.060	0.700	0.692	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	325	570	780	0	0	0	-1
normalized size	1	1.00	1.14	1.99	2.73	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	1.207	0.061	0.696	0.489	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	293	462	627	0	0	0	-1
normalized size	1	1.00	1.42	2.24	3.04	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.875	0.056	0.563	0.577	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	448	1186	0	0	0	0	-1
normalized size	1	1.00	1.26	3.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.813	0.833	1.527	0.000	0.625	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	479	1270	0	0	0	0	-1
normalized size	1	1.00	1.33	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.776	0.641	1.939	0.000	0.859	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	461	1358	0	0	0	0	-1
normalized size	1	1.00	1.20	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.788	1.024	1.763	0.000	0.576	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	569	1337	0	0	0	0	-1
normalized size	1	1.00	1.44	3.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.927	0.733	2.279	0.000	0.511	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	343	646	813	0	0	0	-1
normalized size	1	1.00	1.27	2.38	3.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.805	0.081	0.964	0.632	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	372	691	783	0	0	0	-1
normalized size	1	1.00	1.06	1.96	2.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	1.204	0.082	0.953	0.605	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	402	736	961	0	0	0	-1
normalized size	1	1.00	0.84	1.54	2.01	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	1.394	0.080	0.978	0.691	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	347	1298	0	0	0	0	-1
normalized size	1	1.00	1.05	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.840	0.831	2.194	0.000	0.416	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	260	1192	0	0	0	0	-1
normalized size	1	1.00	1.05	4.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	0.517	1.459	0.000	0.530	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	140	5361	0	0	0	0	-1
normalized size	1	1.00	0.81	31.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.306	0.486	0.572	0.000	0.432	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	102	822	0	0	0	0	-1
normalized size	1	1.00	1.21	9.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.209	0.403	0.000	0.534	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	132	1389	0	0	0	0	-1
normalized size	1	1.00	1.71	18.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.389	0.354	0.000	0.409	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	225	7232	0	0	0	0	-1
normalized size	1	1.00	1.39	44.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.412	0.617	0.949	0.000	0.620	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	317	1841	0	0	0	0	-1
normalized size	1	1.00	1.27	7.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	1.036	2.099	0.000	0.490	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	388	2010	0	0	0	0	-1
normalized size	1	1.00	1.16	6.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.976	1.378	3.394	0.000	0.515	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	425	1467	0	0	0	0	-1
normalized size	1	1.00	1.08	3.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	1.706	2.309	0.000	0.797	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	354	1354	0	0	0	0	-1
normalized size	1	1.00	1.07	4.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.625	1.275	1.742	0.000	0.706	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	295	5542	0	0	0	0	-1
normalized size	1	1.00	1.13	21.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.918	0.745	0.000	0.505	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	233	1030	0	0	0	0	-1
normalized size	1	1.00	1.24	5.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.343	0.620	0.348	0.000	0.490	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	124	341	277	101	0	119	97
normalized size	1	1.00	1.16	3.19	2.59	0.94	0.00	1.11	0.91
time (sec)	N/A	0.124	0.139	0.063	0.334	0.458	0.000	0.161	1.251

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	254	1566	0	0	0	0	-1
normalized size	1	1.00	0.86	5.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.640	0.860	0.600	0.000	0.563	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	347	7397	0	0	0	0	-1
normalized size	1	1.00	0.94	19.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.797	1.722	0.997	0.000	0.520	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	452	2009	0	0	0	0	-1
normalized size	1	1.00	0.94	4.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.953	2.170	2.343	0.000	0.676	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	420	1565	0	0	0	0	-1
normalized size	1	1.00	1.03	3.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.811	2.003	1.998	0.000	1.061	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	418	5750	0	0	0	0	-1
normalized size	1	1.00	1.24	17.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.662	1.299	0.957	0.000	0.756	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	310	1241	0	0	0	0	-1
normalized size	1	1.00	1.17	4.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	1.397	0.774	0.000	0.437	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	150	460	429	164	0	226	405
normalized size	1	1.00	0.96	2.93	2.73	1.04	0.00	1.44	2.58
time (sec)	N/A	0.214	0.309	0.074	0.340	0.625	0.000	0.320	2.691

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	183	398	399	156	0	232	373
normalized size	1	1.00	1.17	2.54	2.54	0.99	0.00	1.48	2.38
time (sec)	N/A	0.178	0.136	0.069	0.344	1.134	0.000	0.191	2.164

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	376	1752	0	0	0	0	-1
normalized size	1	1.00	1.04	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.804	1.462	1.013	0.000	0.659	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	479	7593	0	0	0	0	-1
normalized size	1	1.00	1.07	16.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.983	2.352	1.042	0.000	1.627	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	168	386	445	203	0	333	498
normalized size	1	1.00	0.95	2.19	2.53	1.15	0.00	1.89	2.83
time (sec)	N/A	0.218	0.178	0.068	0.348	0.623	0.000	0.472	2.299

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	717	0	0	0	0	-1
normalized size	1	1.00	0.88	10.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.154	0.517	0.000	1.954	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	644	963	0	0	0	0	-1
normalized size	1	1.00	2.10	3.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.658	1.472	0.586	0.000	0.563	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	488	811	0	0	0	0	-1
normalized size	1	1.00	2.03	3.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	0.993	0.504	0.000	0.666	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	334	6440	0	0	0	0	-1
normalized size	1	1.00	1.75	33.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.301	0.501	0.797	0.000	0.709	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	152	1491	0	0	0	0	-1
normalized size	1	1.00	1.37	13.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.291	0.455	0.000	0.697	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	198	1895	529	160	0	172	582
normalized size	1	1.00	1.42	13.63	3.81	1.15	0.00	1.24	4.19
time (sec)	N/A	0.193	0.153	0.698	0.347	0.874	0.000	0.206	2.300

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	215	2752	796	250	0	362	930
normalized size	1	1.00	1.03	13.23	3.83	1.20	0.00	1.74	4.47
time (sec)	N/A	0.371	0.219	0.832	0.356	0.655	0.000	0.287	3.457

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	279	3637	1085	345	0	555	1304
normalized size	1	1.00	1.01	13.23	3.95	1.25	0.00	2.02	4.74
time (sec)	N/A	0.614	0.223	0.890	0.391	0.763	0.000	0.228	4.494

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	172	400	0	0	0	0	-1
normalized size	1	1.00	0.56	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.636	0.350	3.013	0.000	0.746	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	126	833	0	0	0	0	-1
normalized size	1	1.00	0.61	4.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.374	0.271	0.597	0.000	0.662	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	82	703	0	0	0	0	-1
normalized size	1	1.00	0.79	6.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.089	0.283	0.000	0.570	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	86	1217	0	0	0	0	-1
normalized size	1	1.00	0.92	13.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.151	0.343	0.000	0.590	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	86	1217	0	0	0	0	-1
normalized size	1	1.00	0.92	13.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.070	0.197	0.000	0.581	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	154	1451	0	0	0	0	-1
normalized size	1	1.00	0.81	7.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.463	0.292	0.783	0.000	0.589	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	222	664	0	0	0	0	-1
normalized size	1	1.00	0.73	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.748	0.641	4.374	0.000	0.531	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	233	496	0	0	0	0	-1
normalized size	1	1.00	0.61	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	0.413	1.983	0.000	0.586	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	172	454	0	0	0	0	-1
normalized size	1	1.00	0.66	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	0.267	0.420	0.000	0.574	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	112	285	0	0	0	0	-1
normalized size	1	1.00	0.85	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.115	0.355	0.000	0.578	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	843	0	0	0	0	-1
normalized size	1	1.00	0.86	7.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.173	0.455	0.000	0.536	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	843	0	0	0	0	-1
normalized size	1	1.00	0.86	7.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.077	0.332	0.000	0.651	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	172	583	0	0	0	0	-1
normalized size	1	1.00	0.72	2.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	0.494	1.031	0.000	0.475	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	250	858	0	0	0	0	-1
normalized size	1	1.00	0.66	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.956	0.880	2.003	0.000	0.477	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.041	2.397	0.692	0.000	0.501	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	0.115	0.344	0.000	0.672	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.058	0.143	0.353	0.000	0.624	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	1.969	0.707	0.000	0.547	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.021	1.044	0.263	0.000	0.482	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.055	1.282	0.296	0.000	0.559	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	474	381	0	0	0	0	-1
normalized size	1	1.00	1.72	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	6.592	0.073	0.000	0.566	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	394	298	0	0	0	0	-1
normalized size	1	1.00	1.84	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	3.053	0.060	0.000	0.580	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	315	217	0	0	0	0	-1
normalized size	1	1.00	2.02	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	2.471	0.053	0.000	0.635	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	257	148	0	0	0	0	-1
normalized size	1	1.00	2.25	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.239	0.050	0.000	0.695	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	294	210	0	0	0	0	-1
normalized size	1	1.00	1.99	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	1.632	0.062	0.000	0.618	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	360	279	0	0	0	0	-1
normalized size	1	1.00	1.80	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	3.216	0.069	0.000	0.568	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	435	367	0	0	0	0	-1
normalized size	1	1.00	1.67	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	6.087	0.069	0.000	0.589	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	1072	1656	0	0	0	0	-1
normalized size	1	1.00	2.78	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	15.758	4.583	0.000	0.681	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	861	13923	0	0	0	0	-1
normalized size	1	1.00	3.09	49.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	13.550	1.632	0.000	0.453	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	759	1170	0	0	0	0	-1
normalized size	1	1.00	4.04	6.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	10.192	0.368	0.000	0.567	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	818	1799	0	0	0	0	-1
normalized size	1	1.00	2.56	5.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	12.610	0.436	0.000	0.665	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	1010	26776	0	0	0	0	-1
normalized size	1	1.00	2.45	64.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.604	12.754	3.772	0.000	0.612	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	521	1507	0	0	0	0	-1
normalized size	1	1.00	1.89	5.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	9.370	0.183	0.000	0.710	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	0.539	0.801	0.000	0.542	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	73	76	71	335	61
normalized size	1	1.00	1.00	0.93	1.01	1.06	0.99	4.65	0.85
time (sec)	N/A	0.107	0.019	0.025	0.306	0.564	1.980	0.227	0.991

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	65	72	61	54	227	51
normalized size	1	1.00	1.25	1.03	1.14	0.97	0.86	3.60	0.81
time (sec)	N/A	0.081	0.018	0.030	0.306	0.667	1.551	0.210	0.947

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	59	65	68	63	268	53
normalized size	1	1.00	1.00	0.95	1.05	1.10	1.02	4.32	0.85
time (sec)	N/A	0.087	0.016	0.026	0.305	0.542	1.223	0.157	0.908

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	69	57	37	52	46	160	44
normalized size	1	1.00	1.72	1.42	0.92	1.30	1.15	4.00	1.10
time (sec)	N/A	0.022	0.015	0.026	0.303	0.583	0.845	0.293	0.865

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	48	47	53	49	203	40
normalized size	1	1.00	0.73	0.75	0.73	0.83	0.77	3.17	0.62
time (sec)	N/A	0.021	0.010	0.026	0.307	0.477	0.678	0.178	0.844

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	69	89	0	0	0	-1
normalized size	1	1.00	1.25	1.44	1.85	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.018	0.048	0.310	1.431	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	45	36	51	41	145	37
normalized size	1	1.00	1.00	1.18	0.95	1.34	1.08	3.82	0.97
time (sec)	N/A	0.051	0.010	0.033	0.308	0.591	0.849	0.381	0.810

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	68	87	81	0	0	330	-1
normalized size	1	1.00	1.21	1.55	1.45	0.00	0.00	5.89	-0.02
time (sec)	N/A	0.051	0.033	0.052	0.316	0.529	0.000	2.181	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	59	53	64	63	204	49
normalized size	1	1.00	1.00	1.02	0.91	1.10	1.09	3.52	0.84
time (sec)	N/A	0.076	0.016	0.038	0.307	0.592	1.189	0.209	0.850

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	71	59	61	52	46	160	61
normalized size	1	1.00	1.69	1.40	1.45	1.24	1.10	3.81	1.45
time (sec)	N/A	0.030	0.015	0.036	0.302	1.410	0.885	0.181	0.839

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	68	62	73	75	281	59
normalized size	1	1.00	1.00	0.96	0.87	1.03	1.06	3.96	0.83
time (sec)	N/A	0.088	0.017	0.037	0.315	0.524	1.877	0.209	0.876

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	113	225	190	0	0	0	-1
normalized size	1	1.00	0.70	1.39	1.17	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.579	0.958	0.056	0.320	0.589	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	88	205	146	109	114	522	101
normalized size	1	1.00	0.76	1.77	1.26	0.94	0.98	4.50	0.87
time (sec)	N/A	0.440	0.044	0.056	0.330	0.616	2.144	0.173	0.982

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	95	205	173	0	0	0	-1
normalized size	1	1.00	0.69	1.49	1.25	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.413	0.254	0.056	0.320	0.781	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	66	185	74	91	88	305	77
normalized size	1	1.00	0.69	1.95	0.78	0.96	0.93	3.21	0.81
time (sec)	N/A	0.049	0.029	0.056	0.308	0.531	1.279	0.185	0.899

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	71	182	144	0	0	0	-1
normalized size	1	1.00	0.62	1.58	1.25	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.118	0.054	0.330	0.719	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	145	663	0	0	0	0	-1
normalized size	1	1.00	0.99	4.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.044	1.064	0.000	1.026	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	102	170	152	0	0	0	-1
normalized size	1	1.00	1.10	1.83	1.63	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.137	0.061	0.324	1.516	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	174	741	0	0	0	0	-1
normalized size	1	1.00	1.01	4.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.334	0.082	1.686	0.000	0.730	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	93	237	188	0	0	0	-1
normalized size	1	1.00	0.80	2.04	1.62	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.308	0.306	0.070	0.330	0.467	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	82	199	164	108	102	282	246
normalized size	1	1.00	0.92	2.24	1.84	1.21	1.15	3.17	2.76
time (sec)	N/A	0.108	0.036	0.066	0.326	0.871	1.619	0.359	1.358

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	114	258	228	0	0	0	-1
normalized size	1	1.00	0.80	1.80	1.59	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.445	0.622	0.069	0.322	0.590	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	829	0	0	0	0	-1
normalized size	1	1.00	0.85	5.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.297	1.334	0.000	1.333	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	232	251	277	0	0	0	-1
normalized size	1	1.00	1.20	1.30	1.44	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.280	0.057	0.432	0.536	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	0.784	0.653	0.000	0.596	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.014	0.338	0.620	0.000	0.699	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	1.145	0.644	0.000	1.383	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.016	0.645	0.000	0.596	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	1.397	0.615	0.000	0.591	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	1.182	0.635	0.000	0.665	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	1.164	0.621	0.000	0.753	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	87	89	92	100	383	106
normalized size	1	1.00	1.00	0.91	0.93	0.96	1.04	3.99	1.10
time (sec)	N/A	0.189	0.028	0.029	0.304	0.564	3.132	0.199	1.015

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	103	85	88	77	76	240	101
normalized size	1	1.00	1.18	0.98	1.01	0.89	0.87	2.76	1.16
time (sec)	N/A	0.137	0.027	0.029	0.303	0.506	2.534	0.197	1.208

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	79	81	84	90	319	71
normalized size	1	1.00	1.00	0.92	0.94	0.98	1.05	3.71	0.83
time (sec)	N/A	0.161	0.022	0.031	0.303	0.776	2.006	0.232	0.951

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	93	77	46	68	68	176	64
normalized size	1	1.00	1.86	1.54	0.92	1.36	1.36	3.52	1.28
time (sec)	N/A	0.037	0.023	0.027	0.301	0.604	1.517	0.177	0.918

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	71	68	66	72	75	255	60
normalized size	1	1.00	0.68	0.65	0.63	0.69	0.72	2.45	0.58
time (sec)	N/A	0.044	0.017	0.031	0.302	0.594	1.238	0.377	0.907

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	89	106	0	0	0	-1
normalized size	1	1.00	1.04	1.27	1.51	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.072	0.049	0.311	0.613	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	65	57	66	68	249	57
normalized size	1	1.00	1.00	1.02	0.89	1.03	1.06	3.89	0.89
time (sec)	N/A	0.111	0.017	0.035	0.312	0.533	1.501	0.263	0.911

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	80	82	0	0	0	-1
normalized size	1	1.00	1.00	1.29	1.32	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.094	0.062	0.051	0.308	0.633	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	69	66	72	75	274	59
normalized size	1	1.00	1.00	1.01	0.97	1.06	1.10	4.03	0.87
time (sec)	N/A	0.110	0.018	0.038	0.302	0.714	1.527	0.232	0.878

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	84	105	112	0	0	0	-1
normalized size	1	1.00	1.09	1.36	1.45	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.081	0.053	0.305	0.710	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	80	71	81	88	265	70
normalized size	1	1.00	1.00	0.96	0.86	0.98	1.06	3.19	0.84
time (sec)	N/A	0.138	0.025	0.037	0.307	0.553	1.935	0.178	0.888

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	138	259	214	0	0	0	-1
normalized size	1	1.00	0.68	1.28	1.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.025	1.989	0.056	0.330	0.617	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	108	239	170	133	153	683	221
normalized size	1	1.00	0.69	1.53	1.09	0.85	0.98	4.38	1.42
time (sec)	N/A	0.819	0.067	0.056	0.325	0.573	3.419	0.228	1.220

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	121	239	198	0	0	0	-1
normalized size	1	1.00	0.68	1.34	1.11	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.779	1.256	0.055	0.327	0.590	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	82	219	93	116	133	473	111
normalized size	1	1.00	0.59	1.59	0.67	0.84	0.96	3.43	0.80
time (sec)	N/A	0.088	0.055	0.054	0.311	0.678	2.111	0.190	0.995

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	99	216	175	0	0	0	-1
normalized size	1	1.00	0.58	1.26	1.02	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.691	0.054	0.326	0.763	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	191	728	0	0	0	0	-1
normalized size	1	1.00	1.03	3.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.530	0.065	1.798	0.000	0.606	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	182	222	200	0	0	0	-1
normalized size	1	1.00	1.17	1.42	1.28	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.418	0.057	0.064	0.328	0.628	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	183	774	0	0	0	0	-1
normalized size	1	1.00	1.13	4.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.461	0.070	1.876	0.000	0.602	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	153	249	203	0	0	0	-1
normalized size	1	1.00	0.92	1.49	1.22	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.430	0.074	0.069	0.329	2.103	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	238	927	0	0	0	0	-1
normalized size	1	1.00	1.11	4.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	0.367	2.755	0.000	0.694	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	118	272	239	0	0	0	-1
normalized size	1	1.00	0.75	1.73	1.52	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.594	0.817	0.071	0.333	0.683	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	99	233	188	132	148	440	335
normalized size	1	1.00	0.88	2.06	1.66	1.17	1.31	3.89	2.96
time (sec)	N/A	0.194	0.061	0.067	0.332	0.587	2.597	0.221	1.581

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	140	292	254	0	0	0	-1
normalized size	1	1.00	0.77	1.60	1.39	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.816	1.457	0.072	0.333	0.648	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	124	253	204	148	168	651	357
normalized size	1	1.00	0.73	1.49	1.20	0.87	0.99	3.83	2.10
time (sec)	N/A	0.841	0.067	0.071	0.321	0.608	3.988	0.235	2.720

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	183	883	0	0	0	0	-1
normalized size	1	1.00	0.74	3.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.599	2.464	0.000	0.553	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.952	0.784	0.000	0.405	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.493	0.758	0.000	1.329	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	1.462	0.948	0.000	0.636	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.778	0.795	0.000	0.560	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	1.149	0.883	0.000	0.557	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	1.012	0.913	0.000	0.513	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	79	88	82	88	97	303	80
normalized size	1	1.00	0.55	0.61	0.57	0.61	0.67	2.10	0.56
time (sec)	N/A	0.067	0.045	0.028	0.314	0.519	2.040	0.194	0.935

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	124	250	199	0	0	0	-1
normalized size	1	1.00	0.55	1.10	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.171	1.223	0.056	0.328	0.500	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	231	932	0	0	0	0	-1
normalized size	1	1.00	0.68	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	1.100	5.397	0.000	0.457	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	165	120	0	0	0	-1
normalized size	1	1.00	0.69	1.90	1.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.140	0.056	0.318	0.550	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	145	85	56	41	0	82
normalized size	1	1.00	1.00	3.45	2.02	1.33	0.98	0.00	1.95
time (sec)	N/A	0.069	0.038	0.053	0.316	0.548	0.986	0.000	0.956

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	125	125	0	0	0	-1
normalized size	1	1.00	0.81	2.31	2.31	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.051	0.051	0.318	0.534	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	65	22	10	22	23
normalized size	1	1.00	1.00	0.92	5.00	1.69	0.77	1.69	1.77
time (sec)	N/A	0.016	0.004	0.023	0.316	0.629	0.901	0.214	0.872

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	130	132	0	0	0	-1
normalized size	1	1.00	0.93	2.89	2.93	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.067	0.056	0.334	1.506	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	132	82	63	37	0	80
normalized size	1	1.00	1.00	3.22	2.00	1.54	0.90	0.00	1.95
time (sec)	N/A	0.078	0.040	0.059	0.319	0.810	1.416	0.000	1.049

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	209	162	0	0	0	-1
normalized size	1	1.00	0.71	2.49	1.93	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.269	0.067	0.326	0.682	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	112	812	0	0	0	0	-1
normalized size	1	1.00	0.83	6.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	0.119	0.498	0.000	0.517	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	59	5573	200	0	0	0	-1
normalized size	1	1.00	0.79	74.31	2.67	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.185	0.553	0.326	0.396	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	741	0	0	0	0	-1
normalized size	1	1.00	0.87	9.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.071	0.524	0.000	0.523	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	127	22	10	22	68
normalized size	1	1.00	1.00	0.92	9.77	1.69	0.77	1.69	5.23
time (sec)	N/A	0.026	0.005	0.022	0.330	0.540	0.931	0.192	0.953

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	1188	0	0	0	0	-1
normalized size	1	1.00	0.91	18.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.181	0.067	0.451	0.000	0.401	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	4449	237	0	0	0	-1
normalized size	1	1.00	0.92	67.41	3.59	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.215	0.228	0.603	0.333	0.471	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	1360	0	0	0	0	-1
normalized size	1	1.00	0.96	9.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.357	0.366	0.891	0.000	0.613	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	142	248	0	0	0	0	-1
normalized size	1	1.00	0.69	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.469	0.281	3.151	0.000	0.509	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	788	0	0	0	0	-1
normalized size	1	1.00	0.76	7.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.256	0.558	0.000	0.482	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	87	776	0	0	0	0	-1
normalized size	1	1.00	0.81	7.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.069	0.523	0.000	1.024	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	209	22	10	22	90
normalized size	1	1.00	1.00	0.92	16.08	1.69	0.77	1.69	6.92
time (sec)	N/A	0.023	0.005	0.023	0.335	0.754	0.959	0.141	0.903

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	83	1245	0	0	0	0	-1
normalized size	1	1.00	0.91	13.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.073	0.571	0.000	1.461	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	93	826	0	0	0	0	-1
normalized size	1	1.00	1.03	9.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.212	0.624	0.000	0.495	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	165	406	0	0	0	0	-1
normalized size	1	1.00	0.82	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.496	0.460	4.727	0.000	0.585	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	25	14	0	11
normalized size	1	1.00	1.00	0.80	0.00	1.67	0.93	0.00	0.73
time (sec)	N/A	0.025	0.009	0.036	0.000	0.651	1.216	0.000	0.874

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	1.501	0.128	0.000	0.739	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	21	20	7	21	9
normalized size	1	1.00	1.00	1.11	2.33	2.22	0.78	2.33	1.00
time (sec)	N/A	0.028	0.031	0.023	0.308	0.973	0.644	0.221	0.823

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	0.231	0.245	0.000	1.091	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.136	0.124	0.000	0.749	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	22	8	22	23
normalized size	1	1.00	1.00	1.09	2.09	2.00	0.73	2.00	2.09
time (sec)	N/A	0.025	0.005	0.023	0.326	0.496	0.932	0.193	0.781

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	0.149	0.247	0.000	0.599	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.049	0.611	0.125	0.000	0.532	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	42	22	12	22	23
normalized size	1	1.00	1.00	0.92	3.23	1.69	0.92	1.69	1.77
time (sec)	N/A	0.026	0.007	0.024	0.332	0.420	1.116	0.152	0.799

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	0.737	0.247	0.000	0.390	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	84	26	0	33
normalized size	1	1.00	1.00	1.06	0.00	4.94	1.53	0.00	1.94
time (sec)	N/A	0.031	0.012	0.022	0.000	0.511	1.525	0.000	1.053

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	64	203	177	0	0	0	-1
normalized size	1	1.00	0.59	1.86	1.62	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.169	0.064	0.326	0.631	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	169	126	65	0	0	110
normalized size	1	1.00	0.79	2.96	2.21	1.14	0.00	0.00	1.93
time (sec)	N/A	0.063	0.093	0.062	0.316	0.716	0.000	0.000	0.958

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	66	68	62	48	61	154	37
normalized size	1	1.00	1.20	1.24	1.13	0.87	1.11	2.80	0.67
time (sec)	N/A	0.038	0.055	0.036	0.312	0.402	1.312	0.184	0.913

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	169	122	64	0	255	106
normalized size	1	1.00	0.81	3.13	2.26	1.19	0.00	4.72	1.96
time (sec)	N/A	0.022	0.070	0.060	0.314	0.556	0.000	2.012	0.975

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	63	190	206	0	0	0	-1
normalized size	1	1.00	0.69	2.09	2.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.178	0.069	0.337	1.012	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	180	150	118	253	0	132
normalized size	1	1.00	0.94	2.20	1.83	1.44	3.09	0.00	1.61
time (sec)	N/A	0.145	0.142	0.064	0.330	0.903	2.721	0.000	1.153

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	265	233	0	0	0	-1
normalized size	1	1.00	0.67	2.15	1.89	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.358	0.397	0.072	0.334	0.606	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	103	907	0	0	0	0	-1
normalized size	1	1.00	0.64	5.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.182	0.704	0.000	1.074	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	93	1722	273	96	0	0	231
normalized size	1	1.00	0.99	18.32	2.90	1.02	0.00	0.00	2.46
time (sec)	N/A	0.103	0.135	0.813	0.336	0.571	0.000	0.000	1.764

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	43	191	146	66	0	140	198
normalized size	1	1.00	0.52	2.33	1.78	0.80	0.00	1.71	2.41
time (sec)	N/A	0.068	0.055	0.062	0.333	0.621	0.000	0.174	1.157

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	1695	268	95	0	88	213
normalized size	1	1.00	1.06	19.26	3.05	1.08	0.00	1.00	2.42
time (sec)	N/A	0.064	0.095	0.806	0.338	0.540	0.000	2.080	1.500

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	106	1290	0	0	0	0	-1
normalized size	1	1.00	0.78	9.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.183	0.732	0.000	0.551	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	97	4589	406	0	0	0	-1
normalized size	1	1.00	0.68	32.32	2.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	0.324	0.874	0.346	0.715	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	146	3040	0	0	0	0	-1
normalized size	1	1.00	0.71	14.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	1.016	2.260	0.000	0.535	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	139	1015	0	0	0	0	-1
normalized size	1	1.00	0.61	4.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.188	0.704	0.000	1.522	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	72	1771	465	114	0	0	410
normalized size	1	1.00	0.60	14.64	3.84	0.94	0.00	0.00	3.39
time (sec)	N/A	0.134	0.092	0.818	0.346	1.032	0.000	0.000	1.708

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	91	1708	298	97	0	192	239
normalized size	1	1.00	0.76	14.35	2.50	0.82	0.00	1.61	2.01
time (sec)	N/A	0.112	0.060	0.810	0.336	0.457	0.000	0.192	1.757

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	71	1742	459	113	0	122	378
normalized size	1	1.00	0.62	15.15	3.99	0.98	0.00	1.06	3.29
time (sec)	N/A	0.096	0.060	0.815	0.348	1.146	0.000	2.750	1.718

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	135	1387	0	0	0	0	-1
normalized size	1	1.00	0.70	7.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.388	0.198	0.739	0.000	0.591	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	144	442	0	0	0	0	-1
normalized size	1	1.00	0.75	2.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.435	0.354	3.034	0.000	0.648	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	215	672	0	0	0	0	-1
normalized size	1	1.00	0.71	2.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.955	0.811	6.423	0.000	0.630	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.236	180.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	4.933	0.798	0.000	0.525	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	1.707	0.589	0.000	0.621	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	0	58	0	0	-1
normalized size	1	1.00	1.00	0.89	0.00	2.15	0.00	0.00	-0.04
time (sec)	N/A	0.103	0.113	0.200	0.000	0.524	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	38	0	0	-1
normalized size	1	1.00	1.00	0.93	0.00	2.71	0.00	0.00	-0.07
time (sec)	N/A	0.070	0.067	0.205	0.000	0.461	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	0	54	0	0	-1
normalized size	1	1.00	1.00	0.89	0.00	2.00	0.00	0.00	-0.04
time (sec)	N/A	0.066	0.075	0.176	0.000	1.017	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	1.059	0.515	0.000	1.276	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.316	3.402	0.596	0.000	0.426	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	36	0	111	0	0	-1
normalized size	1	1.00	0.95	0.95	0.00	2.92	0.00	0.00	-0.03
time (sec)	N/A	0.133	0.148	0.197	0.000	1.367	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	28	0	106	0	0	-1
normalized size	1	1.00	0.89	0.78	0.00	2.94	0.00	0.00	-0.03
time (sec)	N/A	0.207	0.079	0.204	0.000	0.589	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	102	0	0	-1
normalized size	1	1.00	0.86	1.03	0.00	2.91	0.00	0.00	-0.03
time (sec)	N/A	0.095	0.093	0.176	0.000	0.591	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.342	4.274	0.529	0.000	0.600	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	9.429	0.602	0.000	0.586	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	51	0	131	0	0	-1
normalized size	1	1.00	0.73	0.80	0.00	2.05	0.00	0.00	-0.02
time (sec)	N/A	0.272	0.122	0.201	0.000	0.449	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	43	0	135	0	0	-1
normalized size	1	1.00	0.92	0.60	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.065	0.204	0.000	0.538	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	51	0	122	0	0	-1
normalized size	1	1.00	1.00	0.88	0.00	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.234	0.083	0.178	0.000	0.520	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	3.834	0.546	0.000	0.617	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	68	0	151	0	0	-1
normalized size	1	1.00	0.75	0.70	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.141	0.180	0.000	0.915	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	84	83	0	171	0	0	-1
normalized size	1	1.00	0.70	0.69	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.101	0.192	0.000	0.699	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	101	98	0	200	0	0	-1
normalized size	1	1.00	0.66	0.64	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.131	0.184	0.000	0.788	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	112	113	0	220	0	0	-1
normalized size	1	1.00	0.63	0.64	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.344	0.102	0.183	0.000	0.584	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	128	128	0	249	0	0	-1
normalized size	1	1.00	0.61	0.61	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.136	0.180	0.000	1.079	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	136	99	71	158	239	83
normalized size	1	1.00	1.27	1.77	1.29	0.92	2.05	3.10	1.08
time (sec)	N/A	0.063	0.085	0.043	0.312	0.799	2.412	0.190	1.392

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	61	225	179	95	0	0	150
normalized size	1	1.00	0.61	2.25	1.79	0.95	0.00	0.00	1.50
time (sec)	N/A	0.072	0.089	0.064	0.324	0.511	0.000	0.000	1.195

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	92	82	71	158	239	105
normalized size	1	1.00	1.17	1.23	1.09	0.95	2.11	3.19	1.40
time (sec)	N/A	0.047	0.060	0.035	0.308	0.564	2.410	0.252	1.079

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	65	225	182	97	0	0	154
normalized size	1	1.00	0.69	2.39	1.94	1.03	0.00	0.00	1.64
time (sec)	N/A	0.045	0.069	0.064	0.324	0.810	0.000	0.000	1.368

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	81	234	268	0	0	0	-1
normalized size	1	1.00	0.63	1.81	2.08	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.208	0.071	0.330	0.623	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	94	228	204	161	549	0	183
normalized size	1	1.00	0.76	1.85	1.66	1.31	4.46	0.00	1.49
time (sec)	N/A	0.235	0.171	0.070	0.333	0.866	4.903	0.000	1.388

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	71	297	226	99	0	250	372
normalized size	1	1.00	0.56	2.34	1.78	0.78	0.00	1.97	2.93
time (sec)	N/A	0.184	0.098	0.070	0.328	0.702	0.000	0.175	1.529

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	121	2571	388	136	0	0	350
normalized size	1	1.00	0.74	15.77	2.38	0.83	0.00	0.00	2.15
time (sec)	N/A	0.250	0.121	0.887	0.329	0.620	0.000	0.000	1.971

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	71	247	206	99	0	251	319
normalized size	1	1.00	0.57	1.98	1.65	0.79	0.00	2.01	2.55
time (sec)	N/A	0.094	0.074	0.067	0.326	0.493	0.000	0.253	1.483

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	127	2571	392	137	0	0	358
normalized size	1	1.00	0.84	17.03	2.60	0.91	0.00	0.00	2.37
time (sec)	N/A	0.110	0.110	0.867	0.338	0.538	0.000	0.000	2.058

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	129	1392	0	0	0	0	-1
normalized size	1	1.00	0.66	7.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.449	0.408	0.838	0.000	0.666	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	127	4797	534	0	0	0	-1
normalized size	1	1.00	0.61	22.95	2.56	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	0.593	0.952	0.346	0.657	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	135	2634	437	140	0	341	414
normalized size	1	1.00	0.70	13.72	2.28	0.73	0.00	1.78	2.16
time (sec)	N/A	0.268	0.128	0.903	0.342	1.518	0.000	0.211	3.092

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	107	2646	657	161	0	0	831
normalized size	1	1.00	0.50	12.31	3.06	0.75	0.00	0.00	3.87
time (sec)	N/A	0.359	0.119	0.851	0.347	0.517	0.000	0.000	3.192

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	148	2582	422	140	0	342	414
normalized size	1	1.00	0.79	13.73	2.24	0.74	0.00	1.82	2.20
time (sec)	N/A	0.164	0.091	0.716	0.338	0.457	0.000	0.295	2.693

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	111	2646	663	166	0	0	736
normalized size	1	1.00	0.55	13.03	3.27	0.82	0.00	0.00	3.63
time (sec)	N/A	0.177	0.165	0.923	0.348	0.570	0.000	0.000	2.293

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	189	1533	0	0	0	0	-1
normalized size	1	1.00	0.68	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.626	0.295	0.855	0.000	0.604	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	218	842	0	0	0	0	-1
normalized size	1	1.00	0.78	3.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.690	0.696	3.045	0.000	0.559	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	152	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.504	180.000	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	7.860	0.743	0.000	0.556	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	15.247	0.489	0.000	0.641	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	36	0	118	0	0	-1
normalized size	1	1.00	0.76	0.88	0.00	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.122	0.165	0.255	0.000	0.594	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	102	0	0	-1
normalized size	1	1.00	0.83	0.83	0.00	3.52	0.00	0.00	-0.03
time (sec)	N/A	0.118	0.140	0.178	0.000	0.498	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	0	88	0	0	-1
normalized size	1	1.00	0.81	0.89	0.00	3.26	0.00	0.00	-0.04
time (sec)	N/A	0.113	0.122	0.187	0.000	0.607	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	102	0	0	-1
normalized size	1	1.00	0.83	0.83	0.00	3.52	0.00	0.00	-0.03
time (sec)	N/A	0.094	0.136	0.190	0.000	0.621	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	36	0	118	0	0	-1
normalized size	1	1.00	0.80	0.88	0.00	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.083	0.087	0.185	0.000	0.513	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	1.342	0.618	0.000	0.535	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.905	12.132	0.508	0.000	1.355	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	62	0	232	0	0	-1
normalized size	1	1.00	0.92	1.17	0.00	4.38	0.00	0.00	-0.02
time (sec)	N/A	0.186	0.200	0.247	0.000	0.694	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	76	80	54	0	231	0	0	-1
normalized size	1	1.38	1.45	0.98	0.00	4.20	0.00	0.00	-0.02
time (sec)	N/A	0.521	0.122	0.178	0.000	0.709	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	60	56	38	0	164	0	0	-1
normalized size	1	1.46	1.37	0.93	0.00	4.00	0.00	0.00	-0.02
time (sec)	N/A	0.287	0.213	0.164	0.000	0.616	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	75	54	0	225	0	0	-1
normalized size	1	1.00	1.42	1.02	0.00	4.25	0.00	0.00	-0.02
time (sec)	N/A	0.245	0.112	0.186	0.000	0.538	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	60	0	222	0	0	-1
normalized size	1	1.00	0.88	1.22	0.00	4.53	0.00	0.00	-0.02
time (sec)	N/A	0.121	0.151	0.232	0.000	0.548	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.650	4.686	0.638	0.000	0.614	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	60	90	0	256	0	0	-1
normalized size	1	1.00	0.60	0.90	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.594	0.219	0.254	0.000	0.666	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	160	66	82	0	267	0	0	-1
normalized size	1	1.50	0.62	0.77	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.655	0.263	0.183	0.000	0.596	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	109	56	51	0	193	0	0	-1
normalized size	1	1.27	0.65	0.59	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.586	0.173	0.164	0.000	0.530	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	82	0	256	0	0	-1
normalized size	1	1.00	0.96	0.82	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.464	0.205	0.184	0.000	0.565	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	88	0	241	0	0	-1
normalized size	1	1.00	1.25	1.28	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.199	0.237	0.000	0.975	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.767	5.695	0.650	0.000	0.556	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	108	122	0	272	0	0	-1
normalized size	1	1.00	0.86	0.98	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.500	0.257	0.230	0.000	0.515	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	132	152	0	303	0	0	-1
normalized size	1	1.00	0.78	0.89	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.963	0.175	0.232	0.000	0.593	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	166	182	0	341	0	0	-1
normalized size	1	1.00	0.65	0.71	0.00	1.33	0.00	0.00	-0.00
time (sec)	N/A	1.349	0.327	0.232	0.000	0.517	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	81	281	240	131	0	0	206
normalized size	1	1.00	0.60	2.10	1.79	0.98	0.00	0.00	1.54
time (sec)	N/A	0.074	0.184	0.067	0.323	0.472	0.000	0.000	1.302

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	157	3447	516	179	0	0	493
normalized size	1	1.00	0.73	16.11	2.41	0.84	0.00	0.00	2.30
time (sec)	N/A	0.169	0.258	0.857	0.338	0.529	0.000	0.000	2.490

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	143	3550	871	216	0	0	1041
normalized size	1	1.00	0.49	12.20	2.99	0.74	0.00	0.00	3.58
time (sec)	N/A	0.327	0.153	0.973	0.373	0.711	0.000	0.000	2.970

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	257	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.748	180.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.068	8.517	0.832	0.000	0.859	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	33.235	0.740	0.000	0.547	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	48	0	220	0	0	-1
normalized size	1	1.00	0.73	0.87	0.00	4.00	0.00	0.00	-0.02
time (sec)	N/A	0.145	0.168	0.305	0.000	0.940	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	200	0	0	-1
normalized size	1	1.00	0.77	0.77	0.00	4.65	0.00	0.00	-0.02
time (sec)	N/A	0.144	0.172	0.231	0.000	0.515	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	48	0	216	0	0	-1
normalized size	1	1.00	0.73	0.87	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.155	0.145	0.265	0.000	0.455	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	136	0	0	-1
normalized size	1	1.00	0.83	0.83	0.00	4.69	0.00	0.00	-0.03
time (sec)	N/A	0.125	0.150	0.186	0.000	0.466	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	48	0	216	0	0	-1
normalized size	1	1.00	1.00	0.87	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.142	0.174	0.256	0.000	0.443	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	33	0	200	0	0	-1
normalized size	1	1.00	1.00	0.77	0.00	4.65	0.00	0.00	-0.02
time (sec)	N/A	0.112	0.216	0.227	0.000	0.570	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	48	0	216	0	0	-1
normalized size	1	1.00	0.73	0.87	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.184	0.280	0.000	0.668	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	1.561	0.712	0.000	0.558	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.070	3.143	0.878	0.000	0.533	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	78	0	418	0	0	-1
normalized size	1	1.00	0.84	1.16	0.00	6.24	0.00	0.00	-0.01
time (sec)	N/A	0.307	0.200	0.230	0.000	0.589	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	86	0	413	0	0	-1
normalized size	1	1.00	0.85	1.30	0.00	6.26	0.00	0.00	-0.02
time (sec)	N/A	0.143	0.302	0.275	0.000	0.491	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	73	121	0	447	0	0	-1
normalized size	1	1.00	0.64	1.06	0.00	3.92	0.00	0.00	-0.01
time (sec)	N/A	0.585	0.377	0.236	0.000	0.510	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	131	0	435	0	0	-1
normalized size	1	1.00	0.93	1.47	0.00	4.89	0.00	0.00	-0.01
time (sec)	N/A	0.386	0.203	0.282	0.000	0.713	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	79	120	163	91	0	0	-1
normalized size	1	1.00	0.57	0.86	1.17	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.110	0.445	0.409	0.599	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	160	175	0	0	0	0	-1
normalized size	1	1.00	0.81	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.513	0.470	0.000	0.638	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	99	88	72	0	0	-1
normalized size	1	1.00	0.69	1.14	1.01	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.073	0.376	0.415	0.491	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	125	154	0	0	0	0	-1
normalized size	1	1.00	0.86	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.233	0.407	0.000	0.514	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	81	30	58	0	47	-1
normalized size	1	1.00	0.91	2.53	0.94	1.81	0.00	1.47	-0.03
time (sec)	N/A	0.047	0.034	0.308	0.405	0.469	0.000	0.223	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	76	126	0	0	0	0	-1
normalized size	1	1.00	0.80	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.081	0.450	0.000	0.866	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	99	0	0	0	0	-1
normalized size	1	1.00	0.76	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.100	0.423	0.000	0.569	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	72	51	58	0	111	-1
normalized size	1	1.00	1.14	1.71	1.21	1.38	0.00	2.64	-0.02
time (sec)	N/A	0.076	0.052	0.405	0.409	0.530	0.000	0.266	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	126	141	0	0	0	0	-1
normalized size	1	1.00	0.92	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.736	0.448	0.000	0.871	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	160	175	0	0	0	0	-1
normalized size	1	1.00	0.78	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.431	0.393	0.000	0.664	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	188	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.821	0.389	0.000	0.924	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	151	0	0	0	0	-1
normalized size	1	1.00	0.87	1.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.238	0.329	0.000	0.907	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	119	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.104	0.389	0.000	0.685	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	100	158	0	0	0	0	-1
normalized size	1	1.00	1.47	2.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.091	0.437	0.000	0.502	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	131	0	0	0	0	-1
normalized size	1	1.00	0.85	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.524	0.413	0.000	0.848	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	188	231	0	0	0	0	-1
normalized size	1	1.00	1.24	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.415	1.269	0.456	0.000	0.633	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	215	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	1.006	0.505	0.000	0.534	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	570	0	0	0	0	0	-1
normalized size	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	5.235	0.495	0.000	0.582	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	157	0	0	0	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.193	0.423	0.000	0.592	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	451	0	0	0	0	0	-1
normalized size	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.439	0.414	0.000	0.603	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	146	215	0	0	0	0	-1
normalized size	1	1.00	1.43	2.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.142	0.464	0.000	0.711	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	131	190	0	0	0	0	-1
normalized size	1	1.00	1.34	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.390	0.433	0.000	0.500	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	416	386	0	0	0	0	-1
normalized size	1	1.00	1.56	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	8.972	0.500	0.000	0.504	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	0.472	1.171	0.000	0.826	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	76	144	96	94	0	0	-1
normalized size	1	1.00	1.03	1.95	1.30	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.069	0.415	0.406	0.624	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	190	0	0	0	0	-1
normalized size	1	1.00	0.88	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.252	0.392	0.000	0.541	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	27	66	39	51	0	61	-1
normalized size	1	1.00	0.63	1.53	0.91	1.19	0.00	1.42	-0.02
time (sec)	N/A	0.051	0.039	0.322	0.306	0.534	0.000	0.288	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	27	38	36	47	0	59	-1
normalized size	1	1.00	0.68	0.95	0.90	1.18	0.00	1.48	-0.02
time (sec)	N/A	0.025	0.036	0.266	0.303	0.508	0.000	0.327	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	97	157	0	0	0	0	-1
normalized size	1	1.00	0.87	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.193	0.428	0.000	0.809	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	89	132	84	107	0	155	-1
normalized size	1	1.00	1.09	1.61	1.02	1.30	0.00	1.89	-0.01
time (sec)	N/A	0.171	0.117	0.436	0.306	0.431	0.000	0.315	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	182	205	0	0	0	0	-1
normalized size	1	1.00	1.02	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.402	1.634	0.454	0.000	0.570	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	0.525	1.073	0.000	0.531	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	165	230	0	0	0	0	-1
normalized size	1	1.00	0.89	1.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.310	0.397	0.428	0.000	0.579	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	193	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.347	0.395	0.000	0.400	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	82	62	69	0	0	-1
normalized size	1	1.00	0.50	1.21	0.91	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.057	0.323	0.313	0.507	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	38	49	57	69	0	0	-1
normalized size	1	1.00	0.60	0.78	0.90	1.10	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.041	0.266	0.309	0.505	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	159	232	0	0	0	0	-1
normalized size	1	1.00	1.25	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.341	0.256	0.450	0.000	0.450	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	215	207	0	0	0	0	-1
normalized size	1	1.00	1.26	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.313	1.151	0.456	0.000	0.496	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	266	313	0	0	0	0	-1
normalized size	1	1.00	1.20	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.776	2.847	0.479	0.000	0.461	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.100	0.528	1.087	0.000	0.556	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	249	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.467	0.508	0.000	0.766	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	541	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	1.068	0.467	0.000	0.653	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	45	98	88	91	0	0	-1
normalized size	1	1.00	0.48	1.04	0.94	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.068	0.326	0.319	0.418	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	45	56	86	87	0	0	-1
normalized size	1	1.00	0.51	0.64	0.98	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.055	0.269	0.322	0.473	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	230	305	0	0	0	0	-1
normalized size	1	1.00	1.24	1.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.409	0.369	0.474	0.000	0.535	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	270	282	0	0	0	0	-1
normalized size	1	1.00	1.44	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.425	2.004	0.477	0.000	0.499	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	555	482	0	0	0	0	-1
normalized size	1	1.00	1.54	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.982	7.867	0.544	0.000	0.547	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.124	0.501	1.099	0.000	1.191	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.132	3.199	0.463	0.000	0.559	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	26	0	0	0	0	-1
normalized size	1	1.00	1.00	2.89	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.104	0.075	0.323	0.000	0.452	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.062	0.023	0.276	0.000	0.439	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.118	1.134	0.514	0.000	0.529	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.109	0.657	1.067	0.000	0.442	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.266	3.263	0.489	0.000	0.436	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	90	0	0	0	0	-1
normalized size	1	1.00	0.94	2.50	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.122	0.088	0.346	0.000	0.674	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0	-1
normalized size	1	1.00	0.91	1.77	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.126	0.092	0.326	0.000	0.613	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.417	6.396	0.551	0.000	0.464	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.107	0.998	1.073	0.000	0.647	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	6.387	0.419	0.000	0.561	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	154	0	0	0	0	-1
normalized size	1	1.00	0.63	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.129	0.362	0.000	0.558	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	86	0	0	0	0	-1
normalized size	1	1.00	0.68	1.32	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.163	0.096	0.340	0.000	0.669	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.485	19.624	0.552	0.000	0.611	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	178	195	0	0	0	0	-1
normalized size	1	1.00	0.73	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.722	0.455	0.000	0.537	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	79	120	128	91	0	0	-1
normalized size	1	1.00	0.58	0.88	0.94	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.071	0.393	0.417	0.515	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	160	175	0	0	0	0	-1
normalized size	1	1.00	0.82	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.476	0.396	0.000	0.427	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	99	50	72	0	0	-1
normalized size	1	1.00	0.83	1.68	0.85	1.22	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.046	0.322	0.414	0.559	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	152	0	0	0	0	-1
normalized size	1	1.00	0.82	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.264	0.755	0.000	0.544	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	91	113	0	0	0	0	-1
normalized size	1	1.00	0.91	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.128	0.419	0.000	0.548	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	121	188	0	0	0	0	-1
normalized size	1	1.00	0.93	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.381	0.446	0.000	0.565	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	126	141	0	0	0	0	-1
normalized size	1	1.00	0.93	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.716	0.429	0.000	0.482	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	96	90	75	0	0	-1
normalized size	1	1.00	1.13	1.37	1.29	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.078	0.416	0.407	0.439	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	222	164	0	0	0	0	-1
normalized size	1	1.00	1.16	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	1.742	0.449	0.000	0.498	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	104	116	204	93	0	0	-1
normalized size	1	1.00	0.69	0.77	1.36	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.373	0.137	0.435	0.406	0.667	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	307	183	0	0	0	0	-1
normalized size	1	1.00	1.26	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	3.703	0.472	0.000	0.563	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	268	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.403	1.689	0.605	0.000	0.603	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	175	211	0	0	0	0	-1
normalized size	1	1.00	0.62	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.073	0.704	0.424	0.000	0.495	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	228	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.853	1.228	0.498	0.000	0.491	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	135	175	0	0	0	0	-1
normalized size	1	1.00	0.77	1.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.426	0.350	0.000	0.547	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	187	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.772	0.799	0.000	0.440	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	203	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.393	0.299	0.450	0.000	0.623	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	223	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.383	0.778	0.479	0.000	0.524	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	188	231	0	0	0	0	-1
normalized size	1	1.00	1.25	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.544	1.262	0.456	0.000	0.662	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	177	171	0	0	0	0	-1
normalized size	1	1.00	1.05	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.297	2.063	0.442	0.000	0.441	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	272	215	0	0	0	0	-1
normalized size	1	1.00	0.93	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.818	1.490	0.458	0.000	0.440	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	79	140	163	106	0	0	-1
normalized size	1	1.00	0.42	0.75	0.88	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.571	0.089	0.380	0.412	0.484	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	224	195	0	0	0	0	-1
normalized size	1	1.00	0.92	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	0.973	0.362	0.000	0.566	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	61	120	67	90	0	0	-1
normalized size	1	1.00	0.75	1.48	0.83	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.067	0.306	0.405	0.594	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	176	173	0	0	0	0	-1
normalized size	1	1.00	0.93	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.610	0.960	0.000	0.490	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	143	132	0	0	0	0	-1
normalized size	1	1.00	0.99	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.230	0.411	0.000	0.488	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	168	205	0	0	0	0	-1
normalized size	1	1.00	0.94	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.625	0.442	0.000	0.473	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	158	145	0	0	0	0	-1
normalized size	1	1.00	0.94	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.387	0.999	0.487	0.000	0.490	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	199	220	0	0	0	0	-1
normalized size	1	1.00	1.05	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	1.268	0.470	0.000	0.549	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	282	164	0	0	0	0	-1
normalized size	1	1.00	1.48	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.566	3.833	0.435	0.000	0.503	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	104	116	126	93	0	0	-1
normalized size	1	1.00	1.11	1.23	1.34	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.090	0.413	0.416	0.489	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	474	184	0	0	0	0	-1
normalized size	1	1.00	1.95	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.773	7.437	0.445	0.000	0.492	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	224	193	0	0	0	0	-1
normalized size	1	1.00	0.96	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.121	1.184	1.135	0.000	0.464	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	176	173	0	0	0	0	-1
normalized size	1	1.00	0.93	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.062	0.913	0.000	0.410	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	152	0	0	0	0	-1
normalized size	1	1.00	0.82	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.068	0.750	0.000	0.458	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	59	74	73	0	90	-1
normalized size	1	1.00	0.55	0.66	0.83	0.82	0.00	1.01	-0.01
time (sec)	N/A	0.051	0.052	0.400	0.309	0.489	0.000	0.707	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	65	79	108	99	0	114	-1
normalized size	1	1.00	0.49	0.59	0.81	0.74	0.00	0.86	-0.01
time (sec)	N/A	0.081	0.069	0.420	0.307	0.444	0.000	0.247	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	81	99	140	121	0	138	-1
normalized size	1	1.00	0.46	0.56	0.79	0.68	0.00	0.78	-0.01
time (sec)	N/A	0.113	0.079	0.439	0.329	0.461	0.000	0.231	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	206	345	0	0	0	0	-1
normalized size	1	1.00	0.71	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.676	0.560	0.000	0.419	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	119	319	0	0	0	0	-1
normalized size	1	1.00	0.51	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.102	0.289	0.548	0.000	0.432	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	109	290	0	0	0	0	-1
normalized size	1	1.00	0.60	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.130	0.517	0.000	0.512	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	74	90	54	0	70	-1
normalized size	1	1.00	0.90	1.54	1.88	1.12	0.00	1.46	-0.02
time (sec)	N/A	0.029	0.060	0.471	0.432	0.515	0.000	0.692	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	64	160	90	84	0	111	-1
normalized size	1	1.00	0.61	1.52	0.86	0.80	0.00	1.06	-0.01
time (sec)	N/A	0.066	0.074	0.479	0.343	0.452	0.000	0.315	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	80	250	132	112	0	149	-1
normalized size	1	1.00	0.51	1.59	0.84	0.71	0.00	0.95	-0.01
time (sec)	N/A	0.101	0.091	0.494	0.345	0.498	0.000	0.313	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	187	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.104	0.025	0.000	0.429	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	70	84	304	105	0	0	-1
normalized size	1	1.00	0.50	0.60	2.19	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.084	0.430	0.496	0.515	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	94	118	514	139	0	0	-1
normalized size	1	1.00	0.45	0.57	2.47	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.152	0.097	0.564	0.519	0.496	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	120	152	751	169	0	0	-1
normalized size	1	1.00	0.43	0.55	2.71	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.123	0.694	0.521	0.590	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	569	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	4.305	0.820	0.000	0.590	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	87	105	0	134	0	0	-1
normalized size	1	1.00	0.46	0.55	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.097	0.438	0.000	0.463	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	119	153	0	176	0	0	-1
normalized size	1	1.00	0.41	0.53	0.00	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.115	0.456	0.000	0.473	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	151	201	0	214	0	0	-1
normalized size	1	1.00	0.39	0.52	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.484	0.141	0.574	0.000	0.520	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	1.257	0.797	0.000	0.453	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.179	0.386	0.000	0.582	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.053	0.027	0.280	0.000	0.630	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	21	0	0	0	0	-1
normalized size	1	1.00	0.81	0.78	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.093	0.062	0.331	0.000	0.559	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	0	0	0	0	-1
normalized size	1	1.00	0.76	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.065	0.377	0.000	0.441	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	39	0	0	0	0	-1
normalized size	1	1.00	0.73	0.71	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.123	0.074	0.434	0.000	0.486	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	1.593	0.807	0.000	0.581	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.959	0.398	0.000	0.683	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0	-1
normalized size	1	1.00	0.91	1.77	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.123	0.064	0.330	0.000	0.591	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	120	0	0	0	0	-1
normalized size	1	1.00	0.87	2.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.160	0.142	0.433	0.000	0.541	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	176	0	0	0	0	-1
normalized size	1	1.00	0.85	2.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.177	0.181	0.531	0.000	0.434	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	65	232	0	0	0	0	-1
normalized size	1	1.00	0.81	2.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.219	0.622	0.000	0.711	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	1.825	0.918	0.000	0.570	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	1.142	0.574	0.000	0.428	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	86	0	0	0	0	-1
normalized size	1	1.00	0.68	1.32	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.167	0.119	0.343	0.000	0.533	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	56	180	0	0	0	0	-1
normalized size	1	1.00	0.71	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.362	0.224	0.465	0.000	0.477	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	272	0	0	0	0	-1
normalized size	1	1.00	0.85	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.405	0.249	0.561	0.000	0.588	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	99	364	0	0	0	0	-1
normalized size	1	1.00	0.93	3.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.439	0.227	0.683	0.000	0.606	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	193	1871	0	0	0	0	-1
normalized size	1	1.00	1.58	15.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	0.410	0.696	0.000	0.696	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	213	334	276	248	372	1471	288
normalized size	1	1.00	0.87	1.36	1.13	1.01	1.52	6.00	1.18
time (sec)	N/A	0.180	0.116	0.034	0.309	0.492	5.555	1.245	1.285

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	233	198	178	245	930	190
normalized size	1	1.00	0.89	1.38	1.17	1.05	1.45	5.50	1.12
time (sec)	N/A	0.129	0.084	0.032	0.306	0.521	3.192	0.247	1.378

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	148	131	119	155	527	118
normalized size	1	1.00	0.89	1.35	1.19	1.08	1.41	4.79	1.07
time (sec)	N/A	0.135	0.053	0.033	0.304	0.491	1.735	0.199	1.088

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	76	65	65	73	266	60
normalized size	1	1.00	1.21	1.33	1.14	1.14	1.28	4.67	1.05
time (sec)	N/A	0.067	0.011	0.030	0.317	0.503	0.792	0.210	0.942

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	662	833	406	0	0	0	-1
normalized size	1	1.00	1.54	1.94	0.95	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	1.459	0.624	0.574	0.578	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	746	2346	550	0	0	0	-1
normalized size	1	1.00	1.26	3.98	0.93	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.910	7.786	1.352	0.535	0.606	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	1828	4311	1084	0	0	0	-1
normalized size	1	1.00	2.78	6.56	1.65	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.966	12.938	1.406	0.603	0.602	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	23	22	14	48	15
normalized size	1	1.00	1.00	1.12	1.35	1.29	0.82	2.82	0.88
time (sec)	N/A	0.044	0.059	0.060	0.351	0.488	0.556	0.234	0.984

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	576	176	180	0	0	0	-1
normalized size	1	1.00	3.37	1.03	1.05	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.936	0.063	0.311	0.472	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	203	196	198	0	0	0	-1
normalized size	1	1.00	1.00	0.97	0.98	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	0.042	0.084	0.319	0.652	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	260	110	119	0	0	0	-1
normalized size	1	1.00	3.02	1.28	1.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.091	0.072	0.305	0.549	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	485	606	304	0	0	0	-1
normalized size	1	1.00	1.22	1.53	0.77	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	1.135	0.624	0.517	0.421	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	874	1599	0	0	0	0	-1
normalized size	1	1.00	3.39	6.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	19.321	0.607	0.000	0.878	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	4.255	1.399	0.000	0.456	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	4.102	1.209	0.000	0.482	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	153	356	0	71	-1
normalized size	1	1.00	1.92	0.00	2.47	5.74	0.00	1.15	-0.02
time (sec)	N/A	0.122	0.121	1.151	0.352	0.818	0.000	0.179	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	226	0	223	730	0	135	-1
normalized size	1	1.00	1.77	0.00	1.74	5.70	0.00	1.05	-0.01
time (sec)	N/A	0.375	0.311	1.168	0.418	0.603	0.000	0.207	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	329	0	401	1280	0	218	-1
normalized size	1	1.00	1.64	0.00	2.00	6.40	0.00	1.09	-0.00
time (sec)	N/A	1.082	0.591	1.190	0.433	0.707	0.000	0.217	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	431	0	639	2006	0	349	-1
normalized size	1	1.00	1.52	0.00	2.26	7.09	0.00	1.23	-0.00
time (sec)	N/A	1.511	0.951	1.175	0.465	0.923	0.000	0.228	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	97	229	0	0	0	0	-1
normalized size	1	1.00	0.52	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.321	0.610	0.000	0.488	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	90	210	0	0	0	0	-1
normalized size	1	1.00	0.62	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.107	0.503	0.000	0.517	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	52	63	42	0	54	-1
normalized size	1	1.00	0.81	1.41	1.70	1.14	0.00	1.46	-0.03
time (sec)	N/A	0.025	0.045	0.447	0.420	0.466	0.000	0.196	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	112	67	62	0	86	-1
normalized size	1	1.00	0.54	1.35	0.81	0.75	0.00	1.04	-0.01
time (sec)	N/A	0.053	0.053	0.469	0.325	0.482	0.000	0.188	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	55	176	99	82	0	118	-1
normalized size	1	1.00	0.44	1.42	0.80	0.66	0.00	0.95	-0.01
time (sec)	N/A	0.082	0.063	0.479	0.337	0.560	0.000	0.191	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	236	4757	314	251	338	455	599
normalized size	1	1.00	0.75	15.10	1.00	0.80	1.07	1.44	1.90
time (sec)	N/A	0.776	0.168	6.088	0.341	0.530	15.181	0.482	5.593

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	192	3739	269	196	279	0	851
normalized size	1	1.00	0.85	16.62	1.20	0.87	1.24	0.00	3.78
time (sec)	N/A	0.269	0.160	6.000	0.332	0.649	9.742	0.000	1.699

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	3994	252	200	258	353	515
normalized size	1	1.00	0.74	16.17	1.02	0.81	1.04	1.43	2.09
time (sec)	N/A	0.634	0.135	4.133	0.336	0.676	6.540	0.399	2.805

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	2951	171	139	202	305	557
normalized size	1	1.00	0.92	21.08	1.22	0.99	1.44	2.18	3.98
time (sec)	N/A	0.119	0.114	3.851	0.320	0.631	3.781	0.354	1.420

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	144	2529	178	132	148	223	385
normalized size	1	1.00	1.38	24.32	1.71	1.27	1.42	2.14	3.70
time (sec)	N/A	0.195	0.019	1.481	0.325	0.533	2.265	0.183	1.794

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	0	1638	152	0	0	0	-1
normalized size	1	1.00	0.00	7.58	0.70	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.263	10.975	0.473	0.493	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	94	332	0	0	0	0	0	-1
normalized size	1	0.90	3.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.209	8.293	0.000	0.477	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	152	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.168	26.704	0.000	0.425	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	191	460	0	0	0	0	0	-1
normalized size	1	0.97	2.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.465	0.367	180.000	0.000	0.529	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	299	0	0	0	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.158	180.000	0.000	0.476	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F(-2)	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	250	0	0	0	0	0	0	-1
normalized size	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	0.302	180.000	0.000	0.430	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	1376	10161	0	0	0	0	-1
normalized size	1	1.00	2.69	19.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.763	5.810	2.907	0.000	0.424	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	599	1251	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.797	3.362	2.634	0.000	0.450	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.234	1.820	0.000	1.087	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	613	613	1226	0	0	0	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.739	3.274	2.015	0.000	0.770	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	982	960	0	0	0	0	-1
normalized size	1	1.00	2.09	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.746	4.968	3.589	0.000	0.707	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	122	70	247	226	74	0	93	67
normalized size	1	1.56	0.90	3.17	2.90	0.95	0.00	1.19	0.86
time (sec)	N/A	0.293	0.090	0.073	0.401	0.449	0.000	0.169	1.155

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [110] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	18	0.333
2	A	7	6	1.00	18	0.333
3	A	7	6	1.00	16	0.375
4	A	4	3	1.00	15	0.200
5	A	6	4	1.00	18	0.222
6	A	8	7	1.00	18	0.389
7	A	4	4	1.00	18	0.222
8	A	4	4	1.00	18	0.222
9	A	4	4	1.00	18	0.222
10	A	7	6	1.00	20	0.300
11	A	7	6	1.00	20	0.300
12	A	7	6	1.00	18	0.333
13	A	4	3	1.00	17	0.176
14	A	9	7	1.00	20	0.350
15	A	11	9	1.31	20	0.450
16	A	11	9	1.00	20	0.450
17	A	4	4	1.00	20	0.200
18	A	4	4	1.00	20	0.200
19	A	4	4	1.00	20	0.200
20	A	7	6	1.00	20	0.300
21	A	7	6	1.00	20	0.300
22	A	4	4	1.00	18	0.222
23	A	4	3	1.00	17	0.176
24	A	13	9	1.00	20	0.450
25	A	14	11	1.00	20	0.550
26	A	14	11	1.00	20	0.550
27	A	15	10	1.00	20	0.500
28	A	4	4	1.00	20	0.200
29	A	4	5	1.00	20	0.250
30	A	4	4	1.00	20	0.200
31	A	7	6	1.00	20	0.300
32	A	4	4	1.00	20	0.200
33	A	4	4	1.00	18	0.222
34	A	4	3	1.00	17	0.176
35	A	17	10	1.00	20	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	18	12	1.00	20	0.600
37	A	17	12	1.00	20	0.600
38	A	18	12	1.00	20	0.600
39	A	19	10	1.00	20	0.500
40	A	4	4	1.00	20	0.200
41	A	4	5	1.00	20	0.250
42	A	4	4	1.00	20	0.200
43	A	16	11	1.00	20	0.550
44	A	11	9	1.00	20	0.450
45	A	7	6	1.00	18	0.333
46	A	3	3	1.00	17	0.176
47	A	2	2	1.00	20	0.100
48	A	8	8	1.00	20	0.400
49	A	12	10	1.00	20	0.500
50	A	17	11	1.00	20	0.550
51	A	16	13	1.00	20	0.650
52	A	13	10	1.00	20	0.500
53	A	10	8	1.00	18	0.444
54	A	5	4	1.00	17	0.235
55	A	11	9	1.00	20	0.450
56	A	16	14	1.00	20	0.700
57	A	19	16	1.00	20	0.800
58	A	21	13	1.00	20	0.650
59	A	18	10	1.00	20	0.500
60	A	15	8	1.00	20	0.400
61	A	5	5	1.00	18	0.278
62	A	5	4	1.00	17	0.235
63	A	16	9	1.00	20	0.450
64	A	21	14	1.00	20	0.700
65	A	24	16	1.00	20	0.800
66	A	5	4	1.00	16	0.250
67	A	3	3	1.00	17	0.176
68	A	27	15	1.00	20	0.750
69	A	22	14	1.00	20	0.700
70	A	17	12	1.00	18	0.667
71	A	9	7	1.00	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	13	11	1.00	20	0.550
73	A	12	10	1.00	20	0.500
74	A	14	11	1.00	20	0.550
75	A	18	13	1.00	20	0.650
76	A	43	15	1.00	22	0.682
77	A	36	15	1.00	22	0.682
78	A	28	14	1.00	20	0.700
79	A	12	10	1.00	19	0.526
80	A	19	14	1.00	22	0.636
81	A	17	15	1.00	22	0.682
82	A	20	15	1.00	22	0.682
83	A	14	13	1.00	22	0.591
84	A	62	15	1.00	22	0.682
85	A	52	15	1.00	22	0.682
86	A	38	14	1.00	20	0.700
87	A	16	12	1.00	19	0.632
88	A	28	16	1.00	22	0.727
89	A	23	17	1.00	22	0.773
90	A	25	20	1.00	22	0.909
91	A	28	17	1.00	22	0.773
92	A	18	14	1.00	22	0.636
93	A	22	15	1.00	22	0.682
94	A	29	14	1.00	22	0.636
95	A	26	14	1.00	22	0.636
96	A	16	12	1.00	22	0.546
97	A	9	9	1.00	20	0.450
98	A	3	4	1.00	19	0.210
99	A	3	4	1.00	22	0.182
100	A	8	8	1.00	22	0.364
101	A	17	13	1.00	22	0.591
102	A	26	15	1.00	22	0.682
103	A	33	19	1.00	22	0.864
104	A	24	17	1.00	22	0.773
105	A	18	14	1.00	22	0.636
106	A	13	10	1.00	20	0.500
107	A	8	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	19	13	1.00	22	0.591
109	A	23	17	1.00	22	0.773
110	A	31	22	1.00	22	1.000
111	A	37	17	1.00	22	0.773
112	A	31	14	1.00	22	0.636
113	A	26	10	1.00	22	0.454
114	A	13	7	1.00	20	0.350
115	A	13	6	1.00	19	0.316
116	A	32	13	1.00	22	0.591
117	A	36	17	1.00	22	0.773
118	A	18	6	1.00	18	0.333
119	A	4	5	1.00	20	0.250
120	A	26	15	1.00	18	0.833
121	A	17	13	1.00	18	0.722
122	A	11	10	1.00	16	0.625
123	A	4	5	1.00	18	0.278
124	A	11	6	1.00	18	0.333
125	A	24	6	1.00	18	0.333
126	A	42	6	1.00	18	0.333
127	A	19	13	1.00	18	0.722
128	A	10	9	1.00	16	0.562
129	A	4	5	1.00	15	0.333
130	A	4	5	1.00	18	0.278
131	A	5	6	1.00	19	0.316
132	A	10	8	1.00	18	0.444
133	A	18	10	1.00	18	0.556
134	A	21	10	1.00	19	0.526
135	A	12	8	1.00	17	0.471
136	A	5	5	1.00	16	0.312
137	A	5	5	1.00	19	0.263
138	A	6	6	1.00	20	0.300
139	A	12	10	1.00	19	0.526
140	A	21	11	1.00	19	0.579
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	16	12	1.00	19	0.632
148	A	12	10	1.00	19	0.526
149	A	9	7	1.00	17	0.412
150	A	4	4	1.00	16	0.250
151	A	7	6	1.00	19	0.316
152	A	12	11	1.00	19	0.579
153	A	15	13	1.00	19	0.684
154	A	14	11	1.00	21	0.524
155	A	8	7	1.00	19	0.368
156	A	1	1	1.00	18	0.056
157	A	9	7	1.00	21	0.333
158	A	13	11	1.00	21	0.524
159	A	9	7	1.00	17	0.412
160	A	0	0	0.00	0	0.000
161	A	9	4	1.00	18	0.222
162	A	9	4	1.00	18	0.222
163	A	9	4	1.00	18	0.222
164	A	2	1	1.00	16	0.062
165	A	3	3	1.00	15	0.200
166	A	5	5	1.00	18	0.278
167	A	8	8	1.00	18	0.444
168	A	5	5	1.00	18	0.278
169	A	10	7	1.00	18	0.389
170	A	3	2	1.00	18	0.111
171	A	9	4	1.00	18	0.222
172	A	34	10	1.00	20	0.500
173	A	26	8	1.00	20	0.400
174	A	24	10	1.00	20	0.500
175	A	4	4	1.00	18	0.222
176	A	7	7	1.00	17	0.412
177	A	12	10	1.00	20	0.500
178	A	10	10	1.00	20	0.500
179	A	15	12	1.00	20	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	13	8	1.00	20	0.400
181	A	11	8	1.00	20	0.400
182	A	22	8	1.00	20	0.400
183	A	8	8	1.00	17	0.471
184	A	10	5	1.00	19	0.263
185	A	0	0	0.00	0	0.000
186	A	0	0	0.00	0	0.000
187	A	0	0	0.00	0	0.000
188	A	0	0	0.00	0	0.000
189	A	0	0	0.00	0	0.000
190	A	0	0	0.00	0	0.000
191	A	0	0	0.00	0	0.000
192	A	14	4	1.00	20	0.200
193	A	14	4	1.00	20	0.200
194	A	14	4	1.00	20	0.200
195	A	3	2	1.00	18	0.111
196	A	4	3	1.00	17	0.176
197	A	10	6	1.00	20	0.300
198	A	13	9	1.00	20	0.450
199	A	9	6	1.00	20	0.300
200	A	13	9	1.00	20	0.450
201	A	10	5	1.00	20	0.250
202	A	15	7	1.00	20	0.350
203	A	59	10	1.00	22	0.454
204	A	47	8	1.00	22	0.364
205	A	44	10	1.00	22	0.454
206	A	5	4	1.00	20	0.200
207	A	9	7	1.00	19	0.368
208	A	23	12	1.00	22	0.546
209	A	20	13	1.00	22	0.591
210	A	21	15	1.00	22	0.682
211	A	19	13	1.00	22	0.591
212	A	29	13	1.00	22	0.591
213	A	27	8	1.00	22	0.364
214	A	16	8	1.00	22	0.364
215	A	42	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	56	9	1.00	22	0.409
217	A	12	9	1.00	19	0.474
218	A	0	0	0.00	0	0.000
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	0	0	0.00	0	0.000
224	A	5	3	1.00	17	0.176
225	A	12	8	1.00	19	0.421
226	A	17	9	1.00	19	0.474
227	A	8	8	1.00	20	0.400
228	A	4	4	1.00	20	0.200
229	A	4	4	1.00	18	0.222
230	A	1	1	1.00	17	0.059
231	A	3	3	1.00	20	0.150
232	A	7	7	1.00	20	0.350
233	A	7	7	1.00	20	0.350
234	A	10	9	1.00	22	0.409
235	A	7	7	1.00	22	0.318
236	A	4	5	1.00	20	0.250
237	A	1	1	1.00	19	0.053
238	A	4	5	1.00	22	0.227
239	A	6	6	1.00	22	0.273
240	A	13	11	1.00	22	0.500
241	A	14	11	1.00	22	0.500
242	A	7	7	1.00	22	0.318
243	A	5	6	1.00	20	0.300
244	A	1	1	1.00	19	0.053
245	A	5	6	1.00	22	0.273
246	A	7	7	1.00	22	0.318
247	A	13	9	1.00	22	0.409
248	A	1	1	1.00	21	0.048
249	A	0	0	0.00	0	0.000
250	A	1	1	1.00	19	0.053
251	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
252	A	0	0	0.00	0	0.000
253	A	1	1	1.00	19	0.053
254	A	0	0	0.00	0	0.000
255	A	0	0	0.00	0	0.000
256	A	1	1	1.00	19	0.053
257	A	0	0	0.00	0	0.000
258	A	1	1	1.00	19	0.053
259	A	8	8	1.00	20	0.400
260	A	2	2	1.00	20	0.100
261	A	3	3	1.00	18	0.167
262	A	2	2	1.00	17	0.118
263	A	7	7	1.00	20	0.350
264	A	10	10	1.00	20	0.500
265	A	15	10	1.00	20	0.500
266	A	8	9	1.00	22	0.409
267	A	4	4	1.00	22	0.182
268	A	3	3	1.00	20	0.150
269	A	4	4	1.00	19	0.210
270	A	8	9	1.00	22	0.409
271	A	11	11	1.00	22	0.500
272	A	22	15	1.00	22	0.682
273	A	11	11	1.00	22	0.500
274	A	4	4	1.00	22	0.182
275	A	5	4	1.00	20	0.200
276	A	4	3	1.00	19	0.158
277	A	11	11	1.00	22	0.500
278	A	12	11	1.00	22	0.500
279	A	25	14	1.00	22	0.636
280	A	9	8	1.00	21	0.381
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	4	3	1.00	22	0.136
284	A	4	4	1.00	20	0.200
285	A	4	3	1.00	19	0.158
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	5	5	1.00	22	0.227
289	A	9	5	1.00	20	0.250
290	A	5	5	1.00	19	0.263
291	A	0	0	0.00	0	0.000
292	A	0	0	0.00	0	0.000
293	A	10	6	1.00	22	0.273
294	A	5	5	1.00	20	0.250
295	A	10	6	1.00	19	0.316
296	A	0	0	0.00	0	0.000
297	A	6	6	1.00	19	0.316
298	A	11	7	1.00	19	0.368
299	A	7	6	1.00	19	0.316
300	A	12	7	1.00	19	0.368
301	A	8	6	1.00	19	0.316
302	A	4	3	1.00	20	0.150
303	A	3	3	1.00	20	0.150
304	A	4	3	1.00	18	0.167
305	A	3	3	1.00	17	0.176
306	A	12	7	1.00	20	0.350
307	A	14	11	1.00	20	0.550
308	A	4	4	1.00	22	0.182
309	A	13	6	1.00	22	0.273
310	A	4	4	1.00	20	0.200
311	A	8	5	1.00	19	0.263
312	A	13	10	1.00	22	0.454
313	A	20	12	1.00	22	0.546
314	A	9	7	1.00	22	0.318
315	A	13	6	1.00	22	0.273
316	A	9	5	1.00	20	0.250
317	A	8	5	1.00	19	0.263
318	A	21	12	1.00	22	0.546
319	A	21	13	1.00	22	0.591
320	A	15	7	1.00	21	0.333
321	A	0	0	0.00	0	0.000
322	A	0	0	0.00	0	0.000
323	A	5	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	5	3	1.00	22	0.136
325	A	4	3	1.00	22	0.136
326	A	5	3	1.00	20	0.150
327	A	5	3	1.00	19	0.158
328	A	0	0	0.00	0	0.000
329	A	0	0	0.00	0	0.000
330	A	6	4	1.00	22	0.182
331	A	20	7	1.38	22	0.318
332	A	12	6	1.46	22	0.273
333	A	10	6	1.00	20	0.300
334	A	6	4	1.00	19	0.210
335	A	0	0	0.00	0	0.000
336	A	21	8	1.00	22	0.364
337	A	25	8	1.50	22	0.364
338	A	22	8	1.27	22	0.364
339	A	19	7	1.00	20	0.350
340	A	11	7	1.00	19	0.368
341	A	0	0	0.00	0	0.000
342	A	20	7	1.00	19	0.368
343	A	35	8	1.00	19	0.421
344	A	49	8	1.00	19	0.421
345	A	4	3	1.00	17	0.176
346	A	13	5	1.00	19	0.263
347	A	13	5	1.00	19	0.263
348	A	21	7	1.00	21	0.333
349	A	0	0	0.00	0	0.000
350	A	0	0	0.00	0	0.000
351	A	6	3	1.00	22	0.136
352	A	6	3	1.00	22	0.136
353	A	6	3	1.00	22	0.136
354	A	5	3	1.00	22	0.136
355	A	6	3	1.00	22	0.136
356	A	6	3	1.00	20	0.150
357	A	6	3	1.00	19	0.158
358	A	0	0	0.00	0	0.000
359	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	13	6	1.00	20	0.300
361	A	7	4	1.00	19	0.210
362	A	22	6	1.00	20	0.300
363	A	14	7	1.00	19	0.368
364	A	9	4	1.00	22	0.182
365	A	7	5	1.00	22	0.227
366	A	5	4	1.00	22	0.182
367	A	3	3	1.00	22	0.136
368	A	2	2	1.00	20	0.100
369	A	1	1	1.00	19	0.053
370	A	1	1	1.00	22	0.045
371	A	4	4	1.00	22	0.182
372	A	3	3	1.00	22	0.136
373	A	6	4	1.00	24	0.167
374	A	11	8	1.00	24	0.333
375	A	2	2	1.00	22	0.091
376	A	8	5	1.00	21	0.238
377	A	8	5	1.00	24	0.208
378	A	2	2	1.00	24	0.083
379	A	13	10	1.00	24	0.417
380	A	21	8	1.00	24	0.333
381	A	13	9	1.00	24	0.375
382	A	9	6	1.00	22	0.273
383	A	10	6	1.00	21	0.286
384	A	10	6	1.00	24	0.250
385	A	9	6	1.00	24	0.250
386	A	13	9	1.00	24	0.375
387	A	0	0	0.00	0	0.000
388	A	5	4	1.00	22	0.182
389	A	2	2	1.00	22	0.091
390	A	2	2	1.00	20	0.100
391	A	1	1	1.00	19	0.053
392	A	4	4	1.00	22	0.182
393	A	6	6	1.00	22	0.273
394	A	8	6	1.00	22	0.273
395	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	5	4	1.00	24	0.167
397	A	11	8	1.00	24	0.333
398	A	2	2	1.00	22	0.091
399	A	2	2	1.00	21	0.095
400	A	11	8	1.00	24	0.333
401	A	5	5	1.00	24	0.208
402	A	25	13	1.00	24	0.542
403	A	0	0	0.00	0	0.000
404	A	13	9	1.00	24	0.375
405	A	13	9	1.00	24	0.375
406	A	3	3	1.00	22	0.136
407	A	2	2	1.00	21	0.095
408	A	14	10	1.00	24	0.417
409	A	12	9	1.00	24	0.375
410	A	28	13	1.00	24	0.542
411	A	0	0	0.00	0	0.000
412	A	0	0	0.00	0	0.000
413	A	2	2	1.00	22	0.091
414	A	2	2	1.00	21	0.095
415	A	0	0	0.00	0	0.000
416	A	0	0	0.00	0	0.000
417	A	0	0	0.00	0	0.000
418	A	3	3	1.00	22	0.136
419	A	3	3	1.00	21	0.143
420	A	0	0	0.00	0	0.000
421	A	0	0	0.00	0	0.000
422	A	0	0	0.00	0	0.000
423	A	4	4	1.00	22	0.182
424	A	4	4	1.00	21	0.190
425	A	0	0	0.00	0	0.000
426	A	11	6	1.00	22	0.273
427	A	9	5	1.00	22	0.227
428	A	7	6	1.00	22	0.273
429	A	3	3	1.00	20	0.150
430	A	2	2	1.00	19	0.105
431	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
432	A	6	6	1.00	22	0.273
433	A	5	4	1.00	22	0.182
434	A	5	5	1.00	22	0.227
435	A	9	5	1.00	22	0.227
436	A	21	7	1.00	22	0.318
437	A	14	5	1.00	22	0.227
438	A	45	10	1.00	24	0.417
439	A	21	7	1.00	24	0.292
440	A	29	10	1.00	24	0.417
441	A	3	3	1.00	22	0.136
442	A	10	7	1.00	21	0.333
443	A	11	8	1.00	24	0.333
444	A	11	8	1.00	24	0.333
445	A	22	11	1.00	24	0.458
446	A	6	5	1.00	24	0.208
447	A	27	7	1.00	22	0.318
448	A	24	6	1.00	22	0.273
449	A	19	7	1.00	22	0.318
450	A	4	3	1.00	20	0.150
451	A	3	2	1.00	19	0.105
452	A	7	6	1.00	22	0.273
453	A	9	7	1.00	22	0.318
454	A	9	6	1.00	22	0.273
455	A	12	7	1.00	22	0.318
456	A	15	6	1.00	22	0.273
457	A	6	5	1.00	22	0.227
458	A	24	6	1.00	22	0.273
459	A	4	2	1.00	19	0.105
460	A	3	2	1.00	19	0.105
461	A	2	2	1.00	19	0.105
462	A	2	2	1.00	19	0.105
463	A	3	2	1.00	19	0.105
464	A	4	2	1.00	19	0.105
465	A	4	3	1.00	20	0.150
466	A	3	3	1.00	20	0.150
467	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	1	1	1.00	20	0.050
469	A	2	2	1.00	20	0.100
470	A	3	2	1.00	20	0.100
471	A	10	7	1.00	21	0.333
472	A	5	4	1.00	21	0.190
473	A	9	4	1.00	21	0.190
474	A	14	4	1.00	21	0.190
475	A	12	8	1.00	21	0.381
476	A	5	4	1.00	21	0.190
477	A	9	4	1.00	21	0.190
478	A	14	4	1.00	21	0.190
479	A	0	0	0.00	0	0.000
480	A	0	0	0.00	0	0.000
481	A	2	2	1.00	21	0.095
482	A	5	3	1.00	21	0.143
483	A	6	3	1.00	21	0.143
484	A	7	3	1.00	21	0.143
485	A	0	0	0.00	0	0.000
486	A	0	0	0.00	0	0.000
487	A	3	3	1.00	21	0.143
488	A	6	4	1.00	21	0.190
489	A	7	4	1.00	21	0.190
490	A	8	4	1.00	21	0.190
491	A	0	0	0.00	0	0.000
492	A	0	0	0.00	0	0.000
493	A	4	4	1.00	21	0.190
494	A	12	7	1.00	21	0.333
495	A	14	7	1.00	21	0.333
496	A	16	7	1.00	21	0.333
497	A	7	6	1.00	28	0.214
498	A	4	4	1.00	14	0.286
499	A	4	4	1.00	14	0.286
500	A	5	5	1.00	14	0.357
501	A	5	4	1.00	12	0.333
502	A	17	5	1.00	14	0.357
503	A	25	13	1.00	14	0.929

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	23	11	1.00	14	0.786
505	A	1	1	1.00	20	0.050
506	A	17	5	1.00	14	0.357
507	A	17	5	1.00	16	0.312
508	A	4	4	1.00	10	0.400
509	A	17	5	1.00	12	0.417
510	A	10	7	1.00	15	0.467
511	A	0	0	0.00	0	0.000
512	A	0	0	0.00	0	0.000
513	A	5	6	1.00	16	0.375
514	A	7	9	1.00	16	0.562
515	A	8	9	1.00	16	0.562
516	A	8	9	1.00	16	0.562
517	A	3	3	1.00	15	0.200
518	A	2	2	1.00	15	0.133
519	A	1	1	1.00	15	0.067
520	A	2	2	1.00	15	0.133
521	A	3	2	1.00	15	0.133
522	A	26	15	1.00	27	0.556
523	A	14	11	1.00	27	0.407
524	A	21	15	1.00	27	0.556
525	A	7	8	1.00	25	0.320
526	A	9	8	1.00	24	0.333
527	A	14	9	1.00	27	0.333
528	A	8	8	0.90	27	0.296
529	A	5	6	1.00	27	0.222
530	A	17	16	0.97	27	0.593
531	A	10	7	1.00	27	0.259
532	A	26	18	0.98	27	0.667
533	A	22	17	1.00	22	0.773
534	A	28	12	1.00	21	0.571
535	A	0	0	0.00	0	0.000
536	A	28	14	1.00	24	0.583
537	A	20	17	1.00	24	0.708
538	A	16	7	1.56	20	0.350

Chapter 3

Listing of integrals

3.1 $\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=108

$$\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{9bd\log(1-cx)}{40c^4}-\frac{bd\log(cx+1)}{40c^4}+\frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}b$$

```
[Out] 1/4*b*d*x/c^3+1/10*b*d*x^2/c^2+1/12*b*d*x^3/c+1/20*b*d*x^4+1/4*d*x^4*(a+b*a
rctanh(c*x))+1/5*c*d*x^5*(a+b*arctanh(c*x))+9/40*b*d*ln(-c*x+1)/c^4-1/40*b*
d*ln(c*x+1)/c^4
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {43, 5936, 12, 801, 633, 31}

$$\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bdx^2}{10c^2}+\frac{bdx}{4c^3}+\frac{9bd\log(1-cx)}{40c^4}-\frac{bd\log(cx+1)}{40c^4}+\frac{bdx^3}{12c}+\frac{1}{20}b$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^3) + (b*d*x^2)/(10*c^2) + (b*d*x^3)/(12*c) + (b*d*x^4)/20 + (d
*x^4*(a + b*ArcTanh[c*x]))/4 + (c*d*x^5*(a + b*ArcTanh[c*x]))/5 + (9*b*d*Lo
g[1 - c*x])/(40*c^4) - (b*d*Log[1 + c*x])/(40*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^4(5+4c\ldots}{20\left(1-c^2x\ldots\right.}$$

$$= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\frac{x^4(5+\ldots}{1-c\ldots}$$

$$= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\left(-\frac{5}{c^4}-\ldots\right.$$

$$= \frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\ldots\right.$$

$$= \frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\ldots\right.$$

$$= \frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\ldots\right.$$

Mathematica [A] time = 0.07, size = 97, normalized size = 0.90

$$\frac{d\left(24ac^5x^5+30ac^4x^4+6bc^4x^4+6bc^4x^4(4cx+5)\tanh^{-1}(cx)+10bc^3x^3+12bc^2x^2+30bcx+27b\log(1-cx)-3\ldots\right.}{120c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(30*b*c*x + 12*b*c^2*x^2 + 10*b*c^3*x^3 + 30*a*c^4*x^4 + 6*b*c^4*x^4 + 2
4*a*c^5*x^5 + 6*b*c^4*x^4*(5 + 4*c*x)*ArcTanh[c*x] + 27*b*Log[1 - c*x] - 3*
b*Log[1 + c*x]))/(120*c^4)
```

fricas [A] time = 0.48, size = 114, normalized size = 1.06

$$\frac{24\,ac^5dx^5+6\,(5\,a+b)c^4dx^4+10\,bc^3dx^3+12\,bc^2dx^2+30\,bcdx-3\,bd\log(cx+1)+27\,bd\log(cx-1)+3\left(4\,bc^5\ldots\right.}{120\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*d*x^5 + 6*(5*a + b)*c^4*d*x^4 + 10*b*c^3*d*x^3 + 12*b*c^2*d
*x^2 + 30*b*c*d*x - 3*b*d*log(c*x + 1) + 27*b*d*log(c*x - 1) + 3*(4*b*c^5*d
*x^5 + 5*b*c^4*d*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

giac [B] time = 0.25, size = 491, normalized size = 4.55

$$\frac{1}{15}c\left(\frac{3\left(\frac{10(cx+1)^4bd}{(cx-1)^4}-\frac{5(cx+1)^3bd}{(cx-1)^3}+\frac{15(cx+1)^2bd}{(cx-1)^2}-\frac{5(cx+1)bd}{cx-1}+bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^5}{(cx-1)^5}-\frac{5(cx+1)^4c^5}{(cx-1)^4}+\frac{10(cx+1)^3c^5}{(cx-1)^3}-\frac{10(cx+1)^2c^5}{(cx-1)^2}+\frac{5(cx+1)c^5}{cx-1}-c^5}+\frac{\frac{60(cx+1)^4ad}{(cx-1)^4}-\frac{30(cx+1)^3ad}{(cx-1)^3}+\frac{90(cx+1)^2}{(cx-1)^2}}{\frac{(cx+1)^5c^5}{(cx-1)^5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/15*c*(3*(10*(c*x + 1)^4*b*d/(c*x - 1)^4 - 5*(c*x + 1)^3*b*d/(c*x - 1)^3 +
 15*(c*x + 1)^2*b*d/(c*x - 1)^2 - 5*(c*x + 1)*b*d/(c*x - 1) + b*d)*log(-(c*
x + 1)/(c*x - 1))/((c*x + 1)^5*c^5/(c*x - 1)^5 - 5*(c*x + 1)^4*c^5/(c*x - 1
)^4 + 10*(c*x + 1)^3*c^5/(c*x - 1)^3 - 10*(c*x + 1)^2*c^5/(c*x - 1)^2 + 5*(
c*x + 1)*c^5/(c*x - 1) - c^5) + (60*(c*x + 1)^4*a*d/(c*x - 1)^4 - 30*(c*x +
 1)^3*a*d/(c*x - 1)^3 + 90*(c*x + 1)^2*a*d/(c*x - 1)^2 - 30*(c*x + 1)*a*d/(
c*x - 1) + 6*a*d + 27*(c*x + 1)^4*b*d/(c*x - 1)^4 - 69*(c*x + 1)^3*b*d/(c*x
 - 1)^3 + 79*(c*x + 1)^2*b*d/(c*x - 1)^2 - 47*(c*x + 1)*b*d/(c*x - 1) + 10*
b*d)/((c*x + 1)^5*c^5/(c*x - 1)^5 - 5*(c*x + 1)^4*c^5/(c*x - 1)^4 + 10*(c*x
 + 1)^3*c^5/(c*x - 1)^3 - 10*(c*x + 1)^2*c^5/(c*x - 1)^2 + 5*(c*x + 1)*c^5/
(c*x - 1) - c^5) - 3*b*d*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 3*b*d*log(-(c*
x + 1)/(c*x - 1))/c^5)
```

maple [A] time = 0.03, size = 101, normalized size = 0.94

$$\frac{cda\,x^5}{5}+\frac{da\,x^4}{4}+\frac{cdb\operatorname{arctanh}(cx)\,x^5}{5}+\frac{db\operatorname{arctanh}(cx)\,x^4}{4}+\frac{bd\,x^4}{20}+\frac{bd\,x^3}{12c}+\frac{bd\,x^2}{10c^2}+\frac{bdx}{4c^3}+\frac{9db\ln(cx-1)}{40c^4}-\frac{bd\ln(cx}{40c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c*d*a*x^5+1/4*d*a*x^4+1/5*c*d*b*arctanh(c*x)*x^5+1/4*d*b*arctanh(c*x)*x
^4+1/20*b*d*x^4+1/12*b*d*x^3/c+1/10*b*d*x^2/c^2+1/4*b*d*x/c^3+9/40/c^4*d*b*
ln(c*x-1)-1/40*b*d*ln(c*x+1)/c^4
```

maxima [A] time = 0.31, size = 121, normalized size = 1.12

$$\frac{1}{5}acdx^5+\frac{1}{4}adx^4+\frac{1}{20}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)bcd+\frac{1}{24}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^
2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d + 1/24*(6*x^4*arctanh(c*x) + c*(2*(
c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d
```

mupad [B] time = 0.99, size = 103, normalized size = 0.95

$$\frac{\frac{b\,c\,d\,x}{4}-\frac{d\left(15\,b\,\mathrm{atanh}(c\,x)-6\,b\,\ln\left(c^2\,x^2-1\right)\right)}{60}+\frac{b\,c^2\,d\,x^2}{10}+\frac{b\,c^3\,d\,x^3}{12}}{c^4}+\frac{d\,\left(15\,a\,x^4+3\,b\,x^4+15\,b\,x^4\,\mathrm{atanh}\left(c\,x\right)\right)}{60}+\frac{c\,d\,\left(12\,a\,x^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] ((b*c*d*x)/4 - (d*(15*b*atanh(c*x) - 6*b*log(c^2*x^2 - 1)))/60 + (b*c^2*d*x
^2)/10 + (b*c^3*d*x^3)/12)/c^4 + (d*(15*a*x^4 + 3*b*x^4 + 15*b*x^4*atanh(c*
x)))/60 + (c*d*(12*a*x^5 + 12*b*x^5*atanh(c*x)))/60
```

sympy [A] time = 1.53, size = 124, normalized size = 1.15

$$\begin{cases} \frac{acdx^5}{5} + \frac{adx^4}{4} + \frac{bcdx^5 \operatorname{atanh}(cx)}{5} + \frac{bdx^4 \operatorname{atanh}(cx)}{4} + \frac{bdx^4}{20} + \frac{bdx^3}{12c} + \frac{bdx^2}{10c^2} + \frac{bdx}{4c^3} + \frac{bd \log\left(x - \frac{1}{c}\right)}{5c^4} - \frac{bd \operatorname{atanh}(cx)}{20c^4} & \text{for } c \neq 0 \\ \frac{adx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**5/5 + a*d*x**4/4 + b*c*d*x**5*atanh(c*x)/5 + b*d*x**4*a
tanh(c*x)/4 + b*d*x**4/20 + b*d*x**3/(12*c) + b*d*x**2/(10*c**2) + b*d*x/(4
*c**3) + b*d*log(x - 1/c)/(5*c**4) - b*d*atanh(c*x)/(20*c**4), Ne(c, 0)), (
a*d*x**4/4, True))
```

3.2 $\int x^2(d + cdx)\left(a + b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=96

$$\frac{1}{4}cdx^4\left(a + b\tanh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a + b\tanh^{-1}(cx)\right)+\frac{7bd\log(1-cx)}{24c^3}+\frac{bd\log(cx+1)}{24c^3}+\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3$$

```
[Out] 1/4*b*d*x/c^2+1/6*b*d*x^2/c+1/12*b*d*x^3+1/3*d*x^3*(a+b*arctanh(c*x))+1/4*c
*d*x^4*(a+b*arctanh(c*x))+7/24*b*d*ln(-c*x+1)/c^3+1/24*b*d*ln(c*x+1)/c^3
```

Rubi [A] time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {43, 5936, 12, 801, 633, 31}

$$\frac{1}{4}cdx^4\left(a + b\tanh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a + b\tanh^{-1}(cx)\right)+\frac{bdx}{4c^2}+\frac{7bd\log(1-cx)}{24c^3}+\frac{bd\log(cx+1)}{24c^3}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^2) + (b*d*x^2)/(6*c) + (b*d*x^3)/12 + (d*x^3*(a + b*ArcTanh[c*
x]))/3 + (c*d*x^4*(a + b*ArcTanh[c*x]))/4 + (7*b*d*Log[1 - c*x])/(24*c^3) +
 (b*d*Log[1 + c*x])/(24*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
```

```
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^3(4+3c}{12\left(1-c^2x\right.}$$

$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\frac{x^3(4+}{1-c}$$

$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\left(-\frac{3}{c^3}-\right.$$

$$=\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(\right.$$

$$=\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(\right.$$

$$=\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(\right.$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.91

$$\frac{d\left(6ac^4x^4+8ac^3x^3+2bc^3x^3+2bc^3x^3(3cx+4)\tanh^{-1}(cx)+4bc^2x^2+6bcx+7b\log(1-cx)+b\log(cx+1)\right)}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(6*b*c*x + 4*b*c^2*x^2 + 8*a*c^3*x^3 + 2*b*c^3*x^3 + 6*a*c^4*x^4 + 2*b*c
^3*x^3*(4 + 3*c*x)*ArcTanh[c*x] + 7*b*Log[1 - c*x] + b*Log[1 + c*x]))/(24*c
^3)
```

fricas [A] time = 0.66, size = 102, normalized size = 1.06

$$\frac{6\,ac^4dx^4+2\,(4\,a+b)c^3dx^3+4\,bc^2dx^2+6\,bcdx+bd\log(cx+1)+7\,bd\log(cx-1)+\left(3\,bc^4dx^4+4\,bc^3dx^3\right)\log\left(\right.}{24\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d*x^4 + 2*(4*a + b)*c^3*d*x^3 + 4*b*c^2*d*x^2 + 6*b*c*d*x + b
*d*log(c*x + 1) + 7*b*d*log(c*x - 1) + (3*b*c^4*d*x^4 + 4*b*c^3*d*x^3)*log(
-(c*x + 1)/(c*x - 1)))/c^3
```

giac [B] time = 0.20, size = 394, normalized size = 4.10

$$\frac{1}{3}c\left(\frac{\left(\frac{6(cx+1)^3bd}{(cx-1)^3}-\frac{3(cx+1)^2bd}{(cx-1)^2}+\frac{4(cx+1)bd}{cx-1}-bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^4}{(cx-1)^4}-\frac{4(cx+1)^3c^4}{(cx-1)^3}+\frac{6(cx+1)^2c^4}{(cx-1)^2}-\frac{4(cx+1)c^4}{cx-1}+c^4}+\frac{\frac{12(cx+1)^3ad}{(cx-1)^3}-\frac{6(cx+1)^2ad}{(cx-1)^2}+\frac{8(cx+1)ad}{cx-1}-2\,ad+\frac{5(cx+1)^3bd}{(cx-1)^3}\right.}{\frac{(cx+1)^4c^4}{(cx-1)^4}-\frac{4(cx+1)^3c^4}{(cx-1)^3}+\frac{6(cx+1)^2c^4}{(cx-1)^2}-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*((6*(c*x + 1)^3*b*d/(c*x - 1)^3 - 3*(c*x + 1)^2*b*d/(c*x - 1)^2 + 4*(
c*x + 1)*b*d/(c*x - 1) - b*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^4/(c
```

```
*x - 1)^4 - 4*(c*x + 1)^3*c^4/(c*x - 1)^3 + 6*(c*x + 1)^2*c^4/(c*x - 1)^2 -
 4*(c*x + 1)*c^4/(c*x - 1) + c^4) + (12*(c*x + 1)^3*a*d/(c*x - 1)^3 - 6*(c*
x + 1)^2*a*d/(c*x - 1)^2 + 8*(c*x + 1)*a*d/(c*x - 1) - 2*a*d + 5*(c*x + 1)^
3*b*d/(c*x - 1)^3 - 10*(c*x + 1)^2*b*d/(c*x - 1)^2 + 7*(c*x + 1)*b*d/(c*x -
 1) - 2*b*d)/((c*x + 1)^4*c^4/(c*x - 1)^4 - 4*(c*x + 1)^3*c^4/(c*x - 1)^3 +
 6*(c*x + 1)^2*c^4/(c*x - 1)^2 - 4*(c*x + 1)*c^4/(c*x - 1) + c^4) - b*d*log
(-(c*x + 1)/(c*x - 1) + 1)/c^4 + b*d*log(-(c*x + 1)/(c*x - 1))/c^4)
```

maple [A] time = 0.03, size = 91, normalized size = 0.95

$$\frac{cda\,x^4}{4}+\frac{da\,x^3}{3}+\frac{cdb\operatorname{arctanh}(cx)\,x^4}{4}+\frac{db\operatorname{arctanh}(cx)\,x^3}{3}+\frac{bd\,x^3}{12}+\frac{bd\,x^2}{6c}+\frac{bdx}{4c^2}+\frac{7db\ln(cx-1)}{24c^3}+\frac{bd\ln(cx+1)}{24c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*c*d*a*x^4+1/3*d*a*x^3+1/4*c*d*b*arctanh(c*x)*x^4+1/3*d*b*arctanh(c*x)*x
^3+1/12*b*d*x^3+1/6*b*d*x^2/c+1/4*b*d*x/c^2+7/24/c^3*d*b*ln(c*x-1)+1/24*b*d
*ln(c*x+1)/c^3
```

maxima [A] time = 0.32, size = 110, normalized size = 1.15

$$\frac{1}{4}\,acdx^4+\frac{1}{3}\,adx^3+\frac{1}{24}\left(6\,x^4\operatorname{artanh}(cx)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log(cx+1)}{c^5}+\frac{3\,\log(cx-1)}{c^5}\right)\right)bcd+\frac{1}{6}\left(2\,x^3\operatorname{arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c*d*x^4 + 1/3*a*d*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*
x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d + 1/6*(2*x^3*arcta
nh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d
```

mupad [B] time = 0.94, size = 92, normalized size = 0.96

$$\frac{\frac{b\,c\,d\,x}{4}-\frac{d\left(3\,b\operatorname{atanh}(c\,x)-2\,b\,\ln\left(c^2\,x^2-1\right)\right)}{12}+\frac{b\,c^2\,d\,x^2}{6}}{c^3}+\frac{d\,\left(4\,a\,x^3+b\,x^3+4\,b\,x^3\operatorname{atanh}(c\,x)\right)}{12}+\frac{c\,d\,\left(3\,a\,x^4+3\,b\,x^4\operatorname{atanh}(c\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] ((b*c*d*x)/4 - (d*(3*b*atanh(c*x) - 2*b*log(c^2*x^2 - 1)))/12 + (b*c^2*d*x^
2)/6)/c^3 + (d*(4*a*x^3 + b*x^3 + 4*b*x^3*atanh(c*x)))/12 + (c*d*(3*a*x^4 +
 3*b*x^4*atanh(c*x)))/12
```

sympy [A] time = 1.20, size = 112, normalized size = 1.17

$$\begin{cases}\frac{acdx^4}{4}+\frac{adx^3}{3}+\frac{bcdx^4\operatorname{atanh}(cx)}{4}+\frac{bdx^3\operatorname{atanh}(cx)}{3}+\frac{bdx^3}{12}+\frac{bdx^2}{6c}+\frac{bdx}{4c^2}+\frac{bd\log\left(x-\frac{1}{c}\right)}{3c^3}+\frac{bd\operatorname{atanh}(cx)}{12c^3} & \text{for } c\neq 0\\ \frac{adx^3}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**4/4 + a*d*x**3/3 + b*c*d*x**4*atanh(c*x)/4 + b*d*x**3*a
tanh(c*x)/3 + b*d*x**3/12 + b*d*x**2/(6*c) + b*d*x/(4*c**2) + b*d*log(x - 1
/c)/(3*c**3) + b*d*atanh(c*x)/(12*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

3.3 $\int x(d + cdx) \left(a + b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=84

$$\frac{1}{3}cdx^3 \left(a + b \tanh^{-1}(cx)\right) + \frac{1}{2}dx^2 \left(a + b \tanh^{-1}(cx)\right) + \frac{5bd \log(1 - cx)}{12c^2} - \frac{bd \log(cx + 1)}{12c^2} + \frac{bdx}{2c} + \frac{1}{6}bdx^2$$

```
[Out] 1/2*b*d*x/c+1/6*b*d*x^2+1/2*d*x^2*(a+b*arctanh(c*x))+1/3*c*d*x^3*(a+b*arcta
nh(c*x))+5/12*b*d*ln(-c*x+1)/c^2-1/12*b*d*ln(c*x+1)/c^2
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {43, 5936, 12, 801, 633, 31}

$$\frac{1}{3}cdx^3 \left(a + b \tanh^{-1}(cx)\right) + \frac{1}{2}dx^2 \left(a + b \tanh^{-1}(cx)\right) + \frac{5bd \log(1 - cx)}{12c^2} - \frac{bd \log(cx + 1)}{12c^2} + \frac{bdx}{2c} + \frac{1}{6}bdx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(2*c) + (b*d*x^2)/6 + (d*x^2*(a + b*ArcTanh[c*x]))/2 + (c*d*x^3*(a
+ b*ArcTanh[c*x]))/3 + (5*b*d*Log[1 - c*x])/(12*c^2) - (b*d*Log[1 + c*x])/(
12*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
```

```
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^2(3+2}{6-6c^2}$$
$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bcd)\int\frac{x^2(3+2}{6-6c^2}$$
$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bcd)\int\left(-\frac{1}{2c^2}-\right.$$
$$=\frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-$$
$$=\frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+$$
$$=\frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.94

$$\frac{d\left(4ac^3x^3+6ac^2x^2+2bc^2x^2+2bc^2x^2(2cx+3)\tanh^{-1}(cx)+6bcx+5b\log(1-cx)-b\log(cx+1)\right)}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(6*b*c*x + 6*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 2*b*c^2*x^2*(3 + 2*
c*x)*ArcTanh[c*x] + 5*b*Log[1 - c*x] - b*Log[1 + c*x]))/(12*c^2)
```

fricas [A] time = 0.54, size = 93, normalized size = 1.11

$$\frac{4\,ac^3dx^3+2\,(3\,a+b)c^2dx^2+6\,bcdx-bd\log\left(cx+1\right)+5\,bd\log\left(cx-1\right)+\left(2\,bc^3dx^3+3\,bc^2dx^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*c^3*d*x^3 + 2*(3*a + b)*c^2*d*x^2 + 6*b*c*d*x - b*d*log(c*x + 1)
+ 5*b*d*log(c*x - 1) + (2*b*c^3*d*x^3 + 3*b*c^2*d*x^2)*log(-(c*x + 1)/(c*x
- 1)))/c^2
```

giac [B] time = 0.16, size = 305, normalized size = 3.63

$$\frac{1}{3}c\left(\frac{\left(\frac{6\,(cx+1)^2bd}{(cx-1)^2}-\frac{3\,(cx+1)bd}{cx-1}+bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^3}{(cx-1)^3}-\frac{3\,(cx+1)^2c^3}{(cx-1)^2}+\frac{3\,(cx+1)c^3}{cx-1}-c^3}+\frac{\frac{12\,(cx+1)^2ad}{(cx-1)^2}-\frac{6\,(cx+1)ad}{cx-1}+2\,ad+\frac{5\,(cx+1)^2bd}{(cx-1)^2}-\frac{8\,(cx+1)bd}{cx-1}+3\,bd}{\frac{(cx+1)^3c^3}{(cx-1)^3}-\frac{3\,(cx+1)^2c^3}{(cx-1)^2}+\frac{3\,(cx+1)c^3}{cx-1}-c^3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*((6*(c*x + 1)^2*b*d/(c*x - 1)^2 - 3*(c*x + 1)*b*d/(c*x - 1) + b*d)*lo
g(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^3/(c*x - 1)^3 - 3*(c*x + 1)^2*c^3/(c
*x - 1)^2 + 3*(c*x + 1)*c^3/(c*x - 1) - c^3) + (12*(c*x + 1)^2*a*d/(c*x - 1
```

```
)^2 - 6*(c*x + 1)*a*d/(c*x - 1) + 2*a*d + 5*(c*x + 1)^2*b*d/(c*x - 1)^2 - 8
*(c*x + 1)*b*d/(c*x - 1) + 3*b*d)/((c*x + 1)^3*c^3/(c*x - 1)^3 - 3*(c*x + 1
)^2*c^3/(c*x - 1)^2 + 3*(c*x + 1)*c^3/(c*x - 1) - c^3) - b*d*log(-(c*x + 1)
/(c*x - 1) + 1)/c^3 + b*d*log(-(c*x + 1)/(c*x - 1))/c^3)
```

maple [A] time = 0.03, size = 81, normalized size = 0.96

$$\frac{cda\,x^3}{3}+\frac{da\,x^2}{2}+\frac{cdb\,\mathrm{arctanh}\,(cx)\,x^3}{3}+\frac{db\,\mathrm{arctanh}\,(cx)\,x^2}{2}+\frac{bd\,x^2}{6}+\frac{bdx}{2c}+\frac{5db\ln(cx-1)}{12c^2}-\frac{bd\ln(cx+1)}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/3*c*d*a*x^3+1/2*d*a*x^2+1/3*c*d*b*arctanh(c*x)*x^3+1/2*d*b*arctanh(c*x)*x
^2+1/6*b*d*x^2+1/2*b*d*x/c+5/12/c^2*d*b*ln(c*x-1)-1/12*b*d*ln(c*x+1)/c^2
```

maxima [A] time = 0.31, size = 99, normalized size = 1.18

$$\frac{1}{3}\,acdx^3+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bcd+\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\,(cx+1)}{c^3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c*d*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4
))*b*c*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1
)/c^3 + log(c*x - 1)/c^3))*b*d
```

mupad [B] time = 0.90, size = 83, normalized size = 0.99

$$\frac{d\,\left(3\,a\,x^2+b\,x^2+3\,b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{6}-\frac{\frac{d\left(3\,b\,\mathrm{atanh}(c\,x)-b\,\ln\left(c^2\,x^2-1\right)\right)}{6}-\frac{b\,c\,d\,x}{2}}{c^2}+\frac{c\,d\,\left(2\,a\,x^3+2\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] (d*(3*a*x^2 + b*x^2 + 3*b*x^2*atanh(c*x)))/6 - ((d*(3*b*atanh(c*x) - b*log(
c^2*x^2 - 1)))/6 - (b*c*d*x)/2)/c^2 + (c*d*(2*a*x^3 + 2*b*x^3*atanh(c*x)))/
6
```

sympy [A] time = 0.92, size = 100, normalized size = 1.19

$$\begin{cases} \frac{acdx^3}{3}+\frac{adx^2}{2}+\frac{bcdx^3\,\mathrm{atanh}\,(cx)}{3}+\frac{bdx^2\,\mathrm{atanh}\,(cx)}{2}+\frac{bdx^2}{6}+\frac{bdx}{2c}+\frac{bd\log\left(x-\frac{1}{c}\right)}{3c^2}-\frac{bd\,\mathrm{atanh}\,(cx)}{6c^2} & \text{for } c\neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**3/3 + a*d*x**2/2 + b*c*d*x**3*atanh(c*x)/3 + b*d*x**2*a
tanh(c*x)/2 + b*d*x**2/6 + b*d*x/(2*c) + b*d*log(x - 1/c)/(3*c**2) - b*d*at
anh(c*x)/(6*c**2), Ne(c, 0)), (a*d*x**2/2, True))
```

3.4 $\int (d + cdx) \left(a + b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=44

$$\frac{d(cx+1)^2 \left(a + b \tanh^{-1}(cx)\right)}{2c} + \frac{bd \log(1 - cx)}{c} + \frac{bdx}{2}$$

```
[Out] 1/2*b*d*x+1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))/c+b*d*ln(-c*x+1)/c
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5926, 627, 43}

$$\frac{d(cx+1)^2 \left(a + b \tanh^{-1}(cx)\right)}{2c} + \frac{bd \log(1 - cx)}{c} + \frac{bdx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/2 + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*c) + (b*d*Log[1 - c*x])
/c
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int (d + cdx) \left(a + b \tanh^{-1}(cx)\right) dx &= \frac{d(1+cx)^2 \left(a + b \tanh^{-1}(cx)\right)}{2c} - \frac{b \int \frac{(d+cdx)^2}{1-c^2x^2} dx}{2d} \\
&= \frac{d(1+cx)^2 \left(a + b \tanh^{-1}(cx)\right)}{2c} - \frac{b \int \frac{d+cdx}{\frac{1}{d} - \frac{cx}{d}} dx}{2d} \\
&= \frac{d(1+cx)^2 \left(a + b \tanh^{-1}(cx)\right)}{2c} - \frac{b \int \left(-d^2 - \frac{2d^2}{-1+cx}\right) dx}{2d} \\
&= \frac{bdx}{2} + \frac{d(1+cx)^2 \left(a + b \tanh^{-1}(cx)\right)}{2c} + \frac{bd \log(1 - cx)}{c}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 95, normalized size = 2.16

$$\frac{1}{2}acdx^2+adx+\frac{bd\log\left(1-c^2x^2\right)}{2c}+\frac{1}{2}bcdx^2\tanh^{-1}(cx)+\frac{bd\log(1-cx)}{4c}-\frac{bd\log(cx+1)}{4c}+bdx\tanh^{-1}(cx)+\frac{bdx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] a*d*x + (b*d*x)/2 + (a*c*d*x^2)/2 + b*d*x*ArcTanh[c*x] + (b*c*d*x^2*ArcTanh
[c*x])/2 + (b*d*Log[1 - c*x])/(4*c) - (b*d*Log[1 + c*x])/(4*c) + (b*d*Log[1
 - c^2*x^2])/(2*c)
```

fricas [A] time = 0.59, size = 77, normalized size = 1.75

$$\frac{2\,ac^2dx^2+2\,(2\,a+b)cdx+bd\log\left(cx+1\right)+3\,bd\log\left(cx-1\right)+\left(bc^2dx^2+2\,bcdx\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*d*x^2 + 2*(2*a + b)*c*d*x + b*d*log(c*x + 1) + 3*b*d*log(c*x -
 1) + (b*c^2*d*x^2 + 2*b*c*d*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

giac [B] time = 0.18, size = 211, normalized size = 4.80

$$-c\left(\frac{bd\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{\left(\frac{2\,(cx+1)bd}{cx-1}-bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2c^2}{(cx-1)^2}-\frac{2\,(cx+1)c^2}{cx-1}+c^2}-\frac{bd\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{\frac{4\,(cx+1)ad}{cx-1}-2\,ad+\frac{(cx+1)bd}{cx-1}-bd}{\frac{(cx+1)^2c^2}{(cx-1)^2}-\frac{2\,(cx+1)c^2}{cx-1}+c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -c*(b*d*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - (2*(c*x + 1)*b*d/(c*x - 1) - b*
d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^2/(c*x - 1)^2 - 2*(c*x + 1)*c^2
/(c*x - 1) + c^2) - b*d*log(-(c*x + 1)/(c*x - 1))/c^2 - (4*(c*x + 1)*a*d/(c
*x - 1) - 2*a*d + (c*x + 1)*b*d/(c*x - 1) - b*d)/((c*x + 1)^2*c^2/(c*x - 1)
^2 - 2*(c*x + 1)*c^2/(c*x - 1) + c^2))
```

maple [A] time = 0.03, size = 65, normalized size = 1.48

$$\frac{cda\,x^2}{2}+adx+\frac{cdb\,\mathrm{arctanh}\left(cx\right)x^2}{2}+db\,\mathrm{arctanh}\left(cx\right)x+\frac{bdx}{2}+\frac{3db\ln\left(cx-1\right)}{4c}+\frac{db\ln\left(cx+1\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/2*c*d*a*x^2+a*d*x+1/2*c*d*b*arctanh(c*x)*x^2+d*b*arctanh(c*x)*x+1/2*b*d*x
+3/4/c*d*b*ln(c*x-1)+1/4/c*d*b*ln(c*x+1)
```

maxima [B] time = 0.31, size = 85, normalized size = 1.93

$$\frac{1}{2}\,acdx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\left(cx\right)+c\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+1\right)}{c^3}+\frac{\log\left(cx-1\right)}{c^3}\right)\right)bcd+adx+\frac{\left(2\,cx\,\mathrm{artanh}\left(cx\right)+\log\left(-c^2x^2+\right.\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*b*c*d + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 +
1))*b*d/c
```

mupad [B] time = 0.86, size = 65, normalized size = 1.48

$$\frac{d\,\left(2\,a\,x+b\,x+2\,b\,x\,\mathrm{atanh}\left(c\,x\right)\right)}{2}+\frac{c\,d\,\left(a\,x^2+b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{2}-\frac{d\,\left(b\,\mathrm{atanh}\left(c\,x\right)-b\,\ln\left(c^2\,x^2-1\right)\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] (d*(2*a*x + b*x + 2*b*x*atanh(c*x)))/2 + (c*d*(a*x^2 + b*x^2*atanh(c*x)))/2
 - (d*(b*atanh(c*x) - b*log(c^2*x^2 - 1)))/(2*c)
```

sympy [A] time = 0.58, size = 75, normalized size = 1.70

$$\begin{cases} \frac{acdx^2}{2} + adx + \frac{bcdx^2 \operatorname{atanh}(cx)}{2} + bdx \operatorname{atanh}(cx) + \frac{bdx}{2} + \frac{bd \log\left(x - \frac{1}{c}\right)}{c} + \frac{bd \operatorname{atanh}(cx)}{2c} & \text{for}\: c \neq 0 \\ adx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**2/2 + a*d*x + b*c*d*x**2*atanh(c*x)/2 + b*d*x*atanh(c*x
) + b*d*x/2 + b*d*log(x - 1/c)/c + b*d*atanh(c*x)/(2*c), Ne(c, 0)), (a*d*x,
 True))
```

3.5 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=60

$$acdx + ad\log(x) + \frac{1}{2}bd\log\left(1-c^2x^2\right) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd\text{Li}_2(cx) + bcdx\tanh^{-1}(cx)$$

```
[Out] a*c*d*x+b*c*d*x*arctanh(c*x)+a*d*ln(x)+1/2*b*d*ln(-c^2*x^2+1)-1/2*b*d*polyl
og(2,-c*x)+1/2*b*d*polylog(2,c*x)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5940, 5910, 260, 5912}

$$-\frac{1}{2}bd\text{PolyLog}(2,-cx)+\frac{1}{2}bd\text{PolyLog}(2,cx)+acdx+ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right)+bcdx\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] a*c*d*x + b*c*d*x*ArcTanh[c*x] + a*d*Log[x] + (b*d*Log[1 - c^2*x^2])/2 - (b
*d*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx &= \int \left(cd\left(a+b\tanh^{-1}(cx)\right)+\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}\right)dx\\ &= d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+(cd)\int \left(a+b\tanh^{-1}(cx)\right)dx\\ &= acdx+ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+(bcd)\int \tanh^{-1}(cx)\,dx\\ &= acdx+bcdx\tanh^{-1}(cx)+ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)-\left(bc^2\right.\\ &= acdx+bcdx\tanh^{-1}(cx)+ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right)-\frac{1}{2}bd\text{Li}_2(-cx)\end{aligned}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.90

$$\frac{1}{2}d\left(2acx+2a\log(x)+b\log\left(1-c^2x^2\right)-b\text{Li}_2(-cx)+b\text{Li}_2(cx)+2bcx\tanh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d*(2*a*c*x + 2*b*c*x*ArcTanh[c*x] + 2*a*Log[x] + b*Log[1 - c^2*x^2] - b*Po
lyLog[2, -(c*x)] + b*PolyLog[2, c*x]))/2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{acdx+ad+(bcdx+bd)\,\text{artanh}\,(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x, x)
```

maple [A] time = 0.05, size = 86, normalized size = 1.43

$$da\ln(cx)+acdx+db\,\text{arctanh}\,(cx)\ln(cx)+bcdx\,\text{arctanh}\,(cx)+\frac{db\ln(cx-1)}{2}+\frac{db\ln(cx+1)}{2}-\frac{db\,\text{dilog}\,(cx)}{2}-\frac{db}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x,x)
```

```
[Out] d*a*ln(c*x)+a*c*d*x+d*b*arctanh(c*x)*ln(c*x)+b*c*d*x*arctanh(c*x)+1/2*d*b*l
n(c*x-1)+1/2*d*b*ln(c*x+1)-1/2*d*b*dilog(c*x)-1/2*d*b*dilog(c*x+1)-1/2*d*b*
ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$acdx+\frac{1}{2}\left(2\,cx\,\operatorname{artanh}\left(cx\right)+\log\left(-c^{2}x^{2}+1\right)\right)bd+\frac{1}{2}\,bd\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{x}\,dx+ad\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] a*c*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d + 1/2*b*d*integr
ate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*d*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)\,\left(d+c\,d\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int ac\,dx+\int\frac{a}{x}\,dx+\int bc\,\mathrm{atanh}\left(cx\right)dx+\int\frac{b\,\mathrm{atanh}\left(cx\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x,x)
```

```
[Out] d*(Integral(a*c, x) + Integral(a/x, x) + Integral(b*c*atanh(c*x), x) + Inte
gral(b*atanh(c*x)/x, x))
```

3.6 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+bcd\log(x)$$

```
[Out] -d*(a+b*arctanh(c*x))/x+a*c*d*ln(x)+b*c*d*ln(x)-1/2*b*c*d*ln(-c^2*x^2+1)-1/
2*b*c*d*polylog(2,-c*x)+1/2*b*c*d*polylog(2,c*x)
```

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5940, 5916, 266, 36, 29, 31, 5912}

$$-\frac{1}{2}bcd\text{PolyLog}(2,-cx)+\frac{1}{2}bcd\text{PolyLog}(2,cx)-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)+bcd\log$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] -((d*(a + b*ArcTanh[c*x]))/x) + a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
- c^2*x^2])/2 - (b*c*d*PolyLog[2, -(c*x)])/2 + (b*c*d*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{x}\right)dx$$

$$= d\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+(cd)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\mathrm{Li}_2(-cx)+\frac{1}{2}bcd\mathrm{Li}_2(cx)+(bcd)$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\mathrm{Li}_2(-cx)+\frac{1}{2}bcd\mathrm{Li}_2(cx)+\frac{1}{2}(bcd$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\mathrm{Li}_2(-cx)+\frac{1}{2}bcd\mathrm{Li}_2(cx)+\frac{1}{2}(bcd$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)+bcd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)-\frac{1}{2}b$$

Mathematica [A] time = 0.07, size = 71, normalized size = 1.01

$$acd\log(x)-\frac{ad}{x}+bcd\left(-\frac{1}{2}\log\left(1-c^2x^2\right)+\log(cx)-\frac{\tanh^{-1}(cx)}{cx}\right)+\frac{1}{2}bcd(\mathrm{Li}_2(cx)-\mathrm{Li}_2(-cx))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*c*d*Log[x] + b*c*d*(-(ArcTanh[c*x]/(c*x)) + Log[c*x] - Log[1
 - c^2*x^2]/2) + (b*c*d*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{acdx+ad+(bcdx+bd)\,\mathrm{artanh}\,(cx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)(b\,\mathrm{artanh}\,(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x^2, x)
```

maple [A] time = 0.05, size = 105, normalized size = 1.50

$$cda\ln(cx)-\frac{ad}{x}+cdb\,\mathrm{arctanh}(cx)\ln(cx)-\frac{db\,\mathrm{arctanh}(cx)}{x}+cdb\ln(cx)-\frac{cdb\ln(cx-1)}{2}-\frac{cdb\ln(cx+1)}{2}-\frac{cdb\,\mathrm{di}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] c*d*a*ln(c*x)-a*d/x+c*d*b*arctanh(c*x)*ln(c*x)-d*b*arctanh(c*x)/x+c*d*b*ln(
c*x)-1/2*c*d*b*ln(c*x-1)-1/2*c*d*b*ln(c*x+1)-1/2*c*d*b*dilog(c*x)-1/2*c*d*b
*dilog(c*x+1)-1/2*c*d*b*ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,bcd\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx+acd\log(x)-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}(cx)}{x}\right)bd-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*d*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c*d*log(x) - 1
/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - a*d/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\mathrm{atanh}(c\,x))\,(d+c\,d\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a}{x^2}\,dx+\int\frac{ac}{x}\,dx+\int\frac{b\,\mathrm{atanh}(cx)}{x^2}\,dx+\int\frac{bc\,\mathrm{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d*(Integral(a/x**2, x) + Integral(a*c/x, x) + Integral(b*atanh(c*x)/x**2, x
) + Integral(b*c*atanh(c*x)/x, x))
```

3.7 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=56

$$-\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx)-\frac{bcd}{2x}$$

```
[Out] -1/2*b*c*d/x-1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))/x^2+b*c^2*d*ln(x)-b*c^2*d*l
n(-c*x+1)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = $\{37, 5936, 12, 77\}$

$$-\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx)-\frac{bcd}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d)/(2*x) - (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*x^2) + b*c^2*d*Log
[x] - b*c^2*d*Log[1 - c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - (bc)\int \frac{d(-1-cx)}{2x^2(1-cx)}\,dx$$
$$= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{2}(bcd)\int \frac{-1-cx}{x^2(1-cx)}\,dx$$
$$= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{2}(bcd)\int \left(-\frac{1}{x^2}-\frac{2c}{x}+\frac{2c^2}{-1+cx}\right)dx$$
$$= -\frac{bcd}{2x} - \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} + bc^2d\log(x) - bc^2d\log(1-cx)$$

Mathematica [A] time = 0.06, size = 76, normalized size = 1.36

$$-\frac{d\left(4acx+2a-4bc^2x^2\log(x)+3bc^2x^2\log(1-cx)+bc^2x^2\log(cx+1)+2bcx+2(2bcx+b)\tanh^{-1}(cx)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -1/4*(d*(2*a + 4*a*c*x + 2*b*c*x + 2*(b + 2*b*c*x)*ArcTanh[c*x] - 4*b*c^2*x
^2*Log[x] + 3*b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x]))/x^2
```

fricas [A] time = 0.48, size = 89, normalized size = 1.59

$$-\frac{bc^2dx^2\log(cx+1)+3\,bc^2dx^2\log(cx-1)-4\,bc^2dx^2\log(x)+2\,(2\,a+b)cdx+2\,ad+(2\,bcdx+bd)\log\left(-\frac{cx\cdot}{cx\cdot}\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c^2*d*x^2*log(c*x + 1) + 3*b*c^2*d*x^2*log(c*x - 1) - 4*b*c^2*d*x^2
*log(x) + 2*(2*a + b)*c*d*x + 2*a*d + (2*b*c*d*x + b*d)*log(-(c*x + 1)/(c*x
 - 1)))/x^2
```

giac [B] time = 0.20, size = 192, normalized size = 3.43

$$\left(bcd\log\left(-\frac{cx+1}{cx-1}-1\right)-bcd\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{2\,(cx+1)bcd}{cx-1}+bcd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1}+\frac{\frac{4\,(cx+1)acd}{cx-1}+2\,acd+\frac{(cx+1)bcd}{cx-1}+}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] (b*c*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c*d*log(-(c*x + 1)/(c*x - 1)) + (2
*(c*x + 1)*b*c*d/(c*x - 1) + b*c*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2/
(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1) + (4*(c*x + 1)*a*c*d/(c*x - 1) + 2
*a*c*d + (c*x + 1)*b*c*d/(c*x - 1) + b*c*d)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c
*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 84, normalized size = 1.50

$$-\frac{cda}{x}-\frac{da}{2x^2}-\frac{cdb\operatorname{arctanh}(cx)}{x}-\frac{db\operatorname{arctanh}(cx)}{2x^2}+c^2db\ln(cx)-\frac{bcd}{2x}-\frac{3c^2db\ln(cx-1)}{4}-\frac{c^2db\ln(cx+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] -c*d*a/x-1/2*d*a/x^2-c*d*b*arctanh(c*x)/x-1/2*d*b*arctanh(c*x)/x^2+c^2*d*b*
ln(c*x)-1/2*b*c*d/x-3/4*c^2*d*b*ln(c*x-1)-1/4*c^2*d*b*ln(c*x+1)
```

maxima [A] time = 0.32, size = 89, normalized size = 1.59

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\left(cx\right)}{x}\right)bcd+\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}\left(cx\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d + 1/4*((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d - a*c*d/x -
 1/2*a*d/x^2
```

mupad [B] time = 0.90, size = 75, normalized size = 1.34

$$\frac{d\,\left(b\,c^2\,\text{atanh}\left(c\,x\right)-b\,c^2\,\ln\left(c^2\,x^2-1\right)+2\,b\,c^2\,\ln(x)\right)}{2}-\frac{\frac{d\,(a+b\,\text{atanh}(c\,x))}{2}+\frac{d\,x\,(2\,a\,c+b\,c+2\,b\,c\,\text{atanh}(c\,x))}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^3,x)
```

```
[Out] (d*(b*c^2*atanh(c*x) - b*c^2*log(c^2*x^2 - 1) + 2*b*c^2*log(x)))/2 - ((d*(a
 + b*atanh(c*x)))/2 + (d*x*(2*a*c + b*c + 2*b*c*atanh(c*x)))/2)/x^2
```

sympy [A] time = 1.08, size = 95, normalized size = 1.70

$$\begin{cases} -\frac{acd}{x}-\frac{ad}{2x^2}+bc^2d\log\left(x\right)-bc^2d\log\left(x-\frac{1}{c}\right)-\frac{bc^2d\,\text{atanh}\left(cx\right)}{2}-\frac{bcd\,\text{atanh}\left(cx\right)}{x}-\frac{bcd}{2x}-\frac{bd\,\text{atanh}\left(cx\right)}{2x^2} & \text{for}\ c\neq 0\\ -\frac{ad}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] Piecewise((-a*c*d/x - a*d/(2*x**2) + b*c**2*d*log(x) - b*c**2*d*log(x - 1/c
) - b*c**2*d*atanh(c*x)/2 - b*c*d*atanh(c*x)/x - b*c*d/(2*x) - b*d*atanh(c*
x)/(2*x**2), Ne(c, 0)), (-a*d/(2*x**2), True))
```

3.8 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+\frac{1}{3}bc^3d\log(x)-\frac{5}{12}bc^3d\log(1-cx)+\frac{1}{12}bc^3d\log(cx+1)-\frac{bc^2d}{2x}-\frac{bcd}{6x^2}$$

```
[Out] -1/6*b*c*d/x^2-1/2*b*c^2*d/x-1/3*d*(a+b*arctanh(c*x))/x^3-1/2*c*d*(a+b*arct
anh(c*x))/x^2+1/3*b*c^3*d*ln(x)-5/12*b*c^3*d*ln(-c*x+1)+1/12*b*c^3*d*ln(c*x
+1)
```

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 801}

$$-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{bc^2d}{2x}+\frac{1}{3}bc^3d\log(x)-\frac{5}{12}bc^3d\log(1-cx)+\frac{1}{12}bc^3d\log(cx+1)-\frac{bcd}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d)/(6*x^2) - (b*c^2*d)/(2*x) - (d*(a + b*ArcTanh[c*x]))/(3*x^3) - (c*
d*(a + b*ArcTanh[c*x]))/(2*x^2) + (b*c^3*d*Log[x])/3 - (5*b*c^3*d*Log[1 - c
*x])/12 + (b*c^3*d*Log[1 + c*x])/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - (bc)\int \frac{d(-2-3cx)}{6x^3\left(1-c^2x^2\right)}\,dx$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{6}(bcd)\int \frac{-2-3cx}{x^3\left(1-c^2x^2\right)}\,dx$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{6}(bcd)\int \left(-\frac{2}{x^3} - \frac{3c}{x^2} - \frac{2c}{x}\right.$$
$$= -\frac{bcd}{6x^2} - \frac{bc^2d}{2x} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} + \frac{1}{3}bc^3d\log(x$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.88

$$-\frac{d\left(6acx+4a-4bc^3x^3\log(x)+5bc^3x^3\log(1-cx)-bc^3x^3\log(cx+1)+6bc^2x^2+2bcx+2b(3cx+2)\tanh^{-1}(cx\right.}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/12*(d*(4*a + 6*a*c*x + 2*b*c*x + 6*b*c^2*x^2 + 2*b*(2 + 3*c*x)*ArcTanh[c
*x] - 4*b*c^3*x^3*Log[x] + 5*b*c^3*x^3*Log[1 - c*x] - b*c^3*x^3*Log[1 + c*x
]))/x^3
```

fricas [A] time = 0.45, size = 101, normalized size = 1.03

$$\frac{bc^3dx^3\log(cx+1)-5\,bc^3dx^3\log(cx-1)+4\,bc^3dx^3\log(x)-6\,bc^2dx^2-2\,(3\,a+b)cdx-4\,ad-(3\,bcdx+2\,bd)\,\mathrm{l}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*d*x^3*log(c*x + 1) - 5*b*c^3*d*x^3*log(c*x - 1) + 4*b*c^3*d*x^3
*log(x) - 6*b*c^2*d*x^2 - 2*(3*a + b)*c*d*x - 4*a*d - (3*b*c*d*x + 2*b*d)*l
og(-(c*x + 1)/(c*x - 1)))/x^3
```

giac [B] time = 0.29, size = 306, normalized size = 3.12

$$\frac{1}{3}\left(bc^2d\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^2d\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{6\,(cx+1)^2bc^2d}{(cx-1)^2}+\frac{3\,(cx+1)bc^2d}{cx-1}+bc^2d\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}+\frac{\frac{12\,(cx+1)^2ac^2d}{(cx-1)^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(b*c^2*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^2*d*log(-(c*x + 1)/(c*x -
1)) + (6*(c*x + 1)^2*b*c^2*d/(c*x - 1)^2 + 3*(c*x + 1)*b*c^2*d/(c*x - 1) +
b*c^2*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2
/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^2*a*c^2*d/(c*x -
1)^2 + 6*(c*x + 1)*a*c^2*d/(c*x - 1) + 2*a*c^2*d + 5*(c*x + 1)^2*b*c^2*d/(c
*x - 1)^2 + 8*(c*x + 1)*b*c^2*d/(c*x - 1) + 3*b*c^2*d)/((c*x + 1)^3/(c*x -
1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 95, normalized size = 0.97

$$-\frac{da}{3x^3}-\frac{cda}{2x^2}-\frac{db\,\mathrm{arctanh}\,(cx)}{3x^3}-\frac{cdb\,\mathrm{arctanh}\,(cx)}{2x^2}-\frac{bcd}{6x^2}-\frac{b\,c^2d}{2x}+\frac{c^3db\ln(cx)}{3}-\frac{5c^3db\ln(cx-1)}{12}+\frac{b\,c^3d\ln(cx+1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] -1/3*d*a/x^3-1/2*c*d*a/x^2-1/3*d*b*arctanh(c*x)/x^3-1/2*c*d*b*arctanh(c*x)/
x^2-1/6*b*c*d/x^2-1/2*b*c^2*d/x+1/3*c^3*d*b*ln(c*x)-5/12*c^3*d*b*ln(c*x-1)+
1/12*b*c^3*d*ln(c*x+1)
```

maxima [A] time = 0.31, size = 99, normalized size = 1.01

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)bcd-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d
- 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*d - 1/2*a*c*d/x^2 - 1/3*a*d/x^3
```

mupad [B] time = 0.90, size = 110, normalized size = 1.12

$$\frac{b\,c^3\,d\,\ln(x)}{3}-\frac{a\,c\,d}{2\,x^2}-\frac{b\,c\,d}{6\,x^2}-\frac{b\,d\,\mathrm{atanh}\left(c\,x\right)}{3\,x^3}-\frac{b\,c^3\,d\,\ln\left(c^2\,x^2-1\right)}{6}-\frac{b\,c^2\,d}{2\,x}-\frac{a\,d}{3\,x^3}-\frac{b\,c^4\,d\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{2\,\sqrt{-c^2}}-\frac{b\,c\,d\,\mathrm{atanh}\left(c\right.}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^4,x)
```

```
[Out] (b*c^3*d*log(x))/3 - (a*c*d)/(2*x^2) - (b*c*d)/(6*x^2) - (b*d*atanh(c*x))/(
3*x^3) - (b*c^3*d*log(c^2*x^2 - 1))/6 - (b*c^2*d)/(2*x) - (a*d)/(3*x^3) - (
b*c^4*d*atan((c^2*x)/(-c^2)^(1/2)))/(2*(-c^2)^(1/2)) - (b*c*d*atanh(c*x))/(
2*x^2)
```

sympy [A] time = 1.42, size = 117, normalized size = 1.19

$$\begin{cases} -\frac{acd}{2x^2}-\frac{ad}{3x^3}+\frac{bc^3d\log\left(x\right)}{3}-\frac{bc^3d\log\left(x-\frac{1}{c}\right)}{3}+\frac{bc^3d\,\operatorname{atanh}\left(cx\right)}{6}-\frac{bc^2d}{2x}-\frac{bcd\,\operatorname{atanh}\left(cx\right)}{2x^2}-\frac{bcd}{6x^2}-\frac{bd\,\operatorname{atanh}\left(cx\right)}{3x^3} & \text{for}\ c\neq 0\\ -\frac{ad}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*c*d/(2*x**2) - a*d/(3*x**3) + b*c**3*d*log(x)/3 - b*c**3*d*lo
g(x - 1/c)/3 + b*c**3*d*atanh(c*x)/6 - b*c**2*d/(2*x) - b*c*d*atanh(c*x)/(2
*x**2) - b*c*d/(6*x**2) - b*d*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d/(3*x**3
), True))
```

3.9 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=110

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+\frac{1}{3}bc^4d\log(x)-\frac{7}{24}bc^4d\log(1-cx)-\frac{1}{24}bc^4d\log(cx+1)-\frac{bc^3d}{4x}-\frac{bc^2d}{6x^2}-$$

```
[Out] -1/12*b*c*d/x^3-1/6*b*c^2*d/x^2-1/4*b*c^3*d/x-1/4*d*(a+b*arctanh(c*x))/x^4-
1/3*c*d*(a+b*arctanh(c*x))/x^3+1/3*b*c^4*d*ln(x)-7/24*b*c^4*d*ln(-c*x+1)-1/
24*b*c^4*d*ln(c*x+1)
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 801}

$$-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d}{6x^2}-\frac{bc^3d}{4x}+\frac{1}{3}bc^4d\log(x)-\frac{7}{24}bc^4d\log(1-cx)-\frac{1}{24}bc^4d\log(cx+1)-$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d)/(12*x^3) - (b*c^2*d)/(6*x^2) - (b*c^3*d)/(4*x) - (d*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c*d*(a + b*ArcTanh[c*x]))/(3*x^3) + (b*c^4*d*Log[x])/3 -
 (7*b*c^4*d*Log[1 - c*x])/24 - (b*c^4*d*Log[1 + c*x])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx = -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - (bc)\int \frac{d(-3-4cx)}{12x^4\left(1-c^2x^2\right)}$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{12}(bcd)\int \frac{-3-4cx}{x^4\left(1-c^2x^2\right.}$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{12}(bcd)\int \left(-\frac{3}{x^4} - \frac{4c}{x^3}\right.$$
$$= -\frac{bcd}{12x^3} - \frac{bc^2d}{6x^2} - \frac{bc^3d}{4x} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} +$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.85

$$-\frac{d\left(8acx+6a-8bc^4x^4\log(x)+7bc^4x^4\log(1-cx)+bc^4x^4\log(cx+1)+6bc^3x^3+4bc^2x^2+2bcx+2b(4cx+\right.}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/24*(d*(6*a + 8*a*c*x + 2*b*c*x + 4*b*c^2*x^2 + 6*b*c^3*x^3 + 2*b*(3 + 4*
c*x)*ArcTanh[c*x] - 8*b*c^4*x^4*Log[x] + 7*b*c^4*x^4*Log[1 - c*x] + b*c^4*x
^4*Log[1 + c*x]))/x^4
```

fricas [A] time = 0.46, size = 110, normalized size = 1.00

$$-\frac{bc^4dx^4\log(cx+1)+7\,bc^4dx^4\log(cx-1)-8\,bc^4dx^4\log(x)+6\,bc^3dx^3+4\,bc^2dx^2+2\,(4\,a+b)cdx+6\,ad+}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(b*c^4*d*x^4*log(c*x + 1) + 7*b*c^4*d*x^4*log(c*x - 1) - 8*b*c^4*d*x^
4*log(x) + 6*b*c^3*d*x^3 + 4*b*c^2*d*x^2 + 2*(4*a + b)*c*d*x + 6*a*d + (4*b
*c*d*x + 3*b*d)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

giac [B] time = 0.17, size = 401, normalized size = 3.65

$$\frac{1}{3}\left(bc^3d\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^3d\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{6\,(cx+1)^3bc^3d}{(cx-1)^3}+\frac{3\,(cx+1)^2bc^3d}{(cx-1)^2}+\frac{4\,(cx+1)bc^3d}{cx-1}+bc^3d\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(b*c^3*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^3*d*log(-(c*x + 1)/(c*x -
1)) + (6*(c*x + 1)^3*b*c^3*d/(c*x - 1)^3 + 3*(c*x + 1)^2*b*c^3*d/(c*x - 1)^
2 + 4*(c*x + 1)*b*c^3*d/(c*x - 1) + b*c^3*d)*log(-(c*x + 1)/(c*x - 1))/((c*
x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^
2 + 4*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^3*a*c^3*d/(c*x - 1)^3 + 6*(c
*x + 1)^2*a*c^3*d/(c*x - 1)^2 + 8*(c*x + 1)*a*c^3*d/(c*x - 1) + 2*a*c^3*d +
 5*(c*x + 1)^3*b*c^3*d/(c*x - 1)^3 + 10*(c*x + 1)^2*b*c^3*d/(c*x - 1)^2 + 7
*(c*x + 1)*b*c^3*d/(c*x - 1) + 2*b*c^3*d)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x
 + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1
))*c
```

maple [A] time = 0.04, size = 105, normalized size = 0.95

$$-\frac{cda}{3x^{3}}-\frac{da}{4x^{4}}-\frac{cdb\,\mathrm{arctanh}\,(cx)}{3x^{3}}-\frac{db\,\mathrm{arctanh}\,(cx)}{4x^{4}}-\frac{bcd}{12x^{3}}-\frac{b\,c^{2}d}{6x^{2}}-\frac{b\,c^{3}d}{4x}+\frac{c^{4}db\ln(cx)}{3}-\frac{7c^{4}db\ln(cx-1)}{24}-\frac{b\,c^{4}d\ln(cx+1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -1/3*c*d*a/x^3-1/4*d*a/x^4-1/3*c*d*b*arctanh(c*x)/x^3-1/4*d*b*arctanh(c*x)/
x^4-1/12*b*c*d/x^3-1/6*b*c^2*d/x^2-1/4*b*c^3*d/x+1/3*c^4*d*b*ln(c*x)-7/24*c
^4*d*b*ln(c*x-1)-1/24*b*c^4*d*ln(c*x+1)
```

maxima [A] time = 0.31, size = 114, normalized size = 1.04

$$-\frac{1}{6}\left(\left(c^{2}\log\left(c^{2}x^{2}-1\right)-c^{2}\log\left(x^{2}\right)+\frac{1}{x^{2}}\right)c+\frac{2\,\mathrm{artanh}\,(cx)}{x^{3}}\right)bcd+\frac{1}{24}\left(\left(3\,c^{3}\log\left(cx+1\right)-3\,c^{3}\log\left(cx-1\right)-\frac{2\left(3\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c*d + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)
/x^3)*c - 6*arctanh(c*x)/x^4)*b*d - 1/3*a*c*d/x^3 - 1/4*a*d/x^4
```

mupad [B] time = 0.85, size = 120, normalized size = 1.09

$$\frac{b\,c^{4}\,d\,\ln(x)}{3}-\frac{a\,c\,d}{3\,x^{3}}-\frac{b\,c\,d}{12\,x^{3}}-\frac{b\,d\,\mathrm{atanh}\,(c\,x)}{4\,x^{4}}-\frac{b\,c^{4}\,d\,\ln\left(c^{2}\,x^{2}-1\right)}{6}-\frac{b\,c^{2}\,d}{6\,x^{2}}-\frac{b\,c^{3}\,d}{4\,x}-\frac{a\,d}{4\,x^{4}}-\frac{b\,c^{5}\,d\,\mathrm{atan}\left(\frac{c^{2}\,x}{\sqrt{-c^{2}}}\right)}{4\,\sqrt{-c^{2}}}-\frac{b\,c\,d\,\mathrm{atanh}\,(c\,x)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^5,x)
```

```
[Out] (b*c^4*d*log(x))/3 - (a*c*d)/(3*x^3) - (b*c*d)/(12*x^3) - (b*d*atanh(c*x))/
(4*x^4) - (b*c^4*d*log(c^2*x^2 - 1))/6 - (b*c^2*d)/(6*x^2) - (b*c^3*d)/(4*x
) - (a*d)/(4*x^4) - (b*c^5*d*atan((c^2*x)/(-c^2)^(1/2)))/(4*(-c^2)^(1/2)) -
 (b*c*d*atanh(c*x))/(3*x^3)
```

sympy [A] time = 1.88, size = 129, normalized size = 1.17

$$\begin{cases} -\frac{acd}{3x^{3}}-\frac{ad}{4x^{4}}+\frac{bc^{4}d\log(x)}{3}-\frac{bc^{4}d\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^{4}d\,\mathrm{atanh}\,(cx)}{12}-\frac{bc^{3}d}{4x}-\frac{bc^{2}d}{6x^{2}}-\frac{bcd\,\mathrm{atanh}\,(cx)}{3x^{3}}-\frac{bcd}{12x^{3}}-\frac{bd\,\mathrm{atanh}\,(cx)}{4x^{4}} & \text{for } c\neq 0 \\ -\frac{ad}{4x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c*d/(3*x**3) - a*d/(4*x**4) + b*c**4*d*log(x)/3 - b*c**4*d*lo
g(x - 1/c)/3 - b*c**4*d*atanh(c*x)/12 - b*c**3*d/(4*x) - b*c**2*d/(6*x**2)
- b*c*d*atanh(c*x)/(3*x**3) - b*c*d/(12*x**3) - b*d*atanh(c*x)/(4*x**4), Ne
(c, 0)), (-a*d/(4*x**4), True))
```

3.10 $\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=157

$$\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{49bd^2\log(1-cx)}{120c^4}-\frac{bd^2\log}{12}$$

```
[Out] 5/12*b*d^2*x/c^3+1/5*b*d^2*x^2/c^2+5/36*b*d^2*x^3/c+1/10*b*d^2*x^4+1/30*b*c
*d^2*x^5+1/4*d^2*x^4*(a+b*arctanh(c*x))+2/5*c*d^2*x^5*(a+b*arctanh(c*x))+1/
6*c^2*d^2*x^6*(a+b*arctanh(c*x))+49/120*b*d^2*ln(-c*x+1)/c^4-1/120*b*d^2*ln
(c*x+1)/c^4
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x}{12c^3}+\frac{49bd^2\log}{120c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (5*b*d^2*x)/(12*c^3) + (b*d^2*x^2)/(5*c^2) + (5*b*d^2*x^3)/(36*c) + (b*d^2*
x^4)/10 + (b*c*d^2*x^5)/30 + (d^2*x^4*(a + b*ArcTanh[c*x]))/4 + (2*c*d^2*x^
5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x]))/6 + (49*b*d^
2*Log[1 - c*x])/(120*c^4) - (b*d^2*Log[1 + c*x])/(120*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6\left(a+b\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6\left(a+b\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6\left(a+b\right.$$

$$=\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}\right.$$

$$=\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}\right.$$

$$=\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}\right.$$

Mathematica [A] time = 0.11, size = 125, normalized size = 0.80

$$\frac{d^2\left(60ac^6x^6+144ac^5x^5+90ac^4x^4+12bc^5x^5+36bc^4x^4+50bc^3x^3+72bc^2x^2+6bc^4x^4\left(10c^2x^2+24cx+15\right)\tanh\right.}{360c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(150*b*c*x + 72*b*c^2*x^2 + 50*b*c^3*x^3 + 90*a*c^4*x^4 + 36*b*c^4*x^4
 + 144*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c^4*x^4*(15 + 24*c*x +
 10*c^2*x^2)*ArcTanh[c*x] + 147*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(360*c^
4)
```

fricas [A] time = 0.45, size = 162, normalized size = 1.03

$$\frac{60\,ac^6d^2x^6+12\,(12\,a+b)c^5d^2x^5+18\,(5\,a+2\,b)c^4d^2x^4+50\,bc^3d^2x^3+72\,bc^2d^2x^2+150\,bcd^2x-3\,bd^2\log\left(cx+1\right.}{360\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(60*a*c^6*d^2*x^6 + 12*(12*a + b)*c^5*d^2*x^5 + 18*(5*a + 2*b)*c^4*d^
2*x^4 + 50*b*c^3*d^2*x^3 + 72*b*c^2*d^2*x^2 + 150*b*c*d^2*x - 3*b*d^2*log(c
*x + 1) + 147*b*d^2*log(c*x - 1) + 3*(10*b*c^6*d^2*x^6 + 24*b*c^5*d^2*x^5 +
 15*b*c^4*d^2*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

giac [B] time = 0.28, size = 620, normalized size = 3.95

$$\frac{1}{45}c\left(\frac{6\left(\frac{30\,(cx+1)^5bd^2}{(cx-1)^5}-\frac{30\,(cx+1)^4bd^2}{(cx-1)^4}+\frac{70\,(cx+1)^3bd^2}{(cx-1)^3}-\frac{45\,(cx+1)^2bd^2}{(cx-1)^2}+\frac{18\,(cx+1)bd^2}{cx-1}-3\,bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^5}{(cx-1)^6}-\frac{6\,(cx+1)^5c^5}{(cx-1)^5}+\frac{15\,(cx+1)^4c^5}{(cx-1)^4}-\frac{20\,(cx+1)^3c^5}{(cx-1)^3}+\frac{15\,(cx+1)^2c^5}{(cx-1)^2}-\frac{6\,(cx+1)c^5}{cx-1}+c^5}+\frac{360\,(cx+1)^5ad^2}{(cx-1)^5}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/45*c*(6*(30*(c*x + 1)^5*b*d^2/(c*x - 1)^5 - 30*(c*x + 1)^4*b*d^2/(c*x - 1
)^4 + 70*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 45*(c*x + 1)^2*b*d^2/(c*x - 1)^2 +
 18*(c*x + 1)*b*d^2/(c*x - 1) - 3*b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x +
1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(
c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^
2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) + (360*(c*x + 1)^5*a*d^2/(c*x - 1)^5 -
 360*(c*x + 1)^4*a*d^2/(c*x - 1)^4 + 840*(c*x + 1)^3*a*d^2/(c*x - 1)^3 - 54
0*(c*x + 1)^2*a*d^2/(c*x - 1)^2 + 216*(c*x + 1)*a*d^2/(c*x - 1) - 36*a*d^2
+ 162*(c*x + 1)^5*b*d^2/(c*x - 1)^5 - 531*(c*x + 1)^4*b*d^2/(c*x - 1)^4 + 8
18*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 696*(c*x + 1)^2*b*d^2/(c*x - 1)^2 + 300*
(c*x + 1)*b*d^2/(c*x - 1) - 53*b*d^2)/((c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x
 + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c
^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1)
 + c^5) - 18*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 18*b*d^2*log(-(c*x +
 1)/(c*x - 1))/c^5)
```

maple [A] time = 0.03, size = 159, normalized size = 1.01

$$\frac{c^2d^2a\,x^6}{6}+\frac{2c\,d^2a\,x^5}{5}+\frac{a\,x^4d^2}{4}+\frac{c^2d^2b\operatorname{arctanh}(cx)\,x^6}{6}+\frac{2c\,d^2b\operatorname{arctanh}(cx)\,x^5}{5}+\frac{d^2b\operatorname{arctanh}(cx)\,x^4}{4}+\frac{bc\,d^2x^5}{30}+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*c^2*d^2*a*x^6+2/5*c*d^2*a*x^5+1/4*a*x^4*d^2+1/6*c^2*d^2*b*arctanh(c*x)*
x^6+2/5*c*d^2*b*arctanh(c*x)*x^5+1/4*d^2*b*arctanh(c*x)*x^4+1/30*b*c*d^2*x^
5+1/10*b*d^2*x^4+5/36*b*d^2*x^3/c+1/5*b*d^2*x^2/c^2+5/12*b*d^2*x/c^3+49/120
/c^4*d^2*b*ln(c*x-1)-1/120*b*d^2*ln(c*x+1)/c^4
```

maxima [A] time = 0.33, size = 210, normalized size = 1.34

$$\frac{1}{6}\,ac^2d^2x^6+\frac{2}{5}\,acd^2x^5+\frac{1}{4}\,ad^2x^4+\frac{1}{180}\left(30\,x^6\operatorname{artanh}(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log(cx+1)}{c^7}+\frac{15\,l}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^2*d^2*x^6 + 2/5*a*c*d^2*x^5 + 1/4*a*d^2*x^4 + 1/180*(30*x^6*arctanh
(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*
log(c*x - 1)/c^7))*b*c^2*d^2 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x
^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d^2 + 1/24*(6*x^4*arctanh(c*x) + c*(
2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^2
```

mupad [B] time = 1.04, size = 146, normalized size = 0.93

$$\frac{\frac{b\,c^2\,d^2\,x^2}{5}-\frac{d^2\left(75\,b\,\mathrm{atanh}(c\,x)-36\,b\,\ln\left(c^2\,x^2-1\right)\right)}{180}+\frac{5\,b\,c^3\,d^2\,x^3}{36}+\frac{5\,b\,c\,d^2\,x}{12}}{c^4}+\frac{d^2\,\left(45\,a\,x^4+18\,b\,x^4+45\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{180}+\frac{c^2\,d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] ((b*c^2*d^2*x^2)/5 - (d^2*(75*b*atanh(c*x) - 36*b*log(c^2*x^2 - 1)))/180 +
(5*b*c^3*d^2*x^3)/36 + (5*b*c*d^2*x)/12)/c^4 + (d^2*(45*a*x^4 + 18*b*x^4 +
45*b*x^4*atanh(c*x)))/180 + (c^2*d^2*(30*a*x^6 + 30*b*x^6*atanh(c*x)))/180
+ (c*d^2*(72*a*x^5 + 6*b*x^5 + 72*b*x^5*atanh(c*x)))/180
```

sympy [A] time = 2.16, size = 196, normalized size = 1.25

$$\begin{cases} \frac{ac^2d^2x^6}{6} + \frac{2acd^2x^5}{5} + \frac{ad^2x^4}{4} + \frac{bc^2d^2x^6 \operatorname{atanh}(cx)}{6} + \frac{2bcd^2x^5 \operatorname{atanh}(cx)}{5} + \frac{bcd^2x^5}{30} + \frac{bd^2x^4 \operatorname{atanh}(cx)}{4} + \frac{bd^2x^4}{10} + \frac{5bd^2x^3}{36c} + \frac{bd^2x^2}{5c^2} + \frac{5bd^2}{12c} \\ \frac{ad^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**6/6 + 2*a*c*d**2*x**5/5 + a*d**2*x**4/4 + b*c**2*
d**2*x**6*atanh(c*x)/6 + 2*b*c*d**2*x**5*atanh(c*x)/5 + b*c*d**2*x**5/30 +
b*d**2*x**4*atanh(c*x)/4 + b*d**2*x**4/10 + 5*b*d**2*x**3/(36*c) + b*d**2*x
**2/(5*c**2) + 5*b*d**2*x/(12*c**3) + 2*b*d**2*log(x - 1/c)/(5*c**4) - b*d*
*2*atanh(c*x)/(60*c**4), Ne(c, 0)), (a*d**2*x**4/4, True))
```

3.11 $\int x^2(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=143

$$\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{31bd^2\log(1-cx)}{60c^3}+\frac{bd^2\log}{6}$$

```
[Out] 1/2*b*d^2*x/c^2+4/15*b*d^2*x^2/c+1/6*b*d^2*x^3+1/20*b*c*d^2*x^4+1/3*d^2*x^3
*(a+b*arctanh(c*x))+1/2*c*d^2*x^4*(a+b*arctanh(c*x))+1/5*c^2*d^2*x^5*(a+b*a
rctanh(c*x))+31/60*b*d^2*ln(-c*x+1)/c^3+1/60*b*d^2*ln(c*x+1)/c^3
```

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{bd^2x}{2c^2}+\frac{31bd^2\log(1-cx)}{60c^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d^2*x)/(2*c^2) + (4*b*d^2*x^2)/(15*c) + (b*d^2*x^3)/6 + (b*c*d^2*x^4)/20
 + (d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c*d^2*x^4*(a + b*ArcTanh[c*x]))/2 +
(c^2*d^2*x^5*(a + b*ArcTanh[c*x]))/5 + (31*b*d^2*Log[1 - c*x])/(60*c^3) + (
b*d^2*Log[1 + c*x])/(60*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5\left(a+b\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5\left(a+b\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5\left(a+b\right.$$

$$=\frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c$$

$$=\frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c$$

$$=\frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c$$

Mathematica [A] time = 0.09, size = 115, normalized size = 0.80

$$\frac{d^2\left(12ac^5x^5+30ac^4x^4+20ac^3x^3+3bc^4x^4+10bc^3x^3+16bc^2x^2+2bc^3x^3\left(6c^2x^2+15cx+10\right)\tanh^{-1}(cx)+30bc\right.}{60c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(30*b*c*x + 16*b*c^2*x^2 + 20*a*c^3*x^3 + 10*b*c^3*x^3 + 30*a*c^4*x^4
+ 3*b*c^4*x^4 + 12*a*c^5*x^5 + 2*b*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2)*ArcTan
h[c*x] + 31*b*Log[1 - c*x] + b*Log[1 + c*x]))/(60*c^3)
```

fricas [A] time = 0.50, size = 146, normalized size = 1.02

$$\frac{12\,ac^5d^2x^5+3\,(10\,a+b)c^4d^2x^4+10\,(2\,a+b)c^3d^2x^3+16\,bc^2d^2x^2+30\,bcd^2x+bd^2\log\left(cx+1\right)+31\,bd^2\log\left(cx-\right.}{60\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^2*x^5 + 3*(10*a + b)*c^4*d^2*x^4 + 10*(2*a + b)*c^3*d^2*x^
3 + 16*b*c^2*d^2*x^2 + 30*b*c*d^2*x + b*d^2*log(c*x + 1) + 31*b*d^2*log(c*x
 - 1) + (6*b*c^5*d^2*x^5 + 15*b*c^4*d^2*x^4 + 10*b*c^3*d^2*x^3)*log(-(c*x +
 1)/(c*x - 1)))/c^3
```

giac [B] time = 0.23, size = 525, normalized size = 3.67

$$\frac{4}{15}c\left(\frac{\left(\frac{15(cx+1)^4bd^2}{(cx-1)^4}-\frac{15(cx+1)^3bd^2}{(cx-1)^3}+\frac{20(cx+1)^2bd^2}{(cx-1)^2}-\frac{10(cx+1)bd^2}{cx-1}+2\,bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^4}{(cx-1)^5}-\frac{5(cx+1)^4c^4}{(cx-1)^4}+\frac{10(cx+1)^3c^4}{(cx-1)^3}-\frac{10(cx+1)^2c^4}{(cx-1)^2}+\frac{5(cx+1)c^4}{cx-1}-c^4}+\frac{\frac{30(cx+1)^4ad^2}{(cx-1)^4}-\frac{30(cx+1)^3ad^2}{(cx-1)^3}+40\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 4/15*c*((15*(c*x + 1)^4*b*d^2/(c*x - 1)^4 - 15*(c*x + 1)^3*b*d^2/(c*x - 1)^
3 + 20*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 10*(c*x + 1)*b*d^2/(c*x - 1) + 2*b*d
^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^4/(c*x - 1)^5 - 5*(c*x + 1)^4*
c^4/(c*x - 1)^4 + 10*(c*x + 1)^3*c^4/(c*x - 1)^3 - 10*(c*x + 1)^2*c^4/(c*x
- 1)^2 + 5*(c*x + 1)*c^4/(c*x - 1) - c^4) + (30*(c*x + 1)^4*a*d^2/(c*x - 1)
^4 - 30*(c*x + 1)^3*a*d^2/(c*x - 1)^3 + 40*(c*x + 1)^2*a*d^2/(c*x - 1)^2 -
20*(c*x + 1)*a*d^2/(c*x - 1) + 4*a*d^2 + 13*(c*x + 1)^4*b*d^2/(c*x - 1)^4 -
 36*(c*x + 1)^3*b*d^2/(c*x - 1)^3 + 41*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 23*(
c*x + 1)*b*d^2/(c*x - 1) + 5*b*d^2)/((c*x + 1)^5*c^4/(c*x - 1)^5 - 5*(c*x +
 1)^4*c^4/(c*x - 1)^4 + 10*(c*x + 1)^3*c^4/(c*x - 1)^3 - 10*(c*x + 1)^2*c^4
/(c*x - 1)^2 + 5*(c*x + 1)*c^4/(c*x - 1) - c^4) - 2*b*d^2*log(-(c*x + 1)/(c
*x - 1) + 1)/c^4 + 2*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^4)
```

maple [A] time = 0.03, size = 147, normalized size = 1.03

$$\frac{c^2 d^2 a\,x^5}{5}+\frac{c\,d^2 a\,x^4}{2}+\frac{d^2 a\,x^3}{3}+\frac{c^2 d^2 b\operatorname{arctanh}(cx)\,x^5}{5}+\frac{c\,d^2 b\operatorname{arctanh}(cx)\,x^4}{2}+\frac{d^2 b\operatorname{arctanh}(cx)\,x^3}{3}+\frac{bc\,d^2 x^4}{20}+\frac{b\,d^2 x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c^2*d^2*a*x^5+1/2*c*d^2*a*x^4+1/3*d^2*a*x^3+1/5*c^2*d^2*b*arctanh(c*x)*
x^5+1/2*c*d^2*b*arctanh(c*x)*x^4+1/3*d^2*b*arctanh(c*x)*x^3+1/20*b*c*d^2*x^
4+1/6*b*d^2*x^3+4/15*b*d^2*x^2/c+1/2*b*d^2*x/c^2+31/60/c^3*d^2*b*ln(c*x-1)+
1/60*b*d^2*ln(c*x+1)/c^3
```

maxima [A] time = 0.31, size = 184, normalized size = 1.29

$$\frac{1}{5}\,ac^2d^2x^5+\frac{1}{2}\,acd^2x^4+\frac{1}{20}\left(4\,x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bc^2d^2+\frac{1}{3}\,ad^2x^3+\frac{1}{12}\left(6\,x^4\operatorname{arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^2*d^2*x^5 + 1/2*a*c*d^2*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^
4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/12*
(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log
(c*x - 1)/c^5))*b*c*d^2 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^
2 - 1)/c^4))*b*d^2
```

mupad [B] time = 1.03, size = 134, normalized size = 0.94

$$\frac{\frac{4\,b\,c^2\,d^2\,x^2}{15}-\frac{d^2\,\left(30\,b\,\mathrm{atanh}(c\,x)-16\,b\,\ln\left(c^2\,x^2-1\right)\right)}{60}+\frac{b\,c\,d^2\,x}{2}}{c^3}+\frac{d^2\,\left(20\,a\,x^3+10\,b\,x^3+20\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{60}+\frac{c^2\,d^2\,\left(12\,a\,x^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] ((4*b*c^2*d^2*x^2)/15 - (d^2*(30*b*atanh(c*x) - 16*b*log(c^2*x^2 - 1)))/60
+ (b*c*d^2*x)/2)/c^3 + (d^2*(20*a*x^3 + 10*b*x^3 + 20*b*x^3*atanh(c*x)))/60
 + (c^2*d^2*(12*a*x^5 + 12*b*x^5*atanh(c*x)))/60 + (c*d^2*(30*a*x^4 + 3*b*x
^4 + 30*b*x^4*atanh(c*x)))/60
```

sympy [A] time = 1.71, size = 177, normalized size = 1.24

$$\begin{cases} \frac{ac^2d^2x^5}{5}+\frac{acd^2x^4}{2}+\frac{ad^2x^3}{3}+\frac{bc^2d^2x^5\operatorname{atanh}(cx)}{5}+\frac{bcd^2x^4\operatorname{atanh}(cx)}{2}+\frac{bcd^2x^4}{20}+\frac{bd^2x^3\operatorname{atanh}(cx)}{3}+\frac{bd^2x^3}{6}+\frac{4bd^2x^2}{15c}+\frac{bd^2x}{2c^2}+\frac{8bd}{} \\ \frac{ad^2x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**5/5 + a*c*d**2*x**4/2 + a*d**2*x**3/3 + b*c**2*d*
*2*x**5*atanh(c*x)/5 + b*c*d**2*x**4*atanh(c*x)/2 + b*c*d**2*x**4/20 + b*d*
*2*x**3*atanh(c*x)/3 + b*d**2*x**3/6 + 4*b*d**2*x**2/(15*c) + b*d**2*x/(2*c
**2) + 8*b*d**2*log(x - 1/c)/(15*c**3) + b*d**2*atanh(c*x)/(30*c**3), Ne(c,
 0)), (a*d**2*x**3/3, True))
```

3.12 $\int x(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=129

$$\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{17bd^2\log(1-cx)}{24c^2}-\frac{bd^2\log}{2}$$

```
[Out] 3/4*b*d^2*x/c+1/3*b*d^2*x^2+1/12*b*c*d^2*x^3+1/2*d^2*x^2*(a+b*arctanh(c*x))
+2/3*c*d^2*x^3*(a+b*arctanh(c*x))+1/4*c^2*d^2*x^4*(a+b*arctanh(c*x))+17/24*
b*d^2*ln(-c*x+1)/c^2-1/24*b*d^2*ln(c*x+1)/c^2
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{43, 5936, 12, 1802, 633, 31\}$

$$\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{17bd^2\log(1-cx)}{24c^2}-\frac{bd^2\log}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^2*x)/(4*c) + (b*d^2*x^2)/3 + (b*c*d^2*x^3)/12 + (d^2*x^2*(a + b*ArcT
anh[c*x]))/2 + (2*c*d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c^2*d^2*x^4*(a + b*A
rcTanh[c*x]))/4 + (17*b*d^2*Log[1 - c*x])/(24*c^2) - (b*d^2*Log[1 + c*x])/(
24*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4\left(a+b\,\mathrm{t}\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4\left(a+b\,\mathrm{t}\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4\left(a+b\,\mathrm{t}\right.$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\right.$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\right.$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\right.$$

Mathematica [A] time = 0.09, size = 107, normalized size = 0.83

$$\frac{d^2\left(6ac^4x^4+16ac^3x^3+12ac^2x^2+2bc^3x^3+8bc^2x^2+2bc^2x^2\left(3c^2x^2+8cx+6\right)\tanh^{-1}(cx)+18bcx+17b\log(1-\right.}{24c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(18*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 16*a*c^3*x^3 + 2*b*c^3*x^3 +
6*a*c^4*x^4 + 2*b*c^2*x^2*(6 + 8*c*x + 3*c^2*x^2)*ArcTanh[c*x] + 17*b*Log[1
 - c*x] - b*Log[1 + c*x]))/(24*c^2)
```

fricas [A] time = 0.69, size = 137, normalized size = 1.06

$$\frac{6\,ac^4d^2x^4+2\,(8\,a+b)c^3d^2x^3+4\,(3\,a+2\,b)c^2d^2x^2+18\,bcd^2x-bd^2\log\left(cx+1\right)+17\,bd^2\log\left(cx-1\right)+\left(3\,bc^4d^2x\right.}{24\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d^2*x^4 + 2*(8*a + b)*c^3*d^2*x^3 + 4*(3*a + 2*b)*c^2*d^2*x^2
 + 18*b*c*d^2*x - b*d^2*log(c*x + 1) + 17*b*d^2*log(c*x - 1) + (3*b*c^4*d^2
*x^4 + 8*b*c^3*d^2*x^3 + 6*b*c^2*d^2*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

giac [B] time = 0.41, size = 425, normalized size = 3.29

$$-\frac{1}{3}c\left(\frac{2\,bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3}-\frac{2\left(\frac{6\,(cx+1)^3bd^2}{(cx-1)^3}-\frac{6\,(cx+1)^2bd^2}{(cx-1)^2}+\frac{4\,(cx+1)bd^2}{cx-1}-bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^3}{(cx-1)^4}-\frac{4\,(cx+1)^3c^3}{(cx-1)^3}+\frac{6\,(cx+1)^2c^3}{(cx-1)^2}-\frac{4\,(cx+1)c^3}{cx-1}+c^3}-\frac{2\,bd^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^3}-\frac{24}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/3*c*(2*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 2*(6*(c*x + 1)^3*b*d^2/
(c*x - 1)^3 - 6*(c*x + 1)^2*b*d^2/(c*x - 1)^2 + 4*(c*x + 1)*b*d^2/(c*x - 1)
 - b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^3/(c*x - 1)^4 - 4*(c*x +
 1)^3*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*c^3/(c*x - 1)^2 - 4*(c*x + 1)*c^3/(c*
x - 1) + c^3) - 2*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^3 - (24*(c*x + 1)^3*a*d
^2/(c*x - 1)^3 - 24*(c*x + 1)^2*a*d^2/(c*x - 1)^2 + 16*(c*x + 1)*a*d^2/(c*x
 - 1) - 4*a*d^2 + 10*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 23*(c*x + 1)^2*b*d^2/(
c*x - 1)^2 + 18*(c*x + 1)*b*d^2/(c*x - 1) - 5*b*d^2)/((c*x + 1)^4*c^3/(c*x
- 1)^4 - 4*(c*x + 1)^3*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*c^3/(c*x - 1)^2 - 4*
(c*x + 1)*c^3/(c*x - 1) + c^3))
```

maple [A] time = 0.03, size = 135, normalized size = 1.05

$$\frac{c^2d^2a\,x^4}{4}+\frac{2c\,d^2a\,x^3}{3}+\frac{d^2a\,x^2}{2}+\frac{c^2d^2b\,\mathrm{arctanh}\,(cx)\,x^4}{4}+\frac{2c\,d^2b\,\mathrm{arctanh}\,(cx)\,x^3}{3}+\frac{d^2b\,\mathrm{arctanh}\,(cx)\,x^2}{2}+\frac{bc\,d^2x^3}{12}+\frac{b\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*c^2*d^2*a*x^4+2/3*c*d^2*a*x^3+1/2*d^2*a*x^2+1/4*c^2*d^2*b*arctanh(c*x)*
x^4+2/3*c*d^2*b*arctanh(c*x)*x^3+1/2*d^2*b*arctanh(c*x)*x^2+1/12*b*c*d^2*x^
3+1/3*b*d^2*x^2+3/4*b*d^2*x/c+17/24/c^2*d^2*b*ln(c*x-1)-1/24*b*d^2*ln(c*x+1
)/c^2
```

maxima [A] time = 0.31, size = 179, normalized size = 1.39

$$\frac{1}{4}\,ac^2d^2x^4+\frac{2}{3}\,acd^2x^3+\frac{1}{24}\left(6\,x^4\,\mathrm{artanh}\,(cx)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log\,(cx+1)}{c^5}+\frac{3\,\log\,(cx-1)}{c^5}\right)\right)bc^2d^2+\frac{1}{3}\left(2\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^2*d^2*x^4 + 2/3*a*c*d^2*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*
x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^2*d^2 + 1/3*
(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c*d^2 + 1/2*a*d
^2*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x
- 1)/c^3))*b*d^2
```

mupad [B] time = 0.97, size = 122, normalized size = 0.95

$$\frac{d^2\,\left(6\,a\,x^2+4\,b\,x^2+6\,b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{12}-\frac{\frac{d^2\,\left(9\,b\,\mathrm{atanh}\left(c\,x\right)-4\,b\,\ln\left(c^2\,x^2-1\right)\right)}{12}-\frac{3\,b\,c\,d^2\,x}{4}}{c^2}+\frac{c^2\,d^2\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atanh}\left(c\,x\right.\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] (d^2*(6*a*x^2 + 4*b*x^2 + 6*b*x^2*atanh(c*x)))/12 - ((d^2*(9*b*atanh(c*x) -
 4*b*log(c^2*x^2 - 1)))/12 - (3*b*c*d^2*x)/4)/c^2 + (c^2*d^2*(3*a*x^4 + 3*b
*x^4*atanh(c*x)))/12 + (c*d^2*(8*a*x^3 + b*x^3 + 8*b*x^3*atanh(c*x)))/12
```

sympy [A] time = 1.33, size = 167, normalized size = 1.29

$$\begin{cases}\frac{ac^2d^2x^4}{4}+\frac{2acd^2x^3}{3}+\frac{ad^2x^2}{2}+\frac{bc^2d^2x^4\,\mathrm{atanh}\,(cx)}{4}+\frac{2bcd^2x^3\,\mathrm{atanh}\,(cx)}{3}+\frac{bcd^2x^3}{12}+\frac{bd^2x^2\,\mathrm{atanh}\,(cx)}{2}+\frac{bd^2x^2}{3}+\frac{3bd^2x}{4c}+\frac{2bd^2\log\left(x\right.}{3c^2} \\ \frac{ad^2x^2}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**4/4 + 2*a*c*d**2*x**3/3 + a*d**2*x**2/2 + b*c**2*
d**2*x**4*atanh(c*x)/4 + 2*b*c*d**2*x**3*atanh(c*x)/3 + b*c*d**2*x**3/12 +
b*d**2*x**2*atanh(c*x)/2 + b*d**2*x**2/3 + 3*b*d**2*x/(4*c) + 2*b*d**2*log(
x - 1/c)/(3*c**2) - b*d**2*atanh(c*x)/(12*c**2), Ne(c, 0)), (a*d**2*x**2/2,
 True))
```

3.13 $\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=71

$$\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{bd^2(cx+1)^2}{6c}+\frac{4bd^2\log(1-cx)}{3c}+\frac{2}{3}bd^2x$$

```
[Out] 2/3*b*d^2*x+1/6*b*d^2*(c*x+1)^2/c+1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))/c+4/
3*b*d^2*ln(-c*x+1)/c
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5926, 627, 43}

$$\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{bd^2(cx+1)^2}{6c}+\frac{4bd^2\log(1-cx)}{3c}+\frac{2}{3}bd^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (2*b*d^2*x)/3 + (b*d^2*(1 + c*x)^2)/(6*c) + (d^2*(1 + c*x)^3*(a + b*ArcTanh
[c*x]))/(3*c) + (4*b*d^2*Log[1 - c*x])/(3*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}-\frac{b\int \frac{(d+cdx)^3}{1-c^2x^2}\,dx}{3d}\\
&= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}-\frac{b\int \frac{(d+cdx)^2}{\frac{1}{d}-\frac{cx}{d}}\,dx}{3d}\\
&= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}-\frac{b\int \left(-2d^3+\frac{4d^2}{\frac{1}{d}-\frac{cx}{d}}-d^2(d+cdx)\right)dx}{3d}\\
&= \frac{2}{3}bd^2x+\frac{bd^2(1+cx)^2}{6c}+\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{4bd^2\log(1-}{3c}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 92, normalized size = 1.30

$$\frac{d^2\left(2ac^3x^3+6ac^2x^2+6acx+bc^2x^2+b\log\left(1-c^2x^2\right)+2bcx\left(c^2x^2+3cx+3\right)\tanh^{-1}(cx)+6bcx+6b\log(1-cx\right.}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(6*a*c*x + 6*b*c*x + 6*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + 2*b*c*x*(
3 + 3*c*x + c^2*x^2)*ArcTanh[c*x] + 6*b*Log[1 - c*x] + b*Log[1 - c^2*x^2]))
/(6*c)
```

fricas [A] time = 0.45, size = 114, normalized size = 1.61

$$\frac{2\,ac^3d^2x^3+(6\,a+b)c^2d^2x^2+6\,(a+b)cd^2x+bd^2\log\left(cx+1\right)+7\,bd^2\log\left(cx-1\right)+\left(bc^3d^2x^3+3\,bc^2d^2x^2+3\,bcd\right.}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*d^2*x^3 + (6*a + b)*c^2*d^2*x^2 + 6*(a + b)*c*d^2*x + b*d^2*lo
g(c*x + 1) + 7*b*d^2*log(c*x - 1) + (b*c^3*d^2*x^3 + 3*b*c^2*d^2*x^2 + 3*b*
c*d^2*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

giac [B] time = 0.20, size = 330, normalized size = 4.65

$$-\frac{2}{3}\left(\frac{2\,bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{2\,bd^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{2\left(\frac{3\,(cx+1)^2bd^2}{(cx-1)^2}-\frac{3\,(cx+1)bd^2}{cx-1}+bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^2}{(cx-1)^3}-\frac{3\,(cx+1)^2c^2}{(cx-1)^2}+\frac{3\,(cx+1)c^2}{cx-1}-c^2}-\frac{\frac{12\,(cx+1)^2ad^2}{(cx-1)^2}-12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -2/3*(2*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 2*b*d^2*log(-(c*x + 1)/(c
*x - 1))/c^2 - 2*(3*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 3*(c*x + 1)*b*d^2/(c*x
- 1) + b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c
*x + 1)^2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2) - (12*(c*x + 1
)^2*a*d^2/(c*x - 1)^2 - 12*(c*x + 1)*a*d^2/(c*x - 1) + 4*a*d^2 + 4*(c*x + 1
)^2*b*d^2/(c*x - 1)^2 - 7*(c*x + 1)*b*d^2/(c*x - 1) + 3*b*d^2)/((c*x + 1)^3
*c^2/(c*x - 1)^3 - 3*(c*x + 1)^2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1
) - c^2))*c
```

maple [A] time = 0.03, size = 121, normalized size = 1.70

$$\frac{c^2d^2a\,x^3}{3}+c\,d^2a\,x^2+ax\,d^2+\frac{d^2a}{3c}+\frac{c^2d^2b\operatorname{arctanh}\left(cx\right)x^3}{3}+c\,d^2b\operatorname{arctanh}\left(cx\right)x^2+d^2b\operatorname{arctanh}\left(cx\right)x+\frac{d^2b\operatorname{arctanh}\left(cx\right.}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/3*c^2*d^2*a*x^3+c*d^2*a*x^2+a*x*d^2+1/3/c*d^2*a+1/3*c^2*d^2*b*arctanh(c*x
)*x^3+c*d^2*b*arctanh(c*x)*x^2+d^2*b*arctanh(c*x)*x+1/3/c*d^2*b*arctanh(c*x
)+1/6*c*d^2*b*x^2+b*d^2*x+4/3/c*d^2*b*ln(c*x-1)
```

maxima [B] time = 0.32, size = 147, normalized size = 2.07

$$\frac{1}{3}\,ac^2d^2x^3+\frac{1}{6}\left(2\,x^3\operatorname{artanh}\left(cx\right)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bc^2d^2+acd^2x^2+\frac{1}{2}\left(2\,x^2\operatorname{artanh}\left(cx\right)+c\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+\right.}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c^2*d^2*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)
/c^4))*b*c^2*d^2 + a*c*d^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log
(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctanh(c
*x) + log(-c^2*x^2 + 1))*b*d^2/c
```

mupad [B] time = 0.96, size = 105, normalized size = 1.48

$$\frac{d^2\,\left(6\,a\,x+6\,b\,x+6\,b\,x\,\mathrm{atanh}\left(c\,x\right)\right)}{6}+\frac{c^2\,d^2\,\left(2\,a\,x^3+2\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{6}-\frac{d^2\,\left(6\,b\,\mathrm{atanh}\left(c\,x\right)-4\,b\,\ln\left(c^2\,x^2-1\right)\right.}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] (d^2*(6*a*x + 6*b*x + 6*b*x*atanh(c*x)))/6 + (c^2*d^2*(2*a*x^3 + 2*b*x^3*at
anh(c*x)))/6 - (d^2*(6*b*atanh(c*x) - 4*b*log(c^2*x^2 - 1)))/(6*c) + (c*d^2
*(6*a*x^2 + b*x^2 + 6*b*x^2*atanh(c*x)))/6
```

sympy [A] time = 0.96, size = 131, normalized size = 1.85

$$\begin{cases} \frac{ac^2d^2x^3}{3} + acd^2x^2 + ad^2x + \frac{bc^2d^2x^3\,\mathrm{atanh}\,(cx)}{3} + bcd^2x^2\,\mathrm{atanh}\,(cx) + \frac{bcd^2x^2}{6} + bd^2x\,\mathrm{atanh}\,(cx) + bd^2x + \frac{4bd^2\log\left(x-\frac{1}{c}\right.}{3c} \\ ad^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**3/3 + a*c*d**2*x**2 + a*d**2*x + b*c**2*d**2*x**3
*atanh(c*x)/3 + b*c*d**2*x**2*atanh(c*x) + b*c*d**2*x**2/6 + b*d**2*x*atanh
(c*x) + b*d**2*x + 4*b*d**2*log(x - 1/c)/(3*c) + b*d**2*atanh(c*x)/(3*c), N
e(c, 0)), (a*d**2*x, True))
```

3.14 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=114

$$\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+2acd^2x+ad^2\log(x)+bd^2\log\left(1-c^2x^2\right)-\frac{1}{2}bd^2\text{Li}_2(-cx)+\frac{1}{2}bd^2\text{Li}_2(cx)+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2$$

```
[Out] 2*a*c*d^2*x+1/2*b*c*d^2*x-1/2*b*d^2*arctanh(c*x)+2*b*c*d^2*x*arctanh(c*x)+1
/2*c^2*d^2*x^2*(a+b*arctanh(c*x))+a*d^2*ln(x)+b*d^2*ln(-c^2*x^2+1)-1/2*b*d^
2*polylog(2,-c*x)+1/2*b*d^2*polylog(2,c*x)
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5940, 5910, 260, 5912, 5916, 321, 206}

$$-\frac{1}{2}bd^2\text{PolyLog}(2,-cx)+\frac{1}{2}bd^2\text{PolyLog}(2,cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+2acd^2x+ad^2\log(x)+bd^2\log\left(1-c^2x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 2*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTanh[c*x])/2 + 2*b*c*d^2*x*ArcTanh[
c*x] + (c^2*d^2*x^2*(a + b*ArcTanh[c*x]))/2 + a*d^2*Log[x] + b*d^2*Log[1 -
c^2*x^2] - (b*d^2*PolyLog[2, -(c*x)])/2 + (b*d^2*PolyLog[2, c*x])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(2cd^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+c^2d^2x\left(a+b\tan\right.\right.$$

$$= d^2\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(2cd^2\right)\int \left(a+b\tanh^{-1}(cx)\right)\,dx+\left(c^2d^2\right)\int$$

$$= 2acd^2x+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+ad^2\log(x)-\frac{1}{2}bd^2\mathrm{Li}_2(-cx)+\frac{1}{2}$$

$$= 2acd^2x+\frac{1}{2}bcd^2x+2bcd^2x\tanh^{-1}(cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+$$

$$= 2acd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\tanh^{-1}(cx)+2bcd^2x\tanh^{-1}(cx)+\frac{1}{2}c^2d^2x^2\left(a\right.$$

Mathematica [A] time = 0.10, size = 103, normalized size = 0.90

$$\frac{1}{4}d^2\left(2ac^2x^2+8acx+4a\log(x)+4b\log\left(1-c^2x^2\right)+2bc^2x^2\tanh^{-1}(cx)-2b\mathrm{Li}_2(-cx)+2b\mathrm{Li}_2(cx)+2bcx+b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^2*(8*a*c*x + 2*b*c*x + 2*a*c^2*x^2 + 8*b*c*x*ArcTanh[c*x] + 2*b*c^2*x^2*
ArcTanh[c*x] + 4*a*Log[x] + b*Log[1 - c*x] - b*Log[1 + c*x] + 4*b*Log[1 - c
^2*x^2] - 2*b*PolyLog[2, -(c*x)] + 2*b*PolyLog[2, c*x]))/4
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^2d^2x^2+2\,acd^2x+ad^2+\left(bc^2d^2x^2+2\,bcd^2x+bd^2\right)\mathrm{artanh}\,(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^2(b\,\mathrm{artanh}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x, x)
```

maple [A] time = 0.05, size = 142, normalized size = 1.25

$$\frac{d^2 a\, c^2 x^2}{2}+2ac\, d^2 x+a\, d^2 \ln (cx)+\frac{d^2 b \operatorname{arctanh}(cx)\, c^2 x^2}{2}+2bc\, d^2 x \operatorname{arctanh}(cx)+d^2 b \operatorname{arctanh}(cx)\ln (cx)-\frac{d^2 b \operatorname{dilog}(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x)
```

```
[Out] 1/2*d^2*a*c^2*x^2+2*a*c*d^2*x+a*d^2*ln(c*x)+1/2*d^2*b*arctanh(c*x)*c^2*x^2+
2*b*c*d^2*x*arctanh(c*x)+d^2*b*arctanh(c*x)*ln(c*x)-1/2*d^2*b*dilog(c*x)-1/
2*d^2*b*dilog(c*x+1)-1/2*d^2*b*ln(c*x)*ln(c*x+1)+1/2*b*c*d^2*x+5/4*d^2*b*ln
(c*x-1)+3/4*d^2*b*ln(c*x+1)
```

maxima [A] time = 0.47, size = 173, normalized size = 1.52

$$\frac{1}{4}\, bc^2 d^2 x^2 \log (cx+1)-\frac{1}{4}\, bc^2 d^2 x^2 \log (-cx+1)+\frac{1}{2}\, ac^2 d^2 x^2+2\, acd^2 x+\frac{1}{2}\, bcd^2 x+\left(2\, cx \operatorname{artanh}(cx)+\log \left(-c^2 x^2+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^2*d^2*x^2*log(c*x + 1) - 1/4*b*c^2*d^2*x^2*log(-c*x + 1) + 1/2*a*c^
2*d^2*x^2 + 2*a*c*d^2*x + 1/2*b*c*d^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*b*d^2 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^2 + 1/2*
(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^2 - 1/4*b*d^2*log(c*x + 1) +
1/4*b*d^2*log(c*x - 1) + a*d^2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b \operatorname{atanh}(c\, x))\, (d+c\, d\, x)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \left(\int 2ac\, dx+\int \frac{a}{x}\, dx+\int ac^2 x\, dx+\int 2bc \operatorname{atanh}(cx)\, dx+\int \frac{b \operatorname{atanh}(cx)}{x}\, dx+\int bc^2 x \operatorname{atanh}(cx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**2*(Integral(2*a*c, x) + Integral(a/x, x) + Integral(a*c**2*x, x) + Integ
ral(2*b*c*atanh(c*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(b*c**2*x*
atanh(c*x), x))
```

3.15 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=61

$$\frac{d^2\left(c^2x^2-1\right)\left(a+b\tanh^{-1}(cx)\right)}{x}+cd^2(2a+b)\log(x)-bcd^2\text{Li}_2(-cx)+bcd^2\text{Li}_2(cx)$$

```
[Out] d^2*(c^2*x^2-1)*(a+b*arctanh(c*x))/x+(2*a+b)*c*d^2*ln(x)-b*c*d^2*polylog(2,
-c*x)+b*c*d^2*polylog(2,c*x)
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5940, 5910, 260, 5916, 266, 36, 29, 31, 5912}

$$-bcd^2\text{PolyLog}(2,-cx)+bcd^2\text{PolyLog}(2,cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2x+2acd^2\log(x)+bc^2d^2x\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] a*c^2*d^2*x + b*c^2*d^2*x*ArcTanh[c*x] - (d^2*(a + b*ArcTanh[c*x]))/x + 2*a
*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, -(c*x)] + b*c*d^2*PolyL
og[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(c^2d^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{2cd^2\left(a+b\tanh^{-1}(\right.}{x}\right.$$

$$= d^2\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(2cd^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(c^2d^2\right)\int (a+$$

$$= ac^2d^2x-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)-bcd^2\mathrm{Li}_2(-cx)+bcd^2\mathrm{Li}_2(c$$

$$= ac^2d^2x+bc^2d^2x\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)-bcd^2$$

$$= ac^2d^2x+bc^2d^2x\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)+\frac{1}{2}bc$$

$$= ac^2d^2x+bc^2d^2x\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)+bcd^2$$

Mathematica [A] time = 0.11, size = 73, normalized size = 1.20

$$\frac{d^2\left(ac^2x^2+2acx\log(x)-a+bc^2x^2\tanh^{-1}(cx)-bcx\mathrm{Li}_2(-cx)+bcx\mathrm{Li}_2(cx)+bcx\log(cx)-b\tanh^{-1}(cx)\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-a + a*c^2*x^2 - b*ArcTanh[c*x] + b*c^2*x^2*ArcTanh[c*x] + 2*a*c*x*Lo
g[x] + b*c*x*Log[c*x] - b*c*x*PolyLog[2, -(c*x)] + b*c*x*PolyLog[2, c*x]))/
x
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^2d^2x^2+2\,acd^2x+ad^2+\left(bc^2d^2x^2+2\,bcd^2x+bd^2\right)\mathrm{artanh}\,(cx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x^2, x)
```

giac [B] time = 1.80, size = 410, normalized size = 6.72

$$\frac{1}{6}\left(\frac{6\,ad^2}{\frac{(cx+1)c^2}{cx-1}+c^2}+\frac{5\,bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}+\frac{3\,bd^2\log\left(-\frac{cx+1}{cx-1}-1\right)}{c^2}+\left(\frac{3\,bd^2}{\frac{(cx+1)c^2}{cx-1}+c^2}-\frac{\frac{3\,(cx+1)^2bd^2}{(cx-1)^2}-\frac{12\,(cx+1)bd^2}{cx-1}}{\frac{(cx+1)^3c^2}{(cx-1)^3}-\frac{3\,(cx+1)^2c^2}{(cx-1)^2}+\frac{3\,(cx}{cx}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*a*d^2/((c*x + 1)*c^2/(c*x - 1) + c^2) + 5*b*d^2*log(-(c*x + 1)/(c*x
- 1) + 1)/c^2 + 3*b*d^2*log(-(c*x + 1)/(c*x - 1) - 1)/c^2 + (3*b*d^2/((c*x
+ 1)*c^2/(c*x - 1) + c^2) - (3*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 12*(c*x + 1)
*b*d^2/(c*x - 1) + 5*b*d^2)/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c*x + 1)^2*c^
2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2))*log(-(c*x + 1)/(c*x - 1))
 - 8*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^2 - 2*(3*(c*x + 1)^2*a*d^2/(c*x - 1)
^2 - 12*(c*x + 1)*a*d^2/(c*x - 1) + 5*a*d^2 - (c*x + 1)^2*b*d^2/(c*x - 1)^2
 + (c*x + 1)*b*d^2/(c*x - 1))/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c*x + 1)^2*
c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2))*c^2
```

maple [A] time = 0.05, size = 123, normalized size = 2.02

$$d^2a\,c^2x+2c\,d^2a\ln(cx)-\frac{d^2a}{x}+b\,c^2d^2x\operatorname{arctanh}(cx)+2c\,d^2b\operatorname{arctanh}(cx)\ln(cx)-\frac{d^2b\operatorname{arctanh}(cx)}{x}+c\,d^2b\ln(cx)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] d^2*a*c^2*x+2*c*d^2*a*ln(c*x)-d^2*a/x+b*c^2*d^2*x*arctanh(c*x)+2*c*d^2*b*ar
ctanh(c*x)*ln(c*x)-d^2*b*arctanh(c*x)/x+c*d^2*b*ln(c*x)-c*d^2*b*dilog(c*x)-
c*d^2*b*dilog(c*x+1)-c*d^2*b*ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$ac^2d^2x+\frac{1}{2}\left(2\,cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)bcd^2+bcd^2\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx+2\,acd^2\log(x)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] a*c^2*d^2*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^2 + b*c*d^
2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 2*a*c*d^2*log(x) - 1/2*(
c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^2 - a*d^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\operatorname{atanh}(c\,x))\,(d+c\,d\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int ac^2\,dx+\int \frac{a}{x^2}\,dx+\int \frac{2ac}{x}\,dx+\int bc^2\operatorname{atanh}(cx)\,dx+\int \frac{b\operatorname{atanh}(cx)}{x^2}\,dx+\int \frac{2bc\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(2*a*c/x, x) + In
tegral(b*c**2*atanh(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(2*
b*c*atanh(c*x)/x, x))
```

3.16 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=137

$$-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2\log(x)-\frac{1}{2}bc^2d^2\text{Li}_2(-cx)+\frac{1}{2}bc^2d^2\text{Li}_2(cx)-bc^2d^2\log\left(1\right.$$

```
[Out] -1/2*b*c*d^2/x+1/2*b*c^2*d^2*arctanh(c*x)-1/2*d^2*(a+b*arctanh(c*x))/x^2-2*
c*d^2*(a+b*arctanh(c*x))/x+a*c^2*d^2*ln(x)+2*b*c^2*d^2*ln(x)-b*c^2*d^2*ln(-
c^2*x^2+1)-1/2*b*c^2*d^2*polylog(2,-c*x)+1/2*b*c^2*d^2*polylog(2,c*x)
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912}

$$-\frac{1}{2}bc^2d^2\text{PolyLog}(2,-cx)+\frac{1}{2}bc^2d^2\text{PolyLog}(2,cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2\log($$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^2)/(2*x) + (b*c^2*d^2*ArcTanh[c*x])/2 - (d^2*(a + b*ArcTanh[c*x]))/
(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x]))/x + a*c^2*d^2*Log[x] + 2*b*c^2*d^2
*Log[x] - b*c^2*d^2*Log[1 - c^2*x^2] - (b*c^2*d^2*PolyLog[2, -(c*x)])/2 + (
b*c^2*d^2*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int \left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x^3} + \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x^2} + \frac{c^2d^2\left(a+b\tanh^{-1}(\right.}{x}\right.$$

$$= d^2\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx + \left(2cd^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx + \left(c^2d^2\right)\int \frac{a+}{}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + ac^2d^2\log(x) - \frac{1}{2}bc^2d^2$$

$$= -\frac{bcd^2}{2x} - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + ac^2d^2\log(x) -$$

$$= -\frac{bcd^2}{2x} + \frac{1}{2}bc^2d^2\tanh^{-1}(cx) - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\tanh^{-1}(\right.}{x}$$

$$= -\frac{bcd^2}{2x} + \frac{1}{2}bc^2d^2\tanh^{-1}(cx) - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\tanh^{-1}(\right.}{x}$$

Mathematica [A] time = 0.10, size = 143, normalized size = 1.04

$$\frac{d^2\left(4ac^2x^2\log(x) - 8acx - 2a - 2bc^2x^2\text{Li}_2(-cx) + 2bc^2x^2\text{Li}_2(cx) + 8bc^2x^2\log(cx) - bc^2x^2\log(1-cx) + bc^2x^2\log\right.}{4x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^2*(-2*a - 8*a*c*x - 2*b*c*x - 2*b*ArcTanh[c*x] - 8*b*c*x*ArcTanh[c*x] +
4*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^2*
x^2*Log[1 + c*x] - 4*b*c^2*x^2*Log[1 - c^2*x^2] - 2*b*c^2*x^2*PolyLog[2, -(
c*x)] + 2*b*c^2*x^2*PolyLog[2, c*x]))/(4*x^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^2d^2x^2 + 2\,acd^2x + ad^2 + \left(bc^2d^2x^2 + 2\,bcd^2x + bd^2\right)\text{artanh}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^2(b\,\text{artanh}\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x^3, x)
```

maple [A] time = 0.06, size = 176, normalized size = 1.28

$$c^2d^2a\ln(cx) - \frac{2c\,d^2a}{x} - \frac{a\,d^2}{2x^2} + c^2d^2b\,\text{arctanh}\,(cx)\ln(cx) - \frac{2c\,d^2b\,\text{arctanh}\,(cx)}{x} - \frac{d^2b\,\text{arctanh}\,(cx)}{2x^2} - \frac{bc\,d^2}{2x} + 2c^2d^2b\,\text{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] c^2*d^2*a*ln(c*x)-2*c*d^2*a/x-1/2*a*d^2/x^2+c^2*d^2*b*arctanh(c*x)*ln(c*x)-
2*c*d^2*b*arctanh(c*x)/x-1/2*d^2*b*arctanh(c*x)/x^2-1/2*b*c*d^2/x+2*c^2*d^2
*b*ln(c*x)-5/4*c^2*d^2*b*ln(c*x-1)-3/4*c^2*d^2*b*ln(c*x+1)-1/2*c^2*d^2*b*di
log(c*x)-1/2*c^2*d^2*b*dilog(c*x+1)-1/2*c^2*d^2*b*ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,bc^2d^2\int \frac{\log(cx+1) - \log(-cx+1)}{x}\,dx + ac^2d^2\log(x) - \left(c\left(\log\left(c^2x^2 - 1\right) - \log\left(x^2\right)\right) + \frac{2\,\text{artanh}\,(cx)}{x}\right)bcd^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*d^2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^2*d^2*lo
g(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^2 + 1/4*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^2 - 2*a
*c*d^2/x - 1/2*a*d^2/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\text{atanh}\,(c\,x))\,\left(d + c\,d\,x\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{a}{x^3}\,dx + \int \frac{2ac}{x^2}\,dx + \int \frac{ac^2}{x}\,dx + \int \frac{b\operatorname{atanh}(cx)}{x^3}\,dx + \int \frac{2bc\operatorname{atanh}(cx)}{x^2}\,dx + \int \frac{bc^2\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(2*a*c/x**2, x) + Integral(a*c**2/x, x)
 + Integral(b*atanh(c*x)/x**3, x) + Integral(2*b*c*atanh(c*x)/x**2, x) + In
tegral(b*c**2*atanh(c*x)/x, x))
```

3.17 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+\frac{4}{3}bc^3d^2\log(x)-\frac{4}{3}bc^3d^2\log(1-cx)-\frac{bc^2d^2}{x}-\frac{bcd^2}{6x^2}$$

```
[Out] -1/6*b*c*d^2/x^2-b*c^2*d^2/x-1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))/x^3+4/3*b
*c^3*d^2*ln(x)-4/3*b*c^3*d^2*ln(-c*x+1)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {37, 5936, 12, 88}

$$-\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{bc^2d^2}{x}+\frac{4}{3}bc^3d^2\log(x)-\frac{4}{3}bc^3d^2\log(1-cx)-\frac{bcd^2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^2)/(6*x^2) - (b*c^2*d^2)/x - (d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x]))
/(3*x^3) + (4*b*c^3*d^2*Log[x])/3 - (4*b*c^3*d^2*Log[1 - c*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - (bc)\int \frac{(d+cdx)^2}{3x^3(-1+cx)}\,dx$$
$$= -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{3}(bc)\int \frac{(d+cdx)^2}{x^3(-1+cx)}\,dx$$
$$= -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{3}(bc)\int \left(-\frac{d^2}{x^3} - \frac{3cd^2}{x^2} - \frac{4c^2d^2}{x} + \frac{4c^3d^2}{-1+c\ldots}\right.$$
$$= -\frac{bcd^2}{6x^2} - \frac{bc^2d^2}{x} - \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} + \frac{4}{3}bc^3d^2\log(x) - \frac{4}{3}bc^3d^2\ldots$$

Mathematica [A] time = 0.09, size = 103, normalized size = 1.27

$$-\frac{d^2\left(6ac^2x^2+6acx+2a-8bc^3x^3\log(x)+7bc^3x^3\log(1-cx)+bc^3x^3\log(cx+1)+6bc^2x^2+2b\left(3c^2x^2+3cx+\ldots\right.\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/6*(d^2*(2*a + 6*a*c*x + b*c*x + 6*a*c^2*x^2 + 6*b*c^2*x^2 + 2*b*(1 + 3*c
*x + 3*c^2*x^2)*ArcTanh[c*x] - 8*b*c^3*x^3*Log[x] + 7*b*c^3*x^3*Log[1 - c*x
] + b*c^3*x^3*Log[1 + c*x]))/x^3
```

fricas [A] time = 0.63, size = 128, normalized size = 1.58

$$-\frac{bc^3d^2x^3\log(cx+1)+7\,bc^3d^2x^3\log(cx-1)-8\,bc^3d^2x^3\log(x)+6\,(a+b)c^2d^2x^2+(6\,a+b)cd^2x+2\,ad^2+\left(3\,b\ldots\right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c^3*d^2*x^3*log(c*x + 1) + 7*b*c^3*d^2*x^3*log(c*x - 1) - 8*b*c^3*d
^2*x^3*log(x) + 6*(a + b)*c^2*d^2*x^2 + (6*a + b)*c*d^2*x + 2*a*d^2 + (3*b*
c^2*d^2*x^2 + 3*b*c*d^2*x + b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^3
```

giac [B] time = 0.15, size = 330, normalized size = 4.07

$$\frac{2}{3}\left(2\,bc^2d^2\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,bc^2d^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{2\left(\frac{3\,(cx+1)^2bc^2d^2}{(cx-1)^2}+\frac{3\,(cx+1)bc^2d^2}{cx-1}+bc^2d^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}+\frac{12\,(c\ldots}{(\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(2*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^2*d^2*log(-(c*x + 1)
/(c*x - 1)) + 2*(3*(c*x + 1)^2*b*c^2*d^2/(c*x - 1)^2 + 3*(c*x + 1)*b*c^2*d^
2/(c*x - 1) + b*c^2*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3
 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^2
*a*c^2*d^2/(c*x - 1)^2 + 12*(c*x + 1)*a*c^2*d^2/(c*x - 1) + 4*a*c^2*d^2 + 4
*(c*x + 1)^2*b*c^2*d^2/(c*x - 1)^2 + 7*(c*x + 1)*b*c^2*d^2/(c*x - 1) + 3*b*
c^2*d^2)/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)
/(c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 141, normalized size = 1.74

$$-\frac{c^2d^2a}{x}-\frac{d^2a}{3x^3}-\frac{c\,d^2a}{x^2}-\frac{c^2d^2b\operatorname{arctanh}(cx)}{x}-\frac{d^2b\operatorname{arctanh}(cx)}{3x^3}-\frac{c\,d^2b\operatorname{arctanh}(cx)}{x^2}-\frac{bc\,d^2}{6x^2}-\frac{b\,c^2d^2}{x}+\frac{4c^3d^2b\ln(cx)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] -c^2*d^2*a/x-1/3*d^2*a/x^3-c*d^2*a/x^2-c^2*d^2*b*arctanh(c*x)/x-1/3*d^2*b*a
rctanh(c*x)/x^3-c*d^2*b*arctanh(c*x)/x^2-1/6*b*c*d^2/x^2-b*c^2*d^2/x+4/3*c^
3*d^2*b*ln(c*x)-7/6*c^3*d^2*b*ln(c*x-1)-1/6*c^3*d^2*b*ln(c*x+1)
```

maxima [B] time = 0.32, size = 157, normalized size = 1.94

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\operatorname{artanh}(cx)}{x}\right)bc^2d^2+\frac{1}{2}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\operatorname{artanh}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^2 + 1/2*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^2 - 1
/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b
*d^2 - a*c^2*d^2/x - a*c*d^2/x^2 - 1/3*a*d^2/x^3
```

mupad [B] time = 0.90, size = 116, normalized size = 1.43

$$\frac{d^2\left(6\,b\,c^3\operatorname{atanh}(c\,x)-4\,b\,c^3\ln\left(c^2\,x^2-1\right)+8\,b\,c^3\ln(x)\right)}{6}-\frac{\frac{d^2\,(2\,a+2\,b\operatorname{atanh}(c\,x))}{6}+\frac{d^2\,x\,(6\,a\,c+b\,c+6\,b\,c\operatorname{atanh}(c\,x))}{6}+\frac{d^2\,x^2}{}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^4,x)
```

```
[Out] (d^2*(6*b*c^3*atanh(c*x) - 4*b*c^3*log(c^2*x^2 - 1) + 8*b*c^3*log(x)))/6 -
((d^2*(2*a + 2*b*atanh(c*x)))/6 + (d^2*x*(6*a*c + b*c + 6*b*c*atanh(c*x)))/
6 + (d^2*x^2*(6*a*c^2 + 6*b*c^2 + 6*b*c^2*atanh(c*x)))/6)/x^3
```

sympy [A] time = 1.58, size = 158, normalized size = 1.95

$$\begin{cases}-\frac{ac^2d^2}{x}-\frac{acd^2}{x^2}-\frac{ad^2}{3x^3}+\frac{4bc^3d^2\log(x)}{3}-\frac{4bc^3d^2\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^3d^2\operatorname{atanh}(cx)}{3}-\frac{bc^2d^2\operatorname{atanh}(cx)}{x}-\frac{bc^2d^2}{x}-\frac{bcd^2\operatorname{atanh}(cx)}{x^2}-\frac{bcd^2}{6x^2}- \\ -\frac{ad^2}{3x^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*c**2*d**2/x - a*c*d**2/x**2 - a*d**2/(3*x**3) + 4*b*c**3*d**2
*log(x)/3 - 4*b*c**3*d**2*log(x - 1/c)/3 - b*c**3*d**2*atanh(c*x)/3 - b*c**
2*d**2*atanh(c*x)/x - b*c**2*d**2/x - b*c*d**2*atanh(c*x)/x**2 - b*c*d**2/(
6*x**2) - b*d**2*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d**2/(3*x**3), True))
```

3.18 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=147

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+\frac{2}{3}bc^4d^2\log(x)-\frac{17}{24}bc^4d^2\log(1-cx)+\frac{}{2}$$

```
[Out] -1/12*b*c*d^2/x^3-1/3*b*c^2*d^2/x^2-3/4*b*c^3*d^2/x-1/4*d^2*(a+b*arctanh(c*
x))/x^4-2/3*c*d^2*(a+b*arctanh(c*x))/x^3-1/2*c^2*d^2*(a+b*arctanh(c*x))/x^2
+2/3*b*c^4*d^2*ln(x)-17/24*b*c^4*d^2*ln(-c*x+1)+1/24*b*c^4*d^2*ln(c*x+1)
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d^2}{3x^2}-\frac{3bc^3d^2}{4x}+\frac{2}{3}bc^4d^2\log(x)-\frac{17}{24}bc^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^2)/(12*x^3) - (b*c^2*d^2)/(3*x^2) - (3*b*c^3*d^2)/(4*x) - (d^2*(a +
 b*ArcTanh[c*x]))/(4*x^4) - (2*c*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) - (c^2*d
^2*(a + b*ArcTanh[c*x]))/(2*x^2) + (2*b*c^4*d^2*Log[x])/3 - (17*b*c^4*d^2*L
og[1 - c*x])/24 + (b*c^4*d^2*Log[1 + c*x])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx = -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{bcd^2}{12x^3} - \frac{bc^2d^2}{3x^2} - \frac{3bc^3d^2}{4x} - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}\right.}{3x^3}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 0.78

$$-\frac{d^2\left(12ac^2x^2+16acx+6a-16bc^4x^4\log(x)+17bc^4x^4\log(1-cx)-bc^4x^4\log(cx+1)+18bc^3x^3+8bc^2x^2+2\right.}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/24*(d^2*(6*a + 16*a*c*x + 2*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 18*b*c^
3*x^3 + 2*b*(3 + 8*c*x + 6*c^2*x^2)*ArcTanh[c*x] - 16*b*c^4*x^4*Log[x] + 17
*b*c^4*x^4*Log[1 - c*x] - b*c^4*x^4*Log[1 + c*x]))/x^4
```

fricas [A] time = 0.70, size = 147, normalized size = 1.00

$$\frac{bc^4d^2x^4\log(cx+1) - 17\,bc^4d^2x^4\log(cx-1) + 16\,bc^4d^2x^4\log(x) - 18\,bc^3d^2x^3 - 4\,(3\,a+2\,b)c^2d^2x^2 - 2\,(8\,a+}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(b*c^4*d^2*x^4*log(c*x + 1) - 17*b*c^4*d^2*x^4*log(c*x - 1) + 16*b*c^4
*d^2*x^4*log(x) - 18*b*c^3*d^2*x^3 - 4*(3*a + 2*b)*c^2*d^2*x^2 - 2*(8*a + b
)*c*d^2*x - 6*a*d^2 - (6*b*c^2*d^2*x^2 + 8*b*c*d^2*x + 3*b*d^2)*log(-(c*x +
 1)/(c*x - 1)))/x^4
```

giac [B] time = 0.28, size = 431, normalized size = 2.93

$$\frac{1}{3}\left(2\,bc^3d^2\log\left(-\frac{cx+1}{cx-1}-1\right) - 2\,bc^3d^2\log\left(-\frac{cx+1}{cx-1}\right) + \frac{2\left(\frac{6\,(cx+1)^3bc^3d^2}{(cx-1)^3} + \frac{6\,(cx+1)^2bc^3d^2}{(cx-1)^2} + \frac{4\,(cx+1)bc^3d^2}{cx-1} + bc^3d^2\right)\log}{\frac{(cx+1)^4}{(cx-1)^4} + \frac{4\,(cx+1)^3}{(cx-1)^3} + \frac{6\,(cx+1)^2}{(cx-1)^2} + \frac{4\,(cx+1)}{cx-1} + 1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(2*b*c^3*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^3*d^2*log(-(c*x + 1)
/(c*x - 1)) + 2*(6*(c*x + 1)^3*b*c^3*d^2/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^3*
d^2/(c*x - 1)^2 + 4*(c*x + 1)*b*c^3*d^2/(c*x - 1) + b*c^3*d^2)*log(-(c*x +
1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x
 + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (24*(c*x + 1)^3*a*c^3*d^
2/(c*x - 1)^3 + 24*(c*x + 1)^2*a*c^3*d^2/(c*x - 1)^2 + 16*(c*x + 1)*a*c^3*d
^2/(c*x - 1) + 4*a*c^3*d^2 + 10*(c*x + 1)^3*b*c^3*d^2/(c*x - 1)^3 + 23*(c*x
 + 1)^2*b*c^3*d^2/(c*x - 1)^2 + 18*(c*x + 1)*b*c^3*d^2/(c*x - 1) + 5*b*c^3*
```

```
d^2)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(
c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 153, normalized size = 1.04

$$-\frac{2c\,d^2a}{3x^3}-\frac{c^2d^2a}{2x^2}-\frac{d^2a}{4x^4}-\frac{2c\,d^2b\,\mathrm{arctanh}\,(cx)}{3x^3}-\frac{c^2d^2b\,\mathrm{arctanh}\,(cx)}{2x^2}-\frac{d^2b\,\mathrm{arctanh}\,(cx)}{4x^4}-\frac{bc\,d^2}{12x^3}-\frac{b\,c^2d^2}{3x^2}-\frac{3b\,c^3d^2}{4x}+\frac{2c^4d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -2/3*c*d^2*a/x^3-1/2*c^2*d^2*a/x^2-1/4*d^2*a/x^4-2/3*c*d^2*b*arctanh(c*x)/x
^3-1/2*c^2*d^2*b*arctanh(c*x)/x^2-1/4*d^2*b*arctanh(c*x)/x^4-1/12*b*c*d^2/x
^3-1/3*b*c^2*d^2/x^2-3/4*b*c^3*d^2/x+2/3*c^4*d^2*b*ln(c*x)-17/24*c^4*d^2*b*
ln(c*x-1)+1/24*b*c^4*d^2*ln(c*x+1)
```

maxima [A] time = 0.32, size = 178, normalized size = 1.21

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\,(cx)}{x^2}\right)bc^2d^2-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*
d^2 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c*d^2 + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x
^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^2 - 1/2*a*c^2*d^2/x^2 - 2/3*a*c*d^
2/x^3 - 1/4*a*d^2/x^4
```

mupad [B] time = 1.01, size = 168, normalized size = 1.14

$$\frac{2\,b\,c^4\,d^2\,\ln(x)}{3}-\frac{b\,c^4\,d^2\,\ln\left(c^2\,x^2-1\right)}{3}-\frac{a\,c^2\,d^2}{2\,x^2}-\frac{b\,c^2\,d^2}{3\,x^2}-\frac{3\,b\,c^3\,d^2}{4\,x}-\frac{a\,d^2}{4\,x^4}-\frac{2\,a\,c\,d^2}{3\,x^3}-\frac{b\,c\,d^2}{12\,x^3}-\frac{b\,d^2\,\mathrm{atanh}\,(c\,x)}{4\,x^4}-\frac{3\,b\,c^5\,d^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^5,x)
```

```
[Out] (2*b*c^4*d^2*log(x))/3 - (b*c^4*d^2*log(c^2*x^2 - 1))/3 - (a*c^2*d^2)/(2*x^
2) - (b*c^2*d^2)/(3*x^2) - (3*b*c^3*d^2)/(4*x) - (a*d^2)/(4*x^4) - (2*a*c*d
^2)/(3*x^3) - (b*c*d^2)/(12*x^3) - (b*d^2*atanh(c*x))/(4*x^4) - (3*b*c^5*d^
2*atan((c^2*x)/(-c^2)^(1/2)))/(4*(-c^2)^(1/2)) - (2*b*c*d^2*atanh(c*x))/(3*
x^3) - (b*c^2*d^2*atanh(c*x))/(2*x^2)
```

sympy [A] time = 2.02, size = 189, normalized size = 1.29

$$\begin{cases} -\frac{ac^2d^2}{2x^2}-\frac{2acd^2}{3x^3}-\frac{ad^2}{4x^4}+\frac{2bc^4d^2\log\left(x\right)}{3}-\frac{2bc^4d^2\log\left(x-\frac{1}{c}\right)}{3}+\frac{bc^4d^2\,\mathrm{atanh}\,(cx)}{12}-\frac{3bc^3d^2}{4x}-\frac{bc^2d^2\,\mathrm{atanh}\,(cx)}{2x^2}-\frac{bc^2d^2}{3x^2}-\frac{2bcd^2\,\mathrm{atanh}\,(cx)}{3x^3} \\ -\frac{ad^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c**2*d**2/(2*x**2) - 2*a*c*d**2/(3*x**3) - a*d**2/(4*x**4) +
2*b*c**4*d**2*log(x)/3 - 2*b*c**4*d**2*log(x - 1/c)/3 + b*c**4*d**2*atanh(c
*x)/12 - 3*b*c**3*d**2/(4*x) - b*c**2*d**2*atanh(c*x)/(2*x**2) - b*c**2*d**
2/(3*x**2) - 2*b*c*d**2*atanh(c*x)/(3*x**3) - b*c*d**2/(12*x**3) - b*d**2*a
tanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**2/(4*x**4), True))
```

3.19 $\int \frac{(d+cdx)^2(a+b\tanh^{-1}(cx))}{x^6} dx$

Optimal. Leaf size=161

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4}+\frac{8}{15}bc^5d^2\log(x)-\frac{31}{60}bc^5d^2\log(1-cx)$$

```
[Out] -1/20*b*c*d^2/x^4-1/6*b*c^2*d^2/x^3-4/15*b*c^3*d^2/x^2-1/2*b*c^4*d^2/x-1/5*
d^2*(a+b*arctanh(c*x))/x^5-1/2*c*d^2*(a+b*arctanh(c*x))/x^4-1/3*c^2*d^2*(a+
b*arctanh(c*x))/x^3+8/15*b*c^5*d^2*ln(x)-31/60*b*c^5*d^2*ln(-c*x+1)-1/60*b*
c^5*d^2*ln(c*x+1)
```

Rubi [A] time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{4bc^3d^2}{15x^2}-\frac{bc^2d^2}{6x^3}-\frac{bc^4d^2}{2x}+\frac{8}{15}bc^5d^2\log$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^2)/(20*x^4) - (b*c^2*d^2)/(6*x^3) - (4*b*c^3*d^2)/(15*x^2) - (b*c^4
*d^2)/(2*x) - (d^2*(a + b*ArcTanh[c*x]))/(5*x^5) - (c*d^2*(a + b*ArcTanh[c*
x]))/(2*x^4) - (c^2*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) + (8*b*c^5*d^2*Log[x]
)/15 - (31*b*c^5*d^2*Log[1 - c*x])/60 - (b*c^5*d^2*Log[1 + c*x])/60
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}$$

$$= -\frac{bcd^2}{20x^4} - \frac{bc^2d^2}{6x^3} - \frac{4bc^3d^2}{15x^2} - \frac{bc^4d^2}{2x} - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tan\right.}{2x^4}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.76

$$-\frac{d^2\left(20ac^2x^2+30acx+12a-32bc^5x^5\log(x)+31bc^5x^5\log(1-cx)+bc^5x^5\log(cx+1)+30bc^4x^4+16bc^3x^3+1\right.}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/60*(d^2*(12*a + 30*a*c*x + 3*b*c*x + 20*a*c^2*x^2 + 10*b*c^2*x^2 + 16*b*
c^3*x^3 + 30*b*c^4*x^4 + 2*b*(6 + 15*c*x + 10*c^2*x^2)*ArcTanh[c*x] - 32*b*
c^5*x^5*Log[x] + 31*b*c^5*x^5*Log[1 - c*x] + b*c^5*x^5*Log[1 + c*x]))/x^5
```

fricas [A] time = 0.54, size = 156, normalized size = 0.97

$$-\frac{bc^5d^2x^5\log(cx+1)+31\,bc^5d^2x^5\log(cx-1)-32\,bc^5d^2x^5\log(x)+30\,bc^4d^2x^4+16\,bc^3d^2x^3+10\,(2\,a+b)c^2d^2x}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(b*c^5*d^2*x^5*log(c*x + 1) + 31*b*c^5*d^2*x^5*log(c*x - 1) - 32*b*c^
5*d^2*x^5*log(x) + 30*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a + b)*c^2*d
^2*x^2 + 3*(10*a + b)*c*d^2*x + 12*a*d^2 + (10*b*c^2*d^2*x^2 + 15*b*c*d^2*x
 + 6*b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

giac [B] time = 0.27, size = 532, normalized size = 3.30

$$\frac{4}{15}\left(2\,bc^4d^2\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,bc^4d^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{15\,(cx+1)^4bc^4d^2}{(cx-1)^4}+\frac{15\,(cx+1)^3bc^4d^2}{(cx-1)^3}+\frac{20\,(cx+1)^2bc^4d^2}{(cx-1)^2}+\frac{10\,(cx+1)bc}{cx-1}\right.}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(2*b*c^4*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^4*d^2*log(-(c*x + 1
)/(c*x - 1)) + (15*(c*x + 1)^4*b*c^4*d^2/(c*x - 1)^4 + 15*(c*x + 1)^3*b*c^4
*d^2/(c*x - 1)^3 + 20*(c*x + 1)^2*b*c^4*d^2/(c*x - 1)^2 + 10*(c*x + 1)*b*c^
4*d^2/(c*x - 1) + 2*b*c^4*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x
- 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x +
 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (30*(c*x + 1)^4*a*c^4*d^2/
(c*x - 1)^4 + 30*(c*x + 1)^3*a*c^4*d^2/(c*x - 1)^3 + 40*(c*x + 1)^2*a*c^4*d
^2/(c*x - 1)^2 + 20*(c*x + 1)*a*c^4*d^2/(c*x - 1) + 4*a*c^4*d^2 + 13*(c*x +
```

```
1)^4*b*c^4*d^2/(c*x - 1)^4 + 36*(c*x + 1)^3*b*c^4*d^2/(c*x - 1)^3 + 41*(c*
x + 1)^2*b*c^4*d^2/(c*x - 1)^2 + 23*(c*x + 1)*b*c^4*d^2/(c*x - 1) + 5*b*c^4
*d^2)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3
/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 165, normalized size = 1.02

$$-\frac{c^2d^2a}{3x^3}-\frac{c\,d^2a}{2x^4}-\frac{d^2a}{5x^5}-\frac{c^2d^2b\,\mathrm{arctanh}\,(cx)}{3x^3}-\frac{c\,d^2b\,\mathrm{arctanh}\,(cx)}{2x^4}-\frac{d^2b\,\mathrm{arctanh}\,(cx)}{5x^5}-\frac{bc\,d^2}{20x^4}-\frac{b\,c^2d^2}{6x^3}-\frac{4b\,c^3d^2}{15x^2}-\frac{b\,c^4d^2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -1/3*c^2*d^2*a/x^3-1/2*c*d^2*a/x^4-1/5*d^2*a/x^5-1/3*c^2*d^2*b*arctanh(c*x)
/x^3-1/2*c*d^2*b*arctanh(c*x)/x^4-1/5*d^2*b*arctanh(c*x)/x^5-1/20*b*c*d^2/x
^4-1/6*b*c^2*d^2/x^3-4/15*b*c^3*d^2/x^2-1/2*b*c^4*d^2/x+8/15*c^5*d^2*b*ln(c
*x)-31/60*c^5*d^2*b*ln(c*x-1)-1/60*b*c^5*d^2*ln(c*x+1)
```

maxima [A] time = 0.32, size = 194, normalized size = 1.20

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\,(cx)}{x^3}\right)bc^2d^2+\frac{1}{12}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^2*d^2 + 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2
+ 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 - 1) -
 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^2 - 1/3*
a*c^2*d^2/x^3 - 1/2*a*c*d^2/x^4 - 1/5*a*d^2/x^5
```

mupad [B] time = 0.95, size = 182, normalized size = 1.13

$$-\frac{12\,a\,d^2+12\,b\,d^2\,\mathrm{atanh}\,(c\,x)+20\,a\,c^2\,d^2\,x^2+10\,b\,c^2\,d^2\,x^2+16\,b\,c^3\,d^2\,x^3+30\,b\,c^4\,d^2\,x^4+30\,a\,c\,d^2\,x+3\,b\,c\,d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^6,x)
```

```
[Out] -(12*a*d^2 + 12*b*d^2*atanh(c*x) + 20*a*c^2*d^2*x^2 + 10*b*c^2*d^2*x^2 + 16
*b*c^3*d^2*x^3 + 30*b*c^4*d^2*x^4 + 30*a*c*d^2*x + 3*b*c*d^2*x - 32*b*c^5*d
^2*x^5*log(x) + 20*b*c^2*d^2*x^2*atanh(c*x) + 16*b*c^5*d^2*x^5*log(c^2*x^2
- 1) + 30*b*c*d^2*x*atanh(c*x) - 30*b*c^4*d^2*x^5*atan((c^2*x)/(-c^2)^(1/2)
)*(-c^2)^(1/2))/(60*x^5)
```

sympy [A] time = 2.56, size = 199, normalized size = 1.24

$$\begin{cases}-\frac{ac^2d^2}{3x^3}-\frac{acd^2}{2x^4}-\frac{ad^2}{5x^5}+\frac{8bc^5d^2\log(x)}{15}-\frac{8bc^5d^2\log\left(x-\frac{1}{c}\right)}{15}-\frac{bc^5d^2\,\mathrm{atanh}\,(cx)}{30}-\frac{bc^4d^2}{2x}-\frac{4bc^3d^2}{15x^2}-\frac{bc^2d^2\,\mathrm{atanh}\,(cx)}{3x^3}-\frac{bc^2d^2}{6x^3}-\frac{bcd^2\,a}{} \\ -\frac{ad^2}{5x^5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**2*d**2/(3*x**3) - a*c*d**2/(2*x**4) - a*d**2/(5*x**5) + 8*
b*c**5*d**2*log(x)/15 - 8*b*c**5*d**2*log(x - 1/c)/15 - b*c**5*d**2*atanh(c
*x)/30 - b*c**4*d**2/(2*x) - 4*b*c**3*d**2/(15*x**2) - b*c**2*d**2*atanh(c*
x)/(3*x**3) - b*c**2*d**2/(6*x**3) - b*c*d**2*atanh(c*x)/(2*x**4) - b*c*d**
2/(20*x**4) - b*d**2*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**2/(5*x**5), Tru
e))
```

3.20 $\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=192

$$\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] 3/4*b*d^3*x/c^3+13/35*b*d^3*x^2/c^2+1/4*b*d^3*x^3/c+13/70*b*d^3*x^4+1/10*b*
c*d^3*x^5+1/42*b*c^2*d^3*x^6+1/4*d^3*x^4*(a+b*arctanh(c*x))+3/5*c*d^3*x^5*(
a+b*arctanh(c*x))+1/2*c^2*d^3*x^6*(a+b*arctanh(c*x))+1/7*c^3*d^3*x^7*(a+b*a
rctanh(c*x))+209/280*b*d^3*ln(-c*x+1)/c^4-1/280*b*d^3*ln(c*x+1)/c^4
```

Rubi [A] time = 0.18, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(4*c^3) + (13*b*d^3*x^2)/(35*c^2) + (b*d^3*x^3)/(4*c) + (13*b*d
^3*x^4)/70 + (b*c*d^3*x^5)/10 + (b*c^2*d^3*x^6)/42 + (d^3*x^4*(a + b*ArcTan
h[c*x]))/4 + (3*c*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^3*x^6*(a + b*Arc
Tanh[c*x]))/2 + (c^3*d^3*x^7*(a + b*ArcTanh[c*x]))/7 + (209*b*d^3*Log[1 - c
*x])/(280*c^4) - (b*d^3*Log[1 + c*x])/(280*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\right.$$

$$=\frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x$$

$$=\frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x$$

$$=\frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x$$

Mathematica [A] time = 0.14, size = 151, normalized size = 0.79

$$\frac{d^3\left(120ac^7x^7+420ac^6x^6+504ac^5x^5+210ac^4x^4+20bc^6x^6+84bc^5x^5+156bc^4x^4+210bc^3x^3+312bc^2x^2+6bc^4\right.}{840c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(630*b*c*x + 312*b*c^2*x^2 + 210*b*c^3*x^3 + 210*a*c^4*x^4 + 156*b*c^4
*x^4 + 504*a*c^5*x^5 + 84*b*c^5*x^5 + 420*a*c^6*x^6 + 20*b*c^6*x^6 + 120*a*
c^7*x^7 + 6*b*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*x]
+ 627*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(840*c^4)
```

fricas [A] time = 0.44, size = 190, normalized size = 0.99

$$\frac{120\,ac^7d^3x^7+20\,(21\,a+b)c^6d^3x^6+84\,(6\,a+b)c^5d^3x^5+6\,(35\,a+26\,b)c^4d^3x^4+210\,bc^3d^3x^3+312\,bc^2d^3x^2+630}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/840*(120*a*c^7*d^3*x^7 + 20*(21*a + b)*c^6*d^3*x^6 + 84*(6*a + b)*c^5*d^3
*x^5 + 6*(35*a + 26*b)*c^4*d^3*x^4 + 210*b*c^3*d^3*x^3 + 312*b*c^2*d^3*x^2
+ 630*b*c*d^3*x - 3*b*d^3*log(c*x + 1) + 627*b*d^3*log(c*x - 1) + 3*(20*b*c
^7*d^3*x^7 + 70*b*c^6*d^3*x^6 + 84*b*c^5*d^3*x^5 + 35*b*c^4*d^3*x^4)*log(-(
c*x + 1)/(c*x - 1)))/c^4
```

giac [B] time = 0.27, size = 722, normalized size = 3.76

$$\frac{1}{105}c\left(\frac{6\left(\frac{140\,(cx+1)^6bd^3}{(cx-1)^6}-\frac{210\,(cx+1)^5bd^3}{(cx-1)^5}+\frac{490\,(cx+1)^4bd^3}{(cx-1)^4}-\frac{455\,(cx+1)^3bd^3}{(cx-1)^3}+\frac{273\,(cx+1)^2bd^3}{(cx-1)^2}-\frac{91\,(cx+1)bd^3}{cx-1}+13\,bd^3\right)\log\left(-\frac{cx+}{cx-}\right.}{\frac{(cx+1)^7c^5}{(cx-1)^7}-\frac{7\,(cx+1)^6c^5}{(cx-1)^6}+\frac{21\,(cx+1)^5c^5}{(cx-1)^5}-\frac{35\,(cx+1)^4c^5}{(cx-1)^4}+\frac{35\,(cx+1)^3c^5}{(cx-1)^3}-\frac{21\,(cx+1)^2c^5}{(cx-1)^2}+\frac{7\,(cx+1)c^5}{cx-1}-c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/105*c*(6*(140*(c*x + 1)^6*b*d^3/(c*x - 1)^6 - 210*(c*x + 1)^5*b*d^3/(c*x
- 1)^5 + 490*(c*x + 1)^4*b*d^3/(c*x - 1)^4 - 455*(c*x + 1)^3*b*d^3/(c*x - 1
)^3 + 273*(c*x + 1)^2*b*d^3/(c*x - 1)^2 - 91*(c*x + 1)*b*d^3/(c*x - 1) + 13
*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^7*c^5/(c*x - 1)^7 - 7*(c*x + 1
)^6*c^5/(c*x - 1)^6 + 21*(c*x + 1)^5*c^5/(c*x - 1)^5 - 35*(c*x + 1)^4*c^5/(
c*x - 1)^4 + 35*(c*x + 1)^3*c^5/(c*x - 1)^3 - 21*(c*x + 1)^2*c^5/(c*x - 1)^
2 + 7*(c*x + 1)*c^5/(c*x - 1) - c^5) + (1680*(c*x + 1)^6*a*d^3/(c*x - 1)^6
- 2520*(c*x + 1)^5*a*d^3/(c*x - 1)^5 + 5880*(c*x + 1)^4*a*d^3/(c*x - 1)^4 -
 5460*(c*x + 1)^3*a*d^3/(c*x - 1)^3 + 3276*(c*x + 1)^2*a*d^3/(c*x - 1)^2 -
1092*(c*x + 1)*a*d^3/(c*x - 1) + 156*a*d^3 + 762*(c*x + 1)^6*b*d^3/(c*x - 1
)^6 - 3063*(c*x + 1)^5*b*d^3/(c*x - 1)^5 + 5959*(c*x + 1)^4*b*d^3/(c*x - 1)
^4 - 6694*(c*x + 1)^3*b*d^3/(c*x - 1)^3 + 4344*(c*x + 1)^2*b*d^3/(c*x - 1)^
2 - 1539*(c*x + 1)*b*d^3/(c*x - 1) + 231*b*d^3)/((c*x + 1)^7*c^5/(c*x - 1)^
7 - 7*(c*x + 1)^6*c^5/(c*x - 1)^6 + 21*(c*x + 1)^5*c^5/(c*x - 1)^5 - 35*(c*
x + 1)^4*c^5/(c*x - 1)^4 + 35*(c*x + 1)^3*c^5/(c*x - 1)^3 - 21*(c*x + 1)^2*
c^5/(c*x - 1)^2 + 7*(c*x + 1)*c^5/(c*x - 1) - c^5) - 78*b*d^3*log(-(c*x + 1
)/(c*x - 1) + 1)/c^5 + 78*b*d^3*log(-(c*x + 1)/(c*x - 1))/c^5)
```

maple [A] time = 0.03, size = 199, normalized size = 1.04

$$\frac{c^3d^3a\,x^7}{7}+\frac{c^2d^3a\,x^6}{2}+\frac{3c\,d^3a\,x^5}{5}+\frac{d^3a\,x^4}{4}+\frac{c^3d^3b\,\mathrm{arctanh}\,(cx)\,x^7}{7}+\frac{c^2d^3b\,\mathrm{arctanh}\,(cx)\,x^6}{2}+\frac{3c\,d^3b\,\mathrm{arctanh}\,(cx)\,x^5}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/7*c^3*d^3*a*x^7+1/2*c^2*d^3*a*x^6+3/5*c*d^3*a*x^5+1/4*d^3*a*x^4+1/7*c^3*d
^3*b*arctanh(c*x)*x^7+1/2*c^2*d^3*b*arctanh(c*x)*x^6+3/5*c*d^3*b*arctanh(c*
x)*x^5+1/4*d^3*b*arctanh(c*x)*x^4+1/42*b*c^2*d^3*x^6+1/10*b*c*d^3*x^5+13/70
*b*d^3*x^4+1/4*b*d^3*x^3/c+13/35*b*d^3*x^2/c^2+3/4*b*d^3*x/c^3+209/280/c^4*
d^3*b*ln(c*x-1)-1/280*b*d^3*ln(c*x+1)/c^4
```

maxima [A] time = 0.32, size = 285, normalized size = 1.48

$$\frac{1}{7}\,ac^3d^3x^7+\frac{1}{2}\,ac^2d^3x^6+\frac{3}{5}\,acd^3x^5+\frac{1}{84}\left(12\,x^7\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,c^4x^6+3\,c^2x^4+6\,x^2}{c^6}+\frac{6\,\log\left(c^2x^2-1\right)}{c^8}\right)\right)bc^3d^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^3*d^3*x^7 + 1/2*a*c^2*d^3*x^6 + 3/5*a*c*d^3*x^5 + 1/84*(12*x^7*arct
anh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8)
)*b*c^3*d^3 + 1/4*a*d^3*x^4 + 1/60*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 +
 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*c^2*
d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
 1)/c^6))*b*c*d^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^3
```

mupad [B] time = 1.05, size = 177, normalized size = 0.92

$$\frac{\frac{13\,b\,c^2\,d^3\,x^2}{35}-\frac{d^3\,\left(315\,b\,\mathrm{atanh}(c\,x)-156\,b\,\ln\left(c^2\,x^2-1\right)\right)}{420}+\frac{b\,c^3\,d^3\,x^3}{4}+\frac{3\,b\,c\,d^3\,x}{4}}{c^4}+\frac{d^3\,\left(105\,a\,x^4+78\,b\,x^4+105\,b\,x^4\,\mathrm{atanh}\,(c\,x)\right)}{420}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

```
[Out] ((13*b*c^2*d^3*x^2)/35 - (d^3*(315*b*atanh(c*x) - 156*b*log(c^2*x^2 - 1)))/
420 + (b*c^3*d^3*x^3)/4 + (3*b*c*d^3*x)/4)/c^4 + (d^3*(105*a*x^4 + 78*b*x^4
 + 105*b*x^4*atanh(c*x)))/420 + (c^3*d^3*(60*a*x^7 + 60*b*x^7*atanh(c*x)))/
420 + (c*d^3*(252*a*x^5 + 42*b*x^5 + 252*b*x^5*atanh(c*x)))/420 + (c^2*d^3*
(210*a*x^6 + 10*b*x^6 + 210*b*x^6*atanh(c*x)))/420
```

sympy [A] time = 2.84, size = 243, normalized size = 1.27

$$\begin{cases} \frac{ac^3d^3x^7}{7} + \frac{ac^2d^3x^6}{2} + \frac{3acd^3x^5}{5} + \frac{ad^3x^4}{4} + \frac{bc^3d^3x^7 \operatorname{atanh}(cx)}{7} + \frac{bc^2d^3x^6 \operatorname{atanh}(cx)}{2} + \frac{bc^2d^3x^6}{42} + \frac{3bcd^3x^5 \operatorname{atanh}(cx)}{5} + \frac{bcd^3x^5}{10} + \frac{bd^3x^4 \operatorname{ata}}{4} \\ \frac{ad^3x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**7/7 + a*c**2*d**3*x**6/2 + 3*a*c*d**3*x**5/5 + a*
d**3*x**4/4 + b*c**3*d**3*x**7*atanh(c*x)/7 + b*c**2*d**3*x**6*atanh(c*x)/2
 + b*c**2*d**3*x**6/42 + 3*b*c*d**3*x**5*atanh(c*x)/5 + b*c*d**3*x**5/10 +
b*d**3*x**4*atanh(c*x)/4 + 13*b*d**3*x**4/70 + b*d**3*x**3/(4*c) + 13*b*d**
3*x**2/(35*c**2) + 3*b*d**3*x/(4*c**3) + 26*b*d**3*log(x - 1/c)/(35*c**4) -
 b*d**3*atanh(c*x)/(140*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

3.21 $\int x^2(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=178

$$\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right.$$

```
[Out] 11/12*b*d^3*x/c^2+7/15*b*d^3*x^2/c+11/36*b*d^3*x^3+3/20*b*c*d^3*x^4+1/30*b*
c^2*d^3*x^5+1/3*d^3*x^3*(a+b*arctanh(c*x))+3/4*c*d^3*x^4*(a+b*arctanh(c*x))
+3/5*c^2*d^3*x^5*(a+b*arctanh(c*x))+1/6*c^3*d^3*x^6*(a+b*arctanh(c*x))+37/4
0*b*d^3*ln(-c*x+1)/c^3+1/120*b*d^3*ln(c*x+1)/c^3
```

Rubi [A] time = 0.18, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (11*b*d^3*x)/(12*c^2) + (7*b*d^3*x^2)/(15*c) + (11*b*d^3*x^3)/36 + (3*b*c*d
^3*x^4)/20 + (b*c^2*d^3*x^5)/30 + (d^3*x^3*(a + b*ArcTanh[c*x]))/3 + (3*c*d
^3*x^4*(a + b*ArcTanh[c*x]))/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x]))/6 + (37*b*d^3*Log[1 - c*x])/(40*c^3) + (b
*d^3*Log[1 + c*x])/(120*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)dx=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\,\mathrm{ta}\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\,\mathrm{ta}\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\,\mathrm{ta}\right.$$

Mathematica [A] time = 0.13, size = 142, normalized size = 0.80

$$\frac{d^3\left(60ac^6x^6+216ac^5x^5+270ac^4x^4+120ac^3x^3+12bc^5x^5+54bc^4x^4+110bc^3x^3+168bc^2x^2+6bc^3x^3\left(10c^3x^3+\right.\right.}{360c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(330*b*c*x + 168*b*c^2*x^2 + 120*a*c^3*x^3 + 110*b*c^3*x^3 + 270*a*c^4
*x^4 + 54*b*c^4*x^4 + 216*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c^3
*x^3*(20 + 45*c*x + 36*c^2*x^2 + 10*c^3*x^3)*ArcTanh[c*x] + 333*b*Log[1 - c
*x] + 3*b*Log[1 + c*x]))/(360*c^3)
```

fricas [A] time = 0.44, size = 178, normalized size = 1.00

$$\frac{60\,ac^6d^3x^6+12\,(18\,a+b)c^5d^3x^5+54\,(5\,a+b)c^4d^3x^4+10\,(12\,a+11\,b)c^3d^3x^3+168\,bc^2d^3x^2+330\,bcd^3x+3\,bd^3}{360\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(60*a*c^6*d^3*x^6 + 12*(18*a + b)*c^5*d^3*x^5 + 54*(5*a + b)*c^4*d^3*
x^4 + 10*(12*a + 11*b)*c^3*d^3*x^3 + 168*b*c^2*d^3*x^2 + 330*b*c*d^3*x + 3*
b*d^3*log(c*x + 1) + 333*b*d^3*log(c*x - 1) + 3*(10*b*c^6*d^3*x^6 + 36*b*c^
5*d^3*x^5 + 45*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3)*log(-(c*x + 1)/(c*x - 1)))
/c^3
```

giac [B] time = 0.23, size = 621, normalized size = 3.49

$$-\frac{1}{45}c\left(\frac{42\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^4}-\frac{6\left(\frac{60\,(cx+1)^5bd^3}{(cx-1)^5}-\frac{90\,(cx+1)^4bd^3}{(cx-1)^4}+\frac{140\,(cx+1)^3bd^3}{(cx-1)^3}-\frac{105\,(cx+1)^2bd^3}{(cx-1)^2}+\frac{42\,(cx+1)bd^3}{cx-1}-7\,bd^3\right)\mathrm{lo}}{\frac{(cx+1)^6c^4}{(cx-1)^6}-\frac{6\,(cx+1)^5c^4}{(cx-1)^5}+\frac{15\,(cx+1)^4c^4}{(cx-1)^4}-\frac{20\,(cx+1)^3c^4}{(cx-1)^3}+\frac{15\,(cx+1)^2c^4}{(cx-1)^2}-\frac{6\,(cx+1)c^4}{cx-1}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/45*c*(42*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^4 - 6*(60*(c*x + 1)^5*b*d
^3/(c*x - 1)^5 - 90*(c*x + 1)^4*b*d^3/(c*x - 1)^4 + 140*(c*x + 1)^3*b*d^3/(
c*x - 1)^3 - 105*(c*x + 1)^2*b*d^3/(c*x - 1)^2 + 42*(c*x + 1)*b*d^3/(c*x -
1) - 7*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^4/(c*x - 1)^6 - 6*(c
*x + 1)^5*c^4/(c*x - 1)^5 + 15*(c*x + 1)^4*c^4/(c*x - 1)^4 - 20*(c*x + 1)^3
*c^4/(c*x - 1)^3 + 15*(c*x + 1)^2*c^4/(c*x - 1)^2 - 6*(c*x + 1)*c^4/(c*x -
1) + c^4) - 42*b*d^3*log(-(c*x + 1)/(c*x - 1))/c^4 - (720*(c*x + 1)^5*a*d^3
/(c*x - 1)^5 - 1080*(c*x + 1)^4*a*d^3/(c*x - 1)^4 + 1680*(c*x + 1)^3*a*d^3/
(c*x - 1)^3 - 1260*(c*x + 1)^2*a*d^3/(c*x - 1)^2 + 504*(c*x + 1)*a*d^3/(c*x
 - 1) - 84*a*d^3 + 318*(c*x + 1)^5*b*d^3/(c*x - 1)^5 - 1119*(c*x + 1)^4*b*d
^3/(c*x - 1)^4 + 1742*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 1464*(c*x + 1)^2*b*d^
3/(c*x - 1)^2 + 636*(c*x + 1)*b*d^3/(c*x - 1) - 113*b*d^3)/((c*x + 1)^6*c^4
/(c*x - 1)^6 - 6*(c*x + 1)^5*c^4/(c*x - 1)^5 + 15*(c*x + 1)^4*c^4/(c*x - 1)
^4 - 20*(c*x + 1)^3*c^4/(c*x - 1)^3 + 15*(c*x + 1)^2*c^4/(c*x - 1)^2 - 6*(c
*x + 1)*c^4/(c*x - 1) + c^4))
```

maple [A] time = 0.03, size = 187, normalized size = 1.05

$$\frac{c^3d^3a\,x^6}{6}+\frac{3c^2d^3a\,x^5}{5}+\frac{3c\,d^3a\,x^4}{4}+\frac{d^3a\,x^3}{3}+\frac{c^3d^3b\,\mathrm{arctanh}\,(cx)\,x^6}{6}+\frac{3c^2d^3b\,\mathrm{arctanh}\,(cx)\,x^5}{5}+\frac{3c\,d^3b\,\mathrm{arctanh}\,(cx)\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*c^3*d^3*a*x^6+3/5*c^2*d^3*a*x^5+3/4*c*d^3*a*x^4+1/3*d^3*a*x^3+1/6*c^3*d
^3*b*arctanh(c*x)*x^6+3/5*c^2*d^3*b*arctanh(c*x)*x^5+3/4*c*d^3*b*arctanh(c*
x)*x^4+1/3*d^3*b*arctanh(c*x)*x^3+1/30*b*c^2*d^3*x^5+3/20*b*c*d^3*x^4+11/36
*b*d^3*x^3+7/15*b*d^3*x^2/c+11/12*b*d^3*x/c^2+37/40/c^3*d^3*b*ln(c*x-1)+1/1
20*b*d^3*ln(c*x+1)/c^3
```

maxima [A] time = 0.32, size = 265, normalized size = 1.49

$$\frac{1}{6}\,ac^3d^3x^6+\frac{3}{5}\,ac^2d^3x^5+\frac{3}{4}\,acd^3x^4+\frac{1}{180}\left(30\,x^6\,\mathrm{artanh}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^3*d^3*x^6 + 3/5*a*c^2*d^3*x^5 + 3/4*a*c*d^3*x^4 + 1/180*(30*x^6*arc
tanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 +
 15*log(c*x - 1)/c^7))*b*c^3*d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 +
 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/8*(6*x
^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
 - 1)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
1)/c^4))*b*d^3
```

mupad [B] time = 1.03, size = 165, normalized size = 0.93

$$\frac{\frac{7\,b\,c^2\,d^3\,x^2}{15}-\frac{d^3\,\left(165\,b\,\mathrm{atanh}(c\,x)-84\,b\,\ln\left(c^2\,x^2-1\right)\right)}{180}+\frac{11\,b\,c\,d^3\,x}{12}}{c^3}+\frac{d^3\,\left(60\,a\,x^3+55\,b\,x^3+60\,b\,x^3\,\mathrm{atanh}\,(c\,x)\right)}{180}+\frac{c^3\,d^3\,\left(30\,a\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

[Out] ((7*b*c^2*d^3*x^2)/15 - (d^3*(165*b*atanh(c*x) - 84*b*log(c^2*x^2 - 1)))/18
0 + (11*b*c*d^3*x)/12)/c^3 + (d^3*(60*a*x^3 + 55*b*x^3 + 60*b*x^3*atanh(c*x
)))/180 + (c^3*d^3*(30*a*x^6 + 30*b*x^6*atanh(c*x)))/180 + (c*d^3*(135*a*x^
4 + 27*b*x^4 + 135*b*x^4*atanh(c*x)))/180 + (c^2*d^3*(108*a*x^5 + 6*b*x^5 +
 108*b*x^5*atanh(c*x)))/180

sympy [A] time = 2.29, size = 235, normalized size = 1.32

$$\begin{cases} \frac{ac^3d^3x^6}{6} + \frac{3ac^2d^3x^5}{5} + \frac{3acd^3x^4}{4} + \frac{ad^3x^3}{3} + \frac{bc^3d^3x^6 \operatorname{atanh}(cx)}{6} + \frac{3bc^2d^3x^5 \operatorname{atanh}(cx)}{5} + \frac{bc^2d^3x^5}{30} + \frac{3bcd^3x^4 \operatorname{atanh}(cx)}{4} + \frac{3bcd^3x^4}{20} + \frac{bd^3x^3}{} \\ \frac{ad^3x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**6/6 + 3*a*c**2*d**3*x**5/5 + 3*a*c*d**3*x**4/4 +
a*d**3*x**3/3 + b*c**3*d**3*x**6*atanh(c*x)/6 + 3*b*c**2*d**3*x**5*atanh(c*
x)/5 + b*c**2*d**3*x**5/30 + 3*b*c*d**3*x**4*atanh(c*x)/4 + 3*b*c*d**3*x**4
/20 + b*d**3*x**3*atanh(c*x)/3 + 11*b*d**3*x**3/36 + 7*b*d**3*x**2/(15*c) +
 11*b*d**3*x/(12*c**2) + 14*b*d**3*log(x - 1/c)/(15*c**3) + b*d**3*atanh(c*
x)/(60*c**3), Ne(c, 0)), (a*d**3*x**3/3, True))
```

3.22 $\int x(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=135

$$\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{bd^3(cx+1)^4}{20c^2}+\frac{bd^3(cx+1)^3}{20c^2}+\frac{3bd^3(cx+1)^2}{20c^2}+$$

```
[Out] 3/5*b*d^3*x/c+3/20*b*d^3*(c*x+1)^2/c^2+1/20*b*d^3*(c*x+1)^3/c^2+1/20*b*d^3*
(c*x+1)^4/c^2-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))/c^2+1/5*d^3*(c*x+1)^5*(a
+b*arctanh(c*x))/c^2+6/5*b*d^3*ln(-c*x+1)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 77}

$$\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{bd^3(cx+1)^4}{20c^2}+\frac{bd^3(cx+1)^3}{20c^2}+\frac{3bd^3(cx+1)^2}{20c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(5*c) + (3*b*d^3*(1 + c*x)^2)/(20*c^2) + (b*d^3*(1 + c*x)^3)/(2
0*c^2) + (b*d^3*(1 + c*x)^4)/(20*c^2) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x
]))/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (6*b*d^3*Log
[1 - c*x])/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)dx=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-(bc)$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{b\int(}{}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{b\int(}{}$$

$$=\frac{3bd^3x}{5c}+\frac{3bd^3(1+cx)^2}{20c^2}+\frac{bd^3(1+cx)^3}{20c^2}+\frac{bd^3(1+cx)^4}{20c^2}-\frac{d^3(1+cx)^4(a+}{4c^2}$$

Mathematica [A] time = 0.11, size = 133, normalized size = 0.99

$$\frac{d^3\left(8ac^5x^5+30ac^4x^4+40ac^3x^3+20ac^2x^2+2bc^4x^4+10bc^3x^3+24bc^2x^2+2bc^2x^2\left(4c^3x^3+15c^2x^2+20cx+10\right)\right.}{40c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(50*b*c*x + 20*a*c^2*x^2 + 24*b*c^2*x^2 + 40*a*c^3*x^3 + 10*b*c^3*x^3
+ 30*a*c^4*x^4 + 2*b*c^4*x^4 + 8*a*c^5*x^5 + 2*b*c^2*x^2*(10 + 20*c*x + 15*
c^2*x^2 + 4*c^3*x^3)*ArcTanh[c*x] + 49*b*Log[1 - c*x] - b*Log[1 + c*x]))/(4
0*c^2)
```

fricas [A] time = 0.49, size = 165, normalized size = 1.22

$$\frac{8\,ac^5d^3x^5+2\,(15\,a+b)c^4d^3x^4+10\,(4\,a+b)c^3d^3x^3+4\,(5\,a+6\,b)c^2d^3x^2+50\,bcd^3x-bd^3\log(cx+1)+49\,bd^3\log}{40\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*d^3*x^5 + 2*(15*a + b)*c^4*d^3*x^4 + 10*(4*a + b)*c^3*d^3*x^3
 + 4*(5*a + 6*b)*c^2*d^3*x^2 + 50*b*c*d^3*x - b*d^3*log(c*x + 1) + 49*b*d^3
*log(c*x - 1) + (4*b*c^5*d^3*x^5 + 15*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3 + 10
*b*c^2*d^3*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

giac [B] time = 0.52, size = 527, normalized size = 3.90

$$-\frac{1}{5}\left(\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3}-\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^3}-\frac{2\left(\frac{20\,(cx+1)^4bd^3}{(cx-1)^4}-\frac{30\,(cx+1)^3bd^3}{(cx-1)^3}+\frac{30\,(cx+1)^2bd^3}{(cx-1)^2}-\frac{15\,(cx+1)bd^3}{cx-1}+3\,bd^3\right)}{\frac{(cx+1)^5c^3}{(cx-1)^5}-\frac{5\,(cx+1)^4c^3}{(cx-1)^4}+\frac{10\,(cx+1)^3c^3}{(cx-1)^3}-\frac{10\,(cx+1)^2c^3}{(cx-1)^2}+\frac{5\,(cx+1)c^3}{cx-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/5*(6*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 6*b*d^3*log(-(c*x + 1)/(c
*x - 1))/c^3 - 2*(20*(c*x + 1)^4*b*d^3/(c*x - 1)^4 - 30*(c*x + 1)^3*b*d^3/(
c*x - 1)^3 + 30*(c*x + 1)^2*b*d^3/(c*x - 1)^2 - 15*(c*x + 1)*b*d^3/(c*x - 1
) + 3*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^3/(c*x - 1)^5 - 5*(c*
x + 1)^4*c^3/(c*x - 1)^4 + 10*(c*x + 1)^3*c^3/(c*x - 1)^3 - 10*(c*x + 1)^2*
c^3/(c*x - 1)^2 + 5*(c*x + 1)*c^3/(c*x - 1) - c^3) - (80*(c*x + 1)^4*a*d^3/
(c*x - 1)^4 - 120*(c*x + 1)^3*a*d^3/(c*x - 1)^3 + 120*(c*x + 1)^2*a*d^3/(c*
x - 1)^2 - 60*(c*x + 1)*a*d^3/(c*x - 1) + 12*a*d^3 + 34*(c*x + 1)^4*b*d^3/(
```

```
c*x - 1)^4 - 103*(c*x + 1)^3*b*d^3/(c*x - 1)^3 + 123*(c*x + 1)^2*b*d^3/(c*x
 - 1)^2 - 69*(c*x + 1)*b*d^3/(c*x - 1) + 15*b*d^3)/((c*x + 1)^5*c^3/(c*x -
1)^5 - 5*(c*x + 1)^4*c^3/(c*x - 1)^4 + 10*(c*x + 1)^3*c^3/(c*x - 1)^3 - 10*
(c*x + 1)^2*c^3/(c*x - 1)^2 + 5*(c*x + 1)*c^3/(c*x - 1) - c^3))*c
```

maple [A] time = 0.03, size = 173, normalized size = 1.28

$$\frac{c^3d^3a\,x^5}{5}+\frac{3c^2d^3a\,x^4}{4}+c\,d^3a\,x^3+\frac{d^3a\,x^2}{2}+\frac{c^3d^3b\operatorname{arctanh}(cx)\,x^5}{5}+\frac{3c^2d^3b\operatorname{arctanh}(cx)\,x^4}{4}+c\,d^3b\operatorname{arctanh}(cx)\,x^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c^3*d^3*a*x^5+3/4*c^2*d^3*a*x^4+c*d^3*a*x^3+1/2*d^3*a*x^2+1/5*c^3*d^3*b
*arctanh(c*x)*x^5+3/4*c^2*d^3*b*arctanh(c*x)*x^4+c*d^3*b*arctanh(c*x)*x^3+1
/2*d^3*b*arctanh(c*x)*x^2+1/20*c^2*d^3*b*x^4+1/4*c*d^3*b*x^3+3/5*d^3*b*x^2+
5/4*b*d^3*x/c+49/40/c^2*d^3*b*ln(c*x-1)-1/40/c^2*d^3*b*ln(c*x+1)
```

maxima [B] time = 0.32, size = 244, normalized size = 1.81

$$\frac{1}{5}\,ac^3d^3x^5+\frac{3}{4}\,ac^2d^3x^4+\frac{1}{20}\left(4\,x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bc^3d^3+acd^3x^3+\frac{1}{8}\left(6\,x^4\operatorname{artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^3*d^3*x^5 + 3/4*a*c^2*d^3*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*
x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^3 + a*c*d^3*x^3 + 1/8*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^2*d^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x
^2 - 1)/c^4))*b*c*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^
2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^3
```

mupad [B] time = 0.98, size = 153, normalized size = 1.13

$$\frac{d^3\,\left(10\,a\,x^2+12\,b\,x^2+10\,b\,x^2\,\operatorname{atanh}(c\,x)\right)}{20}-\frac{\frac{d^3\,\left(25\,b\,\operatorname{atanh}(c\,x)-12\,b\,\ln\left(c^2\,x^2-1\right)\right)}{20}-\frac{5\,b\,c\,d^3\,x}{4}}{c^2}+\frac{c^3\,d^3\,\left(4\,a\,x^5+4\,b\,x^5\,\operatorname{atan}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

```
[Out] (d^3*(10*a*x^2 + 12*b*x^2 + 10*b*x^2*atanh(c*x)))/20 - ((d^3*(25*b*atanh(c*
x) - 12*b*log(c^2*x^2 - 1)))/20 - (5*b*c*d^3*x)/4)/c^2 + (c^3*d^3*(4*a*x^5
+ 4*b*x^5*atanh(c*x)))/20 + (c*d^3*(20*a*x^3 + 5*b*x^3 + 20*b*x^3*atanh(c*x
)))/20 + (c^2*d^3*(15*a*x^4 + b*x^4 + 15*b*x^4*atanh(c*x)))/20
```

sympy [A] time = 1.79, size = 211, normalized size = 1.56

$$\begin{cases} \frac{ac^3d^3x^5}{5}+\frac{3ac^2d^3x^4}{4}+acd^3x^3+\frac{ad^3x^2}{2}+\frac{bc^3d^3x^5\operatorname{atanh}(cx)}{5}+\frac{3bc^2d^3x^4\operatorname{atanh}(cx)}{4}+\frac{bc^2d^3x^4}{20}+bcd^3x^3\operatorname{atanh}(cx)+\frac{bcd^3x^3}{4}+ \\ \frac{ad^3x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**5/5 + 3*a*c**2*d**3*x**4/4 + a*c*d**3*x**3 + a*d*
*3*x**2/2 + b*c**3*d**3*x**5*atanh(c*x)/5 + 3*b*c**2*d**3*x**4*atanh(c*x)/4
 + b*c**2*d**3*x**4/20 + b*c*d**3*x**3*atanh(c*x) + b*c*d**3*x**3/4 + b*d**
3*x**2*atanh(c*x)/2 + 3*b*d**3*x**2/5 + 5*b*d**3*x/(4*c) + 6*b*d**3*log(x -
 1/c)/(5*c**2) - b*d**3*atanh(c*x)/(20*c**2), Ne(c, 0)), (a*d**3*x**2/2, Tr
ue))
```

3.23 $\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=84

$$\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}+\frac{bd^3(cx+1)^3}{12c}+\frac{bd^3(cx+1)^2}{4c}+\frac{2bd^3\log(1-cx)}{c}+bd^3x$$

```
[Out] b*d^3*x+1/4*b*d^3*(c*x+1)^2/c+1/12*b*d^3*(c*x+1)^3/c+1/4*d^3*(c*x+1)^4*(a+b
*arctanh(c*x))/c+2*b*d^3*ln(-c*x+1)/c
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5926, 627, 43}

$$\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}+\frac{bd^3(cx+1)^3}{12c}+\frac{bd^3(cx+1)^2}{4c}+\frac{2bd^3\log(1-cx)}{c}+bd^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] b*d^3*x + (b*d^3*(1 + c*x)^2)/(4*c) + (b*d^3*(1 + c*x)^3)/(12*c) + (d^3*(1
+ c*x)^4*(a + b*ArcTanh[c*x]))/(4*c) + (2*b*d^3*Log[1 - c*x])/c
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}-\frac{b\int \frac{(d+cdx)^4}{1-c^2x^2}\,dx}{4d}\\
&= \frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}-\frac{b\int \frac{(d+cdx)^3}{\frac{1}{d}-\frac{cx}{d}}\,dx}{4d}\\
&= \frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}-\frac{b\int \left(-4d^4+\frac{8d^3}{\frac{1}{d}-\frac{cx}{d}}-2d^3(d+cdx)-d^2(\right.}{4d}\\
&= bd^3x+\frac{bd^3(1+cx)^2}{4c}+\frac{bd^3(1+cx)^3}{12c}+\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}+
\end{aligned}$$

Mathematica [A] time = 0.14, size = 115, normalized size = 1.37

$$\frac{d^3\left(6ac^4x^4+24ac^3x^3+36ac^2x^2+24acx+2bc^3x^3+12bc^2x^2+6bcx\left(c^3x^3+4c^2x^2+6cx+4\right)\tanh^{-1}(cx)+42bc\right.}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(24*a*c*x + 42*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*c^3*x^3 + 2*
b*c^3*x^3 + 6*a*c^4*x^4 + 6*b*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3)*ArcTanh
[c*x] + 45*b*Log[1 - c*x] + 3*b*Log[1 + c*x]))/(24*c)
```

fricas [A] time = 0.49, size = 149, normalized size = 1.77

$$\frac{6\,ac^4d^3x^4+2\,(12\,a+b)c^3d^3x^3+12\,(3\,a+b)c^2d^3x^2+6\,(4\,a+7\,b)cd^3x+3\,bd^3\log\left(cx+1\right)+45\,bd^3\log\left(cx-1\right)+}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d^3*x^4 + 2*(12*a + b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^2
 + 6*(4*a + 7*b)*c*d^3*x + 3*b*d^3*log(c*x + 1) + 45*b*d^3*log(c*x - 1) + 3
*(b*c^4*d^3*x^4 + 4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c*d^3*x)*log(-(c*
x + 1)/(c*x - 1)))/c
```

giac [B] time = 0.22, size = 425, normalized size = 5.06

$$-\frac{1}{3}\left(\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{6\left(\frac{4\,(cx+1)^3bd^3}{(cx-1)^3}-\frac{6\,(cx+1)^2bd^3}{(cx-1)^2}+\frac{4\,(cx+1)bd^3}{cx-1}-bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^2}{(cx-1)^4}-\frac{4\,(cx+1)^3c^2}{(cx-1)^3}+\frac{6\,(cx+1)^2c^2}{(cx-1)^2}-\frac{4\,(cx+1)c^2}{cx-1}+c^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/3*(6*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 6*b*d^3*log(-(c*x + 1)/(c
*x - 1))/c^2 - 6*(4*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 6*(c*x + 1)^2*b*d^3/(c*
x - 1)^2 + 4*(c*x + 1)*b*d^3/(c*x - 1) - b*d^3)*log(-(c*x + 1)/(c*x - 1))/(
(c*x + 1)^4*c^2/(c*x - 1)^4 - 4*(c*x + 1)^3*c^2/(c*x - 1)^3 + 6*(c*x + 1)^2
*c^2/(c*x - 1)^2 - 4*(c*x + 1)*c^2/(c*x - 1) + c^2) - (48*(c*x + 1)^3*a*d^3
/(c*x - 1)^3 - 72*(c*x + 1)^2*a*d^3/(c*x - 1)^2 + 48*(c*x + 1)*a*d^3/(c*x -
 1) - 12*a*d^3 + 18*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 45*(c*x + 1)^2*b*d^3/(c
*x - 1)^2 + 38*(c*x + 1)*b*d^3/(c*x - 1) - 11*b*d^3)/((c*x + 1)^4*c^2/(c*x
- 1)^4 - 4*(c*x + 1)^3*c^2/(c*x - 1)^3 + 6*(c*x + 1)^2*c^2/(c*x - 1)^2 - 4*
(c*x + 1)*c^2/(c*x - 1) + c^2))*c
```

maple [B] time = 0.03, size = 162, normalized size = 1.93

$$\frac{c^3d^3a\,x^4}{4}+c^2d^3a\,x^3+\frac{3c\,d^3a\,x^2}{2}+ax\,d^3+\frac{d^3a}{4c}+\frac{c^3d^3b\operatorname{arctanh}\left(cx\right)x^4}{4}+c^2d^3b\operatorname{arctanh}\left(cx\right)x^3+\frac{3c\,d^3b\operatorname{arctanh}\left(cx\right)x^2}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*c^3*d^3*a*x^4+c^2*d^3*a*x^3+3/2*c*d^3*a*x^2+a*x*d^3+1/4/c*d^3*a+1/4*c^3
*d^3*b*arctanh(c*x)*x^4+c^2*d^3*b*arctanh(c*x)*x^3+3/2*c*d^3*b*arctanh(c*x)
*x^2+d^3*b*arctanh(c*x)*x+1/4/c*d^3*b*arctanh(c*x)+1/12*c^2*d^3*b*x^3+1/2*c
*d^3*b*x^2+7/4*b*d^3*x+2/c*d^3*b*ln(c*x-1)
```

maxima [B] time = 0.32, size = 219, normalized size = 2.61

$$\frac{1}{4}\,ac^3d^3x^4+ac^2d^3x^3+\frac{1}{24}\left(6\,x^4\,\operatorname{artanh}\left(cx\right)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bc^3d^3+\frac{1}{2}\left(2\,x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^3*d^3*x^4 + a*c^2*d^3*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^
3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^3*d^3 + 1/2*(2
*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + 3/2*a*c
*d^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*
x - 1)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1
))*b*d^3/c
```

mupad [B] time = 0.96, size = 136, normalized size = 1.62

$$\frac{d^3\,\left(12\,a\,x+21\,b\,x+12\,b\,x\,\mathrm{atanh}\left(c\,x\right)\right)}{12}+\frac{c^3\,d^3\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atanh}\left(c\,x\right)\right)}{12}-\frac{d^3\,\left(21\,b\,\mathrm{atanh}\left(c\,x\right)-12\,b\,\ln\left(c^2\,x^2\right.\right.}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

```
[Out] (d^3*(12*a*x + 21*b*x + 12*b*x*atanh(c*x)))/12 + (c^3*d^3*(3*a*x^4 + 3*b*x^
4*atanh(c*x)))/12 - (d^3*(21*b*atanh(c*x) - 12*b*log(c^2*x^2 - 1)))/(12*c)
+ (c*d^3*(18*a*x^2 + 6*b*x^2 + 18*b*x^2*atanh(c*x)))/12 + (c^2*d^3*(12*a*x^
3 + b*x^3 + 12*b*x^3*atanh(c*x)))/12
```

sympy [A] time = 1.31, size = 182, normalized size = 2.17

$$\begin{cases} \frac{ac^3d^3x^4}{4} + ac^2d^3x^3 + \frac{3acd^3x^2}{2} + ad^3x + \frac{bc^3d^3x^4\operatorname{atanh}\left(cx\right)}{4} + bc^2d^3x^3\operatorname{atanh}\left(cx\right) + \frac{bc^2d^3x^3}{12} + \frac{3bcd^3x^2\operatorname{atanh}\left(cx\right)}{2} + \frac{bcd^3x^2}{2} + \\ ad^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**4/4 + a*c**2*d**3*x**3 + 3*a*c*d**3*x**2/2 + a*d*
*3*x + b*c**3*d**3*x**4*atanh(c*x)/4 + b*c**2*d**3*x**3*atanh(c*x) + b*c**2
*d**3*x**3/12 + 3*b*c*d**3*x**2*atanh(c*x)/2 + b*c*d**3*x**2/2 + b*d**3*x*a
tanh(c*x) + 7*b*d**3*x/4 + 2*b*d**3*log(x - 1/c)/c + b*d**3*atanh(c*x)/(4*c
), Ne(c, 0)), (a*d**3*x, True))
```

3.24 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=152

$$\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+3acd^3x+ad^3\log(x)+\frac{1}{6}bc^2d^3x^2+\frac{5}{3}bd^3\log\left(1-c^2x^2\right)-\frac{1}{2}l$$

```
[Out] 3*a*c*d^3*x+3/2*b*c*d^3*x+1/6*b*c^2*d^3*x^2-3/2*b*d^3*arctanh(c*x)+3*b*c*d^
3*x*arctanh(c*x)+3/2*c^2*d^3*x^2*(a+b*arctanh(c*x))+1/3*c^3*d^3*x^3*(a+b*ar
ctanh(c*x))+a*d^3*ln(x)+5/3*b*d^3*ln(-c^2*x^2+1)-1/2*b*d^3*polylog(2,-c*x)+
1/2*b*d^3*polylog(2,c*x)
```

Rubi [A] time = 0.17, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5940, 5910, 260, 5912, 5916, 321, 206, 266, 43}

$$-\frac{1}{2}bd^3\text{PolyLog}(2,-cx)+\frac{1}{2}bd^3\text{PolyLog}(2,cx)+\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+3acd^3x+$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 3*a*c*d^3*x + (3*b*c*d^3*x)/2 + (b*c^2*d^3*x^2)/6 - (3*b*d^3*ArcTanh[c*x])/
2 + 3*b*c*d^3*x*ArcTanh[c*x] + (3*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/2 + (c^
3*d^3*x^3*(a + b*ArcTanh[c*x]))/3 + a*d^3*Log[x] + (5*b*d^3*Log[1 - c^2*x^2
])/3 - (b*d^3*PolyLog[2, -(c*x)])/2 + (b*d^3*PolyLog[2, c*x])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(3cd^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3c^2d^3x\left(a+b\tan\right.\right.$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(3cd^3\right)\int \left(a+b\tanh^{-1}(cx)\right)dx+\left(3c^2d^3\right)$$

$$= 3acd^3x+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+a$$

$$= 3acd^3x+\frac{3}{2}bcd^3x+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+$$

$$= 3acd^3x+\frac{3}{2}bcd^3x-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2\left(a\right.$$

$$= 3acd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}bc^2d^3x^2-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)+$$

Mathematica [A] time = 0.13, size = 148, normalized size = 0.97

$$\frac{1}{12}d^3\left(4ac^3x^3+18ac^2x^2+36acx+12a\log(x)+4bc^3x^3\tanh^{-1}(cx)+2bc^2x^2+18b\log\left(1-c^2x^2\right)+2b\log\left(c^2x\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^3*(36*a*c*x + 18*b*c*x + 18*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 36*b
*c*x*ArcTanh[c*x] + 18*b*c^2*x^2*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x] +
12*a*Log[x] + 9*b*Log[1 - c*x] - 9*b*Log[1 + c*x] + 18*b*Log[1 - c^2*x^2] +
 2*b*Log[-1 + c^2*x^2] - 6*b*PolyLog[2, -(c*x)] + 6*b*PolyLog[2, c*x]))/12
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^3d^3x^3 + 3\,ac^2d^3x^2 + 3\,acd^3x + ad^3 + \left(bc^3d^3x^3 + 3\,bc^2d^3x^2 + 3\,bcd^3x + bd^3\right)\text{artanh}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3 (b\,\text{artanh}\left(cx\right) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x, x)
```

maple [A] time = 0.05, size = 182, normalized size = 1.20

$$\frac{d^3a\,c^3x^3}{3}+\frac{3d^3a\,c^2x^2}{2}+3ac\,d^3x+d^3a\ln\left(cx\right)+\frac{d^3b\,\text{arctanh}\left(cx\right)c^3x^3}{3}+\frac{3d^3b\,\text{arctanh}\left(cx\right)c^2x^2}{2}+3bc\,d^3x\,\text{arctanh}\left(cx\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x)
```

```
[Out] 1/3*d^3*a*c^3*x^3+3/2*d^3*a*c^2*x^2+3*a*c*d^3*x+d^3*a*ln(c*x)+1/3*d^3*b*arc
tanh(c*x)*c^3*x^3+3/2*d^3*b*arctanh(c*x)*c^2*x^2+3*b*c*d^3*x*arctanh(c*x)+d
^3*b*arctanh(c*x)*ln(c*x)-1/2*d^3*b*dilog(c*x)-1/2*d^3*b*dilog(c*x+1)-1/2*d
^3*b*ln(c*x)*ln(c*x+1)+1/6*b*c^2*d^3*x^2+3/2*b*c*d^3*x+29/12*d^3*b*ln(c*x-1
)+11/12*d^3*b*ln(c*x+1)
```

maxima [A] time = 0.46, size = 228, normalized size = 1.50

$$\frac{1}{3}\,ac^3d^3x^3+\frac{3}{2}\,ac^2d^3x^2+\frac{1}{6}\,bc^2d^3x^2+3\,acd^3x+\frac{3}{2}\,bcd^3x+\frac{3}{2}\left(2\,cx\,\text{artanh}\left(cx\right)+\log\left(-c^2x^2+1\right)\right)bd^3-\frac{1}{2}\left(\log\left(cx\right)\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^3*d^3*x^3 + 3/2*a*c^2*d^3*x^2 + 1/6*b*c^2*d^3*x^2 + 3*a*c*d^3*x + 3
/2*b*c*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d^3 - 1/2*(lo
g(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^3 + 1/2*(log(c*x + 1)*log(-c*x)
 + dilog(c*x + 1))*b*d^3 - 7/12*b*d^3*log(c*x + 1) + 11/12*b*d^3*log(c*x -
1) + a*d^3*log(x) + 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(c*x + 1) -
 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(-c*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\text{atanh}\left(c\,x\right))\,\left(d + c\,d\,x\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3ac\,dx + \int \frac{a}{x}\,dx + \int 3ac^2x\,dx + \int ac^3x^2\,dx + \int 3bc\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x}\,dx + \int 3bc^2x\operatorname{at}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**3*(Integral(3*a*c, x) + Integral(a/x, x) + Integral(3*a*c**2*x, x) + Int
egral(a*c**3*x**2, x) + Integral(3*b*c*atanh(c*x), x) + Integral(b*atanh(c*
x)/x, x) + Integral(3*b*c**2*x*atanh(c*x), x) + Integral(b*c**3*x**2*atanh(
c*x), x))
```

3.25 $\int \frac{(d+cdx)^3(a+b\tanh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=150

$$\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3ac^2d^3x+3acd^3\log(x)+bcd^3\log\left(1-c^2x^2\right)+\frac{1}{2}bc^2d^3x+3bc^2d^3$$

[Out] 3*a*c^2*d^3*x+1/2*b*c^2*d^3*x-1/2*b*c*d^3*arctanh(c*x)+3*b*c^2*d^3*x*arctan h(c*x)-d^3*(a+b*arctanh(c*x))/x+1/2*c^3*d^3*x^2*(a+b*arctanh(c*x))+3*a*c*d^ 3*ln(x)+b*c*d^3*ln(x)+b*c*d^3*ln(-c^2*x^2+1)-3/2*b*c*d^3*polylog(2,-c*x)+3/ 2*b*c*d^3*polylog(2,c*x)

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5940, 5910, 260, 5916, 266, 36, 29, 31, 5912, 321, 206}

$$-\frac{3}{2}bcd^3\text{PolyLog}(2,-cx)+\frac{3}{2}bcd^3\text{PolyLog}(2,cx)+\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3ac^2d^3x+3a$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] 3*a*c^2*d^3*x + (b*c^2*d^3*x)/2 - (b*c*d^3*ArcTanh[c*x])/2 + 3*b*c^2*d^3*x*
ArcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/x + (c^3*d^3*x^2*(a + b*ArcTanh[c
*x]))/2 + 3*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 - c^2*x^2] - (3
*b*c*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c*d^3*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(3c^2d^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{3cd^3\left(a+b\tan\right.}{x}\right.$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(3cd^3\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(3c^2d^3\right)\int$$

$$= 3ac^2d^3x-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)+3acd^3$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\tanh^{-1}(cx)+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+\right.}{}$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\tanh^{-1}(cx)+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+\right.}{}$$

Mathematica [A] time = 0.14, size = 149, normalized size = 0.99

$$\frac{d^3\left(2ac^3x^3+12ac^2x^2+12acx\log(x)-4a+2bc^3x^3\tanh^{-1}(cx)+2bc^2x^2+4bcx\log\left(1-c^2x^2\right)+12bc^2x^2\tanh\right.}{4x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^3*(-4*a + 12*a*c^2*x^2 + 2*b*c^2*x^2 + 2*a*c^3*x^3 - 4*b*ArcTanh[c*x] +
12*b*c^2*x^2*ArcTanh[c*x] + 2*b*c^3*x^3*ArcTanh[c*x] + 12*a*c*x*Log[x] + 4*
b*c*x*Log[c*x] + b*c*x*Log[1 - c*x] - b*c*x*Log[1 + c*x] + 4*b*c*x*Log[1 -
c^2*x^2] - 6*b*c*x*PolyLog[2, -(c*x)] + 6*b*c*x*PolyLog[2, c*x]))/(4*x)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^3d^3x^3 + 3\,ac^2d^3x^2 + 3\,acd^3x + ad^3 + \left(bc^3d^3x^3 + 3\,bc^2d^3x^2 + 3\,bcd^3x + bd^3\right)\text{artanh}\left(cx\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3(b\,\text{artanh}\left(cx\right) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^2, x)
```

maple [A] time = 0.05, size = 189, normalized size = 1.26

$$\frac{d^3a\,c^3x^2}{2} + 3d^3a\,c^2x + 3c\,d^3a\ln\left(cx\right) - \frac{d^3a}{x} + \frac{d^3b\,\text{arctanh}\left(cx\right)c^3x^2}{2} + 3b\,c^2d^3x\,\text{arctanh}\left(cx\right) + 3c\,d^3b\,\text{arctanh}\left(cx\right)\ln\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] 1/2*d^3*a*c^3*x^2+3*d^3*a*c^2*x+3*c*d^3*a*ln(c*x)-d^3*a/x+1/2*d^3*b*arctanh
(c*x)*c^3*x^2+3*b*c^2*d^3*x*arctanh(c*x)+3*c*d^3*b*arctanh(c*x)*ln(c*x)-d^3
*b*arctanh(c*x)/x-3/2*c*d^3*b*dilog(c*x)-3/2*c*d^3*b*dilog(c*x+1)-3/2*c*d^3
*b*ln(c*x)*ln(c*x+1)+1/2*b*c^2*d^3*x+c*d^3*b*ln(c*x)+5/4*c*d^3*b*ln(c*x-1)+
3/4*c*d^3*b*ln(c*x+1)
```

maxima [A] time = 0.49, size = 229, normalized size = 1.53

$$\frac{1}{4}\,bc^3d^3x^2\log\left(cx+1\right) - \frac{1}{4}\,bc^3d^3x^2\log\left(-cx+1\right) + \frac{1}{2}\,ac^3d^3x^2 + 3\,ac^2d^3x + \frac{1}{2}\,bc^2d^3x + \frac{3}{2}\left(2\,cx\,\text{artanh}\left(cx\right) + \log\left(-c^2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^3*d^3*x^2*log(c*x + 1) - 1/4*b*c^3*d^3*x^2*log(-c*x + 1) + 1/2*a*c^
3*d^3*x^2 + 3*a*c^2*d^3*x + 1/2*b*c^2*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log
(-c^2*x^2 + 1))*b*c*d^3 - 3/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*
c*d^3 + 3/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c*d^3 - 1/4*b*c*d^3
*log(c*x + 1) + 1/4*b*c*d^3*log(c*x - 1) + 3*a*c*d^3*log(x) - 1/2*(c*(log(c
^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^3 - a*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c x)) (d + c d x)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3 \left(\int 3ac^2\, dx + \int \frac{a}{x^2}\, dx + \int \frac{3ac}{x}\, dx + \int ac^3x\, dx + \int 3bc^2 \operatorname{atanh}(cx)\, dx + \int \frac{b \operatorname{atanh}(cx)}{x^2}\, dx + \int \frac{3bc \operatorname{ata}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c/x, x) +
Integral(a*c**3*x, x) + Integral(3*b*c**2*atanh(c*x), x) + Integral(b*atanh
(c*x)/x**2, x) + Integral(3*b*c*atanh(c*x)/x, x) + Integral(b*c**3*x*atanh(
c*x), x))
```

3.26 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=160

$$-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^3d^3x+3ac^2d^3\log(x)+bc^3d^3x\tanh^{-1}(cx)-\frac{3}{2}bc^2d^3\text{Li}_2(-cx)+\frac{3}{2}$$

```
[Out] -1/2*b*c*d^3/x+a*c^3*d^3*x+1/2*b*c^2*d^3*arctanh(c*x)+b*c^3*d^3*x*arctanh(c
*x)-1/2*d^3*(a+b*arctanh(c*x))/x^2-3*c*d^3*(a+b*arctanh(c*x))/x+3*a*c^2*d^3
*ln(x)+3*b*c^2*d^3*ln(x)-b*c^2*d^3*ln(-c^2*x^2+1)-3/2*b*c^2*d^3*polylog(2,-
c*x)+3/2*b*c^2*d^3*polylog(2,c*x)
```

Rubi [A] time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5940, 5910, 260, 5916, 325, 206, 266, 36, 29, 31, 5912}

$$-\frac{3}{2}bc^2d^3\text{PolyLog}(2,-cx)+\frac{3}{2}bc^2d^3\text{PolyLog}(2,cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^3d^3x+3ac^2$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^3)/(2*x) + a*c^3*d^3*x + (b*c^2*d^3*ArcTanh[c*x])/2 + b*c^3*d^3*x*A
rcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[
c*x]))/x + 3*a*c^2*d^3*Log[x] + 3*b*c^2*d^3*Log[x] - b*c^2*d^3*Log[1 - c^2*
x^2] - (3*b*c^2*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c^2*d^3*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int \left(c^3d^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}+\frac{3cd^3\left(a+b\tanh\right.}{x^2}\right.$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(3cd^3\right)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(3c^2d^3\right)\int$$

$$= ac^3d^3x-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3ac^2d^3\log$$

$$= -\frac{bcd^3}{2x}+ac^3d^3x+bc^3d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a\right.}{}$$

$$= -\frac{bcd^3}{2x}+ac^3d^3x+\frac{1}{2}bc^2d^3\tanh^{-1}(cx)+bc^3d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\,\mathrm{t}\right.}{2}$$

$$= -\frac{bcd^3}{2x}+ac^3d^3x+\frac{1}{2}bc^2d^3\tanh^{-1}(cx)+bc^3d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\,\mathrm{t}\right.}{2}$$

Mathematica [A] time = 0.14, size = 165, normalized size = 1.03

$$d^3\left(4ac^3x^3+12ac^2x^2\log(x)-12acx-2a+4bc^3x^3\tanh^{-1}(cx)-6bc^2x^2\mathrm{Li}_2(-cx)+6bc^2x^2\mathrm{Li}_2(cx)+12bc^2x^2\,\mathrm{lo}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^3*(-2*a - 12*a*c*x - 2*b*c*x + 4*a*c^3*x^3 - 2*b*ArcTanh[c*x] - 12*b*c*x
*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x] + 12*a*c^2*x^2*Log[x] + 12*b*c^2*x
^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x] - 4*b*c^2*x^2
*Log[1 - c^2*x^2] - 6*b*c^2*x^2*PolyLog[2, -(c*x)] + 6*b*c^2*x^2*PolyLog[2,
 c*x]))/(4*x^2)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^3d^3x^3 + 3\,ac^2d^3x^2 + 3\,acd^3x + ad^3 + \left(bc^3d^3x^3 + 3\,bc^2d^3x^2 + 3\,bcd^3x + bd^3\right)\text{artanh}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3(b\,\text{artanh}\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^3, x)
```

maple [A] time = 0.06, size = 200, normalized size = 1.25

$$a\,c^3d^3x+3c^2d^3a\ln(cx)-\frac{3c\,d^3a}{x}-\frac{d^3a}{2x^2}+b\,c^3d^3x\,\text{arctanh}\,(cx)+3c^2d^3b\,\text{arctanh}\,(cx)\ln(cx)-\frac{3c\,d^3b\,\text{arctanh}\,(cx)}{x}-\frac{d^3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] a*c^3*d^3*x+3*c^2*d^3*a*ln(c*x)-3*c*d^3*a/x-1/2*d^3*a/x^2+b*c^3*d^3*x*arcta
nh(c*x)+3*c^2*d^3*b*arctanh(c*x)*ln(c*x)-3*c*d^3*b*arctanh(c*x)/x-1/2*d^3*b
*arctanh(c*x)/x^2-3/2*c^2*d^3*b*dilog(c*x)-3/2*c^2*d^3*b*dilog(c*x+1)-3/2*c
^2*d^3*b*ln(c*x)*ln(c*x+1)-1/2*b*c*d^3/x+3*c^2*d^3*b*ln(c*x)-5/4*c^2*d^3*b*
ln(c*x-1)-3/4*c^2*d^3*b*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$ac^3d^3x+\frac{1}{2}\left(2\,cx\,\text{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)bc^2d^3+\frac{3}{2}\,bc^2d^3\int\frac{\log(cx+1) - \log(-cx+1)}{x}\,dx+3\,ac^2d^3\log(x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] a*c^3*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^2*d^3 + 3/2*
b*c^2*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 3*a*c^2*d^3*log(
x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^3 + 1/4
*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^3 - 3
*a*c*d^3/x - 1/2*a*d^3/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))\,\,(d + c\,d\,x)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int ac^3\,dx + \int \frac{a}{x^3}\,dx + \int \frac{3ac}{x^2}\,dx + \int \frac{3ac^2}{x}\,dx + \int bc^3\,\mathrm{atanh}\,(cx)\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x^3}\,dx + \int \frac{3bc\,\mathrm{atanh}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**3*(Integral(a*c**3, x) + Integral(a/x**3, x) + Integral(3*a*c/x**2, x) +
 Integral(3*a*c**2/x, x) + Integral(b*c**3*atanh(c*x), x) + Integral(b*atan
h(c*x)/x**3, x) + Integral(3*b*c*atanh(c*x)/x**2, x) + Integral(3*b*c**2*at
anh(c*x)/x, x))
```

3.27 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+ac^3d^3\log(x)-\frac{1}{2}bc^3d^3\text{Li}_2(-cx)+\frac{1}{2}bc^3$$

```
[Out] -1/6*b*c*d^3/x^2-3/2*b*c^2*d^3/x+3/2*b*c^3*d^3*arctanh(c*x)-1/3*d^3*(a+b*ar
ctanh(c*x))/x^3-3/2*c*d^3*(a+b*arctanh(c*x))/x^2-3*c^2*d^3*(a+b*arctanh(c*x
))/x+a*c^3*d^3*ln(x)+10/3*b*c^3*d^3*ln(x)-5/3*b*c^3*d^3*ln(-c^2*x^2+1)-1/2*
b*c^3*d^3*polylog(2,-c*x)+1/2*b*c^3*d^3*polylog(2,c*x)
```

Rubi [A] time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5940, 5916, 266, 44, 325, 206, 36, 29, 31, 5912}

$$-\frac{1}{2}bc^3d^3\text{PolyLog}(2,-cx)+\frac{1}{2}bc^3d^3\text{PolyLog}(2,cx)-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d^3\left(a+b\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^3)/(6*x^2) - (3*b*c^2*d^3)/(2*x) + (3*b*c^3*d^3*ArcTanh[c*x])/2 - (
d^3*(a + b*ArcTanh[c*x]))/(3*x^3) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(2*x^2)
- (3*c^2*d^3*(a + b*ArcTanh[c*x]))/x + a*c^3*d^3*Log[x] + (10*b*c^3*d^3*Log
[x])/3 - (5*b*c^3*d^3*Log[1 - c^2*x^2])/3 - (b*c^3*d^3*PolyLog[2, -(c*x)])/
2 + (b*c^3*d^3*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = \int\left(\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^4}+\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}+\frac{3c^2d^3\left(a+b\tan\right.}{x^2}\right.$$

$$= d^3\int\frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\left(3cd^3\right)\int\frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(3c^2d^3\right)\int$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3c^2d^3\left(a+b\tanh^{-}\right.}{x}$$

$$= -\frac{3bc^2d^3}{2x}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3c^2d^3\left(a\right.}{}$$

$$= -\frac{3bc^2d^3}{2x}+\frac{3}{2}bc^3d^3\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tan\right.}{2x^2}$$

$$= -\frac{bcd^3}{6x^2}-\frac{3bc^2d^3}{2x}+\frac{3}{2}bc^3d^3\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(\right.}{}$$

Mathematica [A] time = 0.13, size = 175, normalized size = 0.99

$$d^3\left(12ac^3x^3\log(x)-36ac^2x^2-18acx-4a-6bc^3x^3\mathrm{Li}_2(-cx)+6bc^3x^3\mathrm{Li}_2(cx)+40bc^3x^3\log(cx)-9bc^3x^3\log(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-4*a - 18*a*c*x - 2*b*c*x - 36*a*c^2*x^2 - 18*b*c^2*x^2 - 4*b*ArcTanh
[c*x] - 18*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 12*a*c^3*x^3*Lo
g[x] + 40*b*c^3*x^3*Log[c*x] - 9*b*c^3*x^3*Log[1 - c*x] + 9*b*c^3*x^3*Log[1
 + c*x] - 20*b*c^3*x^3*Log[1 - c^2*x^2] - 6*b*c^3*x^3*PolyLog[2, -(c*x)] +
6*b*c^3*x^3*PolyLog[2, c*x]))/(12*x^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^3d^3x^3 + 3\,ac^2d^3x^2 + 3\,acd^3x + ad^3 + \left(bc^3d^3x^3 + 3\,bc^2d^3x^2 + 3\,bcd^3x + bd^3\right)\text{artanh}\left(cx\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3(b\,\text{artanh}\left(cx\right) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^4, x)
```

maple [A] time = 0.06, size = 216, normalized size = 1.23

$$c^3d^3a\ln\left(cx\right) - \frac{3c^2d^3a}{x} - \frac{a\,d^3}{3x^3} - \frac{3c\,d^3a}{2x^2} + c^3d^3b\,\text{arctanh}\left(cx\right)\ln\left(cx\right) - \frac{3c^2d^3b\,\text{arctanh}\left(cx\right)}{x} - \frac{d^3b\,\text{arctanh}\left(cx\right)}{3x^3} - \frac{3c\,d^3b\,\text{ar}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] c^3*d^3*a*ln(c*x)-3*c^2*d^3*a/x-1/3*a*d^3/x^3-3/2*c*d^3*a/x^2+c^3*d^3*b*arc
tanh(c*x)*ln(c*x)-3*c^2*d^3*b*arctanh(c*x)/x-1/3*d^3*b*arctanh(c*x)/x^3-3/2
*c*d^3*b*arctanh(c*x)/x^2-1/6*b*c*d^3/x^2-3/2*b*c^2*d^3/x+10/3*c^3*d^3*b*ln
(c*x)-29/12*c^3*d^3*b*ln(c*x-1)-11/12*c^3*d^3*b*ln(c*x+1)-1/2*c^3*d^3*b*dil
og(c*x)-1/2*c^3*d^3*b*dilog(c*x+1)-1/2*c^3*d^3*b*ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,bc^3d^3\int \frac{\log\left(cx+1\right) - \log\left(-cx+1\right)}{x}\,dx + ac^3d^3\log(x) - \frac{3}{2}\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\text{artanh}\left(cx\right)}{x}\right)bc^2d^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^3*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^3*d^3*lo
g(x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^3 +
 3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d
^3 - 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/
x^3)*b*d^3 - 3*a*c^2*d^3/x - 3/2*a*c*d^3/x^2 - 1/3*a*d^3/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}\,(c\,x))\,\left(d+c\,d\,x\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a}{x^4}\,dx+\int \frac{3ac}{x^3}\,dx+\int \frac{3ac^2}{x^2}\,dx+\int \frac{ac^3}{x}\,dx+\int \frac{b\,\mathrm{atanh}\,(cx)}{x^4}\,dx+\int \frac{3bc\,\mathrm{atanh}\,(cx)}{x^3}\,dx+\int \frac{3bc^2\,\mathrm{ata}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(a/x**4, x) + Integral(3*a*c/x**3, x) + Integral(3*a*c**2/x**
2, x) + Integral(a*c**3/x, x) + Integral(b*atanh(c*x)/x**4, x) + Integral(3
*b*c*atanh(c*x)/x**3, x) + Integral(3*b*c**2*atanh(c*x)/x**2, x) + Integral
(b*c**3*atanh(c*x)/x, x))
```

3.28 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=93

$$-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}+2bc^4d^3\log(x)-2bc^4d^3\log(1-cx)-\frac{7bc^3d^3}{4x}-\frac{bc^2d^3}{2x^2}-\frac{bcd^3}{12x^3}$$

```
[Out] -1/12*b*c*d^3/x^3-1/2*b*c^2*d^3/x^2-7/4*b*c^3*d^3/x-1/4*d^3*(c*x+1)^4*(a+b*
arctanh(c*x))/x^4+2*b*c^4*d^3*ln(x)-2*b*c^4*d^3*ln(-c*x+1)
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{37, 5936, 12, 88\}$

$$-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d^3}{2x^2}-\frac{7bc^3d^3}{4x}+2bc^4d^3\log(x)-2bc^4d^3\log(1-cx)-\frac{bcd^3}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^3)/(12*x^3) - (b*c^2*d^3)/(2*x^2) - (7*b*c^3*d^3)/(4*x) - (d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(4*x^4) + 2*b*c^4*d^3*Log[x] - 2*b*c^4*d^3*Lo
g[1 - c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - (bc)\int \frac{(d+cdx)^3}{4x^4(-1+cx)}\,dx$$
$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{1}{4}(bc)\int \frac{(d+cdx)^3}{x^4(-1+cx)}\,dx$$
$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{1}{4}(bc)\int \left(-\frac{d^3}{x^4} - \frac{4cd^3}{x^3} - \frac{7c^2d^3}{x^2} - \frac{8c^3}{x}\right.$$
$$= -\frac{bcd^3}{12x^3} - \frac{bc^2d^3}{2x^2} - \frac{7bc^3d^3}{4x} - \frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} + 2bc^4d^3\log$$

Mathematica [A] time = 0.12, size = 131, normalized size = 1.41

$$-\frac{d^3\left(24ac^3x^3+36ac^2x^2+24acx+6a-48bc^4x^4\log(x)+45bc^4x^4\log(1-cx)+3bc^4x^4\log(cx+1)+42bc^3x^3\right.}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/24*(d^3*(6*a + 24*a*c*x + 2*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*c
^3*x^3 + 42*b*c^3*x^3 + 6*b*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3)*ArcTanh[c*x
] - 48*b*c^4*x^4*Log[x] + 45*b*c^4*x^4*Log[1 - c*x] + 3*b*c^4*x^4*Log[1 + c
*x]))/x^4
```

fricas [A] time = 0.42, size = 163, normalized size = 1.75

$$-\frac{3\,bc^4d^3x^4\log\left(cx+1\right)+45\,bc^4d^3x^4\log\left(cx-1\right)-48\,bc^4d^3x^4\log(x)+6\,(4\,a+7\,b)c^3d^3x^3+12\,(3\,a+b)c^2d^3x^2}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*b*c^4*d^3*x^4*log(c*x + 1) + 45*b*c^4*d^3*x^4*log(c*x - 1) - 48*b*
c^4*d^3*x^4*log(x) + 6*(4*a + 7*b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^2 +
 2*(12*a + b)*c*d^3*x + 6*a*d^3 + 3*(4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*
b*c*d^3*x + b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

giac [B] time = 0.24, size = 431, normalized size = 4.63

$$\frac{1}{3}\left(6\,bc^3d^3\log\left(-\frac{cx+1}{cx-1}-1\right)-6\,bc^3d^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{6\left(\frac{4\,(cx+1)^3bc^3d^3}{(cx-1)^3}+\frac{6\,(cx+1)^2bc^3d^3}{(cx-1)^2}+\frac{4\,(cx+1)bc^3d^3}{cx-1}+bc^3d^3\right)\log}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(6*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 6*b*c^3*d^3*log(-(c*x + 1)
/(c*x - 1)) + 6*(4*(c*x + 1)^3*b*c^3*d^3/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^3*
d^3/(c*x - 1)^2 + 4*(c*x + 1)*b*c^3*d^3/(c*x - 1) + b*c^3*d^3)*log(-(c*x +
1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x
 + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (48*(c*x + 1)^3*a*c^3*d^
3/(c*x - 1)^3 + 72*(c*x + 1)^2*a*c^3*d^3/(c*x - 1)^2 + 48*(c*x + 1)*a*c^3*d
^3/(c*x - 1) + 12*a*c^3*d^3 + 18*(c*x + 1)^3*b*c^3*d^3/(c*x - 1)^3 + 45*(c*
x + 1)^2*b*c^3*d^3/(c*x - 1)^2 + 38*(c*x + 1)*b*c^3*d^3/(c*x - 1) + 11*b*c^
```

```
3*d^3)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2
/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [B] time = 0.04, size = 181, normalized size = 1.95

$$-\frac{c^3d^3a}{x}-\frac{c\,d^3a}{x^3}-\frac{3c^2d^3a}{2x^2}-\frac{d^3a}{4x^4}-\frac{c^3d^3b\operatorname{arctanh}(cx)}{x}-\frac{c\,d^3b\operatorname{arctanh}(cx)}{x^3}-\frac{3c^2d^3b\operatorname{arctanh}(cx)}{2x^2}-\frac{d^3b\operatorname{arctanh}(cx)}{4x^4}-\frac{bc}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -c^3*d^3*a/x-c*d^3*a/x^3-3/2*c^2*d^3*a/x^2-1/4*d^3*a/x^4-c^3*d^3*b*arctanh(
c*x)/x-c*d^3*b*arctanh(c*x)/x^3-3/2*c^2*d^3*b*arctanh(c*x)/x^2-1/4*d^3*b*ar
ctanh(c*x)/x^4-1/12*b*c*d^3/x^3-1/2*b*c^2*d^3/x^2-7/4*b*c^3*d^3/x+2*c^4*d^3
*b*ln(c*x)-15/8*c^4*d^3*b*ln(c*x-1)-1/8*c^4*d^3*b*ln(c*x+1)
```

maxima [B] time = 0.33, size = 228, normalized size = 2.45

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\operatorname{artanh}(cx)}{x}\right)bc^3d^3+\frac{3}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\operatorname{artanh}(cx}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^3 + 3/4*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*d^3 -
 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c*d^3 - a*c^3*d^3/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2
*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^3 - 3/2*a*c^2*d^3/x^2 - a
*c*d^3/x^3 - 1/4*a*d^3/x^4
```

mupad [B] time = 0.95, size = 147, normalized size = 1.58

$$\frac{d^3\,\left(21\,b\,c^4\operatorname{atanh}(c\,x)-12\,b\,c^4\,\ln\left(c^2\,x^2-1\right)+24\,b\,c^4\,\ln(x)\right)}{12}-\frac{\frac{d^3\,(3\,a+3\,b\operatorname{atanh}(c\,x))}{12}+\frac{d^3\,x\,(12\,a\,c+b\,c+12\,b\,c\operatorname{atanh}(c\,x))}{12}+\frac{d^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^5,x)
```

```
[Out] (d^3*(21*b*c^4*atanh(c*x) - 12*b*c^4*log(c^2*x^2 - 1) + 24*b*c^4*log(x)))/1
2 - ((d^3*(3*a + 3*b*atanh(c*x)))/12 + (d^3*x*(12*a*c + b*c + 12*b*c*atanh(
c*x)))/12 + (d^3*x^2*(18*a*c^2 + 6*b*c^2 + 18*b*c^2*atanh(c*x)))/12 + (d^3*
x^3*(12*a*c^3 + 21*b*c^3 + 12*b*c^3*atanh(c*x)))/12)/x^4
```

sympy [A] time = 2.02, size = 207, normalized size = 2.23

$$\begin{cases}-\frac{ac^3d^3}{x}-\frac{3ac^2d^3}{2x^2}-\frac{acd^3}{x^3}-\frac{ad^3}{4x^4}+2bc^4d^3\log(x)-2bc^4d^3\log\left(x-\frac{1}{c}\right)-\frac{bc^4d^3\operatorname{atanh}(cx)}{4}-\frac{bc^3d^3\operatorname{atanh}(cx)}{x}-\frac{7bc^3d^3}{4x}-\frac{3bc^2d^3}{2} \\ -\frac{ad^3}{4x^4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c**3*d**3/x - 3*a*c**2*d**3/(2*x**2) - a*c*d**3/x**3 - a*d**3
/(4*x**4) + 2*b*c**4*d**3*log(x) - 2*b*c**4*d**3*log(x - 1/c) - b*c**4*d**3
*atanh(c*x)/4 - b*c**3*d**3*atanh(c*x)/x - 7*b*c**3*d**3/(4*x) - 3*b*c**2*d
**3*atanh(c*x)/(2*x**2) - b*c**2*d**3/(2*x**2) - b*c*d**3*atanh(c*x)/x**3 -
 b*c*d**3/(12*x**3) - b*d**3*atanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**3/(4*x*
*4), True))
```

3.29 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=137

$$-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\log(1-cx)-\frac{5bc^4d^3}{4x}-$$

```
[Out] -1/20*b*c*d^3/x^4-1/4*b*c^2*d^3/x^3-3/5*b*c^3*d^3/x^2-5/4*b*c^4*d^3/x-1/5*d
^3*(c*x+1)^4*(a+b*arctanh(c*x))/x^5+1/20*c*d^3*(c*x+1)^4*(a+b*arctanh(c*x))
/x^4+6/5*b*c^5*d^3*ln(x)-6/5*b*c^5*d^3*ln(-c*x+1)
```

Rubi [A] time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {45, 37, 5936, 12, 148}

$$\frac{cd^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{3bc^3d^3}{5x^2}-\frac{bc^2d^3}{4x^3}-\frac{5bc^4d^3}{4x}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^3)/(20*x^4) - (b*c^2*d^3)/(4*x^3) - (3*b*c^3*d^3)/(5*x^2) - (5*b*c^
4*d^3)/(4*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/(5*x^5) + (c*d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(20*x^4) + (6*b*c^5*d^3*Log[x])/5 - (6*b*c^5*
d^3*Log[1 - c*x])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
+ b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-(bc)$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-\frac{1}{20}(b$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-\frac{1}{20}(b$$

$$= -\frac{bcd^3}{20x^4}-\frac{bc^2d^3}{4x^3}-\frac{3bc^3d^3}{5x^2}-\frac{5bc^4d^3}{4x}-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3}{}$$

Mathematica [A] time = 0.13, size = 140, normalized size = 1.02

$$-\frac{d^3\left(20ac^3x^3+40ac^2x^2+30acx+8a-48bc^5x^5\log(x)+49bc^5x^5\log(1-cx)-bc^5x^5\log(cx+1)+50bc^4x^4+24\right.}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/40*(d^3*(8*a + 30*a*c*x + 2*b*c*x + 40*a*c^2*x^2 + 10*b*c^2*x^2 + 20*a*c
^3*x^3 + 24*b*c^3*x^3 + 50*b*c^4*x^4 + 2*b*(4 + 15*c*x + 20*c^2*x^2 + 10*c^
3*x^3)*ArcTanh[c*x] - 48*b*c^5*x^5*Log[x] + 49*b*c^5*x^5*Log[1 - c*x] - b*c
^5*x^5*Log[1 + c*x]))/x^5
```

fricas [A] time = 0.45, size = 175, normalized size = 1.28

$$\frac{bc^5d^3x^5\log(cx+1)-49\,bc^5d^3x^5\log(cx-1)+48\,bc^5d^3x^5\log(x)-50\,bc^4d^3x^4-4\,(5\,a+6\,b)c^3d^3x^3-10\,(4\,a+b)}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(b*c^5*d^3*x^5*log(c*x + 1) - 49*b*c^5*d^3*x^5*log(c*x - 1) + 48*b*c^5
*d^3*x^5*log(x) - 50*b*c^4*d^3*x^4 - 4*(5*a + 6*b)*c^3*d^3*x^3 - 10*(4*a +
b)*c^2*d^3*x^2 - 2*(15*a + b)*c*d^3*x - 8*a*d^3 - (10*b*c^3*d^3*x^3 + 20*b*
c^2*d^3*x^2 + 15*b*c*d^3*x + 4*b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

giac [B] time = 0.20, size = 533, normalized size = 3.89

$$\frac{1}{5}\left(6\,bc^4d^3\log\left(-\frac{cx+1}{cx-1}-1\right)-6\,bc^4d^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{2\left(\frac{20\,(cx+1)^4bc^4d^3}{(cx-1)^4}+\frac{30\,(cx+1)^3bc^4d^3}{(cx-1)^3}+\frac{30\,(cx+1)^2bc^4d^3}{(cx-1)^2}+\frac{15\,(cx+1)b}{cx-1}\right.}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/5*(6*b*c^4*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 6*b*c^4*d^3*log(-(c*x + 1)
/(c*x - 1)) + 2*(20*(c*x + 1)^4*b*c^4*d^3/(c*x - 1)^4 + 30*(c*x + 1)^3*b*c^
```

```
4*d^3/(c*x - 1)^3 + 30*(c*x + 1)^2*b*c^4*d^3/(c*x - 1)^2 + 15*(c*x + 1)*b*c
^4*d^3/(c*x - 1) + 3*b*c^4*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x
 - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x
+ 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (80*(c*x + 1)^4*a*c^4*d^3
/(c*x - 1)^4 + 120*(c*x + 1)^3*a*c^4*d^3/(c*x - 1)^3 + 120*(c*x + 1)^2*a*c^
4*d^3/(c*x - 1)^2 + 60*(c*x + 1)*a*c^4*d^3/(c*x - 1) + 12*a*c^4*d^3 + 34*(c
*x + 1)^4*b*c^4*d^3/(c*x - 1)^4 + 103*(c*x + 1)^3*b*c^4*d^3/(c*x - 1)^3 + 1
23*(c*x + 1)^2*b*c^4*d^3/(c*x - 1)^2 + 69*(c*x + 1)*b*c^4*d^3/(c*x - 1) + 1
5*b*c^4*d^3)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x
 + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) +
1))*c
```

maple [A] time = 0.04, size = 193, normalized size = 1.41

$$-\frac{c^2d^3a}{x^3}-\frac{c^3d^3a}{2x^2}-\frac{3c\,d^3a}{4x^4}-\frac{d^3a}{5x^5}-\frac{c^2d^3b\operatorname{arctanh}(cx)}{x^3}-\frac{c^3d^3b\operatorname{arctanh}(cx)}{2x^2}-\frac{3c\,d^3b\operatorname{arctanh}(cx)}{4x^4}-\frac{d^3b\operatorname{arctanh}(cx)}{5x^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -c^2*d^3*a/x^3-1/2*c^3*d^3*a/x^2-3/4*c*d^3*a/x^4-1/5*d^3*a/x^5-c^2*d^3*b*ar
ctanh(c*x)/x^3-1/2*c^3*d^3*b*arctanh(c*x)/x^2-3/4*c*d^3*b*arctanh(c*x)/x^4-
1/5*d^3*b*arctanh(c*x)/x^5-1/20*b*c*d^3/x^4-1/4*b*c^2*d^3/x^3-3/5*b*c^3*d^3
/x^2-5/4*b*c^4*d^3/x+6/5*c^5*d^3*b*ln(c*x)-49/40*c^5*d^3*b*ln(c*x-1)+1/40*c
^5*d^3*b*ln(c*x+1)
```

maxima [B] time = 0.33, size = 250, normalized size = 1.82

$$\frac{1}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\operatorname{artanh}(cx)}{x^2}\right)bc^3d^3-\frac{1}{2}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3*
d^3 - 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c^2*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*
x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^3 - 1/20*((2*c^4*log(c^2*x^2 -
1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^3 -
1/2*a*c^3*d^3/x^2 - a*c^2*d^3/x^3 - 3/4*a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

mupad [B] time = 0.98, size = 233, normalized size = 1.70

$$-\frac{4\,a\,d^3+4\,b\,d^3\operatorname{atanh}(c\,x)+20\,a\,c^2\,d^3\,x^2+10\,a\,c^3\,d^3\,x^3+10\,a\,c^5\,d^3\,x^5+5\,b\,c^2\,d^3\,x^2+12\,b\,c^3\,d^3\,x^3+25\,b\,c^4\,d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^6,x)
```

```
[Out] -(4*a*d^3 + 4*b*d^3*atanh(c*x) + 20*a*c^2*d^3*x^2 + 10*a*c^3*d^3*x^3 + 10*a
*c^5*d^3*x^5 + 5*b*c^2*d^3*x^2 + 12*b*c^3*d^3*x^3 + 25*b*c^4*d^3*x^4 + 12*b
*c^5*d^3*x^5 + 15*a*c*d^3*x + b*c*d^3*x - 24*b*c^5*d^3*x^5*log(x) + 20*b*c^
2*d^3*x^2*atanh(c*x) + 10*b*c^3*d^3*x^3*atanh(c*x) + 12*b*c^5*d^3*x^5*log(c
^2*x^2 - 1) + 15*b*c*d^3*x*atanh(c*x) - 25*b*c^4*d^3*x^5*atan((c^2*x)/(-c^2
)^(1/2))*(-c^2)^(1/2))/(20*x^5)
```

sympy [A] time = 2.62, size = 233, normalized size = 1.70

$$\begin{cases} -\frac{ac^3d^3}{2x^2} - \frac{ac^2d^3}{x^3} - \frac{3acd^3}{4x^4} - \frac{ad^3}{5x^5} + \frac{6bc^5d^3\log(x)}{5} - \frac{6bc^5d^3\log\left(x-\frac{1}{c}\right)}{5} + \frac{bc^5d^3\operatorname{atanh}(cx)}{20} - \frac{5bc^4d^3}{4x} - \frac{bc^3d^3\operatorname{atanh}(cx)}{2x^2} - \frac{3bc^3d^3}{5x^2} - \frac{bc^2d^3 a}{} \\ -\frac{ad^3}{5x^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**3*d**3/(2*x**2) - a*c**2*d**3/x**3 - 3*a*c*d**3/(4*x**4) -
 a*d**3/(5*x**5) + 6*b*c**5*d**3*log(x)/5 - 6*b*c**5*d**3*log(x - 1/c)/5 +
b*c**5*d**3*atanh(c*x)/20 - 5*b*c**4*d**3/(4*x) - b*c**3*d**3*atanh(c*x)/(2
*x**2) - 3*b*c**3*d**3/(5*x**2) - b*c**2*d**3*atanh(c*x)/x**3 - b*c**2*d**3
/(4*x**3) - 3*b*c*d**3*atanh(c*x)/(4*x**4) - b*c*d**3/(20*x**4) - b*d**3*at
anh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**3/(5*x**5), True))
```

3.30 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx$

Optimal. Leaf size=196

$$-\frac{c^3d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{14}{15}bc^6d^3$$

```
[Out] -1/30*b*c*d^3/x^5-3/20*b*c^2*d^3/x^4-11/36*b*c^3*d^3/x^3-7/15*b*c^4*d^3/x^2
-11/12*b*c^5*d^3/x-1/6*d^3*(a+b*arctanh(c*x))/x^6-3/5*c*d^3*(a+b*arctanh(c*
x))/x^5-3/4*c^2*d^3*(a+b*arctanh(c*x))/x^4-1/3*c^3*d^3*(a+b*arctanh(c*x))/x
^3+14/15*b*c^6*d^3*ln(x)-37/40*b*c^6*d^3*ln(-c*x+1)-1/120*b*c^6*d^3*ln(c*x+
1)
```

Rubi [A] time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^3d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{7bc^4d^3}{15x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -(b*c*d^3)/(30*x^5) - (3*b*c^2*d^3)/(20*x^4) - (11*b*c^3*d^3)/(36*x^3) - (7
*b*c^4*d^3)/(15*x^2) - (11*b*c^5*d^3)/(12*x) - (d^3*(a + b*ArcTanh[c*x]))/(
6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(5*x^5) - (3*c^2*d^3*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x]))/(3*x^3) + (14*b*c^6*d^3*Lo
g[x])/15 - (37*b*c^6*d^3*Log[1 - c*x])/40 - (b*c^6*d^3*Log[1 + c*x])/120
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx = -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{3c^2d^3\left(a+b\tanh^{-1}(c\right.}{4x^4}$$
$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{3c^2d^3\left(a+b\tanh^{-1}(c\right.}{4x^4}$$
$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{3c^2d^3\left(a+b\tanh^{-1}(c\right.}{4x^4}$$
$$= -\frac{bcd^3}{30x^5} - \frac{3bc^2d^3}{20x^4} - \frac{11bc^3d^3}{36x^3} - \frac{7bc^4d^3}{15x^2} - \frac{11bc^5d^3}{12x} - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6} -$$

Mathematica [**A**] time = 0.13, size = 149, normalized size = 0.76

$$-\frac{d^3\left(120ac^3x^3+270ac^2x^2+216acx+60a-336bc^6x^6\log(x)+333bc^6x^6\log(1-cx)+3bc^6x^6\log(cx+1)+330\right.}{360x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -1/360*(d^3*(60*a + 216*a*c*x + 12*b*c*x + 270*a*c^2*x^2 + 54*b*c^2*x^2 + 1
20*a*c^3*x^3 + 110*b*c^3*x^3 + 168*b*c^4*x^4 + 330*b*c^5*x^5 + 6*b*(10 + 36
*c*x + 45*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*x] - 336*b*c^6*x^6*Log[x] + 333*b
*c^6*x^6*Log[1 - c*x] + 3*b*c^6*x^6*Log[1 + c*x]))/x^6
```

fricas [**A**] time = 0.48, size = 188, normalized size = 0.96

$$-\frac{3\,bc^6d^3x^6\log(cx+1)+333\,bc^6d^3x^6\log(cx-1)-336\,bc^6d^3x^6\log(x)+330\,bc^5d^3x^5+168\,bc^4d^3x^4+10\,(12\,a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/360*(3*b*c^6*d^3*x^6*log(c*x + 1) + 333*b*c^6*d^3*x^6*log(c*x - 1) - 336
*b*c^6*d^3*x^6*log(x) + 330*b*c^5*d^3*x^5 + 168*b*c^4*d^3*x^4 + 10*(12*a +
11*b)*c^3*d^3*x^3 + 54*(5*a + b)*c^2*d^3*x^2 + 12*(18*a + b)*c*d^3*x + 60*a
*d^3 + 3*(20*b*c^3*d^3*x^3 + 45*b*c^2*d^3*x^2 + 36*b*c*d^3*x + 10*b*d^3)*lo
g(-(c*x + 1)/(c*x - 1)))/x^6
```

giac [**B**] time = 0.16, size = 634, normalized size = 3.23

$$\frac{1}{45}\left(42\,bc^5d^3\log\left(-\frac{cx+1}{cx-1}-1\right)-42\,bc^5d^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{6\left(\frac{60\,(cx+1)^5bc^5d^3}{(cx-1)^5}+\frac{90\,(cx+1)^4bc^5d^3}{(cx-1)^4}+\frac{140\,(cx+1)^3bc^5d^3}{(cx-1)^3}+\frac{105}{}\right.}{\frac{(cx+1)^6}{(cx-1)^6}+\frac{6\,(cx+1)^5}{(cx-1)^5}+\frac{15\,(cx+1)^4}{(cx-1)^4}+\frac{20\,(c}{(cx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")
```

```
[Out] 1/45*(42*b*c^5*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 42*b*c^5*d^3*log(-(c*x +
 1)/(c*x - 1)) + 6*(60*(c*x + 1)^5*b*c^5*d^3/(c*x - 1)^5 + 90*(c*x + 1)^4*b
*c^5*d^3/(c*x - 1)^4 + 140*(c*x + 1)^3*b*c^5*d^3/(c*x - 1)^3 + 105*(c*x + 1
)^2*b*c^5*d^3/(c*x - 1)^2 + 42*(c*x + 1)*b*c^5*d^3/(c*x - 1) + 7*b*c^5*d^3)
*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x -
1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x +
```

```
1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1) + (720*(c*x + 1)^5*a*c^5*d^3/
(c*x - 1)^5 + 1080*(c*x + 1)^4*a*c^5*d^3/(c*x - 1)^4 + 1680*(c*x + 1)^3*a*c
^5*d^3/(c*x - 1)^3 + 1260*(c*x + 1)^2*a*c^5*d^3/(c*x - 1)^2 + 504*(c*x + 1)
*a*c^5*d^3/(c*x - 1) + 84*a*c^5*d^3 + 318*(c*x + 1)^5*b*c^5*d^3/(c*x - 1)^5
 + 1119*(c*x + 1)^4*b*c^5*d^3/(c*x - 1)^4 + 1742*(c*x + 1)^3*b*c^5*d^3/(c*x
 - 1)^3 + 1464*(c*x + 1)^2*b*c^5*d^3/(c*x - 1)^2 + 636*(c*x + 1)*b*c^5*d^3/
(c*x - 1) + 113*b*c^5*d^3)/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x -
1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x +
1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 205, normalized size = 1.05

$$-\frac{c^3d^3a}{3x^3}-\frac{d^3a}{6x^6}-\frac{3c^2d^3a}{4x^4}-\frac{3c\,d^3a}{5x^5}-\frac{c^3d^3b\operatorname{arctanh}(cx)}{3x^3}-\frac{d^3b\operatorname{arctanh}(cx)}{6x^6}-\frac{3c^2d^3b\operatorname{arctanh}(cx)}{4x^4}-\frac{3c\,d^3b\operatorname{arctanh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x)
```

```
[Out] -1/3*c^3*d^3*a/x^3-1/6*d^3*a/x^6-3/4*c^2*d^3*a/x^4-3/5*c*d^3*a/x^5-1/3*c^3*
d^3*b*arctanh(c*x)/x^3-1/6*d^3*b*arctanh(c*x)/x^6-3/4*c^2*d^3*b*arctanh(c*x
)/x^4-3/5*c*d^3*b*arctanh(c*x)/x^5-1/30*b*c*d^3/x^5-3/20*b*c^2*d^3/x^4-11/3
6*b*c^3*d^3/x^3-7/15*b*c^4*d^3/x^2-11/12*b*c^5*d^3/x+14/15*c^6*d^3*b*ln(c*x
)-37/40*c^6*d^3*b*ln(c*x-1)-1/120*b*c^6*d^3*ln(c*x+1)
```

maxima [A] time = 0.33, size = 273, normalized size = 1.39

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\operatorname{artanh}(cx)}{x^3}\right)bc^3d^3+\frac{1}{8}\left(\left(3c^3\log(cx+1)-3c^3\log(cx-1)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^3*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^3 - 3/20*((2*c^4*log(c^2*x^2 - 1)
- 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c*d^3 + 1
/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x^
2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^3 - 1/3*a*c^3*d^3/x^3 - 3/4*a*c^2*
d^3/x^4 - 3/5*a*c*d^3/x^5 - 1/6*a*d^3/x^6
```

mupad [B] time = 1.07, size = 220, normalized size = 1.12

$$\frac{14\,b\,c^6\,d^3\,\ln(x)}{15}-\frac{7\,b\,c^6\,d^3\,\ln\left(c^2\,x^2-1\right)}{15}-\frac{3\,a\,c^2\,d^3}{4\,x^4}-\frac{a\,c^3\,d^3}{3\,x^3}-\frac{3\,b\,c^2\,d^3}{20\,x^4}-\frac{11\,b\,c^3\,d^3}{36\,x^3}-\frac{7\,b\,c^4\,d^3}{15\,x^2}-\frac{11\,b\,c^5\,d^3}{12\,x}-\frac{a\,d^3}{6\,x^6}-\frac{3\,a}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^7,x)
```

```
[Out] (14*b*c^6*d^3*log(x))/15 - (7*b*c^6*d^3*log(c^2*x^2 - 1))/15 - (3*a*c^2*d^3
)/(4*x^4) - (a*c^3*d^3)/(3*x^3) - (3*b*c^2*d^3)/(20*x^4) - (11*b*c^3*d^3)/(
36*x^3) - (7*b*c^4*d^3)/(15*x^2) - (11*b*c^5*d^3)/(12*x) - (a*d^3)/(6*x^6)
- (3*a*c*d^3)/(5*x^5) - (b*c*d^3)/(30*x^5) - (b*d^3*atanh(c*x))/(6*x^6) - (
11*b*c^7*d^3*atan((c^2*x)/(-c^2)^(1/2)))/(12*(-c^2)^(1/2)) - (3*b*c*d^3*ata
nh(c*x))/(5*x^5) - (3*b*c^2*d^3*atanh(c*x))/(4*x^4) - (b*c^3*d^3*atanh(c*x)
)/(3*x^3)
```

sympy [A] time = 3.31, size = 257, normalized size = 1.31

$$\begin{cases} -\frac{ac^3d^3}{3x^3} - \frac{3ac^2d^3}{4x^4} - \frac{3acd^3}{5x^5} - \frac{ad^3}{6x^6} + \frac{14bc^6d^3\log(x)}{15} - \frac{14bc^6d^3\log\left(x-\frac{1}{c}\right)}{15} - \frac{bc^6d^3\operatorname{atanh}(cx)}{60} - \frac{11bc^5d^3}{12x} - \frac{7bc^4d^3}{15x^2} - \frac{bc^3d^3\operatorname{atanh}(cx)}{3x^3} - \frac{11}{3} \\ -\frac{ad^3}{6x^6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**7,x)
```

```
[Out] Piecewise((-a*c**3*d**3/(3*x**3) - 3*a*c**2*d**3/(4*x**4) - 3*a*c*d**3/(5*x
**5) - a*d**3/(6*x**6) + 14*b*c**6*d**3*log(x)/15 - 14*b*c**6*d**3*log(x -
1/c)/15 - b*c**6*d**3*atanh(c*x)/60 - 11*b*c**5*d**3/(12*x) - 7*b*c**4*d**3
/(15*x**2) - b*c**3*d**3*atanh(c*x)/(3*x**3) - 11*b*c**3*d**3/(36*x**3) - 3
*b*c**2*d**3*atanh(c*x)/(4*x**4) - 3*b*c**2*d**3/(20*x**4) - 3*b*c*d**3*ata
nh(c*x)/(5*x**5) - b*c*d**3/(30*x**5) - b*d**3*atanh(c*x)/(6*x**6), Ne(c, 0
)), (-a*d**3/(6*x**6), True))
```

3.31 $\int x^3(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=224

$$\frac{1}{8}c^4d^4x^8\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{7}c^3d^4x^7\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx\right.$$

```
[Out] 11/8*b*d^4*x/c^3+24/35*b*d^4*x^2/c^2+11/24*b*d^4*x^3/c+12/35*b*d^4*x^4+9/40
*b*c*d^4*x^5+2/21*b*c^2*d^4*x^6+1/56*b*c^3*d^4*x^7+1/4*d^4*x^4*(a+b*arctanh
(c*x))+4/5*c*d^4*x^5*(a+b*arctanh(c*x))+c^2*d^4*x^6*(a+b*arctanh(c*x))+4/7*
c^3*d^4*x^7*(a+b*arctanh(c*x))+1/8*c^4*d^4*x^8*(a+b*arctanh(c*x))+769/560*b
*d^4*ln(-c*x+1)/c^4-1/560*b*d^4*ln(c*x+1)/c^4
```

Rubi [A] time = 0.21, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{8}c^4d^4x^8\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{7}c^3d^4x^7\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (11*b*d^4*x)/(8*c^3) + (24*b*d^4*x^2)/(35*c^2) + (11*b*d^4*x^3)/(24*c) + (1
2*b*d^4*x^4)/35 + (9*b*c*d^4*x^5)/40 + (2*b*c^2*d^4*x^6)/21 + (b*c^3*d^4*x^
7)/56 + (d^4*x^4*(a + b*ArcTanh[c*x]))/4 + (4*c*d^4*x^5*(a + b*ArcTanh[c*x]
))/5 + c^2*d^4*x^6*(a + b*ArcTanh[c*x]) + (4*c^3*d^4*x^7*(a + b*ArcTanh[c*x
]))/7 + (c^4*d^4*x^8*(a + b*ArcTanh[c*x]))/8 + (769*b*d^4*Log[1 - c*x])/(56
0*c^4) - (b*d^4*Log[1 + c*x])/(560*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^3(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\ \text{ta}\right.$$

$$= \frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\ \text{ta}\right.$$

$$= \frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\ \text{ta}\right.$$

$$= \frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6+\frac{1}{56}$$

$$= \frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6+\frac{1}{56}$$

$$= \frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6+\frac{1}{56}$$

Mathematica [A] time = 0.16, size = 177, normalized size = 0.79

$$d^4\left(210ac^8x^8+960ac^7x^7+1680ac^6x^6+1344ac^5x^5+420ac^4x^4+30bc^7x^7+160bc^6x^6+378bc^5x^5+576bc^4x^4+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(2310*b*c*x + 1152*b*c^2*x^2 + 770*b*c^3*x^3 + 420*a*c^4*x^4 + 576*b*c
^4*x^4 + 1344*a*c^5*x^5 + 378*b*c^5*x^5 + 1680*a*c^6*x^6 + 160*b*c^6*x^6 +
960*a*c^7*x^7 + 30*b*c^7*x^7 + 210*a*c^8*x^8 + 6*b*c^4*x^4*(70 + 224*c*x +
280*c^2*x^2 + 160*c^3*x^3 + 35*c^4*x^4)*ArcTanh[c*x] + 2307*b*Log[1 - c*x]
- 3*b*Log[1 + c*x]))/(1680*c^4)
```

fricas [A] time = 0.55, size = 222, normalized size = 0.99

$$210\,ac^8d^4x^8+30\,(32\,a+b)c^7d^4x^7+80\,(21\,a+2\,b)c^6d^4x^6+42\,(32\,a+9\,b)c^5d^4x^5+12\,(35\,a+48\,b)c^4d^4x^4+770\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(210*a*c^8*d^4*x^8 + 30*(32*a + b)*c^7*d^4*x^7 + 80*(21*a + 2*b)*c^6
*d^4*x^6 + 42*(32*a + 9*b)*c^5*d^4*x^5 + 12*(35*a + 48*b)*c^4*d^4*x^4 + 770
*b*c^3*d^4*x^3 + 1152*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 3*b*d^4*log(c*x + 1)
 + 2307*b*d^4*log(c*x - 1) + 3*(35*b*c^8*d^4*x^8 + 160*b*c^7*d^4*x^7 + 280*
b*c^6*d^4*x^6 + 224*b*c^5*d^4*x^5 + 70*b*c^4*d^4*x^4)*log(-(c*x + 1)/(c*x -
 1)))/c^4
```

giac [B] time = 0.34, size = 817, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -4/105*c*(36*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 - 12*(35*(c*x + 1)^7*b
*d^4/(c*x - 1)^7 - 70*(c*x + 1)^6*b*d^4/(c*x - 1)^6 + 175*(c*x + 1)^5*b*d^4
/(c*x - 1)^5 - 210*(c*x + 1)^4*b*d^4/(c*x - 1)^4 + 168*(c*x + 1)^3*b*d^4/(c
*x - 1)^3 - 84*(c*x + 1)^2*b*d^4/(c*x - 1)^2 + 24*(c*x + 1)*b*d^4/(c*x - 1)
 - 3*b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^8*c^5/(c*x - 1)^8 - 8*(c*x
 + 1)^7*c^5/(c*x - 1)^7 + 28*(c*x + 1)^6*c^5/(c*x - 1)^6 - 56*(c*x + 1)^5*c
^5/(c*x - 1)^5 + 70*(c*x + 1)^4*c^5/(c*x - 1)^4 - 56*(c*x + 1)^3*c^5/(c*x -
 1)^3 + 28*(c*x + 1)^2*c^5/(c*x - 1)^2 - 8*(c*x + 1)*c^5/(c*x - 1) + c^5) -
 36*b*d^4*log(-(c*x + 1)/(c*x - 1))/c^5 - (840*(c*x + 1)^7*a*d^4/(c*x - 1)^
7 - 1680*(c*x + 1)^6*a*d^4/(c*x - 1)^6 + 4200*(c*x + 1)^5*a*d^4/(c*x - 1)^5
 - 5040*(c*x + 1)^4*a*d^4/(c*x - 1)^4 + 4032*(c*x + 1)^3*a*d^4/(c*x - 1)^3
- 2016*(c*x + 1)^2*a*d^4/(c*x - 1)^2 + 576*(c*x + 1)*a*d^4/(c*x - 1) - 72*a
*d^4 + 384*(c*x + 1)^7*b*d^4/(c*x - 1)^7 - 1830*(c*x + 1)^6*b*d^4/(c*x - 1)
^6 + 4304*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 6031*(c*x + 1)^4*b*d^4/(c*x - 1)^
4 + 5228*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 2782*(c*x + 1)^2*b*d^4/(c*x - 1)^2
 + 836*(c*x + 1)*b*d^4/(c*x - 1) - 109*b*d^4)/((c*x + 1)^8*c^5/(c*x - 1)^8
- 8*(c*x + 1)^7*c^5/(c*x - 1)^7 + 28*(c*x + 1)^6*c^5/(c*x - 1)^6 - 56*(c*x
+ 1)^5*c^5/(c*x - 1)^5 + 70*(c*x + 1)^4*c^5/(c*x - 1)^4 - 56*(c*x + 1)^3*c^
5/(c*x - 1)^3 + 28*(c*x + 1)^2*c^5/(c*x - 1)^2 - 8*(c*x + 1)*c^5/(c*x - 1)
+ c^5))
```

maple [A] time = 0.03, size = 237, normalized size = 1.06

$$\frac{c^4 d^4 a\, x^8}{8}+\frac{4 c^3 d^4 a\, x^7}{7}+c^2 d^4 a\, x^6+\frac{4 c\, d^4 a\, x^5}{5}+\frac{d^4 a\, x^4}{4}+\frac{c^4 d^4 b \,\mathrm{arctanh}\left(c x\right) x^8}{8}+\frac{4 c^3 d^4 b \,\mathrm{arctanh}\left(c x\right) x^7}{7}+c^2 d^4 b \,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/8*c^4*d^4*a*x^8+4/7*c^3*d^4*a*x^7+c^2*d^4*a*x^6+4/5*c*d^4*a*x^5+1/4*d^4*a
*x^4+1/8*c^4*d^4*b*arctanh(c*x)*x^8+4/7*c^3*d^4*b*arctanh(c*x)*x^7+c^2*d^4*
b*arctanh(c*x)*x^6+4/5*c*d^4*b*arctanh(c*x)*x^5+1/4*d^4*b*arctanh(c*x)*x^4+
1/56*b*c^3*d^4*x^7+2/21*b*c^2*d^4*x^6+9/40*b*c*d^4*x^5+12/35*b*d^4*x^4+11/2
4*b*d^4*x^3/c+24/35*b*d^4*x^2/c^2+11/8*b*d^4*x/c^3+769/560/c^4*d^4*b*ln(c*x
-1)-1/560*b*d^4*ln(c*x+1)/c^4
```

maxima [A] time = 0.33, size = 373, normalized size = 1.67

$$\frac{1}{8}\, ac^4 d^4 x^8+\frac{4}{7}\, ac^3 d^4 x^7+ac^2 d^4 x^6+\frac{4}{5}\, acd^4 x^5+\frac{1}{1680}\left(210\, x^8 \,\mathrm{artanh}\left(cx\right)+c\left(\frac{2\left(15\, c^6 x^7+21\, c^4 x^5+35\, c^2 x^3+105\right.}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^4*x^8 + 4/7*a*c^3*d^4*x^7 + a*c^2*d^4*x^6 + 4/5*a*c*d^4*x^5 + 1
/1680*(210*x^8*arctanh(c*x) + c*(2*(15*c^6*x^7 + 21*c^4*x^5 + 35*c^2*x^3 +
105*x)/c^8 - 105*log(c*x + 1)/c^9 + 105*log(c*x - 1)/c^9))*b*c^4*d^4 + 1/21
*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*
x^2 - 1)/c^8))*b*c^3*d^4 + 1/4*a*d^4*x^4 + 1/30*(30*x^6*arctanh(c*x) + c*(2
*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)
/c^7))*b*c^2*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*l
```

```
og(c^2*x^2 - 1)/c^6))*b*c*d^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 +
3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^4
```

mupad [B] time = 1.84, size = 337, normalized size = 1.50

$$\frac{a\,d^4\,x^4}{4}+\frac{12\,b\,d^4\,x^4}{35}+a\,c^2\,d^4\,x^6+\frac{4\,a\,c^3\,d^4\,x^7}{7}+\frac{a\,c^4\,d^4\,x^8}{8}+\frac{11\,b\,d^4\,x^3}{24\,c}+\frac{24\,b\,d^4\,x^2}{35\,c^2}+\frac{2\,b\,c^2\,d^4\,x^6}{21}+\frac{b\,c^3\,d^4\,x^7}{56}+\frac{769\,b\,d^4\,\ln}{560}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] (a*d^4*x^4)/4 + (12*b*d^4*x^4)/35 + a*c^2*d^4*x^6 + (4*a*c^3*d^4*x^7)/7 + (
a*c^4*d^4*x^8)/8 + (11*b*d^4*x^3)/(24*c) + (24*b*d^4*x^2)/(35*c^2) + (2*b*c
^2*d^4*x^6)/21 + (b*c^3*d^4*x^7)/56 + (769*b*d^4*log(c*x - 1))/(560*c^4) -
(b*d^4*log(c*x + 1))/(560*c^4) + (b*d^4*x^4*log(c*x + 1))/8 - (b*d^4*x^4*lo
g(1 - c*x))/8 + (4*a*c*d^4*x^5)/5 + (11*b*d^4*x)/(8*c^3) + (9*b*c*d^4*x^5)/
40 + (b*c^2*d^4*x^6*log(c*x + 1))/2 - (b*c^2*d^4*x^6*log(1 - c*x))/2 + (2*b
*c^3*d^4*x^7*log(c*x + 1))/7 - (2*b*c^3*d^4*x^7*log(1 - c*x))/7 + (b*c^4*d^
4*x^8*log(c*x + 1))/16 - (b*c^4*d^4*x^8*log(1 - c*x))/16 + (2*b*c*d^4*x^5*l
og(c*x + 1))/5 - (2*b*c*d^4*x^5*log(1 - c*x))/5
```

sympy [A] time = 3.76, size = 294, normalized size = 1.31

$$\begin{cases} \frac{ac^4d^4x^8}{8} + \frac{4ac^3d^4x^7}{7} + ac^2d^4x^6 + \frac{4acd^4x^5}{5} + \frac{ad^4x^4}{4} + \frac{bc^4d^4x^8\operatorname{atanh}(cx)}{8} + \frac{4bc^3d^4x^7\operatorname{atanh}(cx)}{7} + \frac{bc^3d^4x^7}{56} + bc^2d^4x^6\operatorname{atanh}(cx) + \\ \frac{ad^4x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**8/8 + 4*a*c**3*d**4*x**7/7 + a*c**2*d**4*x**6 + 4
*a*c*d**4*x**5/5 + a*d**4*x**4/4 + b*c**4*d**4*x**8*atanh(c*x)/8 + 4*b*c**3
*d**4*x**7*atanh(c*x)/7 + b*c**3*d**4*x**7/56 + b*c**2*d**4*x**6*atanh(c*x)
 + 2*b*c**2*d**4*x**6/21 + 4*b*c*d**4*x**5*atanh(c*x)/5 + 9*b*c*d**4*x**5/4
0 + b*d**4*x**4*atanh(c*x)/4 + 12*b*d**4*x**4/35 + 11*b*d**4*x**3/(24*c) +
24*b*d**4*x**2/(35*c**2) + 11*b*d**4*x/(8*c**3) + 48*b*d**4*log(x - 1/c)/(3
5*c**4) - b*d**4*atanh(c*x)/(280*c**4), Ne(c, 0)), (a*d**4*x**4/4, True))
```

3.32 $\int x^2(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=171

$$\frac{d^4(cx+1)^7\left(a+b\tanh^{-1}(cx)\right)}{7c^3}-\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{1}{42}bc^3d^4x^6+\frac{1}{}$$

```
[Out] 5/3*b*d^4*x/c^2+88/105*b*d^4*x^2/c+5/9*b*d^4*x^3+47/140*b*c*d^4*x^4+2/15*b*
c^2*d^4*x^5+1/42*b*c^3*d^4*x^6+1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/c^3-1/3
*d^4*(c*x+1)^6*(a+b*arctanh(c*x))/c^3+1/7*d^4*(c*x+1)^7*(a+b*arctanh(c*x))/
c^3+176/105*b*d^4*ln(-c*x+1)/c^3
```

Rubi [A] time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 5936, 12, 893}

$$\frac{d^4(cx+1)^7\left(a+b\tanh^{-1}(cx)\right)}{7c^3}-\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{1}{42}bc^3d^4x^6+\frac{2}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (5*b*d^4*x)/(3*c^2) + (88*b*d^4*x^2)/(105*c) + (5*b*d^4*x^3)/9 + (47*b*c*d^
4*x^4)/140 + (2*b*c^2*d^4*x^5)/15 + (b*c^3*d^4*x^6)/42 + (d^4*(1 + c*x)^5*(
a + b*ArcTanh[c*x]))/(5*c^3) - (d^4*(1 + c*x)^6*(a + b*ArcTanh[c*x]))/(3*c^
3) + (d^4*(1 + c*x)^7*(a + b*ArcTanh[c*x]))/(7*c^3) + (176*b*d^4*Log[1 - c*
x])/(105*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx = \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3} - \frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3} + \frac{d^4(1-}{}$$

$$= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3} - \frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3} + \frac{d^4(1-}{}$$

$$= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3} - \frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3} + \frac{d^4(1-}{}$$

$$= \frac{5bd^4x}{3c^2} + \frac{88bd^4x^2}{105c} + \frac{5}{9}bd^4x^3 + \frac{47}{140}bcd^4x^4 + \frac{2}{15}bc^2d^4x^5 + \frac{1}{42}bc^3d^4x^6 + \cdots$$

Mathematica [A] time = 0.15, size = 168, normalized size = 0.98

$$\frac{d^4\left(180ac^7x^7 + 840ac^6x^6 + 1512ac^5x^5 + 1260ac^4x^4 + 420ac^3x^3 + 30bc^6x^6 + 168bc^5x^5 + 423bc^4x^4 + 700bc^3x^3 + 1\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(2100*b*c*x + 1056*b*c^2*x^2 + 420*a*c^3*x^3 + 700*b*c^3*x^3 + 1260*a*
c^4*x^4 + 423*b*c^4*x^4 + 1512*a*c^5*x^5 + 168*b*c^5*x^5 + 840*a*c^6*x^6 +
30*b*c^6*x^6 + 180*a*c^7*x^7 + 12*b*c^3*x^3*(35 + 105*c*x + 126*c^2*x^2 + 7
0*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] + 2106*b*Log[1 - c*x] + 6*b*Log[1 + c*
x]))/(1260*c^3)
```

fricas [A] time = 0.52, size = 208, normalized size = 1.22

$$\frac{180\,ac^7d^4x^7 + 30\,(28\,a+b)c^6d^4x^6 + 168\,(9\,a+b)c^5d^4x^5 + 9\,(140\,a+47\,b)c^4d^4x^4 + 140\,(3\,a+5\,b)c^3d^4x^3 + 1056\,b\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1260*(180*a*c^7*d^4*x^7 + 30*(28*a + b)*c^6*d^4*x^6 + 168*(9*a + b)*c^5*d
^4*x^5 + 9*(140*a + 47*b)*c^4*d^4*x^4 + 140*(3*a + 5*b)*c^3*d^4*x^3 + 1056*
b*c^2*d^4*x^2 + 2100*b*c*d^4*x + 6*b*d^4*log(c*x + 1) + 2106*b*d^4*log(c*x
- 1) + 6*(15*b*c^7*d^4*x^7 + 70*b*c^6*d^4*x^6 + 126*b*c^5*d^4*x^5 + 105*b*c
^4*d^4*x^4 + 35*b*c^3*d^4*x^3)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

giac [B] time = 0.19, size = 723, normalized size = 4.23

$$-\frac{4}{315}\left(\frac{132\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^4} - \frac{132\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^4} - \frac{12\left(\frac{105\,(cx+1)^6bd^4}{(cx-1)^6} - \frac{210\,(cx+1)^5bd^4}{(cx-1)^5} + \frac{385\,(cx+1)^4bd^4}{(cx-1)^4} - \frac{385\,(cx+1)^3\cdots}{(cx-1)^3}\cdots\right.}{\frac{(cx+1)^7c^4}{(cx-1)^7} - \frac{7\,(cx+1)^6c^4}{(cx-1)^6} + \frac{21\,(cx+1)^5c^4}{(cx-1)^5} - \frac{35\,(cx+1)^4c^4}{(cx-1)^4}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -4/315*(132*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^4 - 132*b*d^4*log(-(c*x +
 1)/(c*x - 1))/c^4 - 12*(105*(c*x + 1)^6*b*d^4/(c*x - 1)^6 - 210*(c*x + 1)^
5*b*d^4/(c*x - 1)^5 + 385*(c*x + 1)^4*b*d^4/(c*x - 1)^4 - 385*(c*x + 1)^3*b
*d^4/(c*x - 1)^3 + 231*(c*x + 1)^2*b*d^4/(c*x - 1)^2 - 77*(c*x + 1)*b*d^4/(
c*x - 1) + 11*b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^7*c^4/(c*x - 1)^7
```

```
 - 7*(c*x + 1)^6*c^4/(c*x - 1)^6 + 21*(c*x + 1)^5*c^4/(c*x - 1)^5 - 35*(c*x
 + 1)^4*c^4/(c*x - 1)^4 + 35*(c*x + 1)^3*c^4/(c*x - 1)^3 - 21*(c*x + 1)^2*c
^4/(c*x - 1)^2 + 7*(c*x + 1)*c^4/(c*x - 1) - c^4) - (2520*(c*x + 1)^6*a*d^4
/(c*x - 1)^6 - 5040*(c*x + 1)^5*a*d^4/(c*x - 1)^5 + 9240*(c*x + 1)^4*a*d^4/
(c*x - 1)^4 - 9240*(c*x + 1)^3*a*d^4/(c*x - 1)^3 + 5544*(c*x + 1)^2*a*d^4/(
c*x - 1)^2 - 1848*(c*x + 1)*a*d^4/(c*x - 1) + 264*a*d^4 + 1128*(c*x + 1)^6*
b*d^4/(c*x - 1)^6 - 4812*(c*x + 1)^5*b*d^4/(c*x - 1)^5 + 9476*(c*x + 1)^4*b
*d^4/(c*x - 1)^4 - 10631*(c*x + 1)^3*b*d^4/(c*x - 1)^3 + 6933*(c*x + 1)^2*b
*d^4/(c*x - 1)^2 - 2465*(c*x + 1)*b*d^4/(c*x - 1) + 371*b*d^4)/((c*x + 1)^7
*c^4/(c*x - 1)^7 - 7*(c*x + 1)^6*c^4/(c*x - 1)^6 + 21*(c*x + 1)^5*c^4/(c*x
- 1)^5 - 35*(c*x + 1)^4*c^4/(c*x - 1)^4 + 35*(c*x + 1)^3*c^4/(c*x - 1)^3 -
21*(c*x + 1)^2*c^4/(c*x - 1)^2 + 7*(c*x + 1)*c^4/(c*x - 1) - c^4))*c
```

maple [A] time = 0.03, size = 225, normalized size = 1.32

$$\frac{c^4d^4a\,x^7}{7}+\frac{2c^3d^4a\,x^6}{3}+\frac{6c^2d^4a\,x^5}{5}+c\,d^4a\,x^4+\frac{d^4a\,x^3}{3}+\frac{c^4d^4b\operatorname{arctanh}(cx)\,x^7}{7}+\frac{2c^3d^4b\operatorname{arctanh}(cx)\,x^6}{3}+\frac{6c^2d^4b\operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/7*c^4*d^4*a*x^7+2/3*c^3*d^4*a*x^6+6/5*c^2*d^4*a*x^5+c*d^4*a*x^4+1/3*d^4*a
*x^3+1/7*c^4*d^4*b*arctanh(c*x)*x^7+2/3*c^3*d^4*b*arctanh(c*x)*x^6+6/5*c^2*
d^4*b*arctanh(c*x)*x^5+c*d^4*b*arctanh(c*x)*x^4+1/3*d^4*b*arctanh(c*x)*x^3+
1/42*b*c^3*d^4*x^6+2/15*b*c^2*d^4*x^5+47/140*b*c*d^4*x^4+5/9*b*d^4*x^3+88/1
05*b*d^4*x^2/c+5/3*b*d^4*x/c^2+117/70/c^3*d^4*b*ln(c*x-1)+1/210/c^3*d^4*b*l
n(c*x+1)
```

maxima [B] time = 0.33, size = 339, normalized size = 1.98

$$\frac{1}{7}\,ac^4d^4x^7+\frac{2}{3}\,ac^3d^4x^6+\frac{6}{5}\,ac^2d^4x^5+\frac{1}{84}\left(12\,x^7\operatorname{artanh}(cx)+c\left(\frac{2\,c^4x^6+3\,c^2x^4+6\,x^2}{c^6}+\frac{6\,\log\left(c^2x^2-1\right)}{c^8}\right)\right)bc^4d^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^4*x^7 + 2/3*a*c^3*d^4*x^6 + 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7*ar
ctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^
8))*b*c^4*d^4 + a*c*d^4*x^4 + 1/45*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 +
 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*c^3*
d^4 + 3/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
 1)/c^6))*b*c^2*d^4 + 1/3*a*d^4*x^3 + 1/6*(6*x^4*arctanh(c*x) + c*(2*(c^2*x
^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d^4 + 1/6*(2*
x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d^4
```

mupad [B] time = 1.08, size = 196, normalized size = 1.15

$$\frac{\frac{88\,b\,c^2\,d^4\,x^2}{105}-\frac{d^4\,\left(2100\,b\,\mathrm{atanh}(c\,x)-1056\,b\,\ln\left(c^2\,x^2-1\right)\right)}{1260}+\frac{5\,b\,c\,d^4\,x}{3}}{c^3}+\frac{d^4\,\left(420\,a\,x^3+700\,b\,x^3+420\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{1260}+\frac{c^4\,d^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] ((88*b*c^2*d^4*x^2)/105 - (d^4*(2100*b*atanh(c*x) - 1056*b*log(c^2*x^2 - 1)
))/1260 + (5*b*c*d^4*x)/3)/c^3 + (d^4*(420*a*x^3 + 700*b*x^3 + 420*b*x^3*at
anh(c*x)))/1260 + (c^4*d^4*(180*a*x^7 + 180*b*x^7*atanh(c*x)))/1260 + (c*d^
4*(1260*a*x^4 + 423*b*x^4 + 1260*b*x^4*atanh(c*x)))/1260 + (c^3*d^4*(840*a*
x^6 + 30*b*x^6 + 840*b*x^6*atanh(c*x)))/1260 + (c^2*d^4*(1512*a*x^5 + 168*b
*x^5 + 1512*b*x^5*atanh(c*x)))/1260
```

sympy [A] time = 2.99, size = 279, normalized size = 1.63

$$\begin{cases} \frac{ac^4d^4x^7}{7} + \frac{2ac^3d^4x^6}{3} + \frac{6ac^2d^4x^5}{5} + acd^4x^4 + \frac{ad^4x^3}{3} + \frac{bc^4d^4x^7\operatorname{atanh}(cx)}{7} + \frac{2bc^3d^4x^6\operatorname{atanh}(cx)}{3} + \frac{bc^3d^4x^6}{42} + \frac{6bc^2d^4x^5\operatorname{atanh}(cx)}{5} + \frac{2bc \\ \frac{ad^4x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**7/7 + 2*a*c**3*d**4*x**6/3 + 6*a*c**2*d**4*x**5/5
 + a*c*d**4*x**4 + a*d**4*x**3/3 + b*c**4*d**4*x**7*atanh(c*x)/7 + 2*b*c**3
*d**4*x**6*atanh(c*x)/3 + b*c**3*d**4*x**6/42 + 6*b*c**2*d**4*x**5*atanh(c*
x)/5 + 2*b*c**2*d**4*x**5/15 + b*c*d**4*x**4*atanh(c*x) + 47*b*c*d**4*x**4/
140 + b*d**4*x**3*atanh(c*x)/3 + 5*b*d**4*x**3/9 + 88*b*d**4*x**2/(105*c) +
 5*b*d**4*x/(3*c**2) + 176*b*d**4*log(x - 1/c)/(105*c**3) + b*d**4*atanh(c*
x)/(105*c**3), Ne(c, 0)), (a*d**4*x**3/3, True))
```

3.33 $\int x(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=153

$$\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^4(cx+1)^5}{30c^2}+\frac{bd^4(cx+1)^4}{30c^2}+\frac{4bd^4(cx+1)^3}{45c^2}+\ldots$$

```
[Out] 16/15*b*d^4*x/c+4/15*b*d^4*(c*x+1)^2/c^2+4/45*b*d^4*(c*x+1)^3/c^2+1/30*b*d^
4*(c*x+1)^4/c^2+1/30*b*d^4*(c*x+1)^5/c^2-1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x
))/c^2+1/6*d^4*(c*x+1)^6*(a+b*arctanh(c*x))/c^2+32/15*b*d^4*ln(-c*x+1)/c^2
```

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 77}

$$\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^4(cx+1)^5}{30c^2}+\frac{bd^4(cx+1)^4}{30c^2}+\frac{4bd^4(cx+1)^3}{45c^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (16*b*d^4*x)/(15*c) + (4*b*d^4*(1 + c*x)^2)/(15*c^2) + (4*b*d^4*(1 + c*x)^3
)/(45*c^2) + (b*d^4*(1 + c*x)^4)/(30*c^2) + (b*d^4*(1 + c*x)^5)/(30*c^2) -
(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (d^4*(1 + c*x)^6*(a + b*Ar
cTanh[c*x]))/(6*c^2) + (32*b*d^4*Log[1 - c*x])/(15*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx = -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-(bc)$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{b\int}{}$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{b\int\left(\right.}{}$$

$$= \frac{16bd^4x}{15c}+\frac{4bd^4(1+cx)^2}{15c^2}+\frac{4bd^4(1+cx)^3}{45c^2}+\frac{bd^4(1+cx)^4}{30c^2}+\frac{bd^4(1+cx)^5}{30c^2}$$

Mathematica [A] time = 0.14, size = 159, normalized size = 1.04

$$\frac{d^4\left(30ac^6x^6+144ac^5x^5+270ac^4x^4+240ac^3x^3+90ac^2x^2+6bc^5x^5+36bc^4x^4+100bc^3x^3+192bc^2x^2+6bc^2x^2\left(\right.\right.}{180c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(390*b*c*x + 90*a*c^2*x^2 + 192*b*c^2*x^2 + 240*a*c^3*x^3 + 100*b*c^3*
x^3 + 270*a*c^4*x^4 + 36*b*c^4*x^4 + 144*a*c^5*x^5 + 6*b*c^5*x^5 + 30*a*c^6
*x^6 + 6*b*c^2*x^2*(15 + 40*c*x + 45*c^2*x^2 + 24*c^3*x^3 + 5*c^4*x^4)*ArcT
anh[c*x] + 387*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(180*c^2)
```

fricas [A] time = 0.50, size = 198, normalized size = 1.29

$$\frac{30\,ac^6d^4x^6+6\,(24\,a+b)c^5d^4x^5+18\,(15\,a+2\,b)c^4d^4x^4+20\,(12\,a+5\,b)c^3d^4x^3+6\,(15\,a+32\,b)c^2d^4x^2+390\,bcd^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*d^4*x^6 + 6*(24*a + b)*c^5*d^4*x^5 + 18*(15*a + 2*b)*c^4*d^
4*x^4 + 20*(12*a + 5*b)*c^3*d^4*x^3 + 6*(15*a + 32*b)*c^2*d^4*x^2 + 390*b*c
*d^4*x - 3*b*d^4*log(c*x + 1) + 387*b*d^4*log(c*x - 1) + 3*(5*b*c^6*d^4*x^6
 + 24*b*c^5*d^4*x^5 + 45*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 15*b*c^2*d^4*x^
2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

giac [B] time = 0.28, size = 621, normalized size = 4.06

$$-\frac{8}{45}\left(\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3}-\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^3}-\frac{6\left(\frac{15\,(cx+1)^5bd^4}{(cx-1)^5}-\frac{30\,(cx+1)^4bd^4}{(cx-1)^4}+\frac{40\,(cx+1)^3bd^4}{(cx-1)^3}-\frac{30\,(cx+1)^2bd^4}{(cx-1)^2}+12\right.}{\frac{(cx+1)^6c^3}{(cx-1)^6}-\frac{6\,(cx+1)^5c^3}{(cx-1)^5}+\frac{15\,(cx+1)^4c^3}{(cx-1)^4}-\frac{20\,(cx+1)^3c^3}{(cx-1)^3}+15\,(c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -8/45*(12*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 12*b*d^4*log(-(c*x + 1)
/(c*x - 1))/c^3 - 6*(15*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 30*(c*x + 1)^4*b*d^
4/(c*x - 1)^4 + 40*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 30*(c*x + 1)^2*b*d^4/(c*
x - 1)^2 + 12*(c*x + 1)*b*d^4/(c*x - 1) - 2*b*d^4)*log(-(c*x + 1)/(c*x - 1)
)/((c*x + 1)^6*c^3/(c*x - 1)^6 - 6*(c*x + 1)^5*c^3/(c*x - 1)^5 + 15*(c*x +
1)^4*c^3/(c*x - 1)^4 - 20*(c*x + 1)^3*c^3/(c*x - 1)^3 + 15*(c*x + 1)^2*c^3/
(c*x - 1)^2 - 6*(c*x + 1)*c^3/(c*x - 1) + c^3) - (180*(c*x + 1)^5*a*d^4/(c*
```

```
x - 1)^5 - 360*(c*x + 1)^4*a*d^4/(c*x - 1)^4 + 480*(c*x + 1)^3*a*d^4/(c*x -
 1)^3 - 360*(c*x + 1)^2*a*d^4/(c*x - 1)^2 + 144*(c*x + 1)*a*d^4/(c*x - 1) -
 24*a*d^4 + 78*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 294*(c*x + 1)^4*b*d^4/(c*x -
 1)^4 + 472*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 399*(c*x + 1)^2*b*d^4/(c*x - 1)
^2 + 174*(c*x + 1)*b*d^4/(c*x - 1) - 31*b*d^4)/((c*x + 1)^6*c^3/(c*x - 1)^6
 - 6*(c*x + 1)^5*c^3/(c*x - 1)^5 + 15*(c*x + 1)^4*c^3/(c*x - 1)^4 - 20*(c*x
 + 1)^3*c^3/(c*x - 1)^3 + 15*(c*x + 1)^2*c^3/(c*x - 1)^2 - 6*(c*x + 1)*c^3/
(c*x - 1) + c^3))*c
```

maple [A] time = 0.03, size = 215, normalized size = 1.41

$$\frac{c^4d^4a\,x^6}{6}+\frac{4c^3d^4a\,x^5}{5}+\frac{3c^2d^4a\,x^4}{2}+\frac{4c\,d^4a\,x^3}{3}+\frac{d^4a\,x^2}{2}+\frac{c^4d^4b\,\mathrm{arctanh}\,(cx)\,x^6}{6}+\frac{4c^3d^4b\,\mathrm{arctanh}\,(cx)\,x^5}{5}+\frac{3c^2d^4b\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*c^4*d^4*a*x^6+4/5*c^3*d^4*a*x^5+3/2*c^2*d^4*a*x^4+4/3*c*d^4*a*x^3+1/2*d
^4*a*x^2+1/6*c^4*d^4*b*arctanh(c*x)*x^6+4/5*c^3*d^4*b*arctanh(c*x)*x^5+3/2*
c^2*d^4*b*arctanh(c*x)*x^4+4/3*c*d^4*b*arctanh(c*x)*x^3+1/2*d^4*b*arctanh(c
*x)*x^2+1/30*c^3*d^4*b*x^5+1/5*c^2*d^4*b*x^4+5/9*c*d^4*b*x^3+16/15*d^4*b*x^
2+13/6*b*d^4*x/c+43/20/c^2*d^4*b*ln(c*x-1)-1/60/c^2*d^4*b*ln(c*x+1)
```

maxima [B] time = 0.34, size = 326, normalized size = 2.13

$$\frac{1}{6}\,ac^4d^4x^6+\frac{4}{5}\,ac^3d^4x^5+\frac{3}{2}\,ac^2d^4x^4+\frac{1}{180}\left(30\,x^6\,\mathrm{artanh}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^4*x^6 + 4/5*a*c^3*d^4*x^5 + 3/2*a*c^2*d^4*x^4 + 1/180*(30*x^6*a
rctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7
 + 15*log(c*x - 1)/c^7))*b*c^4*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4
+ 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^4 + 4/3*a*c*d^4*x^3 + 1/4*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^2*d^4 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x
^2 - 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^
2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^4
```

mupad [B] time = 1.05, size = 185, normalized size = 1.21

$$\frac{d^4\,\left(45\,a\,x^2+96\,b\,x^2+45\,b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{90}-\frac{\frac{d^4\,\left(195\,b\,\mathrm{atanh}\left(c\,x\right)-96\,b\,\ln\left(c^2\,x^2-1\right)\right)}{90}-\frac{13\,b\,c\,d^4\,x}{6}}{c^2}+\frac{c^4\,d^4\,\left(15\,a\,x^6+15\,b\,x^6\,\mathrm{a}\right.}{90}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] (d^4*(45*a*x^2 + 96*b*x^2 + 45*b*x^2*atanh(c*x)))/90 - ((d^4*(195*b*atanh(c
*x) - 96*b*log(c^2*x^2 - 1)))/90 - (13*b*c*d^4*x)/6)/c^2 + (c^4*d^4*(15*a*x
^6 + 15*b*x^6*atanh(c*x)))/90 + (c*d^4*(120*a*x^3 + 50*b*x^3 + 120*b*x^3*at
anh(c*x)))/90 + (c^3*d^4*(72*a*x^5 + 3*b*x^5 + 72*b*x^5*atanh(c*x)))/90 + (
c^2*d^4*(135*a*x^4 + 18*b*x^4 + 135*b*x^4*atanh(c*x)))/90
```

sympy [A] time = 2.36, size = 269, normalized size = 1.76

$$\begin{cases}\frac{ac^4d^4x^6}{6}+\frac{4ac^3d^4x^5}{5}+\frac{3ac^2d^4x^4}{2}+\frac{4acd^4x^3}{3}+\frac{ad^4x^2}{2}+\frac{bc^4d^4x^6\,\mathrm{atanh}\,(cx)}{6}+\frac{4bc^3d^4x^5\,\mathrm{atanh}\,(cx)}{5}+\frac{bc^3d^4x^5}{30}+\frac{3bc^2d^4x^4\,\mathrm{atanh}\,(cx)}{2}+ \\ \frac{ad^4x^2}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**6/6 + 4*a*c**3*d**4*x**5/5 + 3*a*c**2*d**4*x**4/2
 + 4*a*c*d**4*x**3/3 + a*d**4*x**2/2 + b*c**4*d**4*x**6*atanh(c*x)/6 + 4*b*
c**3*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**5/30 + 3*b*c**2*d**4*x**4*atan
h(c*x)/2 + b*c**2*d**4*x**4/5 + 4*b*c*d**4*x**3*atanh(c*x)/3 + 5*b*c*d**4*x
**3/9 + b*d**4*x**2*atanh(c*x)/2 + 16*b*d**4*x**2/15 + 13*b*d**4*x/(6*c) +
32*b*d**4*log(x - 1/c)/(15*c**2) - b*d**4*atanh(c*x)/(30*c**2), Ne(c, 0)),
(a*d**4*x**2/2, True))
```

3.34 $\int (d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=107

$$\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}+\frac{bd^4(cx+1)^4}{20c}+\frac{2bd^4(cx+1)^3}{15c}+\frac{2bd^4(cx+1)^2}{5c}+\frac{16bd^4\log(1-cx)}{5c}+\frac{8}{5}bd^4x$$

```
[Out] 8/5*b*d^4*x+2/5*b*d^4*(c*x+1)^2/c+2/15*b*d^4*(c*x+1)^3/c+1/20*b*d^4*(c*x+1)
^4/c+1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/c+16/5*b*d^4*ln(-c*x+1)/c
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{5926, 627, 43\}$

$$\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}+\frac{bd^4(cx+1)^4}{20c}+\frac{2bd^4(cx+1)^3}{15c}+\frac{2bd^4(cx+1)^2}{5c}+\frac{16bd^4\log(1-cx)}{5c}+\frac{8}{5}bd^4x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (8*b*d^4*x)/5 + (2*b*d^4*(1 + c*x)^2)/(5*c) + (2*b*d^4*(1 + c*x)^3)/(15*c)
+ (b*d^4*(1 + c*x)^4)/(20*c) + (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c)
 + (16*b*d^4*Log[1 - c*x])/(5*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}-\frac{b\int\frac{(d+cdx)^5}{1-c^2x^2}\,dx}{5d}\\ &= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}-\frac{b\int\frac{(d+cdx)^4}{\frac{1}{d}-\frac{cx}{d}}\,dx}{5d}\\ &= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}-\frac{b\int\left(-8d^5+\frac{16d^4}{\frac{1}{d}-\frac{cx}{d}}-4d^4(d+cdx)-2d^3\right.}{5d}\\ &= \frac{8}{5}bd^4x+\frac{2bd^4(1+cx)^2}{5c}+\frac{2bd^4(1+cx)^3}{15c}+\frac{bd^4(1+cx)^4}{20c}+\frac{d^4(1+cx)^5\left(a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.18, size = 146, normalized size = 1.36

$$\frac{d^4\left(12ac^5x^5+60ac^4x^4+120ac^3x^3+120ac^2x^2+60acx+3bc^4x^4+20bc^3x^3+66bc^2x^2+6b\log\left(1-c^2x^2\right)+12bc\right.}{60c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(60*a*c*x + 180*b*c*x + 120*a*c^2*x^2 + 66*b*c^2*x^2 + 120*a*c^3*x^3 +
 20*b*c^3*x^3 + 60*a*c^4*x^4 + 3*b*c^4*x^4 + 12*a*c^5*x^5 + 12*b*c*x*(5 + 1
0*c*x + 10*c^2*x^2 + 5*c^3*x^3 + c^4*x^4)*ArcTanh[c*x] + 180*b*Log[1 - c*x]
 + 6*b*Log[1 - c^2*x^2]))/(60*c)
```

fricas [A] time = 0.43, size = 177, normalized size = 1.65

$$\frac{12\,ac^5d^4x^5+3\,(20\,a+b)c^4d^4x^4+20\,(6\,a+b)c^3d^4x^3+6\,(20\,a+11\,b)c^2d^4x^2+60\,(a+3\,b)cd^4x+6\,bd^4\log\left(cx+1\right.}{60\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^4*x^5 + 3*(20*a + b)*c^4*d^4*x^4 + 20*(6*a + b)*c^3*d^4*x^
3 + 6*(20*a + 11*b)*c^2*d^4*x^2 + 60*(a + 3*b)*c*d^4*x + 6*b*d^4*log(c*x +
1) + 186*b*d^4*log(c*x - 1) + 6*(b*c^5*d^4*x^5 + 5*b*c^4*d^4*x^4 + 10*b*c^3
*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

giac [B] time = 0.24, size = 526, normalized size = 4.92

$$-\frac{4}{15}\left(\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{12\left(\frac{5\,(cx+1)^4bd^4}{(cx-1)^4}-\frac{10\,(cx+1)^3bd^4}{(cx-1)^3}+\frac{10\,(cx+1)^2bd^4}{(cx-1)^2}-\frac{5\,(cx+1)bd^4}{cx-1}+bd^4\right.}{\frac{(cx+1)^5c^2}{(cx-1)^5}-\frac{5\,(cx+1)^4c^2}{(cx-1)^4}+\frac{10\,(cx+1)^3c^2}{(cx-1)^3}-\frac{10\,(cx+1)^2c^2}{(cx-1)^2}+\frac{5\,(cx+1}{cx-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -4/15*(12*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 12*b*d^4*log(-(c*x + 1)
/(c*x - 1))/c^2 - 12*(5*(c*x + 1)^4*b*d^4/(c*x - 1)^4 - 10*(c*x + 1)^3*b*d^
4/(c*x - 1)^3 + 10*(c*x + 1)^2*b*d^4/(c*x - 1)^2 - 5*(c*x + 1)*b*d^4/(c*x -
 1) + b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^2/(c*x - 1)^5 - 5*(c*
x + 1)^4*c^2/(c*x - 1)^4 + 10*(c*x + 1)^3*c^2/(c*x - 1)^3 - 10*(c*x + 1)^2*
c^2/(c*x - 1)^2 + 5*(c*x + 1)*c^2/(c*x - 1) - c^2) - (120*(c*x + 1)^4*a*d^4
/(c*x - 1)^4 - 240*(c*x + 1)^3*a*d^4/(c*x - 1)^3 + 240*(c*x + 1)^2*a*d^4/(c
*x - 1)^2 - 120*(c*x + 1)*a*d^4/(c*x - 1) + 24*a*d^4 + 48*(c*x + 1)^4*b*d^4
/(c*x - 1)^4 - 156*(c*x + 1)^3*b*d^4/(c*x - 1)^3 + 196*(c*x + 1)^2*b*d^4/(c
*x - 1)^2 - 113*(c*x + 1)*b*d^4/(c*x - 1) + 25*b*d^4)/((c*x + 1)^5*c^2/(c*x
 - 1)^5 - 5*(c*x + 1)^4*c^2/(c*x - 1)^4 + 10*(c*x + 1)^3*c^2/(c*x - 1)^3 -
10*(c*x + 1)^2*c^2/(c*x - 1)^2 + 5*(c*x + 1)*c^2/(c*x - 1) - c^2))*c
```

maple [B] time = 0.03, size = 202, normalized size = 1.89

$$\frac{c^4d^4a\,x^5}{5}+c^3d^4a\,x^4+2c^2d^4a\,x^3+2c\,d^4a\,x^2+xa\,d^4+\frac{d^4a}{5c}+\frac{c^4d^4b\,\mathrm{arctanh}\,(cx)\,x^5}{5}+c^3d^4b\,\mathrm{arctanh}\,(cx)\,x^4+2c^2d^4b\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c^4*d^4*a*x^5+c^3*d^4*a*x^4+2*c^2*d^4*a*x^3+2*c*d^4*a*x^2+x*a*d^4+1/5/c
*d^4*a+1/5*c^4*d^4*b*arctanh(c*x)*x^5+c^3*d^4*b*arctanh(c*x)*x^4+2*c^2*d^4*
```

```
b*arctanh(c*x)*x^3+2*c*d^4*b*arctanh(c*x)*x^2+d^4*b*arctanh(c*x)*x+1/5/c*d^
4*b*arctanh(c*x)+1/20*c^3*d^4*b*x^4+1/3*c^2*d^4*b*x^3+11/10*c*d^4*b*x^2+3*b
*d^4*x+16/5/c*b*ln(c*x-1)*d^4
```

maxima [B] time = 0.33, size = 283, normalized size = 2.64

$$\frac{1}{5}ac^4d^4x^5+ac^3d^4x^4+\frac{1}{20}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)bc^4d^4+2ac^2d^4x^3+\frac{1}{6}\left(6x^4\operatorname{artar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^4*x^5 + a*c^3*d^4*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4
+ 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^4*d^4 + 2*a*c^2*d^4*x^3 + 1/6*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^3*d^4 + (2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
 1)/c^4))*b*c^2*d^4 + 2*a*c*d^4*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 - lo
g(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^4 + a*d^4*x + 1/2*(2*c*x*arctanh(
c*x) + log(-c^2*x^2 + 1))*b*d^4/c
```

mupad [B] time = 1.05, size = 168, normalized size = 1.57

$$\frac{d^4\,\left(60\,a\,x+180\,b\,x+60\,b\,x\,\mathrm{atanh}\left(c\,x\right)\right)}{60}+\frac{c^4\,d^4\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atanh}\left(c\,x\right)\right)}{60}-\frac{d^4\,\left(180\,b\,\mathrm{atanh}\left(c\,x\right)-96\,b\,\ln\right.}{60\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] (d^4*(60*a*x + 180*b*x + 60*b*x*atanh(c*x)))/60 + (c^4*d^4*(12*a*x^5 + 12*b
*x^5*atanh(c*x)))/60 - (d^4*(180*b*atanh(c*x) - 96*b*log(c^2*x^2 - 1)))/(60
*c) + (c*d^4*(120*a*x^2 + 66*b*x^2 + 120*b*x^2*atanh(c*x)))/60 + (c^3*d^4*(
60*a*x^4 + 3*b*x^4 + 60*b*x^4*atanh(c*x)))/60 + (c^2*d^4*(120*a*x^3 + 20*b*
x^3 + 120*b*x^3*atanh(c*x)))/60
```

sympy [A] time = 1.85, size = 226, normalized size = 2.11

$$\begin{cases} \frac{ac^4d^4x^5}{5}+ac^3d^4x^4+2ac^2d^4x^3+2acd^4x^2+ad^4x+\frac{bc^4d^4x^5\operatorname{atanh}(cx)}{5}+bc^3d^4x^4\operatorname{atanh}(cx)+\frac{bc^3d^4x^4}{20}+2bc^2d^4x^3\operatorname{at} \\ ad^4x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**5/5 + a*c**3*d**4*x**4 + 2*a*c**2*d**4*x**3 + 2*a
*c*d**4*x**2 + a*d**4*x + b*c**4*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**4*
atanh(c*x) + b*c**3*d**4*x**4/20 + 2*b*c**2*d**4*x**3*atanh(c*x) + b*c**2*d
**4*x**3/3 + 2*b*c*d**4*x**2*atanh(c*x) + 11*b*c*d**4*x**2/10 + b*d**4*x*at
anh(c*x) + 3*b*d**4*x + 16*b*d**4*log(x - 1/c)/(5*c) + b*d**4*atanh(c*x)/(5
*c), Ne(c, 0)), (a*d**4*x, True))
```

3.35 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=185

$$\frac{1}{4}c^4d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)+4acd^4x+ad^4\log(x)+\frac{1}{12}bc^3$$

[Out] 4*a*c*d^4*x+13/4*b*c*d^4*x+2/3*b*c^2*d^4*x^2+1/12*b*c^3*d^4*x^3-13/4*b*d^4*arctanh(c*x)+4*b*c*d^4*x*arctanh(c*x)+3*c^2*d^4*x^2*(a+b*arctanh(c*x))+4/3*c^3*d^4*x^3*(a+b*arctanh(c*x))+1/4*c^4*d^4*x^4*(a+b*arctanh(c*x))+a*d^4*ln(x)+8/3*b*d^4*ln(-c^2*x^2+1)-1/2*b*d^4*polylog(2,-c*x)+1/2*b*d^4*polylog(2,c*x)

Rubi [A] time = 0.20, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5940, 5910, 260, 5912, 5916, 321, 206, 266, 43, 302}

$$-\frac{1}{2}bd^4\text{PolyLog}(2,-cx)+\frac{1}{2}bd^4\text{PolyLog}(2,cx)+\frac{1}{4}c^4d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+3c^2d^4x^2$$

Antiderivative was successfully verified.

[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]

[Out] 4*a*c*d^4*x + (13*b*c*d^4*x)/4 + (2*b*c^2*d^4*x^2)/3 + (b*c^3*d^4*x^3)/12 - (13*b*d^4*ArcTanh[c*x])/4 + 4*b*c*d^4*x*ArcTanh[c*x] + 3*c^2*d^4*x^2*(a + b*ArcTanh[c*x]) + (4*c^3*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + (c^4*d^4*x^4*(a + b*ArcTanh[c*x]))/4 + a*d^4*Log[x] + (8*b*d^4*Log[1 - c^2*x^2])/3 - (b*d^4*PolyLog[2, -(c*x)])/2 + (b*d^4*PolyLog[2, c*x])/2

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(4cd^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+6c^2d^4x\left(a+b\tan\right.\right.$$

$$= d^4\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(4cd^4\right)\int \left(a+b\tanh^{-1}(cx)\right)dx+\left(6c^2d^4\right)$$

$$= 4acd^4x+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}$$

$$= 4acd^4x+3bcd^4x+4bcd^4x\tanh^{-1}(cx)+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}$$

$$= 4acd^4x+\frac{13}{4}bcd^4x+\frac{1}{12}bc^3d^4x^3-3bd^4\tanh^{-1}(cx)+4bcd^4x\tanh^{-1}(cx)$$

$$= 4acd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}bc^2d^4x^2+\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\tanh^{-1}(cx)+4bcd$$

Mathematica [A] time = 0.17, size = 179, normalized size = 0.97

$$\frac{1}{24}d^4\left(6ac^4x^4+32ac^3x^3+72ac^2x^2+96acx+24a\log(x)+6bc^4x^4\tanh^{-1}(cx)+2bc^3x^3+32bc^3x^3\tanh^{-1}(cx)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^4*(96*a*c*x + 78*b*c*x + 72*a*c^2*x^2 + 16*b*c^2*x^2 + 32*a*c^3*x^3 + 2*
b*c^3*x^3 + 6*a*c^4*x^4 + 96*b*c*x*ArcTanh[c*x] + 72*b*c^2*x^2*ArcTanh[c*x]
 + 32*b*c^3*x^3*ArcTanh[c*x] + 6*b*c^4*x^4*ArcTanh[c*x] + 24*a*Log[x] + 39*
b*Log[1 - c*x] - 39*b*Log[1 + c*x] + 48*b*Log[1 - c^2*x^2] + 16*b*Log[-1 +
c^2*x^2] - 12*b*PolyLog[2, -(c*x)] + 12*b*PolyLog[2, c*x]))/24
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right) a}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^4 (b\,\text{artanh}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x, x)
```

maple [A] time = 0.05, size = 222, normalized size = 1.20

$$\frac{d^4a\,c^4x^4}{4}+\frac{4d^4a\,c^3x^3}{3}+3d^4a\,c^2x^2+4ac\,d^4x+d^4a\ln(cx)+\frac{d^4b\,\text{arctanh}\,(cx)\,c^4x^4}{4}+\frac{4d^4b\,\text{arctanh}\,(cx)\,c^3x^3}{3}+3d^4b\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x)
```

```
[Out] 1/4*d^4*a*c^4*x^4+4/3*d^4*a*c^3*x^3+3*d^4*a*c^2*x^2+4*a*c*d^4*x+d^4*a*ln(c*
x)+1/4*d^4*b*arctanh(c*x)*c^4*x^4+4/3*d^4*b*arctanh(c*x)*c^3*x^3+3*d^4*b*ar
ctanh(c*x)*c^2*x^2+4*b*c*d^4*x*arctanh(c*x)+d^4*b*arctanh(c*x)*ln(c*x)-1/2*
d^4*b*dilog(c*x)-1/2*d^4*b*dilog(c*x+1)-1/2*d^4*b*ln(c*x)*ln(c*x+1)+1/12*b*
c^3*d^4*x^3+2/3*b*c^2*d^4*x^2+13/4*b*c*d^4*x+103/24*d^4*b*ln(c*x-1)+25/24*d
^4*b*ln(c*x+1)
```

maxima [A] time = 0.47, size = 276, normalized size = 1.49

$$\frac{1}{4}\,ac^4d^4x^4+\frac{4}{3}\,ac^3d^4x^3+\frac{1}{12}\,bc^3d^4x^3+3\,ac^2d^4x^2+\frac{2}{3}\,bc^2d^4x^2+4\,acd^4x+\frac{13}{4}\,bcd^4x+2\left(2\,cx\,\text{artanh}\,(cx) + \log\left(-c^2x^2 +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^4*x^4 + 4/3*a*c^3*d^4*x^3 + 1/12*b*c^3*d^4*x^3 + 3*a*c^2*d^4*x^
2 + 2/3*b*c^2*d^4*x^2 + 4*a*c*d^4*x + 13/4*b*c*d^4*x + 2*(2*c*x*arctanh(c*x
) + log(-c^2*x^2 + 1))*b*d^4 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1
))*b*d^4 + 1/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^4 - 23/24*b*d^
```

```
4*log(c*x + 1) + 55/24*b*d^4*log(c*x - 1) + a*d^4*log(x) + 1/24*(3*b*c^4*d^
4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(c*x + 1) - 1/24*(3*b*c^4*d
^4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(-c*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))\,\,(d + c\,d\,x)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^4 \left(\int 4ac\,dx + \int \frac{a}{x}\,dx + \int 6ac^2x\,dx + \int 4ac^3x^2\,dx + \int ac^4x^3\,dx + \int 4bc\,\mathrm{atanh}\,(cx)\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**4*(Integral(4*a*c, x) + Integral(a/x, x) + Integral(6*a*c**2*x, x) + Int
egral(4*a*c**3*x**2, x) + Integral(a*c**4*x**3, x) + Integral(4*b*c*atanh(c
*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(6*b*c**2*x*atanh(c*x), x)
+ Integral(4*b*c**3*x**2*atanh(c*x), x) + Integral(b*c**4*x**3*atanh(c*x),
x))
```

3.36 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=178

$$\frac{1}{3}c^4d^4x^3\left(a+b\tanh^{-1}(cx)\right)+2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+6ac^2d^4x+4acd^4\log(x)+\frac{1}{6}bc^3d^4x$$

```
[Out] 6*a*c^2*d^4*x+2*b*c^2*d^4*x+1/6*b*c^3*d^4*x^2-2*b*c*d^4*arctanh(c*x)+6*b*c^
2*d^4*x*arctanh(c*x)-d^4*(a+b*arctanh(c*x))/x+2*c^3*d^4*x^2*(a+b*arctanh(c*
x))+1/3*c^4*d^4*x^3*(a+b*arctanh(c*x))+4*a*c*d^4*ln(x)+b*c*d^4*ln(x)+8/3*b*
c*d^4*ln(-c^2*x^2+1)-2*b*c*d^4*polylog(2,-c*x)+2*b*c*d^4*polylog(2,c*x)
```

Rubi [A] time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5940, 5910, 260, 5916, 266, 36, 29, 31, 5912, 321, 206, 43}

$$-2bcd^4\text{PolyLog}(2,-cx)+2bcd^4\text{PolyLog}(2,cx)+\frac{1}{3}c^4d^4x^3\left(a+b\tanh^{-1}(cx)\right)+2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] 6*a*c^2*d^4*x + 2*b*c^2*d^4*x + (b*c^3*d^4*x^2)/6 - 2*b*c*d^4*ArcTanh[c*x]
+ 6*b*c^2*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/x + 2*c^3*d^4*x^2
*(a + b*ArcTanh[c*x]) + (c^4*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + 4*a*c*d^4*Lo
g[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 - c^2*x^2])/3 - 2*b*c*d^4*PolyLog[
2, -(c*x)] + 2*b*c*d^4*PolyLog[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(6c^2d^4\left(a+b\tanh^{-1}(cx)\right) + \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2} + \frac{4cd^4\left(a+b\tanh^{-1}\right.}{x}\right.$$

$$= d^4\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx + \left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx + \left(6c^2d^4\right)\int (a$$

$$= 6ac^2d^4x - \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x} + 2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{1}{3}c^4d^4x^3\,($$

$$= 6ac^2d^4x + 2bc^2d^4x + 6bc^2d^4x\tanh^{-1}(cx) - \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x} + 2c^3d^4x$$

$$= 6ac^2d^4x + 2bc^2d^4x - 2bcd^4\tanh^{-1}(cx) + 6bc^2d^4x\tanh^{-1}(cx) - \frac{d^4\left(a+b\,\mathrm{ta}\right.}{x}$$

$$= 6ac^2d^4x + 2bc^2d^4x + \frac{1}{6}bc^3d^4x^2 - 2bcd^4\tanh^{-1}(cx) + 6bc^2d^4x\tanh^{-1}(cx) -$$

Mathematica [A] time = 0.18, size = 194, normalized size = 1.09

$$\frac{d^4\left(2ac^4x^4 + 12ac^3x^3 + 36ac^2x^2 + 24acx\log(x) - 6a + 2bc^4x^4\tanh^{-1}(cx) + bc^3x^3 + 12bc^3x^3\tanh^{-1}(cx) + 12bc^2x\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^4*(-6*a + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 12*a*c^3*x^3 + b*c^3*x^3 + 2*a*c
^4*x^4 - 6*b*ArcTanh[c*x] + 36*b*c^2*x^2*ArcTanh[c*x] + 12*b*c^3*x^3*ArcTan
h[c*x] + 2*b*c^4*x^4*ArcTanh[c*x] + 24*a*c*x*Log[x] + 6*b*c*x*Log[c*x] + 6*
b*c*x*Log[1 - c*x] - 6*b*c*x*Log[1 + c*x] + 15*b*c*x*Log[1 - c^2*x^2] + b*c
*x*Log[-1 + c^2*x^2] - 12*b*c*x*PolyLog[2, -(c*x)] + 12*b*c*x*PolyLog[2, c*
x]))/(6*x)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right)a}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^4(b\,\mathrm{artanh}\,(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^2, x)
```

maple [A] time = 0.05, size = 229, normalized size = 1.29

$$\frac{d^4 a\, c^4 x^3}{3} + 2 d^4 a\, c^3 x^2 + 6 a\, c^2 d^4 x + 4 c\, d^4 a \ln(cx) - \frac{d^4 a}{x} + \frac{d^4 b \operatorname{arctanh}(cx)\, c^4 x^3}{3} + 2 d^4 b \operatorname{arctanh}(cx)\, c^3 x^2 + 6 b\, c^2 d^4 x \operatorname{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] 1/3*d^4*a*c^4*x^3+2*d^4*a*c^3*x^2+6*a*c^2*d^4*x+4*c*d^4*a*ln(c*x)-d^4*a/x+1
/3*d^4*b*arctanh(c*x)*c^4*x^3+2*d^4*b*arctanh(c*x)*c^3*x^2+6*b*c^2*d^4*x*ar
ctanh(c*x)+4*c*d^4*b*arctanh(c*x)*ln(c*x)-d^4*b*arctanh(c*x)/x-2*c*d^4*b*di
log(c*x)-2*c*d^4*b*dilog(c*x+1)-2*c*d^4*b*ln(c*x)*ln(c*x+1)+1/6*b*c^3*d^4*x
^2+2*b*c^2*d^4*x+c*d^4*b*ln(c*x)+11/3*c*d^4*b*ln(c*x-1)+5/3*c*d^4*b*ln(c*x+
1)
```

maxima [A] time = 0.47, size = 281, normalized size = 1.58

$$\frac{1}{3}\, ac^4 d^4 x^3 + 2\, ac^3 d^4 x^2 + \frac{1}{6}\, bc^3 d^4 x^2 + 6\, ac^2 d^4 x + 2\, bc^2 d^4 x + 3\left(2\, cx \operatorname{artanh}(cx) + \log\left(-c^2 x^2 + 1\right)\right) bcd^4 - 2\left(\log(cx)\, \mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^4*x^3 + 2*a*c^3*d^4*x^2 + 1/6*b*c^3*d^4*x^2 + 6*a*c^2*d^4*x + 2
*b*c^2*d^4*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^4 - 2*(log(
c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*c*d^4 + 2*(log(c*x + 1)*log(-c*x) +
 dilog(c*x + 1))*b*c*d^4 - 5/6*b*c*d^4*log(c*x + 1) + 7/6*b*c*d^4*log(c*x -
 1) + 4*a*c*d^4*log(x) - 1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c
*x)/x)*b*d^4 - a*d^4/x + 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(c*x + 1)
 - 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(-c*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\, x))\, (d + c\, d\, x)^4}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^4 \left(\int 6ac^2\, dx + \int \frac{a}{x^2}\, dx + \int \frac{4ac}{x}\, dx + \int 4ac^3 x\, dx + \int ac^4 x^2\, dx + \int 6bc^2 \operatorname{atanh}(cx)\, dx + \int \frac{b \operatorname{atanh}(cx)}{x^2}\, d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**4*(Integral(6*a*c**2, x) + Integral(a/x**2, x) + Integral(4*a*c/x, x) +
Integral(4*a*c**3*x, x) + Integral(a*c**4*x**2, x) + Integral(6*b*c**2*atan
h(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(4*b*c*atanh(c*x)/x,
x) + Integral(4*b*c**3*x*atanh(c*x), x) + Integral(b*c**4*x**2*atanh(c*x),
x))
```

3.37 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=156

$$\frac{1}{2}c^4d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x}+4ac^3d^4x+6ac^2d^4\log(x)+\frac{1}{2}bc^3d^4x+$$

```
[Out] -1/2*b*c*d^4/x+4*a*c^3*d^4*x+1/2*b*c^3*d^4*x+4*b*c^3*d^4*x*arctanh(c*x)-1/2
*d^4*(a+b*arctanh(c*x))/x^2-4*c*d^4*(a+b*arctanh(c*x))/x+1/2*c^4*d^4*x^2*(a
+b*arctanh(c*x))+6*a*c^2*d^4*ln(x)+4*b*c^2*d^4*ln(x)-3*b*c^2*d^4*polylog(2,
-c*x)+3*b*c^2*d^4*polylog(2,c*x)
```

Rubi [A] time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5940, 5910, 260, 5916, 325, 206, 266, 36, 29, 31, 5912, 321}

$$-3bc^2d^4\text{PolyLog}(2,-cx)+3bc^2d^4\text{PolyLog}(2,cx)+\frac{1}{2}c^4d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^4)/(2*x) + 4*a*c^3*d^4*x + (b*c^3*d^4*x)/2 + 4*b*c^3*d^4*x*ArcTanh[
c*x] - (d^4*(a + b*ArcTanh[c*x]))/(2*x^2) - (4*c*d^4*(a + b*ArcTanh[c*x]))/
x + (c^4*d^4*x^2*(a + b*ArcTanh[c*x]))/2 + 6*a*c^2*d^4*Log[x] + 4*b*c^2*d^4
*Log[x] - 3*b*c^2*d^4*PolyLog[2, -(c*x)] + 3*b*c^2*d^4*PolyLog[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int \left(4c^3d^4\left(a+b\tanh^{-1}(cx)\right) + \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^3} + \frac{4cd^4\left(a+b\tanh^{-1}\right.}{x^2}\right.$$

$$= d^4 \int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx + \left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx + \left(6c^2d^4\right)\int \frac{a\,-}{}$$

$$= 4ac^3d^4x - \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}c^4d^4x^2\left(a+\right.$$

$$= -\frac{bcd^4}{2x} + 4ac^3d^4x + \frac{1}{2}bc^3d^4x + 4bc^3d^4x\tanh^{-1}(cx) - \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd^4}{2x} + 4ac^3d^4x + \frac{1}{2}bc^3d^4x + 4bc^3d^4x\tanh^{-1}(cx) - \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd^4}{2x} + 4ac^3d^4x + \frac{1}{2}bc^3d^4x + 4bc^3d^4x\tanh^{-1}(cx) - \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}$$

Mathematica [A] time = 0.17, size = 143, normalized size = 0.92

$$\frac{d^4\left(ac^4x^4 + 8ac^3x^3 + 12ac^2x^2\log(x) - 8acx - a + bc^4x^4\tanh^{-1}(cx) + bc^3x^3 + 8bc^3x^3\tanh^{-1}(cx) - 6bc^2x^2\mathrm{Li}_2(-cx\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^4*(-a - 8*a*c*x - b*c*x + 8*a*c^3*x^3 + b*c^3*x^3 + a*c^4*x^4 - b*ArcTan
h[c*x] - 8*b*c*x*ArcTanh[c*x] + 8*b*c^3*x^3*ArcTanh[c*x] + b*c^4*x^4*ArcTan
h[c*x] + 12*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - 6*b*c^2*x^2*PolyLog[2
, -(c*x)] + 6*b*c^2*x^2*PolyLog[2, c*x]))/(2*x^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right)a}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^4(b\,\mathrm{artanh}\,(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^3, x)
```

maple [A] time = 0.05, size = 210, normalized size = 1.35

$$\frac{c^4d^4a\,x^2}{2} + 4a\,c^3d^4x + 6c^2d^4a\ln(cx) - \frac{4c\,d^4a}{x} - \frac{d^4a}{2x^2} + \frac{c^4d^4b\,\mathrm{arctanh}\,(cx)\,x^2}{2} + 4b\,c^3d^4x\,\mathrm{arctanh}\,(cx) + 6c^2d^4b\,\mathrm{arctanh}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] 1/2*c^4*d^4*a*x^2+4*a*c^3*d^4*x+6*c^2*d^4*a*ln(c*x)-4*c*d^4*a/x-1/2*d^4*a/x
^2+1/2*c^4*d^4*b*arctanh(c*x)*x^2+4*b*c^3*d^4*x*arctanh(c*x)+6*c^2*d^4*b*ar
ctanh(c*x)*ln(c*x)-4*c*d^4*b*arctanh(c*x)/x-1/2*d^4*b*arctanh(c*x)/x^2-3*c^
2*d^4*b*dilog(c*x)-3*c^2*d^4*b*dilog(c*x+1)-3*c^2*d^4*b*ln(c*x)*ln(c*x+1)+1
/2*b*c^3*d^4*x+4*c^2*d^4*b*ln(c*x)-1/2*b*c*d^4/x
```

maxima [B] time = 0.48, size = 293, normalized size = 1.88

$$\frac{1}{4}\,bc^4d^4x^2\log\left(cx+1\right)-\frac{1}{4}\,bc^4d^4x^2\log\left(-cx+1\right)+\frac{1}{2}\,ac^4d^4x^2+4\,ac^3d^4x+\frac{1}{2}\,bc^3d^4x+2\left(2\,cx\,\mathrm{artanh}\left(cx\right)+\log\left(-c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^4*d^4*x^2*log(c*x + 1) - 1/4*b*c^4*d^4*x^2*log(-c*x + 1) + 1/2*a*c^
4*d^4*x^2 + 4*a*c^3*d^4*x + 1/2*b*c^3*d^4*x + 2*(2*c*x*arctanh(c*x) + log(-
c^2*x^2 + 1))*b*c^2*d^4 - 3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*c^
2*d^4 + 3*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c^2*d^4 - 1/4*b*c^2*d
^4*log(c*x + 1) + 1/4*b*c^2*d^4*log(c*x - 1) + 6*a*c^2*d^4*log(x) - 2*(c*(l
og(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^4 + 1/4*((c*log(c*x +
 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^4 - 4*a*c*d^4/x - 1
/2*a*d^4/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)\,{\left(d+c\,d\,x\right)}^4}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{4}\left(\int 4ac^{3}\, dx + \int \frac{a}{x^{3}}\, dx + \int \frac{4ac}{x^{2}}\, dx + \int \frac{6ac^{2}}{x}\, dx + \int ac^{4}x\, dx + \int 4bc^{3}\,\mathrm{atanh}\left(cx\right) dx + \int \frac{b\,\mathrm{atanh}\left(cx\right)}{x^{3}}\, dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**4*(Integral(4*a*c**3, x) + Integral(a/x**3, x) + Integral(4*a*c/x**2, x)
 + Integral(6*a*c**2/x, x) + Integral(a*c**4*x, x) + Integral(4*b*c**3*atan
h(c*x), x) + Integral(b*atanh(c*x)/x**3, x) + Integral(4*b*c*atanh(c*x)/x**
2, x) + Integral(6*b*c**2*atanh(c*x)/x, x) + Integral(b*c**4*x*atanh(c*x),
x))
```

3.38 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}+ac^4d^4x+4ac^3d^4\log(x)+bc^4d^4x\tanh$$

```
[Out] -1/6*b*c*d^4/x^2-2*b*c^2*d^4/x+a*c^4*d^4*x+2*b*c^3*d^4*arctanh(c*x)+b*c^4*d
^4*x*arctanh(c*x)-1/3*d^4*(a+b*arctanh(c*x))/x^3-2*c*d^4*(a+b*arctanh(c*x))
/x^2-6*c^2*d^4*(a+b*arctanh(c*x))/x+4*a*c^3*d^4*ln(x)+19/3*b*c^3*d^4*ln(x)-
8/3*b*c^3*d^4*ln(-c^2*x^2+1)-2*b*c^3*d^4*polylog(2,-c*x)+2*b*c^3*d^4*polylo
g(2,c*x)
```

Rubi [A] time = 0.22, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5940, 5910, 260, 5916, 266, 44, 325, 206, 36, 29, 31, 5912}

$$-2bc^3d^4\text{PolyLog}(2,-cx)+2bc^3d^4\text{PolyLog}(2,cx)-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{d^4\left(a+b\text{ta}\right.}{3:}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^4)/(6*x^2) - (2*b*c^2*d^4)/x + a*c^4*d^4*x + 2*b*c^3*d^4*ArcTanh[c*
x] + b*c^4*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/(3*x^3) - (2*c*d
^4*(a + b*ArcTanh[c*x]))/x^2 - (6*c^2*d^4*(a + b*ArcTanh[c*x]))/x + 4*a*c^3
*d^4*Log[x] + (19*b*c^3*d^4*Log[x])/3 - (8*b*c^3*d^4*Log[1 - c^2*x^2])/3 -
2*b*c^3*d^4*PolyLog[2, -(c*x)] + 2*b*c^3*d^4*PolyLog[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = \int \left(c^4d^4 \left(a+b\tanh^{-1}(cx)\right) + \frac{d^4 \left(a+b\tanh^{-1}(cx)\right)}{x^4} + \frac{4cd^4 \left(a+b\tanh^{-1}(\right.}{x^3}\right.$$

$$= d^4 \int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx + \left(4cd^4\right) \int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx + \left(6c^2d^4\right) \int \frac{a}{}$$

$$= ac^4d^4x - \frac{d^4 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{2cd^4 \left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{6c^2d^4 \left(a+b\,\mathrm{ta}\right.}{x}$$

$$= -\frac{2bc^2d^4}{x} + ac^4d^4x + bc^4d^4x\tanh^{-1}(cx) - \frac{d^4 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{2cd^4 \left(a\right.}{}$$

$$= -\frac{2bc^2d^4}{x} + ac^4d^4x + 2bc^3d^4\tanh^{-1}(cx) + bc^4d^4x\tanh^{-1}(cx) - \frac{d^4 \left(a+b\,\mathrm{ta}\right.}{3x}$$

$$= -\frac{bcd^4}{6x^2} - \frac{2bc^2d^4}{x} + ac^4d^4x + 2bc^3d^4\tanh^{-1}(cx) + bc^4d^4x\tanh^{-1}(cx) - \frac{d^4 (}{}$$

Mathematica [A] time = 0.18, size = 197, normalized size = 1.04

$$d^4 \left(6ac^4x^4 + 24ac^3x^3\log(x) - 36ac^2x^2 - 12acx - 2a + 6bc^4x^4\tanh^{-1}(cx) - 12bc^3x^3\mathrm{Li}_2(-cx) + 12bc^3x^3\mathrm{Li}_2(cx) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] (d^4*(-2*a - 12*a*c*x - b*c*x - 36*a*c^2*x^2 - 12*b*c^2*x^2 + 6*a*c^4*x^4 -
 2*b*ArcTanh[c*x] - 12*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 6*b
*c^4*x^4*ArcTanh[c*x] + 24*a*c^3*x^3*Log[x] + 38*b*c^3*x^3*Log[c*x] - 6*b*c
^3*x^3*Log[1 - c*x] + 6*b*c^3*x^3*Log[1 + c*x] - 16*b*c^3*x^3*Log[1 - c^2*x
^2] - 12*b*c^3*x^3*PolyLog[2, -(c*x)] + 12*b*c^3*x^3*PolyLog[2, c*x]))/(6*x
^3)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right) a}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^4 (b\,\mathrm{artanh}\,(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^4, x)
```

maple [A] time = 0.06, size = 240, normalized size = 1.27

$$a\,c^4d^4x+4c^3d^4a\ln(cx)-\frac{6c^2d^4a}{x}-\frac{d^4a}{3x^3}-\frac{2c\,d^4a}{x^2}+b\,c^4d^4x\operatorname{arctanh}(cx)+4c^3d^4b\operatorname{arctanh}(cx)\ln(cx)-\frac{6c^2d^4b\operatorname{arctan}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] a*c^4*d^4*x+4*c^3*d^4*a*ln(c*x)-6*c^2*d^4*a/x-1/3*d^4*a/x^3-2*c*d^4*a/x^2+b
*c^4*d^4*x*arctanh(c*x)+4*c^3*d^4*b*arctanh(c*x)*ln(c*x)-6*c^2*d^4*b*arctan
h(c*x)/x-1/3*d^4*b*arctanh(c*x)/x^3-2*c*d^4*b*arctanh(c*x)/x^2-2*c^3*d^4*b*
dilog(c*x)-2*c^3*d^4*b*dilog(c*x+1)-2*c^3*d^4*b*ln(c*x)*ln(c*x+1)-1/6*b*c*d
^4/x^2-2*b*c^2*d^4/x+19/3*c^3*d^4*b*ln(c*x)-11/3*c^3*d^4*b*ln(c*x-1)-5/3*c^
3*d^4*b*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$ac^4d^4x+\frac{1}{2}\left(2\,cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)bc^3d^4+2\,bc^3d^4\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx+4\,ac^3d^4\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^4*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^3*d^4 + 2*b*
c^3*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 4*a*c^3*d^4*log(x)
 - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^4 + ((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^4 - 1/6*(
(c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b*d^4
 - 6*a*c^2*d^4/x - 2*a*c*d^4/x^2 - 1/3*a*d^4/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{atanh}(c\,x))\,(d+c\,d\,x)^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^4\left(\int ac^4\,dx+\int\frac{a}{x^4}\,dx+\int\frac{4ac}{x^3}\,dx+\int\frac{6ac^2}{x^2}\,dx+\int\frac{4ac^3}{x}\,dx+\int bc^4\operatorname{atanh}(cx)\,dx+\int\frac{b\operatorname{atanh}(cx)}{x^4}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] d**4*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(4*a*c/x**3, x) +
 Integral(6*a*c**2/x**2, x) + Integral(4*a*c**3/x, x) + Integral(b*c**4*ata
nh(c*x), x) + Integral(b*atanh(c*x)/x**4, x) + Integral(4*b*c*atanh(c*x)/x*
*3, x) + Integral(6*b*c**2*atanh(c*x)/x**2, x) + Integral(4*b*c**3*atanh(c*
x)/x, x))
```

3.39 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{4c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{3c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+ac^4d^4\log$$

```
[Out] -1/12*b*c*d^4/x^3-2/3*b*c^2*d^4/x^2-13/4*b*c^3*d^4/x+13/4*b*c^4*d^4*arctanh
(c*x)-1/4*d^4*(a+b*arctanh(c*x))/x^4-4/3*c*d^4*(a+b*arctanh(c*x))/x^3-3*c^2
*d^4*(a+b*arctanh(c*x))/x^2-4*c^3*d^4*(a+b*arctanh(c*x))/x+a*c^4*d^4*ln(x)+
16/3*b*c^4*d^4*ln(x)-8/3*b*c^4*d^4*ln(-c^2*x^2+1)-1/2*b*c^4*d^4*polylog(2,-
c*x)+1/2*b*c^4*d^4*polylog(2,c*x)
```

Rubi [A] time = 0.23, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5940, 5916, 325, 206, 266, 44, 36, 29, 31, 5912}

$$-\frac{1}{2}bc^4d^4\text{PolyLog}(2,-cx)+\frac{1}{2}bc^4d^4\text{PolyLog}(2,cx)-\frac{3c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{4c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{4cd^4\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^4)/(12*x^3) - (2*b*c^2*d^4)/(3*x^2) - (13*b*c^3*d^4)/(4*x) + (13*b*
c^4*d^4*ArcTanh[c*x])/4 - (d^4*(a + b*ArcTanh[c*x]))/(4*x^4) - (4*c*d^4*(a
+ b*ArcTanh[c*x]))/(3*x^3) - (3*c^2*d^4*(a + b*ArcTanh[c*x]))/x^2 - (4*c^3*
d^4*(a + b*ArcTanh[c*x]))/x + a*c^4*d^4*Log[x] + (16*b*c^4*d^4*Log[x])/3 -
(8*b*c^4*d^4*Log[1 - c^2*x^2])/3 - (b*c^4*d^4*PolyLog[2, -(c*x)])/2 + (b*c^
4*d^4*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx = \int \left(\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}+\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}+\frac{6c^2d^4\left(a+b\tan\right.}{x^3}\right.$$

$$= d^4\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx+\left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\left(6c^2d^4\right)\int$$

$$= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^4\left(a+b\tanh^{-}\right.}{x^2}$$

$$= -\frac{bcd^4}{12x^3}-\frac{3bc^3d^4}{x}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3}{}$$

$$= -\frac{bcd^4}{12x^3}-\frac{13bc^3d^4}{4x}+3bc^4d^4\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4(}{}$$

$$= -\frac{bcd^4}{12x^3}-\frac{2bc^2d^4}{3x^2}-\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(c\right.}{4x^4}$$

Mathematica [A] time = 0.16, size = 206, normalized size = 0.99

$$\frac{d^4\left(24ac^4x^4\log(x)-96ac^3x^3-72ac^2x^2-32acx-6a-12bc^4x^4\text{Li}_2(-cx)+12bc^4x^4\text{Li}_2(cx)+128bc^4x^4\log(cx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] (d^4*(-6*a - 32*a*c*x - 2*b*c*x - 72*a*c^2*x^2 - 16*b*c^2*x^2 - 96*a*c^3*x^
3 - 78*b*c^3*x^3 - 6*b*ArcTanh[c*x] - 32*b*c*x*ArcTanh[c*x] - 72*b*c^2*x^2*
ArcTanh[c*x] - 96*b*c^3*x^3*ArcTanh[c*x] + 24*a*c^4*x^4*Log[x] + 128*b*c^4*
x^4*Log[c*x] - 39*b*c^4*x^4*Log[1 - c*x] + 39*b*c^4*x^4*Log[1 + c*x] - 64*b
*c^4*x^4*Log[1 - c^2*x^2] - 12*b*c^4*x^4*PolyLog[2, -(c*x)] + 12*b*c^4*x^4*
PolyLog[2, c*x]))/(24*x^4)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right) a}{x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^4 (b\,\mathrm{artanh}\,(cx) + a)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^5, x)
```

maple [A] time = 0.06, size = 256, normalized size = 1.22

$$c^4d^4a\ln(cx) - \frac{4c^3d^4a}{x} - \frac{4c\,d^4a}{3x^3} - \frac{3c^2d^4a}{x^2} - \frac{d^4a}{4x^4} + c^4d^4b\,\mathrm{arctanh}\,(cx)\ln(cx) - \frac{4c^3d^4b\,\mathrm{arctanh}\,(cx)}{x} - \frac{4c\,d^4b\,\mathrm{arctanh}\,(cx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] c^4*d^4*a*ln(c*x)-4*c^3*d^4*a/x-4/3*c*d^4*a/x^3-3*c^2*d^4*a/x^2-1/4*d^4*a/x
^4+c^4*d^4*b*arctanh(c*x)*ln(c*x)-4*c^3*d^4*b*arctanh(c*x)/x-4/3*c*d^4*b*ar
ctanh(c*x)/x^3-3*c^2*d^4*b*arctanh(c*x)/x^2-1/4*d^4*b*arctanh(c*x)/x^4-1/12
*b*c*d^4/x^3-2/3*b*c^2*d^4/x^2-13/4*b*c^3*d^4/x+16/3*c^4*d^4*b*ln(c*x)-103/
24*c^4*d^4*b*ln(c*x-1)-25/24*c^4*d^4*b*ln(c*x+1)-1/2*c^4*d^4*b*dilog(c*x)-1
/2*c^4*d^4*b*dilog(c*x+1)-1/2*c^4*d^4*b*ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,bc^4d^4\int\frac{\log(cx+1) - \log(-cx+1)}{x}\,dx + ac^4d^4\log(x) - 2\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\mathrm{artanh}\,(cx)}{x}\right)bc^3d^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^4*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^4*d^4*lo
g(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^4 + 3
```

```
/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*d
^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/
x^3)*b*c*d^4 - 4*a*c^3*d^4/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x -
1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^4 - 3*a*c^2*d^4/x^2
 - 4/3*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(c\,x))\,\left(d+c\,d\,x\right)^4}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^5,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^4\left(\int \frac{a}{x^5}\,dx + \int \frac{4ac}{x^4}\,dx + \int \frac{6ac^2}{x^3}\,dx + \int \frac{4ac^3}{x^2}\,dx + \int \frac{ac^4}{x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x^5}\,dx + \int \frac{4bc\,\mathrm{atanh}\,(cx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] d**4*(Integral(a/x**5, x) + Integral(4*a*c/x**4, x) + Integral(6*a*c**2/x**
3, x) + Integral(4*a*c**3/x**2, x) + Integral(a*c**4/x, x) + Integral(b*ata
nh(c*x)/x**5, x) + Integral(4*b*c*atanh(c*x)/x**4, x) + Integral(6*b*c**2*a
tanh(c*x)/x**3, x) + Integral(4*b*c**3*atanh(c*x)/x**2, x) + Integral(b*c**
4*atanh(c*x)/x, x))
```

3.40 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=109

$$-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(1-cx)-\frac{3bc^4d^4}{x}-\frac{11bc^3d^4}{10x^2}-\frac{bc^2d^4}{3x^3}-\frac{bcd^4}{20x^4}$$

```
[Out] -1/20*b*c*d^4/x^4-1/3*b*c^2*d^4/x^3-11/10*b*c^3*d^4/x^2-3*b*c^4*d^4/x-1/5*d
^4*(c*x+1)^5*(a+b*arctanh(c*x))/x^5+16/5*b*c^5*d^4*ln(x)-16/5*b*c^5*d^4*ln(
-c*x+1)
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {37, 5936, 12, 88}

$$-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{11bc^3d^4}{10x^2}-\frac{bc^2d^4}{3x^3}-\frac{3bc^4d^4}{x}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(1-cx)-\frac{bcd^4}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^4)/(20*x^4) - (b*c^2*d^4)/(3*x^3) - (11*b*c^3*d^4)/(10*x^2) - (3*b*
c^4*d^4)/x - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*x^5) + (16*b*c^5*d^4
*Log[x])/5 - (16*b*c^5*d^4*Log[1 - c*x])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - (bc)\int \frac{(d+cdx)^4}{5x^5(-1+cx)}\,dx$$
$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{1}{5}(bc)\int \frac{(d+cdx)^4}{x^5(-1+cx)}\,dx$$
$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{1}{5}(bc)\int \left(-\frac{d^4}{x^5} - \frac{5cd^4}{x^4} - \frac{11c^2d^4}{x^3} - 15$$
$$= -\frac{bcd^4}{20x^4} - \frac{bc^2d^4}{3x^3} - \frac{11bc^3d^4}{10x^2} - \frac{3bc^4d^4}{x} - \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} +$$

Mathematica [A] time = 0.15, size = 157, normalized size = 1.44

$$-\frac{d^4\left(60ac^4x^4+120ac^3x^3+120ac^2x^2+60acx+12a-192bc^5x^5\log(x)+186bc^5x^5\log(1-cx)+6bc^5x^5\log(cx\right.}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/60*(d^4*(12*a + 60*a*c*x + 3*b*c*x + 120*a*c^2*x^2 + 20*b*c^2*x^2 + 120*
a*c^3*x^3 + 66*b*c^3*x^3 + 60*a*c^4*x^4 + 180*b*c^4*x^4 + 12*b*(1 + 5*c*x +
 10*c^2*x^2 + 10*c^3*x^3 + 5*c^4*x^4)*ArcTanh[c*x] - 192*b*c^5*x^5*Log[x] +
 186*b*c^5*x^5*Log[1 - c*x] + 6*b*c^5*x^5*Log[1 + c*x]))/x^5
```

fricas [A] time = 0.48, size = 191, normalized size = 1.75

$$-\frac{6\,bc^5d^4x^5\log\left(cx+1\right)+186\,bc^5d^4x^5\log\left(cx-1\right)-192\,bc^5d^4x^5\log(x)+60\,(a+3\,b)c^4d^4x^4+6\,(20\,a+11\,b)c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(6*b*c^5*d^4*x^5*log(c*x + 1) + 186*b*c^5*d^4*x^5*log(c*x - 1) - 192*
b*c^5*d^4*x^5*log(x) + 60*(a + 3*b)*c^4*d^4*x^4 + 6*(20*a + 11*b)*c^3*d^4*x
^3 + 20*(6*a + b)*c^2*d^4*x^2 + 3*(20*a + b)*c*d^4*x + 12*a*d^4 + 6*(5*b*c^
4*d^4*x^4 + 10*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x + b*d^4)*log(
-(c*x + 1)/(c*x - 1)))/x^5
```

giac [B] time = 0.23, size = 532, normalized size = 4.88

$$\frac{4}{15}\left(12\,bc^4d^4\log\left(-\frac{cx+1}{cx-1}-1\right)-12\,bc^4d^4\log\left(-\frac{cx+1}{cx-1}\right)+\frac{12\left(\frac{5\,(cx+1)^4bc^4d^4}{(cx-1)^4}+\frac{10\,(cx+1)^3bc^4d^4}{(cx-1)^3}+\frac{10\,(cx+1)^2bc^4d^4}{(cx-1)^2}+5\right.}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx}{(cx-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(12*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 12*b*c^4*d^4*log(-(c*x +
 1)/(c*x - 1)) + 12*(5*(c*x + 1)^4*b*c^4*d^4/(c*x - 1)^4 + 10*(c*x + 1)^3*b
*c^4*d^4/(c*x - 1)^3 + 10*(c*x + 1)^2*b*c^4*d^4/(c*x - 1)^2 + 5*(c*x + 1)*b
*c^4*d^4/(c*x - 1) + b*c^4*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x
 - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x
+ 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (120*(c*x + 1)^4*a*c^4*d^
4/(c*x - 1)^4 + 240*(c*x + 1)^3*a*c^4*d^4/(c*x - 1)^3 + 240*(c*x + 1)^2*a*c
```

```
^4*d^4/(c*x - 1)^2 + 120*(c*x + 1)*a*c^4*d^4/(c*x - 1) + 24*a*c^4*d^4 + 48*
(c*x + 1)^4*b*c^4*d^4/(c*x - 1)^4 + 156*(c*x + 1)^3*b*c^4*d^4/(c*x - 1)^3 +
 196*(c*x + 1)^2*b*c^4*d^4/(c*x - 1)^2 + 113*(c*x + 1)*b*c^4*d^4/(c*x - 1)
+ 25*b*c^4*d^4)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(
c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1)
 + 1))*c
```

maple [B] time = 0.04, size = 221, normalized size = 2.03

$$-\frac{c^4d^4a}{x}-\frac{2c^2d^4a}{x^3}-\frac{2c^3d^4a}{x^2}-\frac{c\,d^4a}{x^4}-\frac{d^4a}{5x^5}-\frac{c^4d^4b\,\mathrm{arctanh}\,(cx)}{x}-\frac{2c^2d^4b\,\mathrm{arctanh}\,(cx)}{x^3}-\frac{2c^3d^4b\,\mathrm{arctanh}\,(cx)}{x^2}-\frac{c\,d^4b\,\mathrm{arct}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -c^4*d^4*a/x-2*c^2*d^4*a/x^3-2*c^3*d^4*a/x^2-c*d^4*a/x^4-1/5*d^4*a/x^5-c^4*
d^4*b*arctanh(c*x)/x-2*c^2*d^4*b*arctanh(c*x)/x^3-2*c^3*d^4*b*arctanh(c*x)/
x^2-c*d^4*b*arctanh(c*x)/x^4-1/5*d^4*b*arctanh(c*x)/x^5-1/20*b*c*d^4/x^4-1/
3*b*c^2*d^4/x^3-11/10*b*c^3*d^4/x^2-3*b*c^4*d^4/x+16/5*c^5*d^4*b*ln(c*x)-31
/10*c^5*d^4*b*ln(c*x-1)-1/10*c^5*d^4*b*ln(c*x+1)
```

maxima [B] time = 0.33, size = 299, normalized size = 2.74

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}\,(cx)}{x}\right)bc^4d^4+\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\,(cx)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^4*d^4 + ((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3*d^4 - ((c
^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b*c^2*d
^4 - a*c^4*d^4/x + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2
*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^4 - 1/20*((2*c^4*log(c^2*x^2 -
 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^4 -
 2*a*c^3*d^4/x^2 - 2*a*c^2*d^4/x^3 - a*c*d^4/x^4 - 1/5*a*d^4/x^5
```

mupad [B] time = 0.96, size = 179, normalized size = 1.64

$$\frac{d^4\,\left(180\,b\,c^5\,\mathrm{atanh}\left(c\,x\right)-96\,b\,c^5\,\ln\left(c^2\,x^2-1\right)+192\,b\,c^5\,\ln(x)\right)}{60}-\frac{\frac{d^4\,(12\,a+12\,b\,\mathrm{atanh}(c\,x))}{60}+\frac{d^4\,x\,(60\,a\,c+3\,b\,c+60\,b\,c\,\mathrm{atanh}(c\,x}{60}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^6,x)
```

```
[Out] (d^4*(180*b*c^5*atanh(c*x) - 96*b*c^5*log(c^2*x^2 - 1) + 192*b*c^5*log(x)))
/60 - ((d^4*(12*a + 12*b*atanh(c*x)))/60 + (d^4*x*(60*a*c + 3*b*c + 60*b*c*
atanh(c*x)))/60 + (d^4*x^2*(120*a*c^2 + 20*b*c^2 + 120*b*c^2*atanh(c*x)))/6
0 + (d^4*x^4*(60*a*c^4 + 180*b*c^4 + 60*b*c^4*atanh(c*x)))/60 + (d^4*x^3*(1
20*a*c^3 + 66*b*c^3 + 120*b*c^3*atanh(c*x)))/60)/x^5
```

sympy [A] time = 2.70, size = 253, normalized size = 2.32

$$\begin{cases} -\frac{ac^4d^4}{x}-\frac{2ac^3d^4}{x^2}-\frac{2ac^2d^4}{x^3}-\frac{acd^4}{x^4}-\frac{ad^4}{5x^5}+\frac{16bc^5d^4\log\left(x\right)}{5}-\frac{16bc^5d^4\log\left(x-\frac{1}{c}\right)}{5}-\frac{bc^5d^4\,\mathrm{atanh}\,(cx)}{5}-\frac{bc^4d^4\,\mathrm{atanh}\,(cx)}{x}-\frac{3bc^4d^4}{x}-\frac{2bc^3}{} \\ -\frac{ad^4}{5x^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**4*d**4/x - 2*a*c**3*d**4/x**2 - 2*a*c**2*d**4/x**3 - a*c*d
**4/x**4 - a*d**4/(5*x**5) + 16*b*c**5*d**4*log(x)/5 - 16*b*c**5*d**4*log(x
 - 1/c)/5 - b*c**5*d**4*atanh(c*x)/5 - b*c**4*d**4*atanh(c*x)/x - 3*b*c**4*
d**4/x - 2*b*c**3*d**4*atanh(c*x)/x**2 - 11*b*c**3*d**4/(10*x**2) - 2*b*c**
2*d**4*atanh(c*x)/x**3 - b*c**2*d**4/(3*x**3) - b*c*d**4*atanh(c*x)/x**4 -
b*c*d**4/(20*x**4) - b*d**4*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**4/(5*x**
5), True))
```

3.41 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}+\frac{32}{15}bc^6d^4\log(x)-\frac{32}{15}bc^6d^4\log(1-cx)-\frac{13bc^5d^4}{6x}$$

```
[Out] -1/30*b*c*d^4/x^5-1/5*b*c^2*d^4/x^4-5/9*b*c^3*d^4/x^3-16/15*b*c^4*d^4/x^2-1
3/6*b*c^5*d^4/x-1/6*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/x^6+1/30*c*d^4*(c*x+1)
^5*(a+b*arctanh(c*x))/x^5+32/15*b*c^6*d^4*ln(x)-32/15*b*c^6*d^4*ln(-c*x+1)
```

Rubi [A] time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {45, 37, 5936, 12, 148}

$$\frac{cd^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{16bc^4d^4}{15x^2}-\frac{5bc^3d^4}{9x^3}-\frac{bc^2d^4}{5x^4}-\frac{13bc^5d^4}{6x}+\frac{32}{15}bc^6d^4\log$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -(b*c*d^4)/(30*x^5) - (b*c^2*d^4)/(5*x^4) - (5*b*c^3*d^4)/(9*x^3) - (16*b*c
^4*d^4)/(15*x^2) - (13*b*c^5*d^4)/(6*x) - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c
*x]))/(6*x^6) + (c*d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(30*x^5) + (32*b*c
^6*d^4*Log[x])/15 - (32*b*c^6*d^4*Log[1 - c*x])/15
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx = -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-(b\ldots$$
$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-\frac{1}{30\ldots}$$
$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-\frac{1}{30\ldots}$$
$$= -\frac{bcd^4}{30x^5}-\frac{bc^2d^4}{5x^4}-\frac{5bc^3d^4}{9x^3}-\frac{16bc^4d^4}{15x^2}-\frac{13bc^5d^4}{6x}-\frac{d^4(1+cx)^5\left(a+b\tan\ldots\right.}{6x^6}$$

Mathematica [A] time = 0.15, size = 166, normalized size = 1.10

$$-\frac{d^4\left(90ac^4x^4+240ac^3x^3+270ac^2x^2+144acx+30a-384bc^6x^6\log(x)+387bc^6x^6\log(1-cx)-3bc^6x^6\log(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -1/180*(d^4*(30*a + 144*a*c*x + 6*b*c*x + 270*a*c^2*x^2 + 36*b*c^2*x^2 + 24
0*a*c^3*x^3 + 100*b*c^3*x^3 + 90*a*c^4*x^4 + 192*b*c^4*x^4 + 390*b*c^5*x^5
+ 6*b*(5 + 24*c*x + 45*c^2*x^2 + 40*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] - 38
4*b*c^6*x^6*Log[x] + 387*b*c^6*x^6*Log[1 - c*x] - 3*b*c^6*x^6*Log[1 + c*x])
)/x^6
```

fricas [A] time = 0.47, size = 208, normalized size = 1.38

$$\frac{3\,bc^6d^4x^6\log(cx+1)-387\,bc^6d^4x^6\log(cx-1)+384\,bc^6d^4x^6\log(x)-390\,bc^5d^4x^5-6\,(15\,a+32\,b)c^4d^4x^4-\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/180*(3*b*c^6*d^4*x^6*log(c*x + 1) - 387*b*c^6*d^4*x^6*log(c*x - 1) + 384*
b*c^6*d^4*x^6*log(x) - 390*b*c^5*d^4*x^5 - 6*(15*a + 32*b)*c^4*d^4*x^4 - 20
*(12*a + 5*b)*c^3*d^4*x^3 - 18*(15*a + 2*b)*c^2*d^4*x^2 - 6*(24*a + b)*c*d^
4*x - 30*a*d^4 - 3*(15*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 45*b*c^2*d^4*x^2
+ 24*b*c*d^4*x + 5*b*d^4)*log(-(c*x + 1)/(c*x - 1)))/x^6
```

giac [B] time = 0.56, size = 634, normalized size = 4.20

$$\frac{8}{45}\left(12\,bc^5d^4\log\left(-\frac{cx+1}{cx-1}-1\right)-12\,bc^5d^4\log\left(-\frac{cx+1}{cx-1}\right)+\frac{6\left(\frac{15\,(cx+1)^5bc^5d^4}{(cx-1)^5}+\frac{30\,(cx+1)^4bc^5d^4}{(cx-1)^4}+\frac{40\,(cx+1)^3bc^5d^4}{(cx-1)^3}+\ldots\right.}{\frac{(cx+1)^6}{(cx-1)^6}+\frac{6\,(cx+1)^5}{(cx-1)^5}+\frac{15\,(cx+1)^4}{(cx-1)^4}+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")
```

```
[Out] 8/45*(12*b*c^5*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 12*b*c^5*d^4*log(-(c*x +
 1)/(c*x - 1)) + 6*(15*(c*x + 1)^5*b*c^5*d^4/(c*x - 1)^5 + 30*(c*x + 1)^4*b
*c^5*d^4/(c*x - 1)^4 + 40*(c*x + 1)^3*b*c^5*d^4/(c*x - 1)^3 + 30*(c*x + 1)^
2*b*c^5*d^4/(c*x - 1)^2 + 12*(c*x + 1)*b*c^5*d^4/(c*x - 1) + 2*b*c^5*d^4)*l
og(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x - 1)
^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x + 1)
^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1) + (180*(c*x + 1)^5*a*c^5*d^4/(c
*x - 1)^5 + 360*(c*x + 1)^4*a*c^5*d^4/(c*x - 1)^4 + 480*(c*x + 1)^3*a*c^5*d
^4/(c*x - 1)^3 + 360*(c*x + 1)^2*a*c^5*d^4/(c*x - 1)^2 + 144*(c*x + 1)*a*c^
5*d^4/(c*x - 1) + 24*a*c^5*d^4 + 78*(c*x + 1)^5*b*c^5*d^4/(c*x - 1)^5 + 294
*(c*x + 1)^4*b*c^5*d^4/(c*x - 1)^4 + 472*(c*x + 1)^3*b*c^5*d^4/(c*x - 1)^3
+ 399*(c*x + 1)^2*b*c^5*d^4/(c*x - 1)^2 + 174*(c*x + 1)*b*c^5*d^4/(c*x - 1)
 + 31*b*c^5*d^4)/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x - 1)^5 + 15*
(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x + 1)^2/(c*x
- 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 233, normalized size = 1.54

$$-\frac{4c^3d^4a}{3x^3}-\frac{c^4d^4a}{2x^2}-\frac{d^4a}{6x^6}-\frac{3c^2d^4a}{2x^4}-\frac{4c\,d^4a}{5x^5}-\frac{4c^3d^4b\,\mathrm{arctanh}\,(cx)}{3x^3}-\frac{c^4d^4b\,\mathrm{arctanh}\,(cx)}{2x^2}-\frac{d^4b\,\mathrm{arctanh}\,(cx)}{6x^6}-\frac{3c^2d^4b\,\mathrm{arcta}}{2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x)
```

```
[Out] -4/3*c^3*d^4*a/x^3-1/2*c^4*d^4*a/x^2-1/6*d^4*a/x^6-3/2*c^2*d^4*a/x^4-4/5*c*
d^4*a/x^5-4/3*c^3*d^4*b*arctanh(c*x)/x^3-1/2*c^4*d^4*b*arctanh(c*x)/x^2-1/6
*d^4*b*arctanh(c*x)/x^6-3/2*c^2*d^4*b*arctanh(c*x)/x^4-4/5*c*d^4*b*arctanh(
c*x)/x^5-1/30*b*c*d^4/x^5-1/5*b*c^2*d^4/x^4-5/9*b*c^3*d^4/x^3-16/15*b*c^4*d
^4/x^2-13/6*b*c^5*d^4/x+32/15*c^6*d^4*b*ln(c*x)-43/20*c^6*d^4*b*ln(c*x-1)+1
/60*c^6*d^4*b*ln(c*x+1)
```

maxima [B] time = 0.33, size = 329, normalized size = 2.18

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\,(cx)}{x^2}\right)bc^4d^4-\frac{2}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^4*
d^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c^3*d^4 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*
x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^4 - 1/5*((2*c^4*log(c^2*x^2 -
 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c*d^4
 - 1/2*a*c^4*d^4/x^2 + 1/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) -
2*(15*c^4*x^4 + 5*c^2*x^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^4 - 4/3*a*
c^3*d^4/x^3 - 3/2*a*c^2*d^4/x^4 - 4/5*a*c*d^4/x^5 - 1/6*a*d^4/x^6
```

mupad [B] time = 1.24, size = 248, normalized size = 1.64

$$\frac{32\,b\,c^6\,d^4\,\ln(x)}{15}-\frac{16\,b\,c^6\,d^4\,\ln\left(c^2\,x^2-1\right)}{15}-\frac{3\,a\,c^2\,d^4}{2\,x^4}-\frac{4\,a\,c^3\,d^4}{3\,x^3}-\frac{a\,c^4\,d^4}{2\,x^2}-\frac{b\,c^2\,d^4}{5\,x^4}-\frac{5\,b\,c^3\,d^4}{9\,x^3}-\frac{16\,b\,c^4\,d^4}{15\,x^2}-\frac{13\,b\,c^5\,d^4}{6\,x}-\frac{a\,c}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^7,x)
```

```
[Out] (32*b*c^6*d^4*log(x))/15 - (16*b*c^6*d^4*log(c^2*x^2 - 1))/15 - (3*a*c^2*d^
4)/(2*x^4) - (4*a*c^3*d^4)/(3*x^3) - (a*c^4*d^4)/(2*x^2) - (b*c^2*d^4)/(5*x
```

```
^4) - (5*b*c^3*d^4)/(9*x^3) - (16*b*c^4*d^4)/(15*x^2) - (13*b*c^5*d^4)/(6*x
) - (a*d^4)/(6*x^6) - (4*a*c*d^4)/(5*x^5) - (b*c*d^4)/(30*x^5) - (b*d^4*ata
nh(c*x))/(6*x^6) - (13*b*c^7*d^4*atan((c^2*x)/(-c^2)^(1/2)))/(6*(-c^2)^(1/2
)) - (4*b*c*d^4*atanh(c*x))/(5*x^5) - (3*b*c^2*d^4*atanh(c*x))/(2*x^4) - (4
*b*c^3*d^4*atanh(c*x))/(3*x^3) - (b*c^4*d^4*atanh(c*x))/(2*x^2)
```

sympy [A] time = 3.36, size = 291, normalized size = 1.93

$$\begin{cases} -\frac{ac^4d^4}{2x^2} - \frac{4ac^3d^4}{3x^3} - \frac{3ac^2d^4}{2x^4} - \frac{4acd^4}{5x^5} - \frac{ad^4}{6x^6} + \frac{32bc^6d^4\log(x)}{15} - \frac{32bc^6d^4\log\left(x-\frac{1}{c}\right)}{15} + \frac{bc^6d^4\operatorname{atanh}(cx)}{30} - \frac{13bc^5d^4}{6x} - \frac{bc^4d^4\operatorname{atanh}(cx)}{2x^2} - \\ -\frac{ad^4}{6x^6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**7,x)
```

```
[Out] Piecewise((-a*c**4*d**4/(2*x**2) - 4*a*c**3*d**4/(3*x**3) - 3*a*c**2*d**4/(
2*x**4) - 4*a*c*d**4/(5*x**5) - a*d**4/(6*x**6) + 32*b*c**6*d**4*log(x)/15
- 32*b*c**6*d**4*log(x - 1/c)/15 + b*c**6*d**4*atanh(c*x)/30 - 13*b*c**5*d*
*4/(6*x) - b*c**4*d**4*atanh(c*x)/(2*x**2) - 16*b*c**4*d**4/(15*x**2) - 4*b
*c**3*d**4*atanh(c*x)/(3*x**3) - 5*b*c**3*d**4/(9*x**3) - 3*b*c**2*d**4*ata
nh(c*x)/(2*x**4) - b*c**2*d**4/(5*x**4) - 4*b*c*d**4*atanh(c*x)/(5*x**5) -
b*c*d**4/(30*x**5) - b*d**4*atanh(c*x)/(6*x**6), Ne(c, 0)), (-a*d**4/(6*x**
6), True))
```

3.42 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=229

$$-\frac{c^4d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7}-\frac{2cd^4\left(a+b\right.}{3}$$

```
[Out] -1/42*b*c*d^4/x^6-2/15*b*c^2*d^4/x^5-47/140*b*c^3*d^4/x^4-5/9*b*c^4*d^4/x^3
-88/105*b*c^5*d^4/x^2-5/3*b*c^6*d^4/x-1/7*d^4*(a+b*arctanh(c*x))/x^7-2/3*c*
d^4*(a+b*arctanh(c*x))/x^6-6/5*c^2*d^4*(a+b*arctanh(c*x))/x^5-c^3*d^4*(a+b*
arctanh(c*x))/x^4-1/3*c^4*d^4*(a+b*arctanh(c*x))/x^3+176/105*b*c^7*d^4*ln(x
)-117/70*b*c^7*d^4*ln(-c*x+1)-1/210*b*c^7*d^4*ln(c*x+1)
```

Rubi [A] time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^4d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}-\frac{d^4\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

```
[Out] -(b*c*d^4)/(42*x^6) - (2*b*c^2*d^4)/(15*x^5) - (47*b*c^3*d^4)/(140*x^4) - (
5*b*c^4*d^4)/(9*x^3) - (88*b*c^5*d^4)/(105*x^2) - (5*b*c^6*d^4)/(3*x) - (d^
4*(a + b*ArcTanh[c*x]))/(7*x^7) - (2*c*d^4*(a + b*ArcTanh[c*x]))/(3*x^6) -
(6*c^2*d^4*(a + b*ArcTanh[c*x]))/(5*x^5) - (c^3*d^4*(a + b*ArcTanh[c*x]))/x
^4 - (c^4*d^4*(a + b*ArcTanh[c*x]))/(3*x^3) + (176*b*c^7*d^4*Log[x])/105 -
(117*b*c^7*d^4*Log[1 - c*x])/70 - (b*c^7*d^4*Log[1 + c*x])/210
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7} - \frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6} - \frac{6c^2d^4\left(a+b\tanh^{-1}\right.}{5x^5}$$

$$= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7} - \frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6} - \frac{6c^2d^4\left(a+b\tanh^{-1}\right.}{5x^5}$$

$$= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7} - \frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6} - \frac{6c^2d^4\left(a+b\tanh^{-1}\right.}{5x^5}$$

$$= -\frac{bcd^4}{42x^6} - \frac{2bc^2d^4}{15x^5} - \frac{47bc^3d^4}{140x^4} - \frac{5bc^4d^4}{9x^3} - \frac{88bc^5d^4}{105x^2} - \frac{5bc^6d^4}{3x} - \frac{d^4\left(a+b\,\mathrm{t}\right.}{7}$$

Mathematica [A] time = 0.18, size = 175, normalized size = 0.76

$$-\frac{d^4\left(420ac^4x^4+1260ac^3x^3+1512ac^2x^2+840acx+180a-2112bc^7x^7\log(x)+2106bc^7x^7\log(1-cx)+6bc^7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

```
[Out] -1/1260*(d^4*(180*a + 840*a*c*x + 30*b*c*x + 1512*a*c^2*x^2 + 168*b*c^2*x^2
 + 1260*a*c^3*x^3 + 423*b*c^3*x^3 + 420*a*c^4*x^4 + 700*b*c^4*x^4 + 1056*b*
c^5*x^5 + 2100*b*c^6*x^6 + 12*b*(15 + 70*c*x + 126*c^2*x^2 + 105*c^3*x^3 +
35*c^4*x^4)*ArcTanh[c*x] - 2112*b*c^7*x^7*Log[x] + 2106*b*c^7*x^7*Log[1 - c
*x] + 6*b*c^7*x^7*Log[1 + c*x]))/x^7
```

fricas [A] time = 0.60, size = 218, normalized size = 0.95

$$-\frac{6\,bc^7d^4x^7\log\left(cx+1\right)+2106\,bc^7d^4x^7\log\left(cx-1\right)-2112\,bc^7d^4x^7\log(x)+2100\,bc^6d^4x^6+1056\,bc^5d^4x^5+14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/1260*(6*b*c^7*d^4*x^7*log(c*x + 1) + 2106*b*c^7*d^4*x^7*log(c*x - 1) - 2
112*b*c^7*d^4*x^7*log(x) + 2100*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*x^5 + 140*(3
*a + 5*b)*c^4*d^4*x^4 + 9*(140*a + 47*b)*c^3*d^4*x^3 + 168*(9*a + b)*c^2*d^
4*x^2 + 30*(28*a + b)*c*d^4*x + 180*a*d^4 + 6*(35*b*c^4*d^4*x^4 + 105*b*c^3
*d^4*x^3 + 126*b*c^2*d^4*x^2 + 70*b*c*d^4*x + 15*b*d^4)*log(-(c*x + 1)/(c*x
 - 1)))/x^7
```

giac [B] time = 0.33, size = 735, normalized size = 3.21

$$\frac{4}{315}\left(132\,bc^6d^4\log\left(-\frac{cx+1}{cx-1}-1\right)-132\,bc^6d^4\log\left(-\frac{cx+1}{cx-1}\right)+\frac{12\left(\frac{105\,(cx+1)^6bc^6d^4}{(cx-1)^6}+\frac{210\,(cx+1)^5bc^6d^4}{(cx-1)^5}+\frac{385\,(cx+1)^4bc}{(cx-1)^4}\right.}{\frac{(cx+1)^7}{(cx-1)^7}+\frac{7\,(cx+1)^6}{(cx-1)^6}+\frac{21\,(c}{(cx-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] 4/315*(132*b*c^6*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 132*b*c^6*d^4*log(-(c*
x + 1)/(c*x - 1)) + 12*(105*(c*x + 1)^6*b*c^6*d^4/(c*x - 1)^6 + 210*(c*x +
1)^5*b*c^6*d^4/(c*x - 1)^5 + 385*(c*x + 1)^4*b*c^6*d^4/(c*x - 1)^4 + 385*(c
*x + 1)^3*b*c^6*d^4/(c*x - 1)^3 + 231*(c*x + 1)^2*b*c^6*d^4/(c*x - 1)^2 + 7
```

```
7*(c*x + 1)*b*c^6*d^4/(c*x - 1) + 11*b*c^6*d^4)*log(-(c*x + 1)/(c*x - 1))/(
(c*x + 1)^7/(c*x - 1)^7 + 7*(c*x + 1)^6/(c*x - 1)^6 + 21*(c*x + 1)^5/(c*x -
 1)^5 + 35*(c*x + 1)^4/(c*x - 1)^4 + 35*(c*x + 1)^3/(c*x - 1)^3 + 21*(c*x +
 1)^2/(c*x - 1)^2 + 7*(c*x + 1)/(c*x - 1) + 1) + (2520*(c*x + 1)^6*a*c^6*d^
4/(c*x - 1)^6 + 5040*(c*x + 1)^5*a*c^6*d^4/(c*x - 1)^5 + 9240*(c*x + 1)^4*a
*c^6*d^4/(c*x - 1)^4 + 9240*(c*x + 1)^3*a*c^6*d^4/(c*x - 1)^3 + 5544*(c*x +
 1)^2*a*c^6*d^4/(c*x - 1)^2 + 1848*(c*x + 1)*a*c^6*d^4/(c*x - 1) + 264*a*c^
6*d^4 + 1128*(c*x + 1)^6*b*c^6*d^4/(c*x - 1)^6 + 4812*(c*x + 1)^5*b*c^6*d^4
/(c*x - 1)^5 + 9476*(c*x + 1)^4*b*c^6*d^4/(c*x - 1)^4 + 10631*(c*x + 1)^3*b
*c^6*d^4/(c*x - 1)^3 + 6933*(c*x + 1)^2*b*c^6*d^4/(c*x - 1)^2 + 2465*(c*x +
 1)*b*c^6*d^4/(c*x - 1) + 371*b*c^6*d^4)/((c*x + 1)^7/(c*x - 1)^7 + 7*(c*x
+ 1)^6/(c*x - 1)^6 + 21*(c*x + 1)^5/(c*x - 1)^5 + 35*(c*x + 1)^4/(c*x - 1)^
4 + 35*(c*x + 1)^3/(c*x - 1)^3 + 21*(c*x + 1)^2/(c*x - 1)^2 + 7*(c*x + 1)/(
c*x - 1) + 1))*c
```

maple [A] time = 0.04, size = 245, normalized size = 1.07

$$-\frac{c^4d^4a}{3x^3}-\frac{d^4a}{7x^7}-\frac{2c\,d^4a}{3x^6}-\frac{c^3d^4a}{x^4}-\frac{6c^2d^4a}{5x^5}-\frac{c^4d^4b\,\mathrm{arctanh}\,(cx)}{3x^3}-\frac{d^4b\,\mathrm{arctanh}\,(cx)}{7x^7}-\frac{2c\,d^4b\,\mathrm{arctanh}\,(cx)}{3x^6}-\frac{c^3d^4b\,\mathrm{arctanh}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x)
```

```
[Out] -1/3*c^4*d^4*a/x^3-1/7*d^4*a/x^7-2/3*c*d^4*a/x^6-c^3*d^4*a/x^4-6/5*c^2*d^4*
a/x^5-1/3*c^4*d^4*b*arctanh(c*x)/x^3-1/7*d^4*b*arctanh(c*x)/x^7-2/3*c*d^4*b
*arctanh(c*x)/x^6-c^3*d^4*b*arctanh(c*x)/x^4-6/5*c^2*d^4*b*arctanh(c*x)/x^5
-1/42*b*c*d^4/x^6-2/15*b*c^2*d^4/x^5-47/140*b*c^3*d^4/x^4-5/9*b*c^4*d^4/x^3
-88/105*b*c^5*d^4/x^2-5/3*b*c^6*d^4/x+176/105*c^7*d^4*b*ln(c*x)-117/70*c^7*
d^4*b*ln(c*x-1)-1/210*b*c^7*d^4*ln(c*x+1)
```

maxima [A] time = 0.33, size = 353, normalized size = 1.54

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\,(cx)}{x^3}\right)bc^4d^4+\frac{1}{6}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\,(3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^4*d^4 + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^3*d^4 - 3/10*((2*c^4*log(c^2*x^2 - 1)
- 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c^2*d^4 +
 1/45*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x
^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*c*d^4 - 1/84*((6*c^6*log(c^2*x^2 -
1) - 6*c^6*log(x^2) + (6*c^4*x^4 + 3*c^2*x^2 + 2)/x^6)*c + 12*arctanh(c*x)/
x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 - a*c^3*d^4/x^4 - 6/5*a*c^2*d^4/x^5 - 2/3*a*
c*d^4/x^6 - 1/7*a*d^4/x^7
```

mupad [B] time = 1.21, size = 260, normalized size = 1.14

$$\frac{176\,b\,c^7\,d^4\,\ln(x)}{105}-\frac{88\,b\,c^7\,d^4\,\ln\left(c^2\,x^2-1\right)}{105}-\frac{6\,a\,c^2\,d^4}{5\,x^5}-\frac{a\,c^3\,d^4}{x^4}-\frac{a\,c^4\,d^4}{3\,x^3}-\frac{2\,b\,c^2\,d^4}{15\,x^5}-\frac{47\,b\,c^3\,d^4}{140\,x^4}-\frac{5\,b\,c^4\,d^4}{9\,x^3}-\frac{88\,b\,c^5\,d^4}{105\,x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^8,x)
```

```
[Out] (176*b*c^7*d^4*log(x))/105 - (88*b*c^7*d^4*log(c^2*x^2 - 1))/105 - (6*a*c^2
*d^4)/(5*x^5) - (a*c^3*d^4)/x^4 - (a*c^4*d^4)/(3*x^3) - (2*b*c^2*d^4)/(15*x
```

```
^5) - (47*b*c^3*d^4)/(140*x^4) - (5*b*c^4*d^4)/(9*x^3) - (88*b*c^5*d^4)/(10
5*x^2) - (5*b*c^6*d^4)/(3*x) - (a*d^4)/(7*x^7) - (2*a*c*d^4)/(3*x^6) - (b*c
*d^4)/(42*x^6) - (b*d^4*atanh(c*x))/(7*x^7) - (5*b*c^8*d^4*atan((c^2*x)/(-c
^2)^(1/2)))/(3*(-c^2)^(1/2)) - (2*b*c*d^4*atanh(c*x))/(3*x^6) - (6*b*c^2*d^
4*atanh(c*x))/(5*x^5) - (b*c^3*d^4*atanh(c*x))/x^4 - (b*c^4*d^4*atanh(c*x))
/(3*x^3)
```

sympy [A] time = 4.19, size = 301, normalized size = 1.31

$$\begin{cases} -\frac{ac^4d^4}{3x^3} - \frac{ac^3d^4}{x^4} - \frac{6ac^2d^4}{5x^5} - \frac{2acd^4}{3x^6} - \frac{ad^4}{7x^7} + \frac{176bc^7d^4\log(x)}{105} - \frac{176bc^7d^4\log\left(x-\frac{1}{c}\right)}{105} - \frac{bc^7d^4\operatorname{atanh}(cx)}{105} - \frac{5bc^6d^4}{3x} - \frac{88bc^5d^4}{105x^2} - \frac{bc^4d^4 \ldots}{} \\ -\frac{ad^4}{7x^7} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**8,x)
```

```
[Out] Piecewise((-a*c**4*d**4/(3*x**3) - a*c**3*d**4/x**4 - 6*a*c**2*d**4/(5*x**5
) - 2*a*c*d**4/(3*x**6) - a*d**4/(7*x**7) + 176*b*c**7*d**4*log(x)/105 - 17
6*b*c**7*d**4*log(x - 1/c)/105 - b*c**7*d**4*atanh(c*x)/105 - 5*b*c**6*d**4
/(3*x) - 88*b*c**5*d**4/(105*x**2) - b*c**4*d**4*atanh(c*x)/(3*x**3) - 5*b*
c**4*d**4/(9*x**3) - b*c**3*d**4*atanh(c*x)/x**4 - 47*b*c**3*d**4/(140*x**4
) - 6*b*c**2*d**4*atanh(c*x)/(5*x**5) - 2*b*c**2*d**4/(15*x**5) - 2*b*c*d**
4*atanh(c*x)/(3*x**6) - b*c*d**4/(42*x**6) - b*d**4*atanh(c*x)/(7*x**7), Ne
(c, 0)), (-a*d**4/(7*x**7), True))
```

3.43 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx$

Optimal. Leaf size=177

$$\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}-\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd}+\frac{ax}{c^3d}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^4d}+\frac{b\tanh^{-1}(cx)}{2c^4d}$$

```
[Out] a*x/c^3/d-1/2*b*x/c^3/d+1/6*b*x^2/c^2/d+1/2*b*arctanh(c*x)/c^4/d+b*x*arctan
h(c*x)/c^3/d-1/2*x^2*(a+b*arctanh(c*x))/c^2/d+1/3*x^3*(a+b*arctanh(c*x))/c/
d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^4/d+2/3*b*ln(-c^2*x^2+1)/c^4/d-1/2*b*p
olylog(2,1-2/(c*x+1))/c^4/d
```

Rubi [A] time = 0.29, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5930, 5916, 266, 43, 321, 206, 5910, 260, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d}-\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd}+\frac{ax}{c^3d}+\frac{bx}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (a*x)/(c^3*d) - (b*x)/(2*c^3*d) + (b*x^2)/(6*c^2*d) + (b*ArcTanh[c*x])/(2*c
^4*d) + (b*x*ArcTanh[c*x])/(c^3*d) - (x^2*(a + b*ArcTanh[c*x]))/(2*c^2*d) +
 (x^3*(a + b*ArcTanh[c*x]))/(3*c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)
])/(c^4*d) + (2*b*Log[1 - c^2*x^2])/(3*c^4*d) - (b*PolyLog[2, 1 - 2/(1 + c*
x)])/(2*c^4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx = -\frac{\int \frac{x^2(a+b\tanh^{-1}(cx))}{d+cdx}\,dx}{c} + \frac{\int x^2\left(a+b\tanh^{-1}(cx)\right)dx}{cd}$$

$$= \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd} + \frac{\int \frac{x(a+b\tanh^{-1}(cx))}{d+cdx}\,dx}{c^2} - \frac{b\int \frac{x^3}{1-c^2x^2}\,dx}{3d} - \frac{\int x\left(a+b\tanh^{-1}(c\right.}{c^2d}$$

$$= -\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd} - \frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c^3} - \frac{b\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)}{c^4d}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} + \frac{bx^2}{6c^2d} + \frac{b\tanh^{-1}(cx)}{2c^4d} + \frac{bx\tanh^{-1}(cx)}{c^3d} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3}{}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} + \frac{bx^2}{6c^2d} + \frac{b\tanh^{-1}(cx)}{2c^4d} + \frac{bx\tanh^{-1}(cx)}{c^3d} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3}{}$$

Mathematica [A] time = 0.39, size = 129, normalized size = 0.73

$$\frac{2ac^3x^3 - 3ac^2x^2 + 6acx - 6a\log(cx+1) + bc^2x^2 + 4b\log\left(1-c^2x^2\right) + b\tanh^{-1}(cx)\left(2c^3x^3 - 3c^2x^2 + 6cx + 6\log\right.}{6c^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (-b + 6*a*c*x - 3*b*c*x - 3*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + b*ArcTanh
[c*x]*(3 + 6*c*x - 3*c^2*x^2 + 2*c^3*x^3 + 6*Log[1 + E^(-2*ArcTanh[c*x])])
- 6*a*Log[1 + c*x] + 4*b*Log[1 - c^2*x^2] - 3*b*PolyLog[2, -E^(-2*ArcTanh[c
*x])])/(6*c^4*d)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^3\,\mathrm{artanh}\,(cx) + ax^3}{cdx+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c*d*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)x^3}{cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d), x)
```

maple [A] time = 0.05, size = 253, normalized size = 1.43

$$\frac{a\,x^3}{3cd} - \frac{a\,x^2}{2c^2d} + \frac{ax}{c^3d} - \frac{a\ln(cx+1)}{c^4d} + \frac{b\,x^3\,\mathrm{arctanh}\,(cx)}{3cd} - \frac{b\,\mathrm{arctanh}\,(cx)\,x^2}{2c^2d} + \frac{bx\,\mathrm{arctanh}\,(cx)}{c^3d} - \frac{b\,\mathrm{arctanh}\,(cx)\ln(cx+1)}{c^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] 1/3/c*a/d*x^3-1/2/c^2*a/d*x^2+a*x/c^3/d-1/c^4*a/d*ln(c*x+1)+1/3/c*b/d*x^3*a
rctanh(c*x)-1/2/c^2*b/d*arctanh(c*x)*x^2+b*x*arctanh(c*x)/c^3/d-1/c^4*b/d*a
rctanh(c*x)*ln(c*x+1)+1/4/c^4*b/d*ln(c*x+1)^2-1/2/c^4*b/d*ln(-1/2*c*x+1/2)*
ln(c*x+1)+1/2/c^4*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2/c^4*b/d*dilog(1/
2+1/2*c*x)+1/6*b*x^2/c^2/d-1/2*b*x/c^3/d-2/3/c^4*b/d+11/12/c^4*b/d*ln(c*x+1
)+5/12/c^4*b/d*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{72}\left(2\,c^4\left(\frac{2\left(c^2x^3+3\,x\right)}{c^7d}-\frac{3\,\log\left(cx+1\right)}{c^8d}+\frac{3\,\log\left(cx-1\right)}{c^8d}\right)+216\,c^4\int\frac{x^4\log\left(cx+1\right)}{6\left(c^5dx^2-c^3d\right)}\,dx-3\,c^3\left(\frac{x^2}{c^5d}+\frac{\log\left(c^2\right.}{c^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/72*(2*c^4*(2*(c^2*x^3 + 3*x)/(c^7*d) - 3*log(c*x + 1)/(c^8*d) + 3*log(c*x
 - 1)/(c^8*d)) + 216*c^4*integrate(1/6*x^4*log(c*x + 1)/(c^5*d*x^2 - c^3*d)
, x) - 3*c^3*(x^2/(c^5*d) + log(c^2*x^2 - 1)/(c^7*d)) - 216*c^3*integrate(1
/6*x^3*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x) + 9*c^2*(2*x/(c^5*d) - log(c*x
+ 1)/(c^6*d) + log(c*x - 1)/(c^6*d)) - 216*c*integrate(1/6*x*log(c*x + 1)/(
c^5*d*x^2 - c^3*d), x) - 6*(2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*log(c*x + 1))
*log(-c*x + 1)/(c^4*d) + 18*log(6*c^5*d*x^2 - 6*c^3*d)/(c^4*d) - 216*integr
ate(1/6*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x))*b + 1/6*a*((2*c^2*x^3 - 3*c*x
^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax^3}{cx+1}\,dx+\int\frac{bx^3\,\mathrm{atanh}\,(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x**3/(c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(c*x + 1), x))/
d
```

3.44 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx$

Optimal. Leaf size=145

$$-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{ax}{c^2d}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^3d}-\frac{b\tanh^{-1}(cx)}{2c^3d}+\frac{bx}{2c^2d}-\frac{bx\tanh^{-1}(cx}{c^2d}$$

```
[Out] -a*x/c^2/d+1/2*b*x/c^2/d-1/2*b*arctanh(c*x)/c^3/d-b*x*arctanh(c*x)/c^2/d+1/
2*x^2*(a+b*arctanh(c*x))/c/d-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^3/d-1/2*b*l
n(-c^2*x^2+1)/c^3/d+1/2*b*polylog(2,1-2/(c*x+1))/c^3/d
```

Rubi [A] time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5930, 5916, 321, 206, 5910, 260, 5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^3d}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{ax}{c^2d}-\frac{b\log\left(1-c^2x^2\right)}{2c^3d}+\frac{bx}{2c^2d}-\frac{bx\,\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] -((a*x)/(c^2*d)) + (b*x)/(2*c^2*d) - (b*ArcTanh[c*x])/(2*c^3*d) - (b*x*ArcT
anh[c*x])/(c^2*d) + (x^2*(a + b*ArcTanh[c*x]))/(2*c*d) - ((a + b*ArcTanh[c*
x])*Log[2/(1 + c*x)])/(c^3*d) - (b*Log[1 - c^2*x^2])/(2*c^3*d) + (b*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*c^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx = -\frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx}{c}+\frac{\int x\left(a+b\tanh^{-1}(cx)\right)\,dx}{cd}$$

$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}+\frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c^2}-\frac{b\int \frac{x^2}{1-c^2x^2}\,dx}{2d}-\frac{\int \left(a+b\tanh^{-1}(cx\right.}{c^2d}$$

$$= -\frac{ax}{c^2d}+\frac{bx}{2c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d}-\frac{b\int \frac{}{1-c}}{2c}$$

$$= -\frac{ax}{c^2d}+\frac{bx}{2c^2d}-\frac{b\tanh^{-1}(cx)}{2c^3d}-\frac{bx\tanh^{-1}(cx)}{c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{(a+b}{}$$

$$= -\frac{ax}{c^2d}+\frac{bx}{2c^2d}-\frac{b\tanh^{-1}(cx)}{2c^3d}-\frac{bx\tanh^{-1}(cx)}{c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{(a+b}{}$$

Mathematica [A] time = 0.25, size = 97, normalized size = 0.67

$$\frac{ac^2x^2-2acx+2a\log(cx+1)-b\log\left(1-c^2x^2\right)+b\tanh^{-1}(cx)\left(c^2x^2-2cx-2\log\left(e^{-2\tanh^{-1}(cx)}+1\right)-1\right)+b}{2c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (-2*a*c*x + b*c*x + a*c^2*x^2 + b*ArcTanh[c*x]*(-1 - 2*c*x + c^2*x^2 - 2*Lo
g[1 + E^(-2*ArcTanh[c*x])]) + 2*a*Log[1 + c*x] - b*Log[1 - c^2*x^2] + b*Pol
yLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^3*d)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\operatorname{artanh}(cx)+ax^2}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c*d*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{artanh}(cx)+a)x^2}{cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d), x)
```

maple [A] time = 0.05, size = 213, normalized size = 1.47

$$\frac{ax^2}{2cd}-\frac{ax}{c^2d}+\frac{a\ln(cx+1)}{c^3d}+\frac{b\operatorname{arctanh}(cx)x^2}{2cd}-\frac{bx\operatorname{arctanh}(cx)}{c^2d}+\frac{b\operatorname{arctanh}(cx)\ln(cx+1)}{c^3d}-\frac{b\ln(cx+1)^2}{4c^3d}+\frac{b\ln\left(-\frac{cx}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] 1/2/c*a/d*x^2-a*x/c^2/d+1/c^3*a/d*ln(c*x+1)+1/2/c*b/d*arctanh(c*x)*x^2-b*x*
arctanh(c*x)/c^2/d+1/c^3*b/d*arctanh(c*x)*ln(c*x+1)-1/4/c^3*b/d*ln(c*x+1)^2
+1/2/c^3*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2/c^3*b/d*ln(-1/2*c*x+1/2)*ln(1/2
+1/2*c*x)-1/2/c^3*b/d*dilog(1/2+1/2*c*x)+1/2*b*x/c^2/d+1/2/c^3*b/d-3/4/c^3*
b/d*ln(c*x+1)-1/4/c^3*b/d*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(c^3\left(\frac{x^2}{c^4d}+\frac{\log\left(c^2x^2-1\right)}{c^6d}\right)+8c^3\int\frac{x^3\log(cx+1)}{2\left(c^4dx^2-c^2d\right)}\,dx-c^2\left(\frac{2x}{c^4d}-\frac{\log(cx+1)}{c^5d}+\frac{\log(cx-1)}{c^5d}\right)-8c^2\int\frac{x^2\log}{2\left(c^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(c^3*(x^2/(c^4*d) + log(c^2*x^2 - 1)/(c^6*d)) + 8*c^3*integrate(1/2*x^3
*log(c*x + 1)/(c^4*d*x^2 - c^2*d), x) - c^2*(2*x/(c^4*d) - log(c*x + 1)/(c^
5*d) + log(c*x - 1)/(c^5*d)) - 8*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^4*d*
x^2 - c^2*d), x) + 8*c*integrate(1/2*x*log(c*x + 1)/(c^4*d*x^2 - c^2*d), x)
 - 2*(c^2*x^2 - 2*c*x + 2*log(c*x + 1))*log(-c*x + 1)/(c^3*d) - 2*log(2*c^4
*d*x^2 - 2*c^2*d)/(c^3*d) + 8*integrate(1/2*log(c*x + 1)/(c^4*d*x^2 - c^2*d
), x))*b + 1/2*a*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{cx+1}\,dx + \int \frac{bx^2 \operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x**2/(c*x + 1), x) + Integral(b*x**2*atanh(c*x)/(c*x + 1), x))/
d
```

3.45 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx$

Optimal. Leaf size=94

$$\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{ax}{cd}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{b\log\left(1-c^2x^2\right)}{2c^2d}+\frac{bx\tanh^{-1}(cx)}{cd}$$

```
[Out] a*x/c/d+b*x*arctanh(c*x)/c/d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^2/d+1/2*b*l
n(-c^2*x^2+1)/c^2/d-1/2*b*polylog(2,1-2/(c*x+1))/c^2/d
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5930, 5910, 260, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{ax}{cd}+\frac{b\log\left(1-c^2x^2\right)}{2c^2d}+\frac{bx\tanh^{-1}(cx)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (a*x)/(c*d) + (b*x*ArcTanh[c*x])/(c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c
*x)])/(c^2*d) + (b*Log[1 - c^2*x^2])/(2*c^2*d) - (b*PolyLog[2, 1 - 2/(1 + c
*x)])/(2*c^2*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
```

```
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx = -\frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c} + \frac{\int \left(a+b\tanh^{-1}(cx)\right)dx}{cd}$$
$$= \frac{ax}{cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d} + \frac{b\int \tanh^{-1}(cx)\,dx}{cd} - \frac{b\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{cd}$$
$$= \frac{ax}{cd} + \frac{bx\tanh^{-1}(cx)}{cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d} - \frac{b\int \frac{x}{1-c^2x^2}\,dx}{d} - \frac{b\,\mathrm{Subst}\left(\right.}{}$$
$$= \frac{ax}{cd} + \frac{bx\tanh^{-1}(cx)}{cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d} + \frac{b\log\left(1-c^2x^2\right)}{2c^2d} - \frac{b\mathrm{Li}_2\left(\right.}{2}$$

Mathematica [A] time = 0.16, size = 75, normalized size = 0.80

$$\frac{2acx-2a\log(cx+1)+b\log\left(1-c^2x^2\right)-b\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)+2b\tanh^{-1}(cx)\left(cx+\log\left(e^{-2\tanh^{-1}(cx)}+1\right)\right)}{2c^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (2*a*c*x + 2*b*ArcTanh[c*x]*(c*x + Log[1 + E^(-2*ArcTanh[c*x])]) - 2*a*Log[
1 + c*x] + b*Log[1 - c^2*x^2] - b*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^2*
d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx\,\mathrm{artanh}\,(cx)+ax}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(c*d*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)x}{cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(c*d*x + d), x)
```

maple [A] time = 0.05, size = 157, normalized size = 1.67

$$\frac{ax}{cd}-\frac{a\ln(cx+1)}{c^2d}-\frac{b\,\mathrm{arctanh}\,(cx)\ln(cx+1)}{c^2d}+\frac{bx\,\mathrm{arctanh}\,(cx)}{cd}+\frac{b\ln(cx+1)^2}{4c^2d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2c^2d}+\frac{b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] a*x/c/d-1/c^2*a/d*ln(c*x+1)-1/c^2*b/d*arctanh(c*x)*ln(c*x+1)+b*x*arctanh(c*
x)/c/d+1/4/c^2*b/d*ln(c*x+1)^2-1/2/c^2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2/c
^2*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2/c^2*b/d*dilog(1/2+1/2*c*x)+1/2/
c^2*b/d*ln((c*x-1)*(c*x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(c^2\left(\frac{2x}{c^3d} - \frac{\log(cx+1)}{c^4d} + \frac{\log(cx-1)}{c^4d}\right) + 2c^2\int \frac{x^2\log(cx+1)}{c^3dx^2 - cd}\,dx - 4c\int \frac{x\log(cx+1)}{c^3dx^2 - cd}\,dx - \frac{2\left(cx - \log(cx + \right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*x/(c^3*d) - log(c*x + 1)/(c^4*d) + log(c*x - 1)/(c^4*d)) + 2*c^
2*integrate(x^2*log(c*x + 1)/(c^3*d*x^2 - c*d), x) - 4*c*integrate(x*log(c*
x + 1)/(c^3*d*x^2 - c*d), x) - 2*(c*x - log(c*x + 1))*log(-c*x + 1)/(c^2*d)
 + log(c^3*d*x^2 - c*d)/(c^2*d) - 2*integrate(log(c*x + 1)/(c^3*d*x^2 - c*d
), x))*b + a*(x/(c*d) - log(c*x + 1)/(c^2*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

```
[Out] int((x*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{cx+1}\,dx + \int \frac{bx\,\mathrm{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x/(c*x + 1), x) + Integral(b*x*atanh(c*x)/(c*x + 1), x))/d
```

3.46 $\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx$

Optimal. Leaf size=51

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}$$

```
[Out] -(a+b*arctanh(c*x))*ln(2/(c*x+1))/c/d+1/2*b*polylog(2,1-2/(c*x+1))/c/d
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c*d)) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c*d)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+cx}\right)}{cd}\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2cd}\end{aligned}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 1.02

$$\frac{2a\log(cx+1)+b\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)-2b\tanh^{-1}(cx)\log\left(e^{-2\tanh^{-1}(cx)}+1\right)}{2cd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

```
[Out] (-2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 2*a*Log[1 + c*x] + b*Poly
Log[2, -E^(-2*ArcTanh[c*x])])/(2*c*d)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx\right)+a}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(cx\right)+a}{cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

maple [B] time = 0.04, size = 112, normalized size = 2.20

$$\frac{a\ln\left(cx+1\right)}{cd}+\frac{b\,\text{arctanh}\left(cx\right)\ln\left(cx+1\right)}{cd}-\frac{b\ln\left(cx+1\right)^{2}}{4cd}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(cx+1\right)}{2cd}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{1}{2}+\frac{cx}{2}\right)}{2cd}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] 1/c*a/d*ln(c*x+1)+1/c*b/d*arctanh(c*x)*ln(c*x+1)-1/4/c*b/d*ln(c*x+1)^2+1/2/
c*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2/c*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)
-1/2/c*b/d*dilog(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(2\,c\int\frac{x\log\left(cx+1\right)}{c^{2}dx^{2}-d}\,dx-\frac{\log\left(cx+1\right)\log\left(-cx+1\right)}{cd}\right)b+\frac{a\log\left(cdx+d\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*integrate(x*log(c*x + 1)/(c^2*d*x^2 - d), x) - log(c*x + 1)*log(-c
*x + 1)/(c*d))*b + a*log(c*d*x + d)/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\text{atanh}\left(c\,x\right)}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + c*d*x),x)
```

```
[Out] int((a + b*atanh(c*x))/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx+1}\, dx + \int \frac{b \operatorname{atanh}(cx)}{cx+1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d),x)
[Out] (Integral(a/(c*x + 1), x) + Integral(b*atanh(c*x)/(c*x + 1), x))/d
```

3.47 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{2d}$$

```
[Out] (a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d-1/2*b*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5932, 2447}

$$\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

```
[Out] ((a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*PolyLog[2, -1 + 2/(1 + c
*x)])/(2*d)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx&=\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{(bc)\int \frac{\log\left(2-\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}\\&=\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 55, normalized size = 1.20

$$\frac{-2a\log(cx+1)+2a\log(x)-b\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)+2b\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

```
[Out] (2*b*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + 2*a*Log[x] - 2*a*Log[1 + c
*x] - b*PolyLog[2, E^(-2*ArcTanh[c*x])])/(2*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx\right)+a}{cdx^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(cx\right)+a}{(cdx+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x), x)
```

maple [B] time = 0.05, size = 156, normalized size = 3.39

$$\frac{a\ln\left(cx\right)}{d}-\frac{a\ln\left(cx+1\right)}{d}-\frac{b\,\text{arctanh}\left(cx\right)\ln\left(cx+1\right)}{d}+\frac{b\,\text{arctanh}\left(cx\right)\ln\left(cx\right)}{d}+\frac{b\ln\left(cx+1\right)^2}{4d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(cx\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d),x)
```

```
[Out] a/d*ln(c*x)-a/d*ln(c*x+1)-b/d*arctanh(c*x)*ln(c*x+1)+b*arctanh(c*x)/d*ln(c*
x)+1/4*b/d*ln(c*x+1)^2-1/2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b/d*ln(-1/2*c
*x+1/2)*ln(1/2+1/2*c*x)+1/2*b/d*dilog(1/2+1/2*c*x)-1/2*b/d*dilog(c*x)-1/2*b
/d*dilog(c*x+1)-1/2*b/d*ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\log\left(cx+1\right)}{d}-\frac{\log(x)}{d}\right)+\frac{1}{2}\,b\int \frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{cdx^2+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(c*x + 1)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x +
 1))/(c*d*x^2 + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\text{atanh}\left(c\,x\right)}{x\,\left(d+c\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx^2+x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{cx^2+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**2 + x), x) + Integral(b*atanh(c*x)/(c*x**2 + x), x))/d
```

3.48 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx$

Optimal. Leaf size=93

$$-\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{bc\log\left(1-c^2x^2\right)}{2d}+\frac{bc\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{2d}+\frac{bc\log(x)}{d}$$

```
[Out] (-a-b*arctanh(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(-c^2*x^2+1)/d-c*(a+b*arctanh
(c*x))*ln(2-2/(c*x+1))/d+1/2*b*c*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5934, 5916, 266, 36, 29, 31, 5932, 2447}

$$\frac{bc\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}-\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{bc\log\left(1-c^2x^2\right)}{2d}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (b*c*Log[1 - c^2*x^2])/(2*
d) - (c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c*PolyLog[2, -1 +
 2/(1 + c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx\right)+\frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{(bc)\int \frac{1}{x(1-c^2x^2)}\,dx}{d}+\frac{\left(bc^2\right)\int}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{bc\mathrm{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}+\frac{(bc)\,\mathrm{Su}}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{bc\mathrm{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}+\frac{(bc)\,\mathrm{Su}}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log\left(1-c^2x^2\right)}{2d}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right.}{d}$$

Mathematica [A] time = 0.20, size = 93, normalized size = 1.00

$$\frac{bcx\mathrm{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)-2\left(acx\log(x)-acx\log(cx+1)+a-bcx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+b\tanh^{-1}(cx)\left(cx\log\left(1-e^{-2\tanh}\right.\right.\right.}{2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

```
[Out] (-2*(a + b*ArcTanh[c*x]*(1 + c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + a*c*x*Log[
x] - a*c*x*Log[1 + c*x] - b*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) + b*c*x*PolyL
og[2, E^(-2*ArcTanh[c*x])])/(2*d*x)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\,(cx)+a}{cdx^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^2), x)
```

maple [B] time = 0.06, size = 225, normalized size = 2.42

$$-\frac{a}{dx} - \frac{ca\ln(cx)}{d} + \frac{ca\ln(cx+1)}{d} - \frac{b\operatorname{arctanh}(cx)}{dx} - \frac{cb\operatorname{arctanh}(cx)\ln(cx)}{d} + \frac{cb\operatorname{arctanh}(cx)\ln(cx+1)}{d} + \frac{cb\ln(cx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d),x)
```

```
[Out] -a/d/x-c*a/d*ln(c*x)+c*a/d*ln(c*x+1)-b/d*arctanh(c*x)/x-c*b*arctanh(c*x)/d*
ln(c*x)+c*b/d*arctanh(c*x)*ln(c*x+1)+c*b/d*ln(c*x)-1/2*c*b/d*ln(c*x-1)-1/2*
c*b/d*ln(c*x+1)+1/2*c*b/d*dilog(c*x)+1/2*c*b/d*dilog(c*x+1)+1/2*c*b/d*ln(c*
x)*ln(c*x+1)-1/4*c*b/d*ln(c*x+1)^2+1/2*c*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2
*c*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2*c*b/d*dilog(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{c\log(cx+1)}{d} - \frac{c\log(x)}{d} - \frac{1}{dx}\right) + \frac{1}{2}b\int \frac{\log(cx+1) - \log(-cx+1)}{cdx^3 + dx^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/2*b*integrate((log(c*x + 1)
 - log(-c*x + 1))/(c*d*x^3 + d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atanh}(c\,x)}{x^2\,(d + c\,d\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx^3+x^2} \, dx + \int \frac{b \operatorname{atanh}(cx)}{cx^3+x^2} \, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c*x**3 + x**2), x)
)/d
```

3.49 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx$

Optimal. Leaf size=146

$$\frac{c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{bc^2\mathrm{Li}_2\left(\frac{2}{cx+1}-1\right)}{2d}+\frac{bc^2\log\left(1-c^2x\right.}{2d}$$

```
[Out] -1/2*b*c/d/x+1/2*b*c^2*arctanh(c*x)/d+1/2*(-a-b*arctanh(c*x))/d/x^2+c*(a+b*
arctanh(c*x))/d/x-b*c^2*ln(x)/d+1/2*b*c^2*ln(-c^2*x^2+1)/d+c^2*(a+b*arctanh
(c*x))*ln(2-2/(c*x+1))/d-1/2*b*c^2*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5934, 5916, 325, 206, 266, 36, 29, 31, 5932, 2447}

$$-\frac{bc^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}+\frac{c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{bc^2\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

```
[Out] -(b*c)/(2*d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x^2
) + (c*(a + b*ArcTanh[c*x]))/(d*x) - (b*c^2*Log[x])/d + (b*c^2*Log[1 - c^2*
x^2])/(2*d) + (c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*c^2*Po
lyLog[2, -1 + 2/(1 + c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx\right)+\frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2dx^2}+c^2\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx-\frac{c\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d}+\frac{(bc)\int \frac{1}{x^2(1-c^2}}{2d}$$

$$= -\frac{bc}{2dx}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+}\right.}{d}$$

$$= -\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{c^2\left(a+b\tanh\right.}{}$$

$$= -\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{c^2\left(a+b\tanh\right.}{}$$

$$= -\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{bc^2\log(x)}{d}+\frac{b}{}$$

Mathematica [A] time = 0.34, size = 133, normalized size = 0.91

$$-\frac{-2ac^2x^2\log(x)+2ac^2x^2\log(cx+1)-2acx+a+bc^2x^2\mathrm{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)+2bc^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)-b\tanh^{-1}(cx}{2dx^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

```
[Out] -1/2*(a - 2*a*c*x + b*c*x - b*ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x^
2*Log[1 - E^(-2*ArcTanh[c*x])]) - 2*a*c^2*x^2*Log[x] + 2*a*c^2*x^2*Log[1 +
c*x] + 2*b*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + b*c^2*x^2*PolyLog[2, E^(-
2*ArcTanh[c*x])])/(d*x^2)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\,(cx)+a}{cdx^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\mathrm{artanh}\,(cx)+a}{(cdx+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^3), x)
```

maple [B] time = 0.06, size = 286, normalized size = 1.96

$$-\frac{a}{2d\,x^2}+\frac{c^2a\ln(cx)}{d}+\frac{ca}{dx}-\frac{c^2a\ln(cx+1)}{d}-\frac{b\,\mathrm{arctanh}\,(cx)}{2d\,x^2}+\frac{c^2b\,\mathrm{arctanh}\,(cx)\ln(cx)}{d}+\frac{cb\,\mathrm{arctanh}\,(cx)}{dx}-\frac{c^2b\,\mathrm{arctanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d),x)
```

```
[Out] -1/2*a/d/x^2+c^2*a/d*ln(c*x)+c*a/d/x-c^2*a/d*ln(c*x+1)-1/2*b/d*arctanh(c*x)
/x^2+c^2*b*arctanh(c*x)/d*ln(c*x)+c*b/d*arctanh(c*x)/x-c^2*b/d*arctanh(c*x)
*ln(c*x+1)-1/2*c^2*b/d*dilog(c*x)-1/2*c^2*b/d*dilog(c*x+1)-1/2*c^2*b/d*ln(c
*x)*ln(c*x+1)+1/4*c^2*b/d*ln(c*x+1)^2-1/2*c^2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1
)+1/2*c^2*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2*c^2*b/d*dilog(1/2+1/2*c*
x)-1/2*b*c/d/x-c^2*b/d*ln(c*x)+1/4*c^2*b/d*ln(c*x-1)+3/4*c^2*b/d*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{2\,c^2\log(cx+1)}{d}-\frac{2\,c^2\log(x)}{d}-\frac{2\,cx-1}{dx^2}\right)a+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{cdx^4+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a + 1/2*
b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^4 + d*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{x^3\,\left(d+c\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx^4+x^3}\,dx+\int \frac{b\,\mathrm{atanh}\left(cx\right)}{cx^4+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c*x**4 + x**3), x)
)/d
```

3.50 $\int \frac{a+b\tanh^{-1}(cx)}{x^4(d+cdx)}\,dx$

Optimal. Leaf size=185

$$-\frac{c^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}+\frac{bc^3\mathrm{Li}_2\left(\frac{}{cx}\right.}{2d}$$

```
[Out] -1/6*b*c/d/x^2+1/2*b*c^2/d/x-1/2*b*c^3*arctanh(c*x)/d+1/3*(-a-b*arctanh(c*x
))/d/x^3+1/2*c*(a+b*arctanh(c*x))/d/x^2-c^2*(a+b*arctanh(c*x))/d/x+4/3*b*c^
3*ln(x)/d-2/3*b*c^3*ln(-c^2*x^2+1)/d-c^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))
/d+1/2*b*c^3*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.35, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5934, 5916, 266, 44, 325, 206, 36, 29, 31, 5932, 2447}

$$\frac{bc^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```

```
[Out] -(b*c)/(6*d*x^2) + (b*c^2)/(2*d*x) - (b*c^3*ArcTanh[c*x])/(2*d) - (a + b*Ar
cTanh[c*x])/(3*d*x^3) + (c*(a + b*ArcTanh[c*x]))/(2*d*x^2) - (c^2*(a + b*Ar
cTanh[c*x]))/(d*x) + (4*b*c^3*Log[x])/(3*d) - (2*b*c^3*Log[1 - c^2*x^2])/(3
*d) - (c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c^3*PolyLog[2,
 -1 + 2/(1 + c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^4(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx\right) + \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{3dx^3} + c^2\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d} + \frac{(bc)\int \frac{1}{x^3(1-c^2x^2)}}{3d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - c^3\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx + \frac{(bc)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{bc^2}{2dx} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx} - \frac{c^3\left(a+b\tan\right.}{}$$

$$= -\frac{bc}{6dx^2} + \frac{bc^2}{2dx} - \frac{bc^3\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\,\mathrm{t}\right.}{}$$

$$= -\frac{bc}{6dx^2} + \frac{bc^2}{2dx} - \frac{bc^3\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\,\mathrm{t}\right.}{}$$

$$= -\frac{bc}{6dx^2} + \frac{bc^2}{2dx} - \frac{bc^3\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\,\mathrm{t}\right.}{}$$

Mathematica [A] time = 0.43, size = 172, normalized size = 0.93

$$\frac{-6ac^3x^3\log(x) + 6ac^3x^3\log(cx+1) - 6ac^2x^2 + 3acx - 2a + 3bc^3x^3\mathrm{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) + bc^3x^3 + 3bc^2x^2 + 8bc^3x^3}{6dx^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```

```
[Out] (-2*a + 3*a*c*x - b*c*x - 6*a*c^2*x^2 + 3*b*c^2*x^2 + b*c^3*x^3 - b*ArcTanh
[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*x^3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTanh[
c*x])]) - 6*a*c^3*x^3*Log[x] + 6*a*c^3*x^3*Log[1 + c*x] + 8*b*c^3*x^3*Log[(
c*x)/Sqrt[1 - c^2*x^2]] + 3*b*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])])/(6*d
*x^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\,(cx)+a}{cdx^5+dx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^5 + d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{artanh}\,(cx)+a}{(cdx+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^4), x)
```

maple [A] time = 0.06, size = 328, normalized size = 1.77

$$-\frac{a}{3d\,x^{3}}-\frac{c^{2}a}{dx}+\frac{ca}{2d\,x^{2}}-\frac{c^{3}a\ln(cx)}{d}+\frac{c^{3}a\ln(cx+1)}{d}-\frac{b\,\mathrm{arctanh}(cx)}{3d\,x^{3}}-\frac{c^{2}b\,\mathrm{arctanh}(cx)}{dx}+\frac{cb\,\mathrm{arctanh}(cx)}{2d\,x^{2}}-\frac{c^{3}b\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^4/(c*d*x+d),x)
```

```
[Out] -1/3*a/d/x^3-c^2*a/d/x+1/2*c*a/d/x^2-c^3*a/d*ln(c*x)+c^3*a/d*ln(c*x+1)-1/3*
b/d*arctanh(c*x)/x^3-c^2*b/d*arctanh(c*x)/x+1/2*c*b/d*arctanh(c*x)/x^2-c^3*
b*arctanh(c*x)/d*ln(c*x)+c^3*b/d*arctanh(c*x)*ln(c*x+1)-1/6*b*c/d/x^2+1/2*b
*c^2/d/x+4/3*c^3*b/d*ln(c*x)-5/12*c^3*b/d*ln(c*x-1)-11/12*c^3*b/d*ln(c*x+1)
+1/2*c^3*b/d*dilog(c*x)+1/2*c^3*b/d*dilog(c*x+1)+1/2*c^3*b/d*ln(c*x)*ln(c*x
+1)-1/4*c^3*b/d*ln(c*x+1)^2+1/2*c^3*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2*c^3*
b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2*c^3*b/d*dilog(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(\frac{6\,c^{3}\log(cx+1)}{d}-\frac{6\,c^{3}\log(x)}{d}-\frac{6\,c^{2}x^{2}-3\,cx+2}{dx^{3}}\right)a+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{cdx^{5}+dx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^3
))*a + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^5 + d*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{atanh}(c\,x)}{x^{4}\,(d+c\,d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^4*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^4*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{cx^{5}+x^{4}}\,dx+\int\frac{b\,\mathrm{atanh}(cx)}{cx^{5}+x^{4}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**4/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**5 + x**4), x) + Integral(b*atanh(c*x)/(c*x**5 + x**4), x)
)/d
```

3.51 $\int \frac{x^3(a+b\tanh^{-1}(cx))}{(d+cdx)^2}\,dx$

Optimal. Leaf size=181

$$\frac{a+b\tanh^{-1}(cx)}{c^4d^2(cx+1)}-\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}-\frac{2ax}{c^3d^2}+\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{b}{2c^4d^2(cx+1)}$$

```
[Out] -2*a*x/c^3/d^2+1/2*b*x/c^3/d^2+1/2*b/c^4/d^2/(c*x+1)-b*arctanh(c*x)/c^4/d^2
-2*b*x*arctanh(c*x)/c^3/d^2+1/2*x^2*(a+b*arctanh(c*x))/c^2/d^2+(a+b*arctanh
(c*x))/c^4/d^2/(c*x+1)-3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^4/d^2-b*ln(-c^2
*x^2+1)/c^4/d^2+3/2*b*polylog(2,1-2/(c*x+1))/c^4/d^2
```

Rubi [A] time = 0.22, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {5940, 5910, 260, 5916, 321, 206, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^4d^2(cx+1)}-\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}-\frac{2ax}{c^3d^2}-\frac{b\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (-2*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(1 + c*x)) - (b*ArcTa
nh[c*x])/(c^4*d^2) - (2*b*x*ArcTanh[c*x])/(c^3*d^2) + (x^2*(a + b*ArcTanh[c
*x]))/(2*c^2*d^2) + (a + b*ArcTanh[c*x])/(c^4*d^2*(1 + c*x)) - (3*(a + b*Ar
cTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^2) - (b*Log[1 - c^2*x^2])/(c^4*d^2) +
(3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3 \left(a + b\tanh^{-1}(cx)\right)}{(d + cdx)^2} dx = \int \left(-\frac{2\left(a + b\tanh^{-1}(cx)\right)}{c^3d^2} + \frac{x\left(a + b\tanh^{-1}(cx)\right)}{c^2d^2} - \frac{a + b\tanh^{-1}(cx)}{c^3d^2(1 + cx)^2} + \frac{3\left(a + b\,\text{ta}\right.}{c^3d^2(1}\right.$$

$$= -\frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2} dx}{c^3d^2} - \frac{2\int \left(a + b\tanh^{-1}(cx)\right) dx}{c^3d^2} + \frac{3\int \frac{a+b\tanh^{-1}(cx)}{1+cx} dx}{c^3d^2} + \frac{\int x\left(a + b\right.}{}$$

$$= -\frac{2ax}{c^3d^2} + \frac{x^2\left(a + b\tanh^{-1}(cx)\right)}{2c^2d^2} + \frac{a + b\tanh^{-1}(cx)}{c^4d^2(1 + cx)} - \frac{3\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+c}\right.}{c^4d^2}$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2\left(a + b\tanh^{-1}(cx)\right)}{2c^2d^2} + \frac{a + b\tanh^{-1}(cx)}{c^4d^2(1 + cx)} - \frac{3}{}$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{b\tanh^{-1}(cx)}{2c^4d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2\left(a + b\tanh^{-1}(cx)\right)}{2c^2d^2} + \frac{a + b}{c^4d}$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(1 + cx)} - \frac{b\tanh^{-1}(cx)}{2c^4d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2\left(a + b\tanh\right.}{2c^2d^2}$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(1 + cx)} - \frac{b\tanh^{-1}(cx)}{c^4d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2\left(a + b\tanh\right.}{2c^2d^2}$$

Mathematica [A] time = 0.70, size = 142, normalized size = 0.78

$$\frac{2ac^2x^2 - 8acx + \frac{4a}{cx+1} + 12a\log(cx + 1) + b\left(-4\log\left(1 - c^2x^2\right) + 2\tanh^{-1}(cx)\left(c^2x^2 - 4cx - 6\log\left(e^{-2\tanh^{-1}(cx)} + \right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (-8*a*c*x + 2*a*c^2*x^2 + (4*a)/(1 + c*x) + 12*a*Log[1 + c*x] + b*(2*c*x +
Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(-2*ArcTanh[c*x
])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[c*x]] - 6*Log[1
 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]))/(4
*c^4*d^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3\,\text{artanh}\,(cx) + ax^3}{c^2d^2x^2 + 2\,cd^2x + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx) + a)x^3}{(cdx + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^2, x)
```

maple [A] time = 0.06, size = 265, normalized size = 1.46

$$\frac{a\,x^{2}}{2c^{2}d^{2}}-\frac{2ax}{c^{3}d^{2}}+\frac{a}{c^{4}d^{2}\left(cx+1\right)}+\frac{3a\ln\left(cx+1\right)}{c^{4}d^{2}}+\frac{b\operatorname{arctanh}\left(cx\right)x^{2}}{2c^{2}d^{2}}-\frac{2bx\operatorname{arctanh}\left(cx\right)}{c^{3}d^{2}}+\frac{b\operatorname{arctanh}\left(cx\right)}{c^{4}d^{2}\left(cx+1\right)}+\frac{3b\operatorname{arctanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] 1/2/c^2*a/d^2*x^2-2*a*x/c^3/d^2+1/c^4*a/d^2/(c*x+1)+3/c^4*a/d^2*ln(c*x+1)+1
/2/c^2*b/d^2*arctanh(c*x)*x^2-2*b*x*arctanh(c*x)/c^3/d^2+1/c^4*b/d^2*arctan
h(c*x)/(c*x+1)+3/c^4*b/d^2*arctanh(c*x)*ln(c*x+1)-3/4/c^4*b/d^2*ln(c*x+1)^2
+3/2/c^4*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/2/c^4*b/d^2*ln(-1/2*c*x+1/2)*ln
(1/2+1/2*c*x)-3/2/c^4*b/d^2*dilog(1/2+1/2*c*x)+1/2*b*x/c^3/d^2+1/2/c^4*b/d^
2+1/2*b/c^4/d^2/(c*x+1)-3/2/c^4*b/d^2*ln(c*x+1)-1/2/c^4*b/d^2*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(c^{4}\left(\frac{2}{c^{9}d^{2}x+c^{8}d^{2}}+\frac{2\left(cx^{2}-2\,x\right)}{c^{7}d^{2}}+\frac{7\,\log\left(cx+1\right)}{c^{8}d^{2}}+\frac{\log\left(cx-1\right)}{c^{8}d^{2}}\right)+16\,c^{4}\int\frac{x^{4}\log\left(cx+1\right)}{2\left(c^{6}d^{2}x^{3}+c^{5}d^{2}x^{2}-c^{4}d^{2}x-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c^4*(2/(c^9*d^2*x + c^8*d^2) + 2*(c*x^2 - 2*x)/(c^7*d^2) + 7*log(c*x
+ 1)/(c^8*d^2) + log(c*x - 1)/(c^8*d^2)) + 16*c^4*integrate(1/2*x^4*log(c*x
 + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c^3*(2/(c^8
*d^2*x + c^7*d^2) - 4*x/(c^6*d^2) + 5*log(c*x + 1)/(c^7*d^2) - log(c*x - 1)
/(c^7*d^2)) - 16*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*
x^2 - c^4*d^2*x - c^3*d^2), x) - 7*c^2*(2/(c^7*d^2*x + c^6*d^2) + 3*log(c*x
 + 1)/(c^6*d^2) + log(c*x - 1)/(c^6*d^2)) + 48*c^2*integrate(1/2*x^2*log(c*
x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c*(2/(c^6*
d^2*x + c^5*d^2) + log(c*x + 1)/(c^5*d^2) - log(c*x - 1)/(c^5*d^2)) + 96*c*
integrate(1/2*x*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d
^2), x) - 4*(c^3*x^3 - 3*c^2*x^2 - 4*c*x + 6*(c*x + 1)*log(c*x + 1) + 2)*lo
g(-c*x + 1)/(c^5*d^2*x + c^4*d^2) + 4/(c^5*d^2*x + c^4*d^2) - 2*log(c*x + 1
)/(c^4*d^2) + 2*log(c*x - 1)/(c^4*d^2) + 48*integrate(1/2*log(c*x + 1)/(c^6
*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x))*b + 1/2*a*(2/(c^5*d^2*x
+ c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1)/(c^4*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3}\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax^{3}}{c^{2}x^{2}+2cx+1}\,dx+\int\frac{bx^{3}\operatorname{atanh}\left(cx\right)}{c^{2}x^{2}+2cx+1}\,dx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(
c**2*x**2 + 2*c*x + 1), x))/d**2
```

3.52 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=149

$$-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(cx+1)}+\frac{2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}+\frac{ax}{c^2d^2}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b}{2c^3d^2(cx+1)}+\frac{b\tanh^{-1}(cx)}{2c^3d^2}+\frac{bx}{}$$

```
[Out] a*x/c^2/d^2-1/2*b/c^3/d^2/(c*x+1)+1/2*b*arctanh(c*x)/c^3/d^2+b*x*arctanh(c*
x)/c^2/d^2+(-a-b*arctanh(c*x))/c^3/d^2/(c*x+1)+2*(a+b*arctanh(c*x))*ln(2/(c
*x+1))/c^3/d^2+1/2*b*ln(-c^2*x^2+1)/c^3/d^2-b*polylog(2,1-2/(c*x+1))/c^3/d^
2
```

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5940, 5910, 260, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(cx+1)}+\frac{2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}+\frac{ax}{c^2d^2}+\frac{b\log\left(1-c^2x^2\right)}{2c^3d^2}-\frac{b}{2c^3d^2(c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (a*x)/(c^2*d^2) - b/(2*c^3*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c^3*d^2) +
(b*x*ArcTanh[c*x])/(c^2*d^2) - (a + b*ArcTanh[c*x])/(c^3*d^2*(1 + c*x)) + (
2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^3*d^2) + (b*Log[1 - c^2*x^2])/(
2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx=\int\left(\frac{a+b\tanh^{-1}(cx)}{c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)^2}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}\right)dx$$

$$=\frac{\int\left(a+b\tanh^{-1}(cx)\right)dx}{c^2d^2}+\frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^2d^2}-\frac{2\int\frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{c^2d^2}$$

$$=\frac{ax}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{b\int\frac{1}{(1+cx)\left(1-c^2x^2\right)}\,dx}{c^2d^2}+$$

$$=\frac{ax}{c^2d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}-\frac{(2b)}{}$$

$$=\frac{ax}{c^2d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{b\,\mathrm{lo}}{}$$

$$=\frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\mathrm{l}}{c^3d^2}$$

$$=\frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{b\tanh^{-1}(cx)}{2c^3d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+\right.}{}$$

Mathematica [A] time = 0.58, size = 121, normalized size = 0.81

$$\frac{4acx - \frac{4a}{cx+1} - 8a\log(cx+1) + b\left(2\log\left(1-c^2x^2\right) - 4\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right) + \sinh\left(2\tanh^{-1}(cx)\right) - \cosh\left(2\tanh\right.\right.}{4c^3d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (4*a*c*x - (4*a)/(1 + c*x) - 8*a*Log[1 + c*x] + b*(-Cosh[2*ArcTanh[c*x]] +
2*Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*
x]] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2*ArcTan
h[c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{artanh}(cx) + ax^2}{c^2d^2x^2 + 2\,cd^2x + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)x^2}{(cdx + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^2, x)
```

maple [A] time = 0.05, size = 216, normalized size = 1.45

$$\frac{ax}{c^2d^2} - \frac{a}{c^3d^2\,(cx+1)} - \frac{2a\ln(cx+1)}{c^3d^2} + \frac{bx \operatorname{arctanh}(cx)}{c^2d^2} - \frac{b \operatorname{arctanh}(cx)}{c^3d^2\,(cx+1)} - \frac{2b \operatorname{arctanh}(cx)\ln(cx+1)}{c^3d^2} + \frac{b\ln(cx+1}{2c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] a*x/c^2/d^2-1/c^3*a/d^2/(c*x+1)-2/c^3*a/d^2*ln(c*x+1)+b*x*arctanh(c*x)/c^2/
d^2-1/c^3*b/d^2*arctanh(c*x)/(c*x+1)-2/c^3*b/d^2*arctanh(c*x)*ln(c*x+1)+1/2
/c^3*b/d^2*ln(c*x+1)^2-1/c^3*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/c^3*b/d^2*l
n(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/c^3*b/d^2*dilog(1/2+1/2*c*x)-1/2*b/c^3/d^
2/(c*x+1)+3/4/c^3*b/d^2*ln(c*x+1)+1/4/c^3*b/d^2*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(c^3\left(\frac{2}{c^7d^2x + c^6d^2} - \frac{4\,x}{c^5d^2} + \frac{5\,\log(cx+1)}{c^6d^2} - \frac{\log(cx-1)}{c^6d^2}\right) - 4\,c^3 \int \frac{x^3\log(cx+1)}{c^5d^2x^3 + c^4d^2x^2 - c^3d^2x - c^2d^2}\,dx - 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(c^3*(2/(c^7*d^2*x + c^6*d^2) - 4*x/(c^5*d^2) + 5*log(c*x + 1)/(c^6*d^
2) - log(c*x - 1)/(c^6*d^2)) - 4*c^3*integrate(x^3*log(c*x + 1)/(c^5*d^2*x^
```

```
3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) - 2*c^2*(2/(c^6*d^2*x + c^5*d^2)
 + 3*log(c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 12*c^2*integrate(x^
2*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) + 16*c
*integrate(x*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2)
, x) + 4*(c^2*x^2 + c*x - 2*(c*x + 1)*log(c*x + 1) - 1)*log(-c*x + 1)/(c^4*
d^2*x + c^3*d^2) + 2/(c^4*d^2*x + c^3*d^2) - log(c*x + 1)/(c^3*d^2) + log(c
*x - 1)/(c^3*d^2) + 8*integrate(log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c
^3*d^2*x - c^2*d^2), x))*b - a*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*d^2) + 2*l
og(c*x + 1)/(c^3*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^2x^2+2cx+1}\,dx + \int \frac{bx^2\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**2*atanh(c*x)/(
c**2*x**2 + 2*c*x + 1), x))/d**2
```

3.53 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=106

$$\frac{a+b\tanh^{-1}(cx)}{c^2d^2(cx+1)}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b}{2c^2d^2(cx+1)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}$$

```
[Out] 1/2*b/c^2/d^2/(c*x+1)-1/2*b*arctanh(c*x)/c^2/d^2+(a+b*arctanh(c*x))/c^2/d^2
/(c*x+1)-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^2/d^2+1/2*b*polylog(2,1-2/(c*x+
1))/c^2/d^2
```

Rubi [A] time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5940, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(cx+1)}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b}{2c^2d^2(cx+1)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] b/(2*c^2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*c^2*d^2) + (a + b*ArcTanh[c*x
])/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^2*d^2)
+ (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx &= \int \left(-\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}\right)dx\\
&= -\frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{cd^2}+\frac{\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{cd^2}\\
&= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}-\frac{b\int \frac{1}{(1+cx)\left(1-c^2x^2\right)}\,dx}{cd^2}+\frac{b\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{cd^2}\\
&= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+cx}\right)}{c^2d^2}-\dots\\
&= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c^2d^2}-\frac{b\int\left(\frac{1}{2(1+cx)^2}\dots\right)}{\dots}\\
&= \frac{b}{2c^2d^2(1+cx)}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c^2d^2}\dots\\
&= \frac{b}{2c^2d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\dots
\end{aligned}$$

Mathematica [A] time = 0.37, size = 99, normalized size = 0.93

$$\frac{\frac{4a}{cx+1}+4a\log(cx+1)+b\left(2\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)+\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\left(-2\dots\right.\right.}{4c^2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] ((4*a)/(1 + c*x) + 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]] + 2*PolyLog[2
, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] - 2*Log[1 +
```

```
E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]))/(4*c^
2*d^2)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx\,\text{artanh}\,(cx)+ax}{c^2d^2x^2+2\,cd^2x+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)x}{(cdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(c*d*x + d)^2, x)
```

maple [A] time = 0.06, size = 192, normalized size = 1.81

$$\frac{a}{c^2d^2\,(cx+1)}+\frac{a\ln{(cx+1)}}{c^2d^2}+\frac{b\,\text{arctanh}\,(cx)}{c^2d^2\,(cx+1)}+\frac{b\,\text{arctanh}\,(cx)\ln{(cx+1)}}{c^2d^2}+\frac{b}{2c^2d^2\,(cx+1)}-\frac{b\ln{(cx+1)}}{4c^2d^2}+\frac{b\ln{(cx}}{4c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] 1/c^2*a/d^2/(c*x+1)+1/c^2*a/d^2*ln(c*x+1)+1/c^2*b/d^2*arctanh(c*x)/(c*x+1)+
1/c^2*b/d^2*arctanh(c*x)*ln(c*x+1)+1/2*b/c^2/d^2/(c*x+1)-1/4/c^2*b/d^2*ln(c
*x+1)+1/4/c^2*b/d^2*ln(c*x-1)-1/4/c^2*b/d^2*ln(c*x+1)^2+1/2/c^2*b/d^2*ln(-1
/2*c*x+1/2)*ln(c*x+1)-1/2/c^2*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2/c^
2*b/d^2*dilog(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(8\,c^2\int\frac{x^2\log{(cx+1)}}{c^4d^2x^3+c^3d^2x^2-c^2d^2x-cd^2}\,dx-c\left(\frac{2}{c^4d^2x+c^3d^2}+\frac{\log{(cx+1)}}{c^3d^2}-\frac{\log{(cx-1)}}{c^3d^2}\right)+4\,c\int\frac{}{c^4d^2x^3+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*c^2*integrate(x^2*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x^2 - c^2*d^2*
x - c*d^2), x) - c*(2/(c^4*d^2*x + c^3*d^2) + log(c*x + 1)/(c^3*d^2) - log(
c*x - 1)/(c^3*d^2)) + 4*c*integrate(x*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x
^2 - c^2*d^2*x - c*d^2), x) - 4*((c*x + 1)*log(c*x + 1) + 1)*log(-c*x + 1)/
(c^3*d^2*x + c^2*d^2) + 2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) +
log(c*x - 1)/(c^2*d^2) + 4*integrate(log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x^
2 - c^2*d^2*x - c*d^2), x))*b + a*(1/(c^3*d^2*x + c^2*d^2) + log(c*x + 1)/(
c^2*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\left(a+b\,\text{atanh}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

```
[Out] int((x*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^2x^2+2cx+1}\,dx + \int \frac{bx \operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x*atanh(c*x)/(c**2*x
**2 + 2*c*x + 1), x))/d**2
```

3.54 $\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{a+b\tanh^{-1}(cx)}{cd^2(cx+1)}-\frac{b}{2cd^2(cx+1)}+\frac{b\tanh^{-1}(cx)}{2cd^2}$$

```
[Out] -1/2*b/c/d^2/(c*x+1)+1/2*b*arctanh(c*x)/c/d^2+(-a-b*arctanh(c*x))/c/d^2/(c*
x+1)
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5926, 627, 44, 207}

$$-\frac{a+b\tanh^{-1}(cx)}{cd^2(cx+1)}-\frac{b}{2cd^2(cx+1)}+\frac{b\tanh^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
```

```
[Out] -b/(2*c*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c*d^2) - (a + b*ArcTanh[c*x])/
(c*d^2*(1 + c*x))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^2}\,dx = -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} + \frac{b\int \frac{1}{(d+cdx)\left(1-c^2x^2\right)}\,dx}{d}$$
$$= -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} + \frac{b\int \frac{1}{\left(\frac{1}{d}-\frac{cx}{d}\right)(d+cdx)^2}\,dx}{d}$$
$$= -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} + \frac{b\int \left(\frac{1}{2d(1+cx)^2}-\frac{1}{2d\left(-1+c^2x^2\right)}\right)dx}{d}$$
$$= -\frac{b}{2cd^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} - \frac{b\int \frac{1}{-1+c^2x^2}\,dx}{2d^2}$$
$$= -\frac{b}{2cd^2(1+cx)} + \frac{b\tanh^{-1}(cx)}{2cd^2} - \frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 1.12

$$\frac{-4a-(bcx+b)\log(1-cx)+b\log(cx+1)+bcx\log(cx+1)-4b\tanh^{-1}(cx)-2b}{4cd^2(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
```

```
[Out] (-4*a - 2*b - 4*b*ArcTanh[c*x] - (b + b*c*x)*Log[1 - c*x] + b*Log[1 + c*x]
+ b*c*x*Log[1 + c*x])/(4*c*d^2*(1 + c*x))
```

fricas [A] time = 1.29, size = 49, normalized size = 0.86

$$\frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)-4\,a-2\,b}{4\left(c^2d^2x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1)) - 4*a - 2*b)/(c^2*d^2*x + c*d^2)
```

giac [A] time = 0.24, size = 63, normalized size = 1.11

$$\frac{1}{4}c\left(\frac{(cx-1)b\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2d^2}+\frac{(cx-1)(2\,a+b)}{(cx+1)c^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*c*((c*x - 1)*b*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2*d^2) + (c*x - 1
)*(2*a + b)/((c*x + 1)*c^2*d^2))
```

maple [A] time = 0.04, size = 84, normalized size = 1.47

$$-\frac{a}{c\,d^2\,(cx+1)}-\frac{b\,\mathrm{arctanh}\,(cx)}{c\,d^2\,(cx+1)}-\frac{b\ln(cx-1)}{4c\,d^2}-\frac{b}{2c\,d^2\,(cx+1)}+\frac{b\ln(cx+1)}{4c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] -1/c*a/d^2/(c*x+1)-1/c*b/d^2*arctanh(c*x)/(c*x+1)-1/4/c*b/d^2*ln(c*x-1)-1/2
*b/c/d^2/(c*x+1)+1/4/c*b/d^2*ln(c*x+1)
```

maxima [A] time = 0.30, size = 96, normalized size = 1.68

$$-\frac{1}{4}\left(c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log(cx+1)}{c^2d^2}+\frac{\log(cx-1)}{c^2d^2}\right)+\frac{4\,\operatorname{artanh}(cx)}{c^2d^2x+cd^2}\right)b-\frac{a}{c^2d^2x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c
^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*b - a/(c^2*d^2*x + c*d^2)
```

mupad [B] time = 1.07, size = 45, normalized size = 0.79

$$-\frac{b\,\mathrm{atanh}(c\,x)-c\,\left(2\,a\,x+b\,x+b\,x\,\mathrm{atanh}(c\,x)\right)}{2\,x\,c^2\,d^2+2\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + c*d*x)^2,x)
```

```
[Out] -(b*atanh(c*x) - c*(2*a*x + b*x + b*x*atanh(c*x)))/(2*c*d^2 + 2*c^2*d^2*x)
```

sympy [A] time = 1.39, size = 121, normalized size = 2.12

$$\begin{cases} -\frac{2a}{2c^2d^2x+2cd^2}+\frac{bcx\,\mathrm{atanh}(cx)}{2c^2d^2x+2cd^2}-\frac{b\,\mathrm{atanh}(cx)}{2c^2d^2x+2cd^2}-\frac{b}{2c^2d^2x+2cd^2} & \text{for } d\neq 0 \\ \tilde{\infty}\left(ax+bx\,\mathrm{atanh}(cx)+\frac{b\log\left(x-\frac{1}{c}\right)}{c}+\frac{b\,\mathrm{atanh}(cx)}{c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] Piecewise((-2*a/(2*c**2*d**2*x + 2*c*d**2) + b*c*x*atanh(c*x)/(2*c**2*d**2*
x + 2*c*d**2) - b*atanh(c*x)/(2*c**2*d**2*x + 2*c*d**2) - b/(2*c**2*d**2*x
+ 2*c*d**2), Ne(d, 0)), (zoo*(a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*a
tanh(c*x)/c), True))
```

3.55 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^2}\,dx$

Optimal. Leaf size=124

$$\frac{a+b\tanh^{-1}(cx)}{d^2(cx+1)}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{a\log(x)}{d^2}-\frac{b\text{Li}_2(-cx)}{2d^2}+\frac{b\text{Li}_2(cx)}{2d^2}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d^2}+\frac{b}{2d^2(cx+1)}-$$

```
[Out] 1/2*b/d^2/(c*x+1)-1/2*b*arctanh(c*x)/d^2+(a+b*arctanh(c*x))/d^2/(c*x+1)+a*l
n(x)/d^2+(a+b*arctanh(c*x))*ln(2/(c*x+1))/d^2-1/2*b*polylog(2,-c*x)/d^2+1/2
*b*polylog(2,c*x)/d^2-1/2*b*polylog(2,1-2/(c*x+1))/d^2
```

Rubi [A] time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5940, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}(2,-cx)}{2d^2}+\frac{b\text{PolyLog}(2,cx)}{2d^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{a+b\tanh^{-1}(cx)}{d^2(cx+1)}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

```
[Out] b/(2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*d^2) + (a + b*ArcTanh[c*x])/(d^2*
(1 + c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 -
 (b*PolyLog[2, -(c*x)])/(2*d^2) + (b*PolyLog[2, c*x])/(2*d^2) - (b*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)^2} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)}\right)dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{d^2}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\mathrm{Li}_2(-cx)}{2d^2} + \frac{b\mathrm{Li}_2(cx}{2d^2}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\mathrm{Li}_2(-cx)}{2d^2} + \frac{b\mathrm{Li}_2(cx}{2d^2}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\mathrm{Li}_2(-cx)}{2d^2} + \frac{b\mathrm{Li}_2(cx}{2d^2}$$

$$= \frac{b}{2d^2(1+cx)} + \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\mathrm{Li}_2(}{2d}$$

$$= \frac{b}{2d^2(1+cx)} - \frac{b\tanh^{-1}(cx)}{2d^2} + \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log}{d^2}$$

Mathematica [A] time = 0.42, size = 101, normalized size = 0.81

$$\frac{\frac{4a}{cx+1} - 4a\log(cx+1) + 4a\log(x) + b\left(-2\mathrm{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) - \sinh\left(2\tanh^{-1}(cx)\right) + \cosh\left(2\tanh^{-1}(cx)\right) + 2\,\mathrm{t}}{4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

```
[Out] ((4*a)/(1 + c*x) + 4*a*Log[x] - 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]]
- 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]]
+ 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c
*x]]))/(4*d^2)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\left(cx\right)+a}{c^{2}d^{2}x^{3}+2\,cd^{2}x^{2}+d^{2}x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{artanh}\left(cx\right)+a}{\left(cdx+d\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x), x)
```

maple [A] time = 0.06, size = 221, normalized size = 1.78

$$\frac{a\ln\left(cx\right)}{d^{2}}+\frac{a}{d^{2}\left(cx+1\right)}-\frac{a\ln\left(cx+1\right)}{d^{2}}+\frac{b\,\mathrm{arctanh}\left(cx\right)\ln\left(cx\right)}{d^{2}}+\frac{b\,\mathrm{arctanh}\left(cx\right)}{d^{2}\left(cx+1\right)}-\frac{b\,\mathrm{arctanh}\left(cx\right)\ln\left(cx+1\right)}{d^{2}}+\frac{b\ln\left(cx+\right.}{4d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x)
```

```
[Out] a/d^2*ln(c*x)+a/d^2/(c*x+1)-a/d^2*ln(c*x+1)+b/d^2*arctanh(c*x)*ln(c*x)+b/d^
2*arctanh(c*x)/(c*x+1)-b/d^2*arctanh(c*x)*ln(c*x+1)+1/4*b/d^2*ln(c*x+1)^2-1
/2*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c
*x)+1/2*b/d^2*dilog(1/2+1/2*c*x)+1/4*b/d^2*ln(c*x-1)+1/2*b/d^2/(c*x+1)-1/4*
b/d^2*ln(c*x+1)-1/2*b/d^2*dilog(c*x)-1/2*b/d^2*dilog(c*x+1)-1/2*b/d^2*ln(c*
x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{1}{cd^{2}x+d^{2}}-\frac{\log\left(cx+1\right)}{d^{2}}+\frac{\log\left(x\right)}{d^{2}}\right)+\frac{1}{2}\,b\int \frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{c^{2}d^{2}x^{3}+2\,cd^{2}x^{2}+d^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/2*b*integrate((lo
g(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{x\,{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))/(x*(d + c*d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^3+2cx^2+x}\,dx + \int \frac{b\,\text{atanh}\,(cx)}{c^2x^3+2cx^2+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b*atanh(c*x)/(c**2*x*
*3 + 2*c*x**2 + x), x))/d**2
```

3.56 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^2}\,dx$

Optimal. Leaf size=171

$$-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{d^2x}-\frac{2c\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2ac\log(x)}{d^2}-\frac{bc\log\left(1-c^2x^2\right)}{2d^2}+\frac{bc\mathrm{Li}_2(\ldots)}{d\ldots}$$

```
[Out] -1/2*b*c/d^2/(c*x+1)+1/2*b*c*arctanh(c*x)/d^2+(-a-b*arctanh(c*x))/d^2/x-c*(
a+b*arctanh(c*x))/d^2/(c*x+1)-2*a*c*ln(x)/d^2+b*c*ln(x)/d^2-2*c*(a+b*arctan
h(c*x))*ln(2/(c*x+1))/d^2-1/2*b*c*ln(-c^2*x^2+1)/d^2+b*c*polylog(2,-c*x)/d^
2-b*c*polylog(2,c*x)/d^2+b*c*polylog(2,1-2/(c*x+1))/d^2
```

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5940, 5916, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{bc\,\mathrm{PolyLog}(2,-cx)}{d^2}-\frac{bc\,\mathrm{PolyLog}(2,cx)}{d^2}+\frac{bc\,\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^2}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{d^2x}-\frac{2c\log\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```

```
[Out] -(b*c)/(2*d^2*(1 + c*x)) + (b*c*ArcTanh[c*x])/(2*d^2) - (a + b*ArcTanh[c*x]
)/(d^2*x) - (c*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*a*c*Log[x])/d^2 +
 (b*c*Log[x])/d^2 - (2*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 - (b*c*
Log[1 - c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, -(c*x)])/d^2 - (b*c*PolyLog[2,
c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + c*x)])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)^2} + \frac{2c^2\left(a+b\tanh\right.}{d^2(1+c.}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{(2c)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} + \frac{c^2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} + \frac{\left(2c^2\right)\int \frac{a+b\tanh^{-1}(cx}{1+cx}}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+}\right.}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+}\right.}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+}\right.}{d^2}$$

$$= -\frac{bc}{2d^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} + \frac{bc\log(x)}{d^2} - \frac{2c}{}$$

$$= -\frac{bc}{2d^2(1+cx)} + \frac{bc\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2}$$

Mathematica [A] time = 0.83, size = 140, normalized size = 0.82

$$\frac{-\frac{4ac}{cx+1} - 8ac\log(x) + 8ac\log(cx+1) - \frac{4a}{x} + bc\left(4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 4\mathrm{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) + \sinh\left(2\tanh^{-1}(cx)\right) - \cos\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```

```
[Out] ((-4*a)/x - (4*a*c)/(1 + c*x) - 8*a*c*Log[x] + 8*a*c*Log[1 + c*x] + b*c*(-C
osh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog[2, E^(-2*A
rcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) - 2*Cosh[2*Ar
cTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTanh[c*x]])))/(4
*d^2)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{c^2d^2x^4+2\,cd^2x^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^4 + 2*c*d^2*x^3 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\,(cx)+a}{(cdx+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^2), x)
```

maple [A] time = 0.06, size = 269, normalized size = 1.57

$$-\frac{a}{d^2x}-\frac{2ca\ln(cx)}{d^2}-\frac{ca}{d^2(cx+1)}+\frac{2ca\ln(cx+1)}{d^2}-\frac{b\operatorname{arctanh}(cx)}{d^2x}-\frac{2cb\operatorname{arctanh}(cx)\ln(cx)}{d^2}-\frac{cb\operatorname{arctanh}(cx)}{d^2(cx+1)}+\frac{2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x)
```

```
[Out] -a/d^2/x-2*c*a/d^2*ln(c*x)-c*a/d^2/(c*x+1)+2*c*a/d^2*ln(c*x+1)-b/d^2*arctan
h(c*x)/x-2*c*b/d^2*arctanh(c*x)*ln(c*x)-c*b/d^2*arctanh(c*x)/(c*x+1)+2*c*b/
d^2*arctanh(c*x)*ln(c*x+1)+c*b/d^2*ln(c*x)-3/4*c*b/d^2*ln(c*x-1)-1/2*b*c/d^
2/(c*x+1)-1/4*c*b/d^2*ln(c*x+1)+c*b/d^2*dilog(c*x)+c*b/d^2*dilog(c*x+1)+c*b
/d^2*ln(c*x)*ln(c*x+1)-1/2*c*b/d^2*ln(c*x+1)^2+c*b/d^2*ln(-1/2*c*x+1/2)*ln(
c*x+1)-c*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-c*b/d^2*dilog(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{2\,cx+1}{cd^2x^2+d^2x}-\frac{2\,c\log(cx+1)}{d^2}+\frac{2\,c\log(x)}{d^2}\right)+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{c^2d^2x^4+2\,cd^2x^3+d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/d^2
) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^4 + 2*c*d^2*x
^3 + d^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{x^2\,{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^2x^4+2cx^3+x^2}\,dx+\int\frac{b\operatorname{atanh}(cx)}{c^2x^4+2cx^3+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c**2
*x**4 + 2*c*x**3 + x**2), x))/d**2
```

3.57 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^2}\,dx$

Optimal. Leaf size=212

$$\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{a+b\tanh^{-1}(cx)}{2d^2x^2}+\frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x}+\frac{3ac^2\log(x)}{d^2}-$$

```
[Out] -1/2*b*c/d^2/x+1/2*b*c^2/d^2/(c*x+1)+1/2*(-a-b*arctanh(c*x))/d^2/x^2+2*c*(a
+b*arctanh(c*x))/d^2/x+c^2*(a+b*arctanh(c*x))/d^2/(c*x+1)+3*a*c^2*ln(x)/d^2
-2*b*c^2*ln(x)/d^2+3*c^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/d^2+b*c^2*ln(-c^2
*x^2+1)/d^2-3/2*b*c^2*polylog(2,-c*x)/d^2+3/2*b*c^2*polylog(2,c*x)/d^2-3/2*
b*c^2*polylog(2,1-2/(c*x+1))/d^2
```

Rubi [A] time = 0.26, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{3bc^2\text{PolyLog}(2,-cx)}{2d^2}+\frac{3bc^2\text{PolyLog}(2,cx)}{2d^2}-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

```
[Out] -(b*c)/(2*d^2*x) + (b*c^2)/(2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])/(2*d^2*
x^2) + (2*c*(a + b*ArcTanh[c*x]))/(d^2*x) + (c^2*(a + b*ArcTanh[c*x]))/(d^2
*(1 + c*x)) + (3*a*c^2*Log[x])/d^2 - (2*b*c^2*Log[x])/d^2 + (3*c^2*(a + b*A
rcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (b*c^2*Log[1 - c^2*x^2])/d^2 - (3*b*c^
2*PolyLog[2, -(c*x)])/(2*d^2) + (3*b*c^2*PolyLog[2, c*x])/(2*d^2) - (3*b*c^
2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x^3} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x^2} + \frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2x} - \frac{c^3\left(a+b\tanh\right.}{d^2(1+c}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2c)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{\left(3c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} - \frac{c^3\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3ac^2\log(x)}{d^2} + \frac{3c}{}$$

$$= -\frac{bc}{2d^2x} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3ac^2\log}{d^2}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-}\right.}{d^2(1+cx}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2}{2d^2(1+cx)} + \frac{bc^2\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c}{}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2}{2d^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(c\right.}{d^2(1+cx)}$$

Mathematica [A] time = 1.11, size = 189, normalized size = 0.89

$$\frac{4ac^2}{cx+1} + 12ac^2\log(x) - 12ac^2\log(cx+1) + \frac{8ac}{x} - \frac{2a}{x^2} - 6bc^2\operatorname{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) - 8bc^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 2b\tanh^{-1}(cx)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

```
[Out] ((-2*a)/x^2 + (8*a*c)/x - (2*b*c)/x + (4*a*c^2)/(1 + c*x) + b*c^2*Cosh[2*Ar
cTanh[c*x]] + 12*a*c^2*Log[x] - 12*a*c^2*Log[1 + c*x] - 8*b*c^2*Log[(c*x)/S
qrt[1 - c^2*x^2]] - 6*b*c^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - b*c^2*Sinh[2*
ArcTanh[c*x]] + 2*b*ArcTanh[c*x]*(c^2 - x^(-2) + (4*c)/x + c^2*Cosh[2*ArcTa
nh[c*x]] + 6*c^2*Log[1 - E^(-2*ArcTanh[c*x])] - c^2*Sinh[2*ArcTanh[c*x]]))/
(4*d^2)
```

fricas [F] time = 1.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx\right)+a}{c^2d^2x^5+2\,cd^2x^4+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(cx\right)+a}{\left(cdx+d\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^3), x)
```

maple [A] time = 0.07, size = 338, normalized size = 1.59

$$-\frac{a}{2d^2x^2}+\frac{2ca}{d^2x}+\frac{3c^2a\ln\left(cx\right)}{d^2}+\frac{c^2a}{d^2\left(cx+1\right)}-\frac{3c^2a\ln\left(cx+1\right)}{d^2}-\frac{b\,\text{arctanh}\left(cx\right)}{2d^2x^2}+\frac{2cb\,\text{arctanh}\left(cx\right)}{d^2x}+\frac{3c^2b\,\text{arctanh}\left(cx\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x)
```

```
[Out] -1/2*a/d^2/x^2+2*c*a/d^2/x+3*c^2*a/d^2*ln(c*x)+c^2*a/d^2/(c*x+1)-3*c^2*a/d^
2*ln(c*x+1)-1/2*b/d^2*arctanh(c*x)/x^2+2*c*b/d^2*arctanh(c*x)/x+3*c^2*b/d^2
*arctanh(c*x)*ln(c*x)+c^2*b/d^2*arctanh(c*x)/(c*x+1)-3*c^2*b/d^2*arctanh(c*
x)*ln(c*x+1)-3/2*c^2*b/d^2*dilog(c*x)-3/2*c^2*b/d^2*dilog(c*x+1)-3/2*c^2*b/
d^2*ln(c*x)*ln(c*x+1)+3/4*c^2*b/d^2*ln(c*x+1)^2-3/2*c^2*b/d^2*ln(-1/2*c*x+1
/2)*ln(c*x+1)+3/2*c^2*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/2*c^2*b/d^2*
dilog(1/2+1/2*c*x)-1/2*b*c/d^2/x-2*c^2*b/d^2*ln(c*x)+c^2*b/d^2*ln(c*x-1)+1/
2*b*c^2/d^2/(c*x+1)+c^2*b/d^2*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{6\,c^2\log\left(cx+1\right)}{d^2}-\frac{6\,c^2\log(x)}{d^2}-\frac{6\,c^2x^2+3\,cx-1}{cd^2x^3+d^2x^2}\right)+\frac{1}{2}\,b\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{c^2d^2x^5+2\,cd^2x^4+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x - 1)
/(c*d^2*x^3 + d^2*x^2)) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c
^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\text{atanh}\left(c\,x\right)}{x^3\,{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^5+2cx^4+x^3}\,dx + \int \frac{b\,\operatorname{atanh}(cx)}{c^2x^5+2cx^4+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c**2
*x**5 + 2*c*x**4 + x**3), x))/d**2
```

3.58 $\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=227

$$\frac{4\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(cx+1)^2}-\frac{6\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}-\frac{3ax}{c^4d^3}+\frac{3b\text{Li}_2}{}$$

```
[Out] -3*a*x/c^4/d^3+1/2*b*x/c^4/d^3-1/8*b/c^5/d^3/(c*x+1)^2+15/8*b/c^5/d^3/(c*x+
1)-19/8*b*arctanh(c*x)/c^5/d^3-3*b*x*arctanh(c*x)/c^4/d^3+1/2*x^2*(a+b*arct
anh(c*x))/c^3/d^3+1/2*(-a-b*arctanh(c*x))/c^5/d^3/(c*x+1)^2+4*(a+b*arctanh(
c*x))/c^5/d^3/(c*x+1)-6*(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^5/d^3-3/2*b*ln(-
c^2*x^2+1)/c^5/d^3+3*b*polylog(2,1-2/(c*x+1))/c^5/d^3
```

Rubi [A] time = 0.29, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {5940, 5910, 260, 5916, 321, 206, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}+\frac{4\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(cx+1)^2}-\frac{6\log\left(\frac{2}{cx+1}\right)\left(a+b\right.}{c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (-3*a*x)/(c^4*d^3) + (b*x)/(2*c^4*d^3) - b/(8*c^5*d^3*(1 + c*x)^2) + (15*b)
/(8*c^5*d^3*(1 + c*x)) - (19*b*ArcTanh[c*x])/(8*c^5*d^3) - (3*b*x*ArcTanh[c
*x])/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*c^3*d^3) - (a + b*ArcTanh[c*
x])/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh[c*x]))/(c^5*d^3*(1 + c*x))
- (6*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^5*d^3) - (3*b*Log[1 - c^2*x^
2])/(2*c^5*d^3) + (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

```
&& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx = \int\left(-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}+\frac{x\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{a+b\tanh^{-1}(cx)}{c^4d^3(1+cx)^3}-\frac{4\left(a+\right.}{c^4d}\right.$$

$$=\frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^4d^3}-\frac{3\int\left(a+b\tanh^{-1}(cx)\right)dx}{c^4d^3}-\frac{4\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^4d^3}+\frac{6\int\frac{a+b\,\mathrm{t}}{}}{c}$$

$$=-\frac{3ax}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(1+cx)^2}+\frac{4\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3(1+cx)}-\frac{6(}{}$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(1+cx)^2}+$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b\tanh^{-1}(cx)}{2c^5d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}-\frac{a+}{2c}$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b}{8c^5d^3(1+cx)^2}+\frac{15b}{8c^5d^3(1+cx)}-\frac{b\tanh^{-1}(cx)}{2c^5d^3}-\frac{3bx\tanh^{-1}(}{c^4d^3}$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b}{8c^5d^3(1+cx)^2}+\frac{15b}{8c^5d^3(1+cx)}-\frac{19b\tanh^{-1}(cx)}{8c^5d^3}-\frac{3bx\tanh}{c^4d^3}$$

Mathematica [A] time = 0.82, size = 189, normalized size = 0.83

$$\frac{16ac^2x^2-96acx+\frac{128a}{cx+1}-\frac{16a}{(cx+1)^2}+192a\log(cx+1)+b\left(-48\log\left(1-c^2x^2\right)+4\tanh^{-1}(cx)\left(4c^2x^2-24cx-48\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (-96*a*c*x + 16*a*c^2*x^2 - (16*a)/(1 + c*x)^2 + (128*a)/(1 + c*x) + 192*a*
Log[1 + c*x] + b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] -
 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcT
anh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x + 4*c^2*x^2
+ 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*ArcTanh
[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^5*d^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4\,\text{artanh}\,(cx)+ax^4}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arctanh(c*x) + a*x^4)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)x^4}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^4/(c*d*x + d)^3, x)
```

maple [A] time = 0.06, size = 319, normalized size = 1.41

$$\frac{a\,x^2}{2c^3d^3}-\frac{3ax}{c^4d^3}-\frac{a}{2c^5d^3\,(cx+1)^2}+\frac{4a}{c^5d^3\,(cx+1)}+\frac{6a\ln(cx+1)}{c^5d^3}+\frac{b\,\mathrm{arctanh}\,(cx)\,x^2}{2c^3d^3}-\frac{3bx\,\mathrm{arctanh}\,(cx)}{c^4d^3}-\frac{b\,\mathrm{arctanh}\,(cx)}{2c^5d^3\,(cx+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] 1/2/c^3*a/d^3*x^2-3*a*x/c^4/d^3-1/2/c^5*a/d^3/(c*x+1)^2+4/c^5*a/d^3/(c*x+1)
+6/c^5*a/d^3*ln(c*x+1)+1/2/c^3*b/d^3*arctanh(c*x)*x^2-3*b*x*arctanh(c*x)/c^
4/d^3-1/2/c^5*b/d^3*arctanh(c*x)/(c*x+1)^2+4/c^5*b/d^3*arctanh(c*x)/(c*x+1)
+6/c^5*b/d^3*arctanh(c*x)*ln(c*x+1)-3/2/c^5*b/d^3*ln(c*x+1)^2+3/c^5*b/d^3*l
n(-1/2*c*x+1/2)*ln(c*x+1)-3/c^5*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/c^
5*b/d^3*dilog(1/2+1/2*c*x)+1/2*b*x/c^4/d^3+1/2/c^5*b/d^3-1/8*b/c^5/d^3/(c*x
+1)^2+15/8*b/c^5/d^3/(c*x+1)-43/16/c^5*b/d^3*ln(c*x+1)-5/16/c^5*b/d^3*ln(c*
x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{32}\left(c^5\left(\frac{2\,(9\,cx+8)}{c^{12}d^3x^2+2\,c^{11}d^3x+c^{10}d^3}+\frac{4\left(cx^2-4\,x\right)}{c^9d^3}+\frac{31\,\log(cx+1)}{c^{10}d^3}+\frac{\log(cx-1)}{c^{10}d^3}\right)+32\,c^5\int\frac{x^5\log}{2\left(c^8d^3x^4+2\,c^7d^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(c^5*(2*(9*c*x + 8)/(c^12*d^3*x^2 + 2*c^11*d^3*x + c^10*d^3) + 4*(c*x^
2 - 4*x)/(c^9*d^3) + 31*log(c*x + 1)/(c^10*d^3) + log(c*x - 1)/(c^10*d^3))
+ 32*c^5*integrate(1/2*x^5*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^
5*d^3*x - c^4*d^3), x) + 3*c^4*(2*(7*c*x + 6)/(c^11*d^3*x^2 + 2*c^10*d^3*x
+ c^9*d^3) - 8*x/(c^8*d^3) + 17*log(c*x + 1)/(c^9*d^3) - log(c*x - 1)/(c^9*
d^3)) - 32*c^4*integrate(1/2*x^4*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3
- 2*c^5*d^3*x - c^4*d^3), x) - 15*c^3*(2*(5*c*x + 4)/(c^10*d^3*x^2 + 2*c^9*
d^3*x + c^8*d^3) + 7*log(c*x + 1)/(c^8*d^3) + log(c*x - 1)/(c^8*d^3)) + 192
*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^
3*x - c^4*d^3), x) + 9*c^2*(2*(3*c*x + 2)/(c^9*d^3*x^2 + 2*c^8*d^3*x + c^7*
d^3) + log(c*x + 1)/(c^7*d^3) - log(c*x - 1)/(c^7*d^3)) + 576*c^2*integrate
(1/2*x^2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^3)
, x) + 9*c*(2*x/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - log(c*x + 1)/(c^6*d
^3) + log(c*x - 1)/(c^6*d^3)) + 576*c*integrate(1/2*x*log(c*x + 1)/(c^8*d^3
*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^3), x) - 8*(c^4*x^4 - 4*c^3*x^3
- 11*c^2*x^2 + 2*c*x + 12*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) + 7)*log(-c*x
+ 1)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + 14*(c*x + 2)/(c^7*d^3*x^2 + 2*
c^6*d^3*x + c^5*d^3) - 7*log(c*x + 1)/(c^5*d^3) + 7*log(c*x - 1)/(c^5*d^3)
+ 192*integrate(1/2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x
 - c^4*d^3), x))*b + 1/2*a*((8*c*x + 7)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^
3) + (c*x^2 - 6*x)/(c^4*d^3) + 12*log(c*x + 1)/(c^5*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] int((x^4*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^4\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.59 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=194

$$-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(cx+1)^2}+\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}+\frac{ax}{c^3d^3}-\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{11b}{8c^4d^3(cx+1}$$

```
[Out] a*x/c^3/d^3+1/8*b/c^4/d^3/(c*x+1)^2-11/8*b/c^4/d^3/(c*x+1)+11/8*b*arctanh(c
*x)/c^4/d^3+b*x*arctanh(c*x)/c^3/d^3+1/2*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)
^2-3*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)+3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/
c^4/d^3+1/2*b*ln(-c^2*x^2+1)/c^4/d^3-3/2*b*polylog(2,1-2/(c*x+1))/c^4/d^3
```

Rubi [A] time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5940, 5910, 260, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(cx+1)^2}+\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}+\frac{ax}{c^3d^3}+\frac{b\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (a*x)/(c^3*d^3) + b/(8*c^4*d^3*(1 + c*x)^2) - (11*b)/(8*c^4*d^3*(1 + c*x))
+ (11*b*ArcTanh[c*x])/(8*c^4*d^3) + (b*x*ArcTanh[c*x])/(c^3*d^3) + (a + b*A
rcTanh[c*x])/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x]))/(c^4*d^3*(1
 + c*x)) + (3*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^3) + (b*Log[1 -
 c^2*x^2])/(2*c^4*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx=\int\left(\frac{a+b\tanh^{-1}(cx)}{c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{c^3d^3(1+cx)^3}+\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)^2}-\ldots\right.$$

$$=\frac{\int\left(a+b\tanh^{-1}(cx)\right)dx}{c^3d^3}-\frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^3d^3}+\frac{3\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^3d^3}-\ldots$$

$$=\frac{ax}{c^3d^3}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2}-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)}+\ldots$$

$$=\frac{ax}{c^3d^3}+\frac{bx\tanh^{-1}(cx)}{c^3d^3}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2}-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)}+\ldots$$

$$=\frac{ax}{c^3d^3}+\frac{bx\tanh^{-1}(cx)}{c^3d^3}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2}-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)}+\ldots$$

$$=\frac{ax}{c^3d^3}+\frac{b}{8c^4d^3(1+cx)^2}-\frac{11b}{8c^4d^3(1+cx)}+\frac{bx\tanh^{-1}(cx)}{c^3d^3}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2}-\ldots$$

$$=\frac{ax}{c^3d^3}+\frac{b}{8c^4d^3(1+cx)^2}-\frac{11b}{8c^4d^3(1+cx)}+\frac{11b\tanh^{-1}(cx)}{8c^4d^3}+\frac{bx\tanh^{-1}(cx)}{c^3d^3}+\ldots$$

Mathematica [A] time = 0.70, size = 167, normalized size = 0.86

$$32acx-\frac{96a}{cx+1}+\frac{16a}{(cx+1)^2}-96a\log(cx+1)+b\left(16\log\left(1-c^2x^2\right)-48\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)+20\sinh\left(2\tanh^{-1}(cx)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (32*a*c*x + (16*a)/(1 + c*x)^2 - (96*a)/(1 + c*x) - 96*a*Log[1 + c*x] + b*(
-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1 - c^2*x^2] - 48*
PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]
*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 24*Log[1 + E^(-2
*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*
ArcTanh[c*x]]))/(32*c^4*d^3)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3\,\text{artanh}\,(cx)+ax^3}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)x^3}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^3, x)
```

maple [A] time = 0.06, size = 270, normalized size = 1.39

$$\frac{ax}{c^3d^3}+\frac{a}{2c^4d^3\,(cx+1)^2}-\frac{3a}{c^4d^3\,(cx+1)}-\frac{3a\ln(cx+1)}{c^4d^3}+\frac{bx\arctan\text{h}\,(cx)}{c^3d^3}+\frac{b\,\text{arctanh}\,(cx)}{2c^4d^3\,(cx+1)^2}-\frac{3b\,\text{arctanh}\,(cx)}{c^4d^3\,(cx+1)}-\frac{3b\,\text{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] a*x/c^3/d^3+1/2/c^4*a/d^3/(c*x+1)^2-3/c^4*a/d^3/(c*x+1)-3/c^4*a/d^3*ln(c*x+
1)+b*x*arctanh(c*x)/c^3/d^3+1/2/c^4*b/d^3*arctanh(c*x)/(c*x+1)^2-3/c^4*b/d^
3*arctanh(c*x)/(c*x+1)-3/c^4*b/d^3*arctanh(c*x)*ln(c*x+1)+3/4/c^4*b/d^3*ln(
c*x+1)^2-3/2/c^4*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/2/c^4*b/d^3*ln(-1/2*c*x
+1/2)*ln(1/2+1/2*c*x)+3/2/c^4*b/d^3*dilog(1/2+1/2*c*x)+1/8*b/c^4/d^3/(c*x+1
)^2-11/8*b/c^4/d^3/(c*x+1)+19/16/c^4*b/d^3*ln(c*x+1)-3/16/c^4*b/d^3*ln(c*x-
1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{32}\left(2\,c^4\left(\frac{2\,(7\,cx+6)}{c^{10}d^3x^2+2\,c^9d^3x+c^8d^3}-\frac{8\,x}{c^7d^3}+\frac{17\,\log(cx+1)}{c^8d^3}-\frac{\log(cx-1)}{c^8d^3}\right)-32\,c^4\int\frac{x^4\log(cx+}{2\left(c^7d^3x^4+2\,c^6d^3x^3-2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(2*c^4*(2*(7*c*x + 6)/(c^10*d^3*x^2 + 2*c^9*d^3*x + c^8*d^3) - 8*x/(c
^7*d^3) + 17*log(c*x + 1)/(c^8*d^3) - log(c*x - 1)/(c^8*d^3)) - 32*c^4*inte
grate(1/2*x^4*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3
*d^3), x) - 6*c^3*(2*(5*c*x + 4)/(c^9*d^3*x^2 + 2*c^8*d^3*x + c^7*d^3) + 7*
log(c*x + 1)/(c^7*d^3) + log(c*x - 1)/(c^7*d^3)) + 128*c^3*integrate(1/2*x^
3*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) +
288*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4
*d^3*x - c^3*d^3), x) + 9*c*(2*x/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - lo
g(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(c^5*d^3)) + 288*c*integrate(1/2*x*log(
c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) + 8*(2*c
^3*x^3 + 4*c^2*x^2 - 4*c*x - 6*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) - 5)*log(
-c*x + 1)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) + 10*(c*x + 2)/(c^6*d^3*x^2
 + 2*c^5*d^3*x + c^4*d^3) - 5*log(c*x + 1)/(c^4*d^3) + 5*log(c*x - 1)/(c^4*
d^3) + 96*integrate(1/2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d
^3*x - c^3*d^3), x))*b - 1/2*a*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^
4*d^3) - 2*x/(c^3*d^3) + 6*log(c*x + 1)/(c^4*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^3\,\mathrm{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.60 $\int \frac{x^2(a+b\tanh^{-1}(cx))}{(d+cdx)^3}\,dx$

Optimal. Leaf size=150

$$\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(cx+1)^2}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{7b}{8c^3d^3(cx+1)}-\frac{b}{8c^3d^3(c}$$

```
[Out] -1/8*b/c^3/d^3/(c*x+1)^2+7/8*b/c^3/d^3/(c*x+1)-7/8*b*arctanh(c*x)/c^3/d^3+1
/2*(-a-b*arctanh(c*x))/c^3/d^3/(c*x+1)^2+2*(a+b*arctanh(c*x))/c^3/d^3/(c*x+
1)-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^3/d^3+1/2*b*polylog(2,1-2/(c*x+1))/c^
3/d^3
```

Rubi [A] time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5940, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(cx+1)^2}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{7b}{8c^3d^3(cx+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] -b/(8*c^3*d^3*(1 + c*x)^2) + (7*b)/(8*c^3*d^3*(1 + c*x)) - (7*b*ArcTanh[c*x
])/(8*c^3*d^3) - (a + b*ArcTanh[c*x])/(2*c^3*d^3*(1 + c*x)^2) + (2*(a + b*A
rcTanh[c*x]))/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])
/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx=\int \left(\frac{a+b\tanh^{-1}(cx)}{c^2d^3(1+cx)^3}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^3(1+cx)}\right)dx$$

$$=\frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^2d^3}+\frac{\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{c^2d^3}-\frac{2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^2d^3}$$

$$=-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b}{}$$

$$=-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b}{}$$

$$=-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b}{}$$

$$=-\frac{b}{8c^3d^3(1+cx)^2}+\frac{7b}{8c^3d^3(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-$$

$$=-\frac{b}{8c^3d^3(1+cx)^2}+\frac{7b}{8c^3d^3(1+cx)}-\frac{7b\tanh^{-1}(cx)}{8c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2(a+}{c^3}$$

Mathematica [A] time = 0.49, size = 145, normalized size = 0.97

$$\frac{64a}{cx+1}-\frac{16a}{(cx+1)^2}+32a\log(cx+1)+b\left(16\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)-12\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(cx)\right)+1\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] ((-16*a)/(1 + c*x)^2 + (64*a)/(1 + c*x) + 32*a*Log[1 + c*x] + b*(12*Cosh[2*
ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])]
- 12*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2
*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])] - 6*
Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^3*d^3)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\,\text{artanh}\,(cx)+ax^2}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx)+a)x^2}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^3, x)
```

maple [A] time = 0.06, size = 246, normalized size = 1.64

$$-\frac{a}{2c^3d^3\,(cx+1)^2}+\frac{2a}{c^3d^3\,(cx+1)}+\frac{a\ln(cx+1)}{c^3d^3}-\frac{b\,\text{arctanh}\,(cx)}{2c^3d^3\,(cx+1)^2}+\frac{2b\,\text{arctanh}\,(cx)}{c^3d^3\,(cx+1)}+\frac{b\,\text{arctanh}\,(cx)\ln(cx+1)}{c^3d^3}-\frac{b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] -1/2/c^3*a/d^3/(c*x+1)^2+2/c^3*a/d^3/(c*x+1)+1/c^3*a/d^3*ln(c*x+1)-1/2/c^3*
b/d^3*arctanh(c*x)/(c*x+1)^2+2/c^3*b/d^3*arctanh(c*x)/(c*x+1)+1/c^3*b/d^3*a
rctanh(c*x)*ln(c*x+1)-1/4/c^3*b/d^3*ln(c*x+1)^2+1/2/c^3*b/d^3*ln(-1/2*c*x+1
/2)*ln(c*x+1)-1/2/c^3*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2/c^3*b/d^3*
dilog(1/2+1/2*c*x)-1/8*b/c^3/d^3/(c*x+1)^2+7/8*b/c^3/d^3/(c*x+1)-7/16/c^3*b
/d^3*ln(c*x+1)+7/16/c^3*b/d^3*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{32}\left(64\,c^3\int\frac{x^3\log(cx+1)}{2\left(c^6d^3x^4+2\,c^5d^3x^3-2\,c^3d^3x-c^2d^3\right)}\,dx-4\,c^2\left(\frac{2\,(3\,cx+2)}{c^7d^3x^2+2\,c^6d^3x+c^5d^3}+\frac{\log(cx+1)}{c^5d^3}-\frac{\log(cx-}{c^5d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(64*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 -
2*c^3*d^3*x - c^2*d^3), x) - 4*c^2*(2*(3*c*x + 2)/(c^7*d^3*x^2 + 2*c^6*d^3*
x + c^5*d^3) + log(c*x + 1)/(c^5*d^3) - log(c*x - 1)/(c^5*d^3)) + 64*c^2*in
tegrate(1/2*x^2*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c
^2*d^3), x) + 7*c*(2*x/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - log(c*x + 1)
/(c^4*d^3) + log(c*x - 1)/(c^4*d^3)) + 96*c*integrate(1/2*x*log(c*x + 1)/(c
^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x) - 8*(4*c*x + 2*(c^2
```

```
*x^2 + 2*c*x + 1)*log(c*x + 1) + 3)*log(-c*x + 1)/(c^5*d^3*x^2 + 2*c^4*d^3*
x + c^3*d^3) + 6*(c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*
x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3) + 32*integrate(1/2*log(c*x + 1)
/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x))*b + 1/2*a*((4*c
*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1)/(c^3*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^3x^3+3c^2x^2+3cx+1}\, dx + \int \frac{bx^2 \operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\, dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.61 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=77

$$\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}-\frac{3b}{8c^2d^3(cx+1)}+\frac{b}{8c^2d^3(cx+1)^2}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}$$

```
[Out] 1/8*b/c^2/d^3/(c*x+1)^2-3/8*b/c^2/d^3/(c*x+1)-1/8*b*arctanh(c*x)/c^2/d^3+1/
2*x^2*(a+b*arctanh(c*x))/d^3/(c*x+1)^2
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {37, 5936, 12, 88, 207}

$$\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}-\frac{3b}{8c^2d^3(cx+1)}+\frac{b}{8c^2d^3(cx+1)^2}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] b/(8*c^2*d^3*(1 + c*x)^2) - (3*b)/(8*c^2*d^3*(1 + c*x)) - (b*ArcTanh[c*x])/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}dx = \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - (bc)\int \frac{x^2}{2(1-cx)(d+cdx)^3}dx$$
$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{1}{2}(bc)\int \frac{x^2}{(1-cx)(d+cdx)^3}dx$$
$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{1}{2}(bc)\int\left(\frac{1}{2c^2d^3(1+cx)^3} - \frac{3}{4c^2d^3(1+cx)^2} - \frac{1}{4c^2d^3\left(-1\right.}\right.$$
$$= \frac{b}{8c^2d^3(1+cx)^2} - \frac{3b}{8c^2d^3(1+cx)} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} + \frac{b\int \frac{1}{-1+c^2x^2}dx}{8cd^3}$$
$$= \frac{b}{8c^2d^3(1+cx)^2} - \frac{3b}{8c^2d^3(1+cx)} - \frac{b\tanh^{-1}(cx)}{8c^2d^3} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}$$

Mathematica [A] time = 0.11, size = 99, normalized size = 1.29

$$-\frac{16acx+8a-3bc^2x^2\log(cx+1)+6bcx-6bcx\log(cx+1)+3b(cx+1)^2\log(1-cx)-3b\log(cx+1)+8(2b}{16c^2d^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] -1/16*(8*a + 4*b + 16*a*c*x + 6*b*c*x + 8*(b + 2*b*c*x)*ArcTanh[c*x] + 3*b*
(1 + c*x)^2*Log[1 - c*x] - 3*b*Log[1 + c*x] - 6*b*c*x*Log[1 + c*x] - 3*b*c^
2*x^2*Log[1 + c*x])/(c^2*d^3*(1 + c*x)^2)
```

fricas [A] time = 0.42, size = 84, normalized size = 1.09

$$-\frac{2\,(8\,a+3\,b)cx-\left(3\,bc^2x^2-2\,bcx-b\right)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\left(c^4d^3x^2+2\,c^3d^3x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(8*a + 3*b)*c*x - (3*b*c^2*x^2 - 2*b*c*x - b)*log(-(c*x + 1)/(c*x
- 1)) + 8*a + 4*b)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

giac [A] time = 0.20, size = 114, normalized size = 1.48

$$\frac{1}{32}c\left(\frac{2\,(cx-1)^2\left(\frac{2\,(cx+1)b}{cx-1}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^3d^3}+\frac{(cx-1)^2\left(\frac{8\,(cx+1)a}{cx-1}+4\,a+\frac{4\,(cx+1)b}{cx-1}+b\right)}{(cx+1)^2c^3d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/32*c*(2*(c*x - 1)^2*(2*(c*x + 1)*b/(c*x - 1) + b)*log(-(c*x + 1)/(c*x - 1
))/((c*x + 1)^2*c^3*d^3) + (c*x - 1)^2*(8*(c*x + 1)*a/(c*x - 1) + 4*a + 4*(
c*x + 1)*b/(c*x - 1) + b)/((c*x + 1)^2*c^3*d^3))
```

maple [A] time = 0.04, size = 136, normalized size = 1.77

$$\frac{a}{2c^2d^3\,(cx+1)^2}-\frac{a}{c^2d^3\,(cx+1)}+\frac{b\,\mathrm{arctanh}\,(cx)}{2c^2d^3\,(cx+1)^2}-\frac{b\,\mathrm{arctanh}\,(cx)}{c^2d^3\,(cx+1)}-\frac{3b\ln(cx-1)}{16c^2d^3}+\frac{b}{8c^2d^3\,(cx+1)^2}-\frac{3b}{8c^2d^3\,(cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] 1/2/c^2*a/d^3/(c*x+1)^2-1/c^2*a/d^3/(c*x+1)+1/2/c^2*b/d^3*arctanh(c*x)/(c*x
+1)^2-1/c^2*b/d^3*arctanh(c*x)/(c*x+1)-3/16/c^2*b/d^3*ln(c*x-1)+1/8*b/c^2/d
^3/(c*x+1)^2-3/8*b/c^2/d^3/(c*x+1)+3/16/c^2*b/d^3*ln(c*x+1)
```

maxima [B] time = 0.33, size = 152, normalized size = 1.97

$$-\frac{1}{16}\left(c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3}-\frac{3\,\log(cx+1)}{c^3d^3}+\frac{3\,\log(cx-1)}{c^3d^3}\right)+\frac{8\,(2\,cx+1)\,\mathrm{artanh}\,(cx)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}\right)b-\frac{(2\,cx+}{2\left(c^4d^3x^2+2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x +
 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c^4
*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*b - 1/2*(2*c*x + 1)*a/(c^4*d^3*x^2 + 2*c
^3*d^3*x + c^2*d^3)
```

mupad [B] time = 1.27, size = 81, normalized size = 1.05

$$\frac{c\,\left(b\,x-2\,b\,x\,\mathrm{atanh}\left(c\,x\right)\right)-b\,\mathrm{atanh}\left(c\,x\right)+c^2\,\left(4\,a\,x^2+2\,b\,x^2+3\,b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{8\,c^4\,d^3\,x^2+16\,c^3\,d^3\,x+8\,c^2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] (c*(b*x - 2*b*x*atanh(c*x)) - b*atanh(c*x) + c^2*(4*a*x^2 + 2*b*x^2 + 3*b*x
^2*atanh(c*x)))/(8*c^2*d^3 + 16*c^3*d^3*x + 8*c^4*d^3*x^2)
```

sympy [A] time = 2.26, size = 306, normalized size = 3.97

$$\begin{cases} -\frac{8acx}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{4a}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}+\frac{3bc^2x^2\,\mathrm{atanh}\,(cx)}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{2bcx\,\mathrm{atanh}\,(cx)}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{3bcx}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{}{8} \\ \tilde{\infty}\left(\frac{ax^2}{2}+\frac{bx^2\,\mathrm{atanh}\,(cx)}{2}+\frac{bx}{2c}-\frac{b\,\mathrm{atanh}\,(cx)}{2c^2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] Piecewise((-8*a*c*x/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 4*a
/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) + 3*b*c**2*x**2*atanh(c*
x)/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 2*b*c*x*atanh(c*x)/(
8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 3*b*c*x/(8*c**4*d**3*x**
2 + 16*c**3*d**3*x + 8*c**2*d**3) - b*atanh(c*x)/(8*c**4*d**3*x**2 + 16*c**
3*d**3*x + 8*c**2*d**3) - 2*b/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d
**3), Ne(d, 0)), (zoo*(a*x**2/2 + b*x**2*atanh(c*x)/2 + b*x/(2*c) - b*atanh
(c*x)/(2*c**2)), True))
```

3.62 $\int \frac{a+b \tanh^{-1}(cx)}{(d+cdx)^3} \, dx$

Optimal. Leaf size=77

$$-\frac{a+b\tanh^{-1}(cx)}{2cd^3(cx+1)^2}-\frac{b}{8cd^3(cx+1)}-\frac{b}{8cd^3(cx+1)^2}+\frac{b\tanh^{-1}(cx)}{8cd^3}$$

```
[Out] -1/8*b/c/d^3/(c*x+1)^2-1/8*b/c/d^3/(c*x+1)+1/8*b*arctanh(c*x)/c/d^3+1/2*(-a
-b*arctanh(c*x))/c/d^3/(c*x+1)^2
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5926, 627, 44, 207}

$$-\frac{a+b\tanh^{-1}(cx)}{2cd^3(cx+1)^2}-\frac{b}{8cd^3(cx+1)}-\frac{b}{8cd^3(cx+1)^2}+\frac{b\tanh^{-1}(cx)}{8cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
```

```
[Out] -b/(8*c*d^3*(1 + c*x)^2) - b/(8*c*d^3*(1 + c*x)) + (b*ArcTanh[c*x])/(8*c*d^
3) - (a + b*ArcTanh[c*x])/(2*c*d^3*(1 + c*x)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^3}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} + \frac{b\int \frac{1}{(d+cdx)^2(1-c^2x^2)}\,dx}{2d} \\
&= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} + \frac{b\int \frac{1}{\left(\frac{1}{d}-\frac{cx}{d}\right)(d+cdx)^3}\,dx}{2d} \\
&= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} + \frac{b\int \left(\frac{1}{2d^2(1+cx)^3}+\frac{1}{4d^2(1+cx)^2}-\frac{1}{4d^2(-1+c^2x^2)}\right)dx}{2d} \\
&= -\frac{b}{8cd^3(1+cx)^2} - \frac{b}{8cd^3(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} - \frac{b\int \frac{1}{-1+c^2x^2}\,dx}{8d^3} \\
&= -\frac{b}{8cd^3(1+cx)^2} - \frac{b}{8cd^3(1+cx)} + \frac{b\tanh^{-1}(cx)}{8cd^3} - \frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 86, normalized size = 1.12

$$
\frac{-8a+bc^2x^2\log(cx+1)-2bcx+2bcx\log(cx+1)-b(cx+1)^2\log(1-cx)+b\log(cx+1)-8b\tanh^{-1}(cx)-4b}{16cd^3(cx+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
```

```
[Out] (-8*a - 4*b - 2*b*c*x - 8*b*ArcTanh[c*x] - b*(1 + c*x)^2*Log[1 - c*x] + b*L
og[1 + c*x] + 2*b*c*x*Log[1 + c*x] + b*c^2*x^2*Log[1 + c*x])/(16*c*d^3*(1 +
 c*x)^2)
```

fricas [A] time = 0.41, size = 75, normalized size = 0.97

$$
-\frac{2\,bcx-\left(bc^2x^2+2\,bcx-3\,b\right)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\left(c^3d^3x^2+2\,c^2d^3x+cd^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b*c*x - (b*c^2*x^2 + 2*b*c*x - 3*b)*log(-(c*x + 1)/(c*x - 1)) + 8*
a + 4*b)/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

giac [A] time = 0.22, size = 118, normalized size = 1.53

$$
\frac{1}{32}c\left(\frac{2\,(cx-1)^2\left(\frac{2\,(cx+1)b}{cx-1}-b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^2d^3}+\frac{(cx-1)^2\left(\frac{8\,(cx+1)a}{cx-1}-4\,a+\frac{4\,(cx+1)b}{cx-1}-b\right)}{(cx+1)^2c^2d^3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/32*c*(2*(c*x - 1)^2*(2*(c*x + 1)*b/(c*x - 1) - b)*log(-(c*x + 1)/(c*x - 1
))/((c*x + 1)^2*c^2*d^3) + (c*x - 1)^2*(8*(c*x + 1)*a/(c*x - 1) - 4*a + 4*(
c*x + 1)*b/(c*x - 1) - b)/((c*x + 1)^2*c^2*d^3))
```

maple [A] time = 0.04, size = 100, normalized size = 1.30

$$
-\frac{a}{2c\,d^3\,(cx+1)^2}-\frac{b\,\mathrm{arctanh}\,(cx)}{2c\,d^3\,(cx+1)^2}-\frac{b\ln(cx-1)}{16c\,d^3}-\frac{b}{8c\,d^3\,(cx+1)^2}-\frac{b}{8c\,d^3\,(cx+1)}+\frac{b\ln(cx+1)}{16c\,d^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] -1/2/c*a/d^3/(c*x+1)^2-1/2/c*b/d^3*arctanh(c*x)/(c*x+1)^2-1/16/c*b/d^3*ln(c
*x-1)-1/8*b/c/d^3/(c*x+1)^2-1/8*b/c/d^3/(c*x+1)+1/16/c*b/d^3*ln(c*x+1)
```

maxima [A] time = 0.32, size = 134, normalized size = 1.74

$$-\frac{1}{16}\left(c\left(\frac{2(cx+2)}{c^4d^3x^2+2c^3d^3x+c^2d^3}-\frac{\log(cx+1)}{c^2d^3}+\frac{\log(cx-1)}{c^2d^3}\right)+\frac{8\operatorname{artanh}(cx)}{c^3d^3x^2+2c^2d^3x+cd^3}\right)b-\frac{a}{2\left(c^3d^3x^2+2c^2d^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)/
(c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2*d
^3*x + c*d^3))*b - 1/2*a/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

mupad [B] time = 1.09, size = 123, normalized size = 1.60

$$\frac{c^2\left(\frac{a\,x^2}{2}+\frac{b\,x^2}{4}-\frac{b\,x^2\,\ln\left(c^2\,x^2-1\right)}{16}+\frac{b\,x^2\,\ln(c\,x+1)}{8}\right)-\frac{b\,\ln\left(c^2\,x^2-1\right)}{16}-\frac{b\,\mathrm{atanh}(c\,x)}{2}+\frac{b\,\ln(c\,x+1)}{8}+c\,\left(a\,x+\frac{3\,b\,x}{8}+\frac{b\,x\,\ln(c\,x+1)}{4}\right.}{c\,d^3\,(c\,x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + c*d*x)^3,x)
```

```
[Out] (c^2*((a*x^2)/2 + (b*x^2)/4 - (b*x^2*log(c^2*x^2 - 1))/16 + (b*x^2*log(c*x
+ 1))/8) - (b*log(c^2*x^2 - 1))/16 - (b*atanh(c*x))/2 + (b*log(c*x + 1))/8
+ c*(a*x + (3*b*x)/8 + (b*x*log(c*x + 1))/4 - (b*x*log(c^2*x^2 - 1))/8))/(c
*d^3*(c*x + 1)^2)
```

sympy [A] time = 2.15, size = 250, normalized size = 3.25

$$\begin{cases} -\frac{4a}{8c^3d^3x^2+16c^2d^3x+8cd^3}+\frac{bc^2x^2\operatorname{atanh}(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3}+\frac{2bcx\operatorname{atanh}(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3}-\frac{bcx}{8c^3d^3x^2+16c^2d^3x+8cd^3}-\frac{3b\operatorname{atanh}(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3}- \\ \tilde{\infty}\left(ax+bx\operatorname{atanh}(cx)+\frac{b\log\left(x-\frac{1}{c}\right)}{c}+\frac{b\operatorname{atanh}(cx)}{c}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] Piecewise((-4*a/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) + b*c**2*x**
2*atanh(c*x)/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) + 2*b*c*x*atanh
(c*x)/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) - b*c*x/(8*c**3*d**3*x
**2 + 16*c**2*d**3*x + 8*c*d**3) - 3*b*atanh(c*x)/(8*c**3*d**3*x**2 + 16*c*
*2*d**3*x + 8*c*d**3) - 2*b/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3),
 Ne(d, 0)), (zoo*(a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)/c)
, True))
```

3.63 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^3}\,dx$

Optimal. Leaf size=161

$$\frac{a+b\tanh^{-1}(cx)}{d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2d^3(cx+1)^2}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{a\log(x)}{d^3}-\frac{b\text{Li}_2(-cx)}{2d^3}+\frac{b\text{Li}_2(cx)}{2d^3}-\frac{b\text{Li}_2\left(1-\frac{2}{cx\ldots}\right.}{2d^3}$$

```
[Out] 1/8*b/d^3/(c*x+1)^2+5/8*b/d^3/(c*x+1)-5/8*b*arctanh(c*x)/d^3+1/2*(a+b*arcta
nh(c*x))/d^3/(c*x+1)^2+(a+b*arctanh(c*x))/d^3/(c*x+1)+a*ln(x)/d^3+(a+b*arct
anh(c*x))*ln(2/(c*x+1))/d^3-1/2*b*polylog(2,-c*x)/d^3+1/2*b*polylog(2,c*x)/
d^3-1/2*b*polylog(2,1-2/(c*x+1))/d^3
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5940, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}(2,-cx)}{2d^3}+\frac{b\text{PolyLog}(2,cx)}{2d^3}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{a+b\tanh^{-1}(cx)}{d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2d^3(cx+1)^2}+\frac{\log\left(\frac{2}{cx+1}\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```

```
[Out] b/(8*d^3*(1 + c*x)^2) + (5*b)/(8*d^3*(1 + c*x)) - (5*b*ArcTanh[c*x])/(8*d^3
) + (a + b*ArcTanh[c*x])/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])/(d^3*(1
 + c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (
b*PolyLog[2, -(c*x)])/(2*d^3) + (b*PolyLog[2, c*x])/(2*d^3) - (b*PolyLog[2,
 1 - 2/(1 + c*x)])/(2*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^3} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^2} - \frac{c\left(a+b\tanh^{-1}\right.}{d^3(1+cx}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{d^3} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^3} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{d^3}$$

$$= \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^3} -$$

$$= \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^3} -$$

$$= \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^3} -$$

$$= \frac{b}{8d^3(1+cx)^2} + \frac{5b}{8d^3(1+cx)} + \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} + \frac{(a}{}$$

$$= \frac{b}{8d^3(1+cx)^2} + \frac{5b}{8d^3(1+cx)} - \frac{5b\tanh^{-1}(cx)}{8d^3} + \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)}$$

Mathematica [A] time = 0.53, size = 147, normalized size = 0.91

$$\frac{\frac{32a}{cx+1}+\frac{16a}{(cx+1)^2}-32a\log(cx+1)+32a\log(x)+b\left(-16\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)-12\sinh\left(2\tanh^{-1}(cx)\right)-\sinh\left(4\tanh^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```

```
[Out] ((16*a)/(1 + c*x)^2 + (32*a)/(1 + c*x) + 32*a*Log[x] - 32*a*Log[1 + c*x] +
b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*PolyLog[2, E^(-2*Arc
Tanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh[c*
x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[c*x])] - 6*Sinh[2*ArcT
anh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{c^3d^3x^4+3\,c^2d^3x^3+3\,cd^3x^2+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^4 + 3*c^2*d^3*x^3 + 3*c*d^3*x^2 +
d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{artanh}\,(cx)+a}{(cdx+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x), x)
```

maple [A] time = 0.06, size = 264, normalized size = 1.64

$$\frac{a\ln(cx)}{d^3}+\frac{a}{2d^3(cx+1)^2}+\frac{a}{d^3(cx+1)}-\frac{a\ln(cx+1)}{d^3}+\frac{b\,\text{arctanh}\,(cx)\ln(cx)}{d^3}+\frac{b\,\text{arctanh}\,(cx)}{2d^3(cx+1)^2}+\frac{b\,\text{arctanh}\,(cx)}{d^3(cx+1)}-\frac{b\,\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x)
```

```
[Out] a/d^3*ln(c*x)+1/2*a/d^3/(c*x+1)^2+a/d^3/(c*x+1)-a/d^3*ln(c*x+1)+b/d^3*arcta
nh(c*x)*ln(c*x)+1/2*b/d^3*arctanh(c*x)/(c*x+1)^2+b/d^3*arctanh(c*x)/(c*x+1)
-b/d^3*arctanh(c*x)*ln(c*x+1)-1/2*b/d^3*dilog(c*x)-1/2*b/d^3*dilog(c*x+1)-1
/2*b/d^3*ln(c*x)*ln(c*x+1)+1/4*b/d^3*ln(c*x+1)^2-1/2*b/d^3*ln(-1/2*c*x+1/2)
*ln(c*x+1)+1/2*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2*b/d^3*dilog(1/2+1
/2*c*x)+5/16*b/d^3*ln(c*x-1)+1/8*b/d^3/(c*x+1)^2+5/8*b/d^3/(c*x+1)-5/16*b/d
^3*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{2\,cx+3}{c^2d^3x^2+2\,cd^3x+d^3}-\frac{2\,\log(cx+1)}{d^3}+\frac{2\,\log(x)}{d^3}\right)+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{c^3d^3x^4+3\,c^2d^3x^3+3\,cd^3x^2+d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3 + 2
*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^4
+ 3*c^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{x\,{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))/(x*(d + c*d*x)^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3x^4+3c^2x^3+3cx^2+x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b*atanh
(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

3.64 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^3}\,dx$

Optimal. Leaf size=218

$$-\frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}-\frac{a+b\tanh^{-1}(cx)}{d^3x}-\frac{3c\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{3ac\log(x)}{d^3}-\frac{b}{}$$

```
[Out] -1/8*b*c/d^3/(c*x+1)^2-9/8*b*c/d^3/(c*x+1)+9/8*b*c*arctanh(c*x)/d^3+(-a-b*a
rctanh(c*x))/d^3/x-1/2*c*(a+b*arctanh(c*x))/d^3/(c*x+1)^2-2*c*(a+b*arctanh(
c*x))/d^3/(c*x+1)-3*a*c*ln(x)/d^3+b*c*ln(x)/d^3-3*c*(a+b*arctanh(c*x))*ln(2
/(c*x+1))/d^3-1/2*b*c*ln(-c^2*x^2+1)/d^3+3/2*b*c*polylog(2,-c*x)/d^3-3/2*b*
c*polylog(2,c*x)/d^3+3/2*b*c*polylog(2,1-2/(c*x+1))/d^3
```

Rubi [A] time = 0.27, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5940, 5916, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{3bc\text{PolyLog}(2,-cx)}{2d^3}-\frac{3bc\text{PolyLog}(2,cx)}{2d^3}+\frac{3bc\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}-\frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```

```
[Out] -(b*c)/(8*d^3*(1 + c*x)^2) - (9*b*c)/(8*d^3*(1 + c*x)) + (9*b*c*ArcTanh[c*x
])/(8*d^3) - (a + b*ArcTanh[c*x])/(d^3*x) - (c*(a + b*ArcTanh[c*x]))/(2*d^3
*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) - (3*a*c*Log[x])
/d^3 + (b*c*Log[x])/d^3 - (3*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 -
 (b*c*Log[1 - c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, -(c*x)])/(2*d^3) - (3*b
*c*PolyLog[2, c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
```

```
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x^2} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^3} + \frac{2c^2\left(a+b\tanh\right.}{d^3(1+cx\right.}$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{(3c)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} + \frac{c^2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{d^3} + \frac{\left(2c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} - \frac{3c\left(a\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} - \frac{3c\left(a\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} - \frac{3c\left(a\right.}{}$$

$$= -\frac{bc}{8d^3(1+cx)^2} - \frac{9bc}{8d^3(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tan\right.}{d^3(1+}$$

$$= -\frac{bc}{8d^3(1+cx)^2} - \frac{9bc}{8d^3(1+cx)} + \frac{9bc\tanh^{-1}(cx)}{8d^3} - \frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(c\right.}{2d^3(1+cx)^2}$$

Mathematica [A] time = 1.24, size = 186, normalized size = 0.85

$$-\frac{64ac}{cx+1} - \frac{16ac}{(cx+1)^2} - 96ac\log(x) + 96ac\log(cx+1) - \frac{32a}{x} + bc\left(32\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 48\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) + 20\sinh\left(2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```

```
[Out] ((-32*a)/x - (16*a*c)/(1 + c*x)^2 - (64*a*c)/(1 + c*x) - 96*a*c*Log[x] + 96
*a*c*Log[1 + c*x] + b*c*(-20*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] +
32*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 48*PolyLog[2, E^(-2*ArcTanh[c*x])] + 20*S
inh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-8/(c*x) - 10*
Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 24*Log[1 - E^(-2*ArcTanh[c*x]
)] + 10*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*d^3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{c^3d^3x^5+3\,c^2d^3x^4+3\,cd^3x^3+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 +
d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\,(cx)+a}{(cdx+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^2), x)
```

maple [A] time = 0.06, size = 319, normalized size = 1.46

$$-\frac{a}{d^3x}-\frac{3ca\ln(cx)}{d^3}-\frac{ca}{2d^3(cx+1)^2}-\frac{2ca}{d^3(cx+1)}+\frac{3ca\ln(cx+1)}{d^3}-\frac{b\,\text{arctanh}(cx)}{d^3x}-\frac{3cb\,\text{arctanh}(cx)\ln(cx)}{d^3}-\frac{cb\,\text{ar}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x)
```

```
[Out] -a/d^3/x-3*c*a/d^3*ln(c*x)-1/2*c*a/d^3/(c*x+1)^2-2*c*a/d^3/(c*x+1)+3*c*a/d^
3*ln(c*x+1)-b/d^3*arctanh(c*x)/x-3*c*b/d^3*arctanh(c*x)*ln(c*x)-1/2*c*b/d^3
*arctanh(c*x)/(c*x+1)^2-2*c*b/d^3*arctanh(c*x)/(c*x+1)+3*c*b/d^3*arctanh(c*
x)*ln(c*x+1)+c*b/d^3*ln(c*x)-17/16*c*b/d^3*ln(c*x-1)-1/8*b*c/d^3/(c*x+1)^2-
9/8*b*c/d^3/(c*x+1)+1/16*c*b/d^3*ln(c*x+1)+3/2*c*b/d^3*dilog(c*x)+3/2*c*b/d
^3*dilog(c*x+1)+3/2*c*b/d^3*ln(c*x)*ln(c*x+1)-3/4*c*b/d^3*ln(c*x+1)^2+3/2*c
*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/2*c*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c
*x)-3/2*c*b/d^3*dilog(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{6c^2x^2+9cx+2}{c^2d^3x^3+2cd^3x^2+d^3x}-\frac{6c\log(cx+1)}{d^3}+\frac{6c\log(x)}{d^3}\right)+\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{c^3d^3x^5+3c^2d^3x^4+3cd^3x^3+d^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*c*l
og(c*x + 1)/d^3 + 6*c*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-c*
x + 1))/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\text{atanh}(c\,x)}{x^2\,(d+c\,d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx+\int\frac{b\,\text{atanh}(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(b*at
anh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

3.65 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^3}\,dx$

Optimal. Leaf size=268

$$\frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}+\frac{6c^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{a+b\tanh^{-1}(cx)}{2d^3x^2}+\frac{3c\left(a+b\,\mathrm{ta}\right.}{d^3}$$

```
[Out] -1/2*b*c/d^3/x+1/8*b*c^2/d^3/(c*x+1)^2+13/8*b*c^2/d^3/(c*x+1)-9/8*b*c^2*arc
tanh(c*x)/d^3+1/2*(-a-b*arctanh(c*x))/d^3/x^2+3*c*(a+b*arctanh(c*x))/d^3/x+
1/2*c^2*(a+b*arctanh(c*x))/d^3/(c*x+1)^2+3*c^2*(a+b*arctanh(c*x))/d^3/(c*x+
1)+6*a*c^2*ln(x)/d^3-3*b*c^2*ln(x)/d^3+6*c^2*(a+b*arctanh(c*x))*ln(2/(c*x+1
))/d^3+3/2*b*c^2*ln(-c^2*x^2+1)/d^3-3*b*c^2*polylog(2,-c*x)/d^3+3*b*c^2*pol
ylog(2,c*x)/d^3-3*b*c^2*polylog(2,1-2/(c*x+1))/d^3
```

Rubi [A] time = 0.32, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{3bc^2\text{PolyLog}(2,-cx)}{d^3}+\frac{3bc^2\text{PolyLog}(2,cx)}{d^3}-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^3}+\frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}+\frac{c^2\left(a+b\,\mathrm{tanl}\right.}{2d^3(cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```

```
[Out] -(b*c)/(2*d^3*x) + (b*c^2)/(8*d^3*(1 + c*x)^2) + (13*b*c^2)/(8*d^3*(1 + c*x
)) - (9*b*c^2*ArcTanh[c*x])/(8*d^3) - (a + b*ArcTanh[c*x])/(2*d^3*x^2) + (3
*c*(a + b*ArcTanh[c*x]))/(d^3*x) + (c^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c
*x)^2) + (3*c^2*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) + (6*a*c^2*Log[x])/d^
3 - (3*b*c^2*Log[x])/d^3 + (6*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^
3 + (3*b*c^2*Log[1 - c^2*x^2])/(2*d^3) - (3*b*c^2*PolyLog[2, -(c*x)])/d^3 +
 (3*b*c^2*PolyLog[2, c*x])/d^3 - (3*b*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/d^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x^3} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x^2} + \frac{6c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3x} - \frac{c^3\left(a+b\tanh\right.}{d^3(1+c}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3c)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^3} + \frac{\left(6c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{c^3\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} + \frac{3c^2\left(a+b\tanh^{-}\right.}{d^3(1+cx)}$$

$$= -\frac{bc}{2d^3x} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} + \frac{3c^2\left(a+\right.}{d^3(}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2\tanh^{-1}(cx)}{2d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-}\right.}{2d^3(1+cx}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2}{8d^3(1+cx)^2} + \frac{13bc^2}{8d^3(1+cx)} + \frac{bc^2\tanh^{-1}(cx)}{2d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\right.}{d}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2}{8d^3(1+cx)^2} + \frac{13bc^2}{8d^3(1+cx)} - \frac{9bc^2\tanh^{-1}(cx)}{8d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\right.}{}$$

Mathematica [A] time = 1.47, size = 220, normalized size = 0.82

$$\frac{\frac{96ac^2}{cx+1} + \frac{16ac^2}{(cx+1)^2} + 192ac^2\log(x) - 192ac^2\log(cx+1) + \frac{96ac}{x} - \frac{16a}{x^2} + bc^2\left(-96\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 4\tanh^{-1}(cx)\left(-\frac{4}{c^2x^2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```

```
[Out] ((-16*a)/x^2 + (96*a*c)/x + (16*a*c^2)/(1 + c*x)^2 + (96*a*c^2)/(1 + c*x) +
 192*a*c^2*Log[x] - 192*a*c^2*Log[1 + c*x] + b*c^2*(-16/(c*x) + 28*Cosh[2*A
```

```
rcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 96*Log[(c*x)/Sqrt[1 - c^2*x^2]] - 96*
PolyLog[2, E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*
(4 - 4/(c^2*x^2) + 24/(c*x) + 14*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]
] + 48*Log[1 - E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcT
anh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx) + a}{c^3d^3x^6 + 3\,c^2d^3x^5 + 3\,cd^3x^4 + d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4 +
d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^3), x)
```

maple [A] time = 0.07, size = 394, normalized size = 1.47

$$\frac{3c^2b\,\text{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{d^3} - \frac{b\,\text{arctanh}\,(cx)}{2d^3x^2} - \frac{3c^2b\,\text{dilog}\,(cx)}{d^3} - \frac{3c^2b\,\text{dilog}\,(cx+1)}{d^3} + \frac{3ca}{d^3x} + \frac{c^2a}{2d^3\,(cx+1)^2} + \frac{3c^2a}{d^3\,(cx+1)} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x)
```

```
[Out] 3*c*a/d^3/x+1/2*c^2*a/d^3/(c*x+1)^2+3*c^2*a/d^3/(c*x+1)+3/2*c^2*b/d^3*ln(c*
x+1)^2+3*c^2*b/d^3*dilog(1/2+1/2*c*x)-6*c^2*a/d^3*ln(c*x+1)+6*c^2*a/d^3*ln(
c*x)-1/2*b/d^3*arctanh(c*x)/x^2+15/16*c^2*b/d^3*ln(c*x+1)+33/16*c^2*b/d^3*l
n(c*x-1)-3*c^2*b/d^3*ln(c*x)-3*c^2*b/d^3*dilog(c*x)-3*c^2*b/d^3*dilog(c*x+1
)-1/2*b*c/d^3/x+1/8*b*c^2/d^3/(c*x+1)^2+13/8*b*c^2/d^3/(c*x+1)-1/2*a/d^3/x^
2-6*c^2*b/d^3*arctanh(c*x)*ln(c*x+1)-3*c^2*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)
-3*c^2*b/d^3*ln(c*x)*ln(c*x+1)+6*c^2*b/d^3*arctanh(c*x)*ln(c*x)+3*c^2*b/d^3
*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3*c*b/d^3*arctanh(c*x)/x+3*c^2*b/d^3*arct
anh(c*x)/(c*x+1)+1/2*c^2*b/d^3*arctanh(c*x)/(c*x+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{12\,c^3x^3 + 18\,c^2x^2 + 4\,cx - 1}{c^2d^3x^4 + 2\,cd^3x^3 + d^3x^2} - \frac{12\,c^2\log\,(cx+1)}{d^3} + \frac{12\,c^2\log(x)}{d^3}\right) + \frac{1}{2}\,b\int \frac{\log\,(cx+1) - \log\,(-cx+1)}{c^3d^3x^6 + 3\,c^2d^3x^5 + 3\,cd^3x^4 + d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((12*c^3*x^3 + 18*c^2*x^2 + 4*c*x - 1)/(c^2*d^3*x^4 + 2*c*d^3*x^3 + d
^3*x^2) - 12*c^2*log(c*x + 1)/d^3 + 12*c^2*log(x)/d^3) + 1/2*b*integrate((l
og(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4 + d
^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{x^3\,{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^{3}x^{6}+3c^{2}x^{5}+3cx^{4}+x^{3}}\, dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{c^{3}x^{6}+3c^{2}x^{5}+3cx^{4}+x^{3}}\, dx}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x) + Integral(b*at
anh(c*x)/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x))/d**3
```

3.66 $\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^4}\,dx$

Optimal. Leaf size=80

$$-\frac{a+b\tanh^{-1}(cx)}{3c(cx+1)^3}-\frac{b}{24c(cx+1)}-\frac{b}{24c(cx+1)^2}-\frac{b}{18c(cx+1)^3}+\frac{b\tanh^{-1}(cx)}{24c}$$

```
[Out] -1/18*b/c/(c*x+1)^3-1/24*b/c/(c*x+1)^2-1/24*b/c/(c*x+1)+1/24*b*arctanh(c*x)
/c+1/3*(-a-b*arctanh(c*x))/c/(c*x+1)^3
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5926, 627, 44, 207}

$$-\frac{a+b\tanh^{-1}(cx)}{3c(cx+1)^3}-\frac{b}{24c(cx+1)}-\frac{b}{24c(cx+1)^2}-\frac{b}{18c(cx+1)^3}+\frac{b\tanh^{-1}(cx)}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

```
[Out] -b/(18*c*(1 + c*x)^3) - b/(24*c*(1 + c*x)^2) - b/(24*c*(1 + c*x)) + (b*ArcT
anh[c*x])/(24*c) - (a + b*ArcTanh[c*x])/(3*c*(1 + c*x)^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^4}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \frac{1}{(1+cx)^3\left(1-c^2x^2\right)}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \frac{1}{(1-cx)(1+cx)^4}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \left(\frac{1}{2(1+cx)^4}+\frac{1}{4(1+cx)^3}+\frac{1}{8(1+cx)^2}-\frac{1}{8\left(-1+c^2x^2\right)}\right)dx\\ &= -\frac{b}{18c(1+cx)^3}-\frac{b}{24c(1+cx)^2}-\frac{b}{24c(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}-\frac{1}{24}b\int \frac{1}{-1+c^2x^2}\,dx\\ &= -\frac{b}{18c(1+cx)^3}-\frac{b}{24c(1+cx)^2}-\frac{b}{24c(1+cx)}+\frac{b\tanh^{-1}(cx)}{24c}-\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}\end{aligned}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.94

$$-\frac{48a+2b\left(3c^2x^2+9cx+10\right)+3b(cx+1)^3\log(1-cx)-3b(cx+1)^3\log(cx+1)+48b\tanh^{-1}(cx)}{144c(cx+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

```
[Out] -1/144*(48*a + 2*b*(10 + 9*c*x + 3*c^2*x^2) + 48*b*ArcTanh[c*x] + 3*b*(1 +
c*x)^3*Log[1 - c*x] - 3*b*(1 + c*x)^3*Log[1 + c*x])/(c*(1 + c*x)^3)
```

fricas [A] time = 0.43, size = 91, normalized size = 1.14

$$-\frac{6\,bc^2x^2+18\,bcx-3\left(bc^3x^3+3\,bc^2x^2+3\,bcx-7\,b\right)\log\left(-\frac{cx+1}{cx-1}\right)+48\,a+20\,b}{144\left(c^4x^3+3\,c^3x^2+3\,c^2x+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/144*(6*b*c^2*x^2 + 18*b*c*x - 3*(b*c^3*x^3 + 3*b*c^2*x^2 + 3*b*c*x - 7*b
)*log(-(c*x + 1)/(c*x - 1)) + 48*a + 20*b)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x +
 c)
```

giac [B] time = 0.20, size = 161, normalized size = 2.01

$$\frac{1}{288}c\left(\frac{6\,(cx-1)^3\left(\frac{3\,(cx+1)^2b}{(cx-1)^2}-\frac{3\,(cx+1)b}{cx-1}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^3c^2}+\frac{(cx-1)^3\left(\frac{36\,(cx+1)^2a}{(cx-1)^2}-\frac{36\,(cx+1)a}{cx-1}+12\,a+\frac{18\,(cx+1)^2b}{(cx-1)^2}-\frac{9\,(cx}{cx}\right.}{(cx+1)^3c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] 1/288*c*(6*(c*x - 1)^3*(3*(c*x + 1)^2*b/(c*x - 1)^2 - 3*(c*x + 1)*b/(c*x -
1) + b)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (c*x - 1)^3*(36*(c*x
+ 1)^2*a/(c*x - 1)^2 - 36*(c*x + 1)*a/(c*x - 1) + 12*a + 18*(c*x + 1)^2*b/(
c*x - 1)^2 - 9*(c*x + 1)*b/(c*x - 1) + 2*b)/((c*x + 1)^3*c^2))
```

maple [A] time = 0.04, size = 95, normalized size = 1.19

$$-\frac{a}{3c\,(cx+1)^3}-\frac{b\,\mathrm{arctanh}\,(cx)}{3c\,(cx+1)^3}-\frac{b\ln(cx-1)}{48c}-\frac{b}{18c\,(cx+1)^3}-\frac{b}{24c\,(cx+1)^2}-\frac{b}{24c\,(cx+1)}+\frac{b\ln(cx+1)}{48c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*x+1)^4,x)
```

```
[Out] -1/3/c*a/(c*x+1)^3-1/3/c*b/(c*x+1)^3*arctanh(c*x)-1/48/c*b*ln(c*x-1)-1/18*b
/c/(c*x+1)^3-1/24*b/c/(c*x+1)^2-1/24*b/c/(c*x+1)+1/48/c*b*ln(c*x+1)
```

maxima [A] time = 0.31, size = 132, normalized size = 1.65

$$-\frac{1}{144}\left(c\left(\frac{2\left(3\,c^2x^2+9\,cx+10\right)}{c^5x^3+3\,c^4x^2+3\,c^3x+c^2}-\frac{3\,\log\left(cx+1\right)}{c^2}+\frac{3\,\log\left(cx-1\right)}{c^2}\right)+\frac{48\,\operatorname{artanh}\left(cx\right)}{c^4x^3+3\,c^3x^2+3\,c^2x+c}\right)b-\frac{}{3\left(c^4x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/144*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2)
 - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*
c^3*x^2 + 3*c^2*x + c))*b - 1/3*a/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

mupad [B] time = 1.10, size = 139, normalized size = 1.74

$$\frac{\frac{b\,c^2\,x^3}{8}-\frac{b\,x}{8}-\frac{b\,\operatorname{atanh}(c\,x)}{3\,c}-\frac{12\,a+5\,b}{36\,c}+\frac{b\,c^3\,x^4}{24}+\frac{c\,x^2\,(24\,a+7\,b)}{72}+\frac{b\,c\,x^2\,\operatorname{atanh}(c\,x)}{3}}{-c^5\,x^5-3\,c^4\,x^4-2\,c^3\,x^3+2\,c^2\,x^2+3\,c\,x+1}-\frac{b\,\ln\left(c^2\,x^2-1\right)}{48\,c}+\frac{b\,\ln\left(c\,x+1\right)}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(c*x + 1)^4,x)
```

```
[Out] ((b*c^2*x^3)/8 - (b*x)/8 - (b*atanh(c*x))/(3*c) - (12*a + 5*b)/(36*c) + (b*
c^3*x^4)/24 + (c*x^2*(24*a + 7*b))/72 + (b*c*x^2*atanh(c*x))/3)/(3*c*x + 2*
c^2*x^2 - 2*c^3*x^3 - 3*c^4*x^4 - c^5*x^5 + 1) - (b*log(c^2*x^2 - 1))/(48*c
) + (b*log(c*x + 1))/(24*c)
```

sympy [A] time = 2.76, size = 294, normalized size = 3.68

$$\begin{cases} -\frac{24a}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{3bc^3x^3\operatorname{atanh}\left(cx\right)}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{9bc^2x^2\operatorname{atanh}\left(cx\right)}{72c^4x^3+216c^3x^2+216c^2x+72c}-\frac{3bc^2x^2}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{}{72c^4} \\ ax \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*x+1)**4,x)
```

```
[Out] Piecewise((-24*a/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 3*b*c
**3*x**3*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 9*
b*c**2*x**2*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) -
 3*b*c**2*x**2/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 9*b*c*x
*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 9*b*c*x/(7
2*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 21*b*atanh(c*x)/(72*c**4
*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 10*b/(72*c**4*x**3 + 216*c**3*
x**2 + 216*c**2*x + 72*c), Ne(c, 0)), (a*x, True))
```

3.67 $\int \frac{\tanh^{-1}(ax)}{cx+acx^2}\,dx$

Optimal. Leaf size=41

$$\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{c}-\frac{\text{Li}_2\left(\frac{2}{ax+1}-1\right)}{2c}$$

```
[Out] arctanh(a*x)*ln(2-2/(a*x+1))/c-1/2*polylog(2,-1+2/(a*x+1))/c
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 5932, 2447}

$$\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{c}-\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - PolyLog[2, -1 + 2/(1 + a*x)]/(2*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{cx+acx^2}\,dx &= \int \frac{\tanh^{-1}(ax)}{x(c+acx)}\,dx\\
&= \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{a\int \frac{\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}\\
&= \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 39, normalized size = 0.95

$$\frac{\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right)}{c}-\frac{\mathrm{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])])/c - PolyLog[2, E^(-2*ArcTanh[a*
x])]/(2*c)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\,(ax)}{acx^2+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{artanh}\,(ax)}{acx^2+cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

maple [B] time = 0.05, size = 126, normalized size = 3.07

$$\frac{\mathrm{arctanh}\,(ax)\ln(ax)}{c}-\frac{\mathrm{arctanh}\,(ax)\ln(ax+1)}{c}+\frac{\ln(ax+1)^2}{4c}-\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{2c}+\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln\left(\frac{1}{2}+\frac{a}{2}\right.}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(a*c*x^2+c*x),x)
```

```
[Out] 1/c*arctanh(a*x)*ln(a*x)-1/c*arctanh(a*x)*ln(a*x+1)+1/4/c*ln(a*x+1)^2-1/2/c
*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/2/c*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/2/c*di
log(1/2+1/2*a*x)-1/2/c*dilog(a*x)-1/2/c*dilog(a*x+1)-1/2/c*ln(a*x)*ln(a*x+1
)
```

maxima [B] time = 0.32, size = 120, normalized size = 2.93

$$\frac{1}{4}a\left(\frac{\log(ax+1)^2}{ac}-\frac{2\left(\log(ax+1)\log\left(-\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}ax+\frac{1}{2}\right)\right)}{ac}-\frac{2\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)}{ac}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] 1/4*a*(log(a*x + 1)^2/(a*c) - 2*(log(a*x + 1)*log(-1/2*a*x + 1/2) + dilog(1
/2*a*x + 1/2))/(a*c) - 2*(log(a*x + 1)*log(x) + dilog(-a*x))/(a*c) + 2*(log
(-a*x + 1)*log(x) + dilog(a*x))/(a*c)) - (log(a*x + 1)/c - log(x)/c)*arctan
h(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{a\,c\,x^2+c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c*x + a*c*x^2),x)
```

```
[Out] int(atanh(a*x)/(c*x + a*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}(ax)}{ax^2+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(a*c*x**2+c*x),x)
```

```
[Out] Integral(atanh(a*x)/(a*x**2 + x), x)/c
```

3.68 $\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=270

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{20c^4}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^4}+\frac{abdx}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{1}{5}cdx^5\left(a+b\tan\right.$$

```
[Out] 1/2*a*b*d*x/c^3+3/10*b^2*d*x/c^3+1/12*b^2*d*x^2/c^2+1/30*b^2*d*x^3/c-3/10*b
^2*d*arctanh(c*x)/c^4+1/2*b^2*d*x*arctanh(c*x)/c^3+1/5*b*d*x^2*(a+b*arctanh
(c*x))/c^2+1/6*b*d*x^3*(a+b*arctanh(c*x))/c+1/10*b*d*x^4*(a+b*arctanh(c*x))
-1/20*d*(a+b*arctanh(c*x))^2/c^4+1/4*d*x^4*(a+b*arctanh(c*x))^2+1/5*c*d*x^5
*(a+b*arctanh(c*x))^2-2/5*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4+1/3*b^2
*d*ln(-c^2*x^2+1)/c^4-1/5*b^2*d*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A] time = 0.65, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5940, 5916, 5980, 266, 43, 5910, 260, 5948, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{abdx}{2c^3}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{20c^4}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tan\right.}{5c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d*x)/(2*c^3) + (3*b^2*d*x)/(10*c^3) + (b^2*d*x^2)/(12*c^2) + (b^2*d*x^
3)/(30*c) - (3*b^2*d*ArcTanh[c*x])/(10*c^4) + (b^2*d*x*ArcTanh[c*x])/(2*c^3
) + (b*d*x^2*(a + b*ArcTanh[c*x]))/(5*c^2) + (b*d*x^3*(a + b*ArcTanh[c*x]))
/(6*c) + (b*d*x^4*(a + b*ArcTanh[c*x]))/10 - (d*(a + b*ArcTanh[c*x])^2)/(20
*c^4) + (d*x^4*(a + b*ArcTanh[c*x])^2)/4 + (c*d*x^5*(a + b*ArcTanh[c*x])^2)
/5 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (b^2*d*Log[1 -
 c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int \left(dx^3\left(a+b\tanh^{-1}(cx)\right)^2+cdx^4\left(a+b\tanh^{-1}(cx)\right)^2\right)dx$$

$$= d\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+(cd)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$

$$= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{2}(bcd)\int \frac{x^4}{}$$

$$= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}(2bd)\int x^3$$

$$= \frac{bdx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{10}bdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\,\mathrm{t}\right.$$

$$= \frac{abdx}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bdx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{10}bdx^4$$

$$= \frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^3}{30c}+\frac{b^2dx\tanh^{-1}(cx)}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}$$

$$= \frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{b^2dx\tanh^{-1}(c}{2c^3}$$

$$= \frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{b^2dx\tanh^{-1}(c}{2c^3}$$

Mathematica [A] time = 0.76, size = 271, normalized size = 1.00

$$d\left(12a^2c^5x^5+15a^2c^4x^4+6abc^4x^4+10abc^3x^3+12abc^2x^2+12ab\log\left(c^2x^2-1\right)+2b\tanh^{-1}(cx)\left(3ac^4x^4(4cx\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(-18*a*b - 5*b^2 + 30*a*b*c*x + 18*b^2*c*x + 12*a*b*c^2*x^2 + 5*b^2*c^2*
x^2 + 10*a*b*c^3*x^3 + 2*b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 12*
a^2*c^5*x^5 + 3*b^2*(-9 + 5*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 2*b*ArcTa
nh[c*x]*(3*a*c^4*x^4*(5 + 4*c*x) + b*(-9 + 15*c*x + 6*c^2*x^2 + 5*c^3*x^3 +
 3*c^4*x^4) - 12*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x] - 15
*a*b*Log[1 + c*x] + 20*b^2*Log[1 - c^2*x^2] + 12*a*b*Log[-1 + c^2*x^2] + 12
*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^4)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2cdx^4 + a^2dx^3 + \left(b^2cdx^4 + b^2dx^3\right)\text{artanh}\,(cx)^2 + 2\left(abcdx^4 + abdx^3\right)\text{artanh}\,(cx)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^4 + a^2*d*x^3 + (b^2*c*d*x^4 + b^2*d*x^3)*arctanh(c*x)^2
 + 2*(a*b*c*d*x^4 + a*b*d*x^3)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx + d)(b\,\text{artanh}\,(cx) + a)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^3, x)
```

maple [A] time = 0.06, size = 422, normalized size = 1.56

$$\frac{11 d\,b^2 \ln(cx+1)}{60 c^4} + \frac{dab\,x^4}{10} + \frac{29 d\,b^2 \ln(cx-1)}{60 c^4} + \frac{c\,a^2 d\,x^5}{5} + \frac{d\,b^2 \ln(cx+1)^2}{80 c^4} + \frac{9 d\,b^2 \ln(cx-1)^2}{80 c^4} + \frac{d\,b^2\,\text{arctanh}\,(cx)^2\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 11/60/c^4*d*b^2*ln(c*x+1)+1/10*d*a*b*x^4+1/4*d*b^2*arctanh(c*x)^2*x^4+1/10*
d*b^2*arctanh(c*x)*x^4+29/60/c^4*d*b^2*ln(c*x-1)-1/5/c^4*d*b^2*dilog(1/2+1/
2*c*x)+1/5*c*a^2*d*x^5+1/80/c^4*d*b^2*ln(c*x+1)^2+9/80/c^4*d*b^2*ln(c*x-1)^
2+1/2*a*b*d*x/c^3+1/2*b^2*d*x*arctanh(c*x)/c^3+3/10*b^2*d*x/c^3+1/12*b^2*d*
x^2/c^2+1/30*b^2*d*x^3/c+1/5/c^2*d*a*b*x^2-1/20/c^4*d*a*b*ln(c*x+1)+9/20/c^
4*d*a*b*ln(c*x-1)+1/6/c*d*a*b*x^3-9/40/c^4*d*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-
1/40/c^4*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/20/c^4*d*b^2*arctanh(c*x)*ln(c*
x+1)+9/20/c^4*d*b^2*arctanh(c*x)*ln(c*x-1)+1/40/c^4*d*b^2*ln(-1/2*c*x+1/2)*
ln(1/2+1/2*c*x)+1/2*d*a*b*arctanh(c*x)*x^4+1/6/c*d*b^2*arctanh(c*x)*x^3+1/5
/c^2*d*b^2*arctanh(c*x)*x^2+1/4*a^2*d*x^4+1/5*c*d*b^2*arctanh(c*x)^2*x^5+2/
5*c*d*a*b*arctanh(c*x)*x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctanh(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*(4*x^
5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*
d - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^10 - 15*log(c*x + 1
)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 + 2*x^2)/c^8 + 2*log(c^2*
x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*log(c*x + 1)/(c^6*x^2 - c^
4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x + 1)/c^9 + 3*log(c*x - 1
)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8) + 900*c^2*(2*x/c^6 - log(
c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*integrate(1/150*x*log(c*x + 1)/
(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x + 1)^2 + (12*c^5*x^5 - 15*c^4*
x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c^5*x^5 + 1)*log(c*x + 1))*log
(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*x + 1)^2 - 10*log(-c*x + 1) +
```

```
2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4*log(-c*x + 1) + 1) + 2000*(c*x
 - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 9000*(c*x - 1)^2*(2*log
(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000*(c*x - 1)*(log(-c*x + 1)^2 - 2*l
og(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6*x^2 - 150*c^4)/c^5 - 540000*integ
rate(1/150*log(c*x + 1)/(c^6*x^2 - c^4), x))*b^2*c*d + 1/12*(6*x^4*arctanh(
c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))
*a*b*d + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
- 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) +
 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{atanh}\,(c\,x))^2\,\,(d+c\,d\,x)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2x^3\,dx + \int a^2cx^4\,dx + \int b^2x^3\,\mathrm{atanh}^2\,(cx)\,dx + \int 2abx^3\,\mathrm{atanh}\,(cx)\,dx + \int b^2cx^4\,\mathrm{atanh}^2\,(cx)\,dx + \int 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x**3, x) + Integral(a**2*c*x**4, x) + Integral(b**2*x**3*a
tanh(c*x)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(b**2*c*x**
4*atanh(c*x)**2, x) + Integral(2*a*b*c*x**4*atanh(c*x), x))
```

3.69 $\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=236

$$\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{abdx}{2c^2}+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}dx^3\left(a+b\tanh^{-1}\right.$$

```
[Out] 1/2*a*b*d*x/c^2+1/3*b^2*d*x/c^2+1/12*b^2*d*x^2/c-1/3*b^2*d*arctanh(c*x)/c^3
+1/2*b^2*d*x*arctanh(c*x)/c^2+1/3*b*d*x^2*(a+b*arctanh(c*x))/c+1/6*b*d*x^3*
(a+b*arctanh(c*x))+1/12*d*(a+b*arctanh(c*x))^2/c^3+1/3*d*x^3*(a+b*arctanh(c
*x))^2+1/4*c*d*x^4*(a+b*arctanh(c*x))^2-2/3*b*d*(a+b*arctanh(c*x))*ln(2/(-c
*x+1))/c^3+1/3*b^2*d*ln(-c^2*x^2+1)/c^3-1/3*b^2*d*polylog(2,1-2/(-c*x+1))/c
^3
```

Rubi [A] time = 0.53, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5940, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 266, 43, 5910, 260, 5948}

$$-\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}+\frac{abdx}{2c^2}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{1}{4}cdx^4\left(a+b\tanh\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d*x)/(2*c^2) + (b^2*d*x)/(3*c^2) + (b^2*d*x^2)/(12*c) - (b^2*d*ArcTanh
[c*x])/(3*c^3) + (b^2*d*x*ArcTanh[c*x])/(2*c^2) + (b*d*x^2*(a + b*ArcTanh[c
*x]))/(3*c) + (b*d*x^3*(a + b*ArcTanh[c*x]))/6 + (d*(a + b*ArcTanh[c*x])^2)
/(12*c^3) + (d*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d*x^4*(a + b*ArcTanh[c*x]
)^2)/4 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) + (b^2*d*Log
[1 - c^2*x^2])/(3*c^3) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

]

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(dx^2\left(a+b\tanh^{-1}(cx)\right)^2+cdx^3\left(a+b\tanh^{-1}(cx)\right)^2\right)dx$$

$$=d\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+(cd)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{3}(2bcd)\int\frac{x^3(}{}$$

$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}(bd)\int x^2\left(a\right.$$

$$=\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d\left(a+b\tanh^{-1}(c\right.}{3c^3}$$

$$=\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right)+$$

$$=\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}-\frac{b^2d\tanh^{-1}(cx)}{3c^3}+\frac{b^2dx\tanh^{-1}(cx)}{2c^2}+\frac{bdx^2\left(a+b\tanh^{-1}\right.}{3c}$$

$$=\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{b^2dx^2}{12c}-\frac{b^2d\tanh^{-1}(cx)}{3c^3}+\frac{b^2dx\tanh^{-1}(cx)}{2c^2}+\frac{bdx^2\left(a+\right.}{}$$

Mathematica [A] time = 0.57, size = 234, normalized size = 0.99

$$\frac{d\left(3a^2c^4x^4+4a^2c^3x^3+2abc^3x^3+4abc^2x^2+4ab\log\left(c^2x^2-1\right)+2b\tanh^{-1}(cx)\left(ac^3x^3(3cx+4)+b\left(c^3x^3+2c^2x\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(-b^2 + 6*a*b*c*x + 4*b^2*c*x + 4*a*b*c^2*x^2 + b^2*c^2*x^2 + 4*a^2*c^3*
x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-7 + 4*c^3*x^3 + 3*c^4*x^4)*ArcT
anh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^3*x^3*(4 + 3*c*x) + b*(-2 + 3*c*x + 2*c^
2*x^2 + c^3*x^3) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] -
 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 - c^2*x^2] + 4*a*b*Log[-1 + c^2*x^2] + 4*
b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(12*c^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2cdx^3+a^2dx^2+\left(b^2cdx^3+b^2dx^2\right)\operatorname{artanh}(cx)^2+2\left(abcdx^3+abdx^2\right)\operatorname{artanh}(cx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^3 + a^2*d*x^2 + (b^2*c*d*x^3 + b^2*d*x^2)*arctanh(c*x)^2
 + 2*(a*b*c*d*x^3 + a*b*d*x^2)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx + d)(b \operatorname{artanh}(cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^2, x)
```

maple [A] time = 0.06, size = 383, normalized size = 1.62

$$\frac{cdab \operatorname{arctanh}(cx) x^4}{2} - \frac{d b^2 \operatorname{dilog}\left(\frac{1}{2} + \frac{cx}{2}\right)}{3c^3} + \frac{d b^2 \operatorname{arctanh}(cx) x^3}{6} + \frac{d b^2 \operatorname{arctanh}(cx)^2 x^3}{3} + \frac{dab x^3}{6} + \frac{7d b^2 \ln (cx -}{48c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/2*c*d*a*b*arctanh(c*x)*x^4+1/6*d*a*b*x^3+7/48/c^3*d*b^2*ln(c*x-1)^2+1/6/c
^3*d*b^2*ln(c*x+1)+1/2/c^3*d*b^2*ln(c*x-1)-1/3/c^3*d*b^2*dilog(1/2+1/2*c*x)
+1/6*d*b^2*arctanh(c*x)*x^3+1/3*d*b^2*arctanh(c*x)^2*x^3-1/48/c^3*d*b^2*ln(
c*x+1)^2+1/4*c*a^2*d*x^4+1/2*a*b*d*x/c^2+1/2*b^2*d*x*arctanh(c*x)/c^2+1/3*b
^2*d*x/c^2+1/12*b^2*d*x^2/c-7/24/c^3*d*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/3/c*
d*b^2*arctanh(c*x)*x^2+1/3/c*d*a*b*x^2+2/3*d*a*b*arctanh(c*x)*x^3+7/12/c^3*
d*a*b*ln(c*x-1)+1/12/c^3*d*a*b*ln(c*x+1)-1/24/c^3*d*b^2*ln(-1/2*c*x+1/2)*ln
(1/2+1/2*c*x)+7/12/c^3*d*b^2*arctanh(c*x)*ln(c*x-1)+1/12/c^3*d*b^2*arctanh(
c*x)*ln(c*x+1)+1/24/c^3*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/4*c*d*b^2*arctan
h(c*x)^2*x^4+1/3*a^2*d*x^3
```

maxima [A] time = 0.66, size = 402, normalized size = 1.70

$$\frac{1}{4} a^2cdx^4 + \frac{1}{3} a^2dx^3 + \frac{1}{12}\left(6x^4 \operatorname{artanh}(cx) + c\left(\frac{2\left(c^2x^3 + 3x\right)}{c^4} - \frac{3 \log(cx+1)}{c^5} + \frac{3 \log(cx-1)}{c^5}\right)\right)abcd + \frac{1}{3}\left(2x^3 a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3
+ 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c*d + 1/3*(2*x^3
*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*d + 1/3*(log(c*x +
1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d/c^3 + 1/6*b^2*d*log(c*
x + 1)/c^3 + 1/2*b^2*d*log(c*x - 1)/c^3 + 1/48*(4*b^2*c^2*d*x^2 + 16*b^2*c*
d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d)*log(c*x + 1)^2 + (3*b^2*c
^4*d*x^4 + 4*b^2*c^3*d*x^3 - 7*b^2*d)*log(-c*x + 1)^2 + 4*(b^2*c^3*d*x^3 +
2*b^2*c^2*d*x^2 + 3*b^2*c*d*x)*log(c*x + 1) - 2*(2*b^2*c^3*d*x^3 + 4*b^2*c^
2*d*x^2 + 6*b^2*c*d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d)*log(c*x
 + 1))*log(-c*x + 1))/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{atanh}(c\,x)\right)^2 \left(d + c\,d\,x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2x^2\,dx + \int a^2cx^3\,dx + \int b^2x^2\operatorname{atanh}^2(cx)\,dx + \int 2abx^2\operatorname{atanh}(cx)\,dx + \int b^2cx^3\operatorname{atanh}^2(cx)\,dx + \int 2ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x**2, x) + Integral(a**2*c*x**3, x) + Integral(b**2*x**2*a
tanh(c*x)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(b**2*c*x**
3*atanh(c*x)**2, x) + Integral(2*a*b*c*x**3*atanh(c*x), x))
```

3.70 $\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=196

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^2}+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx\right.$$

```
[Out] a*b*d*x/c+1/3*b^2*d*x/c-1/3*b^2*d*arctanh(c*x)/c^2+b^2*d*x*arctanh(c*x)/c+1
/3*b*d*x^2*(a+b*arctanh(c*x))-1/6*d*(a+b*arctanh(c*x))^2/c^2+1/2*d*x^2*(a+b
*arctanh(c*x))^2+1/3*c*d*x^3*(a+b*arctanh(c*x))^2-2/3*b*d*(a+b*arctanh(c*x)
)*ln(2/(-c*x+1))/c^2+1/2*b^2*d*ln(-c^2*x^2+1)/c^2-1/3*b^2*d*polylog(2,1-2/(
-c*x+1))/c^2
```

Rubi [A] time = 0.39, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5940, 5916, 5980, 5910, 260, 5948, 321, 206, 5984, 5918, 2402, 2315}

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^2}+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d*x)/c + (b^2*d*x)/(3*c) - (b^2*d*ArcTanh[c*x])/(3*c^2) + (b^2*d*x*Arc
Tanh[c*x])/c + (b*d*x^2*(a + b*ArcTanh[c*x]))/3 - (d*(a + b*ArcTanh[c*x])^2
)/(6*c^2) + (d*x^2*(a + b*ArcTanh[c*x])^2)/2 + (c*d*x^3*(a + b*ArcTanh[c*x]
)^2)/3 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (b^2*d*Log
[1 - c^2*x^2])/(2*c^2) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int \left(dx\left(a+b\tanh^{-1}(cx)\right)^2 + cdx^2\left(a+b\tanh^{-1}(cx)\right)^2\right)dx$$
$$= d\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx + (cd)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$
$$= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2 - (bcd)\int \frac{x^2(a}{}$$
$$= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}(2bd)\int x($$
$$= \frac{abdx}{c} + \frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right) - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2} + \frac{1}{2}dx^2(a+$$
$$= \frac{abdx}{c} + \frac{b^2dx}{3c} + \frac{b^2dx\tanh^{-1}(cx)}{c} + \frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right) - \frac{d(a+}{}$$
$$= \frac{abdx}{c} + \frac{b^2dx}{3c} - \frac{b^2d\tanh^{-1}(cx)}{3c^2} + \frac{b^2dx\tanh^{-1}(cx)}{c} + \frac{1}{3}bdx^2(a+b\tan$$
$$= \frac{abdx}{c} + \frac{b^2dx}{3c} - \frac{b^2d\tanh^{-1}(cx)}{3c^2} + \frac{b^2dx\tanh^{-1}(cx)}{c} + \frac{1}{3}bdx^2(a+b\tan$$

Mathematica [A] time = 0.51, size = 201, normalized size = 1.03

$$\frac{d\left(2a^2c^3x^3 + 3a^2c^2x^2 + 2abc^2x^2 + 2ab\log\left(c^2x^2-1\right) + 2b\tanh^{-1}(cx)\left(ac^2x^2(2cx+3) + b\left(c^2x^2+3cx-1\right) - 2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(6*a*b*c*x + 2*b^2*c*x + 3*a^2*c^2*x^2 + 2*a*b*c^2*x^2 + 2*a^2*c^3*x^3 +
 b^2*(-5 + 3*c^2*x^2 + 2*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^2*
x^2*(3 + 2*c*x) + b*(-1 + 3*c*x + c^2*x^2) - 2*b*Log[1 + E^(-2*ArcTanh[c*x]
)]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*b^2*Log[1 - c^2*x^2] + 2*
a*b*Log[-1 + c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(6*c^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2cdx^2 + a^2dx + \left(b^2cdx^2 + b^2dx\right)\operatorname{artanh}(cx)^2 + 2\left(abcdx^2 + abdx\right)\operatorname{artanh}(cx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^2 + a^2*d*x + (b^2*c*d*x^2 + b^2*d*x)*arctanh(c*x)^2 + 2
*(a*b*c*d*x^2 + a*b*d*x)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx+d)(b\operatorname{artanh}(cx)+a)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x, x)
```

maple [A] time = 0.06, size = 341, normalized size = 1.74

$$\frac{c\,a^2 d\,x^3}{3}+\frac{a^2 d\,x^2}{2}+\frac{cd\,b^2 \operatorname{arctanh}(cx)^2 x^3}{3}+\frac{d\,b^2 \operatorname{arctanh}(cx)^2 x^2}{2}+\frac{d\,b^2 \operatorname{arctanh}(cx)\,x^2}{3}+\frac{b^2 dx \operatorname{arctanh}(cx)}{c}+\frac{5d\,b^2 a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/3*c*a^2*d*x^3+1/2*a^2*d*x^2+1/3*c*d*b^2*arctanh(c*x)^2*x^3+1/2*d*b^2*arct
anh(c*x)^2*x^2+1/3*d*b^2*arctanh(c*x)*x^2+b^2*d*x*arctanh(c*x)/c+5/6/c^2*d*
b^2*arctanh(c*x)*ln(c*x-1)-1/6/c^2*d*b^2*arctanh(c*x)*ln(c*x+1)+5/24/c^2*d*
b^2*ln(c*x-1)^2-1/3/c^2*d*b^2*dilog(1/2+1/2*c*x)-5/12/c^2*d*b^2*ln(c*x-1)*l
n(1/2+1/2*c*x)+1/24/c^2*d*b^2*ln(c*x+1)^2-1/12/c^2*d*b^2*ln(-1/2*c*x+1/2)*l
n(c*x+1)+1/12/c^2*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/3*b^2*d*x/c+2/3/
c^2*d*b^2*ln(c*x-1)+1/3/c^2*d*b^2*ln(c*x+1)+2/3*c*d*a*b*arctanh(c*x)*x^3+d*
a*b*arctanh(c*x)*x^2+1/3*d*a*b*x^2+a*b*d*x/c+5/6/c^2*d*a*b*ln(c*x-1)-1/6/c^
2*d*a*b*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2cdx^3+\frac{1}{2}\,b^2dx^2\operatorname{artanh}(cx)^2+\frac{1}{3}\left(2\,x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)abcd-\frac{1}{216}\left(2\,c^4\left(\frac{2\left(c^2x^3+3\,x\right)}{c^6}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctanh(c*x)^2 + 1/3*(2*x^3*arctanh(c*x) +
c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d - 1/216*(2*c^4*(2*(c^2*x^3 + 3*
x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c*x - 1)/c^7) - 3*c^3*(x^2/c^4 + log(c^
2*x^2 - 1)/c^6) - 648*c^3*integrate(1/9*x^3*log(c*x + 1)/(c^4*x^2 - c^2), x
) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c^5 + log(c*x - 1)/c^5) - 324*c*integrate
(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2), x) - 6*(3*c^3*x^3*log(c*x + 1)^2 + (2*
c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 + 1)*log(c*x + 1))*log(-c*x + 1))/
c^3 - (2*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 27*(c*x -
1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 54*(c*x - 1)*(log(-c*x + 1
)^2 - 2*log(-c*x + 1) + 2))/c^3 + 18*log(9*c^4*x^2 - 9*c^2)/c^3 - 324*integ
rate(1/9*log(c*x + 1)/(c^4*x^2 - c^2), x))*b^2*c*d + 1/2*a^2*d*x^2 + 1/2*(2
*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*
d + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) -
 (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*l
og(c*x - 1))/c^2)*b^2*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int(x*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2x\,dx+\int a^2cx^2\,dx+\int b^2x\operatorname{atanh}^2(cx)\,dx+\int 2abx\operatorname{atanh}(cx)\,dx+\int b^2cx^2\operatorname{atanh}^2(cx)\,dx+\int 2abcx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x, x) + Integral(a**2*c*x**2, x) + Integral(b**2*x*atanh(c
*x)**2, x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(b**2*c*x**2*atanh(c
*x)**2, x) + Integral(2*a*b*c*x**2*atanh(c*x), x))
```

3.71 $\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=112

$$\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+abdx+\frac{b^2d\log\left(1-c^2x^2\right)}{2c}-\frac{b^2d\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c}+$$

[Out] a*b*d*x+b^2*d*x*arctanh(c*x)+1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))^2/c-2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c+1/2*b^2*d*ln(-c^2*x^2+1)/c-b^2*d*polylog(2,1-2/(-c*x+1))/c

Rubi [A] time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5928, 5910, 260, 1586, 5918, 2402, 2315}

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+abdx+\frac{b^2d\log(1}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] a*b*d*x + b^2*d*x*ArcTanh[c*x] + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x])^2)/(2*
c) - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b^2*d*Log[1 - c^2*x
^2])/(2*c) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{b\int\left(-d^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2(d^2+cd^2x}{\ldots}\right.}{d}$$

$$= \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{(2b)\int\frac{(d^2+cd^2x)(a+b\tanh^{-1}(cx))}{1-c^2x^2}\,dx}{d} + (bd)$$

$$= abdx + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{(2b)\int\frac{a+b\tanh^{-1}(cx)}{\frac{1}{d^2}-\frac{cx}{d^2}}\,dx}{d} + \left(b^2d\right)$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd\left(a+b\,\mathrm{ta}\right.}{}$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd\left(a+b\,\mathrm{ta}\right.}{}$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd\left(a+b\,\mathrm{ta}\right.}{}$$

Mathematica [A] time = 0.32, size = 156, normalized size = 1.39

$$\frac{d\left(a^2c^2x^2+2a^2cx+2ab\log\left(1-c^2x^2\right)+2abcx+ab\log(1-cx)-ab\log(cx+1)+2b\tanh^{-1}(cx)\left(cx(acx+2a\ldots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-3 + 2*c*x + c^2*x^2)*ArcTan
h[c*x]^2 + 2*b*ArcTanh[c*x]*(c*x*(2*a + b + a*c*x) - 2*b*Log[1 + E^(-2*ArcT
anh[c*x])]) + a*b*Log[1 - c*x] - a*b*Log[1 + c*x] + 2*a*b*Log[1 - c^2*x^2]
+ b^2*Log[1 - c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(2*c)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2cdx+a^2d+\left(b^2cdx+b^2d\right)\operatorname{artanh}(cx)^2+2\,(abcdx+abd)\operatorname{artanh}(cx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c*
d*x + a*b*d)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx + d)(b\operatorname{artanh}(cx) + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2, x)
```

maple [B] time = 0.06, size = 296, normalized size = 2.64

$$\frac{c\,a^2d\,x^2}{2}+a^2dx+\frac{cd\,b^2\operatorname{arctanh}(cx)^2\,x^2}{2}+d\,b^2\operatorname{arctanh}(cx)^2\,x+b^2dx\operatorname{arctanh}(cx)+\frac{3d\,b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2c}+\frac{d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/2*c*a^2*d*x^2+a^2*d*x+1/2*c*d*b^2*arctanh(c*x)^2*x^2+d*b^2*arctanh(c*x)^2
*x+b^2*d*x*arctanh(c*x)+3/2/c*d*b^2*arctanh(c*x)*ln(c*x-1)+1/2/c*d*b^2*arct
anh(c*x)*ln(c*x+1)-1/8/c*b^2*ln(c*x+1)^2*d+1/4/c*d*b^2*ln(-1/2*c*x+1/2)*ln(
c*x+1)-1/4/c*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/c*d*b^2*dilog(1/2+1/2
*c*x)+1/2/c*d*b^2*ln(c*x-1)+1/2/c*d*b^2*ln(c*x+1)+3/8/c*d*b^2*ln(c*x-1)^2-3
/4/c*d*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+c*d*a*b*arctanh(c*x)*x^2+2*d*a*b*arcta
nh(c*x)*x+a*b*d*x+3/2/c*d*a*b*ln(c*x-1)+1/2/c*d*a*b*ln(c*x+1)
```

maxima [B] time = 0.56, size = 290, normalized size = 2.59

$$\frac{1}{2}a^2cdx^2+\frac{1}{2}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)abcd+a^2dx+\frac{\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2x^2\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*d*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
 log(c*x - 1)/c^3))*a*b*c*d + a^2*d*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a*b*d/c + (log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b
^2*d/c + 1/2*b^2*d*log(c*x + 1)/c + 1/2*b^2*d*log(c*x - 1)/c + 1/8*(4*b^2*c
*d*x*log(c*x + 1) + (b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1)^2 +
(b^2*c^2*d*x^2 + 2*b^2*c*d*x - 3*b^2*d)*log(-c*x + 1)^2 - 2*(2*b^2*c*d*x +
(b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1))*log(-c*x + 1))/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\operatorname{atanh}(c\,x))^2\ (d + c\,d\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2\,dx+\int b^2\operatorname{atanh}^2(cx)\,dx+\int 2ab\operatorname{atanh}(cx)\,dx+\int a^2cx\,dx+\int b^2cx\operatorname{atanh}^2(cx)\,dx+\int 2abcx\operatorname{atanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*ata
nh(c*x), x) + Integral(a**2*c*x, x) + Integral(b**2*c*x*atanh(c*x)**2, x) +
 Integral(2*a*b*c*x*atanh(c*x), x))
```

3.72 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=191

$$-bd\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd\mathrm{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+d\left(a+b\tanh^{-1}(cx)\right)^2+cdx\left(a+b\right.$$

```
[Out] d*(a+b*arctanh(c*x))^2+c*d*x*(a+b*arctanh(c*x))^2-2*d*(a+b*arctanh(c*x))^2*
arctanh(-1+2/(-c*x+1))-2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-b^2*d*polylo
g(2,1-2/(-c*x+1))-b*d*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*d*(a+b*a
rctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*d*polylog(3,1-2/(-c*x+1))-1/2
*b^2*d*polylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.45, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5914, 6052, 5948, 6058, 6610}

$$-bd\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd\mathrm{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-d)\mathrm{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] d*(a + b*ArcTanh[c*x])^2 + c*d*x*(a + b*ArcTanh[c*x])^2 + 2*d*(a + b*ArcTan
h[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 -
c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 - c*x)] - b*d*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)] + b*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x
)] + (b^2*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

```
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(cd\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}\right)dx$$
$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx + (cd)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx$$
$$= cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right) - (4b$$
$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2 \mathrm{ta}$$
$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2 \mathrm{ta}$$
$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2 \mathrm{ta}$$
$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2 \mathrm{ta}$$

Mathematica [C] time = 0.51, size = 228, normalized size = 1.19

$$d\left(a^2cx + a^2\log(cx) + ab\left(\log\left(1-c^2x^2\right) + 2cx\tanh^{-1}(cx)\right) + ab(\mathrm{Li}_2(cx) - \mathrm{Li}_2(-cx)) + b^2\left(\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right) + \right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] d*(a^2*c*x + a^2*Log[c*x] + a*b*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^2]) + b
^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])])
 + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + a*b*(-PolyLog[2, -(c*x)] + PolyLog[2
, c*x]) + b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 +
E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh
[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTa
nh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*
x])]/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2cdx + a^2d + \left(b^2cdx + b^2d\right)\mathrm{artanh}\,(cx)^2 + 2\,(abcdx + abd)\,\mathrm{artanh}\,(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)(b\,\mathrm{artanh}\,(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x, x)
```

maple [C] time = 0.46, size = 3644, normalized size = 19.08

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x)
```

```
[Out] a^2*d*c*x-d*a*b*dilog(c*x)-d*a*b*dilog(c*x+1)+2*d*b^2*arctanh(c*x)*polylog(
2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x)^2*ln(c*x)+d*b^2*arctanh(c
*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d*b^2*arctanh(c*x)*polylog(2,(c*x+
1)/(-c^2*x^2+1)^(1/2))-d*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1)
)-d*b^2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-d*b^2*arctanh(c*x)^2*ln((
c*x+1)^2/(-c^2*x^2+1)-1)-d*b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/
2))-d*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x
)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+d*a*b*ln(c*x+1)+d*a*b*ln(c*x-1)-1/4*I*
d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1
)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*polylog(2
,-(c*x+1)^2/(-c^2*x^2+1))-1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*
x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-d*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^
(1/2))-2*d*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-d*b^2*dilog(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))+a^2*d*ln(c*x)-2*d*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)
^(1/2))+d*b^2*arctanh(c*x)^2-1/2*d*b^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1
/2*d*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c
^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/
2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(
-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
*arctanh(c*x)^2-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1-I*
(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1
))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+
(c*x+1)^2/(-c^2*x^2+1)))*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*
Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+
(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+
1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^
2+1))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-
c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1+
(c*x+1)^2/(-c^2*x^2+1))+2*d*a*b*arctanh(c*x)*c*x-1/4*I*d*b^2*Pi*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,-(c*x+1)^2/(
-c^2*x^2+1))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/
(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1
/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))
-1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2
))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*d*b^2*Pi*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*dilog(1-I*(c*x
+1)/(-c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x)^2*c*x-d*a*b*ln(c*x)*ln(c*x+1)+2*
d*a*b*arctanh(c*x)*ln(c*x)-1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c
```

```
*x)^2+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*Pi*cs
gn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*Pi*
csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x
)*ln(1+(c*x+1)^2/(-c^2*x^2+1))+1/4*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polyl
og(2,-(c*x+1)^2/(-c^2*x^2+1))-1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+
1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(
1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1
)^(1/2))+1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,-(c*x+1)^2/(-c^2*
x^2+1))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)
^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+
1)))*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x
+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)*ln(1+I*(c*x+
1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*cs
gn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,b^2cdx\log\left(-cx+1\right)^2+a^2cdx+\left(2\,cx\,\mathrm{artanh}\left(cx\right)+\log\left(-c^2x^2+1\right)\right)abd+a^2d\log(x)-\int-\frac{\left(b^2c^2dx^2-b^2d\right)\log\left(cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*c*d*x*log(-c*x + 1)^2 + a^2*c*d*x + (2*c*x*arctanh(c*x) + log(-c^2*
x^2 + 1))*a*b*d + a^2*d*log(x) - integrate(-1/4*((b^2*c^2*d*x^2 - b^2*d)*lo
g(c*x + 1)^2 + 4*(a*b*c*d*x - a*b*d)*log(c*x + 1) - 2*(b^2*c^2*d*x^2 + 2*a*
b*c*d*x - 2*a*b*d + (b^2*c^2*d*x^2 - b^2*d)*log(c*x + 1))*log(-c*x + 1))/(c
*x^2 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2c\,dx+\int\frac{a^2}{x}\,dx+\int b^2c\,\mathrm{atanh}^2\left(cx\right)dx+\int\frac{b^2\,\mathrm{atanh}^2\left(cx\right)}{x}\,dx+\int 2abc\,\mathrm{atanh}\left(cx\right)dx+\int\frac{2ab\,\mathrm{atanh}\left(c\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d*(Integral(a**2*c, x) + Integral(a**2/x, x) + Integral(b**2*c*atanh(c*x)**
2, x) + Integral(b**2*atanh(c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x), x)
 + Integral(2*a*b*atanh(c*x)/x, x))
```

3.73 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=201

$$-bcd\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bcd\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+cd\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a\right.}{}$$

```
[Out] c*d*(a+b*arctanh(c*x))^2-d*(a+b*arctanh(c*x))^2/x-2*c*d*(a+b*arctanh(c*x))^
2*arctanh(-1+2/(-c*x+1))+2*b*c*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b*c*d*(
a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*c*d*(a+b*arctanh(c*x))*polylog(
2,-1+2/(-c*x+1))-b^2*c*d*polylog(2,-1+2/(c*x+1))+1/2*b^2*c*d*polylog(3,1-2/
(-c*x+1))-1/2*b^2*c*d*polylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.48, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610}

$$-bcd\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bcd\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-c)d\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] c*d*(a + b*ArcTanh[c*x])^2 - (d*(a + b*ArcTanh[c*x])^2)/x + 2*c*d*(a + b*Ar
cTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 2*b*c*d*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] +
b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d*PolyLog[2
, -1 + 2/(1 + c*x)] + (b^2*c*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*c*d*Po
lyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
```

```
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} + \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx + (cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right) +$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)$$

Mathematica [C] time = 0.52, size = 249, normalized size = 1.24

$$-\frac{d\left(a^2(-c)x\log(x) + a^2 + ab\left(cx\left(\log\left(1-c^2x^2\right) - 2\log(cx)\right) + 2\tanh^{-1}(cx)\right) + abcx(\text{Li}_2(-cx) - \text{Li}_2(cx)) + b^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] -((d*(a^2 - a^2*c*x*Log[x] + a*b*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1
 - c^2*x^2])) + b^2*(ArcTanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 - E
^(-2*ArcTanh[c*x])]) + c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + a*b*c*x*(Poly
Log[2, -(c*x)] - PolyLog[2, c*x]) - b^2*c*x*((I/24)*Pi^3 - (2*ArcTanh[c*x]^
3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 -
 E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcT
anh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/
2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/x)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2cdx + a^2d + \left(b^2cdx + b^2d\right)\text{artanh}\,(cx)^2 + 2\,(abcdx + abd)\,\text{artanh}\,(cx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^2, x)

maple [C] time = 0.98, size = 3104, normalized size = 15.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x)

[Out] -a^2*d/x-1/4*c*d*b^2*dilog((c*x+1)^2/(-c^2*x^2+1))+c*a^2*d*ln(c*x)-c*d*b^2*arctanh(c*x)^2-1/4*c*d*b^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*c*d*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+3/4*c*d*b^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))-1/2*c*d*b^2*polylog(3,(c*x+1)^2/(-c^2*x^2+1))+1/4*c*d*b^2*dilog(1+(c*x+1)^2/(-c^2*x^2+1))-d*b^2*arctanh(c*x)^2/x-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1+(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/8*I*c*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1+(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog((c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))-1/4*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog((c*x+1)^2/(-c^2*x^2+1))-1/2*I*c*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-c*d*a*b*ln(c*x)*ln(c*x+1)+2*c*d*a*b*arctanh(c*x)*ln(c*x)-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/8*I*c*d*b^2*dilog((c*x+1)^2/(-c^2*x^2+1))*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*dilog(1+(c*x+1)^2/(-c^2*x^2+1))+1/2*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,(c*x+1)^2/(-c^2*x^2+1))-2*d*a*b*arctanh(c*x)/x-c*d*a*b*ln(c*x+1)-c*d*a*b*ln(c*x-1)+2*c*d*a*b*ln(c*x)-c*d*a*b*dilog(c*x)-c*d*a*b*dilog(c*x+1)+c*d*b^2*arctanh(c*x)^2*ln(1-(c*x+1)^2/(-c^2*x^2+1))+c*d*b^2*arctanh(c*x)*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+3/2*c*d*b^2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))+c*d*b^2*arctanh(c*x)^2*ln(c*x)-c*d*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-c*d*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-1/4*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*dilog(1+(c*x+1)^2/(-c^2*x^2+1))+1/2*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*polylog(2,(c*x+1)^2/(-c^2*x^2+1))-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*polylog(2,-(c*

```
x+1)^2/(-c^2*x^2+1))-1/8*I*c*d*b^2*dilog((c*x+1)^2/(-c^2*x^2+1))*Pi*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x
+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/4*I*c*d*b^2*Pi*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(
-c^2*x^2+1)))^2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))+1/4*I*c*d*b^2*Pi*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2cd\log(x) - \left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\mathrm{artanh}\,(cx)}{x}\right)abd - \frac{b^2d\log\left(-cx+1\right)^2}{4\,x} - \frac{a^2d}{x} - \int -\frac{\left(b^2c^2dx^2 - b^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] a^2*c*d*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*d
 - 1/4*b^2*d*log(-c*x + 1)^2/x - a^2*d/x - integrate(-1/4*((b^2*c^2*d*x^2 -
 b^2*d)*log(c*x + 1)^2 + 4*(a*b*c^2*d*x^2 - a*b*c*d*x)*log(c*x + 1) - 2*(2*
a*b*c^2*d*x^2 - (2*a*b*c*d + b^2*c*d)*x + (b^2*c^2*d*x^2 - b^2*d)*log(c*x +
 1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^2}{x^2}\, dx + \int \frac{a^2c}{x}\, dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{x^2}\, dx + \int \frac{2ab\,\mathrm{atanh}\,(cx)}{x^2}\, dx + \int \frac{b^2c\,\mathrm{atanh}^2\,(cx)}{x}\, dx + \int \frac{2abc\,\mathrm{atar}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2/x**2, x) + Integral(a**2*c/x, x) + Integral(b**2*atanh(c*x
)**2/x**2, x) + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(b**2*c*atanh(
c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x)/x, x))
```

3.74 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=151

$$\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2+2bc^2d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{cd\left(a+b\tanh^{-1}(cx\right.}{x}$$

```
[Out] -b*c*d*(a+b*arctanh(c*x))/x+3/2*c^2*d*(a+b*arctanh(c*x))^2-1/2*d*(a+b*arcta
nh(c*x))^2/x^2-c*d*(a+b*arctanh(c*x))^2/x+b^2*c^2*d*ln(x)-1/2*b^2*c^2*d*ln(
-c^2*x^2+1)+2*b*c^2*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^2*c^2*d*polylog(
2,-1+2/(c*x+1))
```

Rubi [A] time = 0.37, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5940, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447}

$$-b^2c^2d\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2+2bc^2d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{d\left(a+b\,\text{ta}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*(a + b*ArcTanh[c*x]))/x) + (3*c^2*d*(a + b*ArcTanh[c*x])^2)/2 - (d
*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (c*d*(a + b*ArcTanh[c*x])^2)/x + b^2*c^2
*d*Log[x] - (b^2*c^2*d*Log[1 - c^2*x^2])/2 + 2*b*c^2*d*(a + b*ArcTanh[c*x])
*Log[2 - 2/(1 + c*x)] - b^2*c^2*d*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\right) dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx + (cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (bcd)\int \frac{a+b\tanh^{-1}(c}{x^2\left(1-c^2x^2\right)}$$

$$= c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x} +$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right.}{2x^2}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right.}{2x^2}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right.}{2x^2}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right.}{2x^2}$$

Mathematica [A] time = 0.28, size = 206, normalized size = 1.36

$$-\frac{d\left(2a^2cx + a^2 - 4abc^2x^2\log(cx) + abc^2x^2\log(1-cx) - abc^2x^2\log(cx+1) + 2abc^2x^2\log\left(1-c^2x^2\right) + 2b\tanh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(d*(a^2 + 2*a^2*c*x + 2*a*b*c*x + b^2*(1 + 2*c*x - 3*c^2*x^2)*ArcTanh[
c*x]^2 + 2*b*ArcTanh[c*x]*(a + 2*a*c*x + b*c*x - 2*b*c^2*x^2*Log[1 - E^(-2*
ArcTanh[c*x])]) - 4*a*b*c^2*x^2*Log[c*x] + a*b*c^2*x^2*Log[1 - c*x] - a*b*c
^2*x^2*Log[1 + c*x] - 2*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*b*c^
2*x^2*Log[1 - c^2*x^2] + 2*b^2*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^
2
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2cdx + a^2d + \left(b^2cdx + b^2d\right)\operatorname{artanh}(cx)^2 + 2\left(abcdx + abd\right)\operatorname{artanh}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^3, x)
```

maple [B] time = 0.08, size = 400, normalized size = 2.65

$$-\frac{2cdab\operatorname{arctanh}(cx)}{x}-\frac{c^2d\,b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2}+2c^2d\,b^2\operatorname{arctanh}(cx)\ln(cx)-\frac{d\,b^2\operatorname{arctanh}(cx)^2}{2x^2}-c^2d\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] -2*c*d*a*b*arctanh(c*x)/x-1/2*c^2*d*b^2*ln(c*x+1)+c^2*d*b^2*ln(c*x)-c^2*d*b
^2*dilog(c*x)-1/2*d*b^2*arctanh(c*x)^2/x^2-c*a^2*d/x-c^2*d*b^2*dilog(c*x+1)
-3/8*c^2*d*b^2*ln(c*x-1)^2+c^2*d*b^2*dilog(1/2+1/2*c*x)+1/8*c^2*d*b^2*ln(c*
x+1)^2-1/2*c^2*d*b^2*ln(c*x-1)-d*a*b*arctanh(c*x)/x^2-1/2*a^2*d/x^2-c*d*a*b
/x-c*d*b^2*arctanh(c*x)^2/x-c^2*d*b^2*ln(c*x)*ln(c*x+1)-c*d*b^2*arctanh(c*x
)/x+1/4*c^2*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/2*c^2*d*b^2*arctanh(c*
x)*ln(c*x-1)-1/2*c^2*d*b^2*arctanh(c*x)*ln(c*x+1)+3/4*c^2*d*b^2*ln(c*x-1)*l
n(1/2+1/2*c*x)-1/4*c^2*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+2*c^2*d*b^2*arctanh
(c*x)*ln(c*x)-1/2*c^2*d*a*b*ln(c*x+1)-3/2*c^2*d*a*b*ln(c*x-1)+2*c^2*d*a*b*l
n(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\operatorname{artanh}(cx)}{x}\right)abcd-\frac{1}{4}b^2cd\left(\frac{\log(-cx+1)^2}{x}+\int-\frac{(cx-1)\log(cx+1)^2+2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c*d - 1/4*b^2*c*d
*(log(-c*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*x
- 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) + 1/2*((c*log(c*x + 1)
 - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*d + 1/8*((2*(log(c*x -
 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1) +
8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c*arctanh(c*x))*b
^2*d - a^2*c*d/x - 1/2*b^2*d*arctanh(c*x)^2/x^2 - 1/2*a^2*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a^2}{x^3}\,dx+\int\frac{a^2c}{x^2}\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{x^3}\,dx+\int\frac{2ab\operatorname{atanh}(cx)}{x^3}\,dx+\int\frac{b^2c\operatorname{atanh}^2(cx)}{x^2}\,dx+\int\frac{2abc\operatorname{atan}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d*(Integral(a**2/x**3, x) + Integral(a**2*c/x**2, x) + Integral(b**2*atanh(
c*x)**2/x**3, x) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(b**2*c*ata
nh(c*x)**2/x**2, x) + Integral(2*a*b*c*atanh(c*x)/x**2, x))
```

3.75 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=206

$$\frac{5}{6}c^3d\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bc^3d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\tanh^{-1}\right.}{3x^3}$$

```
[Out] -1/3*b^2*c^2*d/x+1/3*b^2*c^3*d*arctanh(c*x)-1/3*b*c*d*(a+b*arctanh(c*x))/x^
2-b*c^2*d*(a+b*arctanh(c*x))/x+5/6*c^3*d*(a+b*arctanh(c*x))^2-1/3*d*(a+b*ar
ctanh(c*x))^2/x^3-1/2*c*d*(a+b*arctanh(c*x))^2/x^2+b^2*c^3*d*ln(x)-1/2*b^2*
c^3*d*ln(-c^2*x^2+1)+2/3*b*c^3*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-1/3*b^2
*c^3*d*polylog(2,-1+2/(c*x+1))
```

Rubi [A] time = 0.45, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {5940, 5916, 5982, 325, 206, 5988, 5932, 2447, 266, 36, 29, 31, 5948}

$$-\frac{1}{3}b^2c^3d\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{5}{6}c^3d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{2}{3}bc^3d\log\left(2-\frac{2}{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d)/(3*x) + (b^2*c^3*d*ArcTanh[c*x])/3 - (b*c*d*(a + b*ArcTanh[c*x
]))/(3*x^2) - (b*c^2*d*(a + b*ArcTanh[c*x]))/x + (5*c^3*d*(a + b*ArcTanh[c*
x])^2)/6 - (d*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (c*d*(a + b*ArcTanh[c*x])^2
)/(2*x^2) + b^2*c^3*d*Log[x] - (b^2*c^3*d*Log[1 - c^2*x^2])/2 + (2*b*c^3*d*
(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*d*PolyLog[2, -1 + 2
/(1 + c*x)])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^4} + \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\right) dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx + (cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}(2bcd)\int \frac{a+b\tan}{x^3\,(1-}$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}(2bcd)\int \frac{a+b\tan}{x}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{5}{6}c^3d\left(a+b\tanh\right.$$

$$= -\frac{b^2c^2d}{3x} - \frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{5}{6}c^3d\left(a+\right.$$

$$= -\frac{b^2c^2d}{3x} + \frac{1}{3}b^2c^3d\tanh^{-1}(cx) - \frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{bc^2d\left(a+b\tan\right.}{x}$$

$$= -\frac{b^2c^2d}{3x} + \frac{1}{3}b^2c^3d\tanh^{-1}(cx) - \frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{bc^2d\left(a+b\tan\right.}{x}$$

Mathematica [A] time = 0.48, size = 246, normalized size = 1.19

$$-\frac{d\left(3a^2cx + 2a^2 - 4abc^3x^3\log(cx) + 3abc^3x^3\log(1-cx) - 3abc^3x^3\log(cx+1) + 6abc^2x^2 + 2abc^3x^3\log\left(1-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -1/6*(d*(2*a^2 + 3*a^2*c*x + 2*a*b*c*x + 6*a*b*c^2*x^2 + 2*b^2*c^2*x^2 + b^
2*(2 + 3*c*x - 5*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*(2 + 3*c*x)
+ b*c*x*(1 + 3*c*x - c^2*x^2) - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) -
 4*a*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x^3*Log[1 - c*x] - 3*a*b*c^3*x^3*Log[1
+ c*x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*b*c^3*x^3*Log[1 -
 c^2*x^2] + 2*b^2*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^3
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2cdx + a^2d + \left(b^2cdx + b^2d\right)\operatorname{artanh}(cx)^2 + 2\,(abcdx + abd)\operatorname{artanh}(cx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^4, x)
```

maple [B] time = 0.08, size = 440, normalized size = 2.14

$$-\frac{cdab\operatorname{arctanh}(cx)}{x^2}-\frac{c^3d\,b^2\operatorname{dilog}(cx+1)}{3}-\frac{c^3d\,b^2\operatorname{dilog}(cx)}{3}-\frac{d\,b^2\operatorname{arctanh}(cx)^2}{3x^3}+\frac{c^3d\,b^2\operatorname{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{3}-\frac{5c^3d\,b^2 \mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] -c*d*a*b*arctanh(c*x)/x^2-1/3*c^3*d*b^2*dilog(c*x+1)-5/24*c^3*d*b^2*ln(c*x-
1)^2-1/3*c^3*d*b^2*dilog(c*x)+c^3*d*b^2*ln(c*x)-1/3*d*b^2*arctanh(c*x)^2/x^
3+1/3*c^3*d*b^2*dilog(1/2+1/2*c*x)-1/24*c^3*d*b^2*ln(c*x+1)^2-2/3*c^3*d*b^2
*ln(c*x-1)-1/3*c^3*d*b^2*ln(c*x+1)-1/2*c*a^2*d/x^2-1/3*b^2*c^2*d/x-1/3*c^3*
d*b^2*ln(c*x)*ln(c*x+1)-1/3*a^2*d/x^3-1/12*c^3*d*b^2*ln(-1/2*c*x+1/2)*ln(1/
2+1/2*c*x)+2/3*c^3*d*a*b*ln(c*x)+1/6*c^3*d*a*b*ln(c*x+1)-5/6*c^3*d*a*b*ln(c
*x-1)-1/2*c*d*b^2*arctanh(c*x)^2/x^2-c^2*d*b^2*arctanh(c*x)/x-1/3*c*d*b^2*a
rctanh(c*x)/x^2-c^2*d*a*b/x-1/3*c*d*a*b/x^2-2/3*d*a*b*arctanh(c*x)/x^3-5/6*
c^3*d*b^2*arctanh(c*x)*ln(c*x-1)+1/6*c^3*d*b^2*arctanh(c*x)*ln(c*x+1)+5/12*
c^3*d*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/12*c^3*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+
1)+2/3*c^3*d*b^2*arctanh(c*x)*ln(c*x)
```

maxima [B] time = 0.92, size = 417, normalized size = 2.02

$$-\frac{1}{3}\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2c^3d-\frac{1}{3}\left(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1)\right)b^2c^3d+\frac{1}{3}\left(\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d -
1/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d + 1/3*(log(c*x + 1
)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d - 1/3*b^2*c^3*d*log(c*x + 1) - 2/3*
b^2*c^3*d*log(c*x - 1) + b^2*c^3*d*log(x) + 1/2*((c*log(c*x + 1) - c*log(c*
x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d - 1/3*((c^2*log(c^2*x^2 - 1)
- c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d - 1/2*a^2*c*d/x^2 - 1
/3*a^2*d/x^3 - 1/24*(8*b^2*c^2*d*x^2 - (b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2
*d)*log(c*x + 1)^2 - (5*b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(-c*x + 1
)^2 + 4*(3*b^2*c^2*d*x^2 + b^2*c*d*x)*log(c*x + 1) - 2*(6*b^2*c^2*d*x^2 + 2
*b^2*c*d*x - (b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(c*x + 1))*log(-c*x
 + 1))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a^2}{x^4}\,dx+\int\frac{a^2c}{x^3}\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{x^4}\,dx+\int\frac{2ab\operatorname{atanh}(cx)}{x^4}\,dx+\int\frac{b^2c\operatorname{atanh}^2(cx)}{x^3}\,dx+\int\frac{2abc\operatorname{atanh}(}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d*(Integral(a**2/x**4, x) + Integral(a**2*c/x**3, x) + Integral(b**2*atanh(
c*x)**2/x**4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(b**2*c*ata
nh(c*x)**2/x**3, x) + Integral(2*a*b*c*atanh(c*x)/x**3, x))
```

3.76 $\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=356

$$-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{60c^4}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^4}+\frac{5abd^2x}{6c^3}+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2bd^2x^2\left(a+\right.}{}$$

```
[Out] 5/6*a*b*d^2*x/c^3+3/5*b^2*d^2*x/c^3+31/180*b^2*d^2*x^2/c^2+1/15*b^2*d^2*x^3
/c+1/60*b^2*d^2*x^4-3/5*b^2*d^2*arctanh(c*x)/c^4+5/6*b^2*d^2*x*arctanh(c*x)
/c^3+2/5*b*d^2*x^2*(a+b*arctanh(c*x))/c^2+5/18*b*d^2*x^3*(a+b*arctanh(c*x))
/c+1/5*b*d^2*x^4*(a+b*arctanh(c*x))+1/15*b*c*d^2*x^5*(a+b*arctanh(c*x))-1/6
0*d^2*(a+b*arctanh(c*x))^2/c^4+1/4*d^2*x^4*(a+b*arctanh(c*x))^2+2/5*c*d^2*x
^5*(a+b*arctanh(c*x))^2+1/6*c^2*d^2*x^6*(a+b*arctanh(c*x))^2-4/5*b*d^2*(a+b
*arctanh(c*x))*ln(2/(-c*x+1))/c^4+53/90*b^2*d^2*ln(-c^2*x^2+1)/c^4-2/5*b^2*
d^2*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A] time = 1.02, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 266, 43, 5910, 260, 5948, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{5abd^2x}{6c^3}-\frac{d^2\left(a+b\tanh\right.}{60c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (5*a*b*d^2*x)/(6*c^3) + (3*b^2*d^2*x)/(5*c^3) + (31*b^2*d^2*x^2)/(180*c^2)
+ (b^2*d^2*x^3)/(15*c) + (b^2*d^2*x^4)/60 - (3*b^2*d^2*ArcTanh[c*x])/(5*c^4
) + (5*b^2*d^2*x*ArcTanh[c*x])/(6*c^3) + (2*b*d^2*x^2*(a + b*ArcTanh[c*x]))
/(5*c^2) + (5*b*d^2*x^3*(a + b*ArcTanh[c*x]))/(18*c) + (b*d^2*x^4*(a + b*Ar
cTanh[c*x]))/5 + (b*c*d^2*x^5*(a + b*ArcTanh[c*x]))/15 - (d^2*(a + b*ArcTan
h[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 + (2*c*d^2*x^5*(a
+ b*ArcTanh[c*x])^2)/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x])^2)/6 - (4*b*d^2*
(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (53*b^2*d^2*Log[1 - c^2*x^
2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int\left(d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x^5\left(a\right.\right.$$
$$= d^2\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx+$$
$$= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}c^2d^2x^6\left(a\right.$$
$$= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}c^2d^2x^6\left(a\right.$$
$$= \frac{bd^2x^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{5}bd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{15}bcd^2x^5\left(a\right.$$
$$= \frac{abd^2x}{2c^3}+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{5bd^2x^3\left(a+b\tanh^{-1}(cx)\right)}{18c}+\frac{1}{5}bc$$
$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{b^2d^2x^3}{15c}+\frac{b^2d^2x\tanh^{-1}(cx)}{2c^3}+\frac{2bd^2x^2\left(a+b\tanh^{-1}\right.}{5c^2}$$
$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{7b^2d^2x^2}{60c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\tanh^{-1}(cx)}{5c^4}$$
$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\tanh^{-1}(cx)}{5c^4}$$

Mathematica [A] time = 1.10, size = 329, normalized size = 0.92

$$\frac{d^2\left(30a^2c^6x^6+72a^2c^5x^5+45a^2c^4x^4+12abc^5x^5+36abc^4x^4+50abc^3x^3+72abc^2x^2+72ab\log\left(c^2x^2-1\right)+2b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-108*a*b - 34*b^2 + 150*a*b*c*x + 108*b^2*c*x + 72*a*b*c^2*x^2 + 31*b
^2*c^2*x^2 + 50*a*b*c^3*x^3 + 12*b^2*c^3*x^3 + 45*a^2*c^4*x^4 + 36*a*b*c^4*
x^4 + 3*b^2*c^4*x^4 + 72*a^2*c^5*x^5 + 12*a*b*c^5*x^5 + 30*a^2*c^6*x^6 + 3*
b^2*(-49 + 15*c^4*x^4 + 24*c^5*x^5 + 10*c^6*x^6)*ArcTanh[c*x]^2 + 2*b*ArcTa
nh[c*x]*(3*a*c^4*x^4*(15 + 24*c*x + 10*c^2*x^2) + b*(-54 + 75*c*x + 36*c^2*
x^2 + 25*c^3*x^3 + 18*c^4*x^4 + 6*c^5*x^5) - 72*b*Log[1 + E^(-2*ArcTanh[c*x
])]) + 75*a*b*Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 106*b^2*Log[1 - c^2*x^2]
 + 72*a*b*Log[-1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(18
0*c^4)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^2d^2x^5 + 2\,a^2cd^2x^4 + a^2d^2x^3 + \left(b^2c^2d^2x^5 + 2\,b^2cd^2x^4 + b^2d^2x^3\right)\text{artanh}\left(cx\right)^2 + 2\left(abc^2d^2x^5 + 2\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^5 + 2*a^2*c*d^2*x^4 + a^2*d^2*x^3 + (b^2*c^2*d^2*x^5
 + 2*b^2*c*d^2*x^4 + b^2*d^2*x^3)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^5 + 2*a
*b*c*d^2*x^4 + a*b*d^2*x^3)*arctanh(c*x), x)
```

giac [B] time = 4.22, size = 1135, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/63*(84*((c*x + 1)^5*b^2*d^2/(c*x - 1)^5 + (c*x + 1)^4*b^2*d^2/(c*x - 1)^4
 + (c*x + 1)^3*b^2*d^2/(c*x - 1)^3)*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^
8*c^7/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^7/(c*x
 - 1)^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x - 1)^4 -
 56*(c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 - 8*(c*x +
 1)*c^7/(c*x - 1) + c^7) + 2*(168*(c*x + 1)^5*a*b*d^2/(c*x - 1)^5 + 168*(c*
x + 1)^4*a*b*d^2/(c*x - 1)^4 + 168*(c*x + 1)^3*a*b*d^2/(c*x - 1)^3 + 28*(c*
x + 1)^5*b^2*d^2/(c*x - 1)^5 - 35*(c*x + 1)^4*b^2*d^2/(c*x - 1)^4 + 28*(c*x
 + 1)^3*b^2*d^2/(c*x - 1)^3 - 28*(c*x + 1)^2*b^2*d^2/(c*x - 1)^2 + 8*(c*x +
 1)*b^2*d^2/(c*x - 1) - b^2*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^8*c^7
/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^7/(c*x - 1)
^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x - 1)^4 - 56*(
c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 - 8*(c*x + 1)*c
^7/(c*x - 1) + c^7) + (336*(c*x + 1)^5*a^2*d^2/(c*x - 1)^5 + 336*(c*x + 1)^
4*a^2*d^2/(c*x - 1)^4 + 336*(c*x + 1)^3*a^2*d^2/(c*x - 1)^3 + 112*(c*x + 1)
^5*a*b*d^2/(c*x - 1)^5 - 140*(c*x + 1)^4*a*b*d^2/(c*x - 1)^4 + 112*(c*x + 1
)^3*a*b*d^2/(c*x - 1)^3 - 112*(c*x + 1)^2*a*b*d^2/(c*x - 1)^2 + 32*(c*x + 1
)*a*b*d^2/(c*x - 1) - 4*a*b*d^2 - 2*(c*x + 1)^7*b^2*d^2/(c*x - 1)^7 + 15*(c
*x + 1)^6*b^2*d^2/(c*x - 1)^6 - 30*(c*x + 1)^5*b^2*d^2/(c*x - 1)^5 + 34*(c*
x + 1)^4*b^2*d^2/(c*x - 1)^4 - 30*(c*x + 1)^3*b^2*d^2/(c*x - 1)^3 + 15*(c*x
 + 1)^2*b^2*d^2/(c*x - 1)^2 - 2*(c*x + 1)*b^2*d^2/(c*x - 1))/((c*x + 1)^8*c
^7/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^7/(c*x -
1)^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x - 1)^4 - 56
*(c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 - 8*(c*x + 1)
*c^7/(c*x - 1) + c^7) - 2*b^2*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^7 + 2*b^2
*d^2*log(-(c*x + 1)/(c*x - 1))/c^7)*c^2
```

maple [A] time = 0.06, size = 569, normalized size = 1.60

$$\frac{d^2b^2\,\text{arctanh}\,(cx)\,x^4}{5} + \frac{d^2b^2\,\text{arctanh}\,(cx)^2\,x^4}{4} - \frac{2d^2b^2\,\text{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{5c^4} + \frac{13d^2b^2\ln\left(cx+1\right)}{45c^4} + \frac{c^2d^2a^2x^6}{6} + \frac{2c\,d^2a^2x^5}{5} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 13/45/c^4*d^2*b^2*ln(c*x+1)-2/5/c^4*d^2*b^2*dilog(1/2+1/2*c*x)+1/5*d^2*b^2*
arctanh(c*x)*x^4+1/4*d^2*b^2*arctanh(c*x)^2*x^4+1/6*c^2*d^2*a^2*x^6+2/5*c*d
^2*a^2*x^5+1/5*d^2*a*b*x^4+1/240/c^4*d^2*b^2*ln(c*x+1)^2+49/240/c^4*d^2*b^2
*ln(c*x-1)^2+8/9/c^4*d^2*b^2*ln(c*x-1)+5/6*a*b*d^2*x/c^3+5/6*b^2*d^2*x*arct
anh(c*x)/c^3+1/60*b^2*d^2*x^4+3/5*b^2*d^2*x/c^3+31/180*b^2*d^2*x^2/c^2+1/15
*b^2*d^2*x^3/c+4/5*c*d^2*a*b*arctanh(c*x)*x^5+1/3*c^2*d^2*a*b*arctanh(c*x)*
x^6+1/4*d^2*a^2*x^4+1/2*d^2*a*b*arctanh(c*x)*x^4+1/15*c*d^2*b^2*arctanh(c*x
)*x^5+5/18/c*d^2*b^2*arctanh(c*x)*x^3+1/6*c^2*d^2*b^2*arctanh(c*x)^2*x^6+2/
5*c*d^2*b^2*arctanh(c*x)^2*x^5+2/5/c^2*d^2*b^2*arctanh(c*x)*x^2-1/120/c^4*d
^2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/120/c^4*d^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2
+1/2*c*x)+2/5/c^2*d^2*a*b*x^2+1/15*c*d^2*a*b*x^5+5/18/c*d^2*a*b*x^3-49/120/
c^4*d^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+49/60/c^4*d^2*b^2*arctanh(c*x)*ln(c*x
-1)-1/60/c^4*d^2*b^2*arctanh(c*x)*ln(c*x+1)-1/60/c^4*d^2*a*b*ln(c*x+1)+49/6
0/c^4*d^2*a*b*ln(c*x-1)
```

maxima [B] time = 0.69, size = 766, normalized size = 2.15

$$\frac{1}{6}\,a^2c^2d^2x^6+\frac{2}{5}\,a^2cd^2x^5+\frac{1}{4}\,b^2d^2x^4\,\mathrm{artanh}\,(cx)^2+\frac{1}{4}\,a^2d^2x^4+\frac{1}{90}\left(30\,x^6\,\mathrm{artanh}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^2*d^2*x^6 + 2/5*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctanh(c*x)^2 +
1/4*a^2*d^2*x^4 + 1/90*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 +
 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^2 + 1/5*
(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a
*b*c*d^2 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x
+ 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*d^2 + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4
- 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3
*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*
log(c*x - 1))/c^4)*b^2*d^2 + 2/5*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(
1/2*c*x + 1/2))*b^2*d^2/c^4 - 2/45*b^2*d^2*log(c*x + 1)/c^4 + 5/9*b^2*d^2*l
og(c*x - 1)/c^4 + 1/360*(6*b^2*c^4*d^2*x^4 + 24*b^2*c^3*d^2*x^3 + 32*b^2*c^
2*d^2*x^2 + 216*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 + 7
*b^2*d^2)*log(c*x + 1)^2 + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 - 17*b
^2*d^2)*log(-c*x + 1)^2 + 4*(3*b^2*c^5*d^2*x^5 + 9*b^2*c^4*d^2*x^4 + 5*b^2*
c^3*d^2*x^3 + 18*b^2*c^2*d^2*x^2 + 15*b^2*c*d^2*x)*log(c*x + 1) - 2*(6*b^2*
c^5*d^2*x^5 + 18*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 36*b^2*c^2*d^2*x^2
+ 30*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 + 7*b^2*d^2)*l
og(c*x + 1))*log(-c*x + 1))/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2x^3\,dx+\int 2a^2cx^4\,dx+\int a^2c^2x^5\,dx+\int b^2x^3\,\mathrm{atanh}^2\,(cx)\,dx+\int 2abx^3\,\mathrm{atanh}\,(cx)\,dx+\int 2b^2cx^4\,\mathrm{atanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x**3, x) + Integral(2*a**2*c*x**4, x) + Integral(a**2*c
**2*x**5, x) + Integral(b**2*x**3*atanh(c*x)**2, x) + Integral(2*a*b*x**3*a
tanh(c*x), x) + Integral(2*b**2*c*x**4*atanh(c*x)**2, x) + Integral(b**2*c*
*2*x**5*atanh(c*x)**2, x) + Integral(4*a*b*c*x**4*atanh(c*x), x) + Integral
(2*a*b*c**2*x**5*atanh(c*x), x))
```

3.77 $\int x^2(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=312

$$\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^3}+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abd^2x}{c^2}+\frac{1}{2}cd^2x^4\left(a+b\right.$$

```
[Out] a*b*d^2*x/c^2+19/30*b^2*d^2*x/c^2+1/6*b^2*d^2*x^2/c+1/30*b^2*d^2*x^3-19/30*
b^2*d^2*arctanh(c*x)/c^3+b^2*d^2*x*arctanh(c*x)/c^2+8/15*b*d^2*x^2*(a+b*arc
tanh(c*x))/c+1/3*b*d^2*x^3*(a+b*arctanh(c*x))+1/10*b*c*d^2*x^4*(a+b*arctanh
(c*x))+1/30*d^2*(a+b*arctanh(c*x))^2/c^3+1/3*d^2*x^3*(a+b*arctanh(c*x))^2+1
/2*c*d^2*x^4*(a+b*arctanh(c*x))^2+1/5*c^2*d^2*x^5*(a+b*arctanh(c*x))^2-16/1
5*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^3+2/3*b^2*d^2*ln(-c^2*x^2+1)/c^
3-8/15*b^2*d^2*polylog(2,1-2/(-c*x+1))/c^3
```

Rubi [A] time = 0.88, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 266, 43, 5910, 260, 5948, 302}

$$-\frac{8b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abd^2x}{c^2}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1-cx}\right)\left(a\right.}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + (b^2*d^2*x^2)/(6*c) + (b^2*d^2*
x^3)/30 - (19*b^2*d^2*ArcTanh[c*x])/(30*c^3) + (b^2*d^2*x*ArcTanh[c*x])/c^2
 + (8*b*d^2*x^2*(a + b*ArcTanh[c*x]))/(15*c) + (b*d^2*x^3*(a + b*ArcTanh[c*
x]))/3 + (b*c*d^2*x^4*(a + b*ArcTanh[c*x]))/10 + (d^2*(a + b*ArcTanh[c*x])^
2)/(30*c^3) + (d^2*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d^2*x^4*(a + b*ArcTan
h[c*x])^2)/2 + (c^2*d^2*x^5*(a + b*ArcTanh[c*x])^2)/5 - (16*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)])/(15*c^3) + (2*b^2*d^2*Log[1 - c^2*x^2])/(3*c^
3) - (8*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int \left(d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x^4\left(a\right.\right.$$

$$= d^2\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+$$

$$= \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}c^2d^2x^5\left(a\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}c^2d^2x^5\left(a\right.$$

$$= \frac{bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{3}bd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{10}bcd^2x^4\left(a+\right.$$

$$= \frac{abd^2x}{c^2}+\frac{b^2d^2x}{3c^2}+\frac{8bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{3}bd^2x^3\left(a+b\tanh^{-1}(c\right.$$

$$= \frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{1}{30}b^2d^2x^3-\frac{b^2d^2\tanh^{-1}(cx)}{3c^3}+\frac{b^2d^2x\tanh^{-1}(cx)}{c^2}+$$

$$= \frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{b^2d^2x\,t}{}$$

$$= \frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{b^2d^2x\,t}{}$$

Mathematica [A] time = 1.03, size = 297, normalized size = 0.95

$$\frac{d^2\left(6a^2c^5x^5+15a^2c^4x^4+10a^2c^3x^3+3abc^4x^4+10abc^3x^3+16abc^2x^2+16ab\log\left(c^2x^2-1\right)+b\tanh^{-1}(cx)\left(2ac^3\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-9*a*b - 5*b^2 + 30*a*b*c*x + 19*b^2*c*x + 16*a*b*c^2*x^2 + 5*b^2*c^2
*x^2 + 10*a^2*c^3*x^3 + 10*a*b*c^3*x^3 + b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 3*a
*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-31 + 10*c^3*x^3 + 15*c^4*x^4 + 6*c^5*x^5
)*ArcTanh[c*x]^2 + b*ArcTanh[c*x]*(2*a*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2) +
b*(-19 + 30*c*x + 16*c^2*x^2 + 10*c^3*x^3 + 3*c^4*x^4) - 32*b*Log[1 + E^(-2
*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x] - 15*a*b*Log[1 + c*x] + 20*b^2*Log[1
 - c^2*x^2] + 16*a*b*Log[-1 + c^2*x^2] + 16*b^2*PolyLog[2, -E^(-2*ArcTanh[c
*x])]))/(30*c^3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^2d^2x^4 + 2\,a^2cd^2x^3 + a^2d^2x^2 + \left(b^2c^2d^2x^4 + 2\,b^2cd^2x^3 + b^2d^2x^2\right)\text{artanh}\,(cx)^2 + 2\left(abc^2d^2x^4 + 2\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^4 + 2*a^2*c*d^2*x^3 + a^2*d^2*x^2 + (b^2*c^2*d^2*x^4
 + 2*b^2*c*d^2*x^3 + b^2*d^2*x^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^4 + 2*a
*b*c*d^2*x^3 + a*b*d^2*x^2)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx + d)^2(b\,\text{artanh}\,(cx) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2*x^2, x)
```

maple [A] time = 0.06, size = 521, normalized size = 1.67

$$\frac{d^2b^2\,\text{arctanh}\,(cx)\,x^3}{3}+\frac{c\,d^2a^2x^4}{2}+\frac{d^2ab\,x^3}{3}+\frac{d^2b^2\,\text{arctanh}\,(cx)^2\,x^3}{3}-\frac{8d^2b^2\,\text{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{15c^3}+\frac{31d^2b^2\ln(cx-1)^2}{120c^3}+\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/2*c*d^2*a^2*x^4+1/3*d^2*a*b*x^3+31/120/c^3*d^2*b^2*ln(c*x-1)^2+1/3*d^2*b^
2*arctanh(c*x)^2*x^3+7/20/c^3*d^2*b^2*ln(c*x+1)+59/60/c^3*d^2*b^2*ln(c*x-1)
+1/5*c^2*d^2*a^2*x^5-8/15/c^3*d^2*b^2*dilog(1/2+1/2*c*x)-1/120/c^3*d^2*b^2*
ln(c*x+1)^2+1/3*d^2*b^2*arctanh(c*x)*x^3+c*d^2*a*b*arctanh(c*x)*x^4+2/5*c^2
*d^2*a*b*arctanh(c*x)*x^5+1/30*b^2*d^2*x^3+19/30*b^2*d^2*x/c^2+1/6*b^2*d^2*
x^2/c+1/2*c*d^2*b^2*arctanh(c*x)^2*x^4+1/30/c^3*d^2*b^2*arctanh(c*x)*ln(c*x
+1)-31/60/c^3*d^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/30/c^3*d^2*a*b*ln(c*x+1)+
31/30/c^3*d^2*a*b*ln(c*x-1)+8/15/c*d^2*b^2*arctanh(c*x)*x^2+1/10*c*d^2*b^2*
arctanh(c*x)*x^4+1/5*c^2*d^2*b^2*arctanh(c*x)^2*x^5+1/10*c*d^2*a*b*x^4+2/3*
d^2*a*b*arctanh(c*x)*x^3+1/60/c^3*d^2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/60/c
^3*d^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+31/30/c^3*d^2*b^2*arctanh(c*x)*
ln(c*x-1)+a*b*d^2*x/c^2+b^2*d^2*x*arctanh(c*x)/c^2+1/3*d^2*a^2*x^3+8/15/c*d
^2*a*b*x^2
```

maxima [B] time = 0.66, size = 604, normalized size = 1.94

$$\frac{1}{5}\,a^2c^2d^2x^5+\frac{1}{2}\,a^2cd^2x^4+\frac{1}{10}\left(4\,x^5\,\text{artanh}\,(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)abc^2d^2+\frac{1}{3}\,a^2d^2x^3+\frac{1}{6}\left(6\,x^4\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^2*d^2*x^5 + 1/2*a^2*c*d^2*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*((c^
2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2*x^3
 + 1/6*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5
+ 3*log(c*x - 1)/c^5))*a*b*c*d^2 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*d^2 + 8/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + di
log(1/2*c*x + 1/2))*b^2*d^2/c^3 + 7/20*b^2*d^2*log(c*x + 1)/c^3 + 59/60*b^2
*d^2*log(c*x - 1)/c^3 + 1/120*(4*b^2*c^3*d^2*x^3 + 20*b^2*c^2*d^2*x^2 + 76*
b^2*c*d^2*x + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3
+ b^2*d^2)*log(c*x + 1)^2 + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^
2*c^3*d^2*x^3 - 31*b^2*d^2)*log(-c*x + 1)^2 + 2*(3*b^2*c^4*d^2*x^4 + 10*b^2
*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x)*log(c*x + 1) - 2*(3*b^2
*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x + (
6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + b^2*d^2)*log(
c*x + 1))*log(-c*x + 1))/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^2\,\left(d+c\,d\,x\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int a^{2} x^{2}\, dx + \int 2 a^{2} c x^{3}\, dx + \int a^{2} c^{2} x^{4}\, dx + \int b^{2} x^{2} \operatorname{atanh}^{2}{\left(c x \right)}\, dx + \int 2 a b x^{2} \operatorname{atanh}{\left(c x \right)}\, dx + \int 2 b^{2} c x^{3} \operatorname{atanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x**2, x) + Integral(2*a**2*c*x**3, x) + Integral(a**2*c
**2*x**4, x) + Integral(b**2*x**2*atanh(c*x)**2, x) + Integral(2*a*b*x**2*a
tanh(c*x), x) + Integral(2*b**2*c*x**3*atanh(c*x)**2, x) + Integral(b**2*c*
*2*x**4*atanh(c*x)**2, x) + Integral(4*a*b*c*x**3*atanh(c*x), x) + Integral
(2*a*b*c**2*x**4*atanh(c*x), x))
```

3.78 $\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=280

$$\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{12c^2}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^2}+\frac{2}{3}cd^2x^3\left(a+b\tanh\right.$$

```
[Out] 3/2*a*b*d^2*x/c+2/3*b^2*d^2*x/c+1/12*b^2*d^2*x^2-2/3*b^2*d^2*arctanh(c*x)/c
^2+3/2*b^2*d^2*x*arctanh(c*x)/c+2/3*b*d^2*x^2*(a+b*arctanh(c*x))+1/6*b*c*d^
2*x^3*(a+b*arctanh(c*x))-1/12*d^2*(a+b*arctanh(c*x))^2/c^2+1/2*d^2*x^2*(a+b
*arctanh(c*x))^2+2/3*c*d^2*x^3*(a+b*arctanh(c*x))^2+1/4*c^2*d^2*x^4*(a+b*ar
ctanh(c*x))^2-4/3*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2+5/6*b^2*d^2*l
n(-c^2*x^2+1)/c^2-2/3*b^2*d^2*polylog(2,1-2/(-c*x+1))/c^2
```

Rubi [A] time = 0.65, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5940, 5916, 5980, 5910, 260, 5948, 321, 206, 5984, 5918, 2402, 2315, 266, 43}

$$-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}+\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{12c^2}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tan\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (3*a*b*d^2*x)/(2*c) + (2*b^2*d^2*x)/(3*c) + (b^2*d^2*x^2)/12 - (2*b^2*d^2*A
rcTanh[c*x])/(3*c^2) + (3*b^2*d^2*x*ArcTanh[c*x])/(2*c) + (2*b*d^2*x^2*(a +
 b*ArcTanh[c*x]))/3 + (b*c*d^2*x^3*(a + b*ArcTanh[c*x]))/6 - (d^2*(a + b*Ar
cTanh[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTanh[c*x])^2)/2 + (2*c*d^2*x^3
*(a + b*ArcTanh[c*x])^2)/3 + (c^2*d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 - (4*b*
d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (5*b^2*d^2*Log[1 - c^2
*x^2])/(6*c^2) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

```
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(d^2x\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x^3\left(a\right.\right.$$

$$=d^2\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}c^2d^2x^4\left(\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}c^2d^2x^4\left(\right.$$

$$=\frac{abd^2x}{c}+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}bcd^2x^3\left(a+b\tanh^{-1}(cx)\right)+$$

$$=\frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{b^2d^2x\tanh^{-1}(cx)}{c}+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)+$$

$$=\frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{3b^2d^2x\tanh^{-1}(cx)}{2c}+\frac{2}{3}bd^2x^2$$

$$=\frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{1}{12}b^2d^2x^2-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{3b^2d^2x\tanh^{-1}(c}{2c}$$

Mathematica [A] time = 0.78, size = 263, normalized size = 0.94

$$d^2\left(3a^2c^4x^4+8a^2c^3x^3+6a^2c^2x^2+2abc^3x^3+8abc^2x^2+8ab\log\left(c^2x^2-1\right)+2b\tanh^{-1}(cx)\left(ac^2x^2\left(3c^2x^2+8c\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-b^2 + 18*a*b*c*x + 8*b^2*c*x + 6*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + b^2*c
^2*x^2 + 8*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-17 + 6*c^2*x
^2 + 8*c^3*x^3 + 3*c^4*x^4)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^2*x^2*(6
 + 8*c*x + 3*c^2*x^2) + b*(-4 + 9*c*x + 4*c^2*x^2 + c^3*x^3) - 8*b*Log[1 +
E^(-2*ArcTanh[c*x])]) + 9*a*b*Log[1 - c*x] - 9*a*b*Log[1 + c*x] + 10*b^2*Lo
g[1 - c^2*x^2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^(-2*ArcTanh[
c*x])]))/(12*c^2)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^2d^2x^3+2\,a^2cd^2x^2+a^2d^2x+\left(b^2c^2d^2x^3+2\,b^2cd^2x^2+b^2d^2x\right)\text{artanh}\,(cx)^2+2\left(abc^2d^2x^3+2\,abcd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^3 + 2*a^2*c*d^2*x^2 + a^2*d^2*x + (b^2*c^2*d^2*x^3 +
 2*b^2*c*d^2*x^2 + b^2*d^2*x)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^3 + 2*a*b*c
*d^2*x^2 + a*b*d^2*x)*arctanh(c*x), x)
```

giac [B] time = 2.36, size = 761, normalized size = 2.72

$$\frac{2}{45}\left(\frac{30\,(cx+1)^3b^2d^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{\left(\frac{(cx+1)^6c^5}{(cx-1)^6}-\frac{6\,(cx+1)^5c^5}{(cx-1)^5}+\frac{15\,(cx+1)^4c^5}{(cx-1)^4}-\frac{20\,(cx+1)^3c^5}{(cx-1)^3}+\frac{15\,(cx+1)^2c^5}{(cx-1)^2}-\frac{6\,(cx+1)c^5}{cx-1}+c^5\right)(cx-1)^3}+\frac{2\left(\frac{60\,(cx+1)^3abd^2}{(cx-1)^3}+\frac{10\,(\cdots}{\cdots}\right.}{\frac{(cx+1)^6c^5}{(cx-1)^6}-\frac{6\,(cx+1)^5c^5}{(cx-1)^5}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2/45*(30*(c*x + 1)^3*b^2*d^2*log(-(c*x + 1)/(c*x - 1))^2/(((c*x + 1)^6*c^5/
(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(c*x - 1)^
4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^2 - 6*(c*
x + 1)*c^5/(c*x - 1) + c^5)*(c*x - 1)^3) + 2*(60*(c*x + 1)^3*a*b*d^2/(c*x -
 1)^3 + 10*(c*x + 1)^3*b^2*d^2/(c*x - 1)^3 - 15*(c*x + 1)^2*b^2*d^2/(c*x -
1)^2 + 6*(c*x + 1)*b^2*d^2/(c*x - 1) - b^2*d^2)*log(-(c*x + 1)/(c*x - 1))/(
(c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^
4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*
x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) + (120*(c*x + 1)^3*a^2*d^2/(c*x
 - 1)^3 + 40*(c*x + 1)^3*a*b*d^2/(c*x - 1)^3 - 60*(c*x + 1)^2*a*b*d^2/(c*x
- 1)^2 + 24*(c*x + 1)*a*b*d^2/(c*x - 1) - 4*a*b*d^2 - 2*(c*x + 1)^5*b^2*d^2
/(c*x - 1)^5 + 11*(c*x + 1)^4*b^2*d^2/(c*x - 1)^4 - 18*(c*x + 1)^3*b^2*d^2/
(c*x - 1)^3 + 11*(c*x + 1)^2*b^2*d^2/(c*x - 1)^2 - 2*(c*x + 1)*b^2*d^2/(c*x
 - 1))/((c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c
*x + 1)^4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2
*c^5/(c*x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) - 2*b^2*d^2*log(-(c*x +
 1)/(c*x - 1) + 1)/c^5 + 2*b^2*d^2*log(-(c*x + 1)/(c*x - 1))/c^5)*c^2
```

maple [A] time = 0.05, size = 478, normalized size = 1.71

$$\frac{4c\,d^2ab\operatorname{arctanh}(cx)\,x^3}{3}+\frac{c^2d^2ab\operatorname{arctanh}(cx)\,x^4}{2}-\frac{2d^2b^2\operatorname{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{3c^2}+\frac{2d^2b^2\operatorname{arctanh}(cx)\,x^2}{3}+\frac{d^2b^2\operatorname{arctanh}(cx\cdots}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/2*c^2*d^2*a*b*arctanh(c*x)*x^4+4/3*c*d^2*a*b*arctanh(c*x)*x^3+1/48/c^2*d^
2*b^2*ln(c*x+1)^2+1/4*c^2*d^2*a^2*x^4+2/3*c*d^2*a^2*x^3+2/3*d^2*a*b*x^2-2/3
/c^2*d^2*b^2*dilog(1/2+1/2*c*x)+2/3*d^2*b^2*arctanh(c*x)*x^2+1/2*d^2*b^2*ar
ctanh(c*x)^2*x^2+17/48/c^2*d^2*b^2*ln(c*x-1)^2+1/2/c^2*d^2*b^2*ln(c*x+1)+7/
6/c^2*d^2*b^2*ln(c*x-1)+3/2*a*b*d^2*x/c+3/2*b^2*d^2*x*arctanh(c*x)/c+1/12*b
^2*d^2*x^2+1/2*d^2*a^2*x^2+2/3*b^2*d^2*x/c+1/24/c^2*d^2*b^2*ln(-1/2*c*x+1/2
)*ln(1/2+1/2*c*x)+17/12/c^2*d^2*b^2*arctanh(c*x)*ln(c*x-1)+1/6*c*d^2*b^2*ar
ctanh(c*x)*x^3+1/6*c*d^2*a*b*x^3+d^2*a*b*arctanh(c*x)*x^2-1/12/c^2*d^2*b^2*
arctanh(c*x)*ln(c*x+1)-1/12/c^2*d^2*a*b*ln(c*x+1)+17/12/c^2*d^2*a*b*ln(c*x-
1)+2/3*c*d^2*b^2*arctanh(c*x)^2*x^3+1/4*c^2*d^2*b^2*arctanh(c*x)^2*x^4-17/2
4/c^2*d^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/24/c^2*d^2*b^2*ln(-1/2*c*x+1/2)*l
n(c*x+1)
```

maxima [B] time = 0.67, size = 610, normalized size = 2.18

$$\frac{1}{4}\,a^2c^2d^2x^4+\frac{2}{3}\,a^2cd^2x^3+\frac{1}{2}\,b^2d^2x^2\operatorname{artanh}(cx)^2+\frac{1}{12}\left(6\,x^4\operatorname{artanh}(cx)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^2*d^2*x^4 + 2/3*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctanh(c*x)^2 +
1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 +
3*log(c*x - 1)/c^5))*a*b*c^2*d^2 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*c*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arctanh(c*x)
 + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*d^2 + 1/8*(4*c*(2
*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x -
1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^
2)*b^2*d^2 + 2/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*
b^2*d^2/c^2 + 2/3*b^2*d^2*log(c*x - 1)/c^2 + 1/48*(4*b^2*c^2*d^2*x^2 + 32*b
^2*c*d^2*x + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*log(c*x +
1)^2 + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 - 11*b^2*d^2)*log(-c*x + 1)^2
 + 4*(b^2*c^3*d^2*x^3 + 4*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x)*log(c*x + 1) - 2
*(2*b^2*c^3*d^2*x^3 + 8*b^2*c^2*d^2*x^2 + 6*b^2*c*d^2*x + (3*b^2*c^4*d^2*x^
4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{atanh}\,(c\,x))^2\,(d+c\,d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

```
[Out] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2x\,dx + \int 2a^2cx^2\,dx + \int a^2c^2x^3\,dx + \int b^2x\,\mathrm{atanh}^2\,(cx)\,dx + \int 2abx\,\mathrm{atanh}\,(cx)\,dx + \int 2b^2cx^2\,\mathrm{atanl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x, x) + Integral(2*a**2*c*x**2, x) + Integral(a**2*c**2
*x**3, x) + Integral(b**2*x*atanh(c*x)**2, x) + Integral(2*a*b*x*atanh(c*x)
, x) + Integral(2*b**2*c*x**2*atanh(c*x)**2, x) + Integral(b**2*c**2*x**3*a
tanh(c*x)**2, x) + Integral(4*a*b*c*x**2*atanh(c*x), x) + Integral(2*a*b*c*
*2*x**3*atanh(c*x), x))
```

3.79 $\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=175

$$\frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c}-\frac{8bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c}+2abd^2x+\frac{b^2d^2 \text{l}}{}$$

```
[Out] 2*a*b*d^2*x+1/3*b^2*d^2*x-1/3*b^2*d^2*arctanh(c*x)/c+2*b^2*d^2*x*arctanh(c*
x)+1/3*b*c*d^2*x^2*(a+b*arctanh(c*x))+1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))^
2/c-8/3*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c+b^2*d^2*ln(-c^2*x^2+1)/c-
4/3*b^2*d^2*polylog(2,1-2/(-c*x+1))/c
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5928, 5910, 260, 5916, 321, 206, 1586, 5918, 2402, 2315}

$$-\frac{4b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c}+\frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c}-\frac{8bd^2\log\left(\frac{2}{1-cx}\right)\left(a+\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] 2*a*b*d^2*x + (b^2*d^2*x)/3 - (b^2*d^2*ArcTanh[c*x])/(3*c) + 2*b^2*d^2*x*Ar
cTanh[c*x] + (b*c*d^2*x^2*(a + b*ArcTanh[c*x]))/3 + (d^2*(1 + c*x)^3*(a + b
*ArcTanh[c*x])^2)/(3*c) - (8*b*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(
3*c) + (b^2*d^2*Log[1 - c^2*x^2])/c - (4*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)
])/(3*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c} - \frac{(2b)\int\left(-3d^3\left(a+b\tanh^{-1}(cx)\right)-cd\right.}{}$$

$$= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c} - \frac{(8b)\int\frac{\left(d^3+cd^3x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{3d} + \left(2\right.$$

$$= 2abd^2x + \frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c}$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right) +$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x - \frac{b^2d^2\tanh^{-1}(cx)}{3c} + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2\left(a\right.$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x - \frac{b^2d^2\tanh^{-1}(cx)}{3c} + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2\left(a\right.$$

Mathematica [A] time = 0.66, size = 227, normalized size = 1.30

$$\frac{d^2\left(a^2c^3x^3+3a^2c^2x^2+3a^2cx+abc^2x^2+3ab\log\left(1-c^2x^2\right)+ab\log\left(c^2x^2-1\right)+b\tanh^{-1}(cx)\left(2acx\left(c^2x^2+3cx\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(3*a^2*c*x + 6*a*b*c*x + b^2*c*x + 3*a^2*c^2*x^2 + a*b*c^2*x^2 + a^2*c
^3*x^3 + b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTanh[
c*x]*(2*a*c*x*(3 + 3*c*x + c^2*x^2) + b*(-1 + 6*c*x + c^2*x^2) - 8*b*Log[1
+ E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*a*b*L
og[1 - c^2*x^2] + 3*b^2*Log[1 - c^2*x^2] + a*b*Log[-1 + c^2*x^2] + 4*b^2*Po
lyLog[2, -E^(-2*ArcTanh[c*x])]))/(3*c)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^2d^2x^2+2\,a^2cd^2x+a^2d^2+\left(b^2c^2d^2x^2+2\,b^2cd^2x+b^2d^2\right)\text{artanh}\,(cx)^2+2\,\left(abc^2d^2x^2+2\,abcd^2x+ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*b
^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x +
 a*b*d^2)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2, x)
```

maple [B] time = 0.06, size = 372, normalized size = 2.13

$$\frac{b^2d^2x}{3}+2c\,d^2ab\,\text{arctanh}\,(cx)\,x^2+\frac{2c^2d^2ab\,\text{arctanh}\,(cx)\,x^3}{3}-\frac{4d^2b^2\,\text{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{3c}+2b^2d^2x\,\text{arctanh}\,(cx)+d^2b^2\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/3*b^2*d^2*x+2/3*c^2*d^2*a*b*arctanh(c*x)*x^3+2*c*d^2*a*b*arctanh(c*x)*x^2
+c*d^2*a^2*x^2+1/3*c^2*d^2*a^2*x^3+5/6/c*d^2*b^2*ln(c*x+1)+d^2*b^2*arctanh(
c*x)^2*x+7/6/c*d^2*b^2*ln(c*x-1)+1/3/c*d^2*b^2*arctanh(c*x)^2-4/3/c*d^2*b^2
*dilog(1/2+1/2*c*x)+2/3/c*d^2*b^2*ln(c*x-1)^2-1/3/c*d^2*b^2+1/3/c*d^2*a^2+x
*a^2*d^2+2*a*b*d^2*x+2*b^2*d^2*x*arctanh(c*x)+2*d^2*a*b*arctanh(c*x)*x+8/3/
c*d^2*a*b*ln(c*x-1)+8/3/c*d^2*b^2*arctanh(c*x)*ln(c*x-1)-4/3/c*d^2*b^2*ln(c
*x-1)*ln(1/2+1/2*c*x)+2/3/c*d^2*a*b*arctanh(c*x)+1/3*c^2*d^2*b^2*arctanh(c*
x)^2*x^3+c*d^2*b^2*arctanh(c*x)^2*x^2+1/3*c*d^2*b^2*arctanh(c*x)*x^2+1/3*c*
d^2*a*b*x^2
```

maxima [B] time = 0.55, size = 464, normalized size = 2.65

$$\frac{1}{3}\,a^2c^2d^2x^3+\frac{1}{3}\left(2\,x^3\,\text{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)abc^2d^2+a^2cd^2x^2+\left(2\,x^2\,\text{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+\right.}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*d^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
1)/c^4))*a*b*c^2*d^2 + a^2*c*d^2*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 - l
og(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*c*d^2 + a^2*d^2*x + (2*c*x*arctanh
(c*x) + log(-c^2*x^2 + 1))*a*b*d^2/c + 4/3*(log(c*x + 1)*log(-1/2*c*x + 1/2
) + dilog(1/2*c*x + 1/2))*b^2*d^2/c + 5/6*b^2*d^2*log(c*x + 1)/c + 7/6*b^2*
d^2*log(c*x - 1)/c + 1/12*(4*b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2
*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2 + (b^2*c^3*d^2*x^3 + 3*b^2*c
^2*d^2*x^2 + 3*b^2*c*d^2*x - 7*b^2*d^2)*log(-c*x + 1)^2 + 2*(b^2*c^2*d^2*x^
2 + 6*b^2*c*d^2*x)*log(c*x + 1) - 2*(b^2*c^2*d^2*x^2 + 6*b^2*c*d^2*x + (b^2
*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1))*l
og(-c*x + 1))/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atanh}\,(c\,x))^2\,(d + c\,d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2\,dx + \int b^2\,\mathrm{atanh}^2\,(cx)\,dx + \int 2ab\,\mathrm{atanh}\,(cx)\,dx + \int 2a^2cx\,dx + \int a^2c^2x^2\,dx + \int 2b^2cx\,\mathrm{atanh}^2\,(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*
atanh(c*x), x) + Integral(2*a**2*c*x, x) + Integral(a**2*c**2*x**2, x) + In
tegral(2*b**2*c*x*atanh(c*x)**2, x) + Integral(b**2*c**2*x**2*atanh(c*x)**2
, x) + Integral(4*a*b*c*x*atanh(c*x), x) + Integral(2*a*b*c**2*x**2*atanh(c
*x), x))
```

3.80 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=278

$$\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2-bd^2\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^2\mathrm{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+abc$$

```
[Out] a*b*c*d^2*x+b^2*c*d^2*x*arctanh(c*x)+3/2*d^2*(a+b*arctanh(c*x))^2+2*c*d^2*x
*(a+b*arctanh(c*x))^2+1/2*c^2*d^2*x^2*(a+b*arctanh(c*x))^2-2*d^2*(a+b*arcta
nh(c*x))^2*arctanh(-1+2/(-c*x+1))-4*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))
+1/2*b^2*d^2*ln(-c^2*x^2+1)-2*b^2*d^2*polylog(2,1-2/(-c*x+1))-b*d^2*(a+b*ar
ctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*d^2*(a+b*arctanh(c*x))*polylog(2,-1+2
/(-c*x+1))+1/2*b^2*d^2*polylog(3,1-2/(-c*x+1))-1/2*b^2*d^2*polylog(3,-1+2/(
-c*x+1))
```

Rubi [A] time = 0.59, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 260}

$$-bd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-2b^2d^2\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] a*b*c*d^2*x + b^2*c*d^2*x*ArcTanh[c*x] + (3*d^2*(a + b*ArcTanh[c*x])^2)/2 +
 2*c*d^2*x*(a + b*ArcTanh[c*x])^2 + (c^2*d^2*x^2*(a + b*ArcTanh[c*x])^2)/2
+ 2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 4*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)] + (b^2*d^2*Log[1 - c^2*x^2])/2 - 2*b^2*d^2*Pol
yLog[2, 1 - 2/(1 - c*x)] - b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)] + b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^2
*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\tanh\right.\right.$$

$$= d^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx + \left(2cd^2\right)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx + \left(c^2d^2\right)$$

$$= 2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2d^2\left(a+b\tan\right.$$

$$= 2d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^2d^2x^2\left(a+b\tan\right.$$

$$= abcd^2x + \frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^2d^2x^2$$

$$= abcd^2x + b^2cd^2x\tanh^{-1}(cx) + \frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x\left(a+b\tan\right.$$

$$= abcd^2x + b^2cd^2x\tanh^{-1}(cx) + \frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x\left(a+b\tan\right.$$

Mathematica [C] time = 0.71, size = 324, normalized size = 1.17

$$\frac{1}{2}d^2\left(a^2c^2x^2 + 4a^2cx + 2a^2\log(cx) + ab\left(2c^2x^2\tanh^{-1}(cx) + 2cx + \log(1-cx) - \log(cx+1)\right) + 4ab\left(\log\left(1-c^2x\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] (d^2*(4*a^2*c*x + a^2*c^2*x^2 + 2*a^2*Log[c*x] + a*b*(2*c*x + 2*c^2*x^2*Arc
Tanh[c*x] + Log[1 - c*x] - Log[1 + c*x]) + 4*a*b*(2*c*x*ArcTanh[c*x] + Log[
1 - c^2*x^2]) + b^2*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + L
og[1 - c^2*x^2]) + 4*b^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 +
 E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 2*a*b*(-PolyLo
g[2, -(c*x)] + PolyLog[2, c*x]) + 2*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3
 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(
2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[
c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 -
PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/2
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^2d^2x^2 + 2\,a^2cd^2x + a^2d^2 + \left(b^2c^2d^2x^2 + 2\,b^2cd^2x + b^2d^2\right)\text{artanh}\,(cx)^2 + 2\left(abc^2d^2x^2 + 2\,abcd^2x + ab\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x, x)
```

maple [C] time = 1.10, size = 1082, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x)
```

```
[Out] 5/2*d^2*a*b*ln(c*x-1)+3/2*d^2*a*b*ln(c*x+1)+1/2*d^2*a^2*c^2*x^2+2*x*a^2*c*d
^2-d^2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-4*d^2*b^2*arctan
h(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-4*d^2*b^2*arctanh(c*x)*ln(1-I*(c*
x+1)/(-c^2*x^2+1)^(1/2))-d^2*a*b*dilog(c*x)-d^2*a*b*dilog(c*x+1)+d^2*b^2*ar
ctanh(c*x)^2*ln(c*x)-d^2*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+d^
2*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^2*b^2*arctanh(c*x
)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+d^2*b^2*arctanh(c*x)^2*ln(1+(c*x+1)
/(-c^2*x^2+1)^(1/2))+2*d^2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)
^(1/2))+a*b*c*d^2*x+b^2*c*d^2*x*arctanh(c*x)+1/2*I*d^2*b^2*Pi*csgn(I*((c*x+
1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+1/2*I*d^2*b^2*Pi
*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*
x)^2+4*d^2*a*b*arctanh(c*x)*c*x+d^2*a*b*arctanh(c*x)*c^2*x^2-2*d^2*b^2*poly
log(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^2*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)
^(1/2))+1/2*d^2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+d^2*b^2*arctanh(c*x)
-d^2*b^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))-4*d^2*b^2*dilog(1+I*(c*x+1)/(-c^2*x^2
+1)^(1/2))-4*d^2*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+d^2*a^2*ln(c*x)+
3/2*d^2*b^2*arctanh(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-
1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh
(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+2*d^2*a*b*a
rctanh(c*x)*ln(c*x)-d^2*a*b*ln(c*x)*ln(c*x+1)+1/2*d^2*b^2*arctanh(c*x)^2*c^
2*x^2+2*d^2*b^2*arctanh(c*x)^2*c*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2c^2d^2x^2+2a^2cd^2x+2\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)abd^2+a^2d^2\log(x)+\frac{1}{8}\left(b^2c^2d^2x^2+4b^2cd^2x\right)\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + 2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a*b*d^2 + a^2*d^2*log(x) + 1/8*(b^2*c^2*d^2*x^2 + 4*b^2*c*d^2*x)*log(
```

```
-c*x + 1)^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^
2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b*c^2*d^2*x^2 + a*b*
c*d^2*x - a*b*d^2)*log(c*x + 1) - (4*a*b*c*d^2*x - 4*a*b*d^2 + (4*a*b*c^3*d
^2 + b^2*c^3*d^2)*x^3 - 4*(a*b*c^2*d^2 - b^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d^2*
x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1))*log(-c*x + 1))
/(c*x^2 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^2\,(d + c\,d\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int 2a^2c\,dx + \int \frac{a^2}{x}\,dx + \int a^2c^2x\,dx + \int 2b^2c\,\mathrm{atanh}^2\,(cx)\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{x}\,dx + \int 4abc\,\mathrm{atanh}\,(cx)\,dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d**2*(Integral(2*a**2*c, x) + Integral(a**2/x, x) + Integral(a**2*c**2*x, x
) + Integral(2*b**2*c*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2/x, x)
 + Integral(4*a*b*c*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)/x, x) + Inte
gral(b**2*c**2*x*atanh(c*x)**2, x) + Integral(2*a*b*c**2*x*atanh(c*x), x))
```

3.81 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=283

$$c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2-2bcd^2\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+2bcd^2\mathrm{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] 2*c*d^2*(a+b*arctanh(c*x))^2-d^2*(a+b*arctanh(c*x))^2/x+c^2*d^2*x*(a+b*arct
anh(c*x))^2-4*c*d^2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-2*b*c*d^2*(
a+b*arctanh(c*x))*ln(2/(-c*x+1))+2*b*c*d^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1
))-b^2*c*d^2*polylog(2,1-2/(-c*x+1))-2*b*c*d^2*(a+b*arctanh(c*x))*polylog(2
,1-2/(-c*x+1))+2*b*c*d^2*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-b^2*c*
d^2*polylog(2,-1+2/(c*x+1))+b^2*c*d^2*polylog(3,1-2/(-c*x+1))-b^2*c*d^2*pol
ylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.63, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610}

$$-2bcd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+2bcd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-b^2cd^2\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] 2*c*d^2*(a + b*ArcTanh[c*x])^2 - (d^2*(a + b*ArcTanh[c*x])^2)/x + c^2*d^2*x
*(a + b*ArcTanh[c*x])^2 + 4*c*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 -
 c*x)] - 2*b*c*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] + 2*b*c*d^2*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*d^2*PolyLog[2, 1 - 2/(1 - c*x)]
 - 2*b*c*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 2*b*c*d^2*(
a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^2*PolyLog[2, -1
+ 2/(1 + c*x)] + b^2*c*d^2*PolyLog[3, 1 - 2/(1 - c*x)] - b^2*c*d^2*PolyLog[
3, -1 + 2/(1 - c*x)]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} + \frac{2cd^2\left(a+b\,\mathrm{ta}\right.}{x}\right.$$

$$= d^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx + \left(2cd^2\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx + \left(c^2\right.$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2 + 4cd^2\left(a+b\tan\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\tanh\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\tanh\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\tanh\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\tanh\right.$$

Mathematica [C] time = 0.52, size = 341, normalized size = 1.20

$$d^2\left(12a^2c^2x^2 + 24a^2cx\log(x) - 12a^2 + 24abc^2x^2\tanh^{-1}(cx) - 24abcx\mathrm{Li}_2(-cx) + 24abcx\mathrm{Li}_2(cx) + 24abcx\log(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(-12*a^2 + I*b^2*c*Pi^3*x + 12*a^2*c^2*x^2 - 24*a*b*ArcTanh[c*x] + 24*
a*b*c^2*x^2*ArcTanh[c*x] - 12*b^2*ArcTanh[c*x]^2 + 12*b^2*c^2*x^2*ArcTanh[c
*x]^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 24*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-2*A
rcTanh[c*x])] - 24*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b
^2*c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x
]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 24*a*b*c*x*Log[c*x] +
 12*b^2*c*x*(1 + 2*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 12*b^2*
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x]*PolyLog[2, E^
(2*ArcTanh[c*x])] - 24*a*b*c*x*PolyLog[2, -(c*x)] + 24*a*b*c*x*PolyLog[2, c
*x] + 12*b^2*c*x*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c*x*PolyLog[3, E
^(2*ArcTanh[c*x])]))/(12*x)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^2d^2x^2 + 2\,a^2cd^2x + a^2d^2 + \left(b^2c^2d^2x^2 + 2\,b^2cd^2x + b^2d^2\right)\text{artanh}\left(cx\right)^2 + 2\left(abc^2d^2x^2 + 2\,abcd^2x + ab\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^2(b\,\text{artanh}\,(cx) + a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^2, x)
```

maple [C] time = 1.01, size = 6039, normalized size = 21.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^2d^2x - \frac{1}{2}\,b^2c^2d^2\int \log\left(cx+1\right)\log\left(-cx+1\right)\,dx + \frac{1}{4}\,b^2c^2d^2\int \frac{\log\left(cx+1\right)^2}{c^2x^2}\,dx + \left(2\,cx\,\text{artanh}\left(cx\right) + \log\left(-c^2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d^2*x - 1/2*b^2*c^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1), x) +
1/4*b^2*c^2*d^2*integrate(log(c*x + 1)^2/(c^2*x^2), x) + (2*c*x*arctanh(c*x
) + log(-c^2*x^2 + 1))*a*b*c*d^2 + 1/2*(c*x - (c*x - 1)*log(-c*x + 1) - 1)*
b^2*c*d^2 + 1/4*b^2*c*d^2*gamma(3, -log(c*x + 1)) + 1/2*b^2*c*d^2*integrate
(log(c*x + 1)^2/x, x) - b^2*c*d^2*integrate(log(c*x + 1)*log(-c*x + 1)/x, x
) + 2*a*b*c*d^2*integrate(log(c*x + 1)/x, x) - 2*a*b*c*d^2*integrate(log(-c
*x + 1)/x, x) - 1/2*b^2*c*d^2*integrate(log(-c*x + 1)/x, x) + 2*a^2*c*d^2*l
```

```
og(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*d^2 - 1/2*
b^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1)/x^2, x) - a^2*d^2/x + 1/4*(b^2
*c^2*d^2*x^2 - b^2*d^2)*log(-c*x + 1)^2/x
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^2\,\left(d+c\,d\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2c^2\,dx+\int \frac{a^2}{x^2}\,dx+\int \frac{2a^2c}{x}\,dx+\int b^2c^2\,\mathrm{atanh}^2\left(cx\right)dx+\int \frac{b^2\,\mathrm{atanh}^2\left(cx\right)}{x^2}\,dx+\int 2abc^2\,\mathrm{atanh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(2*a**2*c/x
, x) + Integral(b**2*c**2*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2/x
**2, x) + Integral(2*a*b*c**2*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)/x*
*2, x) + Integral(2*b**2*c*atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh(c*x
)/x, x))
```

3.82 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=313

$$-bc^2d^2\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^2d^2\mathrm{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2+$$

```
[Out] -b*c*d^2*(a+b*arctanh(c*x))/x+5/2*c^2*d^2*(a+b*arctanh(c*x))^2-1/2*d^2*(a+b
*arctanh(c*x))^2/x^2-2*c*d^2*(a+b*arctanh(c*x))^2/x-2*c^2*d^2*(a+b*arctanh(
c*x))^2*arctanh(-1+2/(-c*x+1))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(-c^2*x^
2+1)+4*b*c^2*d^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b*c^2*d^2*(a+b*arctanh(
c*x))*polylog(2,1-2/(-c*x+1))+b*c^2*d^2*(a+b*arctanh(c*x))*polylog(2,-1+2/(
-c*x+1))-2*b^2*c^2*d^2*polylog(2,-1+2/(c*x+1))+1/2*b^2*c^2*d^2*polylog(3,1-
2/(-c*x+1))-1/2*b^2*c^2*d^2*polylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.67, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 5914, 6052, 6058, 6610}

$$-bc^2d^2\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^2d^2\mathrm{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-2b^2c^2d^2\mathrm{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d^2*(a + b*ArcTanh[c*x]))/x) + (5*c^2*d^2*(a + b*ArcTanh[c*x])^2)/2
- (d^2*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x])^2)/x
 + 2*c^2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + b^2*c^2*d^2*
Log[x] - (b^2*c^2*d^2*Log[1 - c^2*x^2])/2 + 4*b*c^2*d^2*(a + b*ArcTanh[c*x]
)*Log[2 - 2/(1 + c*x)] - b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1
 - c*x)] + b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 2*
b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 + c*x)] + (b^2*c^2*d^2*PolyLog[3, 1 - 2/(1
 - c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
```

```
e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int\left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}+\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}+\frac{c^2d^2\left(a+b\tanh\right.}{x}\right.$$

$$= d^2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx+\left(2cd^2\right)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+\left(c^2d^2\right)$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2c^2d^2\left(a+b\tanh^{-}\right.$$

$$= 2c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(\right.}{x}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}\right.}{2x^2}$$

Mathematica [C] time = 0.75, size = 370, normalized size = 1.18

$$-\frac{d^2\left(-2a^2c^2x^2\log(x)+4a^2cx+a^2+2abc^2x^2(\mathrm{Li}_2(-cx)-\mathrm{Li}_2(cx))+4abcx\left(cx\left(\log\left(1-c^2x^2\right)-2\log(cx)\right)+2\,\mathrm{ta}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(d^2*(a^2 + 4*a^2*c*x - 2*a^2*c^2*x^2*Log[x] + a*b*(2*ArcTanh[c*x] + c
*x*(2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])) + b^2*(2*c*x*ArcTanh[c*x] + (
1 - c^2*x^2)*ArcTanh[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) + 4*a
```

```
*b*c*x*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])) + 4*b^2*c*x*
(ArcTanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 - E^(-2*ArcTanh[c*x])])
 + c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + 2*a*b*c^2*x^2*(PolyLog[2, -(c*x)]
 - PolyLog[2, c*x]) - 2*b^2*c^2*x^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - A
rcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*Ar
cTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]
*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - Poly
Log[3, E^(2*ArcTanh[c*x])]/2)))/x^2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^2d^2x^2 + 2\,a^2cd^2x + a^2d^2 + \left(b^2c^2d^2x^2 + 2\,b^2cd^2x + b^2d^2\right)\text{artanh}\,(cx)^2 + 2\,\left(abc^2d^2x^2 + 2\,abcd^2x + \right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^2 (b\,\text{artanh}\,(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^3, x)
```

maple [C] time = 1.74, size = 1167, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] -1/2*I*c^2*d^2*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*c^2*d^2*
b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*c^2*d^2*b^2*Pi*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+
1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-2*c*d^2*a^2
/x-1/2*d^2*a^2/x^2+4*c^2*d^2*b^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/
2))-c^2*d^2*a*b*ln(c*x)*ln(c*x+1)+2*c^2*d^2*a*b*arctanh(c*x)*ln(c*x)-4*c*d^
2*a*b*arctanh(c*x)/x-c^2*d^2*a*b*dilog(c*x+1)-5/2*c^2*d^2*a*b*ln(c*x-1)-3/2
*c^2*d^2*a*b*ln(c*x+1)+c^2*d^2*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)
^(1/2))+2*c^2*d^2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-c
*d^2*a*b/x-d^2*a*b*arctanh(c*x)/x^2+4*c^2*d^2*a*b*ln(c*x)-c^2*d^2*b^2*arcta
nh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+c^2*d^2*b^2*arctanh(c*x)^2*ln(c*
x)-c^2*d^2*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+c^2*d^2*b^2*arct
anh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^2*d^2*b^2*arctanh(c*x)*poly
log(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c*d^2*b^2*arctanh(c*x)^2/x-c*d^2*b^2*ar
ctanh(c*x)/x-c^2*d^2*a*b*dilog(c*x)-2*c^2*d^2*b^2*polylog(3,-(c*x+1)/(-c^2*
x^2+1)^(1/2))+1/2*c^2*d^2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-c^2*d^2*b^
2*arctanh(c*x)+c^2*d^2*b^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-4*c^2*d^2*b^2*d
ilog((c*x+1)/(-c^2*x^2+1)^(1/2))+4*c^2*d^2*b^2*dilog(1+(c*x+1)/(-c^2*x^2+1)
^(1/2))+c^2*d^2*b^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)-1/2*d^2*b^2*arctanh(c*
```

```
x)^2/x^2-3/2*c^2*d^2*b^2*arctanh(c*x)^2+c^2*d^2*a^2*ln(c*x)-2*c^2*d^2*b^2*p
olylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*c^2*d^2*b^2*Pi*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^2d^2\log(x)-2\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\operatorname{artanh}(cx)}{x}\right)abcd^2+\frac{1}{2}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d^2*log(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)
*a*b*c*d^2 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x
)/x^2)*a*b*d^2 - 2*a^2*c*d^2/x - 1/2*a^2*d^2/x^2 - 1/8*(4*b^2*c*d^2*x + b^2
*d^2)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*d^2*
x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b*c^2*
d^2*x^2)*log(c*x + 1) - (4*a*b*c^3*d^2*x^3 - b^2*c*d^2*x - 4*(a*b*c^2*d^2 +
 b^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2*x - b^
2*d^2)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{a^2}{x^3}\,dx+\int\frac{2a^2c}{x^2}\,dx+\int\frac{a^2c^2}{x}\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{x^3}\,dx+\int\frac{2ab\operatorname{atanh}(cx)}{x^3}\,dx+\int\frac{2b^2c\operatorname{atanh}^2(cx)}{x^2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c/x**2, x) + Integral(a**2*c
**2/x, x) + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*atanh(c*x
)/x**3, x) + Integral(2*b**2*c*atanh(c*x)**2/x**2, x) + Integral(b**2*c**2*
atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh(c*x)/x**2, x) + Integral(2*a*b
*c**2*atanh(c*x)/x, x))
```

3.83 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=244

$$\frac{8}{3}abc^3d^2\log(x)+\frac{8}{3}bc^3d^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}\right.}{3x^3}$$

```
[Out] -1/3*b^2*c^2*d^2/x+1/3*b^2*c^3*d^2*arctanh(c*x)-1/3*b*c*d^2*(a+b*arctanh(c*
x))/x^2-2*b*c^2*d^2*(a+b*arctanh(c*x))/x-1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x
))^2/x^3+8/3*a*b*c^3*d^2*ln(x)+2*b^2*c^3*d^2*ln(x)+8/3*b*c^3*d^2*(a+b*arcta
nh(c*x))*ln(2/(-c*x+1))-b^2*c^3*d^2*ln(-c^2*x^2+1)-4/3*b^2*c^3*d^2*polylog(
2,-c*x)+4/3*b^2*c^3*d^2*polylog(2,c*x)+4/3*b^2*c^3*d^2*polylog(2,1-2/(-c*x+
1))
```

Rubi [A] time = 0.26, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {37, 5938, 5916, 325, 206, 266, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-\frac{4}{3}b^2c^3d^2\text{PolyLog}(2,-cx)+\frac{4}{3}b^2c^3d^2\text{PolyLog}(2,cx)+\frac{4}{3}b^2c^3d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{8}{3}abc^3d^2\log(x)-\frac{2bc^2d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d^2)/(3*x) + (b^2*c^3*d^2*ArcTanh[c*x])/3 - (b*c*d^2*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (2*b*c^2*d^2*(a + b*ArcTanh[c*x]))/x - (d^2*(1 + c*x)^3
*(a + b*ArcTanh[c*x])^2)/(3*x^3) + (8*a*b*c^3*d^2*Log[x])/3 + 2*b^2*c^3*d^2
*Log[x] + (8*b*c^3*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/3 - b^2*c^3*d
^2*Log[1 - c^2*x^2] - (4*b^2*c^3*d^2*PolyLog[2, -(c*x)])/3 + (4*b^2*c^3*d^2
*PolyLog[2, c*x])/3 + (4*b^2*c^3*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - (2bc)\int\left(-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} -\right.$$

$$= -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}\left(2bcd^2\right)\int\frac{a+b\tanh^{-1}(cx)}{x^3}\,dx -$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x} - \frac{d^2(1+cx)^3\left(a\right.}{}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x} - \frac{d^2(1}{}$$

$$= -\frac{b^2c^2d^2}{3x} + \frac{1}{3}b^2c^3d^2\tanh^{-1}(cx) - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a\right.}{}$$

$$= -\frac{b^2c^2d^2}{3x} + \frac{1}{3}b^2c^3d^2\tanh^{-1}(cx) - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a\right.}{}$$

Mathematica [A] time = 0.63, size = 270, normalized size = 1.11

$$-\frac{d^2\left(3a^2c^2x^2+3a^2cx+a^2-8abc^3x^3\log(cx)+3abc^3x^3\log(1-cx)-3abc^3x^3\log(cx+1)+6abc^2x^2+4abc^3x\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(d^2*(a^2 + 3*a^2*c*x + a*b*c*x + 3*a^2*c^2*x^2 + 6*a*b*c^2*x^2 + b^2*
c^2*x^2 + b^2*(1 + 3*c*x + 3*c^2*x^2 - 7*c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTan
h[c*x]*(b*c*x*(1 + 6*c*x - c^2*x^2) + a*(2 + 6*c*x + 6*c^2*x^2) - 8*b*c^3*x
^3*Log[1 - E^(-2*ArcTanh[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x^3*L
og[1 - c*x] - 3*a*b*c^3*x^3*Log[1 + c*x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 -
 c^2*x^2]] + 4*a*b*c^3*x^3*Log[1 - c^2*x^2] + 4*b^2*c^3*x^3*PolyLog[2, E^(-
2*ArcTanh[c*x])]))/x^3
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^2d^2x^2+2\,a^2cd^2x+a^2d^2+\left(b^2c^2d^2x^2+2\,b^2cd^2x+b^2d^2\right)\mathrm{artanh}\,(cx)^2+2\left(abc^2d^2x^2+2\,abcd^2x+\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^2(b\,\mathrm{artanh}\,(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^4, x)
```

maple [B] time = 0.08, size = 550, normalized size = 2.25

$$-\frac{2c\,d^2ab\,\mathrm{arctanh}\,(cx)}{x^2}-\frac{2c^2d^2ab\,\mathrm{arctanh}\,(cx)}{x}-\frac{5c^3d^2b^2\ln\,(cx+1)}{6}-\frac{7c^3d^2b^2\ln\,(cx-1)^2}{12}-\frac{c^2d^2a^2}{x}-\frac{4c^3d^2b^2\,\mathrm{dilog}\,(}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] -2*c*d^2*a*b*arctanh(c*x)/x^2-2*c^2*d^2*a*b*arctanh(c*x)/x-4/3*c^3*d^2*b^2*
dilog(c*x)-4/3*c^3*d^2*b^2*dilog(c*x+1)-5/6*c^3*d^2*b^2*ln(c*x+1)-7/12*c^3*
d^2*b^2*ln(c*x-1)^2-1/3*d^2*b^2*arctanh(c*x)^2/x^3-c^2*d^2*a^2/x-c*d^2*a^2/
x^2+2*c^3*d^2*b^2*ln(c*x)+4/3*c^3*d^2*b^2*dilog(1/2+1/2*c*x)+1/12*c^3*d^2*b
^2*ln(c*x+1)^2-7/6*c^3*d^2*b^2*ln(c*x-1)-2*c^2*d^2*a*b/x-1/3*b^2*c^2*d^2/x-
1/3*d^2*a^2/x^3-2*c^2*d^2*b^2*arctanh(c*x)/x-c*d^2*b^2*arctanh(c*x)^2/x^2-1
/3*c*d^2*b^2*arctanh(c*x)/x^2-1/3*c^3*d^2*b^2*arctanh(c*x)*ln(c*x+1)+7/6*c^
3*d^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/6*c^3*d^2*b^2*ln(-1/2*c*x+1/2)*ln(c*x
+1)+1/6*c^3*d^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+8/3*c^3*d^2*a*b*ln(c*x
)-7/3*c^3*d^2*a*b*ln(c*x-1)-1/3*c^3*d^2*a*b*ln(c*x+1)-4/3*c^3*d^2*b^2*ln(c*
x)*ln(c*x+1)+8/3*c^3*d^2*b^2*arctanh(c*x)*ln(c*x)-7/3*c^3*d^2*b^2*arctanh(c
*x)*ln(c*x-1)-2/3*d^2*a*b*arctanh(c*x)/x^3-1/3*c*d^2*a*b/x^2-c^2*d^2*b^2*ar
ctanh(c*x)^2/x
```

maxima [B] time = 0.94, size = 555, normalized size = 2.27

$$-\frac{4}{3}\left(\log\,(cx+1)\log\left(-\frac{1}{2}\,cx+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}\,cx+\frac{1}{2}\right)\right)b^2c^3d^2-\frac{4}{3}\left(\log\,(cx)\log\,(-cx+1)+\mathrm{Li}_2\,(-cx+1)\right)b^2c^3d^2+\frac{4}{3}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -4/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d^2
- 4/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d^2 + 4/3*(log(c*x
 + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d^2 - 5/6*b^2*c^3*d^2*log(c*x + 1
) - 7/6*b^2*c^3*d^2*log(c*x - 1) + 2*b^2*c^3*d^2*log(x) - (c*(log(c^2*x^2 -
 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c^2*d^2 + ((c*log(c*x + 1) - c*log(
c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d^2 - 1/3*((c^2*log(c^2*x^2 -
 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d^2 - a^2*c^2*d^2/x
 - a^2*c*d^2/x^2 - 1/3*a^2*d^2/x^3 - 1/12*(4*b^2*c^2*d^2*x^2 + (b^2*c^3*d^2
*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2 - (7*b^2
*c^3*d^2*x^3 - 3*b^2*c^2*d^2*x^2 - 3*b^2*c*d^2*x - b^2*d^2)*log(-c*x + 1)^2
 + 2*(6*b^2*c^2*d^2*x^2 + b^2*c*d^2*x)*log(c*x + 1) - 2*(6*b^2*c^2*d^2*x^2
+ b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*
d^2)*log(c*x + 1))*log(-c*x + 1))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{a^2}{x^4}\,dx+\int \frac{2a^2c}{x^3}\,dx+\int \frac{a^2c^2}{x^2}\,dx+\int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{x^4}\,dx+\int \frac{2ab\,\mathrm{atanh}\,(cx)}{x^4}\,dx+\int \frac{2b^2c\,\mathrm{atanh}^2\,(cx)}{x^3}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d**2*(Integral(a**2/x**4, x) + Integral(2*a**2*c/x**3, x) + Integral(a**2*c
**2/x**2, x) + Integral(b**2*atanh(c*x)**2/x**4, x) + Integral(2*a*b*atanh(
c*x)/x**4, x) + Integral(2*b**2*c*atanh(c*x)**2/x**3, x) + Integral(b**2*c*
*2*atanh(c*x)**2/x**2, x) + Integral(4*a*b*c*atanh(c*x)/x**3, x) + Integral
(2*a*b*c**2*atanh(c*x)/x**2, x))
```

3.84 $\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=415

$$-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{140c^4}-\frac{52bd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{35c^4}+\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3abd^3x}{2c^3}+\frac{1}{2}c^2d^3x^6\left(a\right.$$

```
[Out] 3/2*a*b*d^3*x/c^3+122/105*b^2*d^3*x/c^3+7/20*b^2*d^3*x^2/c^2+44/315*b^2*d^3
*x^3/c+1/20*b^2*d^3*x^4+1/105*b^2*c*d^3*x^5-122/105*b^2*d^3*arctanh(c*x)/c^
4+3/2*b^2*d^3*x*arctanh(c*x)/c^3+26/35*b*d^3*x^2*(a+b*arctanh(c*x))/c^2+1/2
*b*d^3*x^3*(a+b*arctanh(c*x))/c+13/35*b*d^3*x^4*(a+b*arctanh(c*x))+1/5*b*c*
d^3*x^5*(a+b*arctanh(c*x))+1/21*b*c^2*d^3*x^6*(a+b*arctanh(c*x))-1/140*d^3*
(a+b*arctanh(c*x))^2/c^4+1/4*d^3*x^4*(a+b*arctanh(c*x))^2+3/5*c*d^3*x^5*(a+
b*arctanh(c*x))^2+1/2*c^2*d^3*x^6*(a+b*arctanh(c*x))^2+1/7*c^3*d^3*x^7*(a+b
*arctanh(c*x))^2-52/35*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4+11/10*b^
2*d^3*ln(-c^2*x^2+1)/c^4-26/35*b^2*d^3*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A] time = 1.46, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 266, 43, 5910, 260, 5948, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{26b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{35c^4}+\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{21}bc^2d^3x^6\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (3*a*b*d^3*x)/(2*c^3) + (122*b^2*d^3*x)/(105*c^3) + (7*b^2*d^3*x^2)/(20*c^2
) + (44*b^2*d^3*x^3)/(315*c) + (b^2*d^3*x^4)/20 + (b^2*c*d^3*x^5)/105 - (12
2*b^2*d^3*ArcTanh[c*x])/(105*c^4) + (3*b^2*d^3*x*ArcTanh[c*x])/(2*c^3) + (2
6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/(35*c^2) + (b*d^3*x^3*(a + b*ArcTanh[c*x]
))/(2*c) + (13*b*d^3*x^4*(a + b*ArcTanh[c*x]))/35 + (b*c*d^3*x^5*(a + b*Arc
Tanh[c*x]))/5 + (b*c^2*d^3*x^6*(a + b*ArcTanh[c*x]))/21 - (d^3*(a + b*ArcTa
nh[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c*d^3*x^5*(
a + b*ArcTanh[c*x])^2)/5 + (c^2*d^3*x^6*(a + b*ArcTanh[c*x])^2)/2 + (c^3*d^
3*x^7*(a + b*ArcTanh[c*x])^2)/7 - (52*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 -
 c*x)])/(35*c^4) + (11*b^2*d^3*Log[1 - c^2*x^2])/(10*c^4) - (26*b^2*d^3*Pol
yLog[2, 1 - 2/(1 - c*x)])/(35*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+3cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+3c^2d^3x^5\left(a\right.\right.$$

$$=d^3\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3cd^3\right)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx+$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^6\left(a\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^6\left(a\right.$$

$$=\frac{bd^3x^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{3}{10}bd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}bcd^3x^5\left(a+\right.$$

$$=\frac{abd^3x}{2c^3}+\frac{3bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^3x^3\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{13}{35}ba$$

$$=\frac{3abd^3x}{2c^3}+\frac{199b^2d^3x}{210c^3}+\frac{73b^2d^3x^3}{630c}+\frac{1}{105}b^2cd^3x^5+\frac{b^2d^3x\tanh^{-1}(cx)}{2c^3}+$$

$$=\frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{11b^2d^3x^2}{60c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5$$

$$=\frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5$$

$$=\frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5$$

Mathematica [A] time = 1.66, size = 385, normalized size = 0.93

$$d^3\left(180a^2c^7x^7+630a^2c^6x^6+756a^2c^5x^5+315a^2c^4x^4+60abc^6x^6+252abc^5x^5+468abc^4x^4+630abc^3x^3+936ab\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-1464*a*b - 504*b^2 + 1890*a*b*c*x + 1464*b^2*c*x + 936*a*b*c^2*x^2 +
 441*b^2*c^2*x^2 + 630*a*b*c^3*x^3 + 176*b^2*c^3*x^3 + 315*a^2*c^4*x^4 + 46
8*a*b*c^4*x^4 + 63*b^2*c^4*x^4 + 756*a^2*c^5*x^5 + 252*a*b*c^5*x^5 + 12*b^2
*c^5*x^5 + 630*a^2*c^6*x^6 + 60*a*b*c^6*x^6 + 180*a^2*c^7*x^7 + 9*b^2*(-209
 + 35*c^4*x^4 + 84*c^5*x^5 + 70*c^6*x^6 + 20*c^7*x^7)*ArcTanh[c*x]^2 + 6*b*
ArcTanh[c*x]*(3*a*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3) + b*(-244
 + 315*c*x + 156*c^2*x^2 + 105*c^3*x^3 + 78*c^4*x^4 + 42*c^5*x^5 + 10*c^6*x
^6) - 312*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 945*a*b*Log[1 - c*x] - 945*a*b*
Log[1 + c*x] + 1386*b^2*Log[1 - c^2*x^2] + 936*a*b*Log[-1 + c^2*x^2] + 936*
b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(1260*c^4)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^3d^3x^6 + 3\,a^2c^2d^3x^5 + 3\,a^2cd^3x^4 + a^2d^3x^3 + \left(b^2c^3d^3x^6 + 3\,b^2c^2d^3x^5 + 3\,b^2cd^3x^4 + b^2d^3x^3\right)\text{artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^6 + 3*a^2*c^2*d^3*x^5 + 3*a^2*c*d^3*x^4 + a^2*d^3*x^
3 + (b^2*c^3*d^3*x^6 + 3*b^2*c^2*d^3*x^5 + 3*b^2*c*d^3*x^4 + b^2*d^3*x^3)*a
rctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^6 + 3*a*b*c^2*d^3*x^5 + 3*a*b*c*d^3*x^4 +
a*b*d^3*x^3)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx + d)^3 (b\,\text{artanh}\,(cx) + a)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^3, x)
```

maple [A] time = 0.06, size = 662, normalized size = 1.60

$$\frac{3ab\,d^3x}{2c^3} + \frac{2c^3d^3ab\,\text{arctanh}\,(cx)\,x^7}{7} + c^2d^3ab\,\text{arctanh}\,(cx)\,x^6 + \frac{6c\,d^3ab\,\text{arctanh}\,(cx)\,x^5}{5} + \frac{13d^3ab\,x^4}{35} + \frac{353d^3b^2\ln(c}{210c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 3/2*a*b*d^3*x/c^3+3/2*b^2*d^3*x*arctanh(c*x)/c^3+6/5*c*d^3*a*b*arctanh(c*x)
*x^5+2/7*c^3*d^3*a*b*arctanh(c*x)*x^7+c^2*d^3*a*b*arctanh(c*x)*x^6+13/35*d^
3*a*b*x^4-26/35/c^4*d^3*b^2*dilog(1/2+1/2*c*x)+353/210/c^4*d^3*b^2*ln(c*x-1
)+209/560/c^4*d^3*b^2*ln(c*x-1)^2+1/560/c^4*d^3*b^2*ln(c*x+1)^2+109/210/c^4
*d^3*b^2*ln(c*x+1)+13/35*d^3*b^2*arctanh(c*x)*x^4+1/4*d^3*b^2*arctanh(c*x)^
2*x^4+1/7*c^3*d^3*a^2*x^7+1/2*c^2*d^3*a^2*x^6+3/5*c*d^3*a^2*x^5+1/20*b^2*d^
3*x^4+1/280/c^4*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+209/140/c^4*d^3*b^
2*arctanh(c*x)*ln(c*x-1)+1/5*c*d^3*a*b*x^5+1/2/c*d^3*a*b*x^3+26/35/c^2*d^3*
a*b*x^2+1/5*c*d^3*b^2*arctanh(c*x)*x^5+1/2*c^2*d^3*b^2*arctanh(c*x)^2*x^6+1
/7*c^3*d^3*b^2*arctanh(c*x)^2*x^7+26/35/c^2*d^3*b^2*arctanh(c*x)*x^2+3/5*c*
d^3*b^2*arctanh(c*x)^2*x^5+1/21*c^2*d^3*b^2*arctanh(c*x)*x^6+209/140/c^4*d^
3*a*b*ln(c*x-1)-1/140/c^4*d^3*a*b*ln(c*x+1)+1/21*c^2*d^3*a*b*x^6+1/2/c*d^3*
b^2*arctanh(c*x)*x^3-1/140/c^4*d^3*b^2*arctanh(c*x)*ln(c*x+1)+1/2*d^3*a*b*a
rctanh(c*x)*x^4+1/4*d^3*a^2*x^4-209/280/c^4*d^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*
```

```
x)-1/280/c^4*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+122/105*b^2*d^3*x/c^3+7/20*
b^2*d^3*x^2/c^2+44/315*b^2*d^3*x^3/c+1/105*b^2*c*d^3*x^5
```

maxima [B] time = 0.70, size = 928, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*a^2*c^3*d^3*x^7 + 1/2*a^2*c^2*d^3*x^6 + 3/5*a^2*c*d^3*x^5 + 1/4*b^2*d^3
*x^4*arctanh(c*x)^2 + 1/42*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4
 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*x^4 + 1/
30*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(
c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^3 + 3/10*(4*x^5*arctanh(c*x)
 + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*d^3 + 1/12*(6*
x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*
x - 1)/c^5))*a*b*d^3 + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^
5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*
log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*
b^2*d^3 + 26/35*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b
^2*d^3/c^4 + 13/70*b^2*d^3*log(c*x + 1)/c^4 + 283/210*b^2*d^3*log(c*x - 1)/
c^4 + 1/2520*(24*b^2*c^5*d^3*x^5 + 126*b^2*c^4*d^3*x^4 + 352*b^2*c^3*d^3*x^
3 + 672*b^2*c^2*d^3*x^2 + 2928*b^2*c*d^3*x + 9*(10*b^2*c^7*d^3*x^7 + 35*b^2
*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 17*b^2*d^3)*log(c*x + 1)^2 + 9*(10*b^2*
c^7*d^3*x^7 + 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 - 87*b^2*d^3)*log(-c*
x + 1)^2 + 12*(5*b^2*c^6*d^3*x^6 + 21*b^2*c^5*d^3*x^5 + 39*b^2*c^4*d^3*x^4
+ 35*b^2*c^3*d^3*x^3 + 78*b^2*c^2*d^3*x^2 + 105*b^2*c*d^3*x)*log(c*x + 1) -
 6*(10*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 78*b^2*c^4*d^3*x^4 + 70*b^2*c
^3*d^3*x^3 + 156*b^2*c^2*d^3*x^2 + 210*b^2*c*d^3*x + 3*(10*b^2*c^7*d^3*x^7
+ 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 17*b^2*d^3)*log(c*x + 1))*log(-
c*x + 1))/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{3} \left(\int a^{2} x^{3}\, dx + \int 3 a^{2} c x^{4}\, dx + \int 3 a^{2} c^{2} x^{5}\, dx + \int a^{2} c^{3} x^{6}\, dx + \int b^{2} x^{3} \operatorname{atanh}^{2}{\left(c x \right)}\, dx + \int 2 a b x^{3} \operatorname{atanh}{\left(c x \right)}\, dx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x**3, x) + Integral(3*a**2*c*x**4, x) + Integral(3*a**2
*c**2*x**5, x) + Integral(a**2*c**3*x**6, x) + Integral(b**2*x**3*atanh(c*x
)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(3*b**2*c*x**4*atan
h(c*x)**2, x) + Integral(3*b**2*c**2*x**5*atanh(c*x)**2, x) + Integral(b**2
*c**3*x**6*atanh(c*x)**2, x) + Integral(6*a*b*c*x**4*atanh(c*x), x) + Integ
ral(6*a*b*c**2*x**5*atanh(c*x), x) + Integral(2*a*b*c**3*x**6*atanh(c*x), x
))
```

3.85 $\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=377

$$\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{60c^3}-\frac{28bd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^3}+\frac{3}{5}c^2d^3x^5\left(a+b\tan\right.$$

```
[Out] 11/6*a*b*d^3*x/c^2+37/30*b^2*d^3*x/c^2+61/180*b^2*d^3*x^2/c+1/10*b^2*d^3*x^
3+1/60*b^2*c*d^3*x^4-37/30*b^2*d^3*arctanh(c*x)/c^3+11/6*b^2*d^3*x*arctanh(
c*x)/c^2+14/15*b*d^3*x^2*(a+b*arctanh(c*x))/c+11/18*b*d^3*x^3*(a+b*arctanh(
c*x))+3/10*b*c*d^3*x^4*(a+b*arctanh(c*x))+1/15*b*c^2*d^3*x^5*(a+b*arctanh(c
*x))+1/60*d^3*(a+b*arctanh(c*x))^2/c^3+1/3*d^3*x^3*(a+b*arctanh(c*x))^2+3/4
*c*d^3*x^4*(a+b*arctanh(c*x))^2+3/5*c^2*d^3*x^5*(a+b*arctanh(c*x))^2+1/6*c^
3*d^3*x^6*(a+b*arctanh(c*x))^2-28/15*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1)
)/c^3+113/90*b^2*d^3*ln(-c^2*x^2+1)/c^3-14/15*b^2*d^3*polylog(2,1-2/(-c*x+1
))/c^3
```

Rubi [A] time = 1.23, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 266, 43, 5910, 260, 5948, 302}

$$-\frac{14b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{15}bc^2d^3x^5\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (11*a*b*d^3*x)/(6*c^2) + (37*b^2*d^3*x)/(30*c^2) + (61*b^2*d^3*x^2)/(180*c)
 + (b^2*d^3*x^3)/10 + (b^2*c*d^3*x^4)/60 - (37*b^2*d^3*ArcTanh[c*x])/(30*c^
3) + (11*b^2*d^3*x*ArcTanh[c*x])/(6*c^2) + (14*b*d^3*x^2*(a + b*ArcTanh[c*x
]))/(15*c) + (11*b*d^3*x^3*(a + b*ArcTanh[c*x]))/18 + (3*b*c*d^3*x^4*(a + b
*ArcTanh[c*x]))/10 + (b*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/15 + (d^3*(a + b*
ArcTanh[c*x])^2)/(60*c^3) + (d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + (3*c*d^3*x
^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x])^2)/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x])^2)/6 - (28*b*d^3*(a + b*ArcTanh[c*x])*Log[
2/(1 - c*x)])/(15*c^3) + (113*b^2*d^3*Log[1 - c^2*x^2])/(90*c^3) - (14*b^2*
d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+3cd^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+3c^2d^3x^4\right.$$

$$=d^3\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3cd^3\right)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,d$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3x^5$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3x^5$$

$$=\frac{bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{2}bd^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{10}bcd^3x^4$$

$$=\frac{3abd^3x}{2c^2}+\frac{b^2d^3x}{3c^2}+\frac{14bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{11}{18}bd^3x^3\left(a+b\tan\right.$$

$$=\frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{1}{10}b^2d^3x^3-\frac{b^2d^3\tanh^{-1}(cx)}{3c^3}+\frac{3b^2d^3x\tanh^{-1}}{2c^2}$$

$$=\frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{17b^2d^3x^2}{60c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2d^3}{}$$

$$=\frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61b^2d^3x^2}{180c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2d^3}{}$$

Mathematica [A] time = 1.33, size = 356, normalized size = 0.94

$$d^3\left(30a^2c^6x^6+108a^2c^5x^5+135a^2c^4x^4+60a^2c^3x^3+12abc^5x^5+54abc^4x^4+110abc^3x^3+168abc^2x^2+168ab\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

[Out] (d^3*(-162*a*b - 64*b^2 + 330*a*b*c*x + 222*b^2*c*x + 168*a*b*c^2*x^2 + 61*
b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^3 + 135*a^2*c
^4*x^4 + 54*a*b*c^4*x^4 + 3*b^2*c^4*x^4 + 108*a^2*c^5*x^5 + 12*a*b*c^5*x^5
+ 30*a^2*c^6*x^6 + 3*b^2*(-111 + 20*c^3*x^3 + 45*c^4*x^4 + 36*c^5*x^5 + 10*
c^6*x^6)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*c^3*x^3*(20 + 45*c*x + 36*c
^2*x^2 + 10*c^3*x^3) + b*(-111 + 165*c*x + 84*c^2*x^2 + 55*c^3*x^3 + 27*c^4
*x^4 + 6*c^5*x^5) - 168*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 165*a*b*Log[1 - c
*x] - 165*a*b*Log[1 + c*x] + 226*b^2*Log[1 - c^2*x^2] + 168*a*b*Log[-1 + c^
2*x^2] + 168*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(180*c^3)

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^3d^3x^5 + 3\,a^2c^2d^3x^4 + 3\,a^2cd^3x^3 + a^2d^3x^2 + \left(b^2c^3d^3x^5 + 3\,b^2c^2d^3x^4 + 3\,b^2cd^3x^3 + b^2d^3x^2\right)\text{artanh}\,(cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")

[Out] integral(a^2*c^3*d^3*x^5 + 3*a^2*c^2*d^3*x^4 + 3*a^2*c*d^3*x^3 + a^2*d^3*x^
2 + (b^2*c^3*d^3*x^5 + 3*b^2*c^2*d^3*x^4 + 3*b^2*c*d^3*x^3 + b^2*d^3*x^2)*a
rctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^5 + 3*a*b*c^2*d^3*x^4 + 3*a*b*c*d^3*x^3 +
a*b*d^3*x^2)*arctanh(c*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx + d)^3(b\,\text{artanh}\,(cx) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")

[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^2, x)

maple [A] time = 0.06, size = 618, normalized size = 1.64

$$\frac{11ab\,d^3x}{6c^2}+\frac{d^3b^2\,\text{arctanh}\,(cx)^2\,x^3}{3}+\frac{11d^3b^2\,\text{arctanh}\,(cx)\,x^3}{18}-\frac{14d^3b^2\,\text{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{15c^3}+\frac{11d^3ab\,x^3}{18}+\frac{3c\,d^3a^2x^4}{4}-\frac{d^3b^2\ln}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)

[Out] 11/6*a*b*d^3*x/c^2+11/6*b^2*d^3*x*arctanh(c*x)/c^2+11/18*d^3*a*b*x^3+3/4*c*
d^3*a^2*x^4-1/240/c^3*d^3*b^2*ln(c*x+1)^2+1/3*d^3*b^2*arctanh(c*x)^2*x^3+11
/18*d^3*b^2*arctanh(c*x)*x^3+23/36/c^3*d^3*b^2*ln(c*x+1)-14/15/c^3*d^3*b^2*
dilog(1/2+1/2*c*x)+37/80/c^3*d^3*b^2*ln(c*x-1)^2+337/180/c^3*d^3*b^2*ln(c*x
-1)+1/6*c^3*d^3*a^2*x^6+3/5*c^2*d^3*a^2*x^5+6/5*c^2*d^3*a*b*arctanh(c*x)*x^
5+1/3*c^3*d^3*a*b*arctanh(c*x)*x^6+3/2*c*d^3*a*b*arctanh(c*x)*x^4+1/10*b^2*
d^3*x^3+1/60/c^3*d^3*b^2*arctanh(c*x)*ln(c*x+1)+1/60/c^3*d^3*a*b*ln(c*x+1)+
37/20/c^3*d^3*a*b*ln(c*x-1)+1/15*c^2*d^3*b^2*arctanh(c*x)*x^5+2/3*d^3*a*b*a
rctanh(c*x)*x^3+1/15*c^2*d^3*a*b*x^5+3/10*c*d^3*a*b*x^4+14/15/c*d^3*a*b*x^2
+14/15/c*d^3*b^2*arctanh(c*x)*x^2+3/5*c^2*d^3*b^2*arctanh(c*x)^2*x^5+3/4*c*
d^3*b^2*arctanh(c*x)^2*x^4+3/10*c*d^3*b^2*arctanh(c*x)*x^4-37/40/c^3*d^3*b^
2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/120/c^3*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/
120/c^3*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+37/20/c^3*d^3*b^2*arctanh(
c*x)*ln(c*x-1)+1/6*c^3*d^3*b^2*arctanh(c*x)^2*x^6+1/3*d^3*a^2*x^3+37/30*b^2
*d^3*x/c^2+61/180*b^2*d^3*x^2/c+1/60*b^2*c*d^3*x^4

maxima [B] time = 0.70, size = 775, normalized size = 2.06

$$\frac{1}{6}\,a^2c^3d^3x^6+\frac{3}{5}\,a^2c^2d^3x^5+\frac{3}{4}\,a^2cd^3x^4+\frac{1}{90}\left(30\,x^6\,\text{artanh}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")

[Out] 1/6*a^2*c^3*d^3*x^6 + 3/5*a^2*c^2*d^3*x^5 + 3/4*a^2*c*d^3*x^4 + 1/90*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^3*d^3 + 3/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3*x^3 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c*d^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*d^3 + 14/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d^3/c^3 + 23/36*b^2*d^3*log(c*x + 1)/c^3 + 337/180*b^2*d^3*log(c*x - 1)/c^3 + 1/720*(12*b^2*c^4*d^3*x^4 + 72*b^2*c^3*d^3*x^3 + 244*b^2*c^2*d^3*x^2 + 888*b^2*c*d^3*x + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 + b^2*d^3)*log(c*x + 1)^2 + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 - 111*b^2*d^3)*log(-c*x + 1)^2 + 4*(6*b^2*c^5*d^3*x^5 + 27*b^2*c^4*d^3*x^4 + 55*b^2*c^3*d^3*x^3 + 84*b^2*c^2*d^3*x^2 + 165*b^2*c*d^3*x)*log(c*x + 1) - 2*(12*b^2*c^5*d^3*x^5 + 54*b^2*c^4*d^3*x^4 + 110*b^2*c^3*d^3*x^3 + 168*b^2*c^2*d^3*x^2 + 330*b^2*c*d^3*x + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 + b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c^3

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)

[Out] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int a^2x^2\,dx + \int 3a^2cx^3\,dx + \int 3a^2c^2x^4\,dx + \int a^2c^3x^5\,dx + \int b^2x^2\operatorname{atanh}^2(cx)\,dx + \int 2abx^2\operatorname{atanh}(cx)\,d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)

[Out] d**3*(Integral(a**2*x**2, x) + Integral(3*a**2*c*x**3, x) + Integral(3*a**2*c**2*x**4, x) + Integral(a**2*c**3*x**5, x) + Integral(b**2*x**2*atanh(c*x)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(3*b**2*c*x**3*atanh(c*x)**2, x) + Integral(3*b**2*c**2*x**4*atanh(c*x)**2, x) + Integral(b**2*c**3*x**5*atanh(c*x)**2, x) + Integral(6*a*b*c*x**3*atanh(c*x), x) + Integral(6*a*b*c**2*x**4*atanh(c*x), x) + Integral(2*a*b*c**3*x**5*atanh(c*x), x))

3.86 $\int x(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=286

$$\frac{1}{10}bc^2d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}-\frac{12bd^3\log\left(\frac{2}{1-cx}\right.}{}$$

```
[Out] 5/2*a*b*d^3*x/c+13/10*b^2*d^3*x/c+1/4*b^2*d^3*x^2+1/30*b^2*c*d^3*x^3-13/10*
b^2*d^3*arctanh(c*x)/c^2+5/2*b^2*d^3*x*arctanh(c*x)/c+6/5*b*d^3*x^2*(a+b*ar
ctanh(c*x))+1/2*b*c*d^3*x^3*(a+b*arctanh(c*x))+1/10*b*c^2*d^3*x^4*(a+b*arct
anh(c*x))-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/c^2+1/5*d^3*(c*x+1)^5*(a+b
*arctanh(c*x))^2/c^2-12/5*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2+3/2*b
^2*d^3*ln(-c^2*x^2+1)/c^2-6/5*b^2*d^3*polylog(2,1-2/(-c*x+1))/c^2
```

Rubi [A] time = 0.60, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5940, 5928, 5910, 260, 5916, 321, 206, 266, 43, 1586, 5918, 2402, 2315, 302}

$$-\frac{6b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^2}+\frac{1}{10}bc^2d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (5*a*b*d^3*x)/(2*c) + (13*b^2*d^3*x)/(10*c) + (b^2*d^3*x^2)/4 + (b^2*c*d^3*
x^3)/30 - (13*b^2*d^3*ArcTanh[c*x])/(10*c^2) + (5*b^2*d^3*x*ArcTanh[c*x])/(
2*c) + (6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/5 + (b*c*d^3*x^3*(a + b*ArcTanh[c
*x]))/2 + (b*c^2*d^3*x^4*(a + b*ArcTanh[c*x]))/10 - (d^3*(1 + c*x)^4*(a + b
*ArcTanh[c*x])^2)/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x])^2)/(5*c^2
) - (12*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^2) + (3*b^2*d^3*L
og[1 - c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int x(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(-\frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{cd}\right)$$

$$=-\frac{\int(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c}+\frac{\int(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{cd}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{(2}{}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{(3}{}$$

$$=\frac{5abd^3x}{2c}+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}bcd^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{10}$$

$$=\frac{5abd^3x}{2c}+\frac{6b^2d^3x}{5c}+\frac{5b^2d^3x\tanh^{-1}(cx)}{2c}+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}$$

$$=\frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{30}b^2cd^3x^3-\frac{6b^2d^3\tanh^{-1}(cx)}{5c^2}+\frac{5b^2d^3x\tanh^{-1}(c}{2c}$$

$$=\frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{4}b^2d^3x^2+\frac{1}{30}b^2cd^3x^3-\frac{13b^2d^3\tanh^{-1}(cx)}{10c^2}+\frac{5b^2d}{}$$

Mathematica [A] time = 1.21, size = 325, normalized size = 1.14

$$\frac{d^3\left(12a^2c^5x^5+45a^2c^4x^4+60a^2c^3x^3+30a^2c^2x^2+6abc^4x^4+30abc^3x^3+72abc^2x^2+72ab\log\left(c^2x^2-1\right)+6b\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-18*a*b - 15*b^2 + 150*a*b*c*x + 78*b^2*c*x + 30*a^2*c^2*x^2 + 72*a*b
*c^2*x^2 + 15*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 30*a*b*c^3*x^3 + 2*b^2*c^3*x^3
 + 45*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 12*a^2*c^5*x^5 + 3*b^2*(-49 + 10*c^2*x^
2 + 20*c^3*x^3 + 15*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c*x]*
(a*c^2*x^2*(10 + 20*c*x + 15*c^2*x^2 + 4*c^3*x^3) + b*(-13 + 25*c*x + 12*c^
2*x^2 + 5*c^3*x^3 + c^4*x^4) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 75*a*b*
Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 90*b^2*Log[1 - c^2*x^2] + 72*a*b*Log[-
1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^3d^3x^4+3\,a^2c^2d^3x^3+3\,a^2cd^3x^2+a^2d^3x+\left(b^2c^3d^3x^4+3\,b^2c^2d^3x^3+3\,b^2cd^3x^2+b^2d^3x\right)\operatorname{artanh}(cx)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^4 + 3*a^2*c^2*d^3*x^3 + 3*a^2*c*d^3*x^2 + a^2*d^3*x
+ (b^2*c^3*d^3*x^4 + 3*b^2*c^2*d^3*x^3 + 3*b^2*c*d^3*x^2 + b^2*d^3*x)*arcta
nh(c*x)^2 + 2*(a*b*c^3*d^3*x^4 + 3*a*b*c^2*d^3*x^3 + 3*a*b*c*d^3*x^2 + a*b*
d^3*x)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx + d)^3 (b \operatorname{artanh}(cx) + a)^2 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x, x)
```

maple [B] time = 0.06, size = 570, normalized size = 1.99

$$\frac{5ab\,d^3 x}{2c}+\frac{d^3 b^2 \operatorname{arctanh}(cx)^2 x^2}{2}+\frac{6d^3 b^2 \operatorname{arctanh}(cx)\, x^2}{5}-\frac{6d^3 b^2 \operatorname{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{5c^2}+\frac{49d^3 b^2 \ln(cx-1)^2}{80c^2}+\frac{17d^3 b^2 \ln}{20c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 5/2*a*b*d^3*x/c+5/2*b^2*d^3*x*arctanh(c*x)/c+49/80/c^2*d^3*b^2*ln(c*x-1)^2+
17/20/c^2*d^3*b^2*ln(c*x+1)+1/80/c^2*d^3*b^2*ln(c*x+1)^2+1/2*d^3*b^2*arctan
h(c*x)^2*x^2+6/5*d^3*b^2*arctanh(c*x)*x^2+1/5*c^3*d^3*a^2*x^5+3/4*c^2*d^3*a
^2*x^4+c*d^3*a^2*x^3+6/5*d^3*a*b*x^2-6/5/c^2*d^3*b^2*dilog(1/2+1/2*c*x)+43/
20/c^2*d^3*b^2*ln(c*x-1)+3/2*c^2*d^3*a*b*arctanh(c*x)*x^4+2/5*c^3*d^3*a*b*a
rctanh(c*x)*x^5+2*c*d^3*a*b*arctanh(c*x)*x^3+1/4*b^2*d^3*x^2+3/4*c^2*d^3*b^
2*arctanh(c*x)^2*x^4+1/2*c*d^3*b^2*arctanh(c*x)*x^3+1/5*c^3*d^3*b^2*arctanh
(c*x)^2*x^5+1/10*c^2*d^3*b^2*arctanh(c*x)*x^4-49/40/c^2*d^3*b^2*ln(c*x-1)*l
n(1/2+1/2*c*x)-1/40/c^2*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/40/c^2*d^3*b^2
*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+49/20/c^2*d^3*b^2*arctanh(c*x)*ln(c*x-1)-
1/20/c^2*d^3*b^2*arctanh(c*x)*ln(c*x+1)+1/10*c^2*d^3*a*b*x^4+1/2*c*d^3*a*b*
x^3+d^3*a*b*arctanh(c*x)*x^2-1/20/c^2*d^3*a*b*ln(c*x+1)+49/20/c^2*d^3*a*b*l
n(c*x-1)+c*d^3*b^2*arctanh(c*x)^2*x^3+1/2*d^3*a^2*x^2+13/10*b^2*d^3*x/c+1/3
0*b^2*c*d^3*x^3
```

maxima [B] time = 0.70, size = 780, normalized size = 2.73

$$\frac{1}{5}a^2c^3d^3x^5+\frac{3}{4}a^2c^2d^3x^4+\frac{1}{10}\left(4x^5 \operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)abc^3d^3+a^2cd^3x^3+\frac{1}{2}b^2d^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^3*d^3*x^5 + 3/4*a^2*c^2*d^3*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*((
c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^3*d^3 + a^2*c*d^3*x^3
 + 1/2*b^2*d^3*x^2*arctanh(c*x)^2 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3
 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^2*d^3 + (2*x^
3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d^3 + 1/2*a^2*d^
3*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x -
 1)/c^3))*a*b*d^3 + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3
)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(
```

```
c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2*d^3 + 6/5*(log(c*x + 1)*log(-1/2*c*x
+ 1/2) + dilog(1/2*c*x + 1/2))*b^2*d^3/c^2 + 7/20*b^2*d^3*log(c*x + 1)/c^2
+ 33/20*b^2*d^3*log(c*x - 1)/c^2 + 1/240*(8*b^2*c^3*d^3*x^3 + 60*b^2*c^2*d^
3*x^2 + 312*b^2*c*d^3*x + 3*(4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3*x^4 + 20*b^
2*c^3*d^3*x^3 + 9*b^2*d^3)*log(c*x + 1)^2 + 3*(4*b^2*c^5*d^3*x^5 + 15*b^2*c
^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 - 39*b^2*d^3)*log(-c*x + 1)^2 + 12*(b^2*c^4
*d^3*x^4 + 5*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3*x^2 + 15*b^2*c*d^3*x)*log(c*x
 + 1) - 6*(2*b^2*c^4*d^3*x^4 + 10*b^2*c^3*d^3*x^3 + 24*b^2*c^2*d^3*x^2 + 30
*b^2*c*d^3*x + (4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3
 + 9*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{atanh}\,(c\,x))^2\,(d+c\,d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

```
[Out] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int a^2x\,dx+\int 3a^2cx^2\,dx+\int 3a^2c^2x^3\,dx+\int a^2c^3x^4\,dx+\int b^2x\,\mathrm{atanh}^2\,(cx)\,dx+\int 2abx\,\mathrm{atanh}\,(cx)\,dx+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x, x) + Integral(3*a**2*c*x**2, x) + Integral(3*a**2*c*
*2*x**3, x) + Integral(a**2*c**3*x**4, x) + Integral(b**2*x*atanh(c*x)**2,
x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(3*b**2*c*x**2*atanh(c*x)**2
, x) + Integral(3*b**2*c**2*x**3*atanh(c*x)**2, x) + Integral(b**2*c**3*x**
4*atanh(c*x)**2, x) + Integral(6*a*b*c*x**2*atanh(c*x), x) + Integral(6*a*b
*c**2*x**3*atanh(c*x), x) + Integral(2*a*b*c**3*x**4*atanh(c*x), x))
```

3.87 $\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=206

$$\frac{1}{6}bc^2d^3x^3\left(a+b\tanh^{-1}(cx)\right)+bcd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{4bd^3\log\left(\frac{2}{1-cx}\right)\left(a\right.}{c}$$

```
[Out] 7/2*a*b*d^3*x+b^2*d^3*x+1/12*b^2*c*d^3*x^2-b^2*d^3*arctanh(c*x)/c+7/2*b^2*d
^3*x*arctanh(c*x)+b*c*d^3*x^2*(a+b*arctanh(c*x))+1/6*b*c^2*d^3*x^3*(a+b*arc
tanh(c*x))+1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/c-4*b*d^3*(a+b*arctanh(c*
x))*ln(2/(-c*x+1))/c+11/6*b^2*d^3*ln(-c^2*x^2+1)/c-2*b^2*d^3*polylog(2,1-2/
(-c*x+1))/c
```

Rubi [A] time = 0.21, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {5928, 5910, 260, 5916, 321, 206, 266, 43, 1586, 5918, 2402, 2315}

$$-\frac{2b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{1}{6}bc^2d^3x^3\left(a+b\tanh^{-1}(cx)\right)+bcd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^4\left(a+b\tan\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (7*a*b*d^3*x)/2 + b^2*d^3*x + (b^2*c*d^3*x^2)/12 - (b^2*d^3*ArcTanh[c*x])/c
 + (7*b^2*d^3*x*ArcTanh[c*x])/2 + b*c*d^3*x^2*(a + b*ArcTanh[c*x]) + (b*c^2
*d^3*x^3*(a + b*ArcTanh[c*x]))/6 + (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^2)
/(4*c) - (4*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (11*b^2*d^3*Lo
g[1 - c^2*x^2])/(6*c) - (2*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{b\int \left(-7d^4\left(a+b\tanh^{-1}(cx)\right) - 4cd^4\right.}{}$$

$$= \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{(4b)\int \frac{\left(d^4+cd^4x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx}{d} + \frac{1}{2}$$

$$= \frac{7}{2}abd^3x + bcd^3x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{1}{6}bc^2d^3x^3\left(a+b\tanh^{-1}(cx)\right) +$$

$$= \frac{7}{2}abd^3x + b^2d^3x + \frac{7}{2}b^2d^3x\tanh^{-1}(cx) + bcd^3x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{1}{6}$$

$$= \frac{7}{2}abd^3x + b^2d^3x - \frac{b^2d^3\tanh^{-1}(cx)}{c} + \frac{7}{2}b^2d^3x\tanh^{-1}(cx) + bcd^3x^2\left(a+\right.$$

$$= \frac{7}{2}abd^3x + b^2d^3x + \frac{1}{12}b^2cd^3x^2 - \frac{b^2d^3\tanh^{-1}(cx)}{c} + \frac{7}{2}b^2d^3x\tanh^{-1}(cx)$$

Mathematica [A] time = 0.88, size = 293, normalized size = 1.42

$$\frac{d^3\left(3a^2c^4x^4 + 12a^2c^3x^3 + 18a^2c^2x^2 + 12a^2cx + 2abc^3x^3 + 12abc^2x^2 + 12ab\log\left(1-c^2x^2\right) + 12ab\log\left(c^2x^2-1\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-b^2 + 12*a^2*c*x + 42*a*b*c*x + 12*b^2*c*x + 18*a^2*c^2*x^2 + 12*a*b
*c^2*x^2 + b^2*c^2*x^2 + 12*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + 3
*b^2*(-15 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 + c^4*x^4)*ArcTanh[c*x]^2 + 2*b*A
rcTanh[c*x]*(3*a*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3) + b*(-6 + 21*c*x + 6
*c^2*x^2 + c^3*x^3) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 21*a*b*Log[1 - c
*x] - 21*a*b*Log[1 + c*x] + 12*a*b*Log[1 - c^2*x^2] + 22*b^2*Log[1 - c^2*x^
2] + 12*a*b*Log[-1 + c^2*x^2] + 24*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(
12*c)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\text{artanh}(cx)^2 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (b
^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)^
2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arcta
nh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cdx+d)^3 (b\,\text{artanh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2, x)
```

maple [B] time = 0.06, size = 462, normalized size = 2.24

$$b^2d^3x+\frac{c^3d^3ab\operatorname{arctanh}(cx)\,x^4}{2}+d^3b^2\operatorname{arctanh}(cx)^2\,x+\frac{d^3b^2\operatorname{arctanh}(cx)^2}{4c}-\frac{2d^3b^2\operatorname{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{c}+\frac{7b^2d^3x\operatorname{arctanh}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] b^2*d^3*x+1/2*c^3*d^3*a*b*arctanh(c*x)*x^4+2*c^2*d^3*a*b*arctanh(c*x)*x^3+3
*c*d^3*a*b*arctanh(c*x)*x^2+3/2*c*d^3*a^2*x^2+c^2*d^3*a^2*x^3+1/c*d^3*b^2*l
n(c*x-1)^2+1/4*c^3*d^3*a^2*x^4+d^3*b^2*arctanh(c*x)^2*x+7/3/c*d^3*b^2*ln(c*
x-1)+1/4/c*d^3*b^2*arctanh(c*x)^2+4/3/c*d^3*b^2*ln(c*x+1)-2/c*d^3*b^2*dilog
(1/2+1/2*c*x)+7/2*b^2*d^3*x*arctanh(c*x)-13/12/c*d^3*b^2+1/4/c*d^3*a^2+4/c*
d^3*b^2*arctanh(c*x)*ln(c*x-1)+1/2/c*d^3*a*b*arctanh(c*x)+4/c*d^3*a*b*ln(c*
x-1)+c^2*d^3*b^2*arctanh(c*x)^2*x^3+a^2*x*d^3+2*d^3*a*b*arctanh(c*x)*x+1/6*
c^2*d^3*b^2*arctanh(c*x)*x^3+1/6*c^2*d^3*a*b*x^3+c*d^3*a*b*x^2+c*d^3*b^2*ar
ctanh(c*x)*x^2+1/4*c^3*d^3*b^2*arctanh(c*x)^2*x^4+3/2*c*d^3*b^2*arctanh(c*x
)^2*x^2-2/c*d^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+7/2*a*b*d^3*x+1/12*b^2*c*d^3*
x^2
```

maxima [B] time = 0.56, size = 627, normalized size = 3.04

$$\frac{1}{4}a^2c^3d^3x^4+a^2c^2d^3x^3+\frac{1}{12}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2\left(c^2x^3+3x\right)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)abc^3d^3+\left(2x^3\operatorname{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^3*d^3*x^4 + a^2*c^2*d^3*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^
2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^3*d^3 +
(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c^2*d^3 + 3/2
*a^2*c*d^3*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
log(c*x - 1)/c^3))*a*b*c*d^3 + a^2*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x
^2 + 1))*a*b*d^3/c + 2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x +
1/2))*b^2*d^3/c + 4/3*b^2*d^3*log(c*x + 1)/c + 7/3*b^2*d^3*log(c*x - 1)/c +
 1/48*(4*b^2*c^2*d^3*x^2 + 48*b^2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*
d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c*d^3*x + b^2*d^3)*log(c*x + 1)^2 + 3*(
b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c*d^3*x - 1
5*b^2*d^3)*log(-c*x + 1)^2 + 4*(b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 21*b^
2*c*d^3*x)*log(c*x + 1) - 2*(2*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3*x^2 + 42*b^
2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*
b^2*c*d^3*x + b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int a^2\,dx+\int b^2\operatorname{atanh}^2(cx)\,dx+\int 2ab\operatorname{atanh}(cx)\,dx+\int 3a^2cx\,dx+\int 3a^2c^2x^2\,dx+\int a^2c^3x^3\,dx+\int 3b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*
atanh(c*x), x) + Integral(3*a**2*c*x, x) + Integral(3*a**2*c**2*x**2, x) +
Integral(a**2*c**3*x**3, x) + Integral(3*b**2*c*x*atanh(c*x)**2, x) + Integ
ral(3*b**2*c**2*x**2*atanh(c*x)**2, x) + Integral(b**2*c**3*x**3*atanh(c*x)
**2, x) + Integral(6*a*b*c*x*atanh(c*x), x) + Integral(6*a*b*c**2*x**2*atan
h(c*x), x) + Integral(2*a*b*c**3*x**3*atanh(c*x), x))
```

3.88 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=355

$$\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)-bd^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a\right.$$

```
[Out] 3*a*b*c*d^3*x+1/3*b^2*c*d^3*x-1/3*b^2*d^3*arctanh(c*x)+3*b^2*c*d^3*x*arctan
h(c*x)+1/3*b*c^2*d^3*x^2*(a+b*arctanh(c*x))+11/6*d^3*(a+b*arctanh(c*x))^2+3
*c*d^3*x*(a+b*arctanh(c*x))^2+3/2*c^2*d^3*x^2*(a+b*arctanh(c*x))^2+1/3*c^3*
d^3*x^3*(a+b*arctanh(c*x))^2-2*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+
1))-20/3*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))+3/2*b^2*d^3*ln(-c^2*x^2+1)
-10/3*b^2*d^3*polylog(2,1-2/(-c*x+1))-b*d^3*(a+b*arctanh(c*x))*polylog(2,1-
2/(-c*x+1))+b*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*d^3*p
olylog(3,1-2/(-c*x+1))-1/2*b^2*d^3*polylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.81, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 260, 321, 206}

$$-bd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{10}{3}b^2d^3\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] 3*a*b*c*d^3*x + (b^2*c*d^3*x)/3 - (b^2*d^3*ArcTanh[c*x])/3 + 3*b^2*c*d^3*x*
ArcTanh[c*x] + (b*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/3 + (11*d^3*(a + b*ArcT
anh[c*x])^2)/6 + 3*c*d^3*x*(a + b*ArcTanh[c*x])^2 + (3*c^2*d^3*x^2*(a + b*A
rcTanh[c*x])^2)/2 + (c^3*d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + 2*d^3*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - (20*b*d^3*(a + b*ArcTanh[c*x])*Lo
g[2/(1 - c*x)])/3 + (3*b^2*d^3*Log[1 - c^2*x^2])/2 - (10*b^2*d^3*PolyLog[2,
 1 - 2/(1 - c*x)])/3 - b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x
)] + b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^3*Pol
yLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(3cd^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x\left(a+b\tan\right.\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx + \left(3cd^3\right)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx + \left(3c^2d^3\right.$$

$$= 3cd^3x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}c^3d^3x^3\left(a+\right.$$

$$= 3d^3\left(a+b\tanh^{-1}(cx)\right)^2 + 3cd^3x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2}c^2d^3x^2\left(a+b\tan\right.$$

$$= 3abcd^3x + \frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{11}{6}d^3\left(a+b\tanh^{-1}(cx)\right)^2 + 3cd$$

$$= 3abcd^3x + \frac{1}{3}b^2cd^3x + 3b^2cd^3x\tanh^{-1}(cx) + \frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)$$

$$= 3abcd^3x + \frac{1}{3}b^2cd^3x - \frac{1}{3}b^2d^3\tanh^{-1}(cx) + 3b^2cd^3x\tanh^{-1}(cx) + \frac{1}{3}bc^2d^3x^2$$

$$= 3abcd^3x + \frac{1}{3}b^2cd^3x - \frac{1}{3}b^2d^3\tanh^{-1}(cx) + 3b^2cd^3x\tanh^{-1}(cx) + \frac{1}{3}bc^2d^3x^2$$

Mathematica [C] time = 0.83, size = 448, normalized size = 1.26

$$\frac{1}{24}d^3\left(8a^2c^3x^3 + 36a^2c^2x^2 + 72a^2cx + 24a^2\log(cx) + 16abc^3x^3\tanh^{-1}(cx) + 8abc^2x^2 + 72ab\log\left(1-c^2x^2\right) + 8ab\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] (d^3*(I*b^2*Pi^3 + 72*a^2*c*x + 72*a*b*c*x + 8*b^2*c*x + 36*a^2*c^2*x^2 + 8
*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - 8*b^2*ArcTanh[c*x] + 144*a*b*c*x*ArcTanh[c*x
] + 72*b^2*c*x*ArcTanh[c*x] + 72*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^2*A
rcTanh[c*x] + 16*a*b*c^3*x^3*ArcTanh[c*x] - 116*b^2*ArcTanh[c*x]^2 + 72*b^2
*c*x*ArcTanh[c*x]^2 + 36*b^2*c^2*x^2*ArcTanh[c*x]^2 + 8*b^2*c^3*x^3*ArcTanh
[c*x]^2 - 16*b^2*ArcTanh[c*x]^3 - 160*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTan
h[c*x])] - 24*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*ArcT
anh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*Log[c*x] + 36*a*b*Log[1 - c
*x] - 36*a*b*Log[1 + c*x] + 72*a*b*Log[1 - c^2*x^2] + 36*b^2*Log[1 - c^2*x^
2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*(10 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-
2*ArcTanh[c*x])] + 24*b^2*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*
a*b*PolyLog[2, -(c*x)] + 24*a*b*PolyLog[2, c*x] + 12*b^2*PolyLog[3, -E^(-2*
ArcTanh[c*x])] - 12*b^2*PolyLog[3, E^(2*ArcTanh[c*x])]))/24
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\operatorname{artanh}\left(cx\right)^2 + \cdots}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3 (b \operatorname{artanh}(cx) + a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x, x)
```

maple [C] time = 1.53, size = 1186, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x)
```

```
[Out] 3/2*d^3*a^2*c^2*x^2-1/3*d^3*b^2+3*a*b*c*d^3*x+3*b^2*c*d^3*x*arctanh(c*x)-1/
2*I*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*d^3*b^2*Pi*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+29/6*d^
3*a*b*ln(c*x-1)+11/6*d^3*a*b*ln(c*x+1)+1/3*d^3*a^2*c^3*x^3+1/3*b^2*c*d^3*x+
8/3*b^2*d^3*arctanh(c*x)+3*d^3*b^2*arctanh(c*x)^2*c*x+1/3*d^3*a*b*c^2*x^2+3
/2*d^3*b^2*arctanh(c*x)^2*c^2*x^2+1/3*d^3*b^2*arctanh(c*x)^2*c^3*x^3+1/3*d^
3*b^2*arctanh(c*x)*c^2*x^2+2*d^3*a*b*arctanh(c*x)*ln(c*x)-d^3*a*b*ln(c*x)*l
n(c*x+1)-1/2*I*d^3*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+d^3*a^2*ln
```

```
(c*x)+11/6*d^3*b^2*arctanh(c*x)^2+3*x*a^2*c*d^3-d^3*a*b*dilog(c*x+1)+d^3*b^
2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+d^3*b^2*arctanh(c*x)^2*ln
(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c
^2*x^2+1)^(1/2))+d^3*b^2*arctanh(c*x)^2*ln(c*x)-20/3*d^3*b^2*arctanh(c*x)*l
n(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/
(-c^2*x^2+1)^(1/2))-d^3*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+2*d
^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-d^3*b^2*arctanh(
c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-d^3*a*b*dilog(c*x)-20/3*d^3*b^2*dil
og(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*d^3*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2
+1)^(1/2))+1/2*d^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-2*d^3*b^2*polylog
(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^3*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1
/2))-3*d^3*b^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))+6*d^3*a*b*arctanh(c*x)*c*x+3*d^
3*a*b*arctanh(c*x)*c^2*x^2+2/3*d^3*a*b*arctanh(c*x)*c^3*x^3+1/2*I*d^3*b^2*P
i*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c
*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2c^3d^3x^3+\frac{3}{2}a^2c^2d^3x^2+3a^2cd^3x+3\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)abd^3+a^2d^3\log(x)+\frac{1}{24}\left(2b^2c^3d^3x^3+9b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^3*d^3*x^3 + 3/2*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + 3*(2*c*x*arctan
h(c*x) + log(-c^2*x^2 + 1))*a*b*d^3 + a^2*d^3*log(x) + 1/24*(2*b^2*c^3*d^3*
x^3 + 9*b^2*c^2*d^3*x^2 + 18*b^2*c*d^3*x)*log(-c*x + 1)^2 - integrate(-1/12
*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x
 + 1)^2 + 12*(a*b*c^4*d^3*x^4 + 2*a*b*c^3*d^3*x^3 - 3*a*b*c^2*d^3*x^2 + a*b
*c*d^3*x - a*b*d^3)*log(c*x + 1) - (12*a*b*c*d^3*x - 12*a*b*d^3 + 2*(6*a*b*
c^4*d^3 + b^2*c^4*d^3)*x^4 + 3*(8*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 - 18*(2*
a*b*c^2*d^3 - b^2*c^2*d^3)*x^2 + 6*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2
*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^2 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3a^2c\,dx+\int\frac{a^2}{x}\,dx+\int 3a^2c^2x\,dx+\int a^2c^3x^2\,dx+\int 3b^2c\operatorname{atanh}^2(cx)\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{x}\,dx+\int 6ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(3*a**2*c, x) + Integral(a**2/x, x) + Integral(3*a**2*c**2*x,
 x) + Integral(a**2*c**3*x**2, x) + Integral(3*b**2*c*atanh(c*x)**2, x) + I
ntegral(b**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x), x) + Integr
al(2*a*b*atanh(c*x)/x, x) + Integral(3*b**2*c**2*x*atanh(c*x)**2, x) + Inte
gral(b**2*c**3*x**2*atanh(c*x)**2, x) + Integral(6*a*b*c**2*x*atanh(c*x), x
) + Integral(2*a*b*c**3*x**2*atanh(c*x), x))
```

3.89 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=361

$$\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+abc^2d^3x+3c^2d^3x\left(a+b\tanh^{-1}(cx)\right)^2-3bcd^3\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+$$

```
[Out] a*b*c^2*d^3*x+b^2*c^2*d^3*x*arctanh(c*x)+7/2*c*d^3*(a+b*arctanh(c*x))^2-d^3
*(a+b*arctanh(c*x))^2/x+3*c^2*d^3*x*(a+b*arctanh(c*x))^2+1/2*c^3*d^3*x^2*(a
+b*arctanh(c*x))^2-6*c*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-6*b*
c*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))+1/2*b^2*c*d^3*ln(-c^2*x^2+1)+2*b*c*
d^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-3*b^2*c*d^3*polylog(2,1-2/(-c*x+1))-
3*b*c*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+3*b*c*d^3*(a+b*arctanh
(c*x))*polylog(2,-1+2/(-c*x+1))-b^2*c*d^3*polylog(2,-1+2/(c*x+1))+3/2*b^2*c
*d^3*polylog(3,1-2/(-c*x+1))-3/2*b^2*c*d^3*polylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.78, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5980, 260}

$$-3bcd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bcd^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-3b^2cd^3\text{I}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] a*b*c^2*d^3*x + b^2*c^2*d^3*x*ArcTanh[c*x] + (7*c*d^3*(a + b*ArcTanh[c*x])^
2)/2 - (d^3*(a + b*ArcTanh[c*x])^2)/x + 3*c^2*d^3*x*(a + b*ArcTanh[c*x])^2
+ (c^3*d^3*x^2*(a + b*ArcTanh[c*x])^2)/2 + 6*c*d^3*(a + b*ArcTanh[c*x])^2*A
rcTanh[1 - 2/(1 - c*x)] - 6*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] +
 (b^2*c*d^3*Log[1 - c^2*x^2])/2 + 2*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2 - 2/
(1 + c*x)] - 3*b^2*c*d^3*PolyLog[2, 1 - 2/(1 - c*x)] - 3*b*c*d^3*(a + b*Arc
Tanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c*d^3*(a + b*ArcTanh[c*x])*Pol
yLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^3*PolyLog[2, -1 + 2/(1 + c*x)] + (3*b^2
*c*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (3*b^2*c*d^3*PolyLog[3, -1 + 2/(1 -
 c*x)])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

```
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^2} dx = \int \left(3c^2d^3 \left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^2} + \frac{3cd^3 \left(a+b\tan\right.}{x}\right.$$

$$= d^3 \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} dx + \left(3cd^3\right) \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} dx + \left(3c^2d^3\right.$$

$$= -\frac{d^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x \left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^3d^3x^2 \left(a+b\,\mathrm{ta}\right.$$

$$= 4cd^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x \left(a+b\tanh^{-1}\right.$$

$$= abc^2d^3x + \frac{7}{2}cd^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x \left(a\right.$$

$$= abc^2d^3x + b^2c^2d^3x\tanh^{-1}(cx) + \frac{7}{2}cd^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tan\right.}{x}$$

$$= abc^2d^3x + b^2c^2d^3x\tanh^{-1}(cx) + \frac{7}{2}cd^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tan\right.}{x}$$

Mathematica [C] time = 0.64, size = 479, normalized size = 1.33

$$\frac{d^3 \left(4a^2c^3x^3 + 24a^2c^2x^2 + 24a^2cx\log(x) - 8a^2 + 8abc^3x^3\tanh^{-1}(cx) + 8abc^2x^2 + 16abcx\log\left(1-c^2x^2\right) + 48abc^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] (d^3*(-8*a^2 + I*b^2*c*Pi^3*x + 24*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + 4*a^2*c^3*
x^3 - 16*a*b*ArcTanh[c*x] + 48*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^2*Arc
Tanh[c*x] + 8*a*b*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 - 20*b^2*c*x*
ArcTanh[c*x]^2 + 24*b^2*c^2*x^2*ArcTanh[c*x]^2 + 4*b^2*c^3*x^3*ArcTanh[c*x]
^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 16*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-2*ArcT
anh[c*x])] - 48*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b^2*
c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x]^2
*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c*x] + 4*
a*b*c*x*Log[1 - c*x] - 4*a*b*c*x*Log[1 + c*x] + 16*a*b*c*x*Log[1 - c^2*x^2]
 + 4*b^2*c*x*Log[1 - c^2*x^2] + 24*b^2*c*x*(1 + ArcTanh[c*x])*PolyLog[2, -E
^(-2*ArcTanh[c*x])] - 8*b^2*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*b^2*c*
x*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*a*b*c*x*PolyLog[2, -(c*x
)] + 24*a*b*c*x*PolyLog[2, c*x] + 12*b^2*c*x*PolyLog[3, -E^(-2*ArcTanh[c*x]
)] - 12*b^2*c*x*PolyLog[3, E^(2*ArcTanh[c*x])]))/(8*x)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\mathrm{artanh}\,(cx)^2 + 2}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
```

```
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^3 (b \operatorname{artanh}(cx)+a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^2, x)
```

maple [C] time = 1.94, size = 1270, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x)
```

```
[Out] 1/2*d^3*a^2*c^3*x^2+3*x*a^2*c^2*d^3-d^3*b^2*arctanh(c*x)^2/x+3/2*c*d^3*b^2*
arctanh(c*x)^2+c*d^3*b^2*arctanh(c*x)-6*c*d^3*b^2*dilog(1+I*(c*x+1)/(-c^2*x
^2+1)^(1/2))-6*c*d^3*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/2*c*d^3*b^
2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*c*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^
2+1)^(1/2))-6*c*d^3*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c*d^3*a^2*
ln(c*x)-c*d^3*b^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))-2*c*d^3*b^2*dilog((c*x+1)/(-
c^2*x^2+1)^(1/2))+2*c*d^3*b^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+a*b*c^2*d
^3*x+b^2*c^2*d^3*x*arctanh(c*x)+6*c*d^3*a*b*arctanh(c*x)*ln(c*x)-3*c*d^3*a*
b*ln(c*x)*ln(c*x+1)+6*d^3*a*b*arctanh(c*x)*c^2*x+d^3*a*b*arctanh(c*x)*c^3*x
^2+3/2*I*c*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1
)))*arctanh(c*x)^2+1/2*d^3*b^2*arctanh(c*x)^2*c^3*x^2+5/2*c*d^3*a*b*ln(c*x-
1)+3/2*c*d^3*a*b*ln(c*x+1)-3*c*d^3*a*b*dilog(c*x+1)-3*c*d^3*a*b*dilog(c*x)+
2*c*d^3*a*b*ln(c*x)+3*c*d^3*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1
/2))+3*c*d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c*d^3*b^
2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c*d^3*b^2*arctanh(c*
x)^2*ln(c*x)-2*d^3*a*b*arctanh(c*x)/x-6*c*d^3*b^2*arctanh(c*x)*ln(1+I*(c*x+
1)/(-c^2*x^2+1)^(1/2))-6*c*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)
^(1/2))-3*c*d^3*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+6*c*d^3*b^2
*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3*c*d^3*b^2*arctanh(c*
x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+2*c*d^3*b^2*arctanh(c*x)*ln(1+(c*x+1)
/(-c^2*x^2+1)^(1/2))+3*d^3*b^2*arctanh(c*x)^2*c^2*x+3/2*I*c*d^3*b^2*Pi*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-
d^3*a^2/x-3/2*I*c*d^3*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/2*I*c
*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2c^3d^3x^2+3a^2c^2d^3x+3\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)abcd^3+3a^2cd^3\log(x)-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^3*d^3*x^2 + 3*a^2*c^2*d^3*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*a*b*c*d^3 + 3*a^2*c*d^3*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) +
```

```
 2*arctanh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/8*(b^2*c^3*d^3*x^3 + 6*b^2*c^2*d
^3*x^2 - 2*b^2*d^3)*log(-c*x + 1)^2/x - integrate(-1/4*((b^2*c^4*d^3*x^4 +
2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 4*(a*b*c^4*d^
3*x^4 - a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 - 3*a*b*c*d^3*x)*log(c*x + 1) -
 (12*a*b*c^2*d^3*x^2 + (4*a*b*c^4*d^3 + b^2*c^4*d^3)*x^4 - 2*(2*a*b*c^3*d^3
 - 3*b^2*c^3*d^3)*x^3 - 2*(6*a*b*c*d^3 + b^2*c*d^3)*x + 2*(b^2*c^4*d^3*x^4
+ 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))
/(c*x^3 - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^2\,(d + c\,d\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int \frac{3a^2c}{x}\,dx + \int a^2c^3x\,dx + \int 3b^2c^2\,\mathrm{atanh}^2\,(cx)\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{x^2}\,dx + \int 6ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*c
/x, x) + Integral(a**2*c**3*x, x) + Integral(3*b**2*c**2*atanh(c*x)**2, x)
+ Integral(b**2*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*atanh(c*x), x)
 + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(3*b**2*c*atanh(c*x)**2/x,
x) + Integral(b**2*c**3*x*atanh(c*x)**2, x) + Integral(6*a*b*c*atanh(c*x)/x
, x) + Integral(2*a*b*c**3*x*atanh(c*x), x))
```

3.90 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=385

$$c^3d^3x\left(a+b\tanh^{-1}(cx)\right)^2-3bc^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bc^2d^3\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx\right.$$

```
[Out] -b*c*d^3*(a+b*arctanh(c*x))/x+9/2*c^2*d^3*(a+b*arctanh(c*x))^2-1/2*d^3*(a+b
*arctanh(c*x))^2/x^2-3*c*d^3*(a+b*arctanh(c*x))^2/x+c^3*d^3*x*(a+b*arctanh(
c*x))^2-6*c^2*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))+b^2*c^2*d^3*l
n(x)-2*b*c^2*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-1/2*b^2*c^2*d^3*ln(-c^2*
x^2+1)+6*b*c^2*d^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^2*c^2*d^3*polylog(2
,1-2/(-c*x+1))-3*b*c^2*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+3*b*c
^2*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-3*b^2*c^2*d^3*polylog(2,
-1+2/(c*x+1))+3/2*b^2*c^2*d^3*polylog(3,1-2/(-c*x+1))-3/2*b^2*c^2*d^3*polyl
og(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.79, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 5914, 6052, 6058, 6610}

$$-3bc^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bc^2d^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-b^2c^2d$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d^3*(a + b*ArcTanh[c*x]))/x) + (9*c^2*d^3*(a + b*ArcTanh[c*x])^2)/2
- (d^3*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/x
 + c^3*d^3*x*(a + b*ArcTanh[c*x])^2 + 6*c^2*d^3*(a + b*ArcTanh[c*x])^2*ArcT
anh[1 - 2/(1 - c*x)] + b^2*c^2*d^3*Log[x] - 2*b*c^2*d^3*(a + b*ArcTanh[c*x]
)*Log[2/(1 - c*x)] - (b^2*c^2*d^3*Log[1 - c^2*x^2])/2 + 6*b*c^2*d^3*(a + b*
ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)
] - 3*b*c^2*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c^2*
d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 3*b^2*c^2*d^3*PolyL
og[2, -1 + 2/(1 + c*x)] + (3*b^2*c^2*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (
3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(c^3d^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{3cd^3\left(a+b\tanh\right.}{x^2}\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}dx + \left(3cd^3\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}dx + \left(3c^2d^3\right.$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^3d^3x\left(a+b\tanh^{-}\right.$$

$$= 4c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3cd^3\left(a+b\tanh^{-1}(\right.}{x}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}\right.}{2x^2}$$

Mathematica [C] time = 1.02, size = 461, normalized size = 1.20

$$\frac{1}{2}d^3\left(2a^2c^3x + 6a^2c^2\log(x) - \frac{6a^2c}{x} - \frac{a^2}{x^2} - 6abc^2(\mathrm{Li}_2(-cx) - \mathrm{Li}_2(cx)) + 2abc^2\left(\log\left(1-c^2x^2\right) + 2cx\tanh^{-1}(cx)\right) - \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] (d^3*(-(a^2/x^2) - (6*a^2*c)/x + 2*a^2*c^3*x + 6*a^2*c^2*Log[x] - (a*b*(2*A
rcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])))/x^2 + (b^2*(-
2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + 2*c^2*x^2*Log[(c*x)/Sq
rt[1 - c^2*x^2]]))/x^2 + 2*a*b*c^2*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^2])
- (6*a*b*c*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])))/x + 2*b
^2*c^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x
])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + (6*b^2*c*(ArcTanh[c*x]*((-1 + c*
x)*ArcTanh[c*x] + 2*c*x*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*PolyLog[2, E^(-
2*ArcTanh[c*x])]))/x - 6*a*b*c^2*(PolyLog[2, -(c*x)] - PolyLog[2, c*x]) + 6
*b^2*c^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2
*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]
*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*
x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/
2)))/2
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\mathrm{artanh}\,(cx)^2 + 2\right.}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3 (b \operatorname{artanh}(cx) + a)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^3, x)
```

maple [C] time = 1.76, size = 1358, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] 3*c^2*d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c^2*d^3*b^2
*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^2*d^3*b^2*arctanh(c
*x)^2*ln(c*x)-2*c^2*d^3*b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))
-2*c^2*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3*c^2*d^3*b^
2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+6*c^2*d^3*b^2*arctanh(c*x)*po
lylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3*c^2*d^3*b^2*arctanh(c*x)*polylog(2,-
(c*x+1)^2/(-c^2*x^2+1))-3*c^2*d^3*a*b*dilog(c*x+1)-3*c^2*d^3*a*b*dilog(c*x)
+6*c^2*d^3*a*b*ln(c*x)-3/2*c^2*d^3*a*b*ln(c*x+1)-5/2*c^2*d^3*a*b*ln(c*x-1)-
d^3*a*b*arctanh(c*x)/x^2-c*d^3*a*b/x+3/2*I*c^2*d^3*b^2*Pi*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+c^3*d^3*b^2*arctanh(
c*x)^2*x-c*d^3*b^2*arctanh(c*x)/x-3*c*d^3*b^2*arctanh(c*x)^2/x+6*c^2*d^3*b^
2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^2*d^3*b^2*arctanh(c*x)^
2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^3*d^3*a*b*arctanh(c*x)*x-6*c*d^3*a*b
*arctanh(c*x)/x+6*c^2*d^3*a*b*arctanh(c*x)*ln(c*x)-3*c^2*d^3*a*b*ln(c*x)*ln
(c*x+1)-3/2*I*c^2*d^3*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/2*I*c
^2*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/2*I*c^2*d^3*b^2*Pi*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)
^2-1/2*d^3*a^2/x^2+3/2*c^2*d^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*c^2
*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-6*c^2*d^3*b^2*polylog(3,-(c*
x+1)/(-c^2*x^2+1)^(1/2))+3*c^2*d^3*a^2*ln(c*x)-6*c^2*d^3*b^2*dilog((c*x+1)/
(-c^2*x^2+1)^(1/2))+6*c^2*d^3*b^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+c^2*d
^3*b^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+c^2*d^3*b^2*ln((c*x+1)/(-c^2*x^2+1)
^(1/2)-1)+c^3*x*a^2*d^3-3*c*d^3*a^2/x-c^2*d^3*b^2*arctanh(c*x)-2*c^2*d^3*b^
2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*d^3*b^2*arctanh(c*x)^2/x^2-2*c^
2*d^3*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*c^2*d^3*b^2*arctanh(c*x
)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^3d^3x + \left(2\,cx \operatorname{artanh}(cx) + \log\left(-c^2x^2+1\right)\right)abc^2d^3 + 3\,a^2c^2d^3\log(x) - 3\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2 \operatorname{arta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^3*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b*c^2*d^3 + 3*a^
2*c^2*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a
*b*c*d^3 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/
x^2)*a*b*d^3 - 3*a^2*c*d^3/x - 1/2*a^2*d^3/x^2 + 1/8*(2*b^2*c^3*d^3*x^3 - 6
*b^2*c*d^3*x - b^2*d^3)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^4*d^3*
x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 12*(a*b
*c^3*d^3*x^3 - a*b*c^2*d^3*x^2)*log(c*x + 1) - (2*b^2*c^4*d^3*x^4 + 12*a*b*
c^3*d^3*x^3 - b^2*c*d^3*x - 6*(2*a*b*c^2*d^3 + b^2*c^2*d^3)*x^2 + 2*(b^2*c^
4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(
-c*x + 1))/(c*x^4 - x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^2\,\left(d+c\,d\,x\right)^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int a^2c^3\,dx+\int \frac{a^2}{x^3}\,dx+\int \frac{3a^2c}{x^2}\,dx+\int \frac{3a^2c^2}{x}\,dx+\int b^2c^3\,\mathrm{atanh}^2\left(cx\right)dx+\int \frac{b^2\,\mathrm{atanh}^2\left(cx\right)}{x^3}\,dx+\int 2abc^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2*c**3, x) + Integral(a**2/x**3, x) + Integral(3*a**2*c/x
**2, x) + Integral(3*a**2*c**2/x, x) + Integral(b**2*c**3*atanh(c*x)**2, x)
 + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*c**3*atanh(c*x), x
) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(3*b**2*c*atanh(c*x)**2/x*
*2, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(
c*x)/x**2, x) + Integral(6*a*b*c**2*atanh(c*x)/x, x))
```

3.91 $\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^4} \, dx$

Optimal. Leaf size=396

$$-bc^3d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^3d^3\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{29}{6}c^3d^3\left(a+b\tanh^{-1}(cx\right.$$

```
[Out] -1/3*b^2*c^2*d^3/x+1/3*b^2*c^3*d^3*arctanh(c*x)-1/3*b*c*d^3*(a+b*arctanh(c*
x))/x^2-3*b*c^2*d^3*(a+b*arctanh(c*x))/x+29/6*c^3*d^3*(a+b*arctanh(c*x))^2-
1/3*d^3*(a+b*arctanh(c*x))^2/x^3-3/2*c*d^3*(a+b*arctanh(c*x))^2/x^2-3*c^2*d
^3*(a+b*arctanh(c*x))^2/x-2*c^3*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x
+1))+3*b^2*c^3*d^3*ln(x)-3/2*b^2*c^3*d^3*ln(-c^2*x^2+1)+20/3*b*c^3*d^3*(a+b
*arctanh(c*x))*ln(2-2/(c*x+1))-b*c^3*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(
-c*x+1))+b*c^3*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-10/3*b^2*c^3
*d^3*polylog(2,-1+2/(c*x+1))+1/2*b^2*c^3*d^3*polylog(3,1-2/(-c*x+1))-1/2*b^
2*c^3*d^3*polylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.93, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5916, 5982, 325, 206, 5988, 5932, 2447, 266, 36, 29, 31, 5948, 5914, 6052, 6058, 6610}

$$-bc^3d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^3d^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{10}{3}b^2c^3d$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d^3)/(3*x) + (b^2*c^3*d^3*ArcTanh[c*x])/3 - (b*c*d^3*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/x + (29*c^3*d^3*(a +
 b*ArcTanh[c*x])^2)/6 - (d^3*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (3*c*d^3*(a
+ b*ArcTanh[c*x])^2)/(2*x^2) - (3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/x + 2*c^3
*d^3*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 3*b^2*c^3*d^3*Log[x]
 - (3*b^2*c^3*d^3*Log[1 - c^2*x^2])/2 + (20*b*c^3*d^3*(a + b*ArcTanh[c*x])*
Log[2 - 2/(1 + c*x)])/3 - b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)] + b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - (
10*b^2*c^3*d^3*PolyLog[2, -1 + 2/(1 + c*x)])/3 + (b^2*c^3*d^3*PolyLog[3, 1
- 2/(1 - c*x)])/2 - (b^2*c^3*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}dx = \int\left(\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^4} + \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{3c^2d^3\left(a+b\tan\right.}{x^2}\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}dx + \left(3cd^3\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}dx + \left(3c^2d^3\right.$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3c^2d^3\left(a+b\tanh^{-1}\right.}{x}$$

$$= 3c^3d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - \frac{3cd^3\left(a+b\tanh^{-1}(\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{29}{6}c^3d^3\left(a+b\,\mathrm{ta}\right.$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{29}{6}c^3d^3$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{1}{3}b^2c^3d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3\left(a+b\right.}{x}$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{1}{3}b^2c^3d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3\left(a+b\right.}{x}$$

Mathematica [C] time = 0.73, size = 569, normalized size = 1.44

$$\frac{d^3\left(24a^2c^3x^3\log(x) - 72a^2c^2x^2 - 36a^2cx - 8a^2 - 24abc^3x^3\mathrm{Li}_2(-cx) + 24abc^3x^3\mathrm{Li}_2(cx) + 160abc^3x^3\log(cx) - 36\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-8*a^2 - 36*a^2*c*x - 8*a*b*c*x - 72*a^2*c^2*x^2 - 72*a*b*c^2*x^2 - 8
*b^2*c^2*x^2 + I*b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTanh[c*x] - 72*a*b*c*x*ArcTan
h[c*x] - 8*b^2*c*x*ArcTanh[c*x] - 144*a*b*c^2*x^2*ArcTanh[c*x] - 72*b^2*c^2
*x^2*ArcTanh[c*x] + 8*b^2*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 - 36*
b^2*c*x*ArcTanh[c*x]^2 - 72*b^2*c^2*x^2*ArcTanh[c*x]^2 + 116*b^2*c^3*x^3*Ar
cTanh[c*x]^2 - 16*b^2*c^3*x^3*ArcTanh[c*x]^3 + 160*b^2*c^3*x^3*ArcTanh[c*x]
*Log[1 - E^(-2*ArcTanh[c*x])] - 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1 + E^(-2
*ArcTanh[c*x])] + 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])]
 + 24*a^2*c^3*x^3*Log[x] + 160*a*b*c^3*x^3*Log[c*x] - 36*a*b*c^3*x^3*Log[1
- c*x] + 36*a*b*c^3*x^3*Log[1 + c*x] + 72*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^
2*x^2]] - 80*a*b*c^3*x^3*Log[1 - c^2*x^2] + 24*b^2*c^3*x^3*ArcTanh[c*x]*Pol
yLog[2, -E^(-2*ArcTanh[c*x])] - 80*b^2*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x
])] + 24*b^2*c^3*x^3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*a*b*c
^3*x^3*PolyLog[2, -(c*x)] + 24*a*b*c^3*x^3*PolyLog[2, c*x] + 12*b^2*c^3*x^3
*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c^3*x^3*PolyLog[3, E^(2*ArcTanh[
c*x])]))/(24*x^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\mathrm{artanh}\,(cx)^2 + 2}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3 (b \operatorname{artanh}(cx) + a)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^4, x)
```

maple [C] time = 2.28, size = 1337, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] -11/6*c^3*d^3*a*b*ln(c*x+1)-3*c^2*d^3*b^2*arctanh(c*x)/x-3/2*c*d^3*b^2*arct
anh(c*x)^2/x^2-1/3*c*d^3*b^2*arctanh(c*x)/x^2-3*c^2*d^3*b^2*arctanh(c*x)^2/
x-2/3*d^3*a*b*arctanh(c*x)/x^3+c^3*d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^
2*x^2+1)^(1/2))+2*c^3*d^3*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(
1/2))+c^3*d^3*b^2*arctanh(c*x)^2*ln(c*x)-c^3*d^3*b^2*arctanh(c*x)^2*ln((c*x
+1)^2/(-c^2*x^2+1)-1)+2*c^3*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x
^2+1)^(1/2))-c^3*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3*
c^2*d^3*a*b/x-1/3*c*d^3*a*b/x^2-8/3*b^2*c^3*d^3*arctanh(c*x)-1/2*I*c^3*d^3*
b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*c^3*d^3*b^2*Pi*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(
-c^2*x^2+1)))^2*arctanh(c*x)^2+20/3*c^3*d^3*b^2*arctanh(c*x)*ln(1+(c*x+1)/(
-c^2*x^2+1)^(1/2))-1/3*c^3*d^3*b^2/(c*x+1-(-c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^
(1/2)+1/3*c^3*d^3*b^2/((-c^2*x^2+1)^(1/2)+c*x+1)*(-c^2*x^2+1)^(1/2)-c^3*d^3
*a*b*dilog(c*x+1)-c^3*d^3*a*b*dilog(c*x)+20/3*c^3*d^3*a*b*ln(c*x)+c^3*d^3*b
^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-29/6*c^3*d^3*a*b*ln(c*x-
1)+2*c^3*d^3*a*b*arctanh(c*x)*ln(c*x)-c^3*d^3*a*b*ln(c*x)*ln(c*x+1)-3*c*d^3
*a*b*arctanh(c*x)/x^2-6*c^2*d^3*a*b*arctanh(c*x)/x-3*c^2*d^3*a^2/x-3/2*c*d^
3*a^2/x^2-1/3*d^3*b^2*arctanh(c*x)^2/x^3+c^3*d^3*a^2*ln(c*x)-20/3*c^3*d^3*b
^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+20/3*c^3*d^3*b^2*dilog(1+(c*x+1)/(-c^2
*x^2+1)^(1/2))+3*c^3*d^3*b^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^3*d^3*b^2
*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)-11/6*c^3*d^3*b^2*arctanh(c*x)^2+1/2*c^3*d
^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-2*c^3*d^3*b^2*polylog(3,(c*x+1)/(
-c^2*x^2+1)^(1/2))-2*c^3*d^3*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2
*I*c^3*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+
1)))^3*arctanh(c*x)^2+1/2*I*c^3*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1
))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+
(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/3*d^3*a^2/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^3d^3 \log(x) - 3\left(c\left(\log\left(c^2x^2 - 1\right) - \log\left(x^2\right)\right) + \frac{2 \operatorname{artanh}(cx)}{x}\right)abc^2d^3 + \frac{3}{2}\left(\left(c \log(cx + 1) - c \log(cx - 1) - \frac{2}{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] a^2*c^3*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)
*a*b*c^2*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c
*x)/x^2)*a*b*c*d^3 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c +
 2*arctanh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/x - 3/2*a^2*c*d^3/x^2 - 1/3*a^
2*d^3/x^3 - 1/24*(18*b^2*c^2*d^3*x^2 + 9*b^2*c*d^3*x + 2*b^2*d^3)*log(-c*x
+ 1)^2/x^3 - integrate(-1/12*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^
2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 12*(a*b*c^4*d^3*x^4 - a*b*c^3*d^3*x^3
)*log(c*x + 1) - (12*a*b*c^4*d^3*x^4 - 9*b^2*c^2*d^3*x^2 - 2*b^2*c*d^3*x -
6*(2*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 + 6*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*
x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^4),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(c\,x))^2\,(d+c\,d\,x)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int\frac{a^2}{x^4}\,dx+\int\frac{3a^2c}{x^3}\,dx+\int\frac{3a^2c^2}{x^2}\,dx+\int\frac{a^2c^3}{x}\,dx+\int\frac{b^2\,\mathrm{atanh}^2\,(cx)}{x^4}\,dx+\int\frac{2ab\,\mathrm{atanh}\,(cx)}{x^4}\,dx+\int\frac{3b^2c\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(a**2/x**4, x) + Integral(3*a**2*c/x**3, x) + Integral(3*a**2
*c**2/x**2, x) + Integral(a**2*c**3/x, x) + Integral(b**2*atanh(c*x)**2/x**
4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(3*b**2*c*atanh(c*x)**
2/x**3, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**2, x) + Integral(b**2*c*
*3*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x)/x**3, x) + Integral(6*
a*b*c**2*atanh(c*x)/x**2, x) + Integral(2*a*b*c**3*atanh(c*x)/x, x))
```

3.92 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=271

$$4abc^4d^3\log(x)+4bc^4d^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{7bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}-$$

```
[Out] -1/12*b^2*c^2*d^3/x^2-b^2*c^3*d^3/x+b^2*c^4*d^3*arctanh(c*x)-1/6*b*c*d^3*(a
+b*arctanh(c*x))/x^3-b*c^2*d^3*(a+b*arctanh(c*x))/x^2-7/2*b*c^3*d^3*(a+b*ar
ctanh(c*x))/x-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/x^4+4*a*b*c^4*d^3*ln(x
)+11/3*b^2*c^4*d^3*ln(x)+4*b*c^4*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-11/6
*b^2*c^4*d^3*ln(-c^2*x^2+1)-2*b^2*c^4*d^3*polylog(2,-c*x)+2*b^2*c^4*d^3*pol
ylog(2,c*x)+2*b^2*c^4*d^3*polylog(2,1-2/(-c*x+1))
```

Rubi [A] time = 0.31, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {37, 5938, 5916, 266, 44, 325, 206, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-2b^2c^4d^3\text{PolyLog}(2,-cx)+2b^2c^4d^3\text{PolyLog}(2,cx)+2b^2c^4d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```

```
[Out] -(b^2*c^2*d^3)/(12*x^2) - (b^2*c^3*d^3)/x + b^2*c^4*d^3*ArcTanh[c*x] - (b*c
*d^3*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/x^2 -
 (7*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh
[c*x])^2)/(4*x^4) + 4*a*b*c^4*d^3*Log[x] + (11*b^2*c^4*d^3*Log[x])/3 + 4*b*
c^4*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] - (11*b^2*c^4*d^3*Log[1 - c^2
*x^2])/6 - 2*b^2*c^4*d^3*PolyLog[2, -(c*x)] + 2*b^2*c^4*d^3*PolyLog[2, c*x]
 + 2*b^2*c^4*d^3*PolyLog[2, 1 - 2/(1 - c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4} - (2bc)\int\left(-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4} -\right.$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bcd^3\right)\int\frac{a+b\tanh^{-1}(cx)}{x^4}\,dx +$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{7bc^3d^3\left(a+b\,\mathrm{t}\right.}{2x}$$

$$= -\frac{b^2c^3d^3}{x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{7bc^3d}{}$$

$$= -\frac{b^2c^3d^3}{x} + b^2c^4d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\right.}{x}$$

$$= -\frac{b^2c^2d^3}{12x^2} - \frac{b^2c^3d^3}{x} + b^2c^4d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc}{}$$

Mathematica [A] time = 0.80, size = 343, normalized size = 1.27

$$-\frac{d^3\left(12a^2c^3x^3+18a^2c^2x^2+12a^2cx+3a^2-48abc^4x^4\log(cx)+21abc^4x^4\log(1-cx)-21abc^4x^4\log(cx+1)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```

```
[Out] -1/12*(d^3*(3*a^2 + 12*a^2*c*x + 2*a*b*c*x + 18*a^2*c^2*x^2 + 12*a*b*c^2*x^
2 + b^2*c^2*x^2 + 12*a^2*c^3*x^3 + 42*a*b*c^3*x^3 + 12*b^2*c^3*x^3 - b^2*c^
4*x^4 + 3*b^2*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 - 15*c^4*x^4)*ArcTanh[c*x]
^2 + 2*b*ArcTanh[c*x]*(b*c*x*(1 + 6*c*x + 21*c^2*x^2 - 6*c^3*x^3) + 3*a*(1
+ 4*c*x + 6*c^2*x^2 + 4*c^3*x^3) - 24*b*c^4*x^4*Log[1 - E^(-2*ArcTanh[c*x])
]) - 48*a*b*c^4*x^4*Log[c*x] + 21*a*b*c^4*x^4*Log[1 - c*x] - 21*a*b*c^4*x^4
*Log[1 + c*x] - 44*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 24*a*b*c^4*x^
4*Log[1 - c^2*x^2] + 24*b^2*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^4
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^3d^3x^3+3\,a^2c^2d^3x^2+3\,a^2cd^3x+a^2d^3+\left(b^2c^3d^3x^3+3\,b^2c^2d^3x^2+3\,b^2cd^3x+b^2d^3\right)\mathrm{artanh}\,(cx)^2 \right.}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
```

```
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^3(b\,\mathrm{artanh}\,(cx)+a)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^5, x)
```

maple [B] time = 0.08, size = 646, normalized size = 2.38

$$-\frac{c^2d^3ab}{x^2}+\frac{c^4d^3b^2\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{1}{2}+\frac{cx}{2}\right)}{8}+4c^4d^3ab\ln(cx)-\frac{2c^3d^3ab\,\mathrm{arctanh}\,(cx)}{x}-\frac{3c^2d^3ab\,\mathrm{arctanh}\,(cx)}{x^2}-\frac{2c\,d^3ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x)
```

```
[Out] -c^2*d^3*a*b/x^2-1/2*d^3*a*b*arctanh(c*x)/x^4+1/8*c^4*d^3*b^2*ln(-1/2*c*x+1
/2)*ln(1/2+1/2*c*x)+4*c^4*d^3*a*b*ln(c*x)+4*c^4*d^3*b^2*arctanh(c*x)*ln(c*x
)-15/4*c^4*d^3*b^2*arctanh(c*x)*ln(c*x-1)-2*c^3*d^3*a*b*arctanh(c*x)/x-3*c^
2*d^3*a*b*arctanh(c*x)/x^2-2*c*d^3*a*b*arctanh(c*x)/x^3-4/3*c^4*d^3*b^2*ln(
c*x+1)-c^3*d^3*a^2/x-c*d^3*a^2/x^3-3/2*c^2*d^3*a^2/x^2-2*c^4*d^3*b^2*dilog(
c*x)-2*c^4*d^3*b^2*dilog(c*x+1)+1/16*c^4*d^3*b^2*ln(c*x+1)^2+11/3*c^4*d^3*b
^2*ln(c*x)-15/16*c^4*d^3*b^2*ln(c*x-1)^2+2*c^4*d^3*b^2*dilog(1/2+1/2*c*x)-7
/3*c^4*d^3*b^2*ln(c*x-1)-1/4*d^3*b^2*arctanh(c*x)^2/x^4-1/12*b^2*c^2*d^3/x^
2-1/6*c*d^3*a*b/x^3-b^2*c^3*d^3/x-1/4*d^3*a^2/x^4-2*c^4*d^3*b^2*ln(c*x)*ln(
c*x+1)-1/8*c^4*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-7/2*c^3*d^3*a*b/x-c^3*d^3
*b^2*arctanh(c*x)^2/x-1/6*c*d^3*b^2*arctanh(c*x)/x^3+15/8*c^4*d^3*b^2*ln(c*
x-1)*ln(1/2+1/2*c*x)-1/4*c^4*d^3*b^2*arctanh(c*x)*ln(c*x+1)-1/4*c^4*d^3*a*b
*ln(c*x+1)-15/4*c^4*d^3*a*b*ln(c*x-1)-7/2*c^3*d^3*b^2*arctanh(c*x)/x-3/2*c^
2*d^3*b^2*arctanh(c*x)^2/x^2-c*d^3*b^2*arctanh(c*x)^2/x^3-c^2*d^3*b^2*arcta
nh(c*x)/x^2
```

maxima [B] time = 0.96, size = 813, normalized size = 3.00

$$-2\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2c^4d^3-2\left(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1)\right)b^2c^4d^3+2\left(\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] -2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^4*d^3 -
2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^4*d^3 + 2*(log(c*x + 1)*
log(-c*x) + dilog(c*x + 1))*b^2*c^4*d^3 - b^2*c^4*d^3*log(c*x + 1) - 2*b^2*
c^4*d^3*log(c*x - 1) + 3*b^2*c^4*d^3*log(x) - (c*(log(c^2*x^2 - 1) - log(x^
2)) + 2*arctanh(c*x)/x)*a*b*c^3*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x - 1)
 - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^2*d^3 - ((c^2*log(c^2*x^2 - 1) - c^2*
log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c*d^3 - a^2*c^3*d^3/x + 1/12*
((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*ar
ctanh(c*x)/x^4)*a*b*d^3 + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^2
+ 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c*x
 - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3*c
^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2*d^3 - 3/2*a^2*
c^2*d^3/x^2 - a^2*c*d^3/x^3 - 1/4*b^2*d^3*arctanh(c*x)^2/x^4 - 1/4*a^2*d^3/
```

```
x^4 - 1/8*(8*b^2*c^3*d^3*x^2 + (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d^3*x^2 + 3*b^2
*c^2*d^3*x + 2*b^2*c*d^3)*log(c*x + 1)^2 - (7*b^2*c^4*d^3*x^3 - 2*b^2*c^3*d
^3*x^2 - 3*b^2*c^2*d^3*x - 2*b^2*c*d^3)*log(-c*x + 1)^2 + 4*(3*b^2*c^3*d^3*
x^2 + b^2*c^2*d^3*x)*log(c*x + 1) - 2*(6*b^2*c^3*d^3*x^2 + 2*b^2*c^2*d^3*x
+ (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d^3*x^2 + 3*b^2*c^2*d^3*x + 2*b^2*c*d^3)*log
(c*x + 1))*log(-c*x + 1))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^2\,(d + c\,d\,x)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^5,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a^2}{x^5}\,dx + \int \frac{3a^2c}{x^4}\,dx + \int \frac{3a^2c^2}{x^3}\,dx + \int \frac{a^2c^3}{x^2}\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{x^5}\,dx + \int \frac{2ab\,\mathrm{atanh}\,(cx)}{x^5}\,dx + \int \frac{3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**5,x)
```

```
[Out] d**3*(Integral(a**2/x**5, x) + Integral(3*a**2*c/x**4, x) + Integral(3*a**2
*c**2/x**3, x) + Integral(a**2*c**3/x**2, x) + Integral(b**2*atanh(c*x)**2/
x**5, x) + Integral(2*a*b*atanh(c*x)/x**5, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**4, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**3, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c*atanh(c*x)/x**4, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**3, x) + Integral(2*a*b*c**3*atanh(c*x)/x**2, x
))
```

3.93 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^6}\,dx$

Optimal. Leaf size=352

$$\frac{12}{5}abc^5d^3\log(x)+\frac{12}{5}bc^5d^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{5bc^4d^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{6bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{5x^2}$$

```
[Out] -1/30*b^2*c^2*d^3/x^3-1/4*b^2*c^3*d^3/x^2-13/10*b^2*c^4*d^3/x+13/10*b^2*c^5
*d^3*arctanh(c*x)-1/10*b*c*d^3*(a+b*arctanh(c*x))/x^4-1/2*b*c^2*d^3*(a+b*ar
ctanh(c*x))/x^3-6/5*b*c^3*d^3*(a+b*arctanh(c*x))/x^2-5/2*b*c^4*d^3*(a+b*arc
tanh(c*x))/x-1/5*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/x^5+1/20*c*d^3*(c*x+1)^
4*(a+b*arctanh(c*x))^2/x^4+12/5*a*b*c^5*d^3*ln(x)+3*b^2*c^5*d^3*ln(x)+12/5*
b*c^5*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-3/2*b^2*c^5*d^3*ln(-c^2*x^2+1)-
6/5*b^2*c^5*d^3*polylog(2,-c*x)+6/5*b^2*c^5*d^3*polylog(2,c*x)+6/5*b^2*c^5*
d^3*polylog(2,1-2/(-c*x+1))
```

Rubi [A] time = 0.37, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {45, 37, 5938, 5916, 325, 206, 266, 44, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,-cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{6bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```

```
[Out] -(b^2*c^2*d^3)/(30*x^3) - (b^2*c^3*d^3)/(4*x^2) - (13*b^2*c^4*d^3)/(10*x) +
 (13*b^2*c^5*d^3*ArcTanh[c*x])/10 - (b*c*d^3*(a + b*ArcTanh[c*x]))/(10*x^4)
 - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/(2*x^3) - (6*b*c^3*d^3*(a + b*ArcTanh[c
*x]))/(5*x^2) - (5*b*c^4*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4
*(a + b*ArcTanh[c*x])^2)/(5*x^5) + (c*d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^
2)/(20*x^4) + (12*a*b*c^5*d^3*Log[x])/5 + 3*b^2*c^5*d^3*Log[x] + (12*b*c^5*
d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/5 - (3*b^2*c^5*d^3*Log[1 - c^2*x
^2])/2 - (6*b^2*c^5*d^3*PolyLog[2, -(c*x)])/5 + (6*b^2*c^5*d^3*PolyLog[2, c
*x])/5 + (6*b^2*c^5*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^6}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{20x^4}-(2$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{20x^4}+\frac{1}{5}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^3}-\frac{6bc^3d^3\left(a+b\tanh\right.}{5x^2}$$

$$= -\frac{b^2c^2d^3}{30x^3}-\frac{6b^2c^4d^3}{5x}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^3}$$

$$= -\frac{b^2c^2d^3}{30x^3}-\frac{13b^2c^4d^3}{10x}+\frac{6}{5}b^2c^5d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{b}{}$$

$$= -\frac{b^2c^2d^3}{30x^3}-\frac{b^2c^3d^3}{4x^2}-\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh\right.}{10x^4}$$

Mathematica [A] time = 1.20, size = 372, normalized size = 1.06

$$-\frac{d^3\left(30a^2c^3x^3+60a^2c^2x^2+45a^2cx+12a^2-144abc^5x^5\log(cx)+75abc^5x^5\log(1-cx)-75abc^5x^5\log(cx+1)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```

```
[Out] -1/60*(d^3*(12*a^2 + 45*a^2*c*x + 6*a*b*c*x + 60*a^2*c^2*x^2 + 30*a*b*c^2*x
^2 + 2*b^2*c^2*x^2 + 30*a^2*c^3*x^3 + 72*a*b*c^3*x^3 + 15*b^2*c^3*x^3 + 150
*a*b*c^4*x^4 + 78*b^2*c^4*x^4 - 15*b^2*c^5*x^5 + 3*b^2*(4 + 15*c*x + 20*c^2
*x^2 + 10*c^3*x^3 - 49*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c*x]*(a*(4 + 1
```

```
5*c*x + 20*c^2*x^2 + 10*c^3*x^3) + b*c*x*(1 + 5*c*x + 12*c^2*x^2 + 25*c^3*x
^3 - 13*c^4*x^4) - 24*b*c^5*x^5*Log[1 - E^(-2*ArcTanh[c*x])]) - 144*a*b*c^5
*x^5*Log[c*x] + 75*a*b*c^5*x^5*Log[1 - c*x] - 75*a*b*c^5*x^5*Log[1 + c*x] -
 180*b^2*c^5*x^5*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 72*a*b*c^5*x^5*Log[1 - c^2*
x^2] + 72*b^2*c^5*x^5*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^5
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\operatorname{artanh}(cx)^2 -}{x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx+d)^3(b\operatorname{artanh}(cx)+a)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^6, x)
```

maple [B] time = 0.08, size = 691, normalized size = 1.96

$$\frac{6c^5d^3b^2\operatorname{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)}{5}-\frac{d^3b^2\operatorname{arctanh}(cx)^2}{5x^5}-\frac{6c^5d^3b^2\operatorname{dilog}(cx+1)}{5}-\frac{6c^5d^3b^2\operatorname{dilog}(cx)}{5}-\frac{43c^5d^3b^2\ln(cx-1)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x)
```

```
[Out] -c^3*d^3*a*b*arctanh(c*x)/x^2-3/2*c*d^3*a*b*arctanh(c*x)/x^4-2*c^2*d^3*a*b*
arctanh(c*x)/x^3+6/5*c^5*d^3*b^2*dilog(1/2+1/2*c*x)-1/5*d^3*b^2*arctanh(c*x
)^2/x^5-6/5*c^5*d^3*b^2*dilog(c*x+1)-6/5*c^5*d^3*b^2*dilog(c*x)-43/20*c^5*d
^3*b^2*ln(c*x-1)-49/80*c^5*d^3*b^2*ln(c*x-1)^2-17/20*c^5*d^3*b^2*ln(c*x+1)+
3*c^5*d^3*b^2*ln(c*x)-1/80*c^5*d^3*b^2*ln(c*x+1)^2-3/4*c*d^3*a^2/x^4-c^2*d^
3*a^2/x^3-1/2*c^3*d^3*a^2/x^2-1/30*b^2*c^2*d^3/x^3-1/4*b^2*c^3*d^3/x^2-13/1
0*b^2*c^4*d^3/x-1/5*d^3*a^2/x^5+1/20*c^5*d^3*a*b*ln(c*x+1)-49/20*c^5*d^3*a*
b*ln(c*x-1)-3/4*c*d^3*b^2*arctanh(c*x)^2/x^4-1/2*c^2*d^3*b^2*arctanh(c*x)/x
^3-1/10*c*d^3*b^2*arctanh(c*x)/x^4-5/2*c^4*d^3*b^2*arctanh(c*x)/x-1/2*c^3*d
^3*b^2*arctanh(c*x)^2/x^2-c^2*d^3*b^2*arctanh(c*x)^2/x^3-6/5*c^3*d^3*b^2*ar
ctanh(c*x)/x^2-1/2*c^2*d^3*a*b/x^3-1/10*c*d^3*a*b/x^4-5/2*c^4*d^3*a*b/x-6/5
*c^3*d^3*a*b/x^2-2/5*d^3*a*b*arctanh(c*x)/x^5-1/40*c^5*d^3*b^2*ln(-1/2*c*x+
1/2)*ln(1/2+1/2*c*x)-49/20*c^5*d^3*b^2*arctanh(c*x)*ln(c*x-1)+49/40*c^5*d^3
*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/20*c^5*d^3*b^2*arctanh(c*x)*ln(c*x+1)-6/5*
c^5*d^3*b^2*ln(c*x)*ln(c*x+1)+1/40*c^5*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1
2/5*c^5*d^3*a*b*ln(c*x)+12/5*c^5*d^3*b^2*arctanh(c*x)*ln(c*x)
```

maxima [B] time = 0.95, size = 783, normalized size = 2.22

$$-\frac{6}{5}\left(\log(cx+1)\log\left(-\frac{1}{2}\,cx+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}\,cx+\frac{1}{2}\right)\right)b^2c^5d^3-\frac{6}{5}\left(\log(cx)\log(-cx+1)+\operatorname{Li}_2(-cx+1)\right)b^2c^5d^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="maxima")
```

```
[Out] -6/5*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^5*d^3
- 6/5*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^5*d^3 + 6/5*(log(c*x
 + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^5*d^3 - 17/20*b^2*c^5*d^3*log(c*x +
 1) - 43/20*b^2*c^5*d^3*log(c*x - 1) + 3*b^2*c^5*d^3*log(x) + 1/2*((c*log(c
*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^3*d^3 - ((c^2
*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c^2*d
^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)
*c - 6*arctanh(c*x)/x^4)*a*b*c*d^3 - 1/10*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*
log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*d^3 - 1/2*a^2*c
^3*d^3/x^2 - a^2*c^2*d^3/x^3 - 3/4*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^5 - 1/240*
(312*b^2*c^4*d^3*x^4 + 60*b^2*c^3*d^3*x^3 + 8*b^2*c^2*d^3*x^2 - 3*(b^2*c^5*
d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*
d^3)*log(c*x + 1)^2 - 3*(49*b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c
^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*d^3)*log(-c*x + 1)^2 + 12*(25*b^2*c^4*d
^3*x^4 + 12*b^2*c^3*d^3*x^3 + 5*b^2*c^2*d^3*x^2 + b^2*c*d^3*x)*log(c*x + 1)
 - 6*(50*b^2*c^4*d^3*x^4 + 24*b^2*c^3*d^3*x^3 + 10*b^2*c^2*d^3*x^2 + 2*b^2*
c*d^3*x - (b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 - 15*b
^2*c*d^3*x - 4*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^2\,(d + c\,d\,x)^3}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^6,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a^2}{x^6}\,dx + \int \frac{3a^2c}{x^5}\,dx + \int \frac{3a^2c^2}{x^4}\,dx + \int \frac{a^2c^3}{x^3}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^6}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^6}\,dx + \int \frac{3b^2c\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**6,x)
```

```
[Out] d**3*(Integral(a**2/x**6, x) + Integral(3*a**2*c/x**5, x) + Integral(3*a**2
*c**2/x**4, x) + Integral(a**2*c**3/x**3, x) + Integral(b**2*atanh(c*x)**2/
x**6, x) + Integral(2*a*b*atanh(c*x)/x**6, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**5, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**4, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**3, x) + Integral(6*a*b*c*atanh(c*x)/x**5, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**4, x) + Integral(2*a*b*c**3*atanh(c*x)/x**3, x
))
```

3.94 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^7}\,dx$

Optimal. Leaf size=479

$$\frac{28}{15}abc^6d^3\log(x)+\frac{37}{20}bc^6d^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{60}bc^6d^3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{11bc^5d^3}{}$$

```
[Out] -1/60*b^2*c^2*d^3/x^4-1/10*b^2*c^3*d^3/x^3-61/180*b^2*c^4*d^3/x^2-37/30*b^2
*c^5*d^3/x+37/30*b^2*c^6*d^3*arctanh(c*x)-1/15*b*c*d^3*(a+b*arctanh(c*x))/x
^5-3/10*b*c^2*d^3*(a+b*arctanh(c*x))/x^4-11/18*b*c^3*d^3*(a+b*arctanh(c*x))
/x^3-14/15*b*c^4*d^3*(a+b*arctanh(c*x))/x^2-11/6*b*c^5*d^3*(a+b*arctanh(c*x
))/x-1/6*d^3*(a+b*arctanh(c*x))^2/x^6-3/5*c*d^3*(a+b*arctanh(c*x))^2/x^5-3/
4*c^2*d^3*(a+b*arctanh(c*x))^2/x^4-1/3*c^3*d^3*(a+b*arctanh(c*x))^2/x^3+28/
15*a*b*c^6*d^3*ln(x)+113/45*b^2*c^6*d^3*ln(x)+37/20*b*c^6*d^3*(a+b*arctanh(
c*x))*ln(2/(-c*x+1))+1/60*b*c^6*d^3*(a+b*arctanh(c*x))*ln(2/(c*x+1))-113/90
*b^2*c^6*d^3*ln(-c^2*x^2+1)-14/15*b^2*c^6*d^3*polylog(2,-c*x)+14/15*b^2*c^6
*d^3*polylog(2,c*x)+37/40*b^2*c^6*d^3*polylog(2,1-2/(-c*x+1))-1/120*b^2*c^6
*d^3*polylog(2,1-2/(c*x+1))
```

Rubi [A] time = 0.51, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {43, 5938, 5916, 266, 44, 325, 206, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,-cx)+\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,cx)+\frac{37}{40}b^2c^6d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{1}{120}b^2c^6d^3\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```

```
[Out] -(b^2*c^2*d^3)/(60*x^4) - (b^2*c^3*d^3)/(10*x^3) - (61*b^2*c^4*d^3)/(180*x^
2) - (37*b^2*c^5*d^3)/(30*x) + (37*b^2*c^6*d^3*ArcTanh[c*x])/30 - (b*c*d^3*
(a + b*ArcTanh[c*x]))/(15*x^5) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/(10*x^4
) - (11*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(18*x^3) - (14*b*c^4*d^3*(a + b*Arc
Tanh[c*x]))/(15*x^2) - (11*b*c^5*d^3*(a + b*ArcTanh[c*x]))/(6*x) - (d^3*(a
+ b*ArcTanh[c*x])^2)/(6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/(5*x^5) - (
3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x])^2
)/(3*x^3) + (28*a*b*c^6*d^3*Log[x])/15 + (113*b^2*c^6*d^3*Log[x])/45 + (37*
b*c^6*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/20 + (b*c^6*d^3*(a + b*Arc
Tanh[c*x])*Log[2/(1 + c*x)])/60 - (113*b^2*c^6*d^3*Log[1 - c^2*x^2])/90 - (
14*b^2*c^6*d^3*PolyLog[2, -(c*x)])/15 + (14*b^2*c^6*d^3*PolyLog[2, c*x])/15
 + (37*b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/40 - (b^2*c^6*d^3*PolyLog[2
, 1 - 2/(1 + c*x)])/120
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^7}\,dx = -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} - \frac{3c^2d^3\left(a+b\tan\right.}{4x^4}$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} - \frac{3c^2d^3\left(a+b\tan\right.}{4x^4}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4} - \frac{11bc^3d^3\left(a+\right.}{18}$$

$$= -\frac{b^2c^3d^3}{10x^3} - \frac{14b^2c^5d^3}{15x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5} - \frac{3bc^2d^3\left(a+b\tanh^{-1}\right.}{10x^4}$$

$$= -\frac{b^2c^3d^3}{10x^3} - \frac{37b^2c^5d^3}{30x} + \frac{14}{15}b^2c^6d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5}$$

$$= -\frac{b^2c^2d^3}{60x^4} - \frac{b^2c^3d^3}{10x^3} - \frac{61b^2c^4d^3}{180x^2} - \frac{37b^2c^5d^3}{30x} + \frac{37}{30}b^2c^6d^3\tanh^{-1}(cx) - \frac{bc}{}$$

Mathematica [A] time = 1.39, size = 402, normalized size = 0.84

$$-\frac{d^3\left(60a^2c^3x^3 + 135a^2c^2x^2 + 108a^2cx + 30a^2 - 336abc^6x^6\log(cx) + 165abc^6x^6\log(1-cx) - 165abc^6x^6\log(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```

```
[Out] -1/180*(d^3*(30*a^2 + 108*a^2*c*x + 12*a*b*c*x + 135*a^2*c^2*x^2 + 54*a*b*c
^2*x^2 + 3*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^3
+ 168*a*b*c^4*x^4 + 61*b^2*c^4*x^4 + 330*a*b*c^5*x^5 + 222*b^2*c^5*x^5 - 64
*b^2*c^6*x^6 + 3*b^2*(10 + 36*c*x + 45*c^2*x^2 + 20*c^3*x^3 - 111*c^6*x^6)*
ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*(10 + 36*c*x + 45*c^2*x^2 + 20*c^3*x
^3) + b*c*x*(6 + 27*c*x + 55*c^2*x^2 + 84*c^3*x^3 + 165*c^4*x^4 - 111*c^5*x
^5) - 168*b*c^6*x^6*Log[1 - E^(-2*ArcTanh[c*x])]) - 336*a*b*c^6*x^6*Log[c*x
] + 165*a*b*c^6*x^6*Log[1 - c*x] - 165*a*b*c^6*x^6*Log[1 + c*x] - 452*b^2*c
```

```
^6*x^6*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 168*a*b*c^6*x^6*Log[1 - c^2*x^2] + 16
8*b^2*c^6*x^6*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^6
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\text{artanh}\,(cx)^2 + 2}{x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cdx + d)^3 (b\,\text{artanh}\,(cx) + a)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^7, x)
```

maple [A] time = 0.08, size = 736, normalized size = 1.54

$$-\frac{2c^3d^3ab\,\text{arctanh}\,(cx)}{3x^3} - \frac{3c^2d^3ab\,\text{arctanh}\,(cx)}{2x^4} - \frac{6c\,d^3ab\,\text{arctanh}\,(cx)}{5x^5} - \frac{3c^2d^3a^2}{4x^4} - \frac{3c\,d^3a^2}{5x^5} - \frac{14c^6d^3b^2\,\text{dilog}\,(cx)}{15} - \frac{d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x)
```

```
[Out] -2/3*c^3*d^3*a*b*arctanh(c*x)/x^3-3/4*c^2*d^3*a^2/x^4-3/5*c*d^3*a^2/x^5+1/2
40*c^6*d^3*b^2*ln(c*x+1)^2+113/45*c^6*d^3*b^2*ln(c*x)-14/15*c^6*d^3*b^2*dil
og(c*x)-37/80*c^6*d^3*b^2*ln(c*x-1)^2-1/6*d^3*b^2*arctanh(c*x)^2/x^6-1/3*c^
3*d^3*a^2/x^3+14/15*c^6*d^3*b^2*dilog(1/2+1/2*c*x)-337/180*c^6*d^3*b^2*ln(c
*x-1)-23/36*c^6*d^3*b^2*ln(c*x+1)-14/15*c^6*d^3*b^2*dilog(c*x+1)-3/2*c^2*d^
3*a*b*arctanh(c*x)/x^4-6/5*c*d^3*a*b*arctanh(c*x)/x^5-1/60*b^2*c^2*d^3/x^4-
1/10*b^2*c^3*d^3/x^3-61/180*b^2*c^4*d^3/x^2-37/30*b^2*c^5*d^3/x+37/40*c^6*d
^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/6*d^3*a^2/x^6-37/20*c^6*d^3*b^2*arctanh(
c*x)*ln(c*x-1)+28/15*c^6*d^3*a*b*ln(c*x)+28/15*c^6*d^3*b^2*arctanh(c*x)*ln(
c*x)-37/20*c^6*d^3*a*b*ln(c*x-1)-1/60*c^6*d^3*a*b*ln(c*x+1)-3/4*c^2*d^3*b^2
*arctanh(c*x)^2/x^4-11/18*c^3*d^3*b^2*arctanh(c*x)/x^3-3/5*c*d^3*b^2*arctan
h(c*x)^2/x^5-3/10*c^2*d^3*b^2*arctanh(c*x)/x^4-1/15*c*d^3*a*b/x^5-11/6*c^5*
d^3*a*b/x-14/15*c^4*d^3*a*b/x^2-1/3*d^3*a*b*arctanh(c*x)/x^6-1/15*c*d^3*b^2
*arctanh(c*x)/x^5-11/6*c^5*d^3*b^2*arctanh(c*x)/x-1/3*c^3*d^3*b^2*arctanh(c
*x)^2/x^3-14/15*c^4*d^3*b^2*arctanh(c*x)/x^2-1/60*c^6*d^3*b^2*arctanh(c*x)*
ln(c*x+1)-14/15*c^6*d^3*b^2*ln(c*x)*ln(c*x+1)-1/120*c^6*d^3*b^2*ln(-1/2*c*x
+1/2)*ln(c*x+1)+1/120*c^6*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-11/18*c^
3*d^3*a*b/x^3-3/10*c^2*d^3*a*b/x^4
```

maxima [B] time = 0.98, size = 961, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="maxima")
```

```
[Out] -14/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^6*d^
3 - 14/15*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^6*d^3 + 14/15*(l
og(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^6*d^3 - 23/60*b^2*c^6*d^3*log
(c*x + 1) - 97/60*b^2*c^6*d^3*log(c*x - 1) + 2*b^2*c^6*d^3*log(x) - 1/3*((c
^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c^3
*d^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^
3)*c - 6*arctanh(c*x)/x^4)*a*b*c^2*d^3 - 3/10*((2*c^4*log(c^2*x^2 - 1) - 2*
c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*c*d^3 + 1/9
0*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x^2 +
 3)/x^5)*c - 30*arctanh(c*x)/x^6)*a*b*d^3 + 1/360*((184*c^4*log(x) - (15*c^
4*x^4*log(c*x + 1)^2 + 15*c^4*x^4*log(c*x - 1)^2 + 92*c^4*x^4*log(c*x - 1)
+ 32*c^2*x^2 - 2*(15*c^4*x^4*log(c*x - 1) - 46*c^4*x^4)*log(c*x + 1) + 6)/x
^4)*c^2 + 4*(15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*
c^2*x^2 + 3)/x^5)*c*arctanh(c*x))*b^2*d^3 - 1/3*a^2*c^3*d^3/x^3 - 3/4*a^2*c
^2*d^3/x^4 - 3/5*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctanh(c*x)^2/x^6 - 1/6*a^2*d
^3/x^6 - 1/240*(296*b^2*c^5*d^3*x^4 + 60*b^2*c^4*d^3*x^3 + 24*b^2*c^3*d^3*x
^2 + (11*b^2*c^6*d^3*x^5 + 20*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x + 36*b^2*c
*d^3)*log(c*x + 1)^2 - (101*b^2*c^6*d^3*x^5 - 20*b^2*c^3*d^3*x^2 - 45*b^2*c
^2*d^3*x - 36*b^2*c*d^3)*log(-c*x + 1)^2 + 4*(45*b^2*c^5*d^3*x^4 + 28*b^2*c
^4*d^3*x^3 + 15*b^2*c^3*d^3*x^2 + 9*b^2*c^2*d^3*x)*log(c*x + 1) - 2*(90*b^2
*c^5*d^3*x^4 + 56*b^2*c^4*d^3*x^3 + 30*b^2*c^3*d^3*x^2 + 18*b^2*c^2*d^3*x +
 (11*b^2*c^6*d^3*x^5 + 20*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x + 36*b^2*c*d^3
)*log(c*x + 1))*log(-c*x + 1))/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(c\,x))^2\,(d+c\,d\,x)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^7,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a^2}{x^7}\,dx + \int \frac{3a^2c}{x^6}\,dx + \int \frac{3a^2c^2}{x^5}\,dx + \int \frac{a^2c^3}{x^4}\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{x^7}\,dx + \int \frac{2ab\,\mathrm{atanh}\,(cx)}{x^7}\,dx + \int \frac{3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**7,x)
```

```
[Out] d**3*(Integral(a**2/x**7, x) + Integral(3*a**2*c/x**6, x) + Integral(3*a**2
*c**2/x**5, x) + Integral(a**2*c**3/x**4, x) + Integral(b**2*atanh(c*x)**2/
x**7, x) + Integral(2*a*b*atanh(c*x)/x**7, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**6, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**5, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**4, x) + Integral(6*a*b*c*atanh(c*x)/x**6, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**5, x) + Integral(2*a*b*c**3*atanh(c*x)/x**4, x
))
```

3.95 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=329

$$-\frac{b\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}+\frac{11\left(a+b\tanh^{-1}(cx)\right)^2}{6c^4d}-\frac{8b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^4d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\right.}{c^4d}$$

```
[Out] -a*b*x/c^3/d+1/3*b^2*x/c^3/d-1/3*b^2*arctanh(c*x)/c^4/d-b^2*x*arctanh(c*x)/
c^3/d+1/3*b*x^2*(a+b*arctanh(c*x))/c^2/d+11/6*(a+b*arctanh(c*x))^2/c^4/d+x*
(a+b*arctanh(c*x))^2/c^3/d-1/2*x^2*(a+b*arctanh(c*x))^2/c^2/d+1/3*x^3*(a+b*
arctanh(c*x))^2/c/d-8/3*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4/d+(a+b*arct
anh(c*x))^2*ln(2/(c*x+1))/c^4/d-1/2*b^2*ln(-c^2*x^2+1)/c^4/d-4/3*b^2*polylo
g(2,1-2/(-c*x+1))/c^4/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^4/d-1
/2*b^2*polylog(3,1-2/(c*x+1))/c^4/d
```

Rubi [A] time = 0.84, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5930, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 5910, 260, 5948, 6056, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^4d}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d}-\frac{x^2\left(a+b\tanh\right.}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] -((a*b*x)/(c^3*d)) + (b^2*x)/(3*c^3*d) - (b^2*ArcTanh[c*x])/(3*c^4*d) - (b^
2*x*ArcTanh[c*x])/(c^3*d) + (b*x^2*(a + b*ArcTanh[c*x]))/(3*c^2*d) + (11*(a
 + b*ArcTanh[c*x])^2)/(6*c^4*d) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d) - (x^2
*(a + b*ArcTanh[c*x])^2)/(2*c^2*d) + (x^3*(a + b*ArcTanh[c*x])^2)/(3*c*d) -
 (8*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^4*d) + ((a + b*ArcTanh[c*
x])^2*Log[2/(1 + c*x)])/(c^4*d) - (b^2*Log[1 - c^2*x^2])/(2*c^4*d) - (4*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^4*d) - (b*(a + b*ArcTanh[c*x])*PolyLog[2
, 1 - 2/(1 + c*x)])/(c^4*d) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c}+\frac{\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}$$

$$=\frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3cd}+\frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^2}-\frac{(2b)\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{3d}-\int x(\ldots$$

$$=-\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3cd}-\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^3}+\int\left(a+\ldots\right.$$

$$=\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^4d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d}-\frac{x^2\left(a+b\ldots\right.}{}$$

$$=-\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a+b\tanh^{-1}(cx)\right)^2}{6c^4d}+\frac{x\left(a+b\tanh\ldots\right.}{c^3d}$$

$$=-\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a\ldots\right.}{}$$

$$=-\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a\ldots\right.}{}$$

$$=-\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a\ldots\right.}{}$$

Mathematica [A] time = 0.83, size = 347, normalized size = 1.05

$$-\frac{a^2\log(cx+1)}{c^4d}+\frac{a^2x}{c^3d}-\frac{a^2x^2}{2c^2d}+\frac{a^2x^3}{3cd}+\frac{ab\left(-8\log\left(\frac{1}{\sqrt{1-c^2x^2}}\right)+\left(1-c^2x^2\right)\left(-2cx\tanh^{-1}(cx)+3\tanh^{-1}(cx)-1\right)-3\ldots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a^2*x)/(c^3*d) - (a^2*x^2)/(2*c^2*d) + (a^2*x^3)/(3*c*d) - (a^2*Log[1 + c*
x])/(c^4*d) + (a*b*(-3*c*x + 8*c*x*ArcTanh[c*x] + (1 - c^2*x^2)*(-1 + 3*Arc
Tanh[c*x] - 2*c*x*ArcTanh[c*x]) + 6*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x]
)] - 8*Log[1/Sqrt[1 - c^2*x^2]] - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(3*c
^4*d) + (b^2*(2*c*x - 6*c*x*ArcTanh[c*x] - 2*(1 - c^2*x^2)*ArcTanh[c*x] - 8
```

```
*ArcTanh[c*x]^2 + 8*c*x*ArcTanh[c*x]^2 + 3*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 2
*c*x*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 16*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*Log[1/Sqrt[1 - c^
2*x^2]] + (8 - 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 3*PolyLog
[3, -E^(-2*ArcTanh[c*x])]))/(6*c^4*d)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^3\operatorname{artanh}(cx)^2 + 2\,abx^3\operatorname{artanh}(cx) + a^2x^3}{cdx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c*d*x
 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx) + a)^2x^3}{cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d), x)
```

maple [C] time = 2.19, size = 1298, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] -a*b*x/c^3/d-b^2*x*arctanh(c*x)/c^3/d+1/3*b^2*x/c^3/d-4/3*b^2*arctanh(c*x)/
d/c^4-1/2/c^2*a^2/d*x^2+1/c^3*a^2/d*x+1/3/c*a^2/d*x^3+1/c^4*b^2/d*ln(1+(c*x
+1)^2/(-c^2*x^2+1))-2/3/c^4*b^2/d*arctanh(c*x)^3+I/c^4*b^2/d*Pi*csgn(I*(c*x
+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+1/2*
I/c^4*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*arctanh(c*x)^2+1/2*I/c^4*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-
1/2*I/c^4*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+11/6/c^4*b^2/d*arctanh(c*x)
^2-1/2/c^4*b^2/d*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-8/3/c^4*b^2/d*dilog(1+I
*(c*x+1)/(-c^2*x^2+1)^(1/2))-8/3/c^4*b^2/d*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(
1/2))-4/3/c^4*a*b/d-1/c^4*a^2/d*ln(c*x+1)-1/2*I/c^4*b^2/d*Pi*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^
2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/3/c^4*b^2/d-2/c^4*a*b/d*a
rctanh(c*x)*ln(c*x+1)-1/c^4*a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/c^4*a*b/d*ln
(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+2/3/c*a*b/d*x^3*arctanh(c*x)-1/c^2*a*b/d*arc
tanh(c*x)*x^2+2/c^3*a*b/d*arctanh(c*x)*x+1/2*I/c^4*b^2/d*Pi*csgn(I*(c*x+1)^
2/(c^2*x^2-1))^3*arctanh(c*x)^2+1/2*I/c^4*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^
2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/c^4*b^2/d*arctanh(c*x)*
polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/3/c^2*a*b/d*x^2+11/6/c^4*a*b/d*ln(c*x+
1)+1/c^3*b^2/d*arctanh(c*x)^2*x+1/3/c^2*b^2/d*arctanh(c*x)*x^2+1/3/c*b^2/d*
x^3*arctanh(c*x)^2+2/c^4*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)
)-1/c^4*b^2/d*arctanh(c*x)^2*ln(c*x+1)-8/3/c^4*b^2/d*arctanh(c*x)*ln(1-I*(c
*x+1)/(-c^2*x^2+1)^(1/2))+1/c^4*b^2/d*arctanh(c*x)^2*ln(2)+1/2/c^4*a*b/d*ln
(c*x+1)^2+5/6/c^4*a*b/d*ln(c*x-1)+1/c^4*a*b/d*dilog(1/2+1/2*c*x)-8/3/c^4*b^
```

```
2/d*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2/c^2*b^2/d*arctanh(c
*x)^2*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,a^2\left(\frac{2\,c^2x^3-3\,cx^2+6\,x}{c^3d}-\frac{6\,\log\left(cx+1\right)}{c^4d}\right)+\frac{\left(2\,b^2c^3x^3-3\,b^2c^2x^2+6\,b^2cx-6\,b^2\log\left(cx+1\right)\right)\log\left(-cx+1\right)^2}{24\,c^4d}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*a^2*((2*c^2*x^3 - 3*c*x^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d)) + 1/
24*(2*b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 6*b^2*c*x - 6*b^2*log(c*x + 1))*log(-c*
x + 1)^2/(c^4*d) - integrate(-1/12*(3*(b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*x +
 1)^2 + 12*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1) - (3*b^2*c^2*x^2 + 2*(6
*a*b*c^4 + b^2*c^4)*x^4 + 6*b^2*c*x - (12*a*b*c^3 + b^2*c^3)*x^3 + 6*(b^2*c
^4*x^4 - b^2*c^3*x^3 - b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^5*d*x
^2 - c^3*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
```

```
[Out] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{cx+1}\,dx+\int \frac{b^2x^3\,\mathrm{atanh}^2\left(cx\right)}{cx+1}\,dx+\int \frac{2abx^3\,\mathrm{atanh}\left(cx\right)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x**3/(c*x + 1), x) + Integral(b**2*x**3*atanh(c*x)**2/(c*x +
 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c*x + 1), x))/d
```

3.96 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=247

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d}+\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\right.}{c^3d}$$

```
[Out] a*b*x/c^2/d+b^2*x*arctanh(c*x)/c^2/d-3/2*(a+b*arctanh(c*x))^2/c^3/d-x*(a+b*
arctanh(c*x))^2/c^2/d+1/2*x^2*(a+b*arctanh(c*x))^2/c/d+2*b*(a+b*arctanh(c*x
))*ln(2/(-c*x+1))/c^3/d-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^3/d+1/2*b^2*ln
(-c^2*x^2+1)/c^3/d+b^2*polylog(2,1-2/(-c*x+1))/c^3/d+b*(a+b*arctanh(c*x))*p
olylog(2,1-2/(c*x+1))/c^3/d+1/2*b^2*polylog(3,1-2/(c*x+1))/c^3/d
```

Rubi [A] time = 0.53, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5930, 5916, 5980, 5910, 260, 5948, 5984, 5918, 2402, 2315, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^3d}+\frac{abx}{c^2d}-\frac{x\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a*b*x)/(c^2*d) + (b^2*x*ArcTanh[c*x])/(c^2*d) - (3*(a + b*ArcTanh[c*x])^2)
/(2*c^3*d) - (x*(a + b*ArcTanh[c*x])^2)/(c^2*d) + (x^2*(a + b*ArcTanh[c*x])
^2)/(2*c*d) + (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d) - ((a + b
*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d) + (b^2*Log[1 - c^2*x^2])/(2*c^3*
d) + (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^3*d) + (b*(a + b*ArcTanh[c*x])*Po
lyLog[2, 1 - 2/(1 + c*x)])/(c^3*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c
^3*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)^2}{d + cdx} dx = -\frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx} dx}{c} + \frac{\int x \left(a + b \tanh^{-1}(cx)\right)^2 dx}{cd}$$

$$= \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)^2}{2cd} + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx} dx}{c^2} - \frac{b \int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2} dx}{d} - \frac{\int \left(a \right.}{}$$

$$= -\frac{x \left(a + b \tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)^2}{2cd} - \frac{\left(a + b \tanh^{-1}(cx)\right)^2 \log\left(\frac{}{1-}\right.}{c^3 d}$$

$$= \frac{abx}{c^2 d} - \frac{3 \left(a + b \tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x \left(a + b \tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)^2}{2cd}$$

$$= \frac{abx}{c^2 d} + \frac{b^2 x \tanh^{-1}(cx)}{c^2 d} - \frac{3 \left(a + b \tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x \left(a + b \tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2 \left(a\right.}{}$$

$$= \frac{abx}{c^2 d} + \frac{b^2 x \tanh^{-1}(cx)}{c^2 d} - \frac{3 \left(a + b \tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x \left(a + b \tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2 \left(a\right.}{}$$

$$= \frac{abx}{c^2 d} + \frac{b^2 x \tanh^{-1}(cx)}{c^2 d} - \frac{3 \left(a + b \tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x \left(a + b \tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2 \left(a\right.}{}$$

Mathematica [A] time = 0.52, size = 260, normalized size = 1.05

$$\frac{a^2c^2x^2 - 2a^2cx + 2a^2 \log(cx+1) - 2ab \log\left(1 - c^2x^2\right) + 2abc^2x^2 \tanh^{-1}(cx) + 2b\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)\left(a + b \tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (-2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 - 2*a*b*ArcTanh[c*x] - 4*a*b*c*x*ArcT
anh[c*x] + 2*b^2*c*x*ArcTanh[c*x] + 2*a*b*c^2*x^2*ArcTanh[c*x] + b^2*ArcTan
h[c*x]^2 - 2*b^2*c*x*ArcTanh[c*x]^2 + b^2*c^2*x^2*ArcTanh[c*x]^2 - 4*a*b*Ar
cTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 4*b^2*ArcTanh[c*x]*Log[1 + E^(-2*
ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 2*a^2*
Log[1 + c*x] - 2*a*b*Log[1 - c^2*x^2] + b^2*Log[1 - c^2*x^2] + 2*b*(a - b +
 b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -E^(-2*A
rcTanh[c*x])])/(2*c^3*d)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^2 \operatorname{artanh}(cx)^2 + 2\,abx^2 \operatorname{artanh}(cx) + a^2x^2}{cdx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c*d*x
 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2 x^2}{cdx + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d), x)
```

maple [C] time = 1.46, size = 1192, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] -I/c^3*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^
2-1))^2*arctanh(c*x)^2+1/2*I/c^3*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*
I/c^3*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I/c^3*b^2/d*Pi*csgn(I*
(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+
a*b*x/c^2/d+b^2*x*arctanh(c*x)/c^2/d+1/c^3*a*b/d+1/2/c*a^2/d*x^2-1/c^2*a^2/
d*x+1/c^3*a^2/d*ln(c*x+1)+1/c^3*b^2/d*arctanh(c*x)-3/2/c^3*b^2/d*arctanh(c*
x)^2+1/2/c^3*b^2/d*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+2/c^3*b^2/d*dilog(1+I
*(c*x+1)/(-c^2*x^2+1)^(1/2))+2/c^3*b^2/d*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/
2))-1/c^3*b^2/d*ln(1+(c*x+1)^2/(-c^2*x^2+1))+2/3/c^3*b^2/d*arctanh(c*x)^3+1
/2*I/c^3*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*
x)^2-1/2*I/c^3*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))^3*arctanh(c*x)^2-1/2*I/c^3*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*a
rctanh(c*x)^2+1/c*a*b/d*arctanh(c*x)*x^2-2/c^2*a*b/d*arctanh(c*x)*x+2/c^3*a
*b/d*arctanh(c*x)*ln(c*x+1)+1/c^3*a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/c^3*a*
b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+2/c^3*b^2/d*arctanh(c*x)*ln(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))-1/c^3*a*b/d*dilog(1/2+1/2*c*x)-3/2/c^3*a*b/d*ln(c*x+1
)-1/2/c^3*a*b/d*ln(c*x-1)+2/c^3*b^2/d*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2
+1)^(1/2))-2/c^3*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/c^3*
b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/2/c^3*a*b/d*ln(c*x+
1)^2-1/c^2*b^2/d*arctanh(c*x)^2*x+1/2/c*b^2/d*arctanh(c*x)^2*x^2-1/c^3*b^2/
d*arctanh(c*x)^2*ln(2)+1/c^3*b^2/d*arctanh(c*x)^2*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{cx^2-2x}{c^2d}+\frac{2\log(cx+1)}{c^3d}\right)+\frac{\left(b^2c^2x^2-2b^2cx+2b^2\log(cx+1)\right)\log(-cx+1)^2}{8c^3d}-\int-\frac{\left(b^2c^3x^3-b^2c^2x^2\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d)) + 1/8*(b^2*c^2*x^2
 - 2*b^2*c*x + 2*b^2*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d) - integrate(-1/4
*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*x^2
)*log(c*x + 1) + (2*b^2*c*x - (4*a*b*c^3 + b^2*c^3)*x^3 + (4*a*b*c^2 + b^2*
c^2)*x^2 - 2*(b^2*c^3*x^3 - b^2*c^2*x^2 + b^2*c*x + b^2)*log(c*x + 1))*log(
-c*x + 1))/(c^4*d*x^2 - c^2*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{d+c\,d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{cx+1}\,dx + \int \frac{b^2x^2\operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2abx^2\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x**2/(c*x + 1), x) + Integral(b**2*x**2*atanh(c*x)**2/(c*x +
 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c*x + 1), x))/d
```

3.97 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=172

$$-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\,\text{ta}\right.}{c^2d}$$

```
[Out] (a+b*arctanh(c*x))^2/c^2/d+x*(a+b*arctanh(c*x))^2/c/d-2*b*(a+b*arctanh(c*x)
)*ln(2/(-c*x+1))/c^2/d+(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^2/d-b^2*polylog
(2,1-2/(-c*x+1))/c^2/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^2/d-1/
2*b^2*polylog(3,1-2/(c*x+1))/c^2/d
```

Rubi [A] time = 0.31, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5930, 5910, 5984, 5918, 2402, 2315, 5948, 6056, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{\left(a+b\tanh^{-1}(c\right.}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a + b*ArcTanh[c*x])^2/(c^2*d) + (x*(a + b*ArcTanh[c*x])^2)/(c*d) - (2*b*(a
 + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^2*d) + ((a + b*ArcTanh[c*x])^2*Log[
2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^2*d) - (b*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[3, 1 -
 2/(1 + c*x)])/(2*c^2*d)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
```

```
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c} + \frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}$$

$$= \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d} - \frac{(2b)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}}{d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} - \frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c^2d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} - \frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c^2d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} - \frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c^2d}$$

Mathematica [A] time = 0.49, size = 140, normalized size = 0.81

$$\frac{2a\left(acx-a\log(cx+1)+b\log\left(1-c^2x^2\right)\right)-2b\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)-b\right)+4b\tanh^{-1}(cx)\left((a\right.}{2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (2*b^2*ArcTanh[c*x]^2*(-1 + c*x + Log[1 + E^(-2*ArcTanh[c*x])]) + 4*b*ArcTa
nh[c*x]*(a*c*x + (a - b)*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*a*(a*c*x - a*Log
[1 + c*x] + b*Log[1 - c^2*x^2]) - 2*b*(a - b + b*ArcTanh[c*x])*PolyLog[2, -
E^(-2*ArcTanh[c*x])] - b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(2*c^2*d)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 x \operatorname{artanh}(cx)^2 + 2\, abx \operatorname{artanh}(cx) + a^2 x}{cdx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c*d*x + d),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2 x}{cdx + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d), x)
```

maple [C] time = 0.57, size = 5361, normalized size = 31.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{x}{cd} - \frac{\log(cx+1)}{c^2 d}\right) + \frac{\left(b^2 cx - b^2 \log(cx+1)\right)\log(-cx+1)^2}{4\, c^2 d} - \int -\frac{\left(b^2 c^2 x^2 - b^2 cx\right)\log(cx+1)^2 + 4\left(abc^2 x^2 - a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(x/(c*d) - log(c*x + 1)/(c^2*d)) + 1/4*(b^2*c*x - b^2*log(c*x + 1))*log
(-c*x + 1)^2/(c^2*d) - integrate(-1/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x + 1)
^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*log(c*x + 1) - 2*((2*a*b*c^2 + b^2*c^2)*x^2
- (2*a*b*c - b^2*c)*x + (b^2*c^2*x^2 - 2*b^2*c*x - b^2)*log(c*x + 1))*log(-
c*x + 1))/(c^3*d*x^2 - c*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a + b\,\mathrm{atanh}(c\,x))^2}{d + c\,d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
```

```
[Out] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{cx+1}\,dx + \int \frac{b^2x\operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2abx\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x/(c*x + 1), x) + Integral(b**2*x*atanh(c*x)**2/(c*x + 1), x
) + Integral(2*a*b*x*atanh(c*x)/(c*x + 1), x))/d
```

3.98 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=84

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{cd}+\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2cd}$$

```
[Out] -(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c/d+b*(a+b*arctanh(c*x))*polylog(2,1-2/
(c*x+1))/c/d+1/2*b^2*polylog(3,1-2/(c*x+1))/c/d
```

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5918, 5948, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c*d)) + (b*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 + c*x)])/(c*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2
*c*d)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{(2b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{cd} - \frac{b^2\int \frac{\mathrm{Li}_2(}{1}}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{cd} + \frac{b^2\mathrm{Li}_3(1}{2c}$$

Mathematica [A] time = 0.21, size = 102, normalized size = 1.21

$$\frac{2a^2\log(cx+1)+2b\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)\right)-4ab\tanh^{-1}(cx)\log\left(e^{-2\tanh^{-1}(cx)}+1\right)+b^2\mathrm{Li}_3\left(-e\right.}{2cd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

```
[Out] (-4*a*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*Lo
g[1 + E^(-2*ArcTanh[c*x])] + 2*a^2*Log[1 + c*x] + 2*b*(a + b*ArcTanh[c*x])*
PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(2
*c*d)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\,(cx)^2+2\,ab\,\mathrm{artanh}\,(cx)+a^2}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)^2}{cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(c*d*x + d), x)
```

maple [C] time = 0.40, size = 822, normalized size = 9.79

$$\frac{a^2\ln(cx+1)}{cd}+\frac{b^2\,\mathrm{arctanh}\,(cx)^2\ln(cx+1)}{cd}-\frac{2b^2\,\mathrm{arctanh}\,(cx)^2\ln\left(\frac{cx+1}{\sqrt{-c^2x^2+1}}\right)}{cd}+\frac{2b^2\,\mathrm{arctanh}\,(cx)^3}{3cd}-\frac{ib^2\pi\mathrm{csgn}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] 1/c*a^2/d*ln(c*x+1)+1/c*b^2/d*arctanh(c*x)^2*ln(c*x+1)-2/c*b^2/d*arctanh(c*
x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/3/c*b^2/d*arctanh(c*x)^3-1/2*I/c*b^2/
```

```
d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arc
tanh(c*x)^2+1/2*I/c*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I/c*b^2/d*Pi*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I/c*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^
2-1))^3*arctanh(c*x)^2-1/2*I/c*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/2*I/c*b^2/d*Pi*csgn(I/(1+(c*x+1)^2
/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-I/c*b^2/d*Pi*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-1/c*b^2/d*ar
ctanh(c*x)^2*ln(2)-1/c*b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1)
)+1/2/c*b^2/d*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+2/c*a*b/d*arctanh(c*x)*ln(
c*x+1)-1/2/c*a*b/d*ln(c*x+1)^2+1/c*a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/c*a*b
/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/c*a*b/d*dilog(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2\log(cx+1)\log(-cx+1)^2}{4\,cd}+\frac{a^2\log(cdx+d)}{cd}-\int-\frac{\left(b^2cx-b^2\right)\log(cx+1)^2+4\,(abcx-ab)\log(cx+1)-4\left(b^2\right.}{4\left(c^2dx^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/(c*d) + a^2*log(c*d*x + d)/(c*d) - int
egrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1
) - 4*(b^2*c*x*log(c*x + 1) + a*b*c*x - a*b)*log(-c*x + 1))/(c^2*d*x^2 - d)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\mathrm{atanh}(c\,x))^2}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + c*d*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(d + c*d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^2}{cx+1}\,dx+\int\frac{b^2\,\mathrm{atanh}^2(cx)}{cx+1}\,dx+\int\frac{2ab\,\mathrm{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(c*x + 1), x) +
Integral(2*a*b*atanh(c*x)/(c*x + 1), x))/d
```

3.99 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx$

Optimal. Leaf size=77

$$-\frac{b\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{b^2\text{Li}_3\left(\frac{2}{cx+1}-1\right)}{2d}$$

```
[Out] (a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-b*(a+b*arctanh(c*x))*polylog(2,-1+2/
(c*x+1))/d-1/2*b^2*polylog(3,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.17, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5932, 5948, 6056, 6610}

$$-\frac{b\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b^2\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

```
[Out] ((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (b*(a + b*ArcTanh[c*x])*P
olyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*PolyLog[3, -1 + 2/(1 + c*x)])/(2*d)
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx = \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{(2bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(-1+\frac{2}{1+cx}\right)}{d} + \frac{\left(b^2c\right)\int}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(-1+\frac{2}{1+cx}\right)}{d} - \frac{b^2\mathrm{Li}_3\left(\right.}{}$$

Mathematica [C] time = 0.39, size = 132, normalized size = 1.71

$$\frac{a^2\log(cx)-a^2\log(cx+1)+ab\left(2\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-\mathrm{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)\right)+b^2\left(\tanh^{-1}(cx)\mathrm{Li}_2\left(e^{2}\right.\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

```
[Out] (a^2*Log[c*x] - a^2*Log[1 + c*x] + a*b*(2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTan
h[c*x])] - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^2*((I/24)*Pi^3 - (2*ArcTanh
[c*x]^3)/3 + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*Poly
Log[2, E^(2*ArcTanh[c*x])] - PolyLog[3, E^(2*ArcTanh[c*x])]/2))/d
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\,(cx)^2+2\,ab\,\mathrm{artanh}\,(cx)+a^2}{cdx^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^2 + d*x), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)^2}{(cdx+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x), x)
```

maple [C] time = 0.35, size = 1389, normalized size = 18.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d),x)
```

```
[Out] -1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*b^2/d*Pi*csgn(
I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
```

```
2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh
(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+I*b^2/d*Pi*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+1/2*I
*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)*arctanh(c*x)^2-a^2/d*ln(c*x+1)-2/3*b^2/d*arctanh(c*x)^3+a^2/d*ln(c*x)-2*b
^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*polylog(3,-(c*x+1)/(-c^2
*x^2+1)^(1/2))+1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^
2+1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3
*arctanh(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)
^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-1/2*I*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2
*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+b^2/d*arctanh(c*x)^2*ln(1-
(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^
2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln(2)+2*b^2/d*arctanh(c*x)*polylog(2,-(c*x
+1)/(-c^2*x^2+1)^(1/2))-b^2/d*arctanh(c*x)^2*ln(c*x+1)+a*b/d*dilog(1/2+1/2*
c*x)+2*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*a*b/d*ln(c*x
+1)^2-b^2/d*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-a*b/d*dilog(c*x)-a*
b/d*dilog(c*x+1)-2*a*b/d*arctanh(c*x)*ln(c*x+1)-a*b/d*ln(-1/2*c*x+1/2)*ln(c
*x+1)+a*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+2*a*b/d*arctanh(c*x)*ln(c*x)-a
*b/d*ln(c*x)*ln(c*x+1)+b^2/d*arctanh(c*x)^2*ln(c*x)+b^2/d*arctanh(c*x)^2*ln
(1+(c*x+1)/(-c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2\log(cx+1)\log(-cx+1)^2}{4\,d}-a^2\left(\frac{\log(cx+1)}{d}-\frac{\log(x)}{d}\right)+\int\frac{\left(b^2cx-b^2\right)\log(cx+1)^2+4\,(abcx-ab)\log(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/d - a^2*(log(c*x + 1)/d - log(x)/d) +
 integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x
+ 1) - 2*(2*a*b*c*x - 2*a*b - (b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x +
1))/(c^2*d*x^3 - d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\mathrm{atanh}\,(c\,x))^2}{x\,(d+c\,d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^2}{cx^2+x}\,dx+\int\frac{b^2\,\mathrm{atanh}^2\,(cx)}{cx^2+x}\,dx+\int\frac{2ab\,\mathrm{atanh}\,(cx)}{cx^2+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**2 + x), x) + Integral(b**2*atanh(c*x)**2/(c*x**2 + x),
 x) + Integral(2*a*b*atanh(c*x)/(c*x**2 + x), x))/d
```

3.100 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx$

Optimal. Leaf size=162

$$\frac{bc\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}\right.}{d}$$

```
[Out] c*(a+b*arctanh(c*x))^2/d-(a+b*arctanh(c*x))^2/d/x+2*b*c*(a+b*arctanh(c*x))*
ln(2-2/(c*x+1))/d-c*(a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-b^2*c*polylog(2,
-1+2/(c*x+1))/d+b*c*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))/d+1/2*b^2*c*
polylog(3,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.41, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5934, 5916, 5988, 5932, 2447, 5948, 6056, 6610}

$$\frac{bc\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b^2c\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d}+\frac{b^2c\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{c\left(a+b\tanh\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) + (2*b*c*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (c*(a + b*ArcTanh[c*x])^2*Log[2 -
2/(1 + c*x)])/d - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b*c*(a + b*ArcT
anh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c*PolyLog[3, -1 + 2/(1 + c
*x)])/(2*d)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{(2bc)\int \frac{a+b\tanh^{-1}(c}{x\left(1-c^2x^2\right)}}{d}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right.}{d}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+}\right.}{d}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+}\right.}{d}$$

Mathematica [C] time = 0.62, size = 225, normalized size = 1.39

$$a^2(-c)\log(x)+a^2c\log(cx+1)-\frac{a^2}{x}+\frac{ab\left(2cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+cx\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)-2\tanh^{-1}(cx)\left(cx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+1\right)\right)}{x}+b^2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

```
[Out] (-(a^2/x) - a^2*c*Log[x] + a^2*c*Log[1 + c*x] + (a*b*(-2*ArcTanh[c*x]*(1 +
c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]] + c*
x*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x + b^2*c*((-1/24*I)*Pi^3 + ArcTanh[c*x
]^2 - ArcTanh[c*x]^2/(c*x) + (2*ArcTanh[c*x]^3)/3 + 2*ArcTanh[c*x]*Log[1 -
E^(-2*ArcTanh[c*x])] - ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - PolyLog
[2, E^(-2*ArcTanh[c*x])] - ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + Po
lyLog[3, E^(2*ArcTanh[c*x])]/2))/d
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{cdx^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^3 + d*x^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^2), x)
```

maple [C] time = 0.95, size = 7232, normalized size = 44.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{c \log(cx+1)}{d} - \frac{c \log(x)}{d} - \frac{1}{dx}\right) + \frac{\left(b^2 cx \log(cx+1) - b^2\right) \log(-cx+1)^2}{4\,dx} - \int -\frac{\left(b^2 cx - b^2\right) \log(cx+1)^2 + 4\,(ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/4*(b^2*c*x*log(c*x + 1) -
 b^2)*log(-c*x + 1)^2/(d*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1)^
2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(b^2*c^2*x^2 + 2*a*b - (2*a*b*c - b^
2*c)*x - (b^2*c^3*x^3 + b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x
 + 1))/(c^2*d*x^4 - d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^2}{x^2\,(d + c\,d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx^3+x^2}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{cx^3+x^2}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{cx^3+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**3 + x**2), x) + Integral(b**2*atanh(c*x)**2/(c*x**3 +
x**2), x) + Integral(2*a*b*atanh(c*x)/(c*x**3 + x**2), x))/d
```

3.101 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^3(d+cdx)}\,dx$

Optimal. Leaf size=250

$$-\frac{bc^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}+\frac{c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{2bc^2\log\left(2\right.}{}$$

```
[Out] -b*c*(a+b*arctanh(c*x))/d/x-1/2*c^2*(a+b*arctanh(c*x))^2/d-1/2*(a+b*arctanh
(c*x))^2/d/x^2+c*(a+b*arctanh(c*x))^2/d/x+b^2*c^2*ln(x)/d-1/2*b^2*c^2*ln(-c
^2*x^2+1)/d-2*b*c^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d+c^2*(a+b*arctanh(c
*x))^2*ln(2-2/(c*x+1))/d+b^2*c^2*polylog(2,-1+2/(c*x+1))/d-b*c^2*(a+b*arcta
nh(c*x))*polylog(2,-1+2/(c*x+1))/d-1/2*b^2*c^2*polylog(3,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.63, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5934, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 6056, 6610}

$$-\frac{bc^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{b^2c^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}-\frac{c^2\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/(d*x)) - (c^2*(a + b*ArcTanh[c*x])^2)/(2*d) -
(a + b*ArcTanh[c*x])^2/(2*d*x^2) + (c*(a + b*ArcTanh[c*x])^2)/(d*x) + (b^2*
c^2*Log[x])/d - (b^2*c^2*Log[1 - c^2*x^2])/(2*d) - (2*b*c^2*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)])/d + (c^2*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 +
c*x)])/d + (b^2*c^2*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b*c^2*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*c^2*PolyLog[3, -1 + 2/(1 + c*
x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx-\frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d}+\frac{(bc)}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right.}{d}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\,\text{ta}\right.}{c}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\,\text{ta}\right.}{c}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\,\text{ta}\right.}{c}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\,\text{ta}\right.}{c}$$

Mathematica [C] time = 1.04, size = 317, normalized size = 1.27

$$\frac{2a^2c^2\log(x)-2a^2c^2\log(cx+1)+\frac{2a^2c}{x}-\frac{a^2}{x^2}+\frac{2ab\left(-c^2x^2\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)-cx\left(2cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+1\right)+\tanh^{-1}(cx)\left(c^2x^2+2c^2x^2\log\left(1-\right.\right.\right.}{x^2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

```
[Out] (-(a^2/x^2) + (2*a^2*c)/x + 2*a^2*c^2*Log[x] - 2*a^2*c^2*Log[1 + c*x] + (2*
a*b*(ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-2*ArcTanh[c
*x])]) - c*x*(1 + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - c^2*x^2*PolyLog[2,
E^(-2*ArcTanh[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTanh[c*x]/(c*x) -
ArcTanh[c*x]^2/2 - ArcTanh[c*x]^2/(2*c^2*x^2) + ArcTanh[c*x]^2/(c*x) - (2*A
rcTanh[c*x]^3)/3 - 2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + ArcTanh[c*
x]^2*Log[1 - E^(2*ArcTanh[c*x])] + Log[(c*x)/Sqrt[1 - c^2*x^2]] + PolyLog[2
, E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - Poly
Log[3, E^(2*ArcTanh[c*x])]/2))/(2*d)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2}{cdx^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^4 + d*x^3),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^3), x)
```

maple [C] time = 2.10, size = 1841, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x)
```

```
[Out] c^2*b^2/d*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+c^2*a^2/d*ln(c*x)-2*c^2*b^2/d*di
log(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1
)^(1/2))-2*c^2*b^2/d*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^2*b^2/d*dil
og((c*x+1)/(-c^2*x^2+1)^(1/2))-c^2*a^2/d*ln(c*x+1)-2/3*c^2*b^2/d*arctanh(c*
x)^3+c^2*b^2/d*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-c^2*b^2/d*arctanh(c*x)+3/2*
c^2*b^2/d*arctanh(c*x)^2-1/2*b^2/d*arctanh(c*x)^2/x^2+c*a^2/d/x-1/2*I*c^2*b
^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+1/2*I
*c^2*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arct
anh(c*x)^2+I*c^2*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2*arctanh(c*x)^2-1/2*I*c^2*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c
^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2
*arctanh(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(
I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*
c^2*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+
(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I*(c*x+1)
/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-1/2*I*c
^2*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+2*c*a*b/d*arctanh(c*x)/x
-c^2*a*b/d*ln(c*x)*ln(c*x+1)-2*c^2*a*b/d*arctanh(c*x)*ln(c*x+1)-c^2*a*b/d*l
n(-1/2*c*x+1/2)*ln(c*x+1)+c^2*a*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+2*c^2*
a*b/d*arctanh(c*x)*ln(c*x)+1/2*I*c^2*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/2*I*c
^2*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-1/2*a^2/d/x^2+c^
2*b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^2*b^2/d*arctanh
(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*c^2*a*b/d*ln(c*x-1)+3/2*c^
2*a*b/d*ln(c*x+1)+c^2*a*b/d*dilog(1/2+1/2*c*x)+c*b^2/d*arctanh(c*x)^2/x-c*b
^2/d*arctanh(c*x)/x-c^2*b^2/d*arctanh(c*x)^2*ln(c*x+1)+1/2*c^2*a*b/d*ln(c*x
+1)^2-c*a*b/d/x+c^2*b^2/d*arctanh(c*x)^2*ln(2)-c^2*a*b/d*dilog(c*x)-2*c^2*a
*b/d*ln(c*x)+c^2*b^2/d*arctanh(c*x)^2*ln(c*x)+2*c^2*b^2/d*arctanh(c*x)*poly
log(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-c^2*a*b/d*dilog(c*x+1)-c^2*b^2/d*arctanh(
c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+2*c^2*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(
-c^2*x^2+1)^(1/2))-2*c^2*b^2/d*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2)
)-a*b/d*arctanh(c*x)/x^2+c^2*b^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{2\,c^2\log(cx+1)}{d} - \frac{2\,c^2\log(x)}{d} - \frac{2\,cx-1}{dx^2}\right)a^2 - \frac{\left(2\,b^2c^2x^2\log(cx+1) - 2\,b^2cx + b^2\right)\log(-cx+1)^2}{8\,dx^2} + \int \frac{(b^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a^2 - 1/
8*(2*b^2*c^2*x^2*log(c*x + 1) - 2*b^2*c*x + b^2)*log(-c*x + 1)^2/(d*x^2) +
integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x +
 1) - (2*b^2*c^3*x^3 + b^2*c^2*x^2 - 4*a*b + (4*a*b*c - b^2*c)*x - 2*(b^2*c
^4*x^4 + b^2*c^3*x^3 - b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*d*x
^5 - d*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c\, x))^2}{x^3\, (d + c\, d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx^4+x^3}\, dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{cx^4+x^3}\, dx + \int \frac{2ab \operatorname{atanh}(cx)}{cx^4+x^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**4 + x**3), x) + Integral(b**2*atanh(c*x)**2/(c*x**4 +
x**3), x) + Integral(2*a*b*atanh(c*x)/(c*x**4 + x**3), x))/d
```

3.102 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4(d+cdx)}\,dx$

Optimal. Leaf size=334

$$\frac{bc^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{5c^3\left(a+b\tanh^{-1}(cx)\right)^2}{6d}-\frac{c^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}+\frac{8bc^3\log}{}$$

```
[Out] -1/3*b^2*c^2/d/x+1/3*b^2*c^3*arctanh(c*x)/d-1/3*b*c*(a+b*arctanh(c*x))/d/x^
2+b*c^2*(a+b*arctanh(c*x))/d/x+5/6*c^3*(a+b*arctanh(c*x))^2/d-1/3*(a+b*arct
anh(c*x))^2/d/x^3+1/2*c*(a+b*arctanh(c*x))^2/d/x^2-c^2*(a+b*arctanh(c*x))^2
/d/x-b^2*c^3*ln(x)/d+1/2*b^2*c^3*ln(-c^2*x^2+1)/d+8/3*b*c^3*(a+b*arctanh(c*
x))*ln(2-2/(c*x+1))/d-c^3*(a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-4/3*b^2*c^
3*polylog(2,-1+2/(c*x+1))/d+b*c^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1)
)/d+1/2*b^2*c^3*polylog(3,-1+2/(c*x+1))/d
```

Rubi [A] time = 0.98, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5934, 5916, 5982, 325, 206, 5988, 5932, 2447, 266, 36, 29, 31, 5948, 6056, 6610}

$$\frac{bc^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{4b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{3d}+\frac{b^2c^3\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{5c^3\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

```
[Out] -(b^2*c^2)/(3*d*x) + (b^2*c^3*ArcTanh[c*x])/(3*d) - (b*c*(a + b*ArcTanh[c*x
]))/(3*d*x^2) + (b*c^2*(a + b*ArcTanh[c*x]))/(d*x) + (5*c^3*(a + b*ArcTanh[
c*x])^2)/(6*d) - (a + b*ArcTanh[c*x])^2/(3*d*x^3) + (c*(a + b*ArcTanh[c*x])
^2)/(2*d*x^2) - (c^2*(a + b*ArcTanh[c*x])^2)/(d*x) - (b^2*c^3*Log[x])/d + (
b^2*c^3*Log[1 - c^2*x^2])/(2*d) + (8*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(
1 + c*x)])/(3*d) - (c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (4
*b^2*c^3*PolyLog[2, -1 + 2/(1 + c*x)])/(3*d) + (b*c^3*(a + b*ArcTanh[c*x])*
PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c^3*PolyLog[3, -1 + 2/(1 + c*x)])/(2
*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4(d+cdx)}\,dx=-\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx}{d}$$

$$=-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3}+c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx-\frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d}+\ldots$$

$$=-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}-c^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx+\ldots$$

$$=-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{c^3\left(a+b\tanh^{-1}(cx)\right)^2}{3d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3}+\frac{c\left(a+\ldots\right.}{}$$

$$=-\frac{b^2c^2}{3dx}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\left(a+b\tanh^{-1}(cx)\right)\ldots}{6d}$$

$$=-\frac{b^2c^2}{3dx}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\ldots}{}$$

$$=-\frac{b^2c^2}{3dx}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\ldots}{}$$

$$=-\frac{b^2c^2}{3dx}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\ldots}{}$$

Mathematica [C] time = 1.38, size = 388, normalized size = 1.16

$$-24a^2c^3\log(x)+24a^2c^3\log(cx+1)-\frac{24a^2c^2}{x}+\frac{12a^2c}{x^2}-\frac{8a^2}{x^3}-\frac{8ab\left(-3c^3x^3\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)-cx\left(c^2x^2+8c^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+3c\ldots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

```
[Out] ((-8*a^2)/x^3 + (12*a^2*c)/x^2 - (24*a^2*c^2)/x - 24*a^2*c^3*Log[x] + 24*a^
2*c^3*Log[1 + c*x] - (8*a*b*(ArcTanh[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*x^
3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*(-1 + 3*c*x + c^2*x^2 + 8
*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - 3*c^3*x^3*PolyLog[2, E^(-2*ArcTanh
[c*x])]))/x^3 + b^2*c^3*((-I)*Pi^3 - 8/(c*x) + 8*ArcTanh[c*x] - (8*ArcTanh[
c*x])/(c^2*x^2) + (24*ArcTanh[c*x])/(c*x) + 20*ArcTanh[c*x]^2 - (8*ArcTanh[
c*x]^2)/(c^3*x^3) + (12*ArcTanh[c*x]^2)/(c^2*x^2) - (24*ArcTanh[c*x]^2)/(c*
x) + 16*ArcTanh[c*x]^3 + 64*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*
ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*Log[(c*x)/Sqrt[1 - c^2*x^2]
] - 32*PolyLog[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[2, E^(2*Ar
cTanh[c*x])] + 12*PolyLog[3, E^(2*ArcTanh[c*x])]))/(24*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{cdx^5 + dx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^5 + d*x^4),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^4), x)
```

maple [C] time = 3.39, size = 2010, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x)
```

```
[Out] 1/2*c*a^2/d/x^2-c^3*b^2/d*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-c^3*b^2/d*ln((c*
x+1)/(-c^2*x^2+1)^(1/2)-1)-c^3*a^2/d*ln(c*x)+8/3*c^3*b^2/d*dilog(1+(c*x+1)/
(-c^2*x^2+1)^(1/2))+2*c^3*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^3
*b^2/d*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*c^3*b^2/d*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))
)^2*arctanh(c*x)^2-1/2*I*c^3*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2
*I*c^3*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*arctanh(c*x)^2-I*c^3*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+1/2*I*c^3*b^2/d*Pi*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^
2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*c^3*b^2/d*arctanh(c*x)^2*Pi*csgn(I/(1+(c*
x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*
x^2+1)))^2+4/3*b^2*c^3*arctanh(c*x)/d+c^3*a^2/d*ln(c*x+1)-11/6*c^3*b^2/d*ar
ctanh(c*x)^2+2/3*c^3*b^2/d*arctanh(c*x)^3-1/3*b^2/d*arctanh(c*x)^2/x^3-c^2*
a^2/d/x-8/3*c^3*b^2/d*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^3*a*b/d*arctanh
(c*x)*ln(c*x)-2*c^2*a*b/d*arctanh(c*x)/x+c*a*b/d*arctanh(c*x)/x^2+c^3*a*b/d
*ln(c*x)*ln(c*x+1)+2*c^3*a*b/d*arctanh(c*x)*ln(c*x+1)+c^3*a*b/d*ln(-1/2*c*x
+1/2)*ln(c*x+1)-c^3*a*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-5/6*c^3*a*b/d*ln
```

```
(c*x-1)-1/2*c^3*a*b/d*ln(c*x+1)^2-c^3*b^2/d*arctanh(c*x)^2*ln(2)+8/3*c^3*b^
2/d*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^3*b^2/d*arctanh(c*x)^
2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-c^3*b^2/d*arctanh(c*x)^2*ln(c*x)-1/3*c^3*b
^2/d/(c*x+1-(-c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)+1/3*c^3*b^2/d/((-c^2*x^2
+1)^(1/2)+c*x+1)*(-c^2*x^2+1)^(1/2)-c^3*b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(
-c^2*x^2+1)^(1/2))-2*c^3*b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)
^(1/2))-c^3*b^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^3*b^2
/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+c^3*b^2/d*arctanh(c*x
)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+c^3*b^2/d*arctanh(c*x)^2*ln(c*x+1)-c^3*a*b
/d*dilog(1/2+1/2*c*x)-11/6*c^3*a*b/d*ln(c*x+1)+c^3*a*b/d*dilog(c*x+1)+c^3*a
*b/d*dilog(c*x)+8/3*c^3*a*b/d*ln(c*x)-c^2*b^2/d*arctanh(c*x)^2/x+1/2*c*b^2/
d*arctanh(c*x)^2/x^2+c^2*b^2/d*arctanh(c*x)/x-1/3*c*b^2/d*arctanh(c*x)/x^2+
c^2*a*b/d/x-1/3*c*a*b/d/x^2-2/3*a*b/d*arctanh(c*x)/x^3+1/2*I*c^3*b^2/d*Pi*c
sgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/3*a^2/d/x^3-
1/2*I*c^3*b^2/d*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2
/(-c^2*x^2+1)))-1/2*I*c^3*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-1/2*I*c^3*b^2/d*Pi*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-1/2*I*c^3*b^2/d*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(\frac{6\,c^3\log\left(cx+1\right)}{d}-\frac{6\,c^3\log(x)}{d}-\frac{6\,c^2x^2-3\,cx+2}{dx^3}\right)a^2+\frac{\left(6\,b^2c^3x^3\log\left(cx+1\right)-6\,b^2c^2x^2+3\,b^2cx-2\,b^2\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^3
))*a^2 + 1/24*(6*b^2*c^3*x^3*log(c*x + 1) - 6*b^2*c^2*x^2 + 3*b^2*c*x - 2*b
^2)*log(-c*x + 1)^2/(d*x^3) - integrate(-1/12*(3*(b^2*c*x - b^2)*log(c*x +
1)^2 + 12*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 3*b^2*c^3*x^3 - b
^2*c^2*x^2 + 12*a*b - 2*(6*a*b*c - b^2*c)*x - 6*(b^2*c^5*x^5 + b^2*c^4*x^4
+ b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*d*x^6 - d*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{x^4\,\left(d+c\,d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^4*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^4*(d + c*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx^5+x^4}\, dx + \int \frac{b^2\,\mathrm{atanh}^2\left(cx\right)}{cx^5+x^4}\, dx + \int \frac{2ab\,\mathrm{atanh}\left(cx\right)}{cx^5+x^4}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**4/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**5 + x**4), x) + Integral(b**2*atanh(c*x)**2/(c*x**5 +
x**4), x) + Integral(2*a*b*atanh(c*x)/(c*x**5 + x**4), x))/d
```

3.103 $\int \frac{x^4(a+b\tanh^{-1}(cx))^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=394

$$-\frac{4b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^2(cx+1)}+\frac{29\left(a+b\tanh^{-1}(cx)\right)^2}{6c^5d^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2(cx+1)}-\frac{20b\,\mathrm{lc}}{}$$

```
[Out] -2*a*b*x/c^4/d^2+1/3*b^2*x/c^4/d^2-1/2*b^2/c^5/d^2/(c*x+1)+1/6*b^2*arctanh(
c*x)/c^5/d^2-2*b^2*x*arctanh(c*x)/c^4/d^2+1/3*b*x^2*(a+b*arctanh(c*x))/c^3/
d^2-b*(a+b*arctanh(c*x))/c^5/d^2/(c*x+1)+29/6*(a+b*arctanh(c*x))^2/c^5/d^2+
3*x*(a+b*arctanh(c*x))^2/c^4/d^2-x^2*(a+b*arctanh(c*x))^2/c^3/d^2+1/3*x^3*(
a+b*arctanh(c*x))^2/c^2/d^2-(a+b*arctanh(c*x))^2/c^5/d^2/(c*x+1)-20/3*b*(a+
b*arctanh(c*x))*ln(2/(-c*x+1))/c^5/d^2+4*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))
/c^5/d^2-b^2*ln(-c^2*x^2+1)/c^5/d^2-10/3*b^2*polylog(2,1-2/(-c*x+1))/c^5/d^
2-4*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^5/d^2-2*b^2*polylog(3,1-2
/(c*x+1))/c^5/d^2
```

Rubi [A] time = 0.84, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 19, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 321, 206, 5928, 5926, 627, 44, 207, 6056, 6610}

$$-\frac{4b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2}-\frac{10b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^5d^2}-\frac{2b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^2}+\frac{x^3\left(a+b\,\mathrm{t}\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (-2*a*b*x)/(c^4*d^2) + (b^2*x)/(3*c^4*d^2) - b^2/(2*c^5*d^2*(1 + c*x)) + (b
^2*ArcTanh[c*x])/(6*c^5*d^2) - (2*b^2*x*ArcTanh[c*x])/(c^4*d^2) + (b*x^2*(a
 + b*ArcTanh[c*x]))/(3*c^3*d^2) - (b*(a + b*ArcTanh[c*x]))/(c^5*d^2*(1 + c*
x)) + (29*(a + b*ArcTanh[c*x])^2)/(6*c^5*d^2) + (3*x*(a + b*ArcTanh[c*x])^2
)/(c^4*d^2) - (x^2*(a + b*ArcTanh[c*x])^2)/(c^3*d^2) + (x^3*(a + b*ArcTanh[
c*x])^2)/(3*c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^5*d^2*(1 + c*x)) - (20*b*(
a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^5*d^2) + (4*(a + b*ArcTanh[c*x])
^2*Log[2/(1 + c*x)])/(c^5*d^2) - (b^2*Log[1 - c^2*x^2])/(c^5*d^2) - (10*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^5*d^2) - (4*b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/(c^5*d^2) - (2*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^
5*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
```

```
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int\left(\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} +\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^4d^2} + \frac{3\int\left(a+b\tanh^{-1}(cx)\right)^2 dx}{c^4d^2} - \frac{4\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{c^4d^2} - 2$$

$$= \frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c^2d^2} - \frac{(a}{}$$

$$= \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^2} + \frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^3}{}$$

$$= -\frac{2abx}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2(1+cx)} + \frac{29\left(a+b\tanh^{-1}(cx\right.}{6c^5d^2}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} - \frac{b\left(a+b\tanh^{-1}\right.}{c^5d^2(1+cx}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2\tanh^{-1}(cx)}{3c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} -$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2}{2c^5d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{3c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a\right.}{}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2}{2c^5d^2(1+cx)} + \frac{b^2\tanh^{-1}(cx)}{6c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a\right.}{}$$

Mathematica [A] time = 1.71, size = 425, normalized size = 1.08

$$4a^2c^3x^3 - 12a^2c^2x^2 + 36a^2cx - \frac{12a^2}{cx+1} - 48a^2\log(cx+1) + 2ab\left(2c^2x^2 + 20\log\left(1-c^2x^2\right) + 2\tanh^{-1}(cx)\left(2c^3x\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (36*a^2*c*x - 12*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(1 + c*x) - 48*a^2*
Log[1 + c*x] + b^2*(4*c*x - 4*ArcTanh[c*x] - 24*c*x*ArcTanh[c*x] + 4*c^2*x^
2*ArcTanh[c*x] - 28*ArcTanh[c*x]^2 + 36*c*x*ArcTanh[c*x]^2 - 12*c^2*x^2*Arc
Tanh[c*x]^2 + 4*c^3*x^3*ArcTanh[c*x]^2 - 3*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh
[c*x]*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 80*Arc
Tanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 48*ArcTanh[c*x]^2*Log[1 + E^(-2*Ar
cTanh[c*x])] - 12*Log[1 - c^2*x^2] - 8*(-5 + 6*ArcTanh[c*x])*PolyLog[2, -E^
(-2*ArcTanh[c*x])] - 24*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh
[c*x]] + 6*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Sinh[2*ArcT
anh[c*x]]) + 2*a*b*(-2 - 12*c*x + 2*c^2*x^2 - 3*Cosh[2*ArcTanh[c*x]] + 20*L
og[1 - c^2*x^2] - 24*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*
x]] + 2*ArcTanh[c*x]*(6 + 18*c*x - 6*c^2*x^2 + 2*c^3*x^3 - 3*Cosh[2*ArcTanh
[c*x]] + 24*Log[1 + E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*x]])))/(12*c^
5*d^2)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2x^4\,\mathrm{artanh}(cx)^2 + 2\,abx^4\,\mathrm{artanh}(cx) + a^2x^4}{c^2d^2x^2 + 2\,cd^2x + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx)+a)^2x^4}{(cdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^2, x)
```

maple [C] time = 2.31, size = 1467, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] 2*I/c^5*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn
(I*(c*x+1)^2/(c^2*x^2-1))+4*I/c^5*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-2*I/c^5*b^2/d^2*arctanh
(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
*x+1)^2/(-c^2*x^2+1)))^2+2*I/c^5*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1
)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2-2*a*b*x/d^2/c^4-2*b^2*x*arctanh(c*x)/d^2/c^4+1/3*b^2*x/d^2/c^4-1/4*b^2/c
^5/d^2/(c*x+1)-7/3*b^2*arctanh(c*x)/c^5/d^2+2*I/c^5*b^2/d^2*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+2*I/c^5*b^2/d
^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/3/c^5*b^2/d^2+1/3/c^
2*a^2/d^2*x^3-1/c^3*a^2/d^2*x^2+3/c^4*a^2/d^2*x-1/c^5*a^2/d^2/(c*x+1)-4/c^5
*a^2/d^2*ln(c*x+1)+29/6/c^5*b^2/d^2*arctanh(c*x)^2+2/c^5*b^2/d^2*ln(1+(c*x+
1)^2/(-c^2*x^2+1))-8/3/c^5*b^2/d^2*arctanh(c*x)^3-20/3/c^5*b^2/d^2*dilog(1-
I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3/c^5*b^2/d^2*dilog(1+I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-2/c^5*b^2/d^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-7/3/c^5*a*b/d^2-
2*I/c^5*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I
*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))+6/c^4*a*b/d^2*arctanh(c*x)*x+2/3/c^2*a*b/d^2*arctanh(c*x)*x^3-2/c^3*a
*b/d^2*arctanh(c*x)*x^2+1/2/c^4*b^2/d^2*arctanh(c*x)/(c*x+1)*x-2/c^5*a*b/d^
2*arctanh(c*x)/(c*x+1)-8/c^5*a*b/d^2*arctanh(c*x)*ln(c*x+1)-4/c^5*a*b/d^2*l
n(-1/2*c*x+1/2)*ln(c*x+1)+4/c^5*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-4/
c^5*b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+29/6/c^5*a*b/d^2*ln(c*x+1)+11/6/c^5*a*
b/d^2*ln(c*x-1)-1/c^3*b^2/d^2*arctanh(c*x)^2*x^2+3/c^4*b^2/d^2*arctanh(c*x)
^2*x+1/3/c^3*b^2/d^2*arctanh(c*x)*x^2+4/c^5*b^2/d^2*arctanh(c*x)^2*ln(2)+2/
c^5*a*b/d^2*ln(c*x+1)^2+4/c^5*a*b/d^2*dilog(1/2+1/2*c*x)+1/3/c^2*b^2/d^2*ar
ctanh(c*x)^2*x^3-1/c^5*b^2/d^2*arctanh(c*x)^2/(c*x+1)-1/2/c^5*b^2/d^2*arcta
nh(c*x)/(c*x+1)-20/3/c^5*b^2/d^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(
1/2))+8/c^5*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+4/c^5*b^2
/d^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-20/3/c^5*b^2/d^2*arcta
nh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/c^5*a*b/d^2/(c*x+1)+1/4/c^4*b^
2/d^2/(c*x+1)*x+1/3/c^3*a*b/d^2*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a^2\left(\frac{3}{c^6d^2x+c^5d^2}-\frac{c^2x^3-3\,cx^2+9\,x}{c^4d^2}+\frac{12\,\log(cx+1)}{c^5d^2}\right)+\frac{\left(b^2c^4x^4-2\,b^2c^3x^3+6\,b^2c^2x^2+9\,b^2cx-3\,b^2-12\,(\right.}{12\,\left(c^6d^2x+c^5d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^2*(3/(c^6*d^2*x + c^5*d^2) - (c^2*x^3 - 3*c*x^2 + 9*x)/(c^4*d^2) + 1
2*log(c*x + 1)/(c^5*d^2)) + 1/12*(b^2*c^4*x^4 - 2*b^2*c^3*x^3 + 6*b^2*c^2*x
^2 + 9*b^2*c*x - 3*b^2 - 12*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(
c^6*d^2*x + c^5*d^2) - integrate(-1/12*(3*(b^2*c^5*x^5 - b^2*c^4*x^4)*log(c
*x + 1)^2 + 12*(a*b*c^5*x^5 - a*b*c^4*x^4)*log(c*x + 1) - 2*(4*b^2*c^3*x^3
+ 15*b^2*c^2*x^2 + (6*a*b*c^5 + b^2*c^5)*x^5 - (6*a*b*c^4 + b^2*c^4)*x^4 +
6*b^2*c*x - 3*b^2 + 3*(b^2*c^5*x^5 - b^2*c^4*x^4 - 4*b^2*c^2*x^2 - 8*b^2*c*
x - 4*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^7*d^2*x^3 + c^6*d^2*x^2 - c^5*d^
2*x - c^4*d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(a+b\,\mathrm{atanh}\,(c\,x))^2}{(d+c\,d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

```
[Out] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^4}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^4\,\mathrm{atanh}^2\,(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^4\,\mathrm{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**4/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**4*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**4*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

3.104 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=331

$$\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2(cx+1)}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}+\frac{4b\log\left(\right.}{}$$

```
[Out] a*b*x/c^3/d^2+1/2*b^2/c^4/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/c^4/d^2+b^2*x*ar
ctanh(c*x)/c^3/d^2+b*(a+b*arctanh(c*x))/c^4/d^2/(c*x+1)-3*(a+b*arctanh(c*x)
)^2/c^4/d^2-2*x*(a+b*arctanh(c*x))^2/c^3/d^2+1/2*x^2*(a+b*arctanh(c*x))^2/c
^2/d^2+(a+b*arctanh(c*x))^2/c^4/d^2/(c*x+1)+4*b*(a+b*arctanh(c*x))*ln(2/(-c
*x+1))/c^4/d^2-3*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^4/d^2+1/2*b^2*ln(-c^2
*x^2+1)/c^4/d^2+2*b^2*polylog(2,1-2/(-c*x+1))/c^4/d^2+3*b*(a+b*arctanh(c*x)
)*polylog(2,1-2/(c*x+1))/c^4/d^2+3/2*b^2*polylog(3,1-2/(c*x+1))/c^4/d^2
```

Rubi [A] time = 0.63, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 5928, 5926, 627, 44, 207, 6056, 6610}

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}+\frac{2b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^2}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{x^2\left(a+b\tan\right.}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (a*b*x)/(c^3*d^2) + b^2/(2*c^4*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^4*d
^2) + (b^2*x*ArcTanh[c*x])/(c^3*d^2) + (b*(a + b*ArcTanh[c*x]))/(c^4*d^2*(1
 + c*x)) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^2) - (2*x*(a + b*ArcTanh[c*x])
^2)/(c^3*d^2) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^2*d^2) + (a + b*ArcTanh[c
*x])^2/(c^4*d^2*(1 + c*x)) + (4*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c
^4*d^2) - (3*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^2) + (b^2*Log[
1 - c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^4*d^2) +
 (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^4*d^2) + (3*b^2*
PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

```
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \left(a + b \tanh^{-1}(cx)\right)^2}{(d + cdx)^2} dx = \int \left(-\frac{2 \left(a + b \tanh^{-1}(cx)\right)^2}{c^3 d^2} + \frac{x \left(a + b \tanh^{-1}(cx)\right)^2}{c^2 d^2} - \frac{\left(a + b \tanh^{-1}(cx)\right)^2}{c^3 d^2 (1 + cx)^2} + \right.$$

$$= -\frac{\int \frac{\left(a+b \tanh^{-1}(cx)\right)^2}{(1+cx)^2} dx}{c^3 d^2} - \frac{2 \int \left(a + b \tanh^{-1}(cx)\right)^2 dx}{c^3 d^2} + \frac{3 \int \frac{\left(a+b \tanh^{-1}(cx)\right)^2}{1+cx} dx}{c^3 d^2} +$$

$$= -\frac{2x \left(a + b \tanh^{-1}(cx)\right)^2}{c^3 d^2} + \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)^2}{2c^2 d^2} + \frac{\left(a + b \tanh^{-1}(cx)\right)^2}{c^4 d^2 (1 + cx)} - \frac{3 (}{}$$

$$= -\frac{2 \left(a + b \tanh^{-1}(cx)\right)^2}{c^4 d^2} - \frac{2x \left(a + b \tanh^{-1}(cx)\right)^2}{c^3 d^2} + \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)^2}{2c^2 d^2} + \frac{(}{}$$

$$= \frac{abx}{c^3 d^2} + \frac{b \left(a + b \tanh^{-1}(cx)\right)}{c^4 d^2 (1 + cx)} - \frac{3 \left(a + b \tanh^{-1}(cx)\right)^2}{c^4 d^2} - \frac{2x \left(a + b \tanh^{-1}(cx)\right)^2}{c^3 d^2}$$

$$= \frac{abx}{c^3 d^2} + \frac{b^2 x \tanh^{-1}(cx)}{c^3 d^2} + \frac{b \left(a + b \tanh^{-1}(cx)\right)}{c^4 d^2 (1 + cx)} - \frac{3 \left(a + b \tanh^{-1}(cx)\right)^2}{c^4 d^2} - \frac{2x (a}{}$$

$$= \frac{abx}{c^3 d^2} + \frac{b^2 x \tanh^{-1}(cx)}{c^3 d^2} + \frac{b \left(a + b \tanh^{-1}(cx)\right)}{c^4 d^2 (1 + cx)} - \frac{3 \left(a + b \tanh^{-1}(cx)\right)^2}{c^4 d^2} - \frac{2x (a}{}$$

$$= \frac{abx}{c^3 d^2} + \frac{b^2}{2c^4 d^2 (1 + cx)} + \frac{b^2 x \tanh^{-1}(cx)}{c^3 d^2} + \frac{b \left(a + b \tanh^{-1}(cx)\right)}{c^4 d^2 (1 + cx)} - \frac{3 \left(a + b \tanh\right.}{c^4 d^2}$$

$$= \frac{abx}{c^3 d^2} + \frac{b^2}{2c^4 d^2 (1 + cx)} - \frac{b^2 \tanh^{-1}(cx)}{2c^4 d^2} + \frac{b^2 x \tanh^{-1}(cx)}{c^3 d^2} + \frac{b \left(a + b \tanh^{-1}(cx)\right.}{c^4 d^2 (1 + cx)}$$

Mathematica [A] time = 1.27, size = 354, normalized size = 1.07

$$2a^2c^2x^2 - 8a^2cx + \frac{4a^2}{cx+1} + 12a^2 \log(cx + 1) + 2ab \left(-4 \log \left(1 - c^2x^2\right) + 2 \tanh^{-1}(cx) \left(c^2x^2 - 4cx - 6 \log \left(e^{-2 \tan}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (-8*a^2*c*x + 2*a^2*c^2*x^2 + (4*a^2)/(1 + c*x) + 12*a^2*Log[1 + c*x] + 2*a
*b*(2*c*x + Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(-2
*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[c*x
]] - 6*Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTan
h[c*x]]) + b^2*(4*c*x*ArcTanh[c*x] + 6*ArcTanh[c*x]^2 - 8*c*x*ArcTanh[c*x]^
2 + 2*c^2*x^2*ArcTanh[c*x]^2 + Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Cosh[2
*ArcTanh[c*x]] + 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 16*ArcTanh[c*x]*Lo
g[1 + E^(-2*ArcTanh[c*x])] - 12*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])]
 + 2*Log[1 - c^2*x^2] + 4*(-2 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c
*x])] + 6*PolyLog[3, -E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]] - 2*ArcTa
nh[c*x]*Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(4*c
^4*d^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^3 \operatorname{artanh}(cx)^2 + 2\,abx^3 \operatorname{artanh}(cx) + a^2x^3}{c^2d^2x^2 + 2\,cd^2x + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2 x^3}{(cdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^2, x)
```

maple [C] time = 1.74, size = 1354, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] 1/2/c^2*a^2/d^2*x^2-2/c^3*a^2/d^2*x+1/c^4*a^2/d^2/(c*x+1)+4/c^4*b^2/d^2*dil
og(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+4/c^4*b^2/d^2*dilog(1+I*(c*x+1)/(-c^2*x^
2+1)^(1/2))+3/2/c^4*b^2/d^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+3/c^4*a^2/d^
2*ln(c*x+1)-3/c^4*b^2/d^2*arctanh(c*x)^2-1/c^4*b^2/d^2*ln(1+(c*x+1)^2/(-c^2
*x^2+1))+2/c^4*b^2/d^2*arctanh(c*x)^3+a*b*x/c^3/d^2+b^2*x*arctanh(c*x)/c^3/
d^2+3/2*I/c^4*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*
csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c
^2*x^2+1)))+1/4*b^2/c^4/d^2/(c*x+1)+b^2*arctanh(c*x)/d^2/c^4+3/2*I/c^4*b^2/
d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3*I/c^4*b^2/d^2*arctanh(c*x)^2*Pi*csgn(
I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-3/2*I/c^4*b^2
/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^
2/(c^2*x^2-1))-3/2*I/c^4*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^
2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/2*I
/c^4*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*
arctanh(c*x)^2-3/2*I/c^4*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^
2-1))^3-4/c^3*a*b/d^2*arctanh(c*x)*x+1/c^2*a*b/d^2*arctanh(c*x)*x^2-1/2/c^3
*b^2/d^2*arctanh(c*x)/(c*x+1)*x+1/c^4*b^2/d^2*arctanh(c*x)^2/(c*x+1)+1/2/c^
4*b^2/d^2*arctanh(c*x)/(c*x+1)+4/c^4*b^2/d^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-
c^2*x^2+1)^(1/2))-6/c^4*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2
))-3/c^4*b^2/d^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+4/c^4*b^2/
d^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/c^4*b^2/d^2*arctanh(c
*x)^2*ln(c*x+1)-3/c^4*b^2/d^2*arctanh(c*x)^2*ln(2)-3/2/c^4*a*b/d^2*ln(c*x+1
)^2-3/c^4*a*b/d^2*dilog(1/2+1/2*c*x)-3/c^4*a*b/d^2*ln(c*x+1)+1/c^4*a*b/d^2/
(c*x+1)-1/4/c^3*b^2/d^2/(c*x+1)*x+1/c^4*a*b/d^2-1/c^4*a*b/d^2*ln(c*x-1)-2/c
^3*b^2/d^2*arctanh(c*x)^2*x+1/2/c^2*b^2/d^2*arctanh(c*x)^2*x^2+2/c^4*a*b/d^
2*arctanh(c*x)/(c*x+1)+6/c^4*a*b/d^2*arctanh(c*x)*ln(c*x+1)+3/c^4*a*b/d^2*l
n(-1/2*c*x+1/2)*ln(c*x+1)-3/c^4*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{2}{c^5d^2x + c^4d^2} + \frac{cx^2 - 4x}{c^3d^2} + \frac{6\log(cx+1)}{c^4d^2}\right) + \frac{\left(b^2c^3x^3 - 3b^2c^2x^2 - 4b^2cx + 2b^2 + 6\left(b^2cx + b^2\right)\log(cx+1)\right)}{8\left(c^5d^2x + c^4d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2/(c^5*d^2*x + c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1)
/(c^4*d^2)) + 1/8*(b^2*c^3*x^3 - 3*b^2*c^2*x^2 - 4*b^2*c*x + 2*b^2 + 6*(b^2
*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^5*d^2*x + c^4*d^2) - integrate
(-1/4*((b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*b*c^
3*x^3)*log(c*x + 1) + (7*b^2*c^2*x^2 - (4*a*b*c^4 + b^2*c^4)*x^4 + 2*b^2*c*
x + 2*(2*a*b*c^3 + b^2*c^3)*x^3 - 2*b^2 - 2*(b^2*c^4*x^4 - b^2*c^3*x^3 + 3*
b^2*c^2*x^2 + 6*b^2*c*x + 3*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^6*d^2*x^3
+ c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^2}{\left(d+c\,d\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^3\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^3\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**3*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

3.105 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{2b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(cx+1)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}-\frac{2b\log}{}$$

```
[Out] -1/2*b^2/c^3/d^2/(c*x+1)+1/2*b^2*arctanh(c*x)/c^3/d^2-b*(a+b*arctanh(c*x))/
c^3/d^2/(c*x+1)+3/2*(a+b*arctanh(c*x))^2/c^3/d^2+x*(a+b*arctanh(c*x))^2/c^2
/d^2-(a+b*arctanh(c*x))^2/c^3/d^2/(c*x+1)-2*b*(a+b*arctanh(c*x))*ln(2/(-c*x
+1))/c^3/d^2+2*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^3/d^2-b^2*polylog(2,1-2
/(-c*x+1))/c^3/d^2-2*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^3/d^2-b^
2*polylog(3,1-2/(c*x+1))/c^3/d^2
```

Rubi [A] time = 0.48, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5928, 5926, 627, 44, 207, 5948, 6056, 6610}

$$-\frac{2b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d^2}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b\left(a+b\tanh^{-1}\right.}{c^3d^2(cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] -b^2/(2*c^3*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c^3*d^2) - (b*(a + b*Arc
Tanh[c*x]))/(c^3*d^2*(1 + c*x)) + (3*(a + b*ArcTanh[c*x])^2)/(2*c^3*d^2) +
(x*(a + b*ArcTanh[c*x])^2)/(c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^3*d^2*(1 +
 c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d^2) + (2*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d^2) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(c^3*d^2) - (2*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
3*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
```

```
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)^2}-\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}\right)dx$$

$$=\frac{\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^2d^2}+\frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^2d^2}-\frac{2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{c^2d^2}$$

$$=\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)}+\frac{2\left(a+b\tan\right.}{}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\,\mathrm{t}\right.}{c^3d}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\,\mathrm{t}\right.}{c^3d}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\,\mathrm{t}\right.}{c^3d}$$

$$=-\frac{b^2}{2c^3d^2(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}\right.}{c^2d^2}$$

$$=-\frac{b^2}{2c^3d^2(1+cx)}+\frac{b^2\tanh^{-1}(cx)}{2c^3d^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}-$$

Mathematica [A] time = 0.92, size = 295, normalized size = 1.13

$$4a^2cx-\frac{4a^2}{cx+1}-8a^2\log(cx+1)+2ab\left(2\log\left(1-c^2x^2\right)-4\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)+\sinh\left(2\tanh^{-1}(cx)\right)-\cosh\left(2\tan\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (4*a^2*c*x - (4*a^2)/(1 + c*x) - 8*a^2*Log[1 + c*x] + b^2*(-4*ArcTanh[c*x]^
2 + 4*c*x*ArcTanh[c*x]^2 - Cosh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Cosh[2*Arc
Tanh[c*x]] - 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 8*ArcTanh[c*x]*Log[1 +
 E^(-2*ArcTanh[c*x])] + 8*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + (4
- 8*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 4*PolyLog[3, -E^(-2*Ar
cTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] +
 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 2*a*b*(-Cosh[2*ArcTanh[c*x]] + 2*
```

```
Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]
] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2*ArcTanh[
c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^2 \operatorname{artanh}(cx)^2 + 2\,abx^2 \operatorname{artanh}(cx) + a^2x^2}{c^2d^2x^2 + 2\,cd^2x + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2 x^2}{(cdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^2, x)
```

maple [C] time = 0.74, size = 5542, normalized size = 21.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\frac{1}{c^4d^2x + c^3d^2} - \frac{x}{c^2d^2} + \frac{2\log(cx+1)}{c^3d^2}\right) + \frac{\left(b^2c^2x^2 + b^2cx - b^2 - 2\left(b^2cx + b^2\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(c^4d^2x + c^3d^2\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*d^2) + 2*log(c*x + 1)/(c^3*d^2)) + 1
/4*(b^2*c^2*x^2 + b^2*c*x - b^2 - 2*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*x
+ 1)^2/(c^4*d^2*x + c^3*d^2) - integrate(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*
log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*x^2)*log(c*x + 1) - 2*((2*a*b*c^3
 + b^2*c^3)*x^3 - 2*(a*b*c^2 - b^2*c^2)*x^2 - b^2 + (b^2*c^3*x^3 - 3*b^2*c^
2*x^2 - 4*b^2*c*x - 2*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^5*d^2*x^3 + c^4*
d^2*x^2 - c^3*d^2*x - c^2*d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a + b\,\mathrm{atanh}(c\,x)\right)^2}{\left(d + c\,d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^2\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^2\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**2*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

3.106 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=188

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}-\frac{\log\left(\frac{2}{cx+}\right.}{}$$

```
[Out] 1/2*b^2/c^2/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/c^2/d^2+b*(a+b*arctanh(c*x))/c
^2/d^2/(c*x+1)-1/2*(a+b*arctanh(c*x))^2/c^2/d^2+(a+b*arctanh(c*x))^2/c^2/d^
2/(c*x+1)-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^2/d^2+b*(a+b*arctanh(c*x))*p
olylog(2,1-2/(c*x+1))/c^2/d^2+1/2*b^2*polylog(3,1-2/(c*x+1))/c^2/d^2
```

Rubi [A] time = 0.34, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5940, 5928, 5926, 627, 44, 207, 5948, 5918, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx\right.}{c^2d^2(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] b^2/(2*c^2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^2*d^2) + (b*(a + b*ArcT
anh[c*x]))/(c^2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*c^2*d^2) + (a +
b*ArcTanh[c*x])^2/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(1 +
c*x)])/(c^2*d^2) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}dx = \int \left(-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{cd^2}+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}dx}{cd^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d^2}-\frac{(2b)\int\left(\frac{a+b\tanh^{-1}(cx)}{2(1+cx)^2}-\right.}{cd^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}}{c^2d^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{\left(a+b\tan\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{\left(a+b\tan\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{\left(a+b\tan\right.}{}$$

$$= \frac{b^2}{2c^2d^2(1+cx)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx\right.}{c^2d^2(1+cx)}$$

$$= \frac{b^2}{2c^2d^2(1+cx)}-\frac{b^2\tanh^{-1}(cx)}{2c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+$$

Mathematica [A] time = 0.62, size = 233, normalized size = 1.24

$$\frac{4a^2}{cx+1}+4a^2\log(cx+1)+2ab\left(2\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)+\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] ((4*a^2)/(1 + c*x) + 4*a^2*Log[1 + c*x] + 2*a*b*(Cosh[2*ArcTanh[c*x]] + 2*P
olyLog[2, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] - 2*
Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]
) + b^2*(Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 2*Arc
Tanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c
*x])] + 4*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*PolyLog[3, -E^(
-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c*
x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(4*c^2*d^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2x\,\mathrm{artanh}\,(cx)^2+2\,abx\,\mathrm{artanh}\,(cx)+a^2x}{c^2d^2x^2+2\,cd^2x+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c^2*d^2*x^2
 + 2*c*d^2*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx) + a)^2 x}{(cdx + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d)^2, x)
```

maple [C] time = 0.35, size = 1030, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] -1/2*I/c^2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1
)))^3*arctanh(c*x)^2+1/2*I/c^2*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I/c^2*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*
x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-I/c^2*b^2/d^2*arct
anh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(
1/2))+1/2*I/c^2*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I/c^2*b^2/d^2*a
rctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/c^2*a^2/d^2/(c*x+1)+1/c^2*a^2/d^2*ln(
c*x+1)-1/2/c^2*b^2/d^2*arctanh(c*x)^2+2/3/c^2*b^2/d^2*arctanh(c*x)^3+1/2/c^
2*b^2/d^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+1/4*b^2/c^2/d^2/(c*x+1)-1/2/c*
b^2/d^2*arctanh(c*x)/(c*x+1)*x+2/c^2*a*b/d^2*arctanh(c*x)*ln(c*x+1)+1/c^2*a
*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/c^2*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2
*c*x)+2/c^2*a*b/d^2*arctanh(c*x)/(c*x+1)-1/2*I/c^2*b^2/d^2*arctanh(c*x)^2*P
i*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/c^2*b^2/d^2*arctanh(c*x)^2*ln(2)-1/2/c^
2*a*b/d^2*ln(c*x+1)^2-1/c^2*a*b/d^2*dilog(1/2+1/2*c*x)-1/2/c^2*a*b/d^2*ln(c
*x+1)+1/2/c^2*a*b/d^2*ln(c*x-1)+1/c^2*b^2/d^2*arctanh(c*x)^2/(c*x+1)+1/2/c^
2*b^2/d^2*arctanh(c*x)/(c*x+1)-2/c^2*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^
2*x^2+1)^(1/2))-1/c^2*b^2/d^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1
))+1/c^2*a*b/d^2/(c*x+1)-1/4/c*b^2/d^2/(c*x+1)*x+1/c^2*b^2/d^2*arctanh(c*x)
^2*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{1}{c^3d^2x + c^2d^2} + \frac{\log(cx+1)}{c^2d^2}\right) + \frac{\left(b^2 + \left(b^2cx + b^2\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(c^3d^2x + c^2d^2\right)} - \int -\frac{\left(b^2c^2x^2 - b^2cx\right)\log(cx+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a^2*(1/(c^3*d^2*x + c^2*d^2) + log(c*x + 1)/(c^2*d^2)) + 1/4*(b^2 + (b^2*c*
x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d^2*x + c^2*d^2) - integrate(-1
/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x + 1)^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*log(
c*x + 1) - 2*(2*a*b*c^2*x^2 + b^2 - (2*a*b*c - b^2*c)*x + (2*b^2*c^2*x^2 +
b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^4*d^2*x^3 + c^3*d^2*x^2 - c^
2*d^2*x - c*d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{atanh}\,(c\,x))^2}{(d+c\,d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

```
[Out] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x\,\mathrm{atanh}^2\,(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx\,\mathrm{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x*atanh(c*x)**
2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x*atanh(c*x)/(c**2*x**2 + 2*
c*x + 1), x))/d**2
```

3.107 $\int \frac{\left(a+b \tanh^{-1}(cx)\right)^2}{(d+cdx)^2} \, dx$

Optimal. Leaf size=107

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{b^2}{2cd^2(cx+1)}+\frac{b^2\tanh^{-1}(cx)}{2cd^2}$$

```
[Out] -1/2*b^2/c/d^2/(c*x+1)+1/2*b^2*arctanh(c*x)/c/d^2-b*(a+b*arctanh(c*x))/c/d^
2/(c*x+1)+1/2*(a+b*arctanh(c*x))^2/c/d^2-(a+b*arctanh(c*x))^2/c/d^2/(c*x+1)
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{b^2}{2cd^2(cx+1)}+\frac{b^2\tanh^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

```
[Out] -b^2/(2*c*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c*d^2) - (b*(a + b*ArcTanh
[c*x]))/(c*d^2*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(2*c*d^2) - (a + b*ArcTa
nh[c*x])^2/(c*d^2*(1 + c*x))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
```

```
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{(2b)\int\left(\frac{a+b\tanh^{-1}(cx)}{2d(1+cx)^2}-\frac{a+b\tanh^{-1}(cx)}{2d\left(-1+c^2x^2\right)}\right)dx}{d}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} - \frac{b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{d^2}$$
$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b^2\int\frac{}{(1+cx)(}}{d}$$
$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b^2\int\frac{1}{(1-cx)(}}{d^2}$$
$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b^2\int\left(\frac{1}{2(1+c}\right.}{}$$
$$= -\frac{b^2}{2cd^2(1+cx)} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right.}{cd^2(1+cx)}$$
$$= -\frac{b^2}{2cd^2(1+cx)} + \frac{b^2\tanh^{-1}(cx)}{2cd^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \left(\right.$$

Mathematica [A] time = 0.14, size = 124, normalized size = 1.16

$$\frac{-4a^2+2ab\log(cx+1)+2abcx\log(cx+1)-b(2a+b)(cx+1)\log(1-cx)-4b(2a+b)\tanh^{-1}(cx)-4ab+b^2}{4cd^2(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

```
[Out] (-4*a^2 - 4*a*b - 2*b^2 - 4*b*(2*a + b)*ArcTanh[c*x] + 2*b^2*(-1 + c*x)*Arc
Tanh[c*x]^2 - b*(2*a + b)*(1 + c*x)*Log[1 - c*x] + 2*a*b*Log[1 + c*x] + b^2
*Log[1 + c*x] + 2*a*b*c*x*Log[1 + c*x] + b^2*c*x*Log[1 + c*x])/(4*c*d^2*(1
+ c*x))
```

fricas [A] time = 0.46, size = 101, normalized size = 0.94

$$\frac{\left(b^2cx-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-8\,a^2-8\,ab-4\,b^2+2\left(\left(2\,ab+b^2\right)cx-2\,ab-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{8\left(c^2d^2x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b^2*c*x - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 8*a^2 - 8*a*b - 4*b^2 +
2*((2*a*b + b^2)*c*x - 2*a*b - b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^2*d^2*x +
 c*d^2)
```

giac [A] time = 0.16, size = 119, normalized size = 1.11

$$\frac{1}{8}c\left(\frac{(cx-1)b^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2d^2}+\frac{2\left(2\,ab+b^2\right)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2d^2}+\frac{2\left(2\,a^2+2\,ab+b^2\right)(cx-1)}{(cx+1)c^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/8*c*((c*x - 1)*b^2*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2*d^2) + 2*(2
*a*b + b^2)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2*d^2) + 2*(2*
a^2 + 2*a*b + b^2)*(c*x - 1)/((c*x + 1)*c^2*d^2))
```

maple [B] time = 0.06, size = 341, normalized size = 3.19

$$-\frac{a^2}{c\,d^2\,(cx+1)}-\frac{b^2\operatorname{arctanh}(cx)^2}{c\,d^2\,(cx+1)}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2c\,d^2}-\frac{b^2\operatorname{arctanh}(cx)}{c\,d^2\,(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2c\,d^2}-\frac{b^2\ln(}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] -1/c*a^2/d^2/(c*x+1)-1/c*b^2/d^2*arctanh(c*x)^2/(c*x+1)-1/2/c*b^2/d^2*arcta
nh(c*x)*ln(c*x-1)-1/c*b^2/d^2*arctanh(c*x)/(c*x+1)+1/2/c*b^2/d^2*arctanh(c*
x)*ln(c*x+1)-1/8/c*b^2/d^2*ln(c*x-1)^2+1/4/c*b^2/d^2*ln(c*x-1)*ln(1/2+1/2*c
*x)-1/4/c*b^2/d^2*ln(c*x-1)-1/2*b^2/c/d^2/(c*x+1)+1/4/c*b^2/d^2*ln(c*x+1)-1
/8/c*b^2/d^2*ln(c*x+1)^2+1/4/c*b^2/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/4/c*b^2
/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-2/c*a*b/d^2*arctanh(c*x)/(c*x+1)-1/2/
c*a*b/d^2*ln(c*x-1)-1/c*a*b/d^2/(c*x+1)+1/2/c*a*b/d^2*ln(c*x+1)
```

maxima [B] time = 0.33, size = 277, normalized size = 2.59

$$-\frac{1}{2}\left(c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log(cx+1)}{c^2d^2}+\frac{\log(cx-1)}{c^2d^2}\right)+\frac{4\operatorname{artanh}(cx)}{c^2d^2x+cd^2}\right)ab-\frac{1}{8}\left(4\,c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log(cx+1)}{c^2d^2}+\frac{\mathrm{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c
^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*a*b - 1/8*(4*c*(2/(c^3*d^2*x
 + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c^2*d^2))*arctanh(c*x)
 + ((c*x + 1)*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1
)*log(c*x - 1) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(c^4*d
^2*x + c^3*d^2))*b^2 - b^2*arctanh(c*x)^2/(c^2*d^2*x + c*d^2) - a^2/(c^2*d^
2*x + c*d^2)
```

mupad [B] time = 1.25, size = 97, normalized size = 0.91

$$\frac{b^2\operatorname{atanh}(c\,x)^2+b^2\operatorname{atanh}(c\,x)+2\,a\,b\operatorname{atanh}(c\,x)}{2\,c\,d^2}-\frac{2\,a^2+4\,a\,b\operatorname{atanh}(c\,x)+2\,a\,b+2\,b^2\operatorname{atanh}(c\,x)^2+2\,b^2\operatorname{atanl}}{2\,x\,c^2\,d^2+2\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + c*d*x)^2,x)
```

```
[Out] (b^2*atanh(c*x)^2 + b^2*atanh(c*x) + 2*a*b*atanh(c*x))/(2*c*d^2) - (2*b^2*a
tanh(c*x)^2 + 2*a*b + 2*b^2*atanh(c*x) + 2*a^2 + b^2 + 4*a*b*atanh(c*x))/(2
*c*d^2 + 2*c^2*d^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^2+2cx+1}\,dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2ab \operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(c
**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**2 + 2*c*x +
1), x))/d**2
```

3.108 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^2}\,dx$

Optimal. Leaf size=295

$$-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{b\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{b(a}{}$$

```
[Out] 1/2*b^2/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/d^2+b*(a+b*arctanh(c*x))/d^2/(c*x+
1)-1/2*(a+b*arctanh(c*x))^2/d^2+(a+b*arctanh(c*x))^2/d^2/(c*x+1)-2*(a+b*arc
tanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^2+(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/
d^2-b*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2+b*(a+b*arctanh(c*x))*p
olylog(2,-1+2/(-c*x+1))/d^2-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d^2
+1/2*b^2*polylog(3,1-2/(-c*x+1))/d^2-1/2*b^2*polylog(3,-1+2/(-c*x+1))/d^2-1
/2*b^2*polylog(3,1-2/(c*x+1))/d^2
```

Rubi [A] time = 0.64, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```

```
[Out] b^2/(2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*d^2) + (b*(a + b*ArcTanh[c*x]
))/(d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*d^2) + (a + b*ArcTanh[c*x])^
2/(d^2*(1 + c*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2
 + ((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)])/d^2 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
2/(1 - c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^
2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) - (b^2*PolyLog[3, -1 + 2/(1 -
 c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^2}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)^2} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^2} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{d^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)}{d^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)}{d^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}\right.}{}$$

$$= \frac{b^2}{2d^2(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} +$$

$$= \frac{b^2}{2d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{2d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\right.}{d^2}$$

Mathematica [C] time = 0.86, size = 254, normalized size = 0.86

$$\frac{\frac{24a^2}{cx+1} + 24a^2\log(cx) - 24a^2\log(cx+1) + 12ab\left(-2\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) - \sinh\left(2\tanh^{-1}(cx)\right) + \cosh\left(2\tanh^{-1}(cx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```

```
[Out] ((24*a^2)/(1 + c*x) + 24*a^2*Log[c*x] - 24*a^2*Log[1 + c*x] + 12*a*b*(Cosh[
2*ArcTanh[c*x]] - 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[
2*ArcTanh[c*x]] + 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) -
Sinh[2*ArcTanh[c*x]]) + b^2*(I*Pi^3 - 16*ArcTanh[c*x]^3 + 6*Cosh[2*ArcTanh[
```

```
c*x]] + 12*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 12*ArcTanh[c*x]^2*Cosh[2*Arc
Tanh[c*x]] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*ArcTanh[c*x
]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])] - 6*Si
nh[2*ArcTanh[c*x]] - 12*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 12*ArcTanh[c*x]
^2*Sinh[2*ArcTanh[c*x]]))/(24*d^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2}{c^2d^2x^3+2\,cd^2x^2+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^3 + 2*c
*d^2*x^2 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x), x)
```

maple [C] time = 0.60, size = 1566, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x)
```

```
[Out] 1/2*I*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1
)))^3*arctanh(c*x)^2+1/2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*
x^2-1))^3+1/2*I*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*
x^2+1)))^3*arctanh(c*x)^2-1/2*b^2/d^2*arctanh(c*x)/(c*x+1)*c*x-a*b/d^2*dilo
g(c*x)-a*b/d^2*dilog(c*x+1)+b^2/d^2*arctanh(c*x)^2*ln(c*x)-b^2/d^2*arctanh(
c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2/d^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c
^2*x^2+1)^(1/2))+2*b^2/d^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/
2))+2*b^2/d^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d^2*ar
ctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+a^2/d^2/(c*x+1)-a^2/d^2*ln(c*
x+1)-1/2*b^2/d^2*arctanh(c*x)^2-2/3*b^2/d^2*arctanh(c*x)^3+a^2/d^2*ln(c*x)-
2*b^2/d^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d^2*polylog(3,-(c*x+1
)/(-c^2*x^2+1)^(1/2))+2*a*b/d^2*arctanh(c*x)*ln(c*x)-a*b/d^2*ln(c*x)*ln(c*x
+1)-1/4*b^2/d^2/(c*x+1)*c*x+2*a*b/d^2*arctanh(c*x)/(c*x+1)-2*a*b/d^2*arctan
h(c*x)*ln(c*x+1)-a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+a*b/d^2*ln(-1/2*c*x+1/2
)*ln(1/2+1/2*c*x)+1/4*b^2/d^2/(c*x+1)+1/2*I*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/2*I*b^2/d^2*arctanh(c
*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/2*I*b^2/d^2*arcta
nh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)
^2+a*b/d^2/(c*x+1)+b^2/d^2*arctanh(c*x)^2/(c*x+1)+1/2*b^2/d^2*arctanh(c*x)/
(c*x+1)+2*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d^2*arc
```

```
tanh(c*x)^2*ln(2)-b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+1/2*a*b/d^2*ln(c*x+1)^2+
a*b/d^2*dilog(1/2+1/2*c*x)-1/2*a*b/d^2*ln(c*x+1)+1/2*a*b/d^2*ln(c*x-1)-1/2*
I*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*b^2/d^2*arctanh(c
*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))
-1/2*I*b^2/d^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+I*b^2/d^2*arctanh(
c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{1}{cd^2x+d^2}-\frac{\log(cx+1)}{d^2}+\frac{\log(x)}{d^2}\right)+\frac{\left(b^2-\left(b^2cx+b^2\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(cd^2x+d^2\right)}+\int\frac{\left(b^2cx-b^2\right)\log(cx+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a^2*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/4*(b^2 - (b^2*c
*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x + d^2) + integrate(1/4*((b
^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) - 2*(b^2*c^2*
x^2 - 2*a*b + (2*a*b*c + b^2*c)*x - (b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2)*log
(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^4 + c^2*d^2*x^3 - c*d^2*x^2 - d^2*x),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\mathrm{atanh}(c\,x))^2}{x\,(d+c\,d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^2}{c^2x^3+2cx^2+x}\,dx+\int\frac{b^2\,\mathrm{atanh}^2(cx)}{c^2x^3+2cx^2+x}\,dx+\int\frac{2ab\,\mathrm{atanh}(cx)}{c^2x^3+2cx^2+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b**2*atanh(c*x)**2
/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**3 + 2*
c*x**2 + x), x))/d**2
```

3.109 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^2}\,dx$

Optimal. Leaf size=371

$$\frac{2bc\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2bc\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{2bc\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(c\right.}{d^2}$$

```
[Out] -1/2*b^2*c/d^2/(c*x+1)+1/2*b^2*c*arctanh(c*x)/d^2-b*c*(a+b*arctanh(c*x))/d^
2/(c*x+1)+3/2*c*(a+b*arctanh(c*x))^2/d^2-(a+b*arctanh(c*x))^2/d^2/x-c*(a+b*
arctanh(c*x))^2/d^2/(c*x+1)+4*c*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))
/d^2-2*c*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x))*ln
(2-2/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2-2*b*
c*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x))*
polylog(2,1-2/(c*x+1))/d^2-b^2*c*polylog(2,-1+2/(c*x+1))/d^2-b^2*c*polylog(
3,1-2/(-c*x+1))/d^2+b^2*c*polylog(3,-1+2/(-c*x+1))/d^2+b^2*c*polylog(3,1-2/
(c*x+1))/d^2
```

Rubi [A] time = 0.80, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$\frac{2bc\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2bc\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{2bc\text{PolyLog}\left(2,1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

```
[Out] -(b^2*c)/(2*d^2*(1 + c*x)) + (b^2*c*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*Arc
Tanh[c*x]))/(d^2*(1 + c*x)) + (3*c*(a + b*ArcTanh[c*x])^2)/(2*d^2) - (a + b
*ArcTanh[c*x])^2/(d^2*x) - (c*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c*x)) - (4*
c*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (2*c*(a + b*ArcTan
h[c*x])^2*Log[2/(1 + c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1
+ c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2
 - (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (2*b*c*(
a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1
 + 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/d^2 + (b^2*c*Pol
yLog[3, -1 + 2/(1 - c*x)])/d^2 + (b^2*c*PolyLog[3, 1 - 2/(1 + c*x)])/d^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
```

```
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^2}dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)^2} + \frac{2c^2\left(a\right.}{}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}dx}{d^2} - \frac{(2c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}dx}{d^2} + \frac{c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{d^2} + \frac{\left(2c^2\right)\int}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} - \frac{4c\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\right.}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} - \frac{4c\left(a+b\tan\right.}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\,\mathrm{ta}\right.}{d^2(1}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\,\mathrm{ta}\right.}{d^2(1}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\,\mathrm{ta}\right.}{d^2(1}$$

$$= -\frac{b^2c}{2d^2(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right.}{d^2x}$$

$$= -\frac{b^2c}{2d^2(1+cx)} + \frac{b^2c\tanh^{-1}(cx)}{2d^2} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} -$$

Mathematica [C] time = 1.72, size = 347, normalized size = 0.94

$$\frac{-\frac{12a^2c}{cx+1} - 24a^2c\log(x) + 24a^2c\log(cx+1) - \frac{12a^2}{x} + 6abc\left(4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 4\operatorname{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) + \sinh\left(2\tanh^{-1}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

```
[Out] ((-12*a^2)/x - (12*a^2*c)/(1 + c*x) - 24*a^2*c*Log[x] + 24*a^2*c*Log[1 + c*
x] + b^2*c*((-I)*Pi^3 + 12*ArcTanh[c*x]^2 - (12*ArcTanh[c*x]^2)/(c*x) + 16*
ArcTanh[c*x]^3 - 3*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x
]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 24*ArcTanh[c*x]*Log[1 - E^(-2*
ArcTanh[c*x])] - 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 12*PolyLog
[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] +
 12*PolyLog[3, E^(2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x
]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 6*a*b*c*(
-Cosh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog[2, E^(-2
*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) - 2*Cosh[2*
ArcTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTanh[c*x]])))/
(12*d^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\,(cx)^2 + 2\,ab\,\mathrm{artanh}\,(cx) + a^2}{c^2d^2x^4 + 2\,cd^2x^3 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^4 + 2*c
*d^2*x^3 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^2), x)
```

maple [C] time = 1.00, size = 7397, normalized size = 19.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\frac{2\,cx+1}{cd^2x^2+d^2x} - \frac{2\,c\log(cx+1)}{d^2} + \frac{2\,c\log(x)}{d^2}\right) - \frac{\left(2\,b^2cx + b^2 - 2\left(b^2c^2x^2 + b^2cx\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(cd^2x^2 + d^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/d
^2) - 1/4*(2*b^2*c*x + b^2 - 2*(b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(-c
*x + 1)^2/(c*d^2*x^2 + d^2*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1
)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(2*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 2
*a*b - (2*a*b*c - b^2*c)*x - (2*b^2*c^4*x^4 + 4*b^2*c^3*x^3 + 2*b^2*c^2*x^2
 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^5 + c^2*d^2*x^4 -
 c*d^2*x^3 - d^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^2}{x^2\,(d + c\,d\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^4+2cx^3+x^2}\, dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{c^2x^4+2cx^3+x^2}\, dx + \int \frac{2ab \operatorname{atanh}(cx)}{c^2x^4+2cx^3+x^2}\, dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b**2*atanh(c*x)
**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**
4 + 2*c*x**3 + x**2), x))/d**2
```

3.110 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)^2}\,dx$

Optimal. Leaf size=480

$$-\frac{3bc^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{3bc^2\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{3bc^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh\right.}{d^2}$$

```
[Out] 1/2*b^2*c^2/d^2/(c*x+1)-1/2*b^2*c^2*arctanh(c*x)/d^2-b*c*(a+b*arctanh(c*x))
/d^2/x+b*c^2*(a+b*arctanh(c*x))/d^2/(c*x+1)-2*c^2*(a+b*arctanh(c*x))^2/d^2-
1/2*(a+b*arctanh(c*x))^2/d^2/x^2+2*c*(a+b*arctanh(c*x))^2/d^2/x+c^2*(a+b*ar
ctanh(c*x))^2/d^2/(c*x+1)-6*c^2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))
/d^2+b^2*c^2*ln(x)/d^2+3*c^2*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2-1/2*b^2
*c^2*ln(-c^2*x^2+1)/d^2-4*b*c^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d^2-3*b*
c^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2+3*b*c^2*(a+b*arctanh(c*x
))*polylog(2,-1+2/(-c*x+1))/d^2-3*b*c^2*(a+b*arctanh(c*x))*polylog(2,1-2/(c
*x+1))/d^2+2*b^2*c^2*polylog(2,-1+2/(c*x+1))/d^2+3/2*b^2*c^2*polylog(3,1-2/
(-c*x+1))/d^2-3/2*b^2*c^2*polylog(3,-1+2/(-c*x+1))/d^2-3/2*b^2*c^2*polylog(
3,1-2/(c*x+1))/d^2
```

Rubi [A] time = 0.95, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 22, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5940, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 5914, 6052, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{3bc^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{3bc^2\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```

```
[Out] (b^2*c^2)/(2*d^2*(1 + c*x)) - (b^2*c^2*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*
ArcTanh[c*x]))/(d^2*x) + (b*c^2*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*
c^2*(a + b*ArcTanh[c*x])^2)/d^2 - (a + b*ArcTanh[c*x])^2/(2*d^2*x^2) + (2*c
*(a + b*ArcTanh[c*x])^2)/(d^2*x) + (c^2*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c
*x)) + (6*c^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 + (b^2*c
^2*Log[x])/d^2 + (3*c^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b^2
*c^2*Log[1 - c^2*x^2])/(2*d^2) - (4*b*c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1
 + c*x)])/d^2 - (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/
d^2 + (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 - (3*
b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 + (2*b^2*c^2*Po
lyLog[2, -1 + 2/(1 + c*x)])/d^2 + (3*b^2*c^2*PolyLog[3, 1 - 2/(1 - c*x)])/(
2*d^2) - (3*b^2*c^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^2) - (3*b^2*c^2*Poly
Log[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
```

```
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)^2}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x^3} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x^2} + \frac{3c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c^3\left(a\right.}{}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d^2} - \frac{(2c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} + \frac{\left(3c^2\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} - \frac{c^3\int \frac{(a}{}}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{6c^2\left(a+\right.}{}$$

$$= -\frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+\right.}{d^2}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\right.}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\right.}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\right.}{}$$

$$= \frac{b^2c^2}{2d^2(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(c\right.}{d^2}$$

$$= \frac{b^2c^2}{2d^2(1+cx)} - \frac{b^2c^2\tanh^{-1}(cx)}{2d^2} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2}{}$$

Mathematica [C] time = 2.17, size = 452, normalized size = 0.94

$$\frac{\frac{8a^2c^2}{cx+1} + 24a^2c^2\log(x) - 24a^2c^2\log(cx+1) + \frac{16a^2c}{x} - \frac{4a^2}{x^2} + \frac{4ab\left(-6c^2x^2\operatorname{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right)+cx\left(-8cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)-cx\sinh\left(2\tanh^{-1}\right.\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```

```
[Out] ((-4*a^2)/x^2 + (16*a^2*c)/x + (8*a^2*c^2)/(1 + c*x) + 24*a^2*c^2*Log[x] -
24*a^2*c^2*Log[1 + c*x] + b^2*c^2*(I*Pi^3 - (8*ArcTanh[c*x])/(c*x) - 12*Arc
Tanh[c*x]^2 - (4*ArcTanh[c*x]^2)/(c^2*x^2) + (16*ArcTanh[c*x]^2)/(c*x) - 16
*ArcTanh[c*x]^3 + 2*Cosh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*Cosh[2*ArcTanh[c*
x]] + 4*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 32*ArcTanh[c*x]*Log[1 - E^(-2
*ArcTanh[c*x])] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 8*Log[(c*
x)/Sqrt[1 - c^2*x^2]] + 16*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*ArcTanh[c*x
]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])] - 2*Si
nh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]^2
*Sinh[2*ArcTanh[c*x]]) + (4*a*b*(-6*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]
 + c*x*(-2 + c*x*Cosh[2*ArcTanh[c*x]] - 8*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]
- c*x*Sinh[2*ArcTanh[c*x]]) + 2*ArcTanh[c*x]*(-1 + 4*c*x + c^2*x^2 + c^2*x^
2*Cosh[2*ArcTanh[c*x]] + 6*c^2*x^2*Log[1 - E^(-2*ArcTanh[c*x])] - c^2*x^2*S
inh[2*ArcTanh[c*x]])))/x^2)/(8*d^2)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{c^2d^2x^5 + 2\,cd^2x^4 + d^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^5 + 2*c
*d^2*x^4 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^3), x)
```

maple [C] time = 2.34, size = 2009, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x)
```

```
[Out] 3*I*c^2*b^2/d^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2
*x^2-1))^2*arctanh(c*x)^2-3/2*I*c^2*b^2/d^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^
2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arct
anh(c*x)^2-3/2*I*c^2*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/2*
I*c^2*b^2/d^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*arctanh(c*x)^2-3/2*I*c^2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3
/2*I*c^2*b^2/d^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/4*b^2*c^2/d^2/(c*x+
1)-b^2*c^2*arctanh(c*x)/d^2+4*c*a*b/d^2*arctanh(c*x)/x-1/2*c^3*b^2/d^2*arct
anh(c*x)/(c*x+1)*x-3*c^2*a*b/d^2*ln(c*x)*ln(c*x+1)+2*c^2*a*b/d^2*arctanh(c*
```

```
x)/(c*x+1)-6*c^2*a*b/d^2*arctanh(c*x)*ln(c*x+1)-3*c^2*a*b/d^2*ln(-1/2*c*x+1
/2)*ln(c*x+1)+3*c^2*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+6*c^2*a*b/d^2*
arctanh(c*x)*ln(c*x)+3/2*I*c^2*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(
c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-3/2*I*c^2*b^2/d^2*Pi*csgn(I/(1+(c*x+
1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+4*c^2*b^2/d^2*dilog((c*x+1)/
(-c^2*x^2+1)^(1/2))-3*c^2*a^2/d^2*ln(c*x+1)+3/2*I*c^2*b^2/d^2*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+3/2*I*c^2*b
^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*a
rctanh(c*x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(
c*x)^2+6*c^2*b^2/d^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c
^2*a*b/d^2*ln(c*x+1)+2*c^2*a*b/d^2*ln(c*x-1)+3*c^2*b^2/d^2*arctanh(c*x)^2*l
n(c*x)-3*c^2*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3*c^2*b^2/
d^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c^2*b^2/d^2*arctanh(c
*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/2*c^2*a*b/d^2*ln(c*x+1)^2-1/4*
c^3*b^2/d^2/(c*x+1)*x-3*c^2*b^2/d^2*arctanh(c*x)^2*ln(c*x+1)-4*c^2*a*b/d^2*
ln(c*x)-4*c^2*b^2/d^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c*b^2
/d^2*arctanh(c*x)^2/x+3*c^2*b^2/d^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1
)^(1/2))+c^2*b^2/d^2*arctanh(c*x)^2/(c*x+1)+1/2*c^2*b^2/d^2*arctanh(c*x)/(c
*x+1)-c*b^2/d^2*arctanh(c*x)/x+6*c^2*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^
2*x^2+1)^(1/2))+3*c^2*a*b/d^2*dilog(1/2+1/2*c*x)+3*c^2*b^2/d^2*arctanh(c*x)
^2*ln(2)+c^2*a*b/d^2/(c*x+1)-c*a*b/d^2/x-a*b/d^2*arctanh(c*x)/x^2-3*c^2*a*b
/d^2*dilog(c*x)-3*c^2*a*b/d^2*dilog(c*x+1)-1/2*a^2/d^2/x^2+c^2*b^2/d^2*ln((
c*x+1)/(-c^2*x^2+1)^(1/2)-1)+c^2*b^2/d^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2
*c^2*b^2/d^2*arctanh(c*x)^2-1/2*b^2/d^2*arctanh(c*x)^2/x^2+c^2*a^2/d^2/(c*x
+1)+3*c^2*a^2/d^2*ln(c*x)-6*c^2*b^2/d^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2
))-6*c^2*b^2/d^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-4*c^2*b^2/d^2*dilog
(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*b^2/d^2*arctanh(c*x)^3+2*c*a^2/d^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a^2\left(\frac{6c^2\log(cx+1)}{d^2}-\frac{6c^2\log(x)}{d^2}-\frac{6c^2x^2+3cx-1}{cd^2x^3+d^2x^2}\right)+\frac{\left(6b^2c^2x^2+3b^2cx-b^2-6\left(b^2c^3x^3+b^2c^2x^2\right)\log(cx-\right.}{8\left(cd^2x^3+d^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x -
1)/(c*d^2*x^3 + d^2*x^2)) + 1/8*(6*b^2*c^2*x^2 + 3*b^2*c*x - b^2 - 6*(b^2*c
^3*x^3 + b^2*c^2*x^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x^3 + d^2*x^2) +
 integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x
+ 1) - (6*b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 2*b^2*c^2*x^2 - 4*a*b + (4*a*b*c -
b^2*c)*x - 2*(3*b^2*c^5*x^5 + 6*b^2*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x + b^2
)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^6 + c^2*d^2*x^5 - c*d^2*x^4 - d^2
*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2}{x^3\,(d+c\,d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^5+2cx^4+x^3}\,dx+\int \frac{b^2\operatorname{atanh}^2(cx)}{c^2x^5+2cx^4+x^3}\,dx+\int \frac{2ab\operatorname{atanh}(cx)}{c^2x^5+2cx^4+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b**2*atanh(c*x)
**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**
5 + 2*c*x**4 + x**3), x))/d**2
```

3.111 $\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=408

$$\frac{6b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(cx+1)^2}+\frac{4\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^3(cx+1)}-\frac{(a+b}{2c^5}$$

```
[Out] a*b*x/c^4/d^3-1/16*b^2/c^5/d^3/(c*x+1)^2+29/16*b^2/c^5/d^3/(c*x+1)-29/16*b^
2*arctanh(c*x)/c^5/d^3+b^2*x*arctanh(c*x)/c^4/d^3-1/4*b*(a+b*arctanh(c*x))/
c^5/d^3/(c*x+1)^2+15/4*b*(a+b*arctanh(c*x))/c^5/d^3/(c*x+1)-43/8*(a+b*arcta
nh(c*x))^2/c^5/d^3-3*x*(a+b*arctanh(c*x))^2/c^4/d^3+1/2*x^2*(a+b*arctanh(c*
x))^2/c^3/d^3-1/2*(a+b*arctanh(c*x))^2/c^5/d^3/(c*x+1)^2+4*(a+b*arctanh(c*x
))^2/c^5/d^3/(c*x+1)+6*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^5/d^3-6*(a+b*a
rctanh(c*x))^2*ln(2/(c*x+1))/c^5/d^3+1/2*b^2*ln(-c^2*x^2+1)/c^5/d^3+3*b^2*p
olylog(2,1-2/(-c*x+1))/c^5/d^3+6*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1)
)/c^5/d^3+3*b^2*polylog(3,1-2/(c*x+1))/c^5/d^3
```

Rubi [A] time = 0.81, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 5928, 5926, 627, 44, 207, 6056, 6610}

$$\frac{6b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3}+\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^5d^3}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{x^2\left(a+b\tanh\right.}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (a*b*x)/(c^4*d^3) - b^2/(16*c^5*d^3*(1 + c*x)^2) + (29*b^2)/(16*c^5*d^3*(1
+ c*x)) - (29*b^2*ArcTanh[c*x])/(16*c^5*d^3) + (b^2*x*ArcTanh[c*x])/(c^4*d^
3) - (b*(a + b*ArcTanh[c*x]))/(4*c^5*d^3*(1 + c*x)^2) + (15*b*(a + b*ArcTan
h[c*x]))/(4*c^5*d^3*(1 + c*x)) - (43*(a + b*ArcTanh[c*x])^2)/(8*c^5*d^3) -
(3*x*(a + b*ArcTanh[c*x])^2)/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^
3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh
[c*x])^2)/(c^5*d^3*(1 + c*x)) + (6*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])
/(c^5*d^3) - (6*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^5*d^3) + (b^2*L
og[1 - c^2*x^2])/(2*c^5*d^3) + (3*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^5*d^3
) + (6*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3) + (3*b
^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

```
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \int \left(-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(1+cx)^3} - \right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}dx}{c^4d^3} - \frac{3\int \left(a+b\tanh^{-1}(cx)\right)^2 dx}{c^4d^3} - \frac{4\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{c^4d^3} + \frac{6}{}$$

$$= -\frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5d^3(1+cx)^2} + \frac{4(}{}$$

$$= -\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^3} - \frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3} - \frac{(}{}$$

$$= \frac{abx}{c^4d^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2} + \frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)} - \frac{43\left(a+b\tanh^{-1}(cx)\right)}{8c^5d^3}$$

$$= \frac{abx}{c^4d^3} + \frac{b^2x\tanh^{-1}(cx)}{c^4d^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2} + \frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)} - \frac{43(}{}$$

$$= \frac{abx}{c^4d^3} + \frac{b^2x\tanh^{-1}(cx)}{c^4d^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2} + \frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)} - \frac{43(}{}$$

$$= \frac{abx}{c^4d^3} - \frac{b^2}{16c^5d^3(1+cx)^2} + \frac{29b^2}{16c^5d^3(1+cx)} + \frac{b^2x\tanh^{-1}(cx)}{c^4d^3} - \frac{b\left(a+b\tanh^{-1}(\right.}{4c^5d^3(1+cx)}$$

$$= \frac{abx}{c^4d^3} - \frac{b^2}{16c^5d^3(1+cx)^2} + \frac{29b^2}{16c^5d^3(1+cx)} - \frac{29b^2\tanh^{-1}(cx)}{16c^5d^3} + \frac{b^2x\tanh^{-1}(cx)}{c^4d^3}$$

Mathematica [A] time = 2.00, size = 420, normalized size = 1.03

$$8a^2c^2x^2 - 48a^2cx + \frac{64a^2}{cx+1} - \frac{8a^2}{(cx+1)^2} + 96a^2\log(cx+1) + ab\left(-48\log\left(1-c^2x^2\right) + 4\tanh^{-1}(cx)\left(4c^2x^2 - 24cx - \right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (-48*a^2*c*x + 8*a^2*c^2*x^2 - (8*a^2)/(1 + c*x)^2 + (64*a^2)/(1 + c*x) + 9
6*a^2*Log[1 + c*x] + a*b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh
[c*x]] - 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*Sin
h[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x + 4*
c^2*x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2
*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])) + 16*b^2
*((-3 + 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + (56*Cosh[2*ArcTa
nh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[1 - c^2*x^2] + 192*PolyLog[3, -E^(
-2*ArcTanh[c*x])] - 56*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcT
anh[c*x]*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 96*Log[
1 + E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])
+ 8*ArcTanh[c*x]^2*(20 - 24*c*x + 4*c^2*x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cos
h[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x
]] + Sinh[4*ArcTanh[c*x]]))/64))/(16*c^5*d^3)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^4\,\text{artanh}\,(cx)^2 + 2\,abx^4\,\text{artanh}\,(cx) + a^2x^4}{c^3d^3x^3 + 3\,c^2d^3x^2 + 3\,cd^3x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2 x^4}{(cdx + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^3, x)
```

maple [C] time = 2.00, size = 1565, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] -6*I/c^5*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^2-3*I/c^5*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
))^2+3*I/c^5*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3*I/c^5*b^2/d^3*arctan
h(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-
1))+a*b*x/c^4/d^3+b^2*x*arctanh(c*x)/c^4/d^3-1/64*b^2/c^5/d^3/(c*x+1)^2+7/8
*b^2/c^5/d^3/(c*x+1)+b^2*arctanh(c*x)/c^5/d^3+1/2/c^3*a^2/d^3*x^2-3/c^4*a^2
/d^3*x+4/c^5*a^2/d^3/(c*x+1)-1/2/c^5*a^2/d^3/(c*x+1)^2+6/c^5*a^2/d^3*ln(c*x
+1)-43/8/c^5*b^2/d^3*arctanh(c*x)^2+3/c^5*b^2/d^3*polylog(3,-(c*x+1)^2/(-c^
2*x^2+1))-1/c^5*b^2/d^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))+6/c^5*b^2/d^3*dilog(1+
I*(c*x+1)/(-c^2*x^2+1)^(1/2))+6/c^5*b^2/d^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^
(1/2))+4/c^5*b^2/d^3*arctanh(c*x)^3+1/c^5*a*b/d^3+3*I/c^5*b^2/d^3*arctanh(c
*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+8/c^5*a*b/d^3*arcta
nh(c*x)/(c*x+1)-7/4/c^4*b^2/d^3*arctanh(c*x)/(c*x+1)*x-1/16/c^3*b^2/d^3*arc
tanh(c*x)/(c*x+1)^2*x^2+1/8/c^4*b^2/d^3*arctanh(c*x)/(c*x+1)^2*x+1/c^3*a*b/
d^3*arctanh(c*x)*x^2-6/c^4*a*b/d^3*arctanh(c*x)*x-1/c^5*a*b/d^3*arctanh(c*x
)/(c*x+1)^2+12/c^5*a*b/d^3*arctanh(c*x)*ln(c*x+1)+6/c^5*a*b/d^3*ln(-1/2*c*x
+1/2)*ln(c*x+1)-6/c^5*a*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-6/c^5*b^2/d^
3*arctanh(c*x)^2*ln(2)-3/c^5*a*b/d^3*ln(c*x+1)^2-6/c^5*a*b/d^3*dilog(1/2+1/
2*c*x)-43/8/c^5*a*b/d^3*ln(c*x+1)-5/8/c^5*a*b/d^3*ln(c*x-1)+1/2/c^3*b^2/d^3
*arctanh(c*x)^2*x^2-3/c^4*b^2/d^3*arctanh(c*x)^2*x+4/c^5*b^2/d^3*arctanh(c*
x)^2/(c*x+1)-1/2/c^5*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2-6/c^5*b^2/d^3*arctanh
(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+6/c^5*b^2/d^3*arctanh(c*x)*ln(1+I*
(c*x+1)/(-c^2*x^2+1)^(1/2))+6/c^5*b^2/d^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2
*x^2+1)^(1/2))-12/c^5*b^2/d^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))
-1/64/c^3*b^2/d^3/(c*x+1)^2*x^2+1/32/c^4*b^2/d^3/(c*x+1)^2*x-7/8/c^4*b^2/d^
3/(c*x+1)*x-1/4/c^5*a*b/d^3/(c*x+1)^2+15/4/c^5*a*b/d^3/(c*x+1)+6/c^5*b^2/d^
3*arctanh(c*x)^2*ln(c*x+1)+7/4/c^5*b^2/d^3*arctanh(c*x)/(c*x+1)-1/16/c^5*b^
2/d^3*arctanh(c*x)/(c*x+1)^2-3*I/c^5*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^3-3*I/c^5*b^2/d^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{8\,cx+7}{c^7d^3x^2+2\,c^6d^3x+c^5d^3}+\frac{cx^2-6\,x}{c^4d^3}+\frac{12\,\log\left(cx+1\right)}{c^5d^3}\right)+\frac{\left(b^2c^4x^4-4\,b^2c^3x^3-11\,b^2c^2x^2+2\,b^2cx+7\,b^2+\right.}{8\left(c^7d^3x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((8*c*x + 7)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + (c*x^2 - 6*x)/
(c^4*d^3) + 12*log(c*x + 1)/(c^5*d^3)) + 1/8*(b^2*c^4*x^4 - 4*b^2*c^3*x^3 -
 11*b^2*c^2*x^2 + 2*b^2*c*x + 7*b^2 + 12*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*lo
g(c*x + 1))*log(-c*x + 1)^2/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - integra
te(-1/4*((b^2*c^5*x^5 - b^2*c^4*x^4)*log(c*x + 1)^2 + 4*(a*b*c^5*x^5 - a*b*
c^4*x^4)*log(c*x + 1) + (15*b^2*c^3*x^3 + 9*b^2*c^2*x^2 - (4*a*b*c^5 + b^2*
c^5)*x^5 + (4*a*b*c^4 + 3*b^2*c^4)*x^4 - 9*b^2*c*x - 7*b^2 - 2*(b^2*c^5*x^5
 - b^2*c^4*x^4 + 6*b^2*c^3*x^3 + 18*b^2*c^2*x^2 + 18*b^2*c*x + 6*b^2)*log(c
*x + 1))*log(-c*x + 1))/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^
3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

```
[Out] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^4}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int \frac{b^2x^4\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int \frac{2abx^4\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**4*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.112 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=337

$$-\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}-\frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(cx+1)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(cx+1)^2}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(cx+1)}+\frac{(a+}{2}$$

```
[Out] 1/16*b^2/c^4/d^3/(c*x+1)^2-21/16*b^2/c^4/d^3/(c*x+1)+21/16*b^2*arctanh(c*x)
/c^4/d^3+1/4*b*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)^2-11/4*b*(a+b*arctanh(c*x
))/c^4/d^3/(c*x+1)+19/8*(a+b*arctanh(c*x))^2/c^4/d^3+x*(a+b*arctanh(c*x))^2
/c^3/d^3+1/2*(a+b*arctanh(c*x))^2/c^4/d^3/(c*x+1)^2-3*(a+b*arctanh(c*x))^2/
c^4/d^3/(c*x+1)-2*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4/d^3+3*(a+b*arctan
h(c*x))^2*ln(2/(c*x+1))/c^4/d^3-b^2*polylog(2,1-2/(-c*x+1))/c^4/d^3-3*b*(a+
b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^4/d^3-3/2*b^2*polylog(3,1-2/(c*x+1
))/c^4/d^3
```

Rubi [A] time = 0.66, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5928, 5926, 627, 44, 207, 5948, 6056, 6610}

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^3}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{11b\left(a+b\tanh\right.}{4c^4d^3(c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] b^2/(16*c^4*d^3*(1 + c*x)^2) - (21*b^2)/(16*c^4*d^3*(1 + c*x)) + (21*b^2*Ar
cTanh[c*x])/(16*c^4*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)^2)
 - (11*b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)) + (19*(a + b*ArcTanh[c
*x])^2)/(8*c^4*d^3) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d^3) + (a + b*ArcTanh
[c*x])^2/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^3*(1 +
 c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^4*d^3) + (3*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^3) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(c^4*d^3) - (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
4*d^3) - (3*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)^3} + \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)^2} - \frac{3\left(a+b\right.}{c^3d}\right.$$

$$= \frac{\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^3d^3} - \frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{c^3d^3} + \frac{3\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^3d^3} - 3\int\frac{(a}{}$$

$$= \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(1+cx)} + \frac{3\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tan\right.}{c^4d^3(1}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)} + \frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3} + \frac{x\left(a+\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)} + \frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3} + \frac{x\left(a+\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)} + \frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3} + \frac{x\left(a+\right.}{}$$

$$= \frac{b^2}{16c^4d^3(1+cx)^2} - \frac{21b^2}{16c^4d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-1}(cx)\right.}{4c^4d^3(1+cx)}$$

$$= \frac{b^2}{16c^4d^3(1+cx)^2} - \frac{21b^2}{16c^4d^3(1+cx)} + \frac{21b^2\tanh^{-1}(cx)}{16c^4d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11}{}$$

Mathematica [A] time = 1.30, size = 418, normalized size = 1.24

$$64a^2cx - \frac{192a^2}{cx+1} + \frac{32a^2}{(cx+1)^2} - 192a^2\log(cx+1) + 4ab\left(16\log\left(1-c^2x^2\right) - 48\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right) + 20\sinh\left(2\tanh^{-1}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (64*a^2*c*x + (32*a^2)/(1 + c*x)^2 - (192*a^2)/(1 + c*x) - 192*a^2*Log[1 +
c*x] + 4*a*b*(-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1 -
c^2*x^2] - 48*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]] +
```

```
4*ArcTanh[c*x]*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 24
*Log[1 + E^(-2*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*
x]]) - Sinh[4*ArcTanh[c*x]]) + b^2*(-64*ArcTanh[c*x]^2 + 64*c*x*ArcTanh[c*x
]^2 - 40*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] - 80*A
rcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*
Cosh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Cosh[4*ArcTanh[c*x]] - 128*ArcTanh[
c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 192*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTan
h[c*x])] - 64*(-1 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 96*P
olyLog[3, -E^(-2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]
*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] - Sinh[4*Arc
Tanh[c*x]] - 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 8*ArcTanh[c*x]^2*Sinh[4*
ArcTanh[c*x]]))/(64*c^4*d^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^3\,\text{artanh}\,(cx)^2+2\,abx^3\,\text{artanh}\,(cx)+a^2x^3}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)^2x^3}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^3, x)
```

maple [C] time = 0.96, size = 5750, normalized size = 17.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a^2\left(\frac{6\,cx+5}{c^6d^3x^2+2\,c^5d^3x+c^4d^3}-\frac{2\,x}{c^3d^3}+\frac{6\,\log\,(cx+1)}{c^4d^3}\right)+\frac{\left(2\,b^2c^3x^3+4\,b^2c^2x^2-4\,b^2cx-5\,b^2-6\,\left(b^2c^2x^2+2\right.\right.}{8\,\left(c^6d^3x^2+2\,c^5d^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - 2*x/(c^3*d^3)
 + 6*log(c*x + 1)/(c^4*d^3)) + 1/8*(2*b^2*c^3*x^3 + 4*b^2*c^2*x^2 - 4*b^2*c
*x - 5*b^2 - 6*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^
2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - integrate(-1/4*((b^2*c^4*x^4 - b^
2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1) - (2
*(2*a*b*c^4 + b^2*c^4)*x^4 - 9*b^2*c*x - 2*(2*a*b*c^3 - 3*b^2*c^3)*x^3 - 5*
b^2 + 2*(b^2*c^4*x^4 - 4*b^2*c^3*x^3 - 9*b^2*c^2*x^2 - 9*b^2*c*x - 3*b^2)*l
```

```
og(c*x + 1))*log(-c*x + 1))/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^
3*d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 (a + b \operatorname{atanh}(c x))^2}{(d + c d x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^3x^3+3c^2x^2+3cx+1} dx + \int \frac{b^2x^3 \operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1} dx + \int \frac{2abx^3 \operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1} dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**3*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.113 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=265

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(cx+1)}-\frac{(a+}{2c}$$

```
[Out] -1/16*b^2/c^3/d^3/(c*x+1)^2+13/16*b^2/c^3/d^3/(c*x+1)-13/16*b^2*arctanh(c*x
)/c^3/d^3-1/4*b*(a+b*arctanh(c*x))/c^3/d^3/(c*x+1)^2+7/4*b*(a+b*arctanh(c*x
))/c^3/d^3/(c*x+1)-7/8*(a+b*arctanh(c*x))^2/c^3/d^3-1/2*(a+b*arctanh(c*x))^
2/c^3/d^3/(c*x+1)^2+2*(a+b*arctanh(c*x))^2/c^3/d^3/(c*x+1)-(a+b*arctanh(c*x
))^2*ln(2/(c*x+1))/c^3/d^3+b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^3/
d^3+1/2*b^2*polylog(3,1-2/(c*x+1))/c^3/d^3
```

Rubi [A] time = 0.54, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5940, 5928, 5926, 627, 44, 207, 5948, 5918, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(\right.}{4c^3d^3(cx+1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] -b^2/(16*c^3*d^3*(1 + c*x)^2) + (13*b^2)/(16*c^3*d^3*(1 + c*x)) - (13*b^2*A
rcTanh[c*x])/(16*c^3*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)^2
) + (7*b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)) - (7*(a + b*ArcTanh[c*
x])^2)/(8*c^3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^3*d^3*(1 + c*x)^2) + (2*(a
 + b*ArcTanh[c*x])^2)/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(
1 + c*x)])/(c^3*d^3) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])
/(c^3*d^3) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)^3} - \frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}dx}{c^2d^3} + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}dx}{c^2d^3} - \frac{2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{c^2d^3}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3(1+cx)^2} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right.}{c^3d^3}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3(1+cx)^2} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right.}{c^3d^3}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)} - \frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3} - \frac{(a+}{2c}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)} - \frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3} - \frac{(a+}{2c}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)} - \frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3} - \frac{(a+}{2c}$$

$$= -\frac{b^2}{16c^3d^3(1+cx)^2} + \frac{13b^2}{16c^3d^3(1+cx)} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh^{-1}\right.}{4c^3d^3(1+cx}$$

$$= -\frac{b^2}{16c^3d^3(1+cx)^2} + \frac{13b^2}{16c^3d^3(1+cx)} - \frac{13b^2\tanh^{-1}(cx)}{16c^3d^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2}$$

Mathematica [A] time = 1.40, size = 310, normalized size = 1.17

$$\frac{32a^2}{cx+1} - \frac{8a^2}{(cx+1)^2} + 16a^2\log(cx+1) + ab\left(16\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right) - 12\sinh\left(2\tanh^{-1}(cx)\right) + \sinh\left(4\tanh^{-1}(cx)\right) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] ((-8*a^2)/(1 + c*x)^2 + (32*a^2)/(1 + c*x) + 16*a^2*Log[1 + c*x] + 16*b^2*(
ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c
*x])]/2 + ((-Cosh[2*ArcTanh[c*x]] + Sinh[2*ArcTanh[c*x]])*(-24 + Cosh[2*Arc
Tanh[c*x]] + 4*ArcTanh[c*x]*(-12 + Cosh[2*ArcTanh[c*x]] - Sinh[2*ArcTanh[c*
x]]) - Sinh[2*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*(-6 + Cosh[2*ArcTanh[c*x]]*(
1 + 8*Log[1 + E^(-2*ArcTanh[c*x])]) + (-1 + 8*Log[1 + E^(-2*ArcTanh[c*x])])
*Sinh[2*ArcTanh[c*x]])))/64) + a*b*(12*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTan
h[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + S
inh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTa
nh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])] - 6*Sinh[2*ArcTanh[c*x]] + Sinh[4
*ArcTanh[c*x]])))/(16*c^3*d^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^2\,\text{artanh}(cx)^2 + 2\,abx^2\,\text{artanh}(cx) + a^2x^2}{c^3d^3x^3 + 3\,c^2d^3x^2 + 3\,cd^3x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")

[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2 x^2}{(cdx + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^3, x)

maple [C] time = 0.77, size = 1241, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)

[Out] -1/64/c*b^2/d^3/(c*x+1)^2*x^2-1/c^3*a*b/d^3*arctanh(c*x)/(c*x+1)^2+2/c^3*a*b/d^3*arctanh(c*x)*ln(c*x+1)+1/c^3*a*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/c^3*a*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+4/c^3*a*b/d^3*arctanh(c*x)/(c*x+1)-1/16/c*b^2/d^3*arctanh(c*x)/(c*x+1)^2*x^2-3/4/c^2*b^2/d^3*arctanh(c*x)/(c*x+1)*x+1/32/c^2*b^2/d^3/(c*x+1)^2*x-3/8/c^2*b^2/d^3/(c*x+1)*x-1/4/c^3*a*b/d^3/(c*x+1)^2+7/4/c^3*a*b/d^3/(c*x+1)-1/16/c^3*b^2/d^3*arctanh(c*x)/(c*x+1)^2-1/c^3*b^2/d^3*arctanh(c*x)^2*ln(2)-1/2/c^3*a*b/d^3*ln(c*x+1)^2-1/c^3*a*b/d^3*dilog(1/2+1/2*c*x)-7/8/c^3*a*b/d^3*ln(c*x+1)+7/8/c^3*a*b/d^3*ln(c*x-1)+2/c^3*b^2/d^3*arctanh(c*x)^2/(c*x+1)-1/2/c^3*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2-1/c^3*b^2/d^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-2/c^3*b^2/d^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+1/c^3*b^2/d^3*arctanh(c*x)^2*ln(c*x+1)+3/4/c^3*b^2/d^3*arctanh(c*x)/(c*x+1)+1/8/c^2*b^2/d^3*arctanh(c*x)/(c*x+1)^2*x+1/2*I/c^3*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))+2/c^3*a^2/d^3/(c*x+1)-1/2/c^3*a^2/d^3/(c*x+1)^2+1/c^3*a^2/d^3*ln(c*x+1)-7/8/c^3*b^2/d^3*arctanh(c*x)^2+1/2/c^3*b^2/d^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+2/3/c^3*b^2/d^3*arctanh(c*x)^3-1/2*I/c^3*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2-1/2*I/c^3*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I/c^3*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/2*I/c^3*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-I/c^3*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I/c^3*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/64*b^2/c^3/d^3/(c*x+1)^2+3/8*b^2/c^3/d^3/(c*x+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^2 \left(\frac{4 cx + 3}{c^5 d^3 x^2 + 2 c^4 d^3 x + c^3 d^3} + \frac{2 \log(cx+1)}{c^3 d^3} \right) + \frac{\left(4 b^2 cx + 3 b^2 + 2 \left(b^2 c^2 x^2 + 2 b^2 cx + b^2\right) \log(cx+1)\right) \log(-cx}{8 \left(c^5 d^3 x^2 + 2 c^4 d^3 x + c^3 d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")

[Out] 1/2*a^2*((4*c*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1)/(c^3*d^3)) + 1/8*(4*b^2*c*x + 3*b^2 + 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*lo

```
g(c*x + 1))*log(-c*x + 1)^2/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - integra
te(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*
c^2*x^2)*log(c*x + 1) - (4*a*b*c^3*x^3 + 7*b^2*c*x - 4*(a*b*c^2 - b^2*c^2)*
x^2 + 3*b^2 + 2*(2*b^2*c^3*x^3 + 2*b^2*c^2*x^2 + 3*b^2*c*x + b^2)*log(c*x +
 1))*log(-c*x + 1))/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} x^{2}}{c^{3} x^{3} + 3 c^{2} x^{2} + 3 c x + 1}\, dx + \int \frac{b^{2} x^{2} \operatorname{atanh}^{2}{\left(c x \right)}}{c^{3} x^{3} + 3 c^{2} x^{2} + 3 c x + 1}\, dx + \int \frac{2 a b x^{2} \operatorname{atanh}{\left(c x \right)}}{c^{3} x^{3} + 3 c^{2} x^{2} + 3 c x + 1}\, dx}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**2*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.114 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(cx+1)^2}-\frac{5b^2}{16c^2d^3(cx+1)}+\frac{}{16c}$$

```
[Out] 1/16*b^2/c^2/d^3/(c*x+1)^2-5/16*b^2/c^2/d^3/(c*x+1)+5/16*b^2*arctanh(c*x)/c
^2/d^3+1/4*b*(a+b*arctanh(c*x))/c^2/d^3/(c*x+1)^2-3/4*b*(a+b*arctanh(c*x))/
c^2/d^3/(c*x+1)-1/8*(a+b*arctanh(c*x))^2/c^2/d^3+1/2*x^2*(a+b*arctanh(c*x))
^2/d^3/(c*x+1)^2
```

Rubi [A] time = 0.21, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {37, 5938, 5926, 627, 44, 207, 5948}

$$-\frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(cx+1)^2}-\frac{5b^2}{16c^2d^3(cx+1)}+\frac{}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] b^2/(16*c^2*d^3*(1 + c*x)^2) - (5*b^2)/(16*c^2*d^3*(1 + c*x)) + (5*b^2*ArcT
anh[c*x])/(16*c^2*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)^2) -
 (3*b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*d^3*(1 + c*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
```

```
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - (2bc)\int\left(\frac{a+b\tanh^{-1}(cx)}{4c^2d^3(1+cx)^3} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{8c^2d^3(1+cx)^2} - \ldots\right.$$

$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{4cd^3} - \frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{2cd^3} + \frac{(3b)\int \ldots}{4\ldots}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+\ldots\right.}{2d^3\ldots}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+\ldots\right.}{2d^3\ldots}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+\ldots\right.}{2d^3\ldots}$$

$$= \frac{b^2}{16c^2d^3(1+cx)^2} - \frac{5b^2}{16c^2d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx\ldots\right.}{4c^2d^3(1+cx)}$$

$$= \frac{b^2}{16c^2d^3(1+cx)^2} - \frac{5b^2}{16c^2d^3(1+cx)} + \frac{5b^2\tanh^{-1}(cx)}{16c^2d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \ldots$$

Mathematica [A] time = 0.31, size = 150, normalized size = 0.96

$$\frac{-2\left(16a^2+12ab+5b^2\right)(cx+1)+2\left(8a^2+4ab+b^2\right)-b(12a+5b)(cx+1)^2\log(1-cx)+b(12a+5b)(cx+1)\ldots}{32c^2d^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (2*(8*a^2 + 4*a*b + b^2) - 2*(16*a^2 + 12*a*b + 5*b^2)*(1 + c*x) - 8*b*(b*(
2 + 3*c*x) + a*(4 + 8*c*x))*ArcTanh[c*x] + 4*b^2*(-1 - 2*c*x + 3*c^2*x^2)*A
rcTanh[c*x]^2 - b*(12*a + 5*b)*(1 + c*x)^2*Log[1 - c*x] + b*(12*a + 5*b)*(1
 + c*x)^2*Log[1 + c*x])/(32*c^2*d^3*(1 + c*x)^2)
```

fricas [A] time = 0.63, size = 164, normalized size = 1.04

$$-\frac{2\left(16\,a^2+12\,ab+5\,b^2\right)cx-\left(3\,b^2c^2x^2-2\,b^2cx-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2+16\,a^2+16\,ab+8\,b^2-\left(\left(12\,ab+5\,b^2\right)c^2x^2-\right.}{32\left(c^4d^3x^2+2\,c^3d^3x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(16*a^2 + 12*a*b + 5*b^2)*c*x - (3*b^2*c^2*x^2 - 2*b^2*c*x - b^2)*
log(-(c*x + 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((12*a*b + 5*b^2)*c
^2*x^2 - 2*(4*a*b + b^2)*c*x - 4*a*b - 3*b^2)*log(-(c*x + 1)/(c*x - 1)))/(c
^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

giac [A] time = 0.32, size = 226, normalized size = 1.44

$$\frac{1}{64}c\left(\frac{2\left(\frac{2(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^3d^3}+\frac{2\left(\frac{8(cx+1)ab}{cx-1}+4\,ab+\frac{4(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^3d^3}+\frac{\left(\frac{16(cx\right.}{cx}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/64*c*(2*(2*(c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x
 - 1))^2/((c*x + 1)^2*c^3*d^3) + 2*(8*(c*x + 1)*a*b/(c*x - 1) + 4*a*b + 4*(
c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))/((c*x +
 1)^2*c^3*d^3) + (16*(c*x + 1)*a^2/(c*x - 1) + 8*a^2 + 16*(c*x + 1)*a*b/(c*
x - 1) + 4*a*b + 8*(c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2/((c*x + 1)^2*
c^3*d^3))
```

maple [B] time = 0.07, size = 460, normalized size = 2.93

$$-\frac{a^2}{c^2d^3\,(cx+1)}+\frac{a^2}{2c^2d^3\,(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)^2}{c^2d^3\,(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)^2}{2c^2d^3\,(cx+1)^2}-\frac{3b^2\operatorname{arctanh}(cx)\ln(cx-1)}{8c^2d^3}+\frac{b^2\operatorname{arctanh}(c}{4c^2d^3\,(cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] -1/c^2*a^2/d^3/(c*x+1)+1/2/c^2*a^2/d^3/(c*x+1)^2-1/c^2*b^2/d^3*arctanh(c*x)
^2/(c*x+1)+1/2/c^2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2-3/8/c^2*b^2/d^3*arctanh
(c*x)*ln(c*x-1)+1/4/c^2*b^2/d^3*arctanh(c*x)/(c*x+1)^2-3/4/c^2*b^2/d^3*arct
anh(c*x)/(c*x+1)+3/8/c^2*b^2/d^3*arctanh(c*x)*ln(c*x+1)-3/32/c^2*b^2/d^3*ln
(c*x-1)^2+3/16/c^2*b^2/d^3*ln(c*x-1)*ln(1/2+1/2*c*x)-3/32/c^2*b^2/d^3*ln(c*
x+1)^2+3/16/c^2*b^2/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/16/c^2*b^2/d^3*ln(-1/2
*c*x+1/2)*ln(1/2+1/2*c*x)-5/32/c^2*b^2/d^3*ln(c*x-1)+1/16*b^2/c^2/d^3/(c*x+
1)^2-5/16*b^2/c^2/d^3/(c*x+1)+5/32/c^2*b^2/d^3*ln(c*x+1)-2/c^2*a*b/d^3*arct
anh(c*x)/(c*x+1)+1/c^2*a*b/d^3*arctanh(c*x)/(c*x+1)^2-3/8/c^2*a*b/d^3*ln(c*
x-1)+1/4/c^2*a*b/d^3/(c*x+1)^2-3/4/c^2*a*b/d^3/(c*x+1)+3/8/c^2*a*b/d^3*ln(c
*x+1)
```

maxima [B] time = 0.34, size = 429, normalized size = 2.73

$$-\frac{(2\,cx+1)b^2\operatorname{artanh}(cx)^2}{2\left(c^4d^3x^2+2\,c^3d^3x+c^2d^3\right)}-\frac{1}{8}\left(c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3}-\frac{3\,\log(cx+1)}{c^3d^3}+\frac{3\,\log(cx-1)}{c^3d^3}\right)+\frac{8\,(2\,cx+1)\operatorname{art}}{c^4d^3x^2+2\,c^3d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x + 1)*b^2*arctanh(c*x)^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) -
 1/8*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x +
1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c^4*
d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*a*b - 1/32*(4*c*(2*(3*c*x + 2)/(c^5*d^3*x
^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^
3*d^3))*arctanh(c*x) + (3*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + 3*(c^2*x^2
 + 2*c*x + 1)*log(c*x - 1)^2 + 10*c*x - (5*c^2*x^2 + 10*c*x + 6*(c^2*x^2 +
2*c*x + 1)*log(c*x - 1) + 5)*log(c*x + 1) + 5*(c^2*x^2 + 2*c*x + 1)*log(c*x
 - 1) + 8)*c^2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3))*b^2 - 1/2*(2*c*x + 1)
*a^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

mupad [B] time = 2.69, size = 405, normalized size = 2.58

$$-\frac{16\,a\,b+17\,b^2\,\ln\left(c\,x+1\right)-17\,b^2\,\ln\left(1-c\,x\right)+b^2\ln\left(c\,x+1\right)^2+b^2\ln\left(1-c\,x\right)^2-28\,b^2\,\mathrm{atanh}\left(c\,x\right)+16\,a^2+8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

```
[Out] -(16*a*b + 17*b^2*log(c*x + 1) - 17*b^2*log(1 - c*x) + b^2*log(c*x + 1)^2 +
 b^2*log(1 - c*x)^2 - 28*b^2*atanh(c*x) + 16*a^2 + 8*b^2 + 16*a*b*log(c*x +
 1) - 16*a*b*log(1 - c*x) - 2*b^2*log(c*x + 1)*log(1 - c*x) - 24*a*b*atanh(
c*x) + 32*a^2*c*x + 10*b^2*c*x + 30*b^2*c*x*log(c*x + 1) - 30*b^2*c*x*log(1
 - c*x) - 3*b^2*c^2*x^2*log(c*x + 1)^2 - 3*b^2*c^2*x^2*log(1 - c*x)^2 - 28*
b^2*c^2*x^2*atanh(c*x) + 2*b^2*c*x*log(c*x + 1)^2 + 2*b^2*c*x*log(1 - c*x)^
2 - 56*b^2*c*x*atanh(c*x) + 9*b^2*c^2*x^2*log(c*x + 1) - 9*b^2*c^2*x^2*log(
1 - c*x) + 24*a*b*c*x + 32*a*b*c*x*log(c*x + 1) - 32*a*b*c*x*log(1 - c*x) -
 4*b^2*c*x*log(c*x + 1)*log(1 - c*x) - 24*a*b*c^2*x^2*atanh(c*x) - 48*a*b*c
*x*atanh(c*x) + 6*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x))/(32*c^2*d^3*(c*x +
 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} x}{c^{3} x^{3} + 3 c^{2} x^{2} + 3 c x + 1}\, dx + \int \frac{b^{2} x \operatorname{atanh}^{2}{\left(c x \right)}}{c^{3} x^{3} + 3 c^{2} x^{2} + 3 c x + 1}\, dx + \int \frac{2 a b x \operatorname{atanh}{\left(c x \right)}}{c^{3} x^{3} + 3 c^{2} x^{2} + 3 c x + 1}\, dx}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2*
x*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*b*
x*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.115 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(cx+1)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{3b^2}{16cd^3(cx+1)}-\frac{b^2}{16cd^3(cx$$

```
[Out] -1/16*b^2/c/d^3/(c*x+1)^2-3/16*b^2/c/d^3/(c*x+1)+3/16*b^2*arctanh(c*x)/c/d^
3-1/4*b*(a+b*arctanh(c*x))/c/d^3/(c*x+1)^2-1/4*b*(a+b*arctanh(c*x))/c/d^3/(
c*x+1)+1/8*(a+b*arctanh(c*x))^2/c/d^3-1/2*(a+b*arctanh(c*x))^2/c/d^3/(c*x+1
)^2
```

Rubi [A] time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(cx+1)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{3b^2}{16cd^3(cx+1)}-\frac{b^2}{16cd^3(cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```

```
[Out] -b^2/(16*c*d^3*(1 + c*x)^2) - (3*b^2)/(16*c*d^3*(1 + c*x)) + (3*b^2*ArcTanh
[c*x])/(16*c*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)^2) - (b*(a
+ b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(8*c*d^3) -
 (a + b*ArcTanh[c*x])^2/(2*c*d^3*(1 + c*x)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
```

```
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}+\frac{b\int\left(\frac{a+b\tanh^{-1}(cx)}{2d^2(1+cx)^3}+\frac{a+b\tanh^{-1}(cx)}{4d^2(1+cx)^2}-\frac{a+b\tanh^{-1}(cx)}{4d^2\left(-1+c^2x^2\right)}\right)dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}+\frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{4d^3}-\frac{b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{4d^3}+\frac{b\int\frac{a+b\tanh^{-1}}{(1+cx)^3}}{2d^3}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{\left(a+b\tan\right.}{2cd^3(1}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{\left(a+b\tan\right.}{2cd^3(1}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{\left(a+b\tan\right.}{2cd^3(1}$$

$$= -\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+$$

$$= -\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}+\frac{3b^2\tanh^{-1}(cx)}{16cd^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a\right.}{}$$

Mathematica [A] time = 0.14, size = 183, normalized size = 1.17

$$\frac{-8a^2-4ab-b^2}{16cd^3(cx+1)^2}+\frac{\left(-4ab-3b^2\right)\log(1-cx)}{32cd^3}+\frac{\left(4ab+3b^2\right)\log(cx+1)}{32cd^3}-\frac{b(4a+3b)}{16cd^3(cx+1)}-\frac{b\tanh^{-1}(cx)(4a+bcx+}{4cd^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```

```
[Out] (-8*a^2 - 4*a*b - b^2)/(16*c*d^3*(1 + c*x)^2) - (b*(4*a + 3*b))/(16*c*d^3*(
1 + c*x)) - (b*(4*a + 2*b + b*c*x)*ArcTanh[c*x])/(4*c*d^3*(1 + c*x)^2) + (b
^2*(-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^2)/(8*c*d^3*(1 + c*x)^2) + ((-4*a*b
- 3*b^2)*Log[1 - c*x])/(32*c*d^3) + ((4*a*b + 3*b^2)*Log[1 + c*x])/(32*c*d^
3)
```

fricas [A] time = 1.13, size = 156, normalized size = 0.99

$$-\frac{2\left(4\,ab+3\,b^2\right)cx-\left(b^2c^2x^2+2\,b^2cx-3\,b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2+16\,a^2+16\,ab+8\,b^2-\left(\left(4\,ab+3\,b^2\right)c^2x^2+2\left(4\,ab\right.\right.}{32\left(c^3d^3x^2+2\,c^2d^3x+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(4*a*b + 3*b^2)*c*x - (b^2*c^2*x^2 + 2*b^2*c*x - 3*b^2)*log(-(c*x
+ 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((4*a*b + 3*b^2)*c^2*x^2 + 2*
(4*a*b + b^2)*c*x - 12*a*b - 5*b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^3*d^3*x^2
 + 2*c^2*d^3*x + c*d^3)
```

giac [A] time = 0.19, size = 232, normalized size = 1.48

$$\frac{1}{64}c\left(\frac{2\left(\frac{2(cx+1)b^2}{cx-1}-b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^2d^3}+\frac{2\left(\frac{8(cx+1)ab}{cx-1}-4ab+\frac{4(cx+1)b^2}{cx-1}-b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^2d^3}+\frac{\left(\frac{16(cx\right.}{cx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/64*c*(2*(2*(c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x
 - 1))^2/((c*x + 1)^2*c^2*d^3) + 2*(8*(c*x + 1)*a*b/(c*x - 1) - 4*a*b + 4*(
c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))/((c*x +
 1)^2*c^2*d^3) + (16*(c*x + 1)*a^2/(c*x - 1) - 8*a^2 + 16*(c*x + 1)*a*b/(c*
x - 1) - 4*a*b + 8*(c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2/((c*x + 1)^2*
c^2*d^3))
```

maple [B] time = 0.07, size = 398, normalized size = 2.54

$$-\frac{a^2}{2c\,d^3\,(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)^2}{2c\,d^3\,(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{8c\,d^3}-\frac{b^2\operatorname{arctanh}(cx)}{4c\,d^3\,(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)}{4c\,d^3\,(cx+1)}+\frac{b^2\operatorname{arctanh}(cx}{8c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] -1/2/c*a^2/d^3/(c*x+1)^2-1/2/c*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2-1/8/c*b^2/d
^3*arctanh(c*x)*ln(c*x-1)-1/4/c*b^2/d^3*arctanh(c*x)/(c*x+1)^2-1/4/c*b^2/d^
3*arctanh(c*x)/(c*x+1)+1/8/c*b^2/d^3*arctanh(c*x)*ln(c*x+1)-1/32/c*b^2/d^3*
ln(c*x-1)^2+1/16/c*b^2/d^3*ln(c*x-1)*ln(1/2+1/2*c*x)-1/32/c*b^2/d^3*ln(c*x+
1)^2+1/16/c*b^2/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/16/c*b^2/d^3*ln(-1/2*c*x+1
/2)*ln(1/2+1/2*c*x)-3/32/c*b^2/d^3*ln(c*x-1)-1/16*b^2/c/d^3/(c*x+1)^2-3/16*
b^2/c/d^3/(c*x+1)+3/32/c*b^2/d^3*ln(c*x+1)-1/c*a*b/d^3*arctanh(c*x)/(c*x+1)
^2-1/8/c*a*b/d^3*ln(c*x-1)-1/4/c*a*b/d^3/(c*x+1)^2-1/4/c*a*b/d^3/(c*x+1)+1/
8/c*a*b/d^3*ln(c*x+1)
```

maxima [B] time = 0.34, size = 399, normalized size = 2.54

$$-\frac{1}{8}\left(c\left(\frac{2(cx+2)}{c^4d^3x^2+2c^3d^3x+c^2d^3}-\frac{\log(cx+1)}{c^2d^3}+\frac{\log(cx-1)}{c^2d^3}\right)+\frac{8\,\operatorname{artanh}(cx)}{c^3d^3x^2+2c^2d^3x+cd^3}\right)ab-\frac{1}{32}\left(4c\left(\frac{2(cx}{c^4d^3x^2+2c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)/(
c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2*d^
3*x + c*d^3))*a*b - 1/32*(4*c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2
*d^3) - log(c*x + 1)/(c^2*d^3) + log(c*x - 1)/(c^2*d^3))*arctanh(c*x) + ((c
^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 +
 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log
(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*d^3*x^2 + 2*
c^4*d^3*x + c^3*d^3))*b^2 - 1/2*b^2*arctanh(c*x)^2/(c^3*d^3*x^2 + 2*c^2*d^3
*x + c*d^3) - 1/2*a^2/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

mupad [B] time = 2.16, size = 373, normalized size = 2.38

$$11\,b^2\,\ln(1-c\,x)-11\,b^2\,\ln(c\,x+1)-16\,a\,b-3\,b^2\ln(c\,x+1)^2-3\,b^2\ln(1-c\,x)^2+12\,b^2\,\mathrm{atanh}(c\,x)-16\,a^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + c*d*x)^3,x)
```

```
[Out] (11*b^2*log(1 - c*x) - 11*b^2*log(c*x + 1) - 16*a*b - 3*b^2*log(c*x + 1)^2
- 3*b^2*log(1 - c*x)^2 + 12*b^2*atanh(c*x) - 16*a^2 - 8*b^2 - 16*a*b*log(c*
x + 1) + 16*a*b*log(1 - c*x) + 6*b^2*log(c*x + 1)*log(1 - c*x) + 8*a*b*atan
h(c*x) - 6*b^2*c*x - 10*b^2*c*x*log(c*x + 1) + 10*b^2*c*x*log(1 - c*x) + b^
2*c^2*x^2*log(c*x + 1)^2 + b^2*c^2*x^2*log(1 - c*x)^2 + 12*b^2*c^2*x^2*atan
h(c*x) + 2*b^2*c*x*log(c*x + 1)^2 + 2*b^2*c*x*log(1 - c*x)^2 + 24*b^2*c*x*a
tanh(c*x) - 3*b^2*c^2*x^2*log(c*x + 1) + 3*b^2*c^2*x^2*log(1 - c*x) - 8*a*b
*c*x - 4*b^2*c*x*log(c*x + 1)*log(1 - c*x) + 8*a*b*c^2*x^2*atanh(c*x) + 16*
a*b*c*x*atanh(c*x) - 2*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x))/(32*c*d^3*(c*
x + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int \frac{b^2\,\mathrm{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int \frac{2ab\,\mathrm{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2*at
anh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*b*atan
h(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

3.116 $\int \frac{\left(a+b \tanh^{-1}(cx)\right)^2}{x(d+cdx)^3} \, dx$

Optimal. Leaf size=362

$$-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{b\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{5b(}{}$$

```
[Out] 1/16*b^2/d^3/(c*x+1)^2+11/16*b^2/d^3/(c*x+1)-11/16*b^2*arctanh(c*x)/d^3+1/4
*b*(a+b*arctanh(c*x))/d^3/(c*x+1)^2+5/4*b*(a+b*arctanh(c*x))/d^3/(c*x+1)-5/
8*(a+b*arctanh(c*x))^2/d^3+1/2*(a+b*arctanh(c*x))^2/d^3/(c*x+1)^2+(a+b*arct
anh(c*x))^2/d^3/(c*x+1)-2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^3+(
a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^3-b*(a+b*arctanh(c*x))*polylog(2,1-2/(-
c*x+1))/d^3+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^3-b*(a+b*arctan
h(c*x))*polylog(2,1-2/(c*x+1))/d^3+1/2*b^2*polylog(3,1-2/(-c*x+1))/d^3-1/2*
b^2*polylog(3,-1+2/(-c*x+1))/d^3-1/2*b^2*polylog(3,1-2/(c*x+1))/d^3
```

Rubi [A] time = 0.80, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

```
[Out] b^2/(16*d^3*(1 + c*x)^2) + (11*b^2)/(16*d^3*(1 + c*x)) - (11*b^2*ArcTanh[c*
x])/(16*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) + (5*b*(a + b*A
rcTanh[c*x]))/(4*d^3*(1 + c*x)) - (5*(a + b*ArcTanh[c*x])^2)/(8*d^3) + (a +
 b*ArcTanh[c*x])^2/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])^2/(d^3*(1 + c
*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 + ((a + b*Ar
cTanh[c*x])^2*Log[2/(1 + c*x)])/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1
- 2/(1 - c*x)])/d^3 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])
/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 + (b^2*Poly
Log[3, 1 - 2/(1 - c*x)])/(2*d^3) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^
3) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
```

```
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^3} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^2} - \frac{c\left(a+b\,\mathrm{t}\right.}{d^3(}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(\right.}{1+cx}}{d^3}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right.}{d^3}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right.}{d^3}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{\left(a+b\tanh\right.}{2d^3(1+}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{\left(a+b\tanh\right.}{2d^3(1+}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{\left(a+b\tanh\right.}{2d^3(1+}$$

$$= \frac{b^2}{16d^3(1+cx)^2} + \frac{11b^2}{16d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - 5\left(a\right.$$

$$= \frac{b^2}{16d^3(1+cx)^2} + \frac{11b^2}{16d^3(1+cx)} - \frac{11b^2\tanh^{-1}(cx)}{16d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\right.}{4d^3(}$$

Mathematica [C] time = 1.46, size = 376, normalized size = 1.04

$$\frac{192a^2}{cx+1} + \frac{96a^2}{(cx+1)^2} + 192a^2\log(cx) - 192a^2\log(cx+1) + 12ab\left(-16\mathrm{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right) - 12\sinh\left(2\tanh^{-1}(cx)\right) - \sinh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

```
[Out] ((96*a^2)/(1 + c*x)^2 + (192*a^2)/(1 + c*x) + 192*a^2*Log[c*x] - 192*a^2*Lo
g[1 + c*x] + 12*a*b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*Po
lyLog[2, E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6
*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[c*x]
)] - 6*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]])
 + b^2*((8*I)*Pi^3 - 128*ArcTanh[c*x]^3 + 72*Cosh[2*ArcTanh[c*x]] + 144*Arc
Tanh[c*x]*Cosh[2*ArcTanh[c*x]] + 144*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] +
3*Cosh[4*ArcTanh[c*x]] + 12*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] + 24*ArcTanh[
c*x]^2*Cosh[4*ArcTanh[c*x]] + 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])
] + 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 96*PolyLog[3, E^(2*Ar
cTanh[c*x])] - 72*Sinh[2*ArcTanh[c*x]] - 144*ArcTanh[c*x]*Sinh[2*ArcTanh[c*
x]] - 144*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] - 3*Sinh[4*ArcTanh[c*x]] - 12
*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 24*ArcTanh[c*x]^2*Sinh[4*ArcTanh[c*x]]
))/(192*d^3)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{c^3d^3x^4 + 3\,c^2d^3x^3 + 3\,cd^3x^2 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^4 + 3*c
^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)^3 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x), x)
```

maple [C] time = 1.01, size = 1752, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x)
```

```
[Out] 1/2*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1
)^2/(-c^2*x^2+1)))^3-3/4*b^2/d^3*arctanh(c*x)/(c*x+1)*c*x+1/2*I*b^2/d^3*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1
/2*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/16*b^2/d^3
*arctanh(c*x)/(c*x+1)^2*c^2*x^2-1/8*b^2/d^3*arctanh(c*x)/(c*x+1)^2*c*x-b^2/
d^3*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2/d^3*arctanh(c*x)^2*ln(c
*x)+2*b^2/d^3*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d^3*ar
ctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-a*b/d^3*dilog(c*x)-a*b/d^3*di
log(c*x+1)+2*b^2/d^3*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^
2/d^3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(3,(
c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))
-1/2*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/2*I*b^2/d^3*arctanh(c*x)^2
*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I*b^2/d^3*
```

```
arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/64*b^2/d
^3/(c*x+1)^2+3/8*b^2/d^3/(c*x+1)+1/2*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^2-5/8*b^2/d^3*arctanh(c*x)^2-2/3*b^2/d^3*arctanh(c*x)^3+a^2/d^3/(c*x
+1)+1/2*a^2/d^3/(c*x+1)^2+a^2/d^3*ln(c*x)-a^2/d^3*ln(c*x+1)+2*a*b/d^3*arcta
nh(c*x)*ln(c*x)-a*b/d^3*ln(c*x)*ln(c*x+1)+b^2/d^3*arctanh(c*x)^2/(c*x+1)+1/
2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2+b^2/d^3*arctanh(c*x)^2*ln(2)+2*b^2/d^3*a
rctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-b^2/d^3*arctanh(c*x)^2*ln(c*x+
1)+3/4*b^2/d^3*arctanh(c*x)/(c*x+1)+1/16*b^2/d^3*arctanh(c*x)/(c*x+1)^2+1/4
*a*b/d^3/(c*x+1)^2+5/4*a*b/d^3/(c*x+1)+1/2*a*b/d^3*ln(c*x+1)^2+a*b/d^3*dilo
g(1/2+1/2*c*x)-5/8*a*b/d^3*ln(c*x+1)+5/8*a*b/d^3*ln(c*x-1)+2*a*b/d^3*arctan
h(c*x)/(c*x+1)+a*b/d^3*arctanh(c*x)/(c*x+1)^2-2*a*b/d^3*arctanh(c*x)*ln(c*x
+1)-a*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+a*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*
c*x)+1/64*b^2/d^3/(c*x+1)^2*c^2*x^2-1/32*b^2/d^3/(c*x+1)^2*c*x-3/8*b^2/d^3/
(c*x+1)*c*x-1/2*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-
1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b
^2/d^3*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/2*I*b^2/d^3*arctanh(c*x)
^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+I*
b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^2\left(\frac{2\,cx+3}{c^2d^3x^2+2\,cd^3x+d^3}-\frac{2\,\log\left(cx+1\right)}{d^3}+\frac{2\,\log(x)}{d^3}\right)+\frac{\left(2\,b^2cx+3\,b^2-2\left(b^2c^2x^2+2\,b^2cx+b^2\right)\log\left(cx+1\right)\right)\mathrm{l}}{8\left(c^2d^3x^2+2\,cd^3x+d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3 +
 2*log(x)/d^3) + 1/8*(2*b^2*c*x + 3*b^2 - 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)
*log(c*x + 1))*log(-c*x + 1)^2/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) + integrate(
1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) - (2*b
^2*c^3*x^3 + 5*b^2*c^2*x^2 - 4*a*b + (4*a*b*c + 3*b^2*c)*x - 2*(b^2*c^4*x^4
 + 3*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^4*d
^3*x^5 + 2*c^3*d^3*x^4 - 2*c*d^3*x^2 - d^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^2}{x\,\left(d+c\,d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3x^4+3c^2x^3+3cx^2+x}\,dx+\int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx+\int \frac{2ab\,\mathrm{atanh}\,(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b**2
*atanh(c*x)**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(2*a*
b*atanh(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

3.117 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^3}\,dx$

Optimal. Leaf size=448

$$\frac{3bc\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{3bc\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{3bc\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(c\right.}{d^3}$$

```
[Out] -1/16*b^2*c/d^3/(c*x+1)^2-19/16*b^2*c/d^3/(c*x+1)+19/16*b^2*c*arctanh(c*x)/
d^3-1/4*b*c*(a+b*arctanh(c*x))/d^3/(c*x+1)^2-9/4*b*c*(a+b*arctanh(c*x))/d^3
/(c*x+1)+17/8*c*(a+b*arctanh(c*x))^2/d^3-(a+b*arctanh(c*x))^2/d^3/x-1/2*c*(
a+b*arctanh(c*x))^2/d^3/(c*x+1)^2-2*c*(a+b*arctanh(c*x))^2/d^3/(c*x+1)+6*c*
(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^3-3*c*(a+b*arctanh(c*x))^2*ln
(2/(c*x+1))/d^3+2*b*c*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d^3+3*b*c*(a+b*arc
tanh(c*x))*polylog(2,1-2/(-c*x+1))/d^3-3*b*c*(a+b*arctanh(c*x))*polylog(2,-
1+2/(-c*x+1))/d^3+3*b*c*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d^3-b^2*c
*polylog(2,-1+2/(c*x+1))/d^3-3/2*b^2*c*polylog(3,1-2/(-c*x+1))/d^3+3/2*b^2*
c*polylog(3,-1+2/(-c*x+1))/d^3+3/2*b^2*c*polylog(3,1-2/(c*x+1))/d^3
```

Rubi [A] time = 0.98, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$\frac{3bc\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{3bc\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{3bc\text{PolyLog}\left(2,1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```

```
[Out] -(b^2*c)/(16*d^3*(1 + c*x)^2) - (19*b^2*c)/(16*d^3*(1 + c*x)) + (19*b^2*c*A
rcTanh[c*x])/(16*d^3) - (b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) - (9
*b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)) + (17*c*(a + b*ArcTanh[c*x])^2
)/(8*d^3) - (a + b*ArcTanh[c*x])^2/(d^3*x) - (c*(a + b*ArcTanh[c*x])^2)/(2*
d^3*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x])^2)/(d^3*(1 + c*x)) - (6*c*(a +
 b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 - (3*c*(a + b*ArcTanh[c*x]
)^2*Log[2/(1 + c*x)])/d^3 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)
])/d^3 + (3*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^3 - (3*
b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^3 + (3*b*c*(a + b*
ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 - (b^2*c*PolyLog[2, -1 + 2/(
1 + c*x)])/d^3 - (3*b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*P
olyLog[3, -1 + 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*PolyLog[3, 1 - 2/(1 + c*x)]
)/(2*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x^2} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^3} + \frac{2c^2\left(a\right.}{}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}dx}{d^3} - \frac{(3c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}dx}{d^3} + \frac{c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}dx}{d^3} + \frac{\left(2c^2\right)\int \frac{(}{}}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} - \frac{6c\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tan\right.}{d^3(1+}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\right.}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\right.}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\right.}{}$$

$$= -\frac{b^2c}{16d^3(1+cx)^2} - \frac{19b^2c}{16d^3(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} +$$

$$= -\frac{b^2c}{16d^3(1+cx)^2} - \frac{19b^2c}{16d^3(1+cx)} + \frac{19b^2c\tanh^{-1}(cx)}{16d^3} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a\right.}{}$$

Mathematica [C] time = 2.35, size = 479, normalized size = 1.07

$$\frac{-\frac{128a^2c}{cx+1} - \frac{32a^2c}{(cx+1)^2} - 192a^2c\log(x) + 192a^2c\log(cx+1) - \frac{64a^2}{x} + \frac{4ab\left(cx\left(32\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+20\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(\right.\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```

```
[Out] ((-64*a^2)/x - (32*a^2*c)/(1 + c*x)^2 - (128*a^2*c)/(1 + c*x) - 192*a^2*c*L
og[x] + 192*a^2*c*Log[1 + c*x] + b^2*c*((-8*I)*Pi^3 + 64*ArcTanh[c*x]^2 - (
64*ArcTanh[c*x]^2)/(c*x) + 128*ArcTanh[c*x]^3 - 40*Cosh[2*ArcTanh[c*x]] - 8
0*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]
] - Cosh[4*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] - 8*ArcTanh[
c*x]^2*Cosh[4*ArcTanh[c*x]] + 128*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])]
 - 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 64*PolyLog[2, E^(-2*Arc
Tanh[c*x])] - 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 96*PolyLog[
3, E^(2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]*Sinh[2*A
rcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]
] + 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Sinh[4*ArcTanh[c
*x]]) + (4*a*b*(48*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*x*(-20*Cosh[2*Ar
cTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 20*S
inh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]]) - 4*ArcTanh[c*x]*(8 + 10*c*x*Co
sh[2*ArcTanh[c*x]] + c*x*Cosh[4*ArcTanh[c*x]] + 24*c*x*Log[1 - E^(-2*ArcTan
h[c*x])] - 10*c*x*Sinh[2*ArcTanh[c*x]] - c*x*Sinh[4*ArcTanh[c*x]])))/x)/(64
*d^3)
```

fricas [F] time = 1.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2}{c^3d^3x^5+3\,c^2d^3x^4+3\,cd^3x^3+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^5 + 3*c
^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x^2), x)
```

maple [C] time = 1.04, size = 7593, normalized size = 16.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a^2\left(\frac{6\,c^2x^2+9\,cx+2}{c^2d^3x^3+2\,cd^3x^2+d^3x}-\frac{6\,c\log\,(cx+1)}{d^3}+\frac{6\,c\log(x)}{d^3}\right)-\frac{\left(6\,b^2c^2x^2+9\,b^2cx+2\,b^2-6\,\left(b^2c^3x^3+2\,b^2c^2x\right.\right.}{8\,\left(c^2d^3x^3+2\,cd^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*c
*log(c*x + 1)/d^3 + 6*c*log(x)/d^3) - 1/8*(6*b^2*c^2*x^2 + 9*b^2*c*x + 2*b^
2 - 6*(b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(-c*x + 1)^2
/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - integrate(-1/4*((b^2*c*x - b^2)*log(
c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 15*b^2*c^3*x
^3 + 11*b^2*c^2*x^2 + 4*a*b - 2*(2*a*b*c - b^2*c)*x - 2*(3*b^2*c^5*x^5 + 9*
b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*
log(-c*x + 1))/(c^4*d^3*x^6 + 2*c^3*d^3*x^5 - 2*c*d^3*x^3 - d^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\text{atanh}\,(c\,x))^2}{x^2\,(d+c\,d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(b
**2*atanh(c*x)**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integra
l(2*a*b*atanh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

3.118 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^4}\,dx$

Optimal. Leaf size=176

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(cx+1)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{24c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(cx+1)^3}$$

```
[Out] -1/54*b^2/c/(c*x+1)^3-5/144*b^2/c/(c*x+1)^2-11/144*b^2/c/(c*x+1)+11/144*b^2
*arctanh(c*x)/c-1/9*b*(a+b*arctanh(c*x))/c/(c*x+1)^3-1/12*b*(a+b*arctanh(c*
x))/c/(c*x+1)^2-1/12*b*(a+b*arctanh(c*x))/c/(c*x+1)+1/24*(a+b*arctanh(c*x))
^2/c-1/3*(a+b*arctanh(c*x))^2/c/(c*x+1)^3
```

Rubi [A] time = 0.22, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(cx+1)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{24c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(cx+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```

```
[Out] -b^2/(54*c*(1 + c*x)^3) - (5*b^2)/(144*c*(1 + c*x)^2) - (11*b^2)/(144*c*(1
+ c*x)) + (11*b^2*ArcTanh[c*x])/(144*c) - (b*(a + b*ArcTanh[c*x]))/(9*c*(1
+ c*x)^3) - (b*(a + b*ArcTanh[c*x]))/(12*c*(1 + c*x)^2) - (b*(a + b*ArcTanh
[c*x]))/(12*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(24*c) - (a + b*ArcTanh[c
*x])^2/(3*c*(1 + c*x)^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(1+cx)^3}+\frac{1}{3}(2b)\int\left(\frac{a+b\tanh^{-1}(cx)}{2(1+cx)^4}+\frac{a+b\tanh^{-1}(cx)}{4(1+cx)^3}+\frac{a+b\tanh^{-1}}{8(1+}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(1+cx)^3}+\frac{1}{12}b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx-\frac{1}{12}b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx+$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{\left(a+b\tanh^{-1}\right.}{24c}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{\left(a+b\tanh^{-1}\right.}{24c}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{\left(a+b\tanh^{-1}\right.}{24c}$$

$$= -\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tan\right.}{12c(1+}$$

$$= -\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)}+\frac{11b^2\tanh^{-1}(cx)}{144c}-\frac{b\left(a+b\tanh^{-1}(\right.}{9c(1+cx)^3}$$

Mathematica [A] time = 0.18, size = 168, normalized size = 0.95

$$-\frac{16\left(18a^2+6ab+b^2\right)+24b\tanh^{-1}(cx)\left(24a+b\left(3c^2x^2+9cx+10\right)\right)+6b(12a+11b)(cx+1)^2+6b(12a+5b)(c}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```

```
[Out] -1/864*(16*(18*a^2 + 6*a*b + b^2) + 6*b*(12*a + 5*b)*(1 + c*x) + 6*b*(12*a
+ 11*b)*(1 + c*x)^2 + 24*b*(24*a + b*(10 + 9*c*x + 3*c^2*x^2))*ArcTanh[c*x]
 - 36*b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + 3*b*(12*a + 1
1*b)*(1 + c*x)^3*Log[1 - c*x] - 3*b*(12*a + 11*b)*(1 + c*x)^3*Log[1 + c*x])
/(c*(1 + c*x)^3)
```

fricas [A] time = 0.62, size = 203, normalized size = 1.15

$$-\frac{6\left(12\,ab+11\,b^2\right)c^2x^2+54\left(4\,ab+3\,b^2\right)cx-9\left(b^2c^3x^3+3\,b^2c^2x^2+3\,b^2cx-7\,b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2+288\,a^2+240\,a}{864\left(c^4x^3+3\,c^3x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/864*(6*(12*a*b + 11*b^2)*c^2*x^2 + 54*(4*a*b + 3*b^2)*c*x - 9*(b^2*c^3*x
^3 + 3*b^2*c^2*x^2 + 3*b^2*c*x - 7*b^2)*log(-(c*x + 1)/(c*x - 1))^2 + 288*a
^2 + 240*a*b + 112*b^2 - 3*((12*a*b + 11*b^2)*c^3*x^3 + 3*(12*a*b + 7*b^2)*
c^2*x^2 + 3*(12*a*b - b^2)*c*x - 84*a*b - 29*b^2)*log(-(c*x + 1)/(c*x - 1))
)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

giac [B] time = 0.47, size = 333, normalized size = 1.89

$$\frac{1}{1728}c\left(\frac{18\left(\frac{3(cx+1)^2b^2}{(cx-1)^2}-\frac{3(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^3c^2}+\frac{6\left(\frac{36(cx+1)^2ab}{(cx-1)^2}-\frac{36(cx+1)ab}{cx-1}+12\,ab+\frac{18(cx+1)^2b^2}{(cx-1)^2}\right.}{(cx+1)^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] 1/1728*c*(18*(3*(c*x + 1)^2*b^2/(c*x - 1)^2 - 3*(c*x + 1)*b^2/(c*x - 1) + b
^2)*(c*x - 1)^3*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^3*c^2) + 6*(36*(c*x
+ 1)^2*a*b/(c*x - 1)^2 - 36*(c*x + 1)*a*b/(c*x - 1) + 12*a*b + 18*(c*x + 1)
^2*b^2/(c*x - 1)^2 - 9*(c*x + 1)*b^2/(c*x - 1) + 2*b^2)*(c*x - 1)^3*log(-(c
*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (216*(c*x + 1)^2*a^2/(c*x - 1)^2 - 2
16*(c*x + 1)*a^2/(c*x - 1) + 72*a^2 + 216*(c*x + 1)^2*a*b/(c*x - 1)^2 - 108
*(c*x + 1)*a*b/(c*x - 1) + 24*a*b + 108*(c*x + 1)^2*b^2/(c*x - 1)^2 - 27*(c
*x + 1)*b^2/(c*x - 1) + 4*b^2)*(c*x - 1)^3/((c*x + 1)^3*c^2))
```

maple [B] time = 0.07, size = 386, normalized size = 2.19

$$-\frac{a^2}{3c(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)^2}{3c(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{24c}-\frac{b^2\operatorname{arctanh}(cx)}{9c(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)}{12c(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx}{12c(cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*x+1)^4,x)
```

```
[Out] -1/3/c*a^2/(c*x+1)^3-1/3/c*b^2/(c*x+1)^3*arctanh(c*x)^2-1/24/c*b^2*arctanh(
c*x)*ln(c*x-1)-1/9/c*b^2*arctanh(c*x)/(c*x+1)^3-1/12/c*b^2*arctanh(c*x)/(c*
x+1)^2-1/12/c*b^2*arctanh(c*x)/(c*x+1)+1/24/c*b^2*arctanh(c*x)*ln(c*x+1)-1/
96/c*b^2*ln(c*x-1)^2+1/48/c*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/96/c*b^2*ln(c*x
+1)^2+1/48/c*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/48/c*b^2*ln(-1/2*c*x+1/2)*ln(
1/2+1/2*c*x)-11/288/c*b^2*ln(c*x-1)-1/54*b^2/c/(c*x+1)^3-5/144*b^2/c/(c*x+1
)^2-11/144*b^2/c/(c*x+1)+11/288/c*b^2*ln(c*x+1)-2/3/c*a*b*arctanh(c*x)/(c*x
+1)^3-1/24/c*a*b*ln(c*x-1)-1/9/c*a*b/(c*x+1)^3-1/12/c*a*b/(c*x+1)^2-1/12/c*
a*b/(c*x+1)+1/24/c*a*b*ln(c*x+1)
```

maxima [B] time = 0.35, size = 445, normalized size = 2.53

$$-\frac{1}{72}\left(c\left(\frac{2\left(3\,c^2x^2+9\,cx+10\right)}{c^5x^3+3\,c^4x^2+3\,c^3x+c^2}-\frac{3\,\log(cx+1)}{c^2}+\frac{3\,\log(cx-1)}{c^2}\right)+\frac{48\,\operatorname{artanh}(cx)}{c^4x^3+3\,c^3x^2+3\,c^2x+c}\right)ab-\frac{1}{864}\left(12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/72*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2)
- 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*c
^3*x^2 + 3*c^2*x + c))*a*b - 1/864*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x
^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2)*
arctanh(c*x) + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x +
```

```
1)^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11
*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*
x - 1) + 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x -
1) + 112)*c^2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*b^2 - 1/3*b^2*arctanh(
c*x)^2/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/3*a^2/(c^4*x^3 + 3*c^3*x^2 +
 3*c^2*x + c)
```

mupad [B] time = 2.30, size = 498, normalized size = 2.83

$$\ln(1-c\,x)\left(\ln(c\,x+1)\left(\frac{b^2}{3\,c\,\left(2\,c^3\,x^3+6\,c^2\,x^2+6\,c\,x+2\right)}-\frac{b^2\,\left(c^3\,x^3+3\,c^2\,x^2+3\,c\,x+1\right)}{24\,c\,\left(2\,c^3\,x^3+6\,c^2\,x^2+6\,c\,x+2\right)}\right)+\frac{}{3\,c\,\left(6\,c^3\,x^3+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(c*x + 1)^4,x)
```

```
[Out] log(1 - c*x)*(log(c*x + 1)*(b^2/(3*c*(6*c*x + 6*c^2*x^2 + 2*c^3*x^3 + 2)) -
 (b^2*(3*c*x + 3*c^2*x^2 + c^3*x^3 + 1))/(24*c*(6*c*x + 6*c^2*x^2 + 2*c^3*x
^3 + 2))) + b^2/(3*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6)) + (b*(6*a - b))
/(3*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6)) + (b^2*(69*c*x + 45*c^2*x^2 +
11*c^3*x^3 + 51))/(48*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6))) - (x*(36*a*
b + 27*b^2) + x^2*(11*b^2*c + 12*a*b*c) + (8*(15*a*b + 18*a^2 + 7*b^2))/(3*
c))/(432*c*x + 432*c^2*x^2 + 144*c^3*x^3 + 144) + log(c*x + 1)^2*(b^2/(96*c
) - b^2/(12*c^2*(3*x + 3*c*x^2 + 1/c + c^2*x^3))) + log(1 - c*x)^2*(b^2/(96
*c) - b^2/(3*c*(12*c*x + 12*c^2*x^2 + 4*c^3*x^3 + 4))) - (log(c*x + 1)*((7*
b^2)/(96*c^2) + (5*b^2*x^2)/32 + (23*b^2*x)/(96*c) + (11*b^2*c*x^3)/288 + (
b*(16*a + 5*b))/(48*c^2)))/(3*x + 3*c*x^2 + 1/c + c^2*x^3) - (b*atan(c*x*1i
)*(6*a + 11*b)*1i)/(72*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atanh}(cx))^2}{(cx+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*x+1)**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(c*x + 1)**4, x)
```

3.119 $\int \frac{\tanh^{-1}(ax)^2}{cx-acx^2}\,dx$

Optimal. Leaf size=67

$$-\frac{\text{Li}_3\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\frac{\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{c}$$

```
[Out] arctanh(a*x)^2*ln(2-2/(-a*x+1))/c+arctanh(a*x)*polylog(2,-1+2/(-a*x+1))/c-1
/2*polylog(3,-1+2/(-a*x+1))/c
```

Rubi [A] time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1593, 5932, 5948, 6058, 6610}

$$-\frac{\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2c}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}+\frac{\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*Log[2 - 2/(1 - a*x)])/c + (ArcTanh[a*x]*PolyLog[2, -1 + 2/(
1 - a*x)])/c - PolyLog[3, -1 + 2/(1 - a*x)]/(2*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{cx-acx^2}\,dx = \int \frac{\tanh^{-1}(ax)^2}{x(c-acx)}\,dx$$
$$= \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(2a)\int \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$= \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{a\int \frac{\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$= \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{\mathrm{Li}_3\left(-1+\frac{2}{1-ax}\right)}{2c}$$

Mathematica [A] time = 0.15, size = 59, normalized size = 0.88

$$\frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)}{2c} + \frac{\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
[Out] (ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])])/c + (ArcTanh[a*x]*PolyLog[2, E
^(2*ArcTanh[a*x])])/c - PolyLog[3, E^(2*ArcTanh[a*x])]/(2*c)
```

fricas [F] time = 1.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\mathrm{artanh}\,(ax)^2}{acx^2-cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="fricas")
[Out] integral(-arctanh(a*x)^2/(a*c*x^2 - c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\mathrm{artanh}\,(ax)^2}{acx^2-cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="giac")
[Out] integrate(-arctanh(a*x)^2/(a*c*x^2 - c*x), x)
```

maple [C] time = 0.52, size = 717, normalized size = 10.70

$$\frac{\mathrm{arctanh}\,(ax)^2\ln(ax)}{c} - \frac{\mathrm{arctanh}\,(ax)^2\ln(ax-1)}{c} - \frac{\mathrm{arctanh}\,(ax)^2\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{c} + \frac{\mathrm{arctanh}\,(ax)^2\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a*c*x^2+c*x),x)
```

```
[Out] 1/c*arctanh(a*x)^2*ln(a*x)-1/c*arctanh(a*x)^2*ln(a*x-1)-1/c*arctanh(a*x)^2*
ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))+2/c*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2/c*polylog(3,
(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1
/2))+2/c*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2/c*polylog(3,
-(a*x+1)/(-a^2*x^2+1)^(1/2))+I/c*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x
^2+1)))^3*Pi+I/c*arctanh(a*x)^2*Pi+1/2*I/c*arctanh(a*x)^2*csgn(I/(1+(a*x+1)
^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^
2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*Pi-1/2*I/c*arctanh(a*x)^2*csgn(I/(1
+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-
a^2*x^2+1)))^2*Pi+1/2*I/c*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-I/c*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2*Pi-1/2*I/c*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*
csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+1/c*arct
anh(a*x)^2*ln(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(-ax+1\right)^{3}}{12\,c}+\frac{1}{4}\int -\frac{\log\left(ax+1\right)^{2}-2\,\log\left(ax+1\right)\log\left(-ax+1\right)}{acx^{2}-cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(-a*x + 1)^3/c + 1/4*integrate(-(log(a*x + 1)^2 - 2*log(a*x + 1)*l
og(-a*x + 1))/(a*c*x^2 - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(a\,x\right)^2}{c\,x-a\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(c*x - a*c*x^2),x)
```

```
[Out] int(atanh(a*x)^2/(c*x - a*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^{2}{\left(ax \right)}}{ax^{2}-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a*c*x**2+c*x),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a*x**2 - x), x)/c
```

3.120 $\int (1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=306

$$-\frac{6b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{11b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+3ab^2x+\frac{1}{4}$$

```
[Out] 3*a*b^2*x+1/4*b^3*x-1/4*b^3*arctanh(c*x)/c+3*b^3*x*arctanh(c*x)+1/4*b^2*c*x
^2*(a+b*arctanh(c*x))+4*b*(a+b*arctanh(c*x))^2/c+21/4*b*x*(a+b*arctanh(c*x)
)^2+3/2*b*c*x^2*(a+b*arctanh(c*x))^2+1/4*b*c^2*x^3*(a+b*arctanh(c*x))^2+1/4
*(c*x+1)^4*(a+b*arctanh(c*x))^3/c-11*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/
c-6*b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c+3/2*b^3*ln(-c^2*x^2+1)/c-11/2*b
^3*polylog(2,1-2/(-c*x+1))/c-6*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1
))/c+3*b^3*polylog(3,1-2/(-c*x+1))/c
```

Rubi [A] time = 0.66, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 321, 206, 1586, 6058, 6610}

$$-\frac{6b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{11b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}+\frac{1}{4}b^2cx^2\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] 3*a*b^2*x + (b^3*x)/4 - (b^3*ArcTanh[c*x])/(4*c) + 3*b^3*x*ArcTanh[c*x] + (
b^2*c*x^2*(a + b*ArcTanh[c*x]))/4 + (4*b*(a + b*ArcTanh[c*x])^2)/c + (21*b*
x*(a + b*ArcTanh[c*x])^2)/4 + (3*b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + (b*c^2
*x^3*(a + b*ArcTanh[c*x])^2)/4 + ((1 + c*x)^4*(a + b*ArcTanh[c*x])^3)/(4*c)
 - (11*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c - (6*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c + (3*b^3*Log[1 - c^2*x^2])/(2*c) - (11*b^3*PolyL
og[2, 1 - 2/(1 - c*x)])/(2*c) - (6*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 -
2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^3\,dx=\frac{(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^3}{4c}-\frac{1}{4}(3b)\int\left(-7\left(a+b\tanh^{-1}(cx)\right)^2-4cx\left(a+\right.\right.$$

$$=\frac{(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^3}{4c}+\frac{1}{4}(21b)\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx-(6b)\int$$

$$=\frac{21}{4}bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}bc^2x^3\left(a+b\tanh\right.$$

$$=\frac{21b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}+\frac{21}{4}bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}bcx^2\left(a+b\tanh^{-1}\right.$$

$$=3ab^2x+\frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4b\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{21}{4}bx\left(a+b\right.$$

$$=3ab^2x+\frac{b^3x}{4}+3b^3x\tanh^{-1}(cx)+\frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4b\left(a+b\tan\right.}{c}$$

$$=3ab^2x+\frac{b^3x}{4}-\frac{b^3\tanh^{-1}(cx)}{4c}+3b^3x\tanh^{-1}(cx)+\frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx\right.$$

$$=3ab^2x+\frac{b^3x}{4}-\frac{b^3\tanh^{-1}(cx)}{4c}+3b^3x\tanh^{-1}(cx)+\frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx\right.$$

Mathematica [B] time = 1.47, size = 644, normalized size = 2.10

$$\frac{2a^3c^4x^4+8a^3c^3x^3+12a^3c^2x^2+8a^3cx+6a^2bc^4x^4\tanh^{-1}(cx)+2a^2bc^3x^3+24a^2bc^3x^3\tanh^{-1}(cx)+12a^2bc^2x^2+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-2*a*b^2 + 8*a^3*c*x + 42*a^2*b*c*x + 24*a*b^2*c*x + 2*b^3*c*x + 12*a^3*c^
2*x^2 + 12*a^2*b*c^2*x^2 + 2*a*b^2*c^2*x^2 + 8*a^3*c^3*x^3 + 2*a^2*b*c^3*x^
3 + 2*a^3*c^4*x^4 - 24*a*b^2*ArcTanh[c*x] - 2*b^3*ArcTanh[c*x] + 24*a^2*b*c
```

```
*x*ArcTanh[c*x] + 84*a*b^2*c*x*ArcTanh[c*x] + 24*b^3*c*x*ArcTanh[c*x] + 36*
a^2*b*c^2*x^2*ArcTanh[c*x] + 24*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^2*x^2*
ArcTanh[c*x] + 24*a^2*b*c^3*x^3*ArcTanh[c*x] + 4*a*b^2*c^3*x^3*ArcTanh[c*x]
 + 6*a^2*b*c^4*x^4*ArcTanh[c*x] - 90*a*b^2*ArcTanh[c*x]^2 - 56*b^3*ArcTanh[
c*x]^2 + 24*a*b^2*c*x*ArcTanh[c*x]^2 + 42*b^3*c*x*ArcTanh[c*x]^2 + 36*a*b^2
*c^2*x^2*ArcTanh[c*x]^2 + 12*b^3*c^2*x^2*ArcTanh[c*x]^2 + 24*a*b^2*c^3*x^3*
ArcTanh[c*x]^2 + 2*b^3*c^3*x^3*ArcTanh[c*x]^2 + 6*a*b^2*c^4*x^4*ArcTanh[c*x
]^2 - 30*b^3*ArcTanh[c*x]^3 + 8*b^3*c*x*ArcTanh[c*x]^3 + 12*b^3*c^2*x^2*Arc
Tanh[c*x]^3 + 8*b^3*c^3*x^3*ArcTanh[c*x]^3 + 2*b^3*c^4*x^4*ArcTanh[c*x]^3 -
 96*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 88*b^3*ArcTanh[c*x]*L
og[1 + E^(-2*ArcTanh[c*x])] - 48*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c
*x])] + 45*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 44*a*b^2*Log[1 - c^2
*x^2] + 12*b^3*Log[1 - c^2*x^2] + 4*b^2*(12*a + 11*b + 12*b*ArcTanh[c*x])*P
olyLog[2, -E^(-2*ArcTanh[c*x])] + 24*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/
(8*c)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3c^3x^3 + 3\,a^3c^2x^2 + 3\,a^3cx + \left(b^3c^3x^3 + 3\,b^3c^2x^2 + 3\,b^3cx + b^3\right)\text{artanh}\left(cx\right)^3 + a^3 + 3\left(ab^2c^3x^3 + 3\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c^3*x^3 + 3*a^3*c^2*x^2 + 3*a^3*c*x + (b^3*c^3*x^3 + 3*b^3*c^2
*x^2 + 3*b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c^3*x^3 + 3*a*b^2*c
^2*x^2 + 3*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3*(a^2*b*c^3*x^3 + 3*a^2*b*c
^2*x^2 + 3*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx+1)^3 (b\,\text{artanh}\,(cx) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)^3*(b*arctanh(c*x) + a)^3, x)
```

maple [C] time = 0.59, size = 963, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)^3*(a+b*arctanh(c*x))^3,x)
```

```
[Out] -1/4/c*b^3+1/4/c*a^3+21/4*x*a^2*b+b^3*arctanh(c*x)^3*x+21/4*b^3*arctanh(c*x
)^2*x+1/4*c^3*x^4*a^3+c^2*x^3*a^3+3/2*c*x^2*a^3+1/4/c*b^3*arctanh(c*x)^3+4/
c*b^3*arctanh(c*x)^2-11/c*b^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-11/c*b^
3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3/c*b^3*ln(1+(c*x+1)^2/(-c^2*x^2+1)
)+3/c*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-13/4/c*a*b^2+1/4*c*x^2*a*b^2+1
/4*c^2*a^2*b*x^3+c^2*b^3*arctanh(c*x)^3*x^3+1/4*c^3*b^3*arctanh(c*x)^3*x^4+
1/4*c^2*b^3*arctanh(c*x)^2*x^3+3/2*c*b^3*arctanh(c*x)^2*x^2-11/c*b^3*arctan
h(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6/c*b^3*arctanh(c*x)*polylog(2,-(
c*x+1)^2/(-c^2*x^2+1))-11/c*b^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1
/2))+6/c*b^3*arctanh(c*x)^2*ln(c*x-1)+12/c*a*b^2*arctanh(c*x)*ln(c*x-1)-6/c
*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+3/4*c^3*a*b^2*arctanh(c*x)^2*x^4+3*c^2*a*b
^2*arctanh(c*x)^2*x^3+1/2*c^2*a*b^2*arctanh(c*x)*x^3+3/4*c^3*a^2*b*arctanh(
c*x)*x^4+3*c^2*a^2*b*arctanh(c*x)*x^3+3*c*a*b^2*arctanh(c*x)*x^2+9/2*c*a^2*
b*arctanh(c*x)*x^2-6*I/c*b^3*Pi*arctanh(c*x)^2+11/4*b^3*arctanh(c*x)/c+3*b^
```

```
3*x*arctanh(c*x)+3*x*a*b^2-6*I/c*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^
3*arctanh(c*x)^2+6*I/c*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(
c*x)^2+3/4/c*a^2*b*arctanh(c*x)+6/c*a^2*b*ln(c*x-1)+3/4/c*a*b^2*arctanh(c*x
)^2+7/c*a*b^2*ln(c*x-1)+4/c*a*b^2*ln(c*x+1)+3/c*a*b^2*ln(c*x-1)^2-6/c*a*b^2
*dilog(1/2+1/2*c*x)-6/c*b^3*arctanh(c*x)^2*ln(2)+1/4*c*b^3*arctanh(c*x)*x^2
+3/2*c*b^3*arctanh(c*x)^3*x^2+3*a*b^2*arctanh(c*x)^2*x+3*a^2*b*arctanh(c*x)
*x+21/2*a*b^2*arctanh(c*x)*x+3/2*c*a^2*b*x^2+9/2*c*a*b^2*arctanh(c*x)^2*x^2
+a^3*x+1/4*b^3*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}a^3c^3x^4+a^3c^2x^3+\frac{1}{8}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2\left(c^2x^3+3x\right)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)a^2bc^3+\frac{3}{2}\left(2x^3\operatorname{artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*c^3*x^4 + a^3*c^2*x^3 + 1/8*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3
*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b*c^3 + 3/2*(2*x^3*
arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a^2*b*c^2 + 3/2*a^3*c*x^
2 + 9/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/
c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/
c - 1/32*((b^3*c^4*x^4 + 4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x - 15*b^3
)*log(-c*x + 1)^3 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3)*x^3 + 12*
(3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(b^3*c^4*x^4 +
4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x + b^3)*log(c*x + 1))*log(-c*x + 1
)^2)/c - integrate(-1/16*(2*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 - 2*b^3*c*x - b^3)
*log(c*x + 1)^3 + 12*(a*b^2*c^4*x^4 + 2*a*b^2*c^3*x^3 - 2*a*b^2*c*x - a*b^2
)*log(c*x + 1)^2 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3)*x^3 + 12*(
3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 - 2*b^3*c*x - b
^3)*log(c*x + 1)^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(6*b^3*c^2*x^2 + (8*a*b^
2*c^4 + b^3*c^4)*x^4 + 4*(4*a*b^2*c^3 + b^3*c^3)*x^3 - 8*a*b^2 + b^3 - 4*(4
*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x + 1))/(c*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{atanh}(c\,x))^3\,(c\,x+1)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(c*x + 1)^3,x)
```

```
[Out] int((a + b*atanh(c*x))^3*(c*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{atanh}(cx))^3\,(cx+1)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1)**3, x)
```

3.121 $\int (1+cx)^2 \left(a+b\tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=240

$$-\frac{4b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{6b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+ab^2x+\frac{1}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2+3$$

```
[Out] a*b^2*x+b^3*x*arctanh(c*x)+5/2*b*(a+b*arctanh(c*x))^2/c+3*b*x*(a+b*arctanh(
c*x))^2+1/2*b*c*x^2*(a+b*arctanh(c*x))^2+1/3*(c*x+1)^3*(a+b*arctanh(c*x))^3
/c-6*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c-4*b*(a+b*arctanh(c*x))^2*ln(2/
(-c*x+1))/c+1/2*b^3*ln(-c^2*x^2+1)/c-3*b^3*polylog(2,1-2/(-c*x+1))/c-4*b^2*
(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+2*b^3*polylog(3,1-2/(-c*x+1))/
c
```

Rubi [A] time = 0.44, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 1586, 6058, 6610}

$$-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{2b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}-\frac{6b^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] a*b^2*x + b^3*x*ArcTanh[c*x] + (5*b*(a + b*ArcTanh[c*x])^2)/(2*c) + 3*b*x*(
a + b*ArcTanh[c*x])^2 + (b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + ((1 + c*x)^3*(
a + b*ArcTanh[c*x])^3)/(3*c) - (6*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)]
)/c - (4*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c + (b^3*Log[1 - c^2*x^
2])/(2*c) - (3*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c - (4*b^2*(a + b*ArcTanh[c
*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (2*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)^2 \left(a+b\tanh^{-1}(cx)\right)^3 \, dx = \frac{(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3}{3c} - b\int \left(-3\left(a+b\tanh^{-1}(cx)\right)^2 - cx\left(a+b\right.\right.$$

$$= \frac{(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3}{3c} + (3b)\int \left(a+b\tanh^{-1}(cx)\right)^2 \, dx - (4b)\int$$

$$= 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^3\left(a+b\,\mathrm{t}\right.}{3c}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\tanh^{-1}\right.$$

$$= ab^2x + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\right.$$

$$= ab^2x + b^3x\tanh^{-1}(cx) + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(cx)\right.$$

$$= ab^2x + b^3x\tanh^{-1}(cx) + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(cx)\right.$$

Mathematica [B] time = 0.99, size = 488, normalized size = 2.03

$$2a^3c^3x^3 + 6a^3c^2x^2 + 6a^3cx + 6a^2bc^3x^3\tanh^{-1}(cx) + 3a^2bc^2x^2 + 18a^2bc^2x^2\tanh^{-1}(cx) + 18a^2bcx + 21a^2b\log($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (6*a^3*c*x + 18*a^2*b*c*x + 6*a*b^2*c*x + 6*a^3*c^2*x^2 + 3*a^2*b*c^2*x^2 +
 2*a^3*c^3*x^3 - 6*a*b^2*ArcTanh[c*x] + 18*a^2*b*c*x*ArcTanh[c*x] + 36*a*b^
2*c*x*ArcTanh[c*x] + 6*b^3*c*x*ArcTanh[c*x] + 18*a^2*b*c^2*x^2*ArcTanh[c*x]
 + 6*a*b^2*c^2*x^2*ArcTanh[c*x] + 6*a^2*b*c^3*x^3*ArcTanh[c*x] - 42*a*b^2*A
rcTanh[c*x]^2 - 21*b^3*ArcTanh[c*x]^2 + 18*a*b^2*c*x*ArcTanh[c*x]^2 + 18*b^
3*c*x*ArcTanh[c*x]^2 + 18*a*b^2*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^2*x^2*ArcT
anh[c*x]^2 + 6*a*b^2*c^3*x^3*ArcTanh[c*x]^2 - 14*b^3*ArcTanh[c*x]^3 + 6*b^3
*c*x*ArcTanh[c*x]^3 + 6*b^3*c^2*x^2*ArcTanh[c*x]^3 + 2*b^3*c^3*x^3*ArcTanh[
c*x]^3 - 48*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 36*b^3*ArcTan
h[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*A
rcTanh[c*x])] + 21*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 18*a*b^2*Log
[1 - c^2*x^2] + 3*b^3*Log[1 - c^2*x^2] + 6*b^2*(4*a + 3*b + 4*b*ArcTanh[c*x
])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 12*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x]
)])/(6*c)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(a^3c^2x^2 + 2\,a^3cx + \left(b^3c^2x^2 + 2\,b^3cx + b^3\right)\mathrm{artanh}\,(cx)^3 + a^3 + 3\left(ab^2c^2x^2 + 2\,ab^2cx + ab^2\right)\mathrm{artanh}\,(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c^2*x^2 + 2*a^3*c*x + (b^3*c^2*x^2 + 2*b^3*c*x + b^3)*arctanh(
c*x)^3 + a^3 + 3*(a*b^2*c^2*x^2 + 2*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3*(
a^2*b*c^2*x^2 + 2*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx+1)^2(b\,\text{artanh}\,(cx)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)^2*(b*arctanh(c*x) + a)^3, x)
```

maple [C] time = 0.50, size = 811, normalized size = 3.38

$$\frac{a^3}{3c}+b^3\,\text{arctanh}\,(cx)^3\,x+3b^3\,\text{arctanh}\,(cx)^2\,x+\frac{b^3\,\text{arctanh}\,(cx)^3}{3c}+\frac{5b^3\,\text{arctanh}\,(cx)^2}{2c}+\frac{2b^3\,\text{polylog}\left(3,-\frac{(cx+1)^2}{-c^2x^2+1}\right)}{c}+\frac{b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)^2*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 1/3/c*a^3+3*x*a^2*b+b^3*arctanh(c*x)^3*x+3*b^3*arctanh(c*x)^2*x+1/3*c^2*x^3
*a^3+c*x^2*a^3+1/3/c*b^3*arctanh(c*x)^3+5/2/c*b^3*arctanh(c*x)^2-6/c*b^3*di
log(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6/c*b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^
(1/2))-1/c*b^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))+2/c*b^3*polylog(3,-(c*x+1)^2/(-
c^2*x^2+1))-1/c*a*b^2+1/3*c^2*b^3*arctanh(c*x)^3*x^3+1/2*c*b^3*arctanh(c*x)
^2*x^2-6/c*b^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-4/c*b^3*arct
anh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-6/c*b^3*arctanh(c*x)*ln(1-I*(c*
x+1)/(-c^2*x^2+1)^(1/2))+4/c*b^3*arctanh(c*x)^2*ln(c*x-1)+8/c*a*b^2*arctanh
(c*x)*ln(c*x-1)-4/c*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+c^2*a*b^2*arctanh(c*x)^
2*x^3+c^2*a^2*b*arctanh(c*x)*x^3+c*a*b^2*arctanh(c*x)*x^2+3*c*a^2*b*arctanh
(c*x)*x^2+b^3*arctanh(c*x)/c+b^3*x*arctanh(c*x)+x*a*b^2-4*I/c*b^3*Pi*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+4*I/c*b^3*Pi*csgn(I/(1+(c*x+1
)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/c*a^2*b*arctanh(c*x)+4/c*a^2*b*ln(c*x
-1)+1/c*a*b^2*arctanh(c*x)^2+7/2/c*a*b^2*ln(c*x-1)+5/2/c*a*b^2*ln(c*x+1)+2/
c*a*b^2*ln(c*x-1)^2-4/c*a*b^2*dilog(1/2+1/2*c*x)-4/c*b^3*arctanh(c*x)^2*ln(
2)+c*b^3*arctanh(c*x)^3*x^2+3*a*b^2*arctanh(c*x)^2*x+3*a^2*b*arctanh(c*x)*x
+6*a*b^2*arctanh(c*x)*x+1/2*c*a^2*b*x^2+3*c*a*b^2*arctanh(c*x)^2*x^2-4*I/c*
b^3*Pi*arctanh(c*x)^2+a^3*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^3c^2x^3+\frac{1}{2}\left(2\,x^3\,\text{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)a^2bc^2+a^3cx^2+\frac{3}{2}\left(2\,x^2\,\text{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\,(cx+1)}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*c^2*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c
^4))*a^2*b*c^2 + a^3*c*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x
 + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) +
log(-c^2*x^2 + 1))*a^2*b/c - 1/24*((b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3*c*x
 - 7*b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 + (6*a*b^2*c^2 + b^3*c^2)*x^
2 + 6*(a*b^2*c + b^3*c)*x + (b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3*c*x + b^3)
*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^3*x^3 + b^3*c^2*
x^2 - b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^3 + a*b^2*c^2*x^2 - a*
```

```
b^2*c*x - a*b^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*(6*a*b^2*c^2 + b^3*c
^2)*x^2 + 3*(b^3*c^3*x^3 + b^3*c^2*x^2 - b^3*c*x - b^3)*log(c*x + 1)^2 + 12
*(a*b^2*c + b^3*c)*x + 2*((6*a*b^2*c^3 + b^3*c^3)*x^3 - 6*a*b^2 + b^3 + 3*(
2*a*b^2*c^2 + b^3*c^2)*x^2 - 3*(2*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*
x + 1))/(c*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\operatorname{atanh}(c\,x))^3\,(c\,x + 1)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(c*x + 1)^2,x)
```

```
[Out] int((a + b*atanh(c*x))^3*(c*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atanh}(cx))^3\,(cx + 1)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)**2*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1)**2, x)
```

3.122 $\int (1+cx)\left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=191

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3b\left(a+b\tan\right.}{2c}$$

```
[Out] 3/2*b*(a+b*arctanh(c*x))^2/c+3/2*b*x*(a+b*arctanh(c*x))^2+1/2*(c*x+1)^2*(a+
b*arctanh(c*x))^3/c-3*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c-3*b*(a+b*arct
anh(c*x))^2*ln(2/(-c*x+1))/c-3/2*b^3*polylog(2,1-2/(-c*x+1))/c-3*b^2*(a+b*a
rctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+3/2*b^3*polylog(3,1-2/(-c*x+1))/c
```

Rubi [A] time = 0.30, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 1586, 5948, 6058, 6610}

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}-\frac{3b^2\log\left(\frac{}{1-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c) + (3*b*x*(a + b*ArcTanh[c*x])^2)/2 + ((1
 + c*x)^2*(a + b*ArcTanh[c*x])^3)/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - (3*b^3*P
olyLog[2, 1 - 2/(1 - c*x)])/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2,
1 - 2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

]

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)\left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{1}{2}(3b)\int\left(-\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{2(1+cx)}{}\right.$$

$$= \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} + \frac{1}{2}(3b)\int\left(a+b\tanh^{-1}(cx)\right)^2 dx - (3b)\int\frac{(1}{}$$

$$= \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - (3b)\int\frac{\left(a+b\tan\right.}{1-}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(\right.}{2c}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(\right.}{2c}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(\right.}{2c}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(\right.}{2c}$$

Mathematica [A] time = 0.50, size = 334, normalized size = 1.75

$$\frac{2a^3c^2x^2 + 4a^3cx + 6a^2bc^2x^2\tanh^{-1}(cx) + 6a^2bcx + 9a^2b\log(1-cx) + 3a^2b\log(cx+1) + 12a^2bcx\tanh^{-1}(cx) + 6}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (4*a^3*c*x + 6*a^2*b*c*x + 2*a^3*c^2*x^2 + 12*a^2*b*c*x*ArcTanh[c*x] + 12*a
*b^2*c*x*ArcTanh[c*x] + 6*a^2*b*c^2*x^2*ArcTanh[c*x] - 18*a*b^2*ArcTanh[c*x
]^2 - 6*b^3*ArcTanh[c*x]^2 + 12*a*b^2*c*x*ArcTanh[c*x]^2 + 6*b^3*c*x*ArcTan
h[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTanh[c*x]^2 - 6*b^3*ArcTanh[c*x]^3 + 4*b^3*c*
x*ArcTanh[c*x]^3 + 2*b^3*c^2*x^2*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x]*Log
[1 + E^(-2*ArcTanh[c*x])] - 12*b^3*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])
] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 9*a^2*b*Log[1 - c*
x] + 3*a^2*b*Log[1 + c*x] + 6*a*b^2*Log[1 - c^2*x^2] + 6*b^2*(2*a + b + 2*b
*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3, -E^(-2*A
rcTanh[c*x])])/(4*c)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3cx + \left(b^3cx + b^3\right)\operatorname{artanh}(cx)^3 + a^3 + 3\left(ab^2cx + ab^2\right)\operatorname{artanh}(cx)^2 + 3\left(a^2bcx + a^2b\right)\operatorname{artanh}(cx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c*x + (b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c*x + a*
b^2)*arctanh(c*x)^2 + 3*(a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx+1)(b\operatorname{artanh}(cx)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)*(b*arctanh(c*x) + a)^3, x)
```

maple [C] time = 0.80, size = 6440, normalized size = 33.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)*(a+b*arctanh(c*x))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^3cx^2+\frac{3}{4}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)a^2bc+a^3x+\frac{3\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2\right.\right.}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*c*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a^2*b/c - 1/16*((b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-c*x + 1)^3 - 3
*(2*a*b^2*c^2*x^2 + 2*(2*a*b^2*c + b^3*c)*x + (b^3*c^2*x^2 + 2*b^3*c*x + b^
3)*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^2*x^2 - b^3)*l
og(c*x + 1)^3 + 6*(a*b^2*c^2*x^2 - a*b^2)*log(c*x + 1)^2 - 3*(2*a*b^2*c^2*x
^2 + (b^3*c^2*x^2 - b^3)*log(c*x + 1)^2 + 2*(2*a*b^2*c + b^3*c)*x + (2*b^3*
c*x - 4*a*b^2 + b^3 + (4*a*b^2*c^2 + b^3*c^2)*x^2)*log(c*x + 1))*log(-c*x +
 1))/(c*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{atanh}(c\,x))^3\,(c\,x+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(c*x + 1),x)
```

```
[Out] int((a + b*atanh(c*x))^3*(c*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{atanh}(cx))^3\,(cx+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1), x)
```

3.123 $\int \frac{\left(a+b \tanh^{-1}(cx)\right)^3}{1+cx} dx$

Optimal. Leaf size=111

$$\frac{3b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^3}{c}+\frac{3b^3}{}$$

```
[Out] -(a+b*arctanh(c*x))^3*ln(2/(c*x+1))/c+3/2*b*(a+b*arctanh(c*x))^2*polylog(2,
1-2/(c*x+1))/c+3/2*b^2*(a+b*arctanh(c*x))*polylog(3,1-2/(c*x+1))/c+3/4*b^3*
polylog(4,1-2/(c*x+1))/c
```

Rubi [A] time = 0.22, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5918, 5948, 6056, 6060, 6610}

$$\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{3b^3\text{PolyLog}\left(4,1-\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)])/c) + (3*b*(a + b*ArcTanh[c*x])^
2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[
3, 1 - 2/(1 + c*x)])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 + c*x)])/(4*c)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{1+cx}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c}-\left(3b^2\right)$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c}+\frac{3b^2(a}{}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c}+\frac{3b^2(a}{}$$

Mathematica [A] time = 0.29, size = 152, normalized size = 1.37

$$4a^3\log(cx+1)-12a^2b\tanh^{-1}(cx)\log\left(e^{-2\tanh^{-1}(cx)}+1\right)+6b^2\text{Li}_3\left(-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)\right)-12ab^2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]
```

```
[Out] (-12*a^2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 12*a*b^2*ArcTanh[c*x
]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 4*b^3*ArcTanh[c*x]^3*Log[1 + E^(-2*ArcTa
nh[c*x])] + 4*a^3*Log[1 + c*x] + 6*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, -E^(
-2*ArcTanh[c*x])] + 6*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -E^(-2*ArcTanh[c*
x])] + 3*b^3*PolyLog[4, -E^(-2*ArcTanh[c*x])])/(4*c)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\,(cx)^3+3\,ab^2\,\text{artanh}\,(cx)^2+3\,a^2b\,\text{artanh}\,(cx)+a^3}{cx+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/(c*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{cx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/(c*x + 1), x)
```

maple [C] time = 0.46, size = 1491, normalized size = 13.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1),x)
```

```
[Out] 3/2*I/c*a*b^2*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
*Pi-3*I/c*a*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*Pi+3/2*I/c*a*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi-3/2
*I/c*a*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))^2*Pi+1/2*I/c*b^3*arctanh(c*x)^3*csgn(I/(1+(c*x+1)^2/(-c^2*x
^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))*Pi-3/2*I/c*a*b^2*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c
^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi+a
^3/c*ln(c*x+1)-3/4/c*b^3*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))+1/2/c*b^3*arcta
nh(c*x)^4-1/2*I/c*b^3*arctanh(c*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1
)^2/(-c^2*x^2+1)))^3*Pi-1/2*I/c*b^3*arctanh(c*x)^3*csgn(I*(c*x+1)^2/(c^2*x^
2-1))^3*Pi-1/c*b^3*arctanh(c*x)^3*ln(2)-3/4/c*a^2*b*ln(c*x+1)^2-3/2/c*a^2*b
*dilog(1/2+1/2*c*x)+2/c*a*b^2*arctanh(c*x)^3+3/2/c*a*b^2*polylog(3,-(c*x+1)
^2/(-c^2*x^2+1))+3/2/c*b^3*arctanh(c*x)*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+
1/c*b^3*ln(c*x+1)*arctanh(c*x)^3-2/c*b^3*arctanh(c*x)^3*ln((c*x+1)/(-c^2*x^
2+1)^(1/2))-3/2/c*b^3*arctanh(c*x)^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2
*I/c*b^3*arctanh(c*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi-1/2*I/c*b^3*arctanh(c*x)^3*csgn(I*
(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi-3/2*I/c*a*b^
2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi-3/2*I/c*a*b^2*arctanh(c
*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi-1/2*I/c
*b^3*arctanh(c*x)^3*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi-I/c*b^3*arctanh(c*x)^3*csgn(I*(c*
x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi-3/c*a*b^2*arcta
nh(c*x)^2*ln(2)+3/c*a^2*b*ln(c*x+1)*arctanh(c*x)+3/2/c*a^2*b*ln(-1/2*c*x+1/
2)*ln(c*x+1)-3/2/c*a^2*b*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/c*a*b^2*ln(c*x+
1)*arctanh(c*x)^2-6/c*a*b^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3
/c*a*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3\log(cx+1)\log(-cx+1)^3}{8c}+\frac{a^3\log(cx+1)}{c}+\int\frac{\left(b^3cx-b^3\right)\log(cx+1)^3+6\left(ab^2cx-ab^2\right)\log(cx+1)^2+6(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*log(c*x + 1)*log(-c*x + 1)^3/c + a^3*log(c*x + 1)/c + integrate(1/
8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 +
6*(b^3*c*x*log(c*x + 1) + a*b^2*c*x - a*b^2)*log(-c*x + 1)^2 + 12*(a^2*b*c*
x - a^2*b)*log(c*x + 1) - 3*(4*a^2*b*c*x - 4*a^2*b + (b^3*c*x - b^3)*log(c*
x + 1)^2 + 4*(a*b^2*c*x - a*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*x^2 - 1)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^3}{c\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1),x)
```

```
[Out] int((a + b*atanh(c*x))^3/(c*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{cx + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1),x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1), x)
```

3.124 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^2}\,dx$

Optimal. Leaf size=139

$$-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(cx+1)}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c}-\frac{\left(a+b\tanh^{-1}(cx\right.}{c(cx+1)}$$

```
[Out] -3/4*b^3/c/(c*x+1)+3/4*b^3*arctanh(c*x)/c-3/2*b^2*(a+b*arctanh(c*x))/c/(c*x
+1)+3/4*b*(a+b*arctanh(c*x))^2/c-3/2*b*(a+b*arctanh(c*x))^2/c/(c*x+1)+1/2*(
a+b*arctanh(c*x))^3/c-(a+b*arctanh(c*x))^3/c/(c*x+1)
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(cx+1)}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c}-\frac{\left(a+b\tanh^{-1}(cx\right.}{c(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

```
[Out] (-3*b^3)/(4*c*(1 + c*x)) + (3*b^3*ArcTanh[c*x])/(4*c) - (3*b^2*(a + b*ArcTa
nh[c*x]))/(2*c*(1 + c*x)) + (3*b*(a + b*ArcTanh[c*x])^2)/(4*c) - (3*b*(a +
b*ArcTanh[c*x])^2)/(2*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(2*c) - (a + b*
ArcTanh[c*x])^3/(c*(1 + c*x))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
```

```
Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + (3b)\int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2\left(-1+c^2x^2\right)}\right)dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \frac{1}{2}(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx - \frac{1}{2}(3b)\int \frac{\left(a+b\tanh^{-1}\right.}{-1+c^2}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \left(3b^2\right)\int$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \frac{1}{2}\left(3b^2\right)$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{(a}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{(a}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{(a}{}$$

$$= -\frac{3b^3}{4c(1+cx)} - \frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}\right.}{2c(1+cx}$$

$$= -\frac{3b^3}{4c(1+cx)} + \frac{3b^3\tanh^{-1}(cx)}{4c} - \frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}$$

Mathematica [A] time = 0.15, size = 198, normalized size = 1.42

$$\frac{-8a^3 - 3b\left(2a^2+2ab+b^2\right)(cx+1)\log(1-cx) - 12b\left(2a^2+2ab+b^2\right)\tanh^{-1}(cx) + 6a^2b\log(cx+1) + 6a^2bc}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

```
[Out] (-8*a^3 - 12*a^2*b - 12*a*b^2 - 6*b^3 - 12*b*(2*a^2 + 2*a*b + b^2)*ArcTanh[
c*x] + 6*b^2*(2*a + b)*(-1 + c*x)*ArcTanh[c*x]^2 + 4*b^3*(-1 + c*x)*ArcTanh
[c*x]^3 - 3*b*(2*a^2 + 2*a*b + b^2)*(1 + c*x)*Log[1 - c*x] + 6*a^2*b*Log[1
+ c*x] + 6*a*b^2*Log[1 + c*x] + 3*b^3*Log[1 + c*x] + 6*a^2*b*c*x*Log[1 + c*
x] + 6*a*b^2*c*x*Log[1 + c*x] + 3*b^3*c*x*Log[1 + c*x])/(8*c*(1 + c*x))
```

fricas [A] time = 0.87, size = 160, normalized size = 1.15

$$\frac{\left(b^3cx-b^3\right)\log\left(-\frac{cx+1}{cx-1}\right)^3-16\,a^3-24\,a^2b-24\,ab^2-12\,b^3-3\left(2\,ab^2+b^3-\left(2\,ab^2+b^3\right)cx\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-6\left(2\,a^2\right.}{16\left(c^2x+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((b^3*c*x - b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 16*a^3 - 24*a^2*b - 24*
a*b^2 - 12*b^3 - 3*(2*a*b^2 + b^3 - (2*a*b^2 + b^3)*c*x)*log(-(c*x + 1)/(c*
x - 1))^2 - 6*(2*a^2*b + 2*a*b^2 + b^3 - (2*a^2*b + 2*a*b^2 + b^3)*c*x)*log
(-(c*x + 1)/(c*x - 1)))/(c^2*x + c)
```

giac [A] time = 0.21, size = 172, normalized size = 1.24

$$\frac{1}{16}\left(\frac{(cx-1)b^3\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)c^2}+\frac{3\left(2\,ab^2+b^3\right)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2}+\frac{6\left(2\,a^2b+2\,ab^2+b^3\right)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*((c*x - 1)*b^3*log(-(c*x + 1)/(c*x - 1))^3/((c*x + 1)*c^2) + 3*(2*a*b^
2 + b^3)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2) + 6*(2*a^2*b
 + 2*a*b^2 + b^3)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2) + 2*(
4*a^3 + 6*a^2*b + 6*a*b^2 + 3*b^3)*(c*x - 1)/((c*x + 1)*c^2))*c
```

maple [C] time = 0.70, size = 1895, normalized size = 13.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^2,x)
```

```
[Out] 3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*
csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*
x)^2*Pi*x-3/8*I/c*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^
2*Pi-3/8*I/c*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^3*arctanh(c*x)^2*Pi-3/4/c*b^3*arctanh(c*x)^2*ln(c*x-1)-3/2/c*a*b^2*a
rctanh(c*x)*ln(c*x-1)+3/4/c*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-3/8*I*b^3/(c*x+
1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*
x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2*Pi*x-3/4*I*b^3/(c*x+1)*csgn(I*(c*x+1)^
2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*Pi*x+3/8
*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2*Pi*x-3/4*I/c*b^3/(c*x+1)*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2
*Pi+3/8*I/c*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2*Pi-3/8*I/c*b^3/(c*x+1)
*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*arctanh
(c*x)^2*Pi-3/8*I/c*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-
c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2*Pi-3/8*I*b
^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))
^2*arctanh(c*x)^2*Pi*x-3/8*b^3/c/(c*x+1)+3/8*I/c*b^3/(c*x+1)*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2*Pi+3/8*b^3/(c*x+1)*x+3/4/c*
a^2*b*ln(c*x+1)-3/2/c*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/c
```

```
*a^3/(c*x+1)-3/c*a*b^2/(c*x+1)*arctanh(c*x)-3/c*a*b^2/(c*x+1)*arctanh(c*x)^
2-3/c*a^2*b/(c*x+1)*arctanh(c*x)-3/2/c*a^2*b/(c*x+1)-3/4/c*a^2*b*ln(c*x-1)-
3/4/c*a*b^2*ln(c*x-1)+3/4/c*a*b^2*ln(c*x+1)-3/8/c*a*b^2*ln(c*x-1)^2-3/4/c*b
^3*arctanh(c*x)^2/(c*x+1)-3/4/c*b^3/(c*x+1)*arctanh(c*x)-3/2/c*a*b^2/(c*x+1
)+3/4*b^3/(c*x+1)*arctanh(c*x)^2*x+3/4*b^3/(c*x+1)*arctanh(c*x)*x+1/2*b^3/(
c*x+1)*arctanh(c*x)^3*x-1/2/c*b^3/(c*x+1)*arctanh(c*x)^3+3/4*I/c*b^3/(c*x+1
)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*Pi-3/4*I/c*b^3/(c*x+1
)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2*Pi-3/8*I*b^3/(c*x+1)*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*P
i*x+3/4*I*b^3/(c*x+1)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*P
i*x-3/4*I*b^3/(c*x+1)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2*P
i*x-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2*Pi*x+3
/2/c*a*b^2*arctanh(c*x)*ln(c*x+1)+3/4/c*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/
4/c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/4*I*b^3/(c*x+1)*arctanh(c*x)^2
*Pi*x+3/4*I/c*b^3/(c*x+1)*arctanh(c*x)^2*Pi+3/4/c*b^3*arctanh(c*x)^2*ln(c*x
+1)-3/8/c*a*b^2*ln(c*x+1)^2
```

maxima [B] time = 0.35, size = 529, normalized size = 3.81

$$-\frac{b^3 \operatorname{artanh}(cx)^3}{c^2x+c} - \frac{3}{4}\left(c\left(\frac{2}{c^3x+c^2} - \frac{\log(cx+1)}{c^2} + \frac{\log(cx-1)}{c^2}\right) + \frac{4 \operatorname{artanh}(cx)}{c^2x+c}\right)a^2b - \frac{3}{8}\left(4c\left(\frac{2}{c^3x+c^2} - \frac{\log(\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="maxima")
```

```
[Out] -b^3*arctanh(c*x)^3/(c^2*x + c) - 3/4*(c*(2/(c^3*x + c^2) - log(c*x + 1)/c^
2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*a^2*b - 3/8*(4*c*(2/(c^
3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c*x + 1)
*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*log(c*x - 1
) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(c^4*x + c^3))*a*b^
2 - 1/16*(12*c*(2/(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arct
anh(c*x)^2 - (((c*x + 1)*log(c*x + 1)^3 - (c*x + 1)*log(c*x - 1)^3 - 3*(c*x
 + (c*x + 1)*log(c*x - 1) + 1)*log(c*x + 1)^2 - 3*(c*x + 1)*log(c*x - 1)^2
+ 3*((c*x + 1)*log(c*x - 1)^2 + 2*c*x + 2*(c*x + 1)*log(c*x - 1) + 2)*log(c
*x + 1) - 6*(c*x + 1)*log(c*x - 1) - 12)*c^2/(c^5*x + c^4) - 6*((c*x + 1)*l
og(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*log(c*x - 1)
+ 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c*arctanh(c*x)/(c^4*x + c
^3))*c)*b^3 - 3*a*b^2*arctanh(c*x)^2/(c^2*x + c) - a^3/(c^2*x + c)
```

mupad [B] time = 2.30, size = 582, normalized size = 4.19

$$\ln(1-c\,x)\left(\ln(c\,x+1)\left(\frac{3\,b^3\,x+\frac{3\,b^2\,(4\,a+b)}{c}}{8\,c\,x+8} - \frac{3\,\left(b^3+a\,b^2\right)}{4\,c} + \frac{6\,b^3}{c\,(8\,c\,x+8)}\right) + \frac{3\,b^3\,x+\frac{3\,b^2\,(4\,a+b)}{c}}{8\,c\,x+8} - \ln(c\,x+1)^2\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1)^2,x)
```

```
[Out] log(1 - c*x)*(log(c*x + 1)*((3*b^3*x + (3*b^2*(4*a + b))/c)/(8*c*x + 8) - (
3*(a*b^2 + b^3))/(4*c) + (6*b^3)/(c*(8*c*x + 8))) + (3*b^3*x + (3*b^2*(4*a
+ b))/c)/(8*c*x + 8) - log(c*x + 1)^2*((3*b^3)/(16*c) - (3*b^3)/(c*(8*c*x +
 8))) - (3*b^3*x + (3*(4*a*b^2 - 4*a^2*b + 5*b^3))/c)/(8*c*x + 8) + (6*b^3)
/(c*(8*c*x + 8)) + (3*(8*c*x + 24)*(a*b^2 + b^3))/(4*c*(8*c*x + 8))) - log(
1 - c*x)^2*((3*b^3)/(c*(8*c*x + 8)) - (3*(a*b^2 + b^3))/(8*c) - log(c*x + 1
)*((3*b^3)/(16*c) - (3*b^3)/(c*(8*c*x + 8))) + (3*b^3*(8*c*x + 24))/(16*c*(
8*c*x + 8)) + (3*b^2*(2*a - b))/(c*(8*c*x + 8))) - log(c*x + 1)^2*(((3*b^3*
x)/(16*c) + (3*b^2*(4*a + 3*b))/(16*c^2))/(x + 1/c) - (3*b^2*(a + b))/(8*c)
) - log(1 - c*x)^3*(b^3/(16*c) - b^3/(c*(8*c*x + 8))) + log(c*x + 1)^3*(b^3
```

```
/(16*c) - b^3/(8*c^2*(x + 1/c))) - (log(c*x + 1)*((3*b*(3*a*b + 2*a^2 + 2*b
^2))/(4*c^2) + (3*b^2*x*(a + b))/(4*c)))/(x + 1/c) - (6*a*b^2 + 6*a^2*b + 4
*a^3 + 3*b^3)/(2*c*(2*c*x + 2)) - (b*atan(c*x*1i)*(4*a*b + 2*a^2 + 3*b^2)*3
i)/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atanh}\,(cx))^3}{(cx + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**2, x)
```

3.125 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)^2}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx}$$

```
[Out] -3/64*b^3/c/(c*x+1)^2-21/64*b^3/c/(c*x+1)+21/64*b^3*arctanh(c*x)/c-3/16*b^2
*(a+b*arctanh(c*x))/c/(c*x+1)^2-9/16*b^2*(a+b*arctanh(c*x))/c/(c*x+1)+9/32*
b*(a+b*arctanh(c*x))^2/c-3/8*b*(a+b*arctanh(c*x))^2/c/(c*x+1)^2-3/8*b*(a+b*
arctanh(c*x))^2/c/(c*x+1)+1/8*(a+b*arctanh(c*x))^3/c-1/2*(a+b*arctanh(c*x))
^3/c/(c*x+1)^2
```

Rubi [A] time = 0.37, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)^2}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```

```
[Out] (-3*b^3)/(64*c*(1 + c*x)^2) - (21*b^3)/(64*c*(1 + c*x)) + (21*b^3*ArcTanh[c
*x])/(64*c) - (3*b^2*(a + b*ArcTanh[c*x]))/(16*c*(1 + c*x)^2) - (9*b^2*(a +
 b*ArcTanh[c*x]))/(16*c*(1 + c*x)) + (9*b*(a + b*ArcTanh[c*x])^2)/(32*c) -
(3*b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)^2) - (3*b*(a + b*ArcTanh[c*x])^
2)/(8*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(8*c) - (a + b*ArcTanh[c*x])^3/
(2*c*(1 + c*x)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c(1+cx)^2}+\frac{1}{2}(3b)\int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4(1+cx)^2}-\frac{(a+}{4}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c(1+cx)^2}+\frac{1}{8}(3b)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx-\frac{1}{8}(3b)\int\frac{\left(a+b\tanh^{-1}(c\right.}{-1+c^2x^2}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{8c}-\frac{(a+b\,\mathrm{ta}}{2c(1}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{8c}-\frac{(a+b\,\mathrm{ta}}{2c(1}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\,(a}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\,(a}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\,(a}{}$$

$$= -\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9}{}$$

$$= -\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}+\frac{21b^3\tanh^{-1}(cx)}{64c}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\,(a+}{16}$$

Mathematica [A] time = 0.22, size = 215, normalized size = 1.03

$$\frac{-6b\left(8a^2+12ab+7b^2\right)(cx+1)-3b\left(8a^2+12ab+7b^2\right)(cx+1)^2\log(1-cx)+3b\left(8a^2+12ab+7b^2\right)(cx+1)^2\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```

```
[Out] (-2*(32*a^3 + 24*a^2*b + 12*a*b^2 + 3*b^3) - 6*b*(8*a^2 + 12*a*b + 7*b^2)*(
1 + c*x) - 24*b*(8*a^2 + 4*a*b*(2 + c*x) + b^2*(4 + 3*c*x))*ArcTanh[c*x] +
12*b^2*(-1 + c*x)*(4*a*(3 + c*x) + b*(5 + 3*c*x))*ArcTanh[c*x]^2 + 16*b^3*(
-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^3 - 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1 + c*
```

```
x)^2*Log[1 - c*x] + 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1 + c*x)^2*Log[1 + c*x])/
(128*c*(1 + c*x)^2)
```

fricas [A] time = 0.66, size = 250, normalized size = 1.20

$$\frac{2\left(b^3c^2x^2+2\,b^3cx-3\,b^3\right)\log\left(-\frac{cx+1}{cx-1}\right)^3-64\,a^3-96\,a^2b-96\,ab^2-48\,b^3-6\left(8\,a^2b+12\,ab^2+7\,b^3\right)cx+3\left(\left(4\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(2*(b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 64
*a^3 - 96*a^2*b - 96*a*b^2 - 48*b^3 - 6*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x +
3*((4*a*b^2 + 3*b^3)*c^2*x^2 - 12*a*b^2 - 5*b^3 + 2*(4*a*b^2 + b^3)*c*x)*lo
g(-(c*x + 1)/(c*x - 1))^2 + 3*((8*a^2*b + 12*a*b^2 + 7*b^3)*c^2*x^2 - 24*a^
2*b - 20*a*b^2 - 9*b^3 + 2*(8*a^2*b + 4*a*b^2 + b^3)*c*x)*log(-(c*x + 1)/(c
*x - 1)))/(c^3*x^2 + 2*c^2*x + c)
```

giac [A] time = 0.29, size = 362, normalized size = 1.74

$$\frac{1}{256}\left(\frac{4\left(\frac{2\,(cx+1)b^3}{cx-1}-b^3\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)^2c^2}+\frac{6\left(\frac{8\,(cx+1)ab^2}{cx-1}-4\,ab^2+\frac{4\,(cx+1)b^3}{cx-1}-b^3\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="giac")
```

```
[Out] 1/256*(4*(2*(c*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(c*x + 1)/(c*x
- 1))^3/((c*x + 1)^2*c^2) + 6*(8*(c*x + 1)*a*b^2/(c*x - 1) - 4*a*b^2 + 4*(c
*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))^2/((c*x
+ 1)^2*c^2) + 6*(16*(c*x + 1)*a^2*b/(c*x - 1) - 8*a^2*b + 16*(c*x + 1)*a*b^
2/(c*x - 1) - 4*a*b^2 + 8*(c*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(
c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^2) + (64*(c*x + 1)*a^3/(c*x - 1) - 32*a^
3 + 96*(c*x + 1)*a^2*b/(c*x - 1) - 24*a^2*b + 96*(c*x + 1)*a*b^2/(c*x - 1)
- 12*a*b^2 + 48*(c*x + 1)*b^3/(c*x - 1) - 3*b^3)*(c*x - 1)^2/((c*x + 1)^2*c
^2))*c
```

maple [C] time = 0.83, size = 2752, normalized size = 13.23

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^3,x)
```

```
[Out] 1/4*b^3/(c*x+1)^2*arctanh(c*x)^3*x+9/16*b^3/(c*x+1)^2*arctanh(c*x)^2*x+3/32
*b^3/(c*x+1)^2*arctanh(c*x)*x-3/16/c*a*b^2/(c*x+1)^2-3/8/c*a^2*b/(c*x+1)^2+
45/256*c*b^3/(c*x+1)^2*x^2-3/8/c*b^3/(c*x+1)^2*arctanh(c*x)^3-27/64/c*b^3/(
c*x+1)^2*arctanh(c*x)-3/32/c*b^3*arctanh(c*x)^2/(c*x+1)^2-3/16/c*b^3*arctan
h(c*x)^2*ln(c*x-1)-3/8/c*a*b^2*arctanh(c*x)*ln(c*x-1)+3/16/c*a*b^2*ln(c*x-1
)*ln(1/2+1/2*c*x)-51/256*b^3/c/(c*x+1)^2-3/4/c*a*b^2/(c*x+1)^2*arctanh(c*x)
+3/16/c*a^2*b*ln(c*x+1)-3/8/c*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1
/2))+3/128*b^3/(c*x+1)^2*x-1/2/c*a^3/(c*x+1)^2+21/64*c*b^3/(c*x+1)^2*arctan
h(c*x)*x^2-3/4/c*a*b^2/(c*x+1)*arctanh(c*x)-3/2/c*a*b^2/(c*x+1)^2*arctanh(c
*x)^2-3/8/c*a^2*b/(c*x+1)-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))*x-3/32*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*x^2+3/16*I*c*b^3/(c*x+1)^2*arctanh(c*x)
```

```
^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*x^2-3/16*I*c*b^3/(c*x+1)^2*arcta
nh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^2-3/32*I*c*b^3/(c*x+1)^
2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^2-3/16*I/c*b^3/(c*x+1
)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(
c^2*x^2-1))^2+3/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/32*I
/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csg
n(I*(c*x+1)^2/(c^2*x^2-1))-3/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I/
(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2
*x^2+1)))^2-3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2
-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x+3/16*I*b^3/(c*x+1)^2*arctanh(c*
x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))^2*x-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x-3/16/c*a^2*b*ln(c*
x-1)-9/32/c*a*b^2*ln(c*x-1)+9/32/c*a*b^2*ln(c*x+1)-3/32/c*a*b^2*ln(c*x-1)^2
-3/8/c*b^3*arctanh(c*x)^2/(c*x+1)-9/16/c*a*b^2/(c*x+1)+3/32*I*c*b^3/(c*x+1)
^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^
2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*x^2+3/1
6*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2+3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi
*x-3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2*x-3/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))
^3-3/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
+(c*x+1)^2/(-c^2*x^2+1)))^3+3/16*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/16*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/16*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*
x^2-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*
x-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
*x+1)^2/(-c^2*x^2+1)))^3*x+3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))^3*x+3/8/c*a*b^2*arctanh(c*x)*ln(c*x+1)+3/16/c*a*b^
2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/16/c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+
3/16/c*b^3*arctanh(c*x)^2*ln(c*x+1)-3/32/c*a*b^2*ln(c*x+1)^2+1/8*c*b^3/(c*x
+1)^2*arctanh(c*x)^3*x^2-3/32*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^2-3/2/c*a^2*b/(
c*x+1)^2*arctanh(c*x)-3/16*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^2+3/16*I*b^3/(c*x+
1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*x+3/3
2*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2
*x^2+1)))+9/32*c*b^3/(c*x+1)^2*arctanh(c*x)^2*x^2+3/32*I*c*b^3/(c*x+1)^2*ar
ctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^2-3/32*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1
)^2/(-c^2*x^2+1)))*x^2
```

maxima [B] time = 0.36, size = 796, normalized size = 3.83

$$-\frac{b^3\operatorname{artanh}(cx)^3}{2\left(c^3x^2+2c^2x+c\right)}-\frac{3}{16}\left(c\left(\frac{2(cx+2)}{c^4x^2+2c^3x+c^2}-\frac{\log(cx+1)}{c^2}+\frac{\log(cx-1)}{c^2}\right)+\frac{8\operatorname{artanh}(cx)}{c^3x^2+2c^2x+c}\right)a^2b-\frac{3}{32}\left(4c\left(\frac{}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*arctanh(c*x)^3/(c^3*x^2 + 2*c^2*x + c) - 3/16*(c*(2*(c*x + 2)/(c^4
*x^2 + 2*c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*arctanh(c*
x)/(c^3*x^2 + 2*c^2*x + c))*a^2*b - 3/32*(4*c*(2*(c*x + 2)/(c^4*x^2 + 2*c^3
*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c^2*x^2 +
 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 + 6*c*x -
 (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c*x + 1
```

```
) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*x^2 + 2*c^4*x + c^3)
)*a*b^2 - 1/128*(24*c*(2*(c*x + 2)/(c^4*x^2 + 2*c^3*x + c^2) - log(c*x + 1)
/c^2 + log(c*x - 1)/c^2)*arctanh(c*x)^2 - ((2*(c^2*x^2 + 2*c*x + 1)*log(c*x
 + 1)^3 - 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^3 - 3*(3*c^2*x^2 + 6*c*x + 2
*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c*x + 1)^2 - 9*(c^2*x^2 + 2*c*
x + 1)*log(c*x - 1)^2 - 42*c*x + 3*(7*c^2*x^2 + 2*(c^2*x^2 + 2*c*x + 1)*log
(c*x - 1)^2 + 14*c*x + 6*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 7)*log(c*x +
1) - 21*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) - 48)*c^2/(c^6*x^2 + 2*c^5*x + c
^4) - 12*((c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(
c*x - 1)^2 + 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x -
 1) + 3)*log(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c*arctanh
(c*x)/(c^5*x^2 + 2*c^4*x + c^3))*c)*b^3 - 3/2*a*b^2*arctanh(c*x)^2/(c^3*x^2
 + 2*c^2*x + c) - 1/2*a^3/(c^3*x^2 + 2*c^2*x + c)
```

mupad [B] time = 3.46, size = 930, normalized size = 4.47

$$\frac{102\,b^3\,\ln(1-c\,x) - 102\,b^3\,\ln(c\,x+1) - 96\,a\,b^2 - 96\,a^2\,b - 15\,b^3\,\ln(c\,x+1)^2 - 6\,b^3\,\ln(c\,x+1)^3 - 15\,b^3\,\ln(1-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1)^3,x)
```

```
[Out] (102*b^3*log(1 - c*x) - 102*b^3*log(c*x + 1) - 96*a*b^2 - 96*a^2*b - 15*b^3
*log(c*x + 1)^2 - 6*b^3*log(c*x + 1)^3 - 15*b^3*log(1 - c*x)^2 + 6*b^3*log(
1 - c*x)^3 + 150*b^3*atanh(c*x) - 64*a^3 - 48*b^3 + 144*a*b^2*atanh(c*x) +
48*a^2*b*atanh(c*x) + 30*b^3*log(c*x + 1)*log(1 - c*x) - 132*a*b^2*log(c*x
+ 1) - 96*a^2*b*log(c*x + 1) + 132*a*b^2*log(1 - c*x) + 96*a^2*b*log(1 - c*
x) - 18*b^3*log(c*x + 1)*log(1 - c*x)^2 + 18*b^3*log(c*x + 1)^2*log(1 - c*x
) - 36*a*b^2*log(c*x + 1)^2 - 36*a*b^2*log(1 - c*x)^2 - 42*b^3*c*x - 144*b^
3*c*x*log(c*x + 1) + 144*b^3*c*x*log(1 - c*x) + 9*b^3*c^2*x^2*log(c*x + 1)^
2 + 2*b^3*c^2*x^2*log(c*x + 1)^3 + 9*b^3*c^2*x^2*log(1 - c*x)^2 - 2*b^3*c^2
*x^2*log(1 - c*x)^3 + 150*b^3*c^2*x^2*atanh(c*x) - 72*a*b^2*c*x - 48*a^2*b*
c*x + 6*b^3*c*x*log(c*x + 1)^2 + 4*b^3*c*x*log(c*x + 1)^3 + 6*b^3*c*x*log(1
 - c*x)^2 - 4*b^3*c*x*log(1 - c*x)^3 + 72*a*b^2*log(c*x + 1)*log(1 - c*x) +
 300*b^3*c*x*atanh(c*x) - 54*b^3*c^2*x^2*log(c*x + 1) + 54*b^3*c^2*x^2*log(
1 - c*x) - 12*b^3*c*x*log(c*x + 1)*log(1 - c*x) - 36*a*b^2*c^2*x^2*log(c*x
+ 1) + 36*a*b^2*c^2*x^2*log(1 - c*x) + 6*b^3*c^2*x^2*log(c*x + 1)*log(1 - c
*x)^2 - 6*b^3*c^2*x^2*log(c*x + 1)^2*log(1 - c*x) - 120*a*b^2*c*x*log(c*x +
 1) + 120*a*b^2*c*x*log(1 - c*x) + 12*b^3*c*x*log(c*x + 1)*log(1 - c*x)^2 -
 12*b^3*c*x*log(c*x + 1)^2*log(1 - c*x) + 12*a*b^2*c^2*x^2*log(c*x + 1)^2 +
 12*a*b^2*c^2*x^2*log(1 - c*x)^2 + 144*a*b^2*c^2*x^2*atanh(c*x) + 48*a^2*b*
c^2*x^2*atanh(c*x) + 24*a*b^2*c*x*log(c*x + 1)^2 + 24*a*b^2*c*x*log(1 - c*x
)^2 - 18*b^3*c^2*x^2*log(c*x + 1)*log(1 - c*x) + 288*a*b^2*c*x*atanh(c*x) +
 96*a^2*b*c*x*atanh(c*x) - 24*a*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x) - 48*
a*b^2*c*x*log(c*x + 1)*log(1 - c*x))/(128*c*(c*x + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{(cx+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**3, x)
```

3.126 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^4}\,dx$

Optimal. Leaf size=275

$$-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)^2}-\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(cx+1)^3}+\frac{11b\left(a+b\tanh^{-1}(cx)\right)^2}{96c}-\frac{b\left(a+b\tanh\right.}{8c(cx+}$$

```
[Out] -1/108*b^3/c/(c*x+1)^3-19/576*b^3/c/(c*x+1)^2-85/576*b^3/c/(c*x+1)+85/576*b
^3*arctanh(c*x)/c-1/18*b^2*(a+b*arctanh(c*x))/c/(c*x+1)^3-5/48*b^2*(a+b*arc
tanh(c*x))/c/(c*x+1)^2-11/48*b^2*(a+b*arctanh(c*x))/c/(c*x+1)+11/96*b*(a+b*
arctanh(c*x))^2/c-1/6*b*(a+b*arctanh(c*x))^2/c/(c*x+1)^3-1/8*b*(a+b*arctanh
(c*x))^2/c/(c*x+1)^2-1/8*b*(a+b*arctanh(c*x))^2/c/(c*x+1)+1/24*(a+b*arctanh
(c*x))^3/c-1/3*(a+b*arctanh(c*x))^3/c/(c*x+1)^3
```

Rubi [A] time = 0.61, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)^2}-\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(cx+1)^3}+\frac{11b\left(a+b\tanh^{-1}(cx)\right)^2}{96c}-\frac{b\left(a+b\tanh\right.}{8c(cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

```
[Out] -b^3/(108*c*(1 + c*x)^3) - (19*b^3)/(576*c*(1 + c*x)^2) - (85*b^3)/(576*c*(
1 + c*x)) + (85*b^3*ArcTanh[c*x])/(576*c) - (b^2*(a + b*ArcTanh[c*x]))/(18*
c*(1 + c*x)^3) - (5*b^2*(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)^2) - (11*b^2*
(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)) + (11*b*(a + b*ArcTanh[c*x])^2)/(96*
c) - (b*(a + b*ArcTanh[c*x])^2)/(6*c*(1 + c*x)^3) - (b*(a + b*ArcTanh[c*x])
^2)/(8*c*(1 + c*x)^2) - (b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)) + (a + b
*ArcTanh[c*x])^3/(24*c) - (a + b*ArcTanh[c*x])^3/(3*c*(1 + c*x)^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c(1+cx)^3} + b\int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^4} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4(1+cx)^3} + \frac{\left(a+\right.}{8}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c(1+cx)^3} + \frac{1}{8}b\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx - \frac{1}{8}b\int \frac{\left(a+b\tanh^{-1}(cx)\right)}{-1+c^2x^2}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{6c(1+cx)^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)} + \frac{\left(a+b\right.}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{6c(1+cx)^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)} + \frac{\left(a+b\right.}{}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2} - \frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)} + \frac{11b}{}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2} - \frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)} + \frac{11b}{}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2} - \frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)} + \frac{11b}{}$$

$$= -\frac{b^3}{108c(1+cx)^3} - \frac{19b^3}{576c(1+cx)^2} - \frac{85b^3}{576c(1+cx)} - \frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a\right.}{4}$$

$$= -\frac{b^3}{108c(1+cx)^3} - \frac{19b^3}{576c(1+cx)^2} - \frac{85b^3}{576c(1+cx)} + \frac{85b^3\tanh^{-1}(cx)}{576c} - \frac{b^2\left(a+b\,\mathrm{ta}\right.}{18c(1-}$$

Mathematica [A] time = 0.22, size = 279, normalized size = 1.01

$$-\frac{24b\tanh^{-1}(cx)\left(144a^2+12ab\left(3c^2x^2+9cx+10\right)+b^2\left(33c^2x^2+81cx+56\right)\right)+6b\left(72a^2+132ab+85b^2\right)(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

```
[Out] -1/3456*(32*(36*a^3 + 18*a^2*b + 6*a*b^2 + b^3) + 6*b*(72*a^2 + 60*a*b + 19
*b^2)*(1 + c*x) + 6*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^2 + 24*b*(144*a
```

```
^2 + 12*a*b*(10 + 9*c*x + 3*c^2*x^2) + b^2*(56 + 81*c*x + 33*c^2*x^2))*ArcT
anh[c*x] - 36*b^2*(-1 + c*x)*(12*a*(7 + 4*c*x + c^2*x^2) + b*(29 + 32*c*x +
 11*c^2*x^2))*ArcTanh[c*x]^2 - 144*b^3*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*A
rcTanh[c*x]^3 + 3*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 - c*x] -
3*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 + c*x])/(c*(1 + c*x)^3)
```

fricas [A] time = 0.76, size = 345, normalized size = 1.25

$$-\frac{6\left(72\,a^2b+132\,ab^2+85\,b^3\right)c^2x^2-18\left(b^3c^3x^3+3\,b^3c^2x^2+3\,b^3cx-7\,b^3\right)\log\left(-\frac{cx+1}{cx-1}\right)^3+1152\,a^3+1440\,a^2b+1\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/3456*(6*(72*a^2*b + 132*a*b^2 + 85*b^3)*c^2*x^2 - 18*(b^3*c^3*x^3 + 3*b^
3*c^2*x^2 + 3*b^3*c*x - 7*b^3)*log(-(c*x + 1)/(c*x - 1))^3 + 1152*a^3 + 144
0*a^2*b + 1344*a*b^2 + 656*b^3 + 162*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x - 9*(
(12*a*b^2 + 11*b^3)*c^3*x^3 + 3*(12*a*b^2 + 7*b^3)*c^2*x^2 - 84*a*b^2 - 29*
b^3 + 3*(12*a*b^2 - b^3)*c*x)*log(-(c*x + 1)/(c*x - 1))^2 - 3*((72*a^2*b +
132*a*b^2 + 85*b^3)*c^3*x^3 + 3*(72*a^2*b + 84*a*b^2 + 41*b^3)*c^2*x^2 - 50
4*a^2*b - 348*a*b^2 - 139*b^3 + 3*(72*a^2*b - 12*a*b^2 - 23*b^3)*c*x)*log(-
(c*x + 1)/(c*x - 1)))/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

giac [B] time = 0.23, size = 555, normalized size = 2.02

$$\frac{1}{6912}\left(\frac{36\left(\frac{3\,(cx+1)^2b^3}{(cx-1)^2}-\frac{3\,(cx+1)b^3}{cx-1}+b^3\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)^3c^2}+\frac{18\left(\frac{36\,(cx+1)^2ab^2}{(cx-1)^2}-\frac{36\,(cx+1)ab^2}{cx-1}+12\,ab^2+\frac{18\,(cx+1)^2b^3}{(cx-1)^2}\ldots\right.}{(cx+1)^3c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] 1/6912*(36*(3*(c*x + 1)^2*b^3/(c*x - 1)^2 - 3*(c*x + 1)*b^3/(c*x - 1) + b^3
)*(c*x - 1)^3*log(-(c*x + 1)/(c*x - 1))^3/((c*x + 1)^3*c^2) + 18*(36*(c*x +
 1)^2*a*b^2/(c*x - 1)^2 - 36*(c*x + 1)*a*b^2/(c*x - 1) + 12*a*b^2 + 18*(c*x
 + 1)^2*b^3/(c*x - 1)^2 - 9*(c*x + 1)*b^3/(c*x - 1) + 2*b^3)*(c*x - 1)^3*lo
g(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^3*c^2) + 6*(216*(c*x + 1)^2*a^2*b/(c*x
 - 1)^2 - 216*(c*x + 1)*a^2*b/(c*x - 1) + 72*a^2*b + 216*(c*x + 1)^2*a*b^2/
(c*x - 1)^2 - 108*(c*x + 1)*a*b^2/(c*x - 1) + 24*a*b^2 + 108*(c*x + 1)^2*b^
3/(c*x - 1)^2 - 27*(c*x + 1)*b^3/(c*x - 1) + 4*b^3)*(c*x - 1)^3*log(-(c*x +
 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (864*(c*x + 1)^2*a^3/(c*x - 1)^2 - 864*(
c*x + 1)*a^3/(c*x - 1) + 288*a^3 + 1296*(c*x + 1)^2*a^2*b/(c*x - 1)^2 - 648
*(c*x + 1)*a^2*b/(c*x - 1) + 144*a^2*b + 1296*(c*x + 1)^2*a*b^2/(c*x - 1)^2
 - 324*(c*x + 1)*a*b^2/(c*x - 1) + 48*a*b^2 + 648*(c*x + 1)^2*b^3/(c*x - 1)
^2 - 81*(c*x + 1)*b^3/(c*x - 1) + 8*b^3)*(c*x - 1)^3/((c*x + 1)^3*c^2))*c
```

maple [C] time = 0.89, size = 3637, normalized size = 13.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^4,x)
```

```
[Out] -139/576/c*b^3/(c*x+1)^3*arctanh(c*x)+41/192*c*b^3/(c*x+1)^3*arctanh(c*x)*x
^2+1/24*c^2*b^3/(c*x+1)^3*arctanh(c*x)^3*x^3+1/8*c*b^3/(c*x+1)^3*arctanh(c*
x)^3*x^2+11/96*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*x^3+85/576*c^2*b^3/(c*x+1)^
3*arctanh(c*x)*x^3+11/32*c*b^3/(c*x+1)^3*arctanh(c*x)^2*x^2-1/c*a*b^2/(c*x+
```

```
1)^3*arctanh(c*x)^2-1/c*a^2*b/(c*x+1)^3*arctanh(c*x)-1/3/c*a*b^2/(c*x+1)^3*
arctanh(c*x)-5/96/c*b^3*arctanh(c*x)^2/(c*x+1)^3-5/48/c*a*b^2/(c*x+1)^2-1/8
/c*a^2*b/(c*x+1)^2-1/8/c*b^3*arctanh(c*x)^2/(c*x+1)^2-1/16/c*b^3*arctanh(c*
x)^2*ln(c*x-1)-1/8/c*a*b^2*arctanh(c*x)*ln(c*x-1)+1/16/c*a*b^2*ln(c*x-1)*ln
(1/2+1/2*c*x)-737/6912*b^3/c/(c*x+1)^3+1/16*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*
x)^2+3/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*x-1/4/c*a*b^2/(c*x+1)^2*arctanh
(c*x)+3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*x-3/32*I*b^3/(c*
x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))^2*Pi*x-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*P
i*x-1/32*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3
*Pi*x^3-1/32*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*x^3+1/16*I*c^2*b^3/(c*x+1)^3*arctanh(c*x
)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*x^3-3/32*I*c*b^3/(c*x+1)^3*arct
anh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*x^2-3/32*I*c*b^3/(c*x+1)^3*ar
ctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*
x^2+3/16*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)^3*Pi*x^2-1/16*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^
2*x^2+1)))^2*Pi*x^3-3/16*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)
^2/(-c^2*x^2+1)))^2*Pi*x^2-1/16*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*
(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/32*I/c*b^3/(c
*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/32*I/c*b^3/(c*x+1)^3*Pi*arctanh(c
*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/
32*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/16*I*b^3/(c*x+1
)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))*Pi*x-1/16*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c
^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi*x^3+1/32*I*c^2*b^3/(c*x+
1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*x^3+1/32*I/c*b^3/(c*x+1)^3*Pi*arctanh(
c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/32*I*c^2*b^3/(c*x+1
)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1
)^(1/2))^2*Pi*x^3-181/2304*b^3/(c*x+1)^3*x-1/3/c*a^3/(c*x+1)^3+1/16/c*a^2*b
*ln(c*x+1)-1/8/c*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/4/c*a*
b^2/(c*x+1)*arctanh(c*x)-1/8/c*a^2*b/(c*x+1)-1/16/c*a^2*b*ln(c*x-1)-11/96/c
*a*b^2*ln(c*x-1)+11/96/c*a*b^2*ln(c*x+1)-1/32/c*a*b^2*ln(c*x-1)^2+11/32*b^3
/(c*x+1)^3*arctanh(c*x)^2*x-23/192*b^3/(c*x+1)^3*arctanh(c*x)*x+1/8*b^3/(c*
x+1)^3*arctanh(c*x)^3*x-1/8/c*b^3*arctanh(c*x)^2/(c*x+1)-11/48/c*a*b^2/(c*x
+1)-1/6/c*a^2*b/(c*x+1)^3-1/18/c*a*b^2/(c*x+1)^3+575/6912*c^2*b^3*x^3/(c*x+
1)^3+235/2304*c*b^3/(c*x+1)^3*x^2-7/24/c*b^3/(c*x+1)^3*arctanh(c*x)^3-1/32*
I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*x^3-3/16*I*c*b^3/(
c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))*Pi*x^2+3/32*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)
^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*
Pi*x^2-3/32*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*x^2-3/32*I*c*b^3/(c*x+1)^3*arctanh(
c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1
+(c*x+1)^2/(-c^2*x^2+1)))*Pi*x^2+3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I
*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*x-3/32*I*b^3/(c*x+1)^3*arctanh(
c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*x+3/16
*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*x-3
/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*
x-1/32*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-
1/32*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
```

```
*x+1)^2/(-c^2*x^2+1)))^3+1/16*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I/(1
+(c*x+1)^2/(-c^2*x^2+1)))^3-1/16*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/16*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*x
^3+3/16*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*x^2+1/8/c*a*b^2*arctanh(c*x)*ln
(c*x+1)+1/16/c*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/16/c*a*b^2*ln(-1/2*c*x+1/
2)*ln(1/2+1/2*c*x)-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1))^3*Pi*x+1/16/c*b^3*arctanh(c*x)^2*ln(c*x+1)-1/32/c*a*b^2*ln(c*x+1)
^2+3/32*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-
c^2*x^2+1)))*Pi*x^2+1/32*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*x^3
```

maxima [B] time = 0.39, size = 1085, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*arctanh(c*x)^3/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/48*(c*(2*(3
*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x +
1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2
*x + c))*a^2*b - 1/288*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x
^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2)*arctanh(c*x)
 + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^2 + 9*(c^
3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11*c^3*x^3 + 3
3*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 11)
*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 112)*c^
2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*a*b^2 - 1/3456*(72*c*(2*(3*c^2*x^2
 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 +
 3*log(c*x - 1)/c^2)*arctanh(c*x)^2 + ((510*c^2*x^2 - 18*(c^3*x^3 + 3*c^2*x
^2 + 3*c*x + 1)*log(c*x + 1)^3 + 18*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c
*x - 1)^3 + 9*(11*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 +
3*c*x + 1)*log(c*x - 1) + 11)*log(c*x + 1)^2 + 99*(c^3*x^3 + 3*c^2*x^2 + 3*
c*x + 1)*log(c*x - 1)^2 + 1134*c*x - 3*(85*c^3*x^3 + 255*c^2*x^2 + 18*(c^3*
x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 255*c*x + 66*(c^3*x^3 + 3*c^2
*x^2 + 3*c*x + 1)*log(c*x - 1) + 85)*log(c*x + 1) + 255*(c^3*x^3 + 3*c^2*x^
2 + 3*c*x + 1)*log(c*x - 1) + 656)*c^2/(c^7*x^3 + 3*c^6*x^2 + 3*c^5*x + c^4
) + 12*(66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^2 + 9
*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11*c^3*x^3
 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) +
 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 112
)*c*arctanh(c*x)/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*c)*b^3 - a*b^2*arct
anh(c*x)^2/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/3*a^3/(c^4*x^3 + 3*c^3*x
^2 + 3*c^2*x + c)
```

mupad [B] time = 4.49, size = 1304, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1)^4,x)
```

```
[Out] (1398*b^3*log(1 - c*x) - 1398*b^3*log(c*x + 1) - 1344*a*b^2 - 1440*a^2*b -
261*b^3*log(c*x + 1)^2 - 126*b^3*log(c*x + 1)^3 - 261*b^3*log(1 - c*x)^2 +
126*b^3*log(1 - c*x)^3 + 1962*b^3*atanh(c*x) - 1152*a^3 - 656*b^3 + 1584*a*
b^2*atanh(c*x) + 432*a^2*b*atanh(c*x) + 522*b^3*log(c*x + 1)*log(1 - c*x) -
 1836*a*b^2*log(c*x + 1) - 1728*a^2*b*log(c*x + 1) + 1836*a*b^2*log(1 - c*x
) + 1728*a^2*b*log(1 - c*x) - 378*b^3*log(c*x + 1)*log(1 - c*x)^2 + 378*b^3
```

```
*log(c*x + 1)^2*log(1 - c*x) - 510*b^3*c^2*x^2 - 756*a*b^2*log(c*x + 1)^2 -
 756*a*b^2*log(1 - c*x)^2 - 1134*b^3*c*x - 3150*b^3*c*x*log(c*x + 1) + 3150
*b^3*c*x*log(1 - c*x) - 792*a*b^2*c^2*x^2 - 432*a^2*b*c^2*x^2 + 189*b^3*c^2
*x^2*log(c*x + 1)^2 + 54*b^3*c^2*x^2*log(c*x + 1)^3 + 189*b^3*c^2*x^2*log(1
 - c*x)^2 - 54*b^3*c^2*x^2*log(1 - c*x)^3 + 99*b^3*c^3*x^3*log(c*x + 1)^2 +
 18*b^3*c^3*x^3*log(c*x + 1)^3 + 99*b^3*c^3*x^3*log(1 - c*x)^2 - 18*b^3*c^3
*x^3*log(1 - c*x)^3 + 5886*b^3*c^2*x^2*atanh(c*x) + 1962*b^3*c^3*x^3*atanh(
c*x) - 1944*a*b^2*c*x - 1296*a^2*b*c*x - 27*b^3*c*x*log(c*x + 1)^2 + 54*b^3
*c*x*log(c*x + 1)^3 - 27*b^3*c*x*log(1 - c*x)^2 - 54*b^3*c*x*log(1 - c*x)^3
 + 1512*a*b^2*log(c*x + 1)*log(1 - c*x) + 5886*b^3*c*x*atanh(c*x) - 2574*b^
3*c^2*x^2*log(c*x + 1) + 2574*b^3*c^2*x^2*log(1 - c*x) - 726*b^3*c^3*x^3*lo
g(c*x + 1) + 726*b^3*c^3*x^3*log(1 - c*x) + 54*b^3*c*x*log(c*x + 1)*log(1 -
 c*x) - 1620*a*b^2*c^2*x^2*log(c*x + 1) + 1620*a*b^2*c^2*x^2*log(1 - c*x) -
 396*a*b^2*c^3*x^3*log(c*x + 1) + 396*a*b^2*c^3*x^3*log(1 - c*x) + 162*b^3*
c^2*x^2*log(c*x + 1)*log(1 - c*x)^2 - 162*b^3*c^2*x^2*log(c*x + 1)^2*log(1
- c*x) + 54*b^3*c^3*x^3*log(c*x + 1)*log(1 - c*x)^2 - 54*b^3*c^3*x^3*log(c*
x + 1)^2*log(1 - c*x) - 2484*a*b^2*c*x*log(c*x + 1) + 2484*a*b^2*c*x*log(1
- c*x) + 162*b^3*c*x*log(c*x + 1)*log(1 - c*x)^2 - 162*b^3*c*x*log(c*x + 1)
^2*log(1 - c*x) + 324*a*b^2*c^2*x^2*log(c*x + 1)^2 + 324*a*b^2*c^2*x^2*log(
1 - c*x)^2 + 108*a*b^2*c^3*x^3*log(c*x + 1)^2 + 108*a*b^2*c^3*x^3*log(1 - c
*x)^2 + 4752*a*b^2*c^2*x^2*atanh(c*x) + 1296*a^2*b*c^2*x^2*atanh(c*x) + 158
4*a*b^2*c^3*x^3*atanh(c*x) + 432*a^2*b*c^3*x^3*atanh(c*x) + 324*a*b^2*c*x*l
og(c*x + 1)^2 + 324*a*b^2*c*x*log(1 - c*x)^2 - 378*b^3*c^2*x^2*log(c*x + 1)
*log(1 - c*x) - 198*b^3*c^3*x^3*log(c*x + 1)*log(1 - c*x) + 4752*a*b^2*c*x*
atanh(c*x) + 1296*a^2*b*c*x*atanh(c*x) - 648*a*b^2*c^2*x^2*log(c*x + 1)*log
(1 - c*x) - 216*a*b^2*c^3*x^3*log(c*x + 1)*log(1 - c*x) - 648*a*b^2*c*x*log
(c*x + 1)*log(1 - c*x))/(3456*c*(c*x + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atanh}\,(cx))^3}{(cx+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**4, x)
```

3.127 $\int \frac{x^2 \tanh^{-1}(ax)^3}{c+acx} \, dx$

Optimal. Leaf size=309

$$-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^3c}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{3\text{Li}_4\left(1-\frac{2}{ax+1}\right)}{4a^3c}+\frac{3\text{Li}_2\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{2a^3c}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3c}+\cdots$$

```
[Out] 3/2*arctanh(a*x)^2/a^3/c+3/2*x*arctanh(a*x)^2/a^2/c-3/2*arctanh(a*x)^3/a^3/
c-x*arctanh(a*x)^3/a^2/c+1/2*x^2*arctanh(a*x)^3/a/c-3*arctanh(a*x)*ln(2/(-a
*x+1))/a^3/c+3*arctanh(a*x)^2*ln(2/(-a*x+1))/a^3/c-arctanh(a*x)^3*ln(2/(a*x
+1))/a^3/c-3/2*polylog(2,1-2/(-a*x+1))/a^3/c+3*arctanh(a*x)*polylog(2,1-2/(
-a*x+1))/a^3/c+3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))/a^3/c-3/2*polylog(
3,1-2/(-a*x+1))/a^3/c+3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a^3/c+3/4*pol
ylog(4,1-2/(a*x+1))/a^3/c
```

Rubi [A] time = 0.64, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5930, 5916, 5980, 5910, 5984, 5918, 2402, 2315, 5948, 6058, 6610, 6056, 6060}

$$-\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^3c}-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a^3c}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a^3c}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (3*ArcTanh[a*x]^2)/(2*a^3*c) + (3*x*ArcTanh[a*x]^2)/(2*a^2*c) - (3*ArcTanh[
a*x]^3)/(2*a^3*c) - (x*ArcTanh[a*x]^3)/(a^2*c) + (x^2*ArcTanh[a*x]^3)/(2*a*
c) - (3*ArcTanh[a*x]*Log[2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]^2*Log[2/(1
 - a*x)])/(a^3*c) - (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^3*c) - (3*PolyLog[
2, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)
])/(a^3*c) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/(2*a^3*c) - (3*
PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(
1 + a*x)])/(2*a^3*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a^3*c)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
```

```
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{c+acx}\,dx = -\frac{\int \frac{x\tanh^{-1}(ax)^3}{c+acx}\,dx}{a} + \frac{\int x\tanh^{-1}(ax)^3\,dx}{ac}$$

$$= \frac{x^2\tanh^{-1}(ax)^3}{2ac} + \frac{\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx}{a^2} - \frac{3\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{2c} - \frac{\int \tanh^{-1}(ax)^3\,dx}{a^2c}$$

$$= -\frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} - \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^3c} + \frac{3\int \tanh^{-1}(ax)^2\,dx}{2a^2c} - 3\int$$

$$= \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} - \frac{\tanh^{-1}(ax)^3\log\left(\right.}{a^3c}$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} +$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} -$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} -$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} -$$

Mathematica [A] time = 0.35, size = 172, normalized size = 0.56

$$2a^2x^2\tanh^{-1}(ax)^3 + 6\left(\tanh^{-1}(ax)-1\right)^2 \mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 6\left(\tanh^{-1}(ax)-1\right)\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right) + 3\mathrm{Li}_4\left(-e\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]^3 - 4*a*x*ArcTan
h[a*x]^3 + 2*a^2*x^2*ArcTanh[a*x]^3 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh
[a*x])] + 12*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3
*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])^2*PolyLog[2, -E^(-2*A
rcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*
PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a^3*c)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2\,\mathrm{artanh}\,(ax)^3}{acx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{acx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

maple [A] time = 3.01, size = 400, normalized size = 1.29

$$\frac{x^2 \operatorname{arctanh}(ax)^3}{2ac} - \frac{x \operatorname{arctanh}(ax)^3}{a^2 c} + \frac{3x \operatorname{arctanh}(ax)^2}{2a^2 c} - \frac{3 \operatorname{arctanh}(ax)^3}{2a^3 c} + \frac{3 \operatorname{arctanh}(ax)^2}{2a^3 c} + \frac{\operatorname{arctanh}(ax)^4}{2a^3 c} - \text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(a*c*x+c),x)
```

```
[Out] 1/2*x^2*arctanh(a*x)^3/a/c-x*arctanh(a*x)^3/a^2/c+3/2*x*arctanh(a*x)^2/a^2/
c-3/2*arctanh(a*x)^3/a^3/c+3/2*arctanh(a*x)^2/a^3/c+1/2*arctanh(a*x)^4/a^3/
c-1/a^3/c*arctanh(a*x)^3*ln(1+(a*x+1)^2/(-a^2*x^2+1))-3/2/a^3/c*arctanh(a*x
)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2/a^3/c*arctanh(a*x)*polylog(3,-(a
*x+1)^2/(-a^2*x^2+1))-3/4/a^3/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/a^3/c*
arctanh(a*x)*ln(1+(a*x+1)^2/(-a^2*x^2+1))-3/2/a^3/c*polylog(2,-(a*x+1)^2/(-
a^2*x^2+1))+3/a^3/c*arctanh(a*x)^2*ln(1+(a*x+1)^2/(-a^2*x^2+1))+3/a^3/c*arc
tanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2/a^3/c*polylog(3,-(a*x+1)^2
/(-a^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(a^2x^2 - 2\,ax + 2\,\log(ax+1)\right)\log(-ax+1)^3}{16\,a^3c} + \frac{1}{8}\int \frac{2\left(a^3x^3 - a^2x^2\right)\log(ax+1)^3 - 6\left(a^3x^3 - a^2x^2\right)\log(ax +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*(a^2*x^2 - 2*a*x + 2*log(a*x + 1))*log(-a*x + 1)^3/(a^3*c) + 1/8*inte
grate(1/2*(2*(a^3*x^3 - a^2*x^2)*log(a*x + 1)^3 - 6*(a^3*x^3 - a^2*x^2)*log
(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^3 - a^2*x^2 - 2*a*x + 2*(a^3*x^3 - a^2
*x^2 + a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*c*x^2 - a^2*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \operatorname{atanh}(a\,x)^3}{c + a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^3)/(c + a*c*x),x)
```

```
[Out] int((x^2*atanh(a*x)^3)/(c + a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atanh}^3(ax)}{ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(a*c*x+c),x)
[Out] Integral(x**2*atanh(a*x)**3/(a*x + 1), x)/c
```

3.128 $\int \frac{x \tanh^{-1}(ax)^3}{c+acx} dx$

Optimal. Leaf size=205

$$\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^2c}-\frac{3\text{Li}_4\left(1-\frac{2}{ax+1}\right)}{4a^2c}-\frac{3\text{Li}_2\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2c}-\frac{3\text{Li}_3\left(1-\frac{2}{ax+1}\right)\text{ta}}{2a^2c}$$

```
[Out] arctanh(a*x)^3/a^2/c+x*arctanh(a*x)^3/a/c-3*arctanh(a*x)^2*ln(2/(-a*x+1))/a
^2/c+arctanh(a*x)^3*ln(2/(a*x+1))/a^2/c-3*arctanh(a*x)*polylog(2,1-2/(-a*x+
1))/a^2/c-3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))/a^2/c+3/2*polylog(3,1-2
/(-a*x+1))/a^2/c-3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a^2/c-3/4*polylog(
4,1-2/(a*x+1))/a^2/c
```

Rubi [A] time = 0.37, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5930, 5910, 5984, 5918, 5948, 6058, 6610, 6056, 6060}

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2c}-\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a^2c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a^2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] ArcTanh[a*x]^3/(a^2*c) + (x*ArcTanh[a*x]^3)/(a*c) - (3*ArcTanh[a*x]^2*Log[2
/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^2*c) - (3*ArcTa
nh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[2,
 1 - 2/(1 + a*x)])/(2*a^2*c) + (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^2*c) -
(3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*a^2*c) - (3*PolyLog[4, 1 -
2/(1 + a*x)])/(4*a^2*c)
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{c+acx}\,dx = -\frac{\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx}{a}+\frac{\int \tanh^{-1}(ax)^3\,dx}{ac}$$

$$= \frac{x\tanh^{-1}(ax)^3}{ac}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}-\frac{3\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{c}-\frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{ac}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c}+\frac{x\tanh^{-1}(ax)^3}{ac}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}-\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{2a^2c}-$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c}+\frac{x\tanh^{-1}(ax)^3}{ac}-\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}-\frac{3\,\text{ta}}{}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c}+\frac{x\tanh^{-1}(ax)^3}{ac}-\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}-\frac{3\,\text{ta}}{}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c}+\frac{x\tanh^{-1}(ax)^3}{ac}-\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}-\frac{3\,\text{ta}}{}$$

Mathematica [A] time = 0.27, size = 126, normalized size = 0.61

$$-\frac{3}{2}\left(\tanh^{-1}(ax)-2\right)\tanh^{-1}(ax)\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-\frac{3}{2}\left(\tanh^{-1}(ax)-1\right)\text{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right)-\frac{3}{4}\text{Li}_4\left(-e^{-2\tanh^{-1}\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (-ArcTanh[a*x]^3 + a*x*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcT
anh[a*x])] + ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - (3*(-2 + ArcTanh
[a*x])*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])])/2 - (3*(-1 + ArcTanh[
a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])])/2 - (3*PolyLog[4, -E^(-2*ArcTanh[a*
x])])/4)/(a^2*c)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\,\text{artanh}\,(ax)^3}{acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctanh(a*x)^3/(a*c*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\text{artanh}\,(ax)^3}{acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^3/(a*c*x + c), x)
```

maple [C] time = 0.60, size = 833, normalized size = 4.06

$$\frac{x\,\text{arctanh}\,(ax)^3}{ac}-\frac{\text{arctanh}\,(ax)^3\ln(ax+1)}{a^2c}+\frac{2\,\text{arctanh}\,(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a^2c}+\frac{3\,\text{arctanh}\,(ax)^2\,\text{polylog}\left(2,-\frac{(ax+\cdots}{-a^2x\cdots}\right.}{2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(a*c*x+c),x)
```

```
[Out] x*arctanh(a*x)^3/a/c-1/a^2/c*arctanh(a*x)^3*ln(a*x+1)+2/a^2/c*arctanh(a*x)^
3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2/a^2/c*arctanh(a*x)^2*polylog(2,-(a*x+1
)^2/(-a^2*x^2+1))-3/2/a^2/c*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))
+3/4/a^2/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/a^2/c*arctanh(a*x)^2*ln(1+(
a*x+1)^2/(-a^2*x^2+1))-3/a^2/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+
1))+arctanh(a*x)^3/a^2/c-1/2*arctanh(a*x)^4/a^2/c+1/2*I/a^2/c*Pi*csgn(I/(1+
(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^
2+1)))^2*arctanh(a*x)^3+I/a^2/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3+1/2*I/a^2/c*Pi*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/2*I/a^2/c*Pi*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x
^2+1)))^2*arctanh(a*x)^3+1/2*I/a^2/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3+1/a^2/c*ln(2)*arctanh(a*x)^3
```

```
-1/2*I/a^2/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^
2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)
^3+1/2*I/a^2/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+3/2/a^2/c*
polylog(3,-(a*x+1)^2/(-a^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(ax-\log\left(ax+1\right)\right)\log\left(-ax+1\right)^{3}}{8\,a^{2}c}+\frac{1}{8}\int\frac{\left(a^{2}x^{2}-ax\right)\log\left(ax+1\right)^{3}-3\left(a^{2}x^{2}-ax\right)\log\left(ax+1\right)^{2}\log\left(-ax+1\right)+}{a^{3}cx^{2}-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(a*x - log(a*x + 1))*log(-a*x + 1)^3/(a^2*c) + 1/8*integrate(((a^2*x^2
 - a*x)*log(a*x + 1)^3 - 3*(a^2*x^2 - a*x)*log(a*x + 1)^2*log(-a*x + 1) + 3
*(a^2*x^2 + a*x + (a^2*x^2 - 2*a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^3
*c*x^2 - a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\mathrm{atanh}\left(a\,x\right)^3}{c+a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(c + a*c*x),x)
```

```
[Out] int((x*atanh(a*x)^3)/(c + a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\,\mathrm{atanh}^{3}\left(ax\right)}{ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(x*atanh(a*x)**3/(a*x + 1), x)/c
```

3.129 $\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx$

Optimal. Leaf size=104

$$\frac{3\text{Li}_4\left(1-\frac{2}{ax+1}\right)}{4ac}+\frac{3\text{Li}_2\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{2ac}+\frac{3\text{Li}_3\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{2ac}-\frac{\log\left(\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{ac}$$

```
[Out] -arctanh(a*x)^3*ln(2/(a*x+1))/a/c+3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))
/a/c+3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a/c+3/4*polylog(4,1-2/(a*x+1))
/a/c
```

Rubi [A] time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5918, 5948, 6056, 6060, 6610}

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4ac}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{2ac}-\frac{\log\left(\frac{2}{ax+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(c + a*c*x),x]
```

```
[Out] -((ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a*c)) + (3*ArcTanh[a*x]^2*PolyLog[2, 1
 - 2/(1 + a*x)])/(2*a*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*
a*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a*c)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx = -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac} + \frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac} + \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac} - \frac{3\int \frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac} + \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac} + \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(1-\frac{2}{1+ax}\right)}{2ac} - \frac{3}{}$$

$$= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac} + \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac} + \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(1-\frac{2}{1+ax}\right)}{2ac} + \frac{3\mathrm{I}}{}$$

Mathematica [A] time = 0.09, size = 82, normalized size = 0.79

$$\frac{6\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 6\tanh^{-1}(ax)\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right) + 3\mathrm{Li}_4\left(-e^{-2\tanh^{-1}(ax)}\right) - 4\tanh^{-1}(ax)^3\log\left(\right.}{4ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(c + a*c*x),x]
```

```
[Out] (-4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]^2*PolyLog[
2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])]
+ 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a*c)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\,(ax)^3}{acx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\,(ax)^3}{acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a*c*x + c), x)
```

maple [C] time = 0.28, size = 703, normalized size = 6.76

$$\frac{\mathrm{arctanh}\,(ax)^3\ln(ax+1)}{ac} - \frac{2\,\mathrm{arctanh}\,(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{ac} + \frac{\mathrm{arctanh}\,(ax)^4}{2ac} - \frac{3\,\mathrm{arctanh}\,(ax)^2\,\mathrm{polylog}\left(2, -\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(a*c*x+c),x)
```

```
[Out] 1/a/c*arctanh(a*x)^3*ln(a*x+1)-2/a/c*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)
^(1/2))+1/2/a/c*arctanh(a*x)^4-3/2/a/c*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/
(-a^2*x^2+1))+3/2/a/c*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-3/4/a
/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))+1/2*I/a/c*arctanh(a*x)^3*csgn(I/(1+(a
*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*Pi-1/2*I/a/c*arctanh(a*x)^3*csgn(I/(1+(a
*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+
1)))^2*Pi-1/2*I/a/c*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi-I/a/c
*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))*Pi+1/2*I/a/c*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi-1/2*I/a/c*arctanh(a*x)
^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi-1/
2*I/a/c*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+
1)))^3*Pi-1/a/c*arctanh(a*x)^3*ln(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log(ax+1)\log(-ax+1)^3}{8\,ac}+\frac{1}{8}\int\frac{6\,ax\log(ax+1)\log(-ax+1)^2+(ax-1)\log(ax+1)^3-3\,(ax-1)\log(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*log(a*x + 1)*log(-a*x + 1)^3/(a*c) + 1/8*integrate((6*a*x*log(a*x + 1)
*log(-a*x + 1)^2 + (a*x - 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*lo
g(-a*x + 1))/(a^2*c*x^2 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{atanh}(a\,x)^3}{c+a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(c + a*c*x),x)
```

```
[Out] int(atanh(a*x)^3/(c + a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\mathrm{atanh}^3(ax)}{ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x + 1), x)/c
```

3.130 $\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx$

Optimal. Leaf size=93

$$-\frac{3\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}-\frac{3\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}-\frac{3\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{2c}+\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}$$

```
[Out] arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3/2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))
/c-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*polylog(4,-1+2/(a*x+1))/c
```

Rubi [A] time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5932, 5948, 6056, 6060, 6610}

$$-\frac{3\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{\log\left(2-\frac{2}{ax}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c)
- (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx = \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{(3a)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{1-a^2x^2}}{c}$$
$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(-1+\frac{2}{1+}\right.}{2c}$$
$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(-1+\frac{2}{1+}\right.}{2c}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.92

$$\frac{96\tanh^{-1}(ax)^2\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) + 48\mathrm{Li}_4\left(e^{2\tanh^{-1}(ax)}\right) - 32\tanh^{-1}(ax)^4 + 64}{64c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

```
[Out] (Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] +
 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog
[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\,(ax)^3}{acx^2+cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\,(ax)^3}{(acx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x), x)
```

maple [C] time = 0.34, size = 1217, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(a*c*x+c),x)
```

```
[Out] 1/c*ln(2)*arctanh(a*x)^3-1/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+2/
c*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/2/c*arctanh(a*x)^4+6/c*po
lylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)^2*
polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(3,-(a*x+1)/
(-a^2*x^2+1)^(1/2))+1/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c
*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*poly
log(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)
-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctan
h(a*x)^3+1/2*I/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I/(1+
(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arc
tanh(a*x)^3+I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))^2*arctanh(a*x)^3+1/c*arctanh(a*x)^3*ln(a*x)-1/c*arctanh(a*x)^3*ln(
a*x+1)+1/2*I/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^
3*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+
1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*
arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x
+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x
^2+1)))*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2
+1)))*arctanh(a*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(ax+1\right)\log\left(-ax+1\right)^{3}}{8\,c}-\frac{1}{8}\int -\frac{\left(ax-1\right)\log\left(ax+1\right)^{3}-3\left(ax-1\right)\log\left(ax+1\right)^{2}\log\left(-ax+1\right)-3\left(a^{2}x^{2}+1\right)\log}{a^{2}cx^{3}-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1)
^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x + 1
)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(a\,x\right)^3}{x\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(c + a*c*x)),x)
```

```
[Out] int(atanh(a*x)^3/(x*(c + a*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^{3}{\left(ax \right)}}{ax^{2}+x}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

3.131 $\int \frac{\tanh^{-1}(ax)^3}{cx+acx^2}\,dx$

Optimal. Leaf size=93

$$-\frac{3\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}-\frac{3\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}-\frac{3\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{2c}+\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}$$

```
[Out] arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3/2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))
/c-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*polylog(4,-1+2/(a*x+1))/c
```

Rubi [A] time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1593, 5932, 5948, 6056, 6060, 6610}

$$-\frac{3\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{\log\left(2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c)
- (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
```

```
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{cx+acx^2}\,dx = \int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx$$
$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{(3a)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{1-a^2x^2}\,d}{c}$$
$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(-1+\frac{2}{1+ax}\right)}{2c}$$
$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(-1+\frac{2}{1+ax}\right)}{2c}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.92

$$\frac{96\tanh^{-1}(ax)^2\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) + 48\text{Li}_4\left(e^{2\tanh^{-1}(ax)}\right) - 32\tanh^{-1}(ax)^4 + 64\tan}{64c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]
```

```
[Out] (Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] +
 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog
[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(ax)^3}{acx^2+cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}(ax)^3}{acx^2+cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

maple [C] time = 0.20, size = 1217, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(a*c*x^2+c*x),x)
```

```
[Out] 1/c*ln(2)*arctanh(a*x)^3-1/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+2/
c*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/2/c*arctanh(a*x)^4+6/c*po
lylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)^2*
polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(3,-(a*x+1)/
(-a^2*x^2+1)^(1/2))+1/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c
*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*poly
log(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)
-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctan
h(a*x)^3+1/2*I/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I/(1+
(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arc
tanh(a*x)^3+I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))^2*arctanh(a*x)^3+1/c*arctanh(a*x)^3*ln(a*x)-1/c*arctanh(a*x)^3*ln(
a*x+1)+1/2*I/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^
3*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+
1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*
arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x
+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x
^2+1)))*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2
+1)))*arctanh(a*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(ax+1)\log(-ax+1)^3}{8c} - \frac{1}{8}\int -\frac{(ax-1)\log(ax+1)^3 - 3(ax-1)\log(ax+1)^2\log(-ax+1) - 3\left(a^2x^2+1\right.}{a^2cx^3 - cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] 1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1)
^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x + 1
)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{a\,c\,x^2 + c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(c*x + a*c*x^2),x)
```

```
[Out] int(atanh(a*x)^3/(c*x + a*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^3(ax)}{ax^2+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(a*c*x**2+c*x),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

3.132 $\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx$

Optimal. Leaf size=191

$$-\frac{3a\text{Li}_3\left(\frac{2}{ax+1}-1\right)}{2c}+\frac{3a\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}+\frac{3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}-\frac{3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{c}+\frac{3a\text{Li}_3\left(\frac{2}{ax+1}\right.}{}$$

```
[Out] a*arctanh(a*x)^3/c-arctanh(a*x)^3/c/x+3*a*arctanh(a*x)^2*ln(2-2/(a*x+1))/c-
a*arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))
/c+3/2*a*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3/2*a*polylog(3,-1+2/(a*x
+1))/c+3/2*a*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3/4*a*polylog(4,-1+2/(a
*x+1))/c
```

Rubi [A] time = 0.46, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5934, 5916, 5988, 5932, 5948, 6056, 6610, 6060}

$$-\frac{3a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{3a\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}+\frac{3a\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3a\tanh^{-1}(ax)\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```

```
[Out] (a*ArcTanh[a*x]^3)/c - ArcTanh[a*x]^3/(c*x) + (3*a*ArcTanh[a*x]^2*Log[2 - 2
/(1 + a*x)])/c - (a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a*ArcTanh[a
*x]*PolyLog[2, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]^2*PolyLog[2, -1 + 2
/(1 + a*x)])/(2*c) - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*ArcTan
h[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*PolyLog[4, -1 + 2/(1 + a*
x)])/(4*c)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx = -\left(a\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx\right) + \frac{\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{cx} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)}\,dx}{c} + \frac{\left(3a^2\right)\int \frac{\tanh^{-1}(ax)^2\log}{1-a^2x}}{c}$$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{3a\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{}{1+}\right.}{2c}$$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+a}\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+a}\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+a}\right.}{c}$$

Mathematica [C] time = 0.29, size = 154, normalized size = 0.81

$$a\left(-\frac{3}{2}\left(\tanh^{-1}(ax)-2\right)\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\left(\tanh^{-1}(ax)-1\right)\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)-\frac{3}{4}\mathrm{Li}_4\left(e^{2\tanh^{-1}(ax)}\right)+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```

```
[Out] (a*((I/8)*Pi^3 - Pi^4/64 - ArcTanh[a*x]^3 - ArcTanh[a*x]^3/(a*x) + ArcTanh[
a*x]^4/2 + 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^3*Lo
g[1 - E^(2*ArcTanh[a*x])] - (3*(-2 + ArcTanh[a*x])*ArcTanh[a*x]*PolyLog[2,
E^(2*ArcTanh[a*x])])/2 + (3*(-1 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh[a*x
])])/2 - (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4))/c
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)^3}{acx^3 + cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^3 + c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{(acx + c)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x^2), x)
```

maple [C] time = 0.78, size = 1451, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(a*c*x+c),x)
```

```
[Out] -I*a/c*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))^2-1/2*I*a/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3-1/2*I*a/c*P
i*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csg
n(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^3+1
/2*I*a/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^3+1
/2*I*a/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+
1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3+1/2*I*a/c*Pi*csgn(I*((a*
x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))^2*arctanh(a*x)^3+1/2*I*a/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3-1/2*I
*a/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*
arctanh(a*x)^3-arctanh(a*x)^3/c/x-6*a/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2
))-6*a/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a/c*polylog(4,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-6*a/c*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a*arctanh
(a*x)^4/c-a*arctanh(a*x)^3/c-1/2*I*a/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)
/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/2*I*a/c*Pi*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/2*I*a/c*arctanh
(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-3*a/c*arctanh(a*x)^2*polylog(2,(
a*x+1)/(-a^2*x^2+1)^(1/2))+6*a/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1
)^(1/2))-2*a/c*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+a/c*arctanh(a*
```

```
x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)-a/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2
+1)^(1/2))-3*a/c*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a/
c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a/c*arctanh(a*x)^3*ln
(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-a/c*ln(2)*arctanh(a*x)^3+3*a/c*arctanh(a*x)^
2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a/c*arctanh(a*x)*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+3*a/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a/
c*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a/c*arctanh(a*x)^3*ln
(a*x)+a/c*arctanh(a*x)^3*ln(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(ax\log\left(ax+1\right)-1\right)\log\left(-ax+1\right)^{3}}{8\,cx}+\frac{1}{8}\int\frac{\left(ax-1\right)\log\left(ax+1\right)^{3}-3\left(ax-1\right)\log\left(ax+1\right)^{2}\log\left(-ax+1\right)-3\left(a^{2}\right.}{a^{2}cx^{4}-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(a*x*log(a*x + 1) - 1)*log(-a*x + 1)^3/(c*x) + 1/8*integrate(((a*x - 1
)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 +
a*x - (a^3*x^3 + a^2*x^2 + a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^2*c*x
^4 - c*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{atanh}\left(a\,x\right)^{3}}{x^{2}\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(c + a*c*x)),x)
```

```
[Out] int(atanh(a*x)^3/(x^2*(c + a*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\operatorname{atanh}^{3}\left(ax\right)}{ax^{3}+x^{2}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**3 + x**2), x)/c
```

3.133 $\int \frac{\tanh^{-1}(ax)^3}{x^3(c+acx)}\,dx$

Optimal. Leaf size=305

$$-\frac{3a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)}{2c}+\frac{3a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)}{2c}-\frac{3a^2\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}-\frac{3a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}+\frac{3a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)}{c}$$

```
[Out] 3/2*a^2*arctanh(a*x)^2/c-3/2*a*arctanh(a*x)^2/c/x-1/2*a^2*arctanh(a*x)^3/c-
1/2*arctanh(a*x)^3/c/x^2+a*arctanh(a*x)^3/c/x+3*a^2*arctanh(a*x)*ln(2-2/(a*
x+1))/c-3*a^2*arctanh(a*x)^2*ln(2-2/(a*x+1))/c+a^2*arctanh(a*x)^3*ln(2-2/(a
*x+1))/c-3/2*a^2*polylog(2,-1+2/(a*x+1))/c+3*a^2*arctanh(a*x)*polylog(2,-1+
2/(a*x+1))/c-3/2*a^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c+3/2*a^2*polyl
og(3,-1+2/(a*x+1))/c-3/2*a^2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*a^2
*polylog(4,-1+2/(a*x+1))/c
```

Rubi [A] time = 0.75, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5934, 5916, 5982, 5988, 5932, 2447, 5948, 6056, 6610, 6060}

$$-\frac{3a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}+\frac{3a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}-\frac{3a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3a^2\tanh^{-1}(ax)^2\text{PolyLog}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```

```
[Out] (3*a^2*ArcTanh[a*x]^2)/(2*c) - (3*a*ArcTanh[a*x]^2)/(2*c*x) - (a^2*ArcTanh[
a*x]^3)/(2*c) - ArcTanh[a*x]^3/(2*c*x^2) + (a*ArcTanh[a*x]^3)/(c*x) + (3*a^
2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - (3*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1
 + a*x)])/c + (a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a^2*PolyLog[
2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*
x)])/c - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2
*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 +
2/(1 + a*x)])/(2*c) - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3(c+acx)}\,dx = -\left(a\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx\right) + \frac{\int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{2cx^2} + a^2\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx - \frac{a\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx}{2c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} + \frac{a^2\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx}{2c} -$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} + \frac{a^2\tanh^{-1}(ax)^3\log}{c}$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c} - \frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} -$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c} - \frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} +$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c} - \frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} +$$

Mathematica [C] time = 0.64, size = 222, normalized size = 0.73

$$a^2\left(-\frac{32\tanh^{-1}(ax)^3}{a^2x^2} + 96\left(\tanh^{-1}(ax)-2\right)\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) - 96\mathrm{Li}_2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```

```
[Out] (a^2*((-8*I)*Pi^3 + Pi^4 + 96*ArcTanh[a*x]^2 - (96*ArcTanh[a*x]^2)/(a*x) +
96*ArcTanh[a*x]^3 - (32*ArcTanh[a*x]^3)/(a^2*x^2) + (64*ArcTanh[a*x]^3)/(a*
x) - 32*ArcTanh[a*x]^4 + 192*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] - 19
2*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1 - E^
(2*ArcTanh[a*x])] - 96*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*(-2 + ArcTanh[a
*x])*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 96*PolyLog[3, E^(2*ArcTa
nh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4,
E^(2*ArcTanh[a*x])]))/(64*c)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)^3}{acx^4+cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^4 + c*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="giac")

[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x^3), x)

maple [B] time = 4.37, size = 664, normalized size = 2.18

$$\frac{a \operatorname{arctanh}(ax)^3}{cx} - \frac{3a \operatorname{arctanh}(ax)^2}{2cx} - \frac{\operatorname{arctanh}(ax)^3}{2c\,x^2} - \frac{a^2 \operatorname{arctanh}(ax)^4}{2c} + \frac{6a^2 \operatorname{polylog}\left(4, \frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c} + \frac{6a^2 \operatorname{polylog}\left(\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)^3/x^3/(a*c*x+c),x)

[Out] a*arctanh(a*x)^3/c/x-3/2*a*arctanh(a*x)^2/c/x-1/2*arctanh(a*x)^3/c/x^2-1/2*a^2*arctanh(a*x)^4/c+6*a^2/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2/c+3*a^2/c*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^3/c+6*a^2/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2/c*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2/c*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2/c*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2\,a^2x^2\log(ax+1) - 2\,ax + 1\right)\log(-ax+1)^3}{16\,cx^2} - \frac{1}{8}\int -\frac{2\,(ax-1)\log(ax+1)^3 - 6\,(ax-1)\log(ax+1)^2\log(-ax}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="maxima")

[Out] 1/16*(2*a^2*x^2*log(a*x + 1) - 2*a*x + 1)*log(-a*x + 1)^3/(c*x^2) - 1/8*integrate(-1/2*(2*(a*x - 1)*log(a*x + 1)^3 - 6*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) + 3*(2*a^3*x^3 + a^2*x^2 - a*x - 2*(a^4*x^4 + a^3*x^3 - a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^2*c*x^5 - c*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a*x)^3/(x^3*(c + a*c*x)),x)

[Out] int(atanh(a*x)^3/(x^3*(c + a*c*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^3(ax)}{ax^4+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)**3/x**3/(a*c*x+c),x)

[Out] Integral(atanh(a*x)**3/(a*x**4 + x**3), x)/c

3.134 $\int \frac{x^2 \tanh^{-1}(ax)^4}{c-acx} dx$

Optimal. Leaf size=384

$$-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{a^3c}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{a^3c}-\frac{3\text{Li}_5\left(1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^3c}+\frac{6\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3c}$$

```
[Out] -2*arctanh(a*x)^3/a^3/c-2*x*arctanh(a*x)^3/a^2/c-1/2*arctanh(a*x)^4/a^3/c-x
*arctanh(a*x)^4/a^2/c-1/2*x^2*arctanh(a*x)^4/a/c+6*arctanh(a*x)^2*ln(2/(-a*
x+1))/a^3/c+4*arctanh(a*x)^3*ln(2/(-a*x+1))/a^3/c+arctanh(a*x)^4*ln(2/(-a*x
+1))/a^3/c+6*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^3/c+6*arctanh(a*x)^2*po
lylog(2,1-2/(-a*x+1))/a^3/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/a^3/c-
3*polylog(3,1-2/(-a*x+1))/a^3/c-6*arctanh(a*x)*polylog(3,1-2/(-a*x+1))/a^3/
c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a^3/c+3*polylog(4,1-2/(-a*x+1))/
a^3/c+3*arctanh(a*x)*polylog(4,1-2/(-a*x+1))/a^3/c-3/2*polylog(5,1-2/(-a*x+
1))/a^3/c
```

Rubi [A] time = 0.86, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {5930, 5916, 5980, 5910, 5984, 5918, 5948, 6058, 6610, 6062}

$$-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^3c}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^3c}-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] (-2*ArcTanh[a*x]^3)/(a^3*c) - (2*x*ArcTanh[a*x]^3)/(a^2*c) - ArcTanh[a*x]^4
/(2*a^3*c) - (x*ArcTanh[a*x]^4)/(a^2*c) - (x^2*ArcTanh[a*x]^4)/(2*a*c) + (6
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(a^3*c) + (4*ArcTanh[a*x]^3*Log[2/(1 - a*
x)])/(a^3*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]*
PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(a^3*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c)
 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(a^3*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 -
 2/(1 - a*x)])/(a^3*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^
3*c) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]*PolyLog[4,
 1 - 2/(1 - a*x)])/(a^3*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^3*c)
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^4}{c-acx}\,dx = \frac{\int \frac{x \tanh^{-1}(ax)^4}{c-acx}\,dx}{a} - \frac{\int x \tanh^{-1}(ax)^4\,dx}{ac}$$
$$= -\frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx}{a^2} + \frac{2\int \frac{x^2 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{c} - \frac{\int \tanh^{-1}(ax)^4\,dx}{a^2c}$$
$$= -\frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\tanh^{-1}(ax)^4 \log\left(\frac{2}{1-ax}\right)}{a^3c} - \frac{2\int \tanh^{-1}(ax)^3\,dx}{a^2c} + \cdots$$
$$= -\frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\tanh^{-1}(ax)^4 \log\cdots}{a^3c}$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} - \cdots$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} - \cdots$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} - \cdots$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} - \cdots$$

Mathematica [A] time = 0.41, size = 233, normalized size = 0.61

$$-\frac{-\frac{1}{2}\left(1-a^2x^2\right)\tanh^{-1}(ax)^4 + 2\left(\tanh^{-1}(ax)^2 + 3\tanh^{-1}(ax) + 3\right)\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -((-2*ArcTanh[a*x]^3 + 2*a*x*ArcTanh[a*x]^3 - ArcTanh[a*x]^4 + a*x*ArcTanh[
a*x]^4 - ((1 - a^2*x^2)*ArcTanh[a*x]^4)/2 - (2*ArcTanh[a*x]^5)/5 - 6*ArcTan
h[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcT
anh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2*ArcTanh[a*x]*(
3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 3*(
1 + ArcTanh[a*x])^2*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*
ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x])] + (3*PolyL
og[5, -E^(-2*ArcTanh[a*x])])/2)/(a^3*c))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{x^2 \operatorname{artanh}(ax)^4}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 \operatorname{artanh}(ax)^4}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

maple [A] time = 1.98, size = 496, normalized size = 1.29

$$-\frac{x^2 \operatorname{arctanh}(ax)^4}{2ac} - \frac{x \operatorname{arctanh}(ax)^4}{a^2c} - \frac{2x \operatorname{arctanh}(ax)^3}{a^2c} - \frac{\operatorname{arctanh}(ax)^4}{2a^3c} - \frac{2 \operatorname{arctanh}(ax)^3}{a^3c} + \frac{\operatorname{arctanh}(ax)^4 \ln\left(1 + \cdots\right.}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^4/(-a*c*x+c),x)
```

```
[Out] -1/2*x^2*arctanh(a*x)^4/a/c-x*arctanh(a*x)^4/a^2/c-2*x*arctanh(a*x)^3/a^2/c
-1/2*arctanh(a*x)^4/a^3/c-2*arctanh(a*x)^3/a^3/c+1/a^3/c*arctanh(a*x)^4*ln(
1+(a*x+1)^2/(-a^2*x^2+1))+2/a^3/c*arctanh(a*x)^3*polylog(2,-(a*x+1)^2/(-a^2
*x^2+1))-3/a^3/c*arctanh(a*x)^2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/a^3/c*
arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/2/a^3/c*polylog(5,-(a*x+1
)^2/(-a^2*x^2+1))+6/a^3/c*arctanh(a*x)^2*ln(1+(a*x+1)^2/(-a^2*x^2+1))+6/a^3
/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/a^3/c*polylog(3,-(a*x+
1)^2/(-a^2*x^2+1))+4/a^3/c*arctanh(a*x)^3*ln(1+(a*x+1)^2/(-a^2*x^2+1))+6/a^
3/c*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-6/a^3/c*arctanh(a*x)*
polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/a^3/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4 \log(-ax+1)^5 + 5\left(2 \log(-ax+1)^4 - 4 \log(-ax+1)^3 + 6 \log(-ax+1)^2 - 6 \log(-ax+1) + 3\right)(ax-1)^2 + \cdots}{320\, a^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/320*(4*log(-a*x + 1)^5 + 5*(2*log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 6*lo
g(-a*x + 1)^2 - 6*log(-a*x + 1) + 3)*(a*x - 1)^2 + 40*(log(-a*x + 1)^4 - 4*
log(-a*x + 1)^3 + 12*log(-a*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))/(a
^3*c) + 1/16*integrate(-(x^2*log(a*x + 1)^4 - 4*x^2*log(a*x + 1)^3*log(-a*x
 + 1) + 6*x^2*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*x^2*log(a*x + 1)*log(-a*x
+ 1)^3)/(a*c*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \operatorname{atanh}(a\,x)^4}{c - a\,c\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^4)/(c - a*c*x),x)
```

```
[Out] int((x^2*atanh(a*x)^4)/(c - a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2 \operatorname{atanh}^4(ax)}{ax-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(x**2*atanh(a*x)**4/(a*x - 1), x)/c
```

3.135 $\int \frac{x \tanh^{-1}(ax)^4}{c-acx} dx$

Optimal. Leaf size=261

$$\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{a^2c}-\frac{3\text{Li}_5\left(1-\frac{2}{1-ax}\right)}{2a^2c}+\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^2c}+\frac{6\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2c}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{a^2c}$$

```
[Out] -arctanh(a*x)^4/a^2/c-x*arctanh(a*x)^4/a/c+4*arctanh(a*x)^3*ln(2/(-a*x+1))/
a^2/c+arctanh(a*x)^4*ln(2/(-a*x+1))/a^2/c+6*arctanh(a*x)^2*polylog(2,1-2/(-
a*x+1))/a^2/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/a^2/c-6*arctanh(a*x)
*polylog(3,1-2/(-a*x+1))/a^2/c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a^2
/c+3*polylog(4,1-2/(-a*x+1))/a^2/c+3*arctanh(a*x)*polylog(4,1-2/(-a*x+1))/a
^2/c-3/2*polylog(5,1-2/(-a*x+1))/a^2/c
```

Rubi [A] time = 0.50, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {5930, 5910, 5984, 5918, 5948, 6058, 6062, 6610}

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^2c}-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2c}+\frac{6\tanh^{-1}(ax)^2\text{PolyLo}}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -(ArcTanh[a*x]^4/(a^2*c)) - (x*ArcTanh[a*x]^4)/(a*c) + (4*ArcTanh[a*x]^3*Lo
g[2/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^2*c) + (6*Ar
cTanh[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) + (2*ArcTanh[a*x]^3*PolyL
og[2, 1 - 2/(1 - a*x)])/(a^2*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x
)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^2*c) + (3*P
olyLog[4, 1 - 2/(1 - a*x)])/(a^2*c) + (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 -
 a*x)])/(a^2*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^2*c)
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^4}{c-acx}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx}{a} - \frac{\int \tanh^{-1}(ax)^4\,dx}{ac}$$

$$= -\frac{x\tanh^{-1}(ax)^4}{ac} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{4\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{c} - \frac{4\int \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{ac}$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2c} -$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + 2$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + 6$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + 6$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + 6$$

Mathematica [A] time = 0.27, size = 172, normalized size = 0.66

$$-\frac{2\left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)+3\left(\tanh^{-1}(ax)+2\right)\tanh^{-1}(ax)\text{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right)+3\text{t}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -((-ArcTanh[a*x]^4 + a*x*ArcTanh[a*x]^4 - (2*ArcTanh[a*x]^5)/5 - 4*ArcTanh[
a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[
a*x])] + 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, -E^(-2*ArcTanh[a*x]
)] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3
*PolyLog[4, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTan
h[a*x])] + (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a^2*c))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x\,\text{artanh}\left(ax\right)^4}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x\,\text{artanh}\left(ax\right)^4}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

maple [C] time = 0.42, size = 454, normalized size = 1.74

$$-\frac{x\arctanh\left(ax\right)^4}{ac}-\frac{\arctanh\left(ax\right)^4\ln\left(ax-1\right)}{a^2c}+\frac{2\arctanh\left(ax\right)^3\text{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{a^2c}-\frac{3\arctanh\left(ax\right)^2\text{polylog}}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^4/(-a*c*x+c),x)
```

```
[Out] -x*arctanh(a*x)^4/a/c-1/a^2/c*arctanh(a*x)^4*ln(a*x-1)+2/a^2/c*arctanh(a*x)
^3*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/a^2/c*arctanh(a*x)^2*polylog(3,-(a*
x+1)^2/(-a^2*x^2+1))+3/a^2/c*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1)
)-3/2/a^2/c*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))+I/a^2/c*Pi*csgn(I/(1+(a*x+1)
^2/(-a^2*x^2+1)))^3*arctanh(a*x)^4-I/a^2/c*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2
+1)))^2*arctanh(a*x)^4-arctanh(a*x)^4/a^2/c+4/a^2/c*arctanh(a*x)^3*ln(1+(a*
x+1)^2/(-a^2*x^2+1))+6/a^2/c*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+
1))-6/a^2/c*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/a^2/c*polylog
(4,-(a*x+1)^2/(-a^2*x^2+1))+1/a^2/c*ln(2)*arctanh(a*x)^4+I/a^2/c*Pi*arctanh
(a*x)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(-ax+1\right)^5+5\left(\log\left(-ax+1\right)^4-4\log\left(-ax+1\right)^3+12\log\left(-ax+1\right)^2-24\log\left(-ax+1\right)+24\right)\left(ax-1\right)}{80\,a^2c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*(log(-a*x + 1)^5 + 5*(log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 12*log(-a
*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))/(a^2*c) + 1/16*integrate(-(x*
log(a*x + 1)^4 - 4*x*log(a*x + 1)^3*log(-a*x + 1) + 6*x*log(a*x + 1)^2*log(
-a*x + 1)^2 - 4*x*log(a*x + 1)*log(-a*x + 1)^3)/(a*c*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\mathrm{atanh}\left(a\,x\right)^4}{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^4)/(c - a*c*x),x)
```

```
[Out] int((x*atanh(a*x)^4)/(c - a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x\,\mathrm{atanh}^4\,(ax)}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(x*atanh(a*x)**4/(a*x - 1), x)/c
```

3.136 $\int \frac{\tanh^{-1}(ax)^4}{c-acx} dx$

Optimal. Leaf size=131

$$-\frac{3\text{Li}_5\left(1-\frac{2}{1-ax}\right)}{2ac}+\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{ac}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{ac}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{ac}+\text{lo}$$

```
[Out] arctanh(a*x)^4*ln(2/(-a*x+1))/a/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/
a/c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a/c+3*arctanh(a*x)*polylog(4,1
-2/(-a*x+1))/a/c-3/2*polylog(5,1-2/(-a*x+1))/a/c
```

Rubi [A] time = 0.21, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5918, 5948, 6058, 6062, 6610}

$$-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2ac}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{ac}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{ac}+\frac{3\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(c - a*c*x),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 -
2/(1 - a*x)])/(a*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a*c)
+ (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 - a*x)])/(a*c) - (3*PolyLog[5, 1 - 2/
(1 - a*x)])/(2*a*c)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx = \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} - \frac{4\int \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{6\int \frac{\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} + \frac{6\int}{}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} + \frac{3\,\text{ta}}{}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} + \frac{3\,\text{ta}}{}$$

Mathematica [A] time = 0.11, size = 112, normalized size = 0.85

$$-\frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)\text{Li}_4\left(-e^{-2\tanh^{-1}(ax)}\right) + \frac{3}{2}\text{Li}_5}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(c - a*c*x),x]
```

```
[Out] -(((-2*ArcTanh[a*x]^5)/5 - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2*
ArcTanh[a*x]^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]^2*PolyLog[
3, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x])]
+ (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a*c))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)^4}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{artanh}\,(ax)^4}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/(a*c*x - c), x)
```

maple [C] time = 0.36, size = 285, normalized size = 2.18

$$-\frac{\operatorname{arctanh}(ax)^4 \ln(ax-1)}{ac}+\frac{i \operatorname{arctanh}(ax)^4 \pi \operatorname{csgn}\left(\frac{i}{1+\frac{(ax+1)^2}{-a^2x^2+1}}\right)^3}{ac}-\frac{i \operatorname{arctanh}(ax)^4 \pi \operatorname{csgn}\left(\frac{i}{1+\frac{(ax+1)^2}{-a^2x^2+1}}\right)^2}{ac}+\frac{i \operatorname{arctanh}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/(-a*c*x+c),x)
```

```
[Out] -1/a/c*arctanh(a*x)^4*ln(a*x-1)+I/a/c*arctanh(a*x)^4*Pi*csgn(I/(1+(a*x+1)^2
/(-a^2*x^2+1)))^3-I/a/c*arctanh(a*x)^4*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1))
)^2+I/a/c*arctanh(a*x)^4*Pi+1/a/c*arctanh(a*x)^4*ln(2)+2/a/c*arctanh(a*x)^3
*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/a/c*arctanh(a*x)^2*polylog(3,-(a*x+1)
^2/(-a^2*x^2+1))+3/a/c*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/2/
a/c*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log(-ax+1)^5}{80\,ac}+\frac{1}{16}\int -\frac{\log(ax+1)^4-4\,\log(ax+1)^3\log(-ax+1)+6\,\log(ax+1)^2\log(-ax+1)^2-4\,\log}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/(a*c) + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x +
 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log
(-a*x + 1)^3)/(a*c*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^4}{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(c - a*c*x),x)
```

```
[Out] int(atanh(a*x)^4/(c - a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x - 1), x)/c
```

3.137 $\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx$

Optimal. Leaf size=118

$$-\frac{3\text{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{3\text{Li}_3\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^2}{c}+\frac{3\text{Li}_4\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\frac{\log\left(2\ldots\right.}{}$$

```
[Out] arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1))
/c-3*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c+3*arctanh(a*x)*polylog(4,-1+
2/(-a*x+1))/c-3/2*polylog(5,-1+2/(-a*x+1))/c
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5932, 5948, 6058, 6062, 6610}

$$-\frac{3\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{c}+\frac{3\tanh^{-1}(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx = \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(4a)\int \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{(6a)\int \frac{\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{1-a^2x^2}}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_3\left(-1+\frac{}{1}\right.}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_3\left(-1+\frac{}{1}\right.}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_3\left(-1+\frac{}{1}\right.}{c}$$

Mathematica [A] time = 0.17, size = 102, normalized size = 0.86

$$\frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{3\tanh^{-1}(ax)\mathrm{Li}_4\left(e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\mathrm{Li}_5\left(e^{2\tanh^{-1}}\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog[
2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x])
])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5, E^
(2*ArcTanh[a*x])])/(2*c)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\mathrm{artanh}\,(ax)^4}{acx^2-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\mathrm{artanh}\,(ax)^4}{(acx-c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x), x)
```

maple [C] time = 0.46, size = 843, normalized size = 7.14

$$\frac{\operatorname{arctanh}(ax)^4\ln(ax)}{c}-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}-\frac{\operatorname{arctanh}(ax)^4\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{c}+\frac{\operatorname{arctanh}(ax)^4\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x/(-a*c*x+c),x)
```

```
[Out] 1/c*arctanh(a*x)^4*ln(a*x)-1/c*arctanh(a*x)^4*ln(a*x-1)-1/c*arctanh(a*x)^4*
ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))+4/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh
(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,(
a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*a
rctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*
x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c
*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/c*arctanh(a*x)^4*csgn(I/(1+(a
*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2
/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*Pi-1/2*I/c*arctanh(a*x)^4*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)
^2/(-a^2*x^2+1)))^2*Pi-1/2*I/c*arctanh(a*x)^4*csgn(I/(1+(a*x+1)^2/(-a^2*x^2
+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I/
c*arctanh(a*x)^4*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi+1/2*I/c*arctanh(a*
x)^4*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-I/c
*arctanh(a*x)^4*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I/c*arctanh(a*x)^4*
Pi+1/c*arctanh(a*x)^4*ln(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log(-ax+1)^5}{80\,c}+\frac{1}{16}\int-\frac{\log(ax+1)^4-4\,\log(ax+1)^3\log(-ax+1)+6\,\log(ax+1)^2\log(-ax+1)^2-4\,\log(}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^
3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-a*
x + 1)^3)/(a*c*x^2 - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{atanh}(a\,x)^4}{x\,(c-a\,c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(x*(c - a*c*x)),x)
```

```
[Out] int(atanh(a*x)^4/(x*(c - a*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\operatorname{atanh}^4(ax)}{ax^2-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

3.138 $\int \frac{\tanh^{-1}(ax)^4}{cx-acx^2} dx$

Optimal. Leaf size=118

$$-\frac{3\text{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{3\text{Li}_3\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^2}{c}+\frac{3\text{Li}_4\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\text{lo}$$

```
[Out] arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1))
/c-3*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c+3*arctanh(a*x)*polylog(4,-1+
2/(-a*x+1))/c-3/2*polylog(5,-1+2/(-a*x+1))/c
```

Rubi [A] time = 0.23, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1593, 5932, 5948, 6058, 6062, 6610}

$$-\frac{3\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{c}+\frac{3\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
```

```
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{cx-acx^2}\,dx = \int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx$$
$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(4a)\int \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{(6a)\int \frac{\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{1-a^2x^2}\,d}{c}$$
$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c}$$
$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c}$$
$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c}$$

Mathematica [A] time = 0.08, size = 102, normalized size = 0.86

$$\frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{3\tanh^{-1}(ax)\text{Li}_4\left(e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\text{Li}_5\left(e^{2\tanh^{-1}(ax)}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog[
2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x])
])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5, E^
(2*ArcTanh[a*x])])/(2*c)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)^4}{acx^2-cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{artanh}\,(ax)^4}{acx^2-cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

maple [C] time = 0.33, size = 843, normalized size = 7.14

$$\frac{\operatorname{arctanh}(ax)^4 \ln(ax)}{c} - \frac{\operatorname{arctanh}(ax)^4 \ln(ax-1)}{c} - \frac{\operatorname{arctanh}(ax)^4 \ln\left(\frac{(ax+1)^2}{-a^2x^2+1} - 1\right)}{c} + \frac{\operatorname{arctanh}(ax)^4 \ln\left(1 - \frac{ax+1}{\sqrt{-a^2x^2}}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/(-a*c*x^2+c*x),x)
```

```
[Out] 1/c*arctanh(a*x)^4*ln(a*x)-1/c*arctanh(a*x)^4*ln(a*x-1)-1/c*arctanh(a*x)^4*
ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))+4/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh
(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,(
a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*a
rctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*
x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c
*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/c*arctanh(a*x)^4*csgn(I/(1+(a
*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2
/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*Pi-1/2*I/c*arctanh(a*x)^4*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)
^2/(-a^2*x^2+1)))^2*Pi-1/2*I/c*arctanh(a*x)^4*csgn(I/(1+(a*x+1)^2/(-a^2*x^2
+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I/
c*arctanh(a*x)^4*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi+1/2*I/c*arctanh(a*
x)^4*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-I/c
*arctanh(a*x)^4*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I/c*arctanh(a*x)^4*
Pi+1/c*arctanh(a*x)^4*ln(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log(-ax+1)^5}{80\,c} + \frac{1}{16}\int -\frac{\log(ax+1)^4 - 4\,\log(ax+1)^3\log(-ax+1) + 6\,\log(ax+1)^2\log(-ax+1)^2 - 4\,\mathrm{lo}}{acx^2 - cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^
3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-a*
x + 1)^3)/(a*c*x^2 - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^4}{c\,x - a\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(c*x - a*c*x^2),x)
```

```
[Out] int(atanh(a*x)^4/(c*x - a*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^2 - x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/(-a*c*x**2+c*x),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

3.139 $\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx$

Optimal. Leaf size=239

$$-\frac{3a\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{c}-\frac{3a\text{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2a\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{6a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{c}-\frac{3a\text{Li}_3\left(\frac{2}{1-}\right.}{}$$

```
[Out] a*arctanh(a*x)^4/c-arctanh(a*x)^4/c/x+a*arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+4
*a*arctanh(a*x)^3*ln(2-2/(a*x+1))/c+2*a*arctanh(a*x)^3*polylog(2,-1+2/(-a*x
+1))/c-6*a*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3*a*arctanh(a*x)^2*poly
log(3,-1+2/(-a*x+1))/c-6*a*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3*a*arcta
nh(a*x)*polylog(4,-1+2/(-a*x+1))/c-3*a*polylog(4,-1+2/(a*x+1))/c-3/2*a*poly
log(5,-1+2/(-a*x+1))/c
```

Rubi [A] time = 0.55, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5934, 5916, 5988, 5932, 5948, 6056, 6060, 6610, 6058, 6062}

$$-\frac{3a\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{c}-\frac{3a\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2a\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{6a\tanh^{-1}(ax)^2\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

```
[Out] (a*ArcTanh[a*x]^4)/c - ArcTanh[a*x]^4/(c*x) + (a*ArcTanh[a*x]^4*Log[2 - 2/(
1 - a*x)])/c + (4*a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a*ArcTanh[a
*x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/c - (3*a*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (6
*a*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]*PolyLog
[4, -1 + 2/(1 - a*x)])/c - (3*a*PolyLog[4, -1 + 2/(1 + a*x)])/c - (3*a*Poly
Log[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx = a\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx + \frac{\int \frac{\tanh^{-1}(ax)^4}{x^2}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{(4a)\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}\,dx}{c} - \frac{\left(4a^2\right)\int \frac{\tanh^{-1}(ax)^3}{1-}}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2a\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\right.}{c}$$

Mathematica [C] time = 0.49, size = 172, normalized size = 0.72

$$-\frac{a\left(-2\left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)^2\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)+3\left(\tanh^{-1}(ax)+2\right)\tanh^{-1}(ax)\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)-3\,\text{ta}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

```
[Out] -((a*(-1/16*Pi^4 + (I/160)*Pi^5 + ArcTanh[a*x]^4 + ArcTanh[a*x]^4/(a*x) - 4
*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2*A
rcTanh[a*x])] - 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, E^(2*ArcTanh
[a*x])] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh[a*x])]
- 3*PolyLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh
[a*x])] + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)^4}{acx^3-cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^3 - c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{artanh}\,(ax)^4}{(acx-c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x^2), x)
```

maple [B] time = 1.03, size = 583, normalized size = 2.44

$$-\frac{a \operatorname{arctanh}(ax)^4}{c}-\frac{\operatorname{arctanh}(ax)^4}{cx}+\frac{a \operatorname{arctanh}(ax)^4 \ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}+\frac{4a \operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2, \frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}-\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x^2/(-a*c*x+c),x)
```

```
[Out] -a*arctanh(a*x)^4/c-arctanh(a*x)^4/c/x+a/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^
2*x^2+1)^(1/2))+4*a/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-
12*a/c*arctanh(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a/c*arctanh(
a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-24*a/c*polylog(5,(a*x+1)/(-a^2*x
^2+1)^(1/2))+a/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a/c*arct
anh(a*x)^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a/c*arctanh(a*x)^2*pol
ylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a/c*arctanh(a*x)*polylog(4,-(a*x+1)/
(-a^2*x^2+1)^(1/2))-24*a/c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a/c*arc
tanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a/c*arctanh(a*x)^2*polylog(
2,(a*x+1)/(-a^2*x^2+1)^(1/2))-24*a/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x
^2+1)^(1/2))+24*a/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a/c*arctanh(a*x
)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a/c*arctanh(a*x)^2*polylog(2,-(a*x+
1)/(-a^2*x^2+1)^(1/2))-24*a/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^
(1/2))+24*a/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ax \log(-ax+1)^5 + 5 \log(-ax+1)^4}{80\, cx}+\frac{1}{16}\int -\frac{\log(ax+1)^4 - 4 \log(ax+1)^3 \log(-ax+1) + 6 \log(ax+1)^2 \log}{acx^3 - cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*(a*x*log(-a*x + 1)^5 + 5*log(-a*x + 1)^4)/(c*x) + 1/16*integrate(-(lo
g(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x +
 1)^2 - 4*(a*x + log(a*x + 1))*log(-a*x + 1)^3)/(a*c*x^3 - c*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^4}{x^2\,\left(c-a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(x^2*(c - a*c*x)),x)
```

```
[Out] int(atanh(a*x)^4/(x^2*(c - a*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^3-x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x**2/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**3 - x**2), x)/c
```

3.140 $\int \frac{\tanh^{-1}(ax)^4}{x^3(c-acx)}\,dx$

Optimal. Leaf size=380

$$-\frac{3a^2\mathrm{Li}_3\left(\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\mathrm{Li}_4\left(\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\mathrm{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2a^2\mathrm{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{6a^2\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)}{c}$$

```
[Out] 2*a^2*arctanh(a*x)^3/c-2*a*arctanh(a*x)^3/c/x+3/2*a^2*arctanh(a*x)^4/c-1/2*
arctanh(a*x)^4/c/x^2-a*arctanh(a*x)^4/c/x+a^2*arctanh(a*x)^4*ln(2-2/(-a*x+1
))/c+6*a^2*arctanh(a*x)^2*ln(2-2/(a*x+1))/c+4*a^2*arctanh(a*x)^3*ln(2-2/(a*
x+1))/c+2*a^2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1))/c-6*a^2*arctanh(a*x)*
polylog(2,-1+2/(a*x+1))/c-6*a^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3*
a^2*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c-3*a^2*polylog(3,-1+2/(a*x+1))
/c-6*a^2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3*a^2*arctanh(a*x)*polylog(
4,-1+2/(-a*x+1))/c-3*a^2*polylog(4,-1+2/(a*x+1))/c-3/2*a^2*polylog(5,-1+2/(
-a*x+1))/c
```

Rubi [A] time = 0.96, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5934, 5916, 5982, 5988, 5932, 5948, 6056, 6610, 6060, 6058, 6062}

$$-\frac{3a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2a^2\tanh^{-1}(ax)^3\text{PolyLog}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

```
[Out] (2*a^2*ArcTanh[a*x]^3)/c - (2*a*ArcTanh[a*x]^3)/(c*x) + (3*a^2*ArcTanh[a*x]
^4)/(2*c) - ArcTanh[a*x]^4/(2*c*x^2) - (a*ArcTanh[a*x]^4)/(c*x) + (a^2*ArcT
anh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (6*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a
*x)])/c + (4*a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a^2*ArcTanh[a*
x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[2, -1 +
2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/c - (
3*a^2*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[3, -1
 + 2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c +
(3*a^2*ArcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[4, -1
+ 2/(1 + a*x)])/c - (3*a^2*PolyLog[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x^3(c-acx)}\,dx = a\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx + \frac{\int \frac{\tanh^{-1}(ax)^4}{x^3}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{2cx^2} + a^2\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx + \frac{a\int \frac{\tanh^{-1}(ax)^4}{x^2}\,dx}{c} + \frac{(2a)\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{(2a)\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c} +$$

$$= -\frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\tanh^{-1}(ax)^4\,\mathrm{lo}}{c}$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} +$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} +$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} +$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} +$$

Mathematica [C] time = 0.88, size = 250, normalized size = 0.66

$$-\frac{a^2\left(\frac{\tanh^{-1}(ax)^4}{2a^2x^2} - 2\left(\tanh^{-1}(ax)^2 + 3\tanh^{-1}(ax) + 3\right)\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) - 3\tanh^{-1}(ax)\mathrm{Li}_4\left(e^{2\tanh^{-1}(}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

```
[Out] -((a^2*((-1/4*I)*Pi^3 - Pi^4/16 + (I/160)*Pi^5 + 2*ArcTanh[a*x]^3 + (2*ArcT
anh[a*x]^3)/(a*x) + ArcTanh[a*x]^4/2 + ArcTanh[a*x]^4/(2*a^2*x^2) + ArcTanh
[a*x]^4/(a*x) - 6*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 4*ArcTanh[a*
x]^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x]
)] - 2*ArcTanh[a*x]*(3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, E^(2*A
rcTanh[a*x])] + 3*(1 + ArcTanh[a*x])^2*PolyLog[3, E^(2*ArcTanh[a*x])] - 3*P
olyLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x]
)] + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\mathrm{artanh}\,(ax)^4}{acx^4 - cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^4 - c*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x^3), x)
```

maple [B] time = 2.00, size = 858, normalized size = 2.26

$$\frac{24a^2\operatorname{arctanh}(ax)\operatorname{polylog}\left(4,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}-\frac{a^2\operatorname{arctanh}(ax)^4}{2c}-\frac{\operatorname{arctanh}(ax)^4}{2c\,x^2}-\frac{24a^2\operatorname{polylog}\left(5,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}-\frac{24a^2\,\mathrm{p}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x^3/(-a*c*x+c),x)
```

```
[Out] -1/2*a^2*arctanh(a*x)^4/c-1/2*arctanh(a*x)^4/c/x^2-2*a^2*arctanh(a*x)^3/c-2
*a*arctanh(a*x)^3/c/x-a*arctanh(a*x)^4/c/x-24*a^2/c*polylog(5,(a*x+1)/(-a^2
*x^2+1)^(1/2))-24*a^2/c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a^2/c*pol
ylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a^2/c*polylog(3,(a*x+1)/(-a^2*x^2+1)
^(1/2))+24*a^2/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a^2/c*polylog(4,-
(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^
(1/2))+4*a^2/c*arctanh(a*x)^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2/
c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a^2/c*arctanh(a*x)*pol
ylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+4*a^2/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
-12*a^2/c*arctanh(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a^2/c*arc
tanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2/c*arctanh(a*x)^3*ln(
1-(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a^2/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+6*a^2/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-12
*a^2/c*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a^2/c*arcta
nh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-24*a^2/c*arctanh(a*x)*polylog
(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^
2+1)^(1/2))+12*a^2/c*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
24*a^2/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a^2/c*arcta
nh(a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,a^2x^2\log(-ax+1)^5+5\,(2\,ax+1)\log(-ax+1)^4}{160\,cx^2}+\frac{1}{16}\int -\frac{\log(ax+1)^4-4\,\log(ax+1)^3\log(-ax+1)+6\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/160*(2*a^2*x^2*log(-a*x + 1)^5 + 5*(2*a*x + 1)*log(-a*x + 1)^4)/(c*x^2)
+ 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*log(
a*x + 1)^2*log(-a*x + 1)^2 - 2*(2*a^2*x^2 + a*x + 2*log(a*x + 1))*log(-a*x
+ 1)^3)/(a*c*x^4 - c*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atanh}(a\,x)^4}{x^3\,\left(c-a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(x^3*(c - a*c*x)),x)
[Out] int(atanh(a*x)^4/(x^3*(c - a*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^4-x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x**3/(-a*c*x+c),x)
[Out] -Integral(atanh(a*x)**4/(a*x**4 - x**3), x)/c
```

3.141 $\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{x}{(acx+c)\tanh^{-1}(ax)},x\right)$$

[Out] Unintegrable(x/(a*c*x+c)/arctanh(a*x),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Int[x/((c + a*c*x)*ArcTanh[a*x]),x]

[Out] Defer[Int][x/((c + a*c*x)*ArcTanh[a*x]), x]

Rubi steps

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx=\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.40, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x/((c + a*c*x)*ArcTanh[a*x]),x]

[Out] Integrate[x/((c + a*c*x)*ArcTanh[a*x]), x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{(acx+c)\,\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")

[Out] integral(x/((a*c*x + c)*arctanh(a*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(acx+c)\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")

[Out] integrate(x/((a*c*x + c)*arctanh(a*x)), x)

maple [A] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{x}{(acx+c)\,\text{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c*x+c)/arctanh(a*x),x)
```

```
[Out] int(x/(a*c*x+c)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(acx + c)\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\mathrm{atanh}\,(a\,x)\,\left(c + a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)*(c + a*c*x)),x)
```

```
[Out] int(x/(atanh(a*x)*(c + a*c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{ax\,\mathrm{atanh}\,(ax) + \mathrm{atanh}\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/atanh(a*x),x)
```

```
[Out] Integral(x/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

3.142 $\int \frac{1}{(c+acx)\tanh^{-1}(ax)} dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{1}{(acx+c)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(a*c*x+c)/arctanh(a*x),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a*c*x)*ArcTanh[a*x]),x]
[Out] Defer[Int][1/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)} dx = \int \frac{1}{(c+acx)\tanh^{-1}(ax)} dx$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a*c*x)*ArcTanh[a*x]),x]
[Out] Integrate[1/((c + a*c*x)*ArcTanh[a*x]), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(acx+c)\,\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(1/((a*c*x + c)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx+c)\,\text{artanh}\,(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

maple [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx+c)\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x+c)/arctanh(a*x),x)
```

```
[Out] int(1/(a*c*x+c)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx+c)\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\mathrm{atanh}\,(a\,x)\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(c + a*c*x)),x)
```

```
[Out] int(1/(atanh(a*x)*(c + a*c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax\,\mathrm{atanh}\,(ax)+\mathrm{atanh}\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/atanh(a*x),x)
```

```
[Out] Integral(1/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

3.143 $\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x(acx+c)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(a*c*x+c)/arctanh(a*x),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
[Out] Defer[Int][1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx = \int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
[Out] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(acx^2+cx\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(1/((a*c*x^2 + c*x)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx+c)x\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(acx+c\right)\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c*x+c)/arctanh(a*x),x)
[Out] int(1/x/(a*c*x+c)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx+c)x\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(c + a*c*x)),x)
[Out] int(1/(x*atanh(a*x)*(c + a*c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^2\operatorname{atanh}\left(ax\right)+x\operatorname{atanh}\left(ax\right)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/atanh(a*x),x)
[Out] Integral(1/(a*x**2*atanh(a*x) + x*atanh(a*x)), x)/c
```

3.144 $\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{x}{(acx+c)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Defer[Int][x/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx=\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x}{(acx+c)\operatorname{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(acx+c)\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

maple [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x}{(acx + c)\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(ax^2 - x\right)}{ac\log\left(ax+1\right) - ac\log\left(-ax+1\right)} + \int -\frac{2\,(2\,ax - 1)}{ac\log\left(ax+1\right) - ac\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x^2 - x)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + integrate(-2*(2*a*x
- 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^2\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(c + a*c*x)),x)
```

```
[Out] int(x/(atanh(a*x)^2*(c + a*c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{ax\operatorname{atanh}^{2}(ax)+\operatorname{atanh}^{2}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(x/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

3.145 $\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{1}{(acx+c)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Defer[Int][1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx = \int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(acx+c)\,\text{artanh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx + c)\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2(ax-1)}{ac\log(ax+1) - ac\log(-ax+1)} + 2\int -\frac{1}{c\log(ax+1) - c\log(-ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x - 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + 2*integrate(-1/(c*log(
a*x + 1) - c*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\operatorname{atanh}(a\,x)^2\,(c + a\,c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(c + a*c*x)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(c + a*c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax\operatorname{atanh}^2(ax)+\operatorname{atanh}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

3.146 $\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x(acx+c)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Defer[Int][1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx=\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(acx^2+cx\right)\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/((a*c*x^2 + c*x)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(acx+c)x\,\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)^2), x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(acx+c\right)\operatorname{arctanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(ax-1\right)}{acx\log\left(ax+1\right)-acx\log\left(-ax+1\right)}+2\int -\frac{1}{acx^{2}\log\left(ax+1\right)-acx^{2}\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x - 1)/(a*c*x*log(a*x + 1) - a*c*x*log(-a*x + 1)) + 2*integrate(-1/(a*
c*x^2*log(a*x + 1) - a*c*x^2*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^{2}\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^2*(c + a*c*x)),x)
```

```
[Out] int(1/(x*atanh(a*x)^2*(c + a*c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^{2}\operatorname{atanh}^{2}\left(ax\right)+x\operatorname{atanh}^{2}\left(ax\right)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(a*x**2*atanh(a*x)**2 + x*atanh(a*x)**2), x)/c
```

3.147 $\int \frac{x^3(a+b\tanh^{-1}(cx))}{d+ex}\,dx$

Optimal. Leaf size=275

$$\frac{d^3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^4}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^4}-\frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2}+\frac{x^3\left(a+b\tanh\right.}{3e}$$

```
[Out] a*d^2*x/e^3-1/2*b*d*x/c/e^2+1/6*b*x^2/c/e+1/2*b*d*arctanh(c*x)/c^2/e^2+b*d^
2*x*arctanh(c*x)/e^3-1/2*d*x^2*(a+b*arctanh(c*x))/e^2+1/3*x^3*(a+b*arctanh(
c*x))/e+d^3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^4-d^3*(a+b*arctanh(c*x))*ln(
2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^4+1/2*b*d^2*ln(-c^2*x^2+1)/c/e^3+1/6*b*ln(-c
^2*x^2+1)/c^3/e-1/2*b*d^3*polylog(2,1-2/(c*x+1))/e^4+1/2*b*d^3*polylog(2,1-
2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^4
```

Rubi [A] time = 0.26, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {5940, 5910, 260, 5916, 321, 206, 266, 43, 5920, 2402, 2315, 2447}

$$-\frac{bd^3\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^4}+\frac{bd^3\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^4}+\frac{d^3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^4}-\frac{d^3\left(a+b\tanh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (a*d^2*x)/e^3 - (b*d*x)/(2*c*e^2) + (b*x^2)/(6*c*e) + (b*d*ArcTanh[c*x])/(2
*c^2*e^2) + (b*d^2*x*ArcTanh[c*x])/e^3 - (d*x^2*(a + b*ArcTanh[c*x]))/(2*e^
2) + (x^3*(a + b*ArcTanh[c*x]))/(3*e) + (d^3*(a + b*ArcTanh[c*x])*Log[2/(1
+ c*x)])/e^4 - (d^3*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1
+ c*x))])/e^4 + (b*d^2*Log[1 - c^2*x^2])/(2*c*e^3) + (b*Log[1 - c^2*x^2])/(
6*c^3*e) - (b*d^3*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^4) + (b*d^3*PolyLog[2,
1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int\left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^3} - \frac{dx\left(a+b\tanh^{-1}(cx)\right)}{e^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{e} - \frac{d^3\left(a\right.}{}\right.$$

$$= \frac{d^2\int\left(a+b\tanh^{-1}(cx)\right)dx}{e^3} - \frac{d^3\int\frac{a+b\tanh^{-1}(cx)}{d+ex}dx}{e^3} - \frac{d\int x\left(a+b\tanh^{-1}(cx)\right)dx}{e^2} +$$

$$= \frac{ad^2x}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3e} + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)\log}{e^4}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3e}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bd\tanh^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a\right.}{}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bx^2}{6ce} + \frac{bd\tanh^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2}$$

Mathematica [C] time = 6.59, size = 474, normalized size = 1.72

$$\frac{-6ad^3\log(d+ex) + 6ad^2ex - 3ade^2x^2 + 2ae^3x^3 - \frac{be^3}{c^3} + \frac{3}{2}i\pi bd^3\log\left(1-c^2x^2\right) + \frac{3bd^2e\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}}{c}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (-((b*e^3)/c^3) + 6*a*d^2*e*x - (3*b*d*e^2*x)/c - 3*a*d*e^2*x^2 + (b*e^3*x^
2)/c + 2*a*e^3*x^3 + (3*b*d*e^2*ArcTanh[c*x])/c^2 - (3*I)*b*d^3*Pi*ArcTanh[
c*x] + 6*b*d^2*e*x*ArcTanh[c*x] - 3*b*d*e^2*x^2*ArcTanh[c*x] + 2*b*e^3*x^3*
ArcTanh[c*x] - 6*b*d^3*ArcTanh[(c*d)/e]*ArcTanh[c*x] + 3*b*d^3*ArcTanh[c*x]
^2 - (3*b*d^2*e*ArcTanh[c*x]^2)/c + (3*b*d^2*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcT
anh[c*x]^2)/(c*E^ArcTanh[(c*d)/e]) + 6*b*d^3*ArcTanh[c*x]*Log[1 + E^(-2*Arc
Tanh[c*x])] + (3*I)*b*d^3*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 6*b*d^3*ArcTanh[
(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*b*d^3*ArcTan
h[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*a*d^3*Log[d +
e*x] + (3*b*d^2*e*Log[1 - c^2*x^2])/c + (b*e^3*Log[1 - c^2*x^2])/c^3 + ((3*
I)/2)*b*d^3*Pi*Log[1 - c^2*x^2] + 6*b*d^3*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTa
nh[(c*d)/e] + ArcTanh[c*x]]] - 3*b*d^3*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3
*b*d^3*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(6*e^4)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{bx^3\operatorname{artanh}(cx) + ax^3}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx) + a)x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(e*x + d), x)
```

maple [A] time = 0.07, size = 381, normalized size = 1.39

$$\frac{a\,x^3}{3e}-\frac{a\,x^2 d}{2e^2}+\frac{a\,d^2 x}{e^3}-\frac{a\,d^3\ln\left(cxe+cd\right)}{e^4}+\frac{b\,\mathrm{arctanh}\left(cx\right)x^3}{3e}-\frac{b\,\mathrm{arctanh}\left(cx\right)x^2 d}{2e^2}+\frac{b\,d^2 x\,\mathrm{arctanh}\left(cx\right)}{e^3}-\frac{b\,\mathrm{arctanh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] 1/3*a*x^3/e-1/2*a/e^2*x^2*d+a*d^2*x/e^3-a*d^3/e^4*ln(c*e*x+c*d)+1/3*b*arcta
nh(c*x)*x^3/e-1/2*b*arctanh(c*x)/e^2*x^2*d+b*d^2*x*arctanh(c*x)/e^3-b*arcta
nh(c*x)*d^3/e^4*ln(c*e*x+c*d)+1/2*b/e^4*d^3*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*
d+e))+1/2*b/e^4*d^3*dilog((c*e*x+e)/(-c*d+e))-1/2*b/e^4*d^3*ln(c*e*x+c*d)*l
n((c*e*x-e)/(-c*d-e))-1/2*b/e^4*d^3*dilog((c*e*x-e)/(-c*d-e))-1/2*b*d*x/c/e
^2-2/3/c*b*d^2/e^3+1/6*b*x^2/c/e+1/2/c*b/e^3*ln(c*e*x+e)*d^2+1/4/c^2*b/e^2*
ln(c*e*x+e)*d+1/6/c^3*b/e*ln(c*e*x+e)+1/2/c*b/e^3*ln(c*e*x-e)*d^2-1/4/c^2*b
/e^2*ln(c*e*x-e)*d+1/6/c^3*b/e*ln(c*e*x-e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\,a\left(\frac{6\,d^3\log\left(ex+d\right)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)+\frac{1}{2}\,b\int\frac{x^3\big(\log\left(cx+1\right)-\log\left(-cx+1\right)\big)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*a*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + 1
/2*b*integrate(x^3*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^3\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + e*x),x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\left(a+b\,\mathrm{atanh}\left(cx\right)\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x))/(d + e*x), x)
```

3.148 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx$

Optimal. Leaf size=214

$$-\frac{d^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{adx}{e^2}-\frac{bd\log(\ldots}{2\ldots}$$

```
[Out] -a*d*x/e^2+1/2*b*x/c/e-1/2*b*arctanh(c*x)/c^2/e-b*d*x*arctanh(c*x)/e^2+1/2*
x^2*(a+b*arctanh(c*x))/e-d^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^3+d^2*(a+b*
arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3-1/2*b*d*ln(-c^2*x^2+1)/c/
e^2+1/2*b*d^2*polylog(2,1-2/(c*x+1))/e^3-1/2*b*d^2*polylog(2,1-2*c*(e*x+d)/
(c*d+e)/(c*x+1))/e^3
```

Rubi [A] time = 0.20, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5940, 5910, 260, 5916, 321, 206, 5920, 2402, 2315, 2447}

$$\frac{bd^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^3}-\frac{bd^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^3}-\frac{d^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}+\frac{d^2\left(a+b\tanh^{-1}(c\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] -((a*d*x)/e^2) + (b*x)/(2*c*e) - (b*ArcTanh[c*x])/(2*c^2*e) - (b*d*x*ArcTan
h[c*x])/e^2 + (x^2*(a + b*ArcTanh[c*x]))/(2*e) - (d^2*(a + b*ArcTanh[c*x])*
Log[2/(1 + c*x)])/e^3 + (d^2*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d
 + e)*(1 + c*x))])/e^3 - (b*d*Log[1 - c^2*x^2])/(2*c*e^2) + (b*d^2*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*e^3) - (b*d^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d
+ e)*(1 + c*x))])/(2*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int \left(-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{e^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)}{e}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int\left(a+b\tanh^{-1}(cx)\right)dx}{e^2}+\frac{d^2\int\frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{e^2}+\frac{\int x\left(a+b\tanh^{-1}(cx)\right)dx}{e}$$

$$= -\frac{adx}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^3}+\frac{d^2\left(a+b\,\mathrm{ta}\right.}{}$$

$$= -\frac{adx}{e^2}+\frac{bx}{2ce}-\frac{bdx\tanh^{-1}(cx)}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^3}$$

$$= -\frac{adx}{e^2}+\frac{bx}{2ce}-\frac{b\tanh^{-1}(cx)}{2c^2e}-\frac{bdx\tanh^{-1}(cx)}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{d^2\left(a\right.}{}$$

Mathematica [C] time = 3.05, size = 394, normalized size = 1.84

$$2ad^2\log(d+ex)-2adex+ae^2x^2-\frac{bde\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}}{c}-\frac{1}{2}i\pi bd^2\log\left(1-c^2x^2\right)-\frac{bde\log\left(1-c^2x^2\right)}{c}-\frac{be^2\tan$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (-2*a*d*e*x + (b*e^2*x)/c + a*e^2*x^2 - (b*e^2*ArcTanh[c*x])/c^2 + I*b*d^2*
Pi*ArcTanh[c*x] - 2*b*d*e*x*ArcTanh[c*x] + b*e^2*x^2*ArcTanh[c*x] + 2*b*d^2
*ArcTanh[(c*d)/e]*ArcTanh[c*x] - b*d^2*ArcTanh[c*x]^2 + (b*d*e*ArcTanh[c*x]
^2)/c - (b*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c*d)/e
]) - 2*b*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - I*b*d^2*Pi*Log[1 +
 E^(2*ArcTanh[c*x])] + 2*b*d^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d
)/e] + ArcTanh[c*x]))] + 2*b*d^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/
e] + ArcTanh[c*x]))] + 2*a*d^2*Log[d + e*x] - (b*d*e*Log[1 - c^2*x^2])/c -
(I/2)*b*d^2*Pi*Log[1 - c^2*x^2] - 2*b*d^2*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTa
nh[(c*d)/e] + ArcTanh[c*x]]] + b*d^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] - b*d
^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(2*e^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\operatorname{artanh}(cx)+ax^2}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{artanh}(cx)+a)x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(e*x + d), x)
```

maple [A] time = 0.06, size = 298, normalized size = 1.39

$$\frac{a\,x^2}{2e}-\frac{adx}{e^2}+\frac{a\,d^2\ln(cxe+cd)}{e^3}+\frac{b\operatorname{arctanh}(cx)\,x^2}{2e}-\frac{bdx\operatorname{arctanh}(cx)}{e^2}+\frac{b\operatorname{arctanh}(cx)\,d^2\ln(cxe+cd)}{e^3}+\frac{bx}{2ce}+\frac{bd}{2c\,e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] 1/2*a*x^2/e-a*d*x/e^2+a*d^2/e^3*ln(c*e*x+c*d)+1/2*b*arctanh(c*x)*x^2/e-b*d*
x*arctanh(c*x)/e^2+b*arctanh(c*x)*d^2/e^3*ln(c*e*x+c*d)+1/2*b*x/c/e+1/2/c*b
*d/e^2-1/2/c*b/e^2*ln(c*e*x+e)*d-1/4/c^2*b/e*ln(c*e*x+e)-1/2/c*b/e^2*ln(c*e
*x-e)*d+1/4/c^2*b/e*ln(c*e*x-e)-1/2*b/e^3*d^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-
c*d+e))-1/2*b/e^3*d^2*dilog((c*e*x+e)/(-c*d+e))+1/2*b/e^3*d^2*ln(c*e*x+c*d)
*ln((c*e*x-e)/(-c*d-e))+1/2*b/e^3*d^2*dilog((c*e*x-e)/(-c*d-e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{2\,d^2\log(ex+d)}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)+\frac{1}{2}b\int\frac{x^2\left(\log(cx+1)-\log(-cx+1)\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/2*b*integrate(x^2*
(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\,\mathrm{atanh}\left(cx\right)\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))/(d + e*x), x)
```

3.149 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx$

Optimal. Leaf size=156

$$\frac{d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}+\frac{ax}{e}+\frac{b\log\left(1-c^2x^2\right)}{2ce}-\frac{bd\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2e^2}+\cdots$$

```
[Out] a*x/e+b*x*arctanh(c*x)/e+d*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^2-d*(a+b*arct
anh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2+1/2*b*ln(-c^2*x^2+1)/c/e-1/2*
b*d*polylog(2,1-2/(c*x+1))/e^2+1/2*b*d*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x
+1))/e^2
```

Rubi [A] time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5940, 5910, 260, 5920, 2402, 2315, 2447}

$$-\frac{bd\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^2}+\frac{bd\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^2}+\frac{d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\cdots}{e^2}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (a*x)/e + (b*x*ArcTanh[c*x])/e + (d*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/
e^2 - (d*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e
^2 + (b*Log[1 - c^2*x^2])/(2*c*e) - (b*d*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^
2) + (b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{e} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{e(d+ex)}\right) dx$$

$$= \frac{\int \left(a+b\tanh^{-1}(cx)\right)dx}{e} - \frac{d\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2} - \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^2} - $$

$$= \frac{ax}{e} + \frac{bx\tanh^{-1}(cx)}{e} + \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2} - \frac{d\left(a+b\tanh^{-1}(cx)\right)\log}{e^2}$$

$$= \frac{ax}{e} + \frac{bx\tanh^{-1}(cx)}{e} + \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2} - \frac{d\left(a+b\tanh^{-1}(cx)\right)\log}{e^2}$$

Mathematica [C] time = 2.47, size = 315, normalized size = 2.02

$$-2ad\log(d+ex) + 2aex + \frac{b\left(e\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)} + \frac{1}{2}i\pi cd\log\left(1-c^2x^2\right) + e\log\left(1-c^2x^2\right) + cd\text{Li}_2\left(e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c*e*x*ArcT
anh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*Arc
Tanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e]
 + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*
ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTa
nh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*Arc
Tanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*PolyLog[2,
 -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))]))/c)/(2*e^2)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx\,\text{artanh}\,(cx)+ax}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)x}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(e*x + d), x)
```

maple [A] time = 0.05, size = 217, normalized size = 1.39

$$\frac{ax}{e} - \frac{ad\ln(cxe + cd)}{e^2} + \frac{bx\operatorname{arctanh}(cx)}{e} - \frac{b\operatorname{arctanh}(cx)\,d\ln(cxe + cd)}{e^2} + \frac{bd\ln(cxe + cd)\ln\left(\frac{cxe+e}{-cd+e}\right)}{2e^2} + \frac{bd\operatorname{dilog}\left(\frac{cxe+e}{-cd+e}\right.}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] a*x/e-a*d/e^2*ln(c*e*x+c*d)+b*x*arctanh(c*x)/e-b*arctanh(c*x)*d/e^2*ln(c*e*
x+c*d)+1/2*b/e^2*d*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/2*b/e^2*d*dilog((
c*e*x+e)/(-c*d+e))-1/2*b/e^2*d*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-1/2*b/e
^2*d*dilog((c*e*x-e)/(-c*d-e))+1/2/c*b/e*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+(c*e*
x+c*d)^2-e^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{x}{e} - \frac{d\log(ex + d)}{e^2}\right) + \frac{1}{2}b\int \frac{x\left(\log(cx + 1) - \log(-cx + 1)\right)}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(x/e - d*log(e*x + d)/e^2) + 1/2*b*integrate(x*(log(c*x + 1) - log(-c*x +
 1))/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a + b\,\mathrm{atanh}\left(c\,x\right)\right)}{d + e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + e*x),x)
```

```
[Out] int((x*(a + b*atanh(c*x)))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a + b\operatorname{atanh}(cx)\right)}{d + ex} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))/(d + e*x), x)
```

3.150 $\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=114

$$\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e}-\frac{b\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2e}$$

```
[Out] -(a+b*arctanh(c*x))*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+
e)/(c*x+1))/e+1/2*b*polylog(2,1-2/(c*x+1))/e-1/2*b*polylog(2,1-2*c*(e*x+d)/
(c*d+e)/(c*x+1))/e
```

Rubi [A] time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5920, 2402, 2315, 2447}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])*Log[(2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(
2*e) - (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{(bc)\int\frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}}{e}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}-\frac{b\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)\ldots}\right.}{2e}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{b\text{Li}_2\left(1-\frac{2}{1+cx}\right.}{2e}$$

Mathematica [C] time = 0.24, size = 257, normalized size = 2.25

$$a\log(d+ex)-\frac{1}{2}ib\left(-\log\left(\frac{2}{\sqrt{1-c^2x^2}}\right)\left(\pi-2i\tanh^{-1}(cx)\right)-i\text{Li}_2\left(e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+i\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh\ldots\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x] + b*ArcTanh[c*x]*(Log[1 - c^2*x^2]/2 + Log[I*Sinh[ArcTanh[(
c*d)/e] + ArcTanh[c*x]]]) - (I/2)*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c*x])^2 +
 I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x])*Log[1 +
E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[1 - E^(-2
*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Log[2/Sqrt
[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I)*Sinh[Arc
Tanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])] - I*Poly
Log[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/e
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\,(cx)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(e*x + d), x)
```

maple [A] time = 0.05, size = 148, normalized size = 1.30

$$\frac{a\ln(cxe+cd)}{e}+\frac{b\ln(cxe+cd)\,\text{arctanh}\,(cx)}{e}-\frac{b\ln(cxe+cd)\ln\left(\frac{cxe+e}{-cd+e}\right)}{2e}-\frac{b\,\text{dilog}\left(\frac{cxe+e}{-cd+e}\right)}{2e}+\frac{b\ln(cxe+cd)\ln\left(\frac{cxe-e}{-cd-e}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e+b*ln(c*e*x+c*d)/e*arctanh(c*x)-1/2*b/e*ln(c*e*x+c*d)*ln((
c*e*x+e)/(-c*d+e))-1/2*b/e*dilog((c*e*x+e)/(-c*d+e))+1/2*b/e*ln(c*e*x+c*d)*
ln((c*e*x-e)/(-c*d-e))+1/2*b/e*dilog((c*e*x-e)/(-c*d-e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}b\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{ex+d}\,dx+\frac{a\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(e*x + d), x) + a*log(e*x +
d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{atanh}\left(cx\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d + e*x), x)
```

3.151 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+ex)}\,dx$

Optimal. Leaf size=148

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{a\log(x)}{d}+\frac{b\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}-\frac{b\text{Li}_2(-cx}{2d}$$

```
[Out] a*ln(x)/d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/d-(a+b*arctanh(c*x))*ln(2*c*(e*x
+d)/(c*d+e)/(c*x+1))/d-1/2*b*polylog(2,-c*x)/d+1/2*b*polylog(2,c*x)/d-1/2*b
*polylog(2,1-2/(c*x+1))/d+1/2*b*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Rubi [A] time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5940, 5912, 5920, 2402, 2315, 2447}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d}-\frac{b\text{PolyLog}(2,-cx)}{2d}+\frac{b\text{PolyLog}(2,cx)}{2d}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]
```

```
[Out] (a*Log[x])/d + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[
c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d - (b*PolyLog[2, -(c*x)]
)/(2*d) + (b*PolyLog[2, c*x])/(2*d) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d)
 + (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
```

```
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{e\left(a+b\tanh^{-1}(cx)\right)}{d(d+ex)}\right)dx$$
$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d}-\frac{e\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d}$$
$$= \frac{a\log(x)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-\frac{bl}{}$$
$$= \frac{a\log(x)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-\frac{bl}{}$$
$$= \frac{a\log(x)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-\frac{bl}{}$$

Mathematica [C] time = 1.63, size = 294, normalized size = 1.99

$$\frac{-2ad\log(d+ex)+2ad\log(x)+\frac{b\left(e\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}+\frac{1}{2}i\pi cd\log\left(1-c^2x^2\right)+cd\,\text{Li}_2\left(e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)-2cd\,\text{ta}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]
```

```
[Out] (2*a*d*Log[x] - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] - 2*c*d*A
rcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*ArcTanh[c*x]^2 + (Sqr
t[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] + 2*c*d*ArcTanh[c
*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 2
*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 2
*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + (I/2)
*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/
e] + ArcTanh[c*x]]] - c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2,
E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c)/(2*d^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}(cx) + a}{(ex + d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x), x)
```

maple [A] time = 0.06, size = 210, normalized size = 1.42

$$\frac{a \ln(cx)}{d} - \frac{a \ln(cxe + cd)}{d} + \frac{b \operatorname{arctanh}(cx) \ln(cx)}{d} - \frac{b \operatorname{arctanh}(cx) \ln(cxe + cd)}{d} + \frac{b \ln(cxe + cd) \ln\left(\frac{cxe+e}{-cd+e}\right)}{2d} + \frac{b \operatorname{dilog}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(e*x+d),x)
```

```
[Out] a/d*ln(c*x)-a/d*ln(c*e*x+c*d)+b*arctanh(c*x)/d*ln(c*x)-b*arctanh(c*x)/d*ln(
c*e*x+c*d)+1/2*b/d*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/2*b/d*dilog((c*e*
x+e)/(-c*d+e))-1/2*b/d*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-1/2*b/d*dilog((
c*e*x-e)/(-c*d-e))-1/2*b/d*dilog(c*x)-1/2*b/d*dilog(c*x+1)-1/2*b/d*ln(c*x)*
ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\log(ex + d)}{d} - \frac{\log(x)}{d}\right) + \frac{1}{2} b \int \frac{\log(cx + 1) - \log(-cx + 1)}{ex^2 + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(e*x + d)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x +
 1))/(e*x^2 + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atanh}(c\, x)}{x\, (d + e\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(cx)}{x (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(x*(d + e*x)), x)
```

3.152 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=200

$$-\frac{e\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^2}-\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{ae\log(x)}{d^2}-\frac{bc\log(}{}$$

```
[Out] (-a-b*arctanh(c*x))/d/x+b*c*ln(x)/d-a*e*ln(x)/d^2-e*(a+b*arctanh(c*x))*ln(2
/(c*x+1))/d^2+e*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2-1/2*
b*c*ln(-c^2*x^2+1)/d+1/2*b*e*polylog(2,-c*x)/d^2-1/2*b*e*polylog(2,c*x)/d^2
+1/2*b*e*polylog(2,1-2/(c*x+1))/d^2-1/2*b*e*polylog(2,1-2*c*(e*x+d)/(c*d+e)
/(c*x+1))/d^2
```

Rubi [A] time = 0.20, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5940, 5916, 266, 36, 29, 31, 5912, 5920, 2402, 2315, 2447}

$$\frac{be\text{PolyLog}(2,-cx)}{2d^2}-\frac{be\text{PolyLog}(2,cx)}{2d^2}+\frac{be\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}-\frac{be\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d^2}-\frac{e\log\left(\frac{2}{cx+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
 b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])*Log[(2*c*(
d + e*x))/((c*d + e)*(1 + c*x))])/d^2 - (b*c*Log[1 - c^2*x^2])/(2*d) + (b*e
*PolyLog[2, -(c*x)])/(2*d^2) - (b*e*PolyLog[2, c*x])/(2*d^2) + (b*e*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*d^2) - (b*e*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+ex)}\,dx = \int\left(\frac{a+b\tanh^{-1}(cx)}{dx^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh^{-1}(c\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh^{-1}(c\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh^{-1}(c\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} + \frac{bc\log(x)}{d} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a\right.}{}$$

Mathematica [C] time = 3.22, size = 360, normalized size = 1.80

$$-\frac{\frac{2ad^2}{x} + 2ade\log(x) - 2ade\log(d+ex) + \frac{be^2\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}}{c} - 2bcd^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + \frac{1}{2}i\pi bde\log\left(1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*((2*a*d^2)/x - I*b*d*e*Pi*ArcTanh[c*x] + (2*b*d^2*ArcTanh[c*x])/x - 2*
b*d*e*ArcTanh[(c*d)/e]*ArcTanh[c*x] + b*d*e*ArcTanh[c*x]^2 - (b*e^2*ArcTanh
[c*x]^2)/c + (b*Sqrt[1 - (c^2*d^2)/e^2]*e^2*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c
*d)/e]) + 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*b*d*e*Pi*Lo
g[1 + E^(2*ArcTanh[c*x])] - 2*b*d*e*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh
[(c*d)/e] + ArcTanh[c*x]))] - 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(
c*d)/e] + ArcTanh[c*x]))] + 2*a*d*e*Log[x] - 2*a*d*e*Log[d + e*x] - 2*b*c*d
^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + (I/2)*b*d*e*Pi*Log[1 - c^2*x^2] + 2*b*d*e
*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - b*d*e*Poly
Log[2, E^(-2*ArcTanh[c*x])] + b*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + Ar
cTanh[c*x]))])/d^3
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\,(cx)+a}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x^2), x)
```

maple [A] time = 0.07, size = 279, normalized size = 1.40

$$-\frac{a}{dx}-\frac{ae\ln(cx)}{d^2}+\frac{ae\ln(cxe+cd)}{d^2}-\frac{b\operatorname{arctanh}(cx)}{dx}-\frac{b\operatorname{arctanh}(cx)e\ln(cx)}{d^2}+\frac{b\operatorname{arctanh}(cx)e\ln(cxe+cd)}{d^2}+\frac{cb\ln(c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(e*x+d),x)
```

```
[Out] -a/d/x-a/d^2*e*ln(c*x)+a/d^2*e*ln(c*e*x+c*d)-b*arctanh(c*x)/d/x-b*arctanh(c
*x)/d^2*e*ln(c*x)+b*arctanh(c*x)/d^2*e*ln(c*e*x+c*d)+c*b/d*ln(c*x)-1/2*c*b/
d*ln(c*x-1)-1/2*c*b/d*ln(c*x+1)+1/2*b/d^2*e*dilog(c*x)+1/2*b/d^2*e*dilog(c*
x+1)+1/2*b/d^2*e*ln(c*x)*ln(c*x+1)-1/2*b/d^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c
*d+e))*e-1/2*b/d^2*dilog((c*e*x+e)/(-c*d+e))*e+1/2*b/d^2*ln(c*e*x+c*d)*ln((
c*e*x-e)/(-c*d-e))*e+1/2*b/d^2*dilog((c*e*x-e)/(-c*d-e))*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{e\log(ex+d)}{d^2}-\frac{e\log(x)}{d^2}-\frac{1}{dx}\right)+\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{ex^3+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*b*integrate((log(c*x
+ 1) - log(-c*x + 1))/(e*x^3 + d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{atanh}(cx)}{x^2(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(x**2*(d + e*x)), x)
```

3.153 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=261

$$\frac{e^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{e^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^3}+\frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x}-\frac{a+b\tanh^{-1}(c}{2dx^2}$$

```
[Out] -1/2*b*c/d/x+1/2*b*c^2*arctanh(c*x)/d+1/2*(-a-b*arctanh(c*x))/d/x^2+e*(a+b*
arctanh(c*x))/d^2/x-b*c*e*ln(x)/d^2+a*e^2*ln(x)/d^3+e^2*(a+b*arctanh(c*x))*
ln(2/(c*x+1))/d^3-e^2*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^
3+1/2*b*c*e*ln(-c^2*x^2+1)/d^2-1/2*b*e^2*polylog(2,-c*x)/d^3+1/2*b*e^2*poly
log(2,c*x)/d^3-1/2*b*e^2*polylog(2,1-2/(c*x+1))/d^3+1/2*b*e^2*polylog(2,1-2
*c*(e*x+d)/(c*d+e)/(c*x+1))/d^3
```

Rubi [A] time = 0.25, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912, 5920, 2402, 2315, 2447}

$$-\frac{be^2\text{PolyLog}(2,-cx)}{2d^3}+\frac{be^2\text{PolyLog}(2,cx)}{2d^3}-\frac{be^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{be^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d^3}+\frac{e^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -(b*c)/(2*d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x^2
) + (e*(a + b*ArcTanh[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])/
d^3 + (e^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (e^2*(a + b*ArcTanh
[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d^3 + (b*c*e*Log[1 - c^2
*x^2])/(2*d^2) - (b*e^2*PolyLog[2, -(c*x)])/(2*d^3) + (b*e^2*PolyLog[2, c*x
])/(2*d^3) - (b*e^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3) + (b*e^2*PolyLog[2
, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx^3} - \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x^2} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{d^3x} - \frac{e^3\left(a+b\tanh\right.}{d^3(d+}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{e^3\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\right.}{d^3}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tanh^{-1}(cx\right.}{d^3}$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \cdots$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} - \frac{bce\log(x)}{d^2} + \cdots$$

Mathematica [C] time = 6.09, size = 435, normalized size = 1.67

$$\frac{ae^2\log(x)}{d^3} - \frac{ae^2\log(d+ex)}{d^3} + \frac{ae}{d^2x} - \frac{a}{2dx^2} - \frac{b\left(\frac{cd^3\left(1-c^2x^2\right)\tanh^{-1}(cx)}{x^2} + \frac{c^2d^3}{x} - e^3\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2 e^{-\tanh^{-1}\left(\frac{cd}{e}\right)} + \cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*a/(d*x^2) + (a*e)/(d^2*x) + (a*e^2*Log[x])/d^3 - (a*e^2*Log[d + e*x])/
d^3 - (b*((c^2*d^3)/x + I*c*d*e^2*Pi*ArcTanh[c*x] - (2*c*d^2*e*ArcTanh[c*x]
)/x + (c*d^3*(1 - c^2*x^2)*ArcTanh[c*x])/x^2 + 2*c*d*e^2*ArcTanh[(c*d)/e]*A
rcTanh[c*x] - c*d*e^2*ArcTanh[c*x]^2 + e^3*ArcTanh[c*x]^2 - (Sqrt[1 - (c^2*
d^2)/e^2]*e^3*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e^2*ArcTanh[c*x]*L
og[1 - E^(-2*ArcTanh[c*x])] - I*c*d*e^2*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*
c*d*e^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]
+ 2*c*d*e^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]
+ I*c*d*e^2*Pi*Log[1/Sqrt[1 - c^2*x^2]] + 2*c^2*d^2*e*Log[(c*x)/Sqrt[1 - c^
2*x^2]] - 2*c*d*e^2*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[
c*x]]] + c*d*e^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - c*d*e^2*PolyLog[2, E^(-2
*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(2*c*d^4)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\,(cx)+a}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x^3), x)
```

maple [A] time = 0.07, size = 367, normalized size = 1.41

$$-\frac{a}{2d\,x^2}+\frac{a\,e^2\ln(cx)}{d^3}+\frac{ae}{d^2x}-\frac{a\,e^2\ln(cxe+cd)}{d^3}-\frac{b\,\mathrm{arctanh}(cx)}{2d\,x^2}+\frac{b\,\mathrm{arctanh}(cx)\,e^2\ln(cx)}{d^3}+\frac{b\,\mathrm{arctanh}(cx)\,e}{d^2x}-\frac{b\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(e*x+d),x)
```

```
[Out] -1/2*a/d/x^2+a/d^3*e^2*ln(c*x)+a/d^2*e/x-a/d^3*e^2*ln(c*e*x+c*d)-1/2*b*arct
anh(c*x)/d/x^2+b*arctanh(c*x)/d^3*e^2*ln(c*x)+b*arctanh(c*x)/d^2*e/x-b*arct
anh(c*x)/d^3*e^2*ln(c*e*x+c*d)-1/2*b/d^3*e^2*dilog(c*x)-1/2*b/d^3*e^2*dilog
(c*x+1)-1/2*b/d^3*e^2*ln(c*x)*ln(c*x+1)+1/2*b/d^3*e^2*ln(c*e*x+c*d)*ln((c*e
*x+e)/(-c*d+e))+1/2*b/d^3*e^2*dilog((c*e*x+e)/(-c*d+e))-1/2*b/d^3*e^2*ln(c*
e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-1/2*b/d^3*e^2*dilog((c*e*x-e)/(-c*d-e))-1/2
*b*c/d/x-c*b/d^2*e*ln(c*x)-1/4*c^2*b/d*ln(c*x-1)+1/2*c*b/d^2*ln(c*x-1)*e+1/
4*c^2*b/d*ln(c*x+1)+1/2*c*b/d^2*ln(c*x+1)*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{2\,e^2\log(ex+d)}{d^3}-\frac{2\,e^2\log(x)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{ex^4+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
+ 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(e*x^4 + d*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{atanh}(c\,x)}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{atanh}(cx)}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(x**3*(d + e*x)), x)
```

3.154 $\int \frac{x^2(a+b\tanh^{-1}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=385

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}+\frac{bd^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}-\frac{bd^2\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e^3}-\frac{d^2 l}{}$$

```
[Out] a*b*x/c/e+b^2*x*arctanh(c*x)/c/e-d*(a+b*arctanh(c*x))^2/c/e^2-1/2*(a+b*arct
anh(c*x))^2/c^2/e-d*x*(a+b*arctanh(c*x))^2/e^2+1/2*x^2*(a+b*arctanh(c*x))^2
/e+2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c/e^2-d^2*(a+b*arctanh(c*x))^2*l
n(2/(c*x+1))/e^3+d^2*(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e
^3+1/2*b^2*ln(-c^2*x^2+1)/c^2/e+b^2*d*polylog(2,1-2/(-c*x+1))/c/e^2+b*d^2*(
a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e^3-b*d^2*(a+b*arctanh(c*x))*polyl
og(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3+1/2*b^2*d^2*polylog(3,1-2/(c*x+1))/
e^3-1/2*b^2*d^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3
```

Rubi [A] time = 0.43, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 5922}

$$\frac{bd^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}-\frac{bd^2\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}+\frac{b^2d^2\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (a*b*x)/(c*e) + (b^2*x*ArcTanh[c*x])/(c*e) - (d*(a + b*ArcTanh[c*x])^2)/(c*
e^2) - (a + b*ArcTanh[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTanh[c*x])^2)/e^2
+ (x^2*(a + b*ArcTanh[c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1
 - c*x)])/(c*e^2) - (d^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e^3 + (d^
2*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^3 +
(b^2*Log[1 - c^2*x^2])/(2*c^2*e) + (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e
^2) + (b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^3 - (b*d^2
*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]
)/e^3 + (b^2*d^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e^3) - (b^2*d^2*PolyLog[3,
 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx = \int\left(-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{e^2(d+ex)}\right.$$

$$= -\frac{d\int\left(a+b\tanh^{-1}(cx)\right)^2dx}{e^2}+\frac{d^2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}dx}{e^2}+\frac{\int x\left(a+b\tanh^{-1}(cx\right)}{e}$$

$$= -\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2\log}{e^3}$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e}-\frac{d}{}$$

$$= \frac{abx}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+$$

$$= \frac{abx}{ce}+\frac{b^2x\tanh^{-1}(cx)}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a\right.}{}$$

$$= \frac{abx}{ce}+\frac{b^2x\tanh^{-1}(cx)}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a\right.}{}$$

Mathematica [C] time = 15.76, size = 1072, normalized size = 2.78

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (-6*a^2*d*e*x + 3*a^2*e^2*x^2 + 6*a^2*d^2*Log[d + e*x] + (6*a*b*(c*e^2*x +
I*c^2*d^2*Pi*ArcTanh[c*x] - 2*c^2*d*e*x*ArcTanh[c*x] + e^2*(-1 + c^2*x^2)*A
rcTanh[c*x] + 2*c^2*d^2*ArcTanh[(c*d)/e]*ArcTanh[c*x] - c^2*d^2*ArcTanh[c*x
]^2 + c*d*e*ArcTanh[c*x]^2 - (c*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)
/E^ArcTanh[(c*d)/e] - 2*c^2*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] -
 I*c^2*d^2*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c^2*d^2*ArcTanh[(c*d)/e]*Log[
1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c^2*d^2*ArcTanh[c*x]*Log[
1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - c*d*e*Log[1 - c^2*x^2] - (I
/2)*c^2*d^2*Pi*Log[1 - c^2*x^2] - 2*c^2*d^2*ArcTanh[(c*d)/e]*Log[I*Sinh[Arc
Tanh[(c*d)/e] + ArcTanh[c*x]]] + c^2*d^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] -
 c^2*d^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c^2 + (b^2*
(6*c*e^2*x*ArcTanh[c*x] + 6*c*d*e*ArcTanh[c*x]^2 - 6*c^2*d*e*x*ArcTanh[c*x]
^2 + 3*e^2*(-1 + c^2*x^2)*ArcTanh[c*x]^2 - 2*c^2*d^2*ArcTanh[c*x]^3 + 2*c*d
*e*ArcTanh[c*x]^3 + 12*c*d*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 6*
c^2*d^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 3*e^2*Log[1 - c^2*x^2
] + 6*c*d*(-e + c*d*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*c^2*
d^2*PolyLog[3, -E^(-2*ArcTanh[c*x])] - (6*c*d*(-(c*d) + e)*(c*d + e)*(-3*c*
d*ArcTanh[c*x]^3 + e*ArcTanh[c*x]^3 - (2*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[
c*x]^3)/E^ArcTanh[(c*d)/e] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x
]) + E^ArcTanh[c*x])/2] + 3*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e]
+ ArcTanh[c*x])] + 3*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTa
nh[c*x])] + 6*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)
/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 3*c*d
*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*
x])))/(2*E^ArcTanh[c*x])] - 3*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 -
 c^2*x^2]] - ((3*I)/2)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 6*c*d*ArcTanh
[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 6*c*d
```

```
*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcT
anh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*PolyLog[3,
 -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*PolyLog[3, E^(ArcTanh[(c*d)/
e] + ArcTanh[c*x])]))/(3*c^2*d^2 - 3*e^2)))/c^2)/(6*e^3)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2x^2\,\mathrm{artanh}\,(cx)^2 + 2\,abx^2\,\mathrm{artanh}\,(cx) + a^2x^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(e*x +
 d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(e*x + d), x)
```

maple [C] time = 4.58, size = 1656, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x)
```

```
[Out] a*b*x/c/e+b^2*x*arctanh(c*x)/c/e-1/2*I*b^2/e^3*d^2*Pi*csgn(I/(1+(c*x+1)^2/(
-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x
^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*b^2/e^3*d^2*Pi*c
sgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*csgn(I
*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2
/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*b^2/e^3*d^2*Pi*csgn(I/(1+(c*x+1)^2/(
-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x
^2+1))))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)
))/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+1/c^2*b^2*arctanh(c*x)/e-1/c^
2*b^2/e*ln(1+(c*x+1)^2/(-c^2*x^2+1))-1/2/c^2*b^2*arctanh(c*x)^2/e+1/2*b^2*a
rctanh(c*x)^2*x^2/e+1/2*b^2*d^2/e^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+a^2*
d^2/e^3*ln(c*e*x+c*d)-a^2*d/e^2*x+1/2*a^2*x^2/e+2/c*b^2/e^2*dilog(1+I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*d+2/c*b^2/e^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))*
d-1/c*b^2/e^2*arctanh(c*x)^2*d-1/2/c^2*a*b/e*ln(c*e*x+e)+1/2/c^2*a*b/e*ln(c
*e*x-e)+a*b*arctanh(c*x)*x^2/e-a*b/e^3*d^2*dilog((c*e*x+e)/(-c*d+e))+a*b/e^
3*d^2*dilog((c*e*x-e)/(-c*d-e))-1/2*b^2*d^2/e^2/(c*d+e)*polylog(3,(c*d+e)*(
c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-b^2*d^2/e^3*arctanh(c*x)^2*ln(((c*x+1)^2/(-
c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))+b^2*arctanh(c*x)^2*d^2/e^3*
ln(c*e*x+c*d)+1/c*a*b*d/e^2-b^2*d^2/e^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(
-c^2*x^2+1))-b^2*arctanh(c*x)^2*d/e^2*x+c*b^2*d^3/e^3/(c*d+e)*arctanh(c*x)^
2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^2*d^3/e^3/(c*d+e)*arcta
nh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/2*I*b^2/e^3*d^
2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(
1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+a*b/e^3*d^2*ln(c*e*x+c*d)*ln((c
*e*x-e)/(-c*d-e))+b^2*d^2/e^2/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2
/(-c^2*x^2+1)/(-c*d+e))+b^2*d^2/e^2/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*
```

```
(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+2*a*b*arctanh(c*x)*d^2/e^3*ln(c*e*x+c*d)-a
*b/e^3*d^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-1/2*c*b^2*d^3/e^3/(c*d+e)*p
olylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/c*a*b/e^2*ln(c*e*x+e)*d
-1/c*a*b/e^2*ln(c*e*x-e)*d+2/c*b^2/e^2*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))*a
rctanh(c*x)*d+2/c*b^2/e^2*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)*d
-2*a*b*arctanh(c*x)*d/e^2*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right)+\frac{\left(b^2ex^2-2b^2dx\right)\log(-cx+1)^2}{8e^2}-\int-\frac{\left(b^2ce^2x^3-b^2e^2x^2\right)\log(cx+1)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/8*(b^2*e*x^2 - 2
*b^2*d*x)*log(-c*x + 1)^2/e^2 - integrate(-1/4*((b^2*c*e^2*x^3 - b^2*e^2*x^
2)*log(c*x + 1)^2 + 4*(a*b*c*e^2*x^3 - a*b*e^2*x^2)*log(c*x + 1) + (2*b^2*c
*d^2*x - (4*a*b*c*e^2 + b^2*c*e^2)*x^3 + (b^2*c*d*e + 4*a*b*e^2)*x^2 - 2*(b
^2*c*e^2*x^3 - b^2*e^2*x^2)*log(c*x + 1))*log(-c*x + 1))/(c*e^3*x^2 - d*e^2
 + (c*d*e^2 - e^3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\operatorname{atanh}(cx)\right)^2}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**2/(d + e*x), x)
```

3.155 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=279

$$-\frac{bd\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}+\frac{bd\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e^2}+\frac{d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right.}{e^2}$$

```
[Out] (a+b*arctanh(c*x))^2/c/e+x*(a+b*arctanh(c*x))^2/e-2*b*(a+b*arctanh(c*x))*ln
(2/(-c*x+1))/c/e+d*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/e^2-d*(a+b*arctanh(c*
x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2-b^2*polylog(2,1-2/(-c*x+1))/c/e-b
*d*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e^2+b*d*(a+b*arctanh(c*x))*pol
ylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2-1/2*b^2*d*polylog(3,1-2/(c*x+1))/
e^2+1/2*b^2*d*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2
```

Rubi [A] time = 0.26, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5922}

$$-\frac{bd\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}+\frac{bd\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}-\frac{b^2d\text{PolyLog}\left(\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (a + b*ArcTanh[c*x])^2/(c*e) + (x*(a + b*ArcTanh[c*x])^2)/e - (2*b*(a + b*A
rcTanh[c*x])*Log[2/(1 - c*x)])/(c*e) + (d*(a + b*ArcTanh[c*x])^2*Log[2/(1 +
 c*x)])/e^2 - (d*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 +
 c*x))])/e^2 - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e) - (b*d*(a + b*ArcTan
h[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^2 + (b*d*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^2 - (b^2*d*PolyLog[3, 1
- 2/(1 + c*x)])/(2*e^2) + (b^2*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*
(1 + c*x))])/(2*e^2)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{e(d+ex)}\right)dx$$

$$=\frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{e}-\frac{d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{e}$$

$$=\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right.}{e^2}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{e^2}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{ce}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{ce}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{ce}$$

Mathematica [C] time = 13.55, size = 861, normalized size = 3.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (3*a^2*e*x - 3*a^2*d*Log[d + e*x] + (3*a*b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c*
e*x*ArcTanh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2
 - e*ArcTanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[
(c*d)/e] + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1
 + E^(2*ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d
)/e] + ArcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e]
 + ArcTanh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2
*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*Po
lyLog[2, -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))]))/c + (b^2*(-3*e*ArcTanh[c*x]^2 + 3*c*e*x*ArcTanh[c*x]^2 + 4
*c*d*ArcTanh[c*x]^3 - 2*e*ArcTanh[c*x]^3 + (2*Sqrt[1 - (c^2*d^2)/e^2]*e*Arc
Tanh[c*x]^3)/E^ArcTanh[(c*d)/e] - 6*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*
x])] + 3*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + (3*I)*c*d*Pi*Arc
Tanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] - 3*c*d*ArcTanh[c*x]^
2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 3*c*d*ArcTanh[c*x]^2*Log[1
 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*
x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)
/e] + ArcTanh[c*x])))] - 3*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x
])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] + 3*c*d*ArcTanh[c*x
]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] + ((3*I)/2)*c*d*Pi*ArcTanh[c*x]*Lo
g[1 - c^2*x^2] + 6*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*
d)/e] + ArcTanh[c*x]]] + 3*(e - c*d*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh
[c*x])] - 6*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x]
)] - 6*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - (
3*c*d*PolyLog[3, -E^(-2*ArcTanh[c*x])])/2 + 6*c*d*PolyLog[3, -E^(ArcTanh[(c
*d)/e] + ArcTanh[c*x])] + 6*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*
x])]))/c)/(3*e^2)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 x \operatorname{artanh}(cx)^2 + 2\,abx \operatorname{artanh}(cx) + a^2 x}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(e*x + d), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2 x}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(e*x + d), x)
```

maple [C] time = 1.63, size = 13923, normalized size = 49.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2x\log(-cx+1)^2}{4e}+a^2\left(\frac{x}{e}-\frac{d\log(ex+d)}{e^2}\right)-\int-\frac{\left(b^2cex^2-b^2ex\right)\log(cx+1)^2+4\left(abcex^2-abex\right)\log(cx+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x*log(-c*x + 1)^2/e + a^2*(x/e - d*log(e*x + d)/e^2) - integrate(-1
/4*((b^2*c*e*x^2 - b^2*e*x)*log(c*x + 1)^2 + 4*(a*b*c*e*x^2 - a*b*e*x)*log(
c*x + 1) - 2*((2*a*b*c*e + b^2*c*e)*x^2 + (b^2*c*d - 2*a*b*e)*x + (b^2*c*e*
x^2 - b^2*e*x)*log(c*x + 1))*log(-c*x + 1))/(c*e^2*x^2 - d*e + (c*d*e - e^2
)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(a+b\,\mathrm{atanh}\,(c\,x))^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + e*x),x)
```

```
[Out] int((x*(a + b*atanh(c*x))^2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+b\,\mathrm{atanh}\,(cx))^2}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))**2/(d + e*x), x)
```

3.156 $\int \frac{(a+b\tanh^{-1}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=188

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(c\right.}{e}$$

```
[Out] -(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(
c*d+e)/(c*x+1))/e+b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e-b*(a+b*arct
anh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*b^2*polylog(3,1-2/
(c*x+1))/e-1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e
```

Rubi [A] time = 0.05, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5922}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e}-\frac{b^2\text{PolyLog}\left(3,1\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/e - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d
+ e*x))/((c*d + e)*(1 + c*x))])/e + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e)
 - (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx=-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{b\left(a+b\,t\right.}{}$$

Mathematica [C] time = 10.19, size = 759, normalized size = 4.04

$$\frac{6a^2\log(d+ex)-6iab\left(-\log\left(\frac{2}{\sqrt{1-c^2x^2}}\right)\left(\pi-2i\tanh^{-1}(cx)\right)-i\text{Li}_2\left(e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+i\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\text{ta}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

```
[Out] (6*a^2*Log[d + e*x] + 6*a*b*ArcTanh[c*x]*(Log[1 - c^2*x^2] + 2*Log[I*Sinh[A
rcTanh[(c*d)/e] + ArcTanh[c*x]]]) - (6*I)*a*b*((-1/4*I)*(Pi - (2*I)*ArcTanh
[c*x])^2 + I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x]
)*Log[1 + E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log
[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*
Log[2/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I
)*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])
] - I*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]]) + (b^2*(-8*c*d*
ArcTanh[c*x]^3 + 4*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[
c*x]^3)/E^ArcTanh[(c*d)/e] - 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x
])] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2]
 + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*
d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcT
anh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-
1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcTanh[c*x]^2*Log[(e
*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x
])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] - (3*I)*c*d
*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Lo
g[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 6*c*d*ArcTanh[c*x]*PolyLog[2,
-E^(-2*ArcTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e]
 + ArcTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + Ar
cTanh[c*x])] + 3*c*d*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*c*d*PolyLog[3, -
E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e
] + ArcTanh[c*x])]))/(c*d))/(6*e)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d), x)
```

maple [C] time = 0.37, size = 1170, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(e*x+d),x)
```

```
[Out] a^2*ln(c*e*x+c*d)/e+b^2*ln(c*e*x+c*d)/e*arctanh(c*x)^2-b^2/e*arctanh(c*x)^2
*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I*b^2/
e*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^
2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
*Pi+1/2*I*b^2/e*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+
(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c
^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*csgn(I/(1+(c*x+1)^2/(-c^2*x
```

```
^2+1)))*Pi-1/2*I*b^2/e*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+
c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(((c*x
+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*Pi+1/2*I*b^2/e*arc
tanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2
+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi-b^2/e*arctanh(c*x)*polylog(2,-(c*x+1
)^2/(-c^2*x^2+1))+1/2*b^2/e*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^2/(c*d+e)*
arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+b^2/(c*d+e)*ar
ctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2*b^2/(c*d+
e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^2/e*d/(c*d+e)*arc
tanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^2/e*d/(c*d+e)
*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2*c*b^2/
e*d/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+2*a*b*ln(c*e
*x+c*d)/e*arctanh(c*x)-a*b/e*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-a*b/e*dil
og((c*e*x+e)/(-c*d+e))+a*b/e*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+a*b/e*dil
og((c*e*x-e)/(-c*d-e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\log(ex+d)}{e} + \int \frac{b^2\left(\log(cx+1) - \log(-cx+1)\right)^2}{4(ex+d)} + \frac{ab\left(\log(cx+1) - \log(-cx+1)\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/(e*
x + d) + a*b*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^2}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}(cx))^2}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(d + e*x), x)
```

3.157 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+ex)}\,dx$

Optimal. Leaf size=319

$$\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}\right.}{d}$$

```
[Out] -2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d+(a+b*arctanh(c*x))^2*ln(2/
(c*x+1))/d-(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d-b*(a+b*ar
ctanh(c*x))*polylog(2,1-2/(-c*x+1))/d+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(
-c*x+1))/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d+b*(a+b*arctanh(c*x
))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d+1/2*b^2*polylog(3,1-2/(-c*x+1
))/d-1/2*b^2*polylog(3,-1+2/(-c*x+1))/d-1/2*b^2*polylog(3,1-2/(c*x+1))/d+1/
2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Rubi [A] time = 0.43, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5922}

$$\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{b\text{PolyLog}\left(2,\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d + ((a + b*ArcTanh[c*x
])^2*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*
d + e)*(1 + c*x))])/d - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)]
)/d + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d - (b*(a + b*A
rcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + (b^2*PolyLog[3, 1 - 2
/(1 - c*x)])/(2*d) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d) - (b^2*PolyLo
g[3, 1 - 2/(1 + c*x)])/(2*d) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))])/(2*d)
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
```

```
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+ex)}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d}-\frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{d}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d}-\frac{(a+b\,\mathrm{ta}}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d}-\frac{(a+b\,\mathrm{ta}}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d}-\frac{(a+b\,\mathrm{ta}}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d}-\frac{(a+b\,\mathrm{ta}}{}$$

Mathematica [C] time = 12.61, size = 818, normalized size = 2.56

$$\frac{24cd\log(x)a^2-24cd\log(d+ex)a^2-24b\left(-\sqrt{1-\frac{c^2d^2}{e^2}}\,ee^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\tanh^{-1}(cx)^2-cd\tanh^{-1}(cx)^2+e\tanh^{-1}(cx)^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] (24*a^2*c*d*Log[x] - 24*a^2*c*d*Log[d + e*x] - 24*a*b*(I*c*d*Pi*ArcTanh[c*x
] + 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] - c*d*ArcTanh[c*x]^2 + e*ArcTanh[c*
x]^2 - (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*
d*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - I*c*d*Pi*Log[1 + E^(2*ArcTanh
[c*x])] + 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[
c*x]))] + 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]
))] - (I/2)*c*d*Pi*Log[1 - c^2*x^2] - 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[Arc
Tanh[(c*d)/e] + ArcTanh[c*x]]] + c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] - c*d*
PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]) + b^2*(I*c*d*Pi^3 + 1
6*c*d*ArcTanh[c*x]^3 - 16*e*ArcTanh[c*x]^3 + (16*Sqrt[1 - (c^2*d^2)/e^2]*e*
ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (24*I)*c*d*Pi*ArcTanh[c*x]*Log[2] + c*
d*ArcTanh[c*x]^2*Log[16777216] + (24*I)*c*d*Pi*ArcTanh[c*x]*Log[E^(-ArcTanh
[c*x]) + E^ArcTanh[c*x]] + 24*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])
] - 24*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 24
*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 48*c*d*A
rcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])
*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] - 24*c*d*ArcTanh[c*x]^2*Lo
g[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/E^ArcTanh[c*
x]] + 24*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] + (12*I)*c
*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] + 48*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*
Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 24*c*d*ArcTanh[c*x]*PolyLog[
2, E^(2*ArcTanh[c*x])] - 48*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e
] + ArcTanh[c*x])] - 48*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + A
rcTanh[c*x])] - 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] + 48*c*d*PolyLog[3, -
E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 48*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e
] + ArcTanh[c*x])]))/(24*c*d^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)^2}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x), x)
```

maple [C] time = 0.44, size = 1799, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(e*x+d),x)
```

```
[Out] 1/2*b^2*c/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+a*b/d*
dilog((c*e*x+e)/(-c*d+e))-a*b/d*dilog((c*e*x-e)/(-c*d-e))-a*b/d*dilog(c*x)-
```

```
a*b/d*dilog(c*x+1)+b^2*arctanh(c*x)^2/d*ln(c*x)-b^2*arctanh(c*x)^2/d*ln(c*e
*x+c*d)+b^2*arctanh(c*x)^2/d*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)
^2/(-c^2*x^2+1)))-b^2*arctanh(c*x)^2/d*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2/d*a
rctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog
(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+
1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+a^2/d
*ln(c*x)-a^2/d*ln(c*e*x+c*d)-2*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-
2*b^2/d*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*b^2/d*Pi*arctanh(c*x)^
2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I*b^2/d*Pi*a
rctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x
^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)
-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))-b^2/d*e/(
c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-b^2/d*e
/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/
2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/
(-c^2*x^2+1)))^3-1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2
+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1/2*
I*b^2/d*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/2*I*b^2/d*Pi*arctanh(c*
x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*
csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*
x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c
^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2+1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*
(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/
(-c^2*x^2+1)))^2-b^2*c/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*
x^2+1)/(-c*d+e))-b^2*c/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c
^2*x^2+1)/(-c*d+e))+1/2*b^2/d*e/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x
^2+1)/(-c*d+e))+2*a*b*arctanh(c*x)/d*ln(c*x)-2*a*b*arctanh(c*x)/d*ln(c*e*x+
c*d)+a*b/d*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-a*b/d*ln(c*e*x+c*d)*ln((c*e
*x-e)/(-c*d-e))-a*b/d*ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\frac{\log(ex+d)}{d} - \frac{\log(x)}{d}\right) + \int \frac{b^2\left(\log(cx+1) - \log(-cx+1)\right)^2}{4\left(ex^2+dx\right)} + \frac{ab\left(\log(cx+1) - \log(-cx+1)\right)}{ex^2+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a^2*(log(e*x + d)/d - log(x)/d) + integrate(1/4*b^2*(log(c*x + 1) - log(-c
*x + 1))^2/(e*x^2 + d*x) + a*b*(log(c*x + 1) - log(-c*x + 1))/(e*x^2 + d*x)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atanh}(cx))^2}{x\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(x*(d + e*x)), x)
```

3.158 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx$

Optimal. Leaf size=412

$$\frac{be\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{be\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{be\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{be}{}$$

```
[Out] c*(a+b*arctanh(c*x))^2/d-(a+b*arctanh(c*x))^2/d/x+2*e*(a+b*arctanh(c*x))^2*
arctanh(-1+2/(-c*x+1))/d^2-e*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2+e*(a+b*
arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x)
)*ln(2-2/(c*x+1))/d+b*e*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2-b*e*
(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^2+b*e*(a+b*arctanh(c*x))*poly
log(2,1-2/(c*x+1))/d^2-b^2*c*polylog(2,-1+2/(c*x+1))/d-b*e*(a+b*arctanh(c*x
))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2-1/2*b^2*e*polylog(3,1-2/(-c
*x+1))/d^2+1/2*b^2*e*polylog(3,-1+2/(-c*x+1))/d^2+1/2*b^2*e*polylog(3,1-2/(
c*x+1))/d^2-1/2*b^2*e*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2
```

Rubi [A] time = 0.60, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5922}

$$\frac{be\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{be\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{be\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) - (2*e*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (e*(a + b*ArcTanh[c*x])^2*Lo
g[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d
+ (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2 - (b*e*(a + b*
ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (b*e*(a + b*ArcTanh[c*x])
*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d
- (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 +
c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[
3, -1 + 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
 - (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^2)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d} - \frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} + \frac{e^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{d^2}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} - \frac{e\left(a+b\tanh^{-1}(\right.}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\right.}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\right.}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\right.}{d^2}$$

Mathematica [C] time = 12.75, size = 1010, normalized size = 2.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] -(a^2/(d*x)) - (a^2*e*Log[x])/d^2 + (a^2*e*Log[d + e*x])/d^2 + (a*b*(I*c*d*
e*Pi*ArcTanh[c*x] - (2*c*d^2*ArcTanh[c*x])/x + 2*c*d*e*ArcTanh[(c*d)/e]*Arc
Tanh[c*x] - c*d*e*ArcTanh[c*x]^2 + e^2*ArcTanh[c*x]^2 - (Sqrt[1 - (c^2*d^2)
/e^2]*e^2*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e*ArcTanh[c*x]*Log[1 -
 E^(-2*ArcTanh[c*x])] - I*c*d*e*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c*d*e*Ar
cTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c*d*e*
ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c^2*d^2*
Log[(c*x)/Sqrt[1 - c^2*x^2]] - (I/2)*c*d*e*Pi*Log[1 - c^2*x^2] - 2*c*d*e*Ar
cTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + c*d*e*PolyLog
[2, E^(-2*ArcTanh[c*x])] - c*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTa
nh[c*x]))]))/(c*d^3) + (b^2*((-I)*c*d*e*Pi^3 + 24*c^2*d^2*ArcTanh[c*x]^2 -
(24*c*d^2*ArcTanh[c*x]^2)/x + 8*c*d*e*ArcTanh[c*x]^3 + 8*e^2*ArcTanh[c*x]^3
 + 48*c^2*d^2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*c*d*e*ArcTanh[
c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*c^2*d^2*PolyLog[2, E^(-2*ArcTanh[c*
x])] - 24*c*d*e*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 12*c*d*e*Poly
Log[3, E^(2*ArcTanh[c*x])]))/(24*c*d^3) + (b^2*(c*d - e)*e*(c*d + e)*(-3*c*
d*ArcTanh[c*x]^3 + e*ArcTanh[c*x]^3 - (2*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[
c*x]^3)/E^ArcTanh[(c*d)/e] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x
]) + E^ArcTanh[c*x])/2] + 3*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e]
+ ArcTanh[c*x])] + 3*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTa
nh[c*x])] + 6*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)
/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 3*c*d
*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*
```

```
x])))/(2*E^ArcTanh[c*x])] - 3*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 -
 c^2*x^2]] - ((3*I)/2)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 6*c*d*ArcTanh
[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 6*c*d
*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcT
anh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*PolyLog[3,
 -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*PolyLog[3, E^(ArcTanh[(c*d)/
e] + ArcTanh[c*x])]))/(d^3*(3*c^3*d^2 - 3*c*e^2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{ex^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x^3 + d*x^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x^2), x)
```

maple [C] time = 3.77, size = 26776, normalized size = 64.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(e*x+d),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{e\log(ex+d)}{d^2} - \frac{e\log(x)}{d^2} - \frac{1}{dx}\right) - \frac{b^2\log(-cx+1)^2}{4\,dx} - \int -\frac{\left(b^2cdx - b^2d\right)\log(cx+1)^2 + 4\,(abcdx - abd)\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
[Out] a^2*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/4*b^2*log(-c*x + 1)^2
/(d*x) - integrate(-1/4*((b^2*c*d*x - b^2*d)*log(c*x + 1)^2 + 4*(a*b*c*d*x
- a*b*d)*log(c*x + 1) + 2*(b^2*c*e*x^2 + 2*a*b*d - (2*a*b*c*d - b^2*c*d)*x
- (b^2*c*d*x - b^2*d)*log(c*x + 1))*log(-c*x + 1))/(c*d*e*x^4 - d^2*x^2 + (
c*d^2 - d*e)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\operatorname{atanh}(c\,x))^2}{x^2\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + e*x)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atanh}\,(cx))^2}{x^2\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(x**2*(d + e*x)), x)
```

3.159 $\int \frac{\tanh^{-1}(cx)^2}{x(d+ex)} \, dx$

Optimal. Leaf size=275

$$\frac{\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}+\frac{\tanh^{-1}(cx)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d}-\frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d}-\frac{\text{Li}_3\left(\frac{2}{1-cx}\right.}{2d}$$

```
[Out] -2*arctanh(c*x)^2*arctanh(-1+2/(-c*x+1))/d+arctanh(c*x)^2*ln(2/(c*x+1))/d-a
rctanh(c*x)^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d-arctanh(c*x)*polylog(2,1-2/
(-c*x+1))/d+arctanh(c*x)*polylog(2,-1+2/(-c*x+1))/d-arctanh(c*x)*polylog(2,
1-2/(c*x+1))/d+arctanh(c*x)*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d+1/2*
polylog(3,1-2/(-c*x+1))/d-1/2*polylog(3,-1+2/(-c*x+1))/d-1/2*polylog(3,1-2/
(c*x+1))/d+1/2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Rubi [A] time = 0.34, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5922}

$$\frac{\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d}+\frac{\tanh^{-1}(cx)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d}-\frac{\text{PolyLog}\left(3,\frac{2}{1-}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

```
[Out] (2*ArcTanh[c*x]^2*ArcTanh[1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]^2*Log[2/(1 +
c*x)])/d - (ArcTanh[c*x]^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d -
(ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]*PolyLog[2, -1
+ 2/(1 - c*x)])/d - (ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/d + (ArcTanh
[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + PolyLog[3,
 1 - 2/(1 - c*x)]/(2*d) - PolyLog[3, -1 + 2/(1 - c*x)]/(2*d) - PolyLog[3, 1
 - 2/(1 + c*x)]/(2*d) + PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x)
)]/(2*d)
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(cx)^2}{x(d+ex)}\,dx = \int \left(\frac{\tanh^{-1}(cx)^2}{dx} - \frac{e\tanh^{-1}(cx)^2}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\tanh^{-1}(cx)^2}{x}\,dx}{d} - \frac{e\int \frac{\tanh^{-1}(cx)^2}{d+ex}\,dx}{d}$$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - $$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - $$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - $$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - $$

Mathematica [C] time = 9.37, size = 521, normalized size = 1.89

$$16e\sqrt{1-\frac{c^2d^2}{e^2}}\,\tanh^{-1}(cx)^3e^{-\tanh^{-1}\left(\frac{cd}{e}\right)} + 24cd\tanh^{-1}(cx)^2\log\left(\frac{c(d+ex)}{\sqrt{1-c^2x^2}}\right) + 12i\pi cd\log\left(1-c^2x^2\right)\tanh^{-1}(cx) - 48c$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

```
[Out] (I*c*d*Pi^3 + 16*c*d*ArcTanh[c*x]^3 - 16*e*ArcTanh[c*x]^3 + (16*Sqrt[1 - (c
^2*d^2)/e^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (24*I)*c*d*Pi*ArcTanh[c
*x]*Log[2] + c*d*ArcTanh[c*x]^2*Log[16777216] + (24*I)*c*d*Pi*ArcTanh[c*x]*
Log[E^(-ArcTanh[c*x]) + E^ArcTanh[c*x]] + 24*c*d*ArcTanh[c*x]^2*Log[1 - E^(
2*ArcTanh[c*x])] - 24*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcT
anh[c*x])] - 24*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*
x])] - 48*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e]
- ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] - 24*c*d*Ar
cTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])
))/E^ArcTanh[c*x]] + 24*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x
^2]] + (12*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] + 48*c*d*ArcTanh[(c*d)/e
]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 24*c*d*ArcTan
h[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 48*c*d*ArcTanh[c*x]*PolyLog[2, -E^(
ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 48*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTa
nh[(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] + 48*c
*d*PolyLog[3, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 48*c*d*PolyLog[3, E^(
ArcTanh[(c*d)/e] + ArcTanh[c*x])])/(24*c*d^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\,(cx)^2}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arctanh(c*x)^2/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\,(cx)^2}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arctanh(c*x)^2/((e*x + d)*x), x)
```

maple [C] time = 0.18, size = 1507, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c*x)^2/x/(e*x+d),x)
```

```
[Out] arctanh(c*x)^2/d*ln(c*x)-arctanh(c*x)^2/d*ln(c*e*x+c*d)+arctanh(c*x)^2/d*ln
(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))-arctanh(c*x)^
2/d*ln((c*x+1)^2/(-c^2*x^2+1)-1)+1/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+
1)^(1/2))+2/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2/d*polylo
g(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+1/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1
)^(1/2))+2/d*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2/d*polylo
g(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*(((c*x+1)
^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*csgn(I*(((c*x+1)^2/(-
c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2-1/2*I/d*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c
*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*csgn(I*(((c*x+1)
^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2
+1)))-1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1
+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I/d*Pi*arctanh(
```

```
c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1
)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I/d*P
i*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(
-c^2*x^2+1)))^2+1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/d*e/(c*d+e)*arctanh(c*x)^2*ln(1-(c
*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/d*e/(c*d+e)*arctanh(c*x)*polylog(2
,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/2/d*e/(c*d+e)*polylog(3,(c*d+e)
*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-c/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*
x+1)^2/(-c^2*x^2+1)/(-c*d+e))-c/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x
+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/2*c/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^
2*x^2+1)/(-c*d+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(cx)^2}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(c*x)^2/((e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(c\,x)^2}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c*x)^2/(x*(d + e*x)),x)
```

```
[Out] int(atanh(c*x)^2/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(cx)}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c*x)**2/x/(e*x+d),x)
```

```
[Out] Integral(atanh(c*x)**2/(x*(d + e*x)), x)
```

3.160 $\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{aex+ad+(bex+bd)\arctan(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arctan(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\,(a+b\arctan(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a+b\,\mathrm{atan}(c\,x))\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*atan(c*x))*(d + e*x)),x)
```

```
[Out] int(1/((a + b*atan(c*x))*(d + e*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{atan}(cx))\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
```

```
[Out] Integral(1/((a + b*atan(c*x))*(d + e*x)), x)
```

3.161 $\int x^4 \left(1-a^2x^2\right) \tanh^{-1}(ax) \, dx$

Optimal. Leaf size=72

$$\frac{x^2}{35a^3}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{35a^5}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)+\frac{x^4}{70a}$$

```
[Out] 1/35*x^2/a^3+1/70*x^4/a-1/42*a*x^6+1/5*x^5*arctanh(a*x)-1/7*a^2*x^7*arctanh
(a*x)+1/35*ln(-a^2*x^2+1)/a^5
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 266, 43}

$$\frac{x^2}{35a^3}+\frac{\log\left(1-a^2x^2\right)}{35a^5}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{ax^6}{42}+\frac{x^4}{70a}+\frac{1}{5}x^5\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*Ar
cTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^4\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx = -\left(a^2\int x^6\tanh^{-1}(ax)\,dx\right)+\int x^4\tanh^{-1}(ax)\,dx$$

$$= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{5}a\int\frac{x^5}{1-a^2x^2}\,dx+\frac{1}{7}a^3\int\frac{x^7}{1-a^2x^2}\,dx$$

$$= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{10}a\,\text{Subst}\left(\int\frac{x^2}{1-a^2x}\,dx,x,x^2\right)+\frac{1}{14}a^3\,\text{S}$$

$$= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{10}a\,\text{Subst}\left(\int\left(-\frac{1}{a^4}-\frac{x}{a^2}-\frac{1}{a^4\left(-1+a^2x\right.}\right.\right.$$

$$= \frac{x^2}{35a^3}+\frac{x^4}{70a}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{35a^5}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.00

$$\frac{x^2}{35a^3}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{35a^5}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)+\frac{x^4}{70a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*Ar
cTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

fricas [A] time = 0.56, size = 76, normalized size = 1.06

$$-\frac{5\,a^6x^6-3\,a^4x^4-6\,a^2x^2+3\left(5\,a^7x^7-7\,a^5x^5\right)\log\left(-\frac{ax+1}{ax-1}\right)-6\,\log\left(a^2x^2-1\right)}{210\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*x^6 - 3*a^4*x^4 - 6*a^2*x^2 + 3*(5*a^7*x^7 - 7*a^5*x^5)*log(-
(a*x + 1)/(a*x - 1)) - 6*log(a^2*x^2 - 1))/a^5
```

giac [B] time = 0.23, size = 335, normalized size = 4.65

$$\frac{2}{105}a\left(\frac{3\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6}-\frac{3\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^6}-\frac{\frac{3\,(ax+1)^5}{(ax-1)^5}+\frac{36\,(ax+1)^4}{(ax-1)^4}+\frac{2\,(ax+1)^3}{(ax-1)^3}+\frac{36\,(ax+1)^2}{(ax-1)^2}+\frac{3\,(ax+1)}{ax-1}}{a^6\left(\frac{ax+1}{ax-1}-1\right)^6}-\frac{3\left(\frac{35\,(ax+1)^5}{(ax-1)^5}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 2/105*a*(3*log(abs(-a*x - 1)/abs(a*x - 1))/a^6 - 3*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^6 - (3*(a*x + 1)^5/(a*x - 1)^5 + 36*(a*x + 1)^4/(a*x - 1)^4 +
2*(a*x + 1)^3/(a*x - 1)^3 + 36*(a*x + 1)^2/(a*x - 1)^2 + 3*(a*x + 1)/(a*x -
 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^6) - 3*(35*(a*x + 1)^5/(a*x - 1)^5 + 35
*(a*x + 1)^4/(a*x - 1)^4 + 70*(a*x + 1)^3/(a*x - 1)^3 + 14*(a*x + 1)^2/(a*x
```

```
 - 1)^2 + 7*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*
x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x
 - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^7))
```

maple [A] time = 0.02, size = 67, normalized size = 0.93

$$-\frac{a^{2}x^{7}\operatorname{arctanh}(ax)}{7}+\frac{x^{5}\operatorname{arctanh}(ax)}{5}-\frac{x^{6}a}{42}+\frac{x^{4}}{70a}+\frac{x^{2}}{35a^{3}}+\frac{\ln(ax-1)}{35a^{5}}+\frac{\ln(ax+1)}{35a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/7*a^2*x^7*arctanh(a*x)+1/5*x^5*arctanh(a*x)-1/42*x^6*a+1/70*x^4/a+1/35*x
^2/a^3+1/35/a^5*ln(a*x-1)+1/35/a^5*ln(a*x+1)
```

maxima [A] time = 0.31, size = 73, normalized size = 1.01

$$-\frac{1}{210}a\left(\frac{5a^{4}x^{6}-3a^{2}x^{4}-6x^{2}}{a^{4}}-\frac{6\log(ax+1)}{a^{6}}-\frac{6\log(ax-1)}{a^{6}}\right)-\frac{1}{35}\left(5a^{2}x^{7}-7x^{5}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 - 6*log(
a*x - 1)/a^6) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)
```

mupad [B] time = 0.99, size = 61, normalized size = 0.85

$$\frac{\frac{\ln\left(a^{2}x^{2}-1\right)}{35}+\frac{a^{2}x^{2}}{35}+\frac{a^{4}x^{4}}{70}}{a^{5}}-\frac{a\,x^{6}}{42}+\frac{x^{5}\operatorname{atanh}(a\,x)}{5}-\frac{a^{2}\,x^{7}\operatorname{atanh}(a\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] (log(a^2*x^2 - 1)/35 + (a^2*x^2)/35 + (a^4*x^4)/70)/a^5 - (a*x^6)/42 + (x^5
*atanh(a*x))/5 - (a^2*x^7*atanh(a*x))/7
```

sympy [A] time = 1.98, size = 71, normalized size = 0.99

$$\begin{cases}-\frac{a^{2}x^{7}\operatorname{atanh}(ax)}{7}-\frac{ax^{6}}{42}+\frac{x^{5}\operatorname{atanh}(ax)}{5}+\frac{x^{4}}{70a}+\frac{x^{2}}{35a^{3}}+\frac{2\log\left(x-\frac{1}{a}\right)}{35a^{5}}+\frac{2\operatorname{atanh}(ax)}{35a^{5}} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**7*atanh(a*x)/7 - a*x**6/42 + x**5*atanh(a*x)/5 + x**4/(
70*a) + x**2/(35*a**3) + 2*log(x - 1/a)/(35*a**5) + 2*atanh(a*x)/(35*a**5),
 Ne(a, 0)), (0, True))
```

3.162 $\int x^3\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}(ax)}{12a^4}+\frac{x}{12a^3}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{ax^5}{30}+\frac{1}{4}x^4\tanh^{-1}(ax)+\frac{x^3}{36a}$$

```
[Out] 1/12*x/a^3+1/36*x^3/a-1/30*a*x^5-1/12*arctanh(a*x)/a^4+1/4*x^4*arctanh(a*x)
-1/6*a^2*x^6*arctanh(a*x)
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 302, 206}

$$-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{x}{12a^3}-\frac{\tanh^{-1}(ax)}{12a^4}-\frac{ax^5}{30}+\frac{x^3}{36a}+\frac{1}{4}x^4\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 - ArcTanh[a*x]/(12*a^4) + (x^4*ArcTanh
[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^3\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx = -\left(a^2\int x^5\tanh^{-1}(ax)\,dx\right)+\int x^3\tanh^{-1}(ax)\,dx$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{1}{4}a\int\frac{x^4}{1-a^2x^2}\,dx+\frac{1}{6}a^3\int\frac{x^6}{1-a^2x^2}\,d$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{1}{4}a\int\left(-\frac{1}{a^4}-\frac{x^2}{a^2}+\frac{1}{a^4\left(1-a^2x^2\right)}\right)dx$$
$$= \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{\int\frac{1}{1-a^2x^2}\,dx}{6a^3}-\int$$
$$= \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}-\frac{\tanh^{-1}(ax)}{12a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)$$

Mathematica [A] time = 0.02, size = 79, normalized size = 1.25

$$\frac{\log(1-ax)}{24a^4}-\frac{\log(ax+1)}{24a^4}+\frac{x}{12a^3}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{ax^5}{30}+\frac{1}{4}x^4\tanh^{-1}(ax)+\frac{x^3}{36a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcT
anh[a*x])/6 + Log[1 - a*x]/(24*a^4) - Log[1 + a*x]/(24*a^4)
```

fricas [A] time = 0.67, size = 61, normalized size = 0.97

$$-\frac{12\,a^5x^5-10\,a^3x^3-30\,ax+15\left(2\,a^6x^6-3\,a^4x^4+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{360\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/360*(12*a^5*x^5 - 10*a^3*x^3 - 30*a*x + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*l
og(-(a*x + 1)/(a*x - 1)))/a^4
```

giac [B] time = 0.21, size = 227, normalized size = 3.60

$$-\frac{1}{45}a\left(\frac{\frac{45(ax+1)^3}{(ax-1)^3}-\frac{25(ax+1)^2}{(ax-1)^2}+\frac{35(ax+1)}{ax-1}-7}{a^5\left(\frac{ax+1}{ax-1}-1\right)^5}+\frac{30\left(\frac{3(ax+1)^4}{(ax-1)^4}+\frac{2(ax+1)^3}{(ax-1)^3}+\frac{3(ax+1)^2}{(ax-1)^2}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^5\left(\frac{ax+1}{ax-1}-1\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/45*a*((45*(a*x + 1)^3/(a*x - 1)^3 - 25*(a*x + 1)^2/(a*x - 1)^2 + 35*(a*x
 + 1)/(a*x - 1) - 7)/(a^5*((a*x + 1)/(a*x - 1) - 1)^5) + 30*(3*(a*x + 1)^4/
(a*x - 1)^4 + 2*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2)*log(-(
a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/
(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^5*((a*x + 1)/(a*x - 1)
- 1)^6))
```

maple [A] time = 0.03, size = 65, normalized size = 1.03

$$-\frac{a^2x^6\operatorname{arctanh}(ax)}{6}+\frac{x^4\operatorname{arctanh}(ax)}{4}-\frac{a\,x^5}{30}+\frac{x^3}{36a}+\frac{x}{12a^3}+\frac{\ln(ax-1)}{24a^4}-\frac{\ln(ax+1)}{24a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/6*a^2*x^6*arctanh(a*x)+1/4*x^4*arctanh(a*x)-1/30*a*x^5+1/36*x^3/a+1/12*x
/a^3+1/24/a^4*ln(a*x-1)-1/24/a^4*ln(a*x+1)
```

maxima [A] time = 0.31, size = 72, normalized size = 1.14

$$-\frac{1}{360}\,a\left(\frac{2\left(6\,a^4x^5-5\,a^2x^3-15\,x\right)}{a^4}+\frac{15\,\log(ax+1)}{a^5}-\frac{15\,\log(ax-1)}{a^5}\right)-\frac{1}{12}\left(2\,a^2x^6-3\,x^4\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/360*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*l
og(a*x - 1)/a^5) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)
```

mupad [B] time = 0.95, size = 51, normalized size = 0.81

$$\frac{\frac{a\,x}{12}-\frac{\operatorname{atanh}(a\,x)}{12}+\frac{a^3\,x^3}{36}}{a^4}-\frac{a\,x^5}{30}+\frac{x^4\operatorname{atanh}(a\,x)}{4}-\frac{a^2\,x^6\operatorname{atanh}(a\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] ((a*x)/12 - atanh(a*x)/12 + (a^3*x^3)/36)/a^4 - (a*x^5)/30 + (x^4*atanh(a*x
))/4 - (a^2*x^6*atanh(a*x))/6
```

sympy [A] time = 1.55, size = 54, normalized size = 0.86

$$\begin{cases}-\frac{a^2x^6\operatorname{atanh}(ax)}{6}-\frac{ax^5}{30}+\frac{x^4\operatorname{atanh}(ax)}{4}+\frac{x^3}{36a}+\frac{x}{12a^3}-\frac{\operatorname{atanh}(ax)}{12a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**6*atanh(a*x)/6 - a*x**5/30 + x**4*atanh(a*x)/4 + x**3/(
36*a) + x/(12*a**3) - atanh(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

3.163 $\int x^2 \left(1-a^2x^2\right) \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=62

$$-\frac{1}{5}a^2x^5\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{15a^3}-\frac{ax^4}{20}+\frac{1}{3}x^3\tanh^{-1}(ax)+\frac{x^2}{15a}$$

```
[Out] 1/15*x^2/a-1/20*a*x^4+1/3*x^3*arctanh(a*x)-1/5*a^2*x^5*arctanh(a*x)+1/15*ln
(-a^2*x^2+1)/a^3
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 266, 43}

$$\frac{\log\left(1-a^2x^2\right)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{ax^4}{20}+\frac{x^2}{15a}+\frac{1}{3}x^3\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5 +
 Log[1 - a^2*x^2]/(15*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^2(1-a^2x^2)\tanh^{-1}(ax)\,dx = -\left(a^2\int x^4\tanh^{-1}(ax)\,dx\right)+\int x^2\tanh^{-1}(ax)\,dx$$
$$= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{1}{3}a\int\frac{x^3}{1-a^2x^2}\,dx+\frac{1}{5}a^3\int\frac{x^5}{1-a^2x^2}\,dx$$
$$= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{1}{6}a\,\text{Subst}\left(\int\frac{x}{1-a^2x}\,dx,x,x^2\right)+\frac{1}{10}a^3\,\text{S}$$
$$= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{1}{6}a\,\text{Subst}\left(\int\left(-\frac{1}{a^2}-\frac{1}{a^2(-1+a^2x)}\right)dx,\right.$$
$$= \frac{x^2}{15a}-\frac{ax^4}{20}+\frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{15a^3}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.00

$$-\frac{1}{5}a^2x^5\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{15a^3}-\frac{ax^4}{20}+\frac{1}{3}x^3\tanh^{-1}(ax)+\frac{x^2}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5 +
 Log[1 - a^2*x^2]/(15*a^3)
```

fricas [A] time = 0.54, size = 68, normalized size = 1.10

$$-\frac{3a^4x^4-4a^2x^2+2(3a^5x^5-5a^3x^3)\log\left(-\frac{ax+1}{ax-1}\right)-4\log(a^2x^2-1)}{60a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/60*(3*a^4*x^4 - 4*a^2*x^2 + 2*(3*a^5*x^5 - 5*a^3*x^3)*log(-(a*x + 1)/(a*
x - 1)) - 4*log(a^2*x^2 - 1))/a^3
```

giac [B] time = 0.16, size = 268, normalized size = 4.32

$$\frac{2}{15}a\left(\frac{\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4}-\frac{\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^4}-\frac{\frac{(ax+1)^3}{(ax-1)^3}+\frac{4(ax+1)^2}{(ax-1)^2}+\frac{ax+1}{ax-1}}{a^4\left(\frac{ax+1}{ax-1}-1\right)^4}-\frac{\left(\frac{15(ax+1)^3}{(ax-1)^3}+\frac{5(ax+1)^2}{(ax-1)^2}+\frac{5(ax+1)}{ax-1}-1\right)\log\left(-\cdots\right)}{a^4\left(\frac{ax+1}{ax-1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 2/15*a*(log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - log(abs(-(a*x + 1)/(a*x - 1)
+ 1))/a^4 - ((a*x + 1)^3/(a*x - 1)^3 + 4*(a*x + 1)^2/(a*x - 1)^2 + (a*x + 1
)/(a*x - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^4) - (15*(a*x + 1)^3/(a*x - 1)^
3 + 5*(a*x + 1)^2/(a*x - 1)^2 + 5*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x +
1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1)
+ 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^5))
```

maple [A] time = 0.03, size = 59, normalized size = 0.95

$$-\frac{a^{2}x^{5}\operatorname{arctanh}\left(ax\right)}{5}+\frac{x^{3}\operatorname{arctanh}\left(ax\right)}{3}-\frac{x^{4}a}{20}+\frac{x^{2}}{15a}+\frac{\ln\left(ax-1\right)}{15a^{3}}+\frac{\ln\left(ax+1\right)}{15a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/5*a^2*x^5*arctanh(a*x)+1/3*x^3*arctanh(a*x)-1/20*x^4*a+1/15*x^2/a+1/15/a
^3*ln(a*x-1)+1/15/a^3*ln(a*x+1)
```

maxima [A] time = 0.31, size = 65, normalized size = 1.05

$$-\frac{1}{60}a\left(\frac{3a^{2}x^{4}-4x^{2}}{a^{2}}-\frac{4\log\left(ax+1\right)}{a^{4}}-\frac{4\log\left(ax-1\right)}{a^{4}}\right)-\frac{1}{15}\left(3a^{2}x^{5}-5x^{3}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/60*a*((3*a^2*x^4 - 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4)
 - 1/15*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)
```

mupad [B] time = 0.91, size = 53, normalized size = 0.85

$$\frac{\frac{\ln\left(a^{2}x^{2}-1\right)}{15}+\frac{a^{2}x^{2}}{15}}{a^{3}}-\frac{a\,x^{4}}{20}+\frac{x^{3}\operatorname{atanh}\left(a\,x\right)}{3}-\frac{a^{2}\,x^{5}\operatorname{atanh}\left(a\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] (log(a^2*x^2 - 1)/15 + (a^2*x^2)/15)/a^3 - (a*x^4)/20 + (x^3*atanh(a*x))/3
- (a^2*x^5*atanh(a*x))/5
```

sympy [A] time = 1.22, size = 63, normalized size = 1.02

$$\begin{cases}-\frac{a^{2}x^{5}\operatorname{atanh}\left(ax\right)}{5}-\frac{ax^{4}}{20}+\frac{x^{3}\operatorname{atanh}\left(ax\right)}{3}+\frac{x^{2}}{15a}+\frac{2\log\left(x-\frac{1}{a}\right)}{15a^{3}}+\frac{2\operatorname{atanh}\left(ax\right)}{15a^{3}} & \text{for}\ a\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**5*atanh(a*x)/5 - a*x**4/20 + x**3*atanh(a*x)/3 + x**2/(
15*a) + 2*log(x - 1/a)/(15*a**3) + 2*atanh(a*x)/(15*a**3), Ne(a, 0)), (0, T
rue))
```

3.164 $\int x\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=40

$$-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{4a^2}-\frac{ax^3}{12}+\frac{x}{4a}$$

```
[Out] 1/4*x/a-1/12*a*x^3-1/4*(-a^2*x^2+1)^2*arctanh(a*x)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5994}

$$-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{4a^2}-\frac{ax^3}{12}+\frac{x}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(4*a) - (a*x^3)/12 - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*a^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx &= -\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{4a^2}+\frac{\int\left(1-a^2x^2\right)\,dx}{4a}\\ &= \frac{x}{4a}-\frac{ax^3}{12}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.72

$$-\frac{1}{4}a^2x^4\tanh^{-1}(ax)+\frac{\log(1-ax)}{8a^2}-\frac{\log(ax+1)}{8a^2}-\frac{ax^3}{12}+\frac{1}{2}x^2\tanh^{-1}(ax)+\frac{x}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(4*a) - (a*x^3)/12 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTanh[a*x])/4 + Lo
g[1 - a*x]/(8*a^2) - Log[1 + a*x]/(8*a^2)
```

fricas [A] time = 0.58, size = 52, normalized size = 1.30

$$-\frac{2\,a^3x^3-6\,ax+3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{24\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/24*(2*a^3*x^3 - 6*a*x + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1)))/a^2
```

giac [B] time = 0.29, size = 160, normalized size = 4.00

$$-\frac{1}{3}a\left(\frac{\frac{3(ax+1)}{ax-1}-1}{a^3\left(\frac{ax+1}{ax-1}-1\right)^3}+\frac{6(ax+1)^2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^2a^3\left(\frac{ax+1}{ax-1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/3*a*((3*(a*x + 1)/(a*x - 1) - 1)/(a^3*((a*x + 1)/(a*x - 1) - 1)^3) + 6*(
a*x + 1)^2*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) +
1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x - 1
)^2*a^3*((a*x + 1)/(a*x - 1) - 1)^4))
```

maple [A] time = 0.03, size = 57, normalized size = 1.42

$$-\frac{a^2\operatorname{arctanh}(ax)x^4}{4}+\frac{\operatorname{arctanh}(ax)x^2}{2}-\frac{x^3a}{12}+\frac{x}{4a}+\frac{\ln(ax-1)}{8a^2}-\frac{\ln(ax+1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/4*a^2*arctanh(a*x)*x^4+1/2*arctanh(a*x)*x^2-1/12*x^3*a+1/4*x/a+1/8/a^2*l
n(a*x-1)-1/8/a^2*ln(a*x+1)
```

maxima [A] time = 0.30, size = 37, normalized size = 0.92

$$-\frac{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)}{4a^2}-\frac{a^2x^3-3x}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*x^2 - 1)^2*arctanh(a*x)/a^2 - 1/12*(a^2*x^3 - 3*x)/a
```

mupad [B] time = 0.86, size = 44, normalized size = 1.10

$$\frac{x^2\operatorname{atanh}(a\,x)}{2}-\frac{\frac{\operatorname{atanh}(a\,x)}{4}-\frac{a\,x}{4}}{a^2}-\frac{a\,x^3}{12}-\frac{a^2\,x^4\operatorname{atanh}(a\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] (x^2*atanh(a*x))/2 - (atanh(a*x)/4 - (a*x)/4)/a^2 - (a*x^3)/12 - (a^2*x^4*a
tanh(a*x))/4
```

sympy [A] time = 0.85, size = 46, normalized size = 1.15

$$\begin{cases} -\frac{a^2x^4\,\mathrm{atanh}\,(ax)}{4} - \frac{ax^3}{12} + \frac{x^2\,\mathrm{atanh}\,(ax)}{2} + \frac{x}{4a} - \frac{\mathrm{atanh}\,(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**4*atanh(a*x)/4 - a*x**3/12 + x**2*atanh(a*x)/2 + x/(4*a
) - atanh(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

3.165 $\int \left(1-a^2x^2\right) \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=64

$$\frac{1-a^2x^2}{6a}+\frac{\log\left(1-a^2x^2\right)}{3a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2}{3}x\tanh^{-1}(ax)$$

```
[Out] 1/6*(-a^2*x^2+1)/a+2/3*x*arctanh(a*x)+1/3*x*(-a^2*x^2+1)*arctanh(a*x)+1/3*l
n(-a^2*x^2+1)/a
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5942, 5910, 260}

$$\frac{1-a^2x^2}{6a}+\frac{\log\left(1-a^2x^2\right)}{3a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2}{3}x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] (1 - a^2*x^2)/(6*a) + (2*x*ArcTanh[a*x])/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x])
/3 + Log[1 - a^2*x^2]/(3*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx &= \frac{1-a^2x^2}{6a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2}{3}\int \tanh^{-1}(ax)\,dx\\ &= \frac{1-a^2x^2}{6a}+\frac{2}{3}x\tanh^{-1}(ax)+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)-\frac{1}{3}(2a)\int \frac{x}{1-a^2x^2}\,dx\\ &= \frac{1-a^2x^2}{6a}+\frac{2}{3}x\tanh^{-1}(ax)+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 0.73

$$-\frac{1}{3}a^2x^3\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{3a}-\frac{ax^2}{6}+x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] -1/6*(a*x^2) + x*ArcTanh[a*x] - (a^2*x^3*ArcTanh[a*x])/3 + Log[1 - a^2*x^2]
/(3*a)
```

fricas [A] time = 0.48, size = 53, normalized size = 0.83

$$-\frac{a^2x^2 + \left(a^3x^3 - 3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) - 2\,\log\left(a^2x^2 - 1\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*x^2 + (a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 2*log(a^2*x^2
 - 1))/a
```

giac [B] time = 0.18, size = 203, normalized size = 3.17

$$\frac{2}{3}\,a\left(\frac{\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2} - \frac{\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2} - \frac{\left(\frac{3\,(ax+1)}{ax-1}-1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^2\left(\frac{ax+1}{ax-1}-1\right)^3} - \frac{ax+1}{(ax-1)a^2\left(\frac{ax+1}{ax-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 2/3*a*(log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - log(abs(-(a*x + 1)/(a*x - 1) +
 1))/a^2 - (3*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((
a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a
*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^3) - (a*x + 1)/((a*x - 1)
*a^2*((a*x + 1)/(a*x - 1) - 1)^2))
```

maple [A] time = 0.03, size = 48, normalized size = 0.75

$$-\frac{a^2\,\mathrm{arctanh}\,(ax)\,x^3}{3} + x\,\mathrm{arctanh}\,(ax) - \frac{a\,x^2}{6} + \frac{\ln\,(ax-1)}{3a} + \frac{\ln\,(ax+1)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/3*a^2*arctanh(a*x)*x^3+x*arctanh(a*x)-1/6*a*x^2+1/3/a*ln(a*x-1)+1/3/a*ln
(a*x+1)
```

maxima [A] time = 0.31, size = 47, normalized size = 0.73

$$-\frac{1}{6}\left(x^2 - \frac{2\,\log\,(ax+1)}{a^2} - \frac{2\,\log\,(ax-1)}{a^2}\right)a - \frac{1}{3}\left(a^2x^3 - 3\,x\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/6*(x^2 - 2*log(a*x + 1)/a^2 - 2*log(a*x - 1)/a^2)*a - 1/3*(a^2*x^3 - 3*x
)*arctanh(a*x)
```

mupad [B] time = 0.84, size = 40, normalized size = 0.62

$$x \operatorname{atanh}(a x) - \frac{a x^2}{6} + \frac{\ln\left(a^2 x^2 - 1\right)}{3 a} - \frac{a^2 x^3 \operatorname{atanh}(a x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] x*atanh(a*x) - (a*x^2)/6 + log(a^2*x^2 - 1)/(3*a) - (a^2*x^3*atanh(a*x))/3
```

sympy [A] time = 0.68, size = 49, normalized size = 0.77

$$\begin{cases} -\frac{a^2 x^3 \operatorname{atanh}(ax)}{3} - \frac{ax^2}{6} + x \operatorname{atanh}(ax) + \frac{2 \log\left(x - \frac{1}{a}\right)}{3a} + \frac{2 \operatorname{atanh}(ax)}{3a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**3*atanh(a*x)/3 - a*x**2/6 + x*atanh(a*x) + 2*log(x - 1/
a)/(3*a) + 2*atanh(a*x)/(3*a), Ne(a, 0)), (0, True))
```

3.166 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}-\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)$$

```
[Out] -1/2*a*x+1/2*arctanh(a*x)-1/2*a^2*x^2*arctanh(a*x)-1/2*polylog(2,-a*x)+1/2*
polylog(2,a*x)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6014, 5912, 5916, 321, 206}

$$-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
```

```
[Out] -(a*x)/2 + ArcTanh[a*x]/2 - (a^2*x^2*ArcTanh[a*x])/2 - PolyLog[2, -(a*x)]/2
 + PolyLog[2, a*x]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{x}\,dx = -\left(a^2\int x\tanh^{-1}(ax)\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x}\,dx$$
$$= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}+\frac{1}{2}a^3\int \frac{x^2}{1-a^2x^2}\,dx$$
$$= -\frac{ax}{2}-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}+\frac{1}{2}a\int \frac{1}{1-a^2x^2}\,dx$$
$$= -\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.25

$$-\frac{1}{2}a^2x^2\tanh^{-1}(ax)+\frac{1}{2}(\text{Li}_2(ax)-\text{Li}_2(-ax))-\frac{ax}{2}-\frac{1}{4}\log(1-ax)+\frac{1}{4}\log(ax+1)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
[Out] -1/2*(a*x) - (a^2*x^2*ArcTanh[a*x])/2 - Log[1 - a*x]/4 + Log[1 + a*x]/4 + (
-PolyLog[2, -(a*x)] + PolyLog[2, a*x])/2
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}\,(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\text{artanh}\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

maple [A] time = 0.05, size = 69, normalized size = 1.44

$$-\frac{a^2x^2\,\text{arctanh}\,(ax)}{2}+\text{arctanh}\,(ax)\ln(ax)-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{\text{dilog}\,(ax)}{2}-\frac{\text{dilog}\,(ax+1)}{2}-\frac{\ln(ax)\ln}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x,x)
[Out] -1/2*a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*a*x-1/4*ln(a*x-1)+1/4*ln
(a*x+1)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

maxima [B] time = 0.31, size = 89, normalized size = 1.85

$$-\frac{1}{4}a\left(2x+\frac{2\left(\log(ax+1)\log(x)+\text{Li}_2(-ax)\right)}{a}-\frac{2\left(\log(-ax+1)\log(x)+\text{Li}_2(ax)\right)}{a}-\frac{\log(ax+1)}{a}+\frac{\log(ax}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*x + 2*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 2*(log(-a*x + 1)*lo
g(x) + dilog(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) - 1/2*(a^2*x^2 - lo
g(x^2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x,x)
```

```
[Out] -int((atanh(a*x)*(a^2*x^2 - 1))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\operatorname{atanh}(ax)}{x}\right) dx - \int a^2x\operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x,x)
```

```
[Out] -Integral(-atanh(a*x)/x, x) - Integral(a**2*x*atanh(a*x), x)
```

3.167 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=38

$$-a\log\left(1-a^2x^2\right)+a^2(-x)\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

[Out] -arctanh(a*x)/x-a^2*x*arctanh(a*x)+a*ln(x)-a*ln(-a^2*x^2+1)

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6014, 5916, 266, 36, 29, 31, 5910, 260}

$$-a\log\left(1-a^2x^2\right)+a^2(-x)\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]

[Out] -(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{x^2}\,dx &= -\left(a^2\int \tanh^{-1}(ax)\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx+a^3\int \frac{x}{1-a^2x^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\left(1-a^2x\right)}\,dx\right.\\ &= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\\ &= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\log(x)-a\log\left(1-a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$-a\log\left(1-a^2x^2\right)+a^2(-x)\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]
[Out] -(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]
```

fricas [A] time = 0.59, size = 51, normalized size = 1.34

$$-\frac{2\,ax\log\left(a^2x^2-1\right)-2\,ax\log(x)+\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="fricas")
[Out] -1/2*(2*a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + (a^2*x^2 + 1)*log(-(a*x + 1)/
(a*x - 1)))/x
```

giac [B] time = 0.38, size = 145, normalized size = 3.82

$$-a\left(\frac{2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{\frac{(ax+1)^2}{(ax-1)^2}-1}+\log\left(\frac{(ax+1)^2}{(ax-1)^2}\right)-\log\left(\left|\frac{(ax+1)^2}{(ax-1)^2}-1\right|\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] -a*(2*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a
*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)^2/(
a*x - 1)^2 - 1) + log((a*x + 1)^2/(a*x - 1)^2) - log(abs((a*x + 1)^2/(a*x -
 1)^2 - 1)))
```

maple [A] time = 0.03, size = 45, normalized size = 1.18

$$-a^{2}x \operatorname{arctanh}(ax) - \frac{\operatorname{arctanh}(ax)}{x} + a\ln(ax) - a\ln(ax-1) - a\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^2,x)
```

```
[Out] -a^2*x*arctanh(a*x)-arctanh(a*x)/x+a*ln(a*x)-a*ln(a*x-1)-a*ln(a*x+1)
```

maxima [A] time = 0.31, size = 36, normalized size = 0.95

$$-a\left(\log(ax+1) + \log(ax-1) - \log(x)\right) - \left(a^{2}x + \frac{1}{x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -a*(log(a*x + 1) + log(a*x - 1) - log(x)) - (a^2*x + 1/x)*arctanh(a*x)
```

mupad [B] time = 0.81, size = 37, normalized size = 0.97

$$a\ \ln(x) - a\ \ln\left(a^{2}\,x^{2}-1\right) - \frac{\operatorname{atanh}(a\,x)}{x} - a^{2}\,x\operatorname{atanh}(a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^2,x)
```

```
[Out] a*log(x) - a*log(a^2*x^2 - 1) - atanh(a*x)/x - a^2*x*atanh(a*x)
```

sympy [A] time = 0.85, size = 41, normalized size = 1.08

$$\begin{cases} -a^{2}x \operatorname{atanh}(ax) + a\log(x) - 2a\log\left(x - \frac{1}{a}\right) - 2a\operatorname{atanh}(ax) - \frac{\operatorname{atanh}(ax)}{x} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**2,x)
```

```
[Out] Piecewise((-a**2*x*atanh(a*x) + a*log(x) - 2*a*log(x - 1/a) - 2*a*atanh(a*x
) - atanh(a*x)/x, Ne(a, 0)), (0, True))
```

3.168 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=56

$$\frac{1}{2}a^2\text{Li}_2(-ax) - \frac{1}{2}a^2\text{Li}_2(ax) + \frac{1}{2}a^2\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

```
[Out] -1/2*a/x+1/2*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1/2*a^2*polylog(2,-a*x)-
1/2*a^2*polylog(2,a*x)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6014, 5916, 325, 206, 5912}

$$\frac{1}{2}a^2\text{PolyLog}(2,-ax) - \frac{1}{2}a^2\text{PolyLog}(2,ax) + \frac{1}{2}a^2\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -a/(2*x) + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*PolyLog[2, -(
a*x)])/2 - (a^2*PolyLog[2, a*x])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
```

```
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^3}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)}{x}\,dx\right) + \int \frac{\tanh^{-1}(ax)}{x^3}\,dx$$
$$= -\frac{\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\text{Li}_2(-ax) - \frac{1}{2}a^2\text{Li}_2(ax) + \frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}\,dx$$
$$= -\frac{a}{2x} - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\text{Li}_2(-ax) - \frac{1}{2}a^2\text{Li}_2(ax) + \frac{1}{2}a^3\int \frac{1}{1-a^2x^2}\,dx$$
$$= -\frac{a}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\text{Li}_2(-ax) - \frac{1}{2}a^2\text{Li}_2(ax)$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.21

$$-\frac{1}{2}a^2(\text{Li}_2(ax) - \text{Li}_2(-ax)) - \frac{1}{4}a^2\log(1-ax) + \frac{1}{4}a^2\log(ax+1) - \frac{\tanh^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*a/x - ArcTanh[a*x]/(2*x^2) - (a^2*Log[1 - a*x])/4 + (a^2*Log[1 + a*x])
/4 - (a^2*(-PolyLog[2, -(a*x)] + PolyLog[2, a*x]))/2
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(a^2x^2-1)\,\text{artanh}\,(ax)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)/x^3, x)
```

giac [B] time = 2.18, size = 330, normalized size = 5.89

$$a^{2}\left(\frac{\log\left(\frac{(ax+1)^{2}}{(ax-1)^{2}}\right)}{a}-\frac{\log\left(\left|\frac{(ax+1)^{2}}{(ax-1)^{2}}-1\right|\right)}{a}+\frac{\frac{(ax+1)^{2}}{(ax-1)^{2}}-2}{a\left(\frac{(ax+1)^{2}}{(ax-1)^{2}}-1\right)}-\frac{2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}-1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}+1}\right)}{a\left(\frac{(ax+1)^{2}}{(ax-1)^{2}}-1\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] a^2*(log((a*x + 1)^2/(a*x - 1)^2)/a - log(abs((a*x + 1)^2/(a*x - 1)^2 - 1))
/a + ((a*x + 1)^2/(a*x - 1)^2 - 2)/(a*((a*x + 1)^2/(a*x - 1)^2 - 1)) - 2*lo
g(-(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(a*x - 1) + 1)/((a*x +
 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x -
1) - a) - 1)) - 1)/(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(a*x -
 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*
x + 1)*a/(a*x - 1) - a) - 1)) + 1))/(a*((a*x + 1)^2/(a*x - 1)^2 - 1)^2))
```

maple [A] time = 0.05, size = 87, normalized size = 1.55

$$-a^{2}\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{2x^{2}}-\frac{a}{2x}-\frac{a^{2}\ln(ax-1)}{4}+\frac{a^{2}\ln(ax+1)}{4}+\frac{a^{2}\operatorname{dilog}(ax)}{2}+\frac{a^{2}\operatorname{dilog}(ax+1)}{2}+\frac{a^{2}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^3,x)
```

```
[Out] -a^2*arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)/x^2-1/2*a/x-1/4*a^2*ln(a*x-1)+1/
4*a^2*ln(a*x+1)+1/2*a^2*dilog(a*x)+1/2*a^2*dilog(a*x+1)+1/2*a^2*ln(a*x)*ln(
a*x+1)
```

maxima [A] time = 0.32, size = 81, normalized size = 1.45

$$\frac{1}{4}\left(2\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)a-2\left(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax)\right)a+a\log(ax+1)-a\log(ax-1)-\frac{2}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + di
log(a*x))*a + a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a - 1/2*(a^2*log(x^2)
+ 1/x^2)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^3,x)
[Out] -int((atanh(a*x)*(a^2*x^2 - 1))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\operatorname{atanh}(ax)}{x^3}\right)dx-\int \frac{a^2\operatorname{atanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**3,x)
[Out] -Integral(-atanh(a*x)/x**3, x) - Integral(a**2*atanh(a*x)/x, x)
```

3.169 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=58

$$-\frac{2}{3}a^3\log(x)+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{3}a^3\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{3x^3}-\frac{a}{6x^2}$$

```
[Out] -1/6*a/x^2-1/3*arctanh(a*x)/x^3+a^2*arctanh(a*x)/x-2/3*a^3*ln(x)+1/3*a^3*ln
(-a^2*x^2+1)
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6014, 5916, 266, 44, 36, 29, 31}

$$\frac{1}{3}a^3\log\left(1-a^2x^2\right)-\frac{2}{3}a^3\log(x)+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/3
 + (a^3*Log[1 - a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{3}a\int \frac{1}{x^3(1-a^2x^2)}\,dx-a^3\int \frac{1}{x(1-a^2x^2)}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{x^2(1-a^2x)}\,dx,x,x^2\right)-\frac{1}{2}a^3\,\text{Su}\\ &= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{6}a\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{a^2}{x}-\frac{a^4}{-1+a^2x}\right)dx,x,x^2\right)\\ &= -\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{2}{3}a^3\log(x)+\frac{1}{3}a^3\log(1-a^2x^2)\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.00

$$-\frac{2}{3}a^3\log(x)+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{3}a^3\log(1-a^2x^2)-\frac{\tanh^{-1}(ax)}{3x^3}-\frac{a}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/3
 + (a^3*Log[1 - a^2*x^2])/3
```

fricas [A] time = 0.59, size = 64, normalized size = 1.10

$$\frac{2\,a^3x^3\log(a^2x^2-1)-4\,a^3x^3\log(x)-ax+(3\,a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*x^3*log(a^2*x^2 - 1) - 4*a^3*x^3*log(x) - a*x + (3*a^2*x^2 - 1)*
log(-(a*x + 1)/(a*x - 1)))/x^3
```

giac [B] time = 0.21, size = 204, normalized size = 3.52

$$\frac{2}{3}\left(a^2\log\left(\frac{|-ax-1|}{|ax-1|}\right)-a^2\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)+\frac{(ax+1)a^2}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^2}-\frac{\left(\frac{3(ax+1)a^2}{ax-1}+a^2\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{\left(\frac{ax+1}{ax-1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(a^2*log(abs(-a*x - 1)/abs(a*x - 1)) - a^2*log(abs(-(a*x + 1)/(a*x - 1)
 - 1)) + (a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2) - (3*(a*x +
1)*a^2/(a*x - 1) + a^2)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x
 - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) -
1))/((a*x + 1)/(a*x - 1) + 1)^3)*a
```

maple [A] time = 0.04, size = 59, normalized size = 1.02

$$\frac{a^2 \operatorname{arctanh}(ax)}{x} - \frac{\operatorname{arctanh}(ax)}{3x^3} - \frac{a}{6x^2} - \frac{2a^3 \ln(ax)}{3} + \frac{a^3 \ln(ax-1)}{3} + \frac{a^3 \ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^4,x)
```

```
[Out] a^2*arctanh(a*x)/x-1/3*arctanh(a*x)/x^3-1/6*a/x^2-2/3*a^3*ln(a*x)+1/3*a^3*l
n(a*x-1)+1/3*a^3*ln(a*x+1)
```

maxima [A] time = 0.31, size = 53, normalized size = 0.91

$$\frac{1}{6}\left(2\,a^2 \log\left(a^2x^2-1\right) - 2\,a^2 \log\left(x^2\right) - \frac{1}{x^2}\right)a + \frac{\left(3\,a^2x^2-1\right)\operatorname{artanh}(ax)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*log(x^2) - 1/x^2)*a + 1/3*(3*a^2*x^2 -
1)*arctanh(a*x)/x^3
```

mupad [B] time = 0.85, size = 49, normalized size = 0.84

$$\frac{a^3\,\ln\left(a^2\,x^2-1\right)}{3} - \frac{a}{6\,x^2} - \frac{\operatorname{atanh}(a\,x)}{3\,x^3} - \frac{2\,a^3\,\ln(x)}{3} + \frac{a^2\operatorname{atanh}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^4,x)
```

```
[Out] (a^3*log(a^2*x^2 - 1))/3 - a/(6*x^2) - atanh(a*x)/(3*x^3) - (2*a^3*log(x))/
3 + (a^2*atanh(a*x))/x
```

sympy [A] time = 1.19, size = 63, normalized size = 1.09

$$\begin{cases} -\frac{2a^3\log(x)}{3} + \frac{2a^3\log\left(x-\frac{1}{a}\right)}{3} + \frac{2a^3\operatorname{atanh}(ax)}{3} + \frac{a^2\operatorname{atanh}(ax)}{x} - \frac{a}{6x^2} - \frac{\operatorname{atanh}(ax)}{3x^3} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**4,x)
```

```
[Out] Piecewise((-2*a**3*log(x)/3 + 2*a**3*log(x - 1/a)/3 + 2*a**3*atanh(a*x)/3 +
 a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3), Ne(a, 0)), (0, True)
)
```

3.170 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=42

$$\frac{a^3}{4x} - \frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4} - \frac{a}{12x^3}$$

```
[Out] -1/12*a/x^3+1/4*a^3/x-1/4*(-a^2*x^2+1)^2*arctanh(a*x)/x^4
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6008, 14}

$$-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4} + \frac{a^3}{4x} - \frac{a}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -a/(12*x^3) + a^3/(4*x) - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^5}\,dx &= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4} + \frac{1}{4}a\int \frac{1-a^2x^2}{x^4}\,dx\\
&= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4} + \frac{1}{4}a\int \left(\frac{1}{x^4} - \frac{a^2}{x^2}\right)dx\\
&= -\frac{a}{12x^3} + \frac{a^3}{4x} - \frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.69

$$\frac{1}{8}a^4\log(1-ax) - \frac{1}{8}a^4\log(ax+1) + \frac{a^3}{4x} + \frac{a^2\tanh^{-1}(ax)}{2x^2} - \frac{\tanh^{-1}(ax)}{4x^4} - \frac{a}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -1/12*a/x^3 + a^3/(4*x) - ArcTanh[a*x]/(4*x^4) + (a^2*ArcTanh[a*x])/(2*x^2)
 + (a^4*Log[1 - a*x])/8 - (a^4*Log[1 + a*x])/8
```

fricas [A] time = 1.41, size = 52, normalized size = 1.24

$$\frac{6\,a^3x^3 - 2\,ax - 3\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 - 2*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1)))/x^4
```

giac [B] time = 0.18, size = 160, normalized size = 3.81

$$-\frac{1}{3}\,a\left(\frac{a^3\left(\frac{3\,(ax+1)}{ax-1}+1\right)}{\left(\frac{ax+1}{ax-1}+1\right)^3}+\frac{6\,(ax+1)^2a^3\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^2\left(\frac{ax+1}{ax-1}+1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] -1/3*a*(a^3*(3*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)/(a*x - 1) + 1)^3 + 6*(a*
x + 1)^2*a^3*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a)
+ 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x -
 1)^2*((a*x + 1)/(a*x - 1) + 1)^4))
```

maple [A] time = 0.04, size = 59, normalized size = 1.40

$$\frac{a^2\operatorname{arctanh}(ax)}{2x^2}-\frac{\operatorname{arctanh}(ax)}{4x^4}+\frac{a^3}{4x}-\frac{a}{12x^3}+\frac{a^4\ln(ax-1)}{8}-\frac{a^4\ln(ax+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^5,x)
```

```
[Out] 1/2*a^2*arctanh(a*x)/x^2-1/4*arctanh(a*x)/x^4+1/4*a^3/x-1/12*a/x^3+1/8*a^4*
ln(a*x-1)-1/8*a^4*ln(a*x+1)
```

maxima [A] time = 0.30, size = 61, normalized size = 1.45

$$-\frac{1}{24}\left(3\,a^3\log(ax+1) - 3\,a^3\log(ax-1) - \frac{2\left(3\,a^2x^2-1\right)}{x^3}\right)a + \frac{\left(2\,a^2x^2-1\right)\operatorname{artanh}(ax)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 - 1)/x^3)*a +
 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/x^4
```

mupad [B] time = 0.84, size = 61, normalized size = 1.45

$$\frac{a^3}{4\,x} - \frac{\operatorname{atanh}(a\,x)}{4\,x^4} - \frac{a}{12\,x^3} + \frac{a^5 \operatorname{atan}\left(\frac{a^2\,x}{\sqrt{-a^2}}\right)}{4\,\sqrt{-a^2}} + \frac{a^2 \operatorname{atanh}(a\,x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^5,x)
```

```
[Out] a^3/(4*x) - atanh(a*x)/(4*x^4) - a/(12*x^3) + (a^5*atan((a^2*x)/(-a^2)^(1/2
)))/(4*(-a^2)^(1/2)) + (a^2*atanh(a*x))/(2*x^2)
```

sympy [A] time = 0.89, size = 46, normalized size = 1.10

$$-\frac{a^4 \operatorname{atanh}(ax)}{4} + \frac{a^3}{4x} + \frac{a^2 \operatorname{atanh}(ax)}{2x^2} - \frac{a}{12x^3} - \frac{\operatorname{atanh}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**5,x)
```

```
[Out] -a**4*atanh(a*x)/4 + a**3/(4*x) + a**2*atanh(a*x)/(2*x**2) - a/(12*x**3) -
atanh(a*x)/(4*x**4)
```

3.171 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=71

$$-\frac{2}{15}a^5\log(x)+\frac{a^3}{15x^2}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{15}a^5\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{5x^5}-\frac{a}{20x^4}$$

```
[Out] -1/20*a/x^4+1/15*a^3/x^2-1/5*arctanh(a*x)/x^5+1/3*a^2*arctanh(a*x)/x^3-2/15
*a^5*ln(x)+1/15*a^5*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 266, 44}

$$\frac{a^3}{15x^2}+\frac{1}{15}a^5\log\left(1-a^2x^2\right)+\frac{a^2\tanh^{-1}(ax)}{3x^3}-\frac{2}{15}a^5\log(x)-\frac{a}{20x^4}-\frac{\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] -a/(20*x^4) + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*x
^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^6}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^6}\,dx$$
$$= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{5}a\int \frac{1}{x^5(1-a^2x^2)}\,dx-\frac{1}{3}a^3\int \frac{1}{x^3(1-a^2x^2)}$$
$$= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{10}a\,\text{Subst}\left(\int \frac{1}{x^3(1-a^2x)}\,dx,x,x^2\right)-\frac{1}{6}a^3\,\text{Su}$$
$$= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{10}a\,\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{a^2}{x^2}+\frac{a^4}{x}-\frac{a^6}{-1+a^2x}\right)dx\right.$$
$$= -\frac{a}{20x^4}+\frac{a^3}{15x^2}-\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}-\frac{2}{15}a^5\log(x)+\frac{1}{15}a^5\log(1-a$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.00

$$-\frac{2}{15}a^5\log(x)+\frac{a^3}{15x^2}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{15}a^5\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{5x^5}-\frac{a}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] -1/20*a/x^4 + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*x
^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

fricas [A] time = 0.52, size = 73, normalized size = 1.03

$$\frac{4a^5x^5\log\left(a^2x^2-1\right)-8a^5x^5\log(x)+4a^3x^3-3ax+2\left(5a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(4*a^5*x^5*log(a^2*x^2 - 1) - 8*a^5*x^5*log(x) + 4*a^3*x^3 - 3*a*x + 2
*(5*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

giac [B] time = 0.21, size = 281, normalized size = 3.96

$$\frac{2}{15}\left(a^4\log\left(\frac{|-ax-1|}{|ax-1|}\right)-a^4\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)+\frac{\frac{(ax+1)^3a^4}{(ax-1)^3}-\frac{4(ax+1)^2a^4}{(ax-1)^2}+\frac{(ax+1)a^4}{ax-1}}{\left(\frac{ax+1}{ax-1}+1\right)^4}-\frac{\left(\frac{15(ax+1)^3a^4}{(ax-1)^3}-\frac{5(ax+1)^2a^4}{(ax-1)^2}+\right.}{\left(\frac{ax}{ax}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(a^4*log(abs(-a*x - 1)/abs(a*x - 1)) - a^4*log(abs(-(a*x + 1)/(a*x - 1
) - 1)) + ((a*x + 1)^3*a^4/(a*x - 1)^3 - 4*(a*x + 1)^2*a^4/(a*x - 1)^2 + (a
*x + 1)*a^4/(a*x - 1))/((a*x + 1)/(a*x - 1) + 1)^4 - (15*(a*x + 1)^3*a^4/(a
*x - 1)^3 - 5*(a*x + 1)^2*a^4/(a*x - 1)^2 + 5*(a*x + 1)*a^4/(a*x - 1) + a^4
```

```
)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a
*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)/(a*x -
1) + 1)^5)*a
```

maple [A] time = 0.04, size = 68, normalized size = 0.96

$$\frac{a^{2}\operatorname{arctanh}(ax)}{3x^{3}}-\frac{\operatorname{arctanh}(ax)}{5x^{5}}-\frac{a}{20x^{4}}+\frac{a^{3}}{15x^{2}}-\frac{2a^{5}\ln(ax)}{15}+\frac{a^{5}\ln(ax-1)}{15}+\frac{a^{5}\ln(ax+1)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^6,x)
```

```
[Out] 1/3*a^2*arctanh(a*x)/x^3-1/5*arctanh(a*x)/x^5-1/20*a/x^4+1/15*a^3/x^2-2/15*
a^5*ln(a*x)+1/15*a^5*ln(a*x-1)+1/15*a^5*ln(a*x+1)
```

maxima [A] time = 0.32, size = 62, normalized size = 0.87

$$\frac{1}{60}\left(4\,a^{4}\log\left(a^{2}x^{2}-1\right)-4\,a^{4}\log\left(x^{2}\right)+\frac{4\,a^{2}x^{2}-3}{x^{4}}\right)a+\frac{\left(5\,a^{2}x^{2}-3\right)\operatorname{artanh}(ax)}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/60*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x^2 - 3)/x^4)*a + 1/
15*(5*a^2*x^2 - 3)*arctanh(a*x)/x^5
```

mupad [B] time = 0.88, size = 59, normalized size = 0.83

$$\frac{a^{5}\,\ln\left(a^{2}\,x^{2}-1\right)}{15}-\frac{\frac{\operatorname{atanh}(a\,x)}{5}+\frac{a\,x}{20}-\frac{a^{3}\,x^{3}}{15}-\frac{a^{2}\,x^{2}\operatorname{atanh}(a\,x)}{3}}{x^{5}}-\frac{2\,a^{5}\,\ln(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^6,x)
```

```
[Out] (a^5*log(a^2*x^2 - 1))/15 - (atanh(a*x)/5 + (a*x)/20 - (a^3*x^3)/15 - (a^2*
x^2*atanh(a*x))/3)/x^5 - (2*a^5*log(x))/15
```

sympy [A] time = 1.88, size = 75, normalized size = 1.06

$$\begin{cases}-\frac{2a^{5}\log(x)}{15}+\frac{2a^{5}\log\left(x-\frac{1}{a}\right)}{15}+\frac{2a^{5}\operatorname{atanh}(ax)}{15}+\frac{a^{3}}{15x^{2}}+\frac{a^{2}\operatorname{atanh}(ax)}{3x^{3}}-\frac{a}{20x^{4}}-\frac{\operatorname{atanh}(ax)}{5x^{5}} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**6,x)
```

```
[Out] Piecewise((-2*a**5*log(x)/15 + 2*a**5*log(x - 1/a)/15 + 2*a**5*atanh(a*x)/1
5 + a**3/(15*x**2) + a**2*atanh(a*x)/(3*x**3) - a/(20*x**4) - atanh(a*x)/(5
*x**5), Ne(a, 0)), (0, True))
```

3.172 $\int x^4 \left(1-a^2x^2\right) \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=162

$$-\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{35a^5}+\frac{2\tanh^{-1}(ax)^2}{35a^5}-\frac{4\tanh^{-1}(ax)}{105a^5}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{35a^5}+\frac{4x}{105a^4}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}-\frac{1}{7}a^2x^7\tan$$

```
[Out] 4/105*x/a^4-2/315*x^3/a^2-1/105*x^5-4/105*arctanh(a*x)/a^5+2/35*x^2*arctanh
(a*x)/a^3+1/35*x^4*arctanh(a*x)/a-1/21*a*x^6*arctanh(a*x)+2/35*arctanh(a*x)
^2/a^5+1/5*x^5*arctanh(a*x)^2-1/7*a^2*x^7*arctanh(a*x)^2-4/35*arctanh(a*x)*
ln(2/(-a*x+1))/a^5-2/35*polylog(2,1-2/(-a*x+1))/a^5
```

Rubi [A] time = 0.58, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6014, 5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a^5}-\frac{2x^3}{315a^2}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{4x}{105a^4}+\frac{2\tanh^{-1}(ax)^2}{35a^5}-\frac{4\tanh^{-1}(ax)}{105a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (4*x)/(105*a^4) - (2*x^3)/(315*a^2) - x^5/105 - (4*ArcTanh[a*x])/(105*a^5)
+ (2*x^2*ArcTanh[a*x])/(35*a^3) + (x^4*ArcTanh[a*x])/(35*a) - (a*x^6*ArcTan
h[a*x])/21 + (2*ArcTanh[a*x]^2)/(35*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (a^2*x^
7*ArcTanh[a*x]^2)/7 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(35*a^5) - (2*PolyL
og[2, 1 - 2/(1 - a*x)])/(35*a^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^4 \left(1-a^2x^2\right) \tanh^{-1}(ax)^2 \, dx = -\left(a^2 \int x^6 \tanh^{-1}(ax)^2 \, dx\right) + \int x^4 \tanh^{-1}(ax)^2 \, dx$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax)^2 - \frac{1}{7}a^2x^7 \tanh^{-1}(ax)^2 - \frac{1}{5}(2a) \int \frac{x^5 \tanh^{-1}(ax)}{1-a^2x^2} \, dx + \frac{1}{7}\left(2a\right.$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax)^2 - \frac{1}{7}a^2x^7 \tanh^{-1}(ax)^2 + \frac{2\int x^3 \tanh^{-1}(ax)\, dx}{5a} - \frac{2\int \frac{x^3 \tanh^{-}}{1-a^2}}{5a}$$
$$= \frac{x^4 \tanh^{-1}(ax)}{10a} - \frac{1}{21}ax^6 \tanh^{-1}(ax) + \frac{1}{5}x^5 \tanh^{-1}(ax)^2 - \frac{1}{7}a^2x^7 \tanh^{-1}(ax)^2$$
$$= \frac{x^2 \tanh^{-1}(ax)}{5a^3} + \frac{x^4 \tanh^{-1}(ax)}{35a} - \frac{1}{21}ax^6 \tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \mathrm{t}$$
$$= \frac{53x}{210a^4} + \frac{11x^3}{630a^2} - \frac{x^5}{105} + \frac{2x^2 \tanh^{-1}(ax)}{35a^3} + \frac{x^4 \tanh^{-1}(ax)}{35a} - \frac{1}{21}ax^6 \tanh^{-1}(a$$
$$= \frac{4x}{105a^4} - \frac{2x^3}{315a^2} - \frac{x^5}{105} - \frac{53 \tanh^{-1}(ax)}{210a^5} + \frac{2x^2 \tanh^{-1}(ax)}{35a^3} + \frac{x^4 \tanh^{-1}(ax)}{35a}$$
$$= \frac{4x}{105a^4} - \frac{2x^3}{315a^2} - \frac{x^5}{105} - \frac{4 \tanh^{-1}(ax)}{105a^5} + \frac{2x^2 \tanh^{-1}(ax)}{35a^3} + \frac{x^4 \tanh^{-1}(ax)}{35a} -$$
$$= \frac{4x}{105a^4} - \frac{2x^3}{315a^2} - \frac{x^5}{105} - \frac{4 \tanh^{-1}(ax)}{105a^5} + \frac{2x^2 \tanh^{-1}(ax)}{35a^3} + \frac{x^4 \tanh^{-1}(ax)}{35a} -$$

Mathematica [A] time = 0.96, size = 113, normalized size = 0.70

$$-\frac{3a^5x^5 + 2a^3x^3 + 9\left(5a^7x^7 - 7a^5x^5 + 2\right)\tanh^{-1}(ax)^2 + 3\tanh^{-1}(ax)\left(5a^6x^6 - 3a^4x^4 - 6a^2x^2 + 12\log\left(e^{-2\tanh}\right.\right.}{315a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/315*(-12*a*x + 2*a^3*x^3 + 3*a^5*x^5 + 9*(2 - 7*a^5*x^5 + 5*a^7*x^7)*Arc
Tanh[a*x]^2 + 3*ArcTanh[a*x]*(4 - 6*a^2*x^2 - 3*a^4*x^4 + 5*a^6*x^6 + 12*Lo
g[1 + E^(-2*ArcTanh[a*x])]) - 18*PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^5
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^2x^6 - x^4\right)\mathrm{artanh}\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^6 - x^4)*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2x^2 - 1\right)x^4 \,\mathrm{artanh}\left(ax\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

[Out] integrate(-(a^2*x^2 - 1)*x^4*arctanh(a*x)^2, x)

maple [A] time = 0.06, size = 225, normalized size = 1.39

$$-\frac{a^2x^7 \operatorname{arctanh}(ax)^2}{7}+\frac{x^5 \operatorname{arctanh}(ax)^2}{5}-\frac{a\,x^6 \operatorname{arctanh}(ax)}{21}+\frac{x^4 \operatorname{arctanh}(ax)}{35a}+\frac{2x^2 \operatorname{arctanh}(ax)}{35a^3}+\frac{2 \operatorname{arctanh}(ax)\,\mathrm{l}}{35a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x)

[Out] -1/7*a^2*x^7*arctanh(a*x)^2+1/5*x^5*arctanh(a*x)^2-1/21*a*x^6*arctanh(a*x)+
1/35*x^4*arctanh(a*x)/a+2/35*x^2*arctanh(a*x)/a^3+2/35/a^5*arctanh(a*x)*ln(
a*x-1)+2/35/a^5*arctanh(a*x)*ln(a*x+1)+1/70/a^5*ln(a*x-1)^2-2/35/a^5*dilog(
1/2+1/2*a*x)-1/35/a^5*ln(a*x-1)*ln(1/2+1/2*a*x)-1/70/a^5*ln(a*x+1)^2-1/35/a
^5*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/35/a^5*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/1
05*x^5-2/315*x^3/a^2+4/105*x/a^4+2/105/a^5*ln(a*x-1)-2/105/a^5*ln(a*x+1)

maxima [A] time = 0.32, size = 190, normalized size = 1.17

$$-\frac{1}{630}a^2\left(\frac{6\,a^5x^5+4\,a^3x^3-24\,ax+9\,\log(ax+1)^2-18\,\log(ax+1)\log(ax-1)-9\,\log(ax-1)^2-12\,\log(ax-}{a^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")

[Out] -1/630*a^2*((6*a^5*x^5 + 4*a^3*x^3 - 24*a*x + 9*log(a*x + 1)^2 - 18*log(a*x
 + 1)*log(a*x - 1) - 9*log(a*x - 1)^2 - 12*log(a*x - 1))/a^7 + 36*(log(a*x
- 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^7 + 12*log(a*x + 1)/a^7)
 - 1/105*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 - 6*lo
g(a*x - 1)/a^6)*arctanh(a*x) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int x^4 \operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^4*atanh(a*x)^2*(a^2*x^2 - 1),x)

[Out] -int(x^4*atanh(a*x)^2*(a^2*x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-x^4 \operatorname{atanh}^2(ax)\right)dx-\int a^2x^6 \operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(-a**2*x**2+1)*atanh(a*x)**2,x)

[Out] -Integral(-x**4*atanh(a*x)**2, x) - Integral(a**2*x**6*atanh(a*x)**2, x)

3.173 $\int x^3 \left(1-a^2x^2\right) \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=116

$$-\frac{\tanh^{-1}(ax)^2}{12a^4}+\frac{x\tanh^{-1}(ax)}{6a^3}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2-\frac{x^2}{180a^2}+\frac{7\log\left(1-a^2x^2\right)}{90a^4}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(a$$

```
[Out] -1/180*x^2/a^2-1/60*x^4+1/6*x*arctanh(a*x)/a^3+1/18*x^3*arctanh(a*x)/a-1/15
*a*x^5*arctanh(a*x)-1/12*arctanh(a*x)^2/a^4+1/4*x^4*arctanh(a*x)^2-1/6*a^2*
x^6*arctanh(a*x)^2+7/90*ln(-a^2*x^2+1)/a^4
```

Rubi [A] time = 0.44, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6014, 5916, 5980, 266, 43, 5910, 260, 5948}

$$-\frac{x^2}{180a^2}+\frac{7\log\left(1-a^2x^2\right)}{90a^4}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2+\frac{x\tanh^{-1}(ax)}{6a^3}-\frac{\tanh^{-1}(ax)^2}{12a^4}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -x^2/(180*a^2) - x^4/60 + (x*ArcTanh[a*x])/(6*a^3) + (x^3*ArcTanh[a*x])/(18
*a) - (a*x^5*ArcTanh[a*x])/15 - ArcTanh[a*x]^2/(12*a^4) + (x^4*ArcTanh[a*x]
^2)/4 - (a^2*x^6*ArcTanh[a*x]^2)/6 + (7*Log[1 - a^2*x^2])/(90*a^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int x^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx &= -\left(a^2\int x^5\tanh^{-1}(ax)^2\,dx\right)+\int x^3\tanh^{-1}(ax)^2\,dx\\
&= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2-\frac{1}{2}a\int\frac{x^4\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\frac{1}{3}a^3\int\frac{x^6\,\mathrm{ta}}{1}\\
&= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2+\frac{\int x^2\tanh^{-1}(ax)\,dx}{2a}-\frac{\int\frac{x^2\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{2a}\\
&= \frac{x^3\tanh^{-1}(ax)}{6a}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2-\\
&= \frac{x\tanh^{-1}(ax)}{2a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{4a^4}+\frac{1}{4}x^4\tanh\\
&= \frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{12a^4}+\frac{1}{4}x^4\tanh\\
&= \frac{x^2}{20a^2}-\frac{x^4}{60}+\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{12a^4}\\
&= -\frac{x^2}{180a^2}-\frac{x^4}{60}+\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(}{12a^4}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.76

$$-\frac{3a^4x^4+a^2x^2-14\log\left(1-a^2x^2\right)+15\left(2a^6x^6-3a^4x^4+1\right)\tanh^{-1}(ax)^2+2ax\left(6a^4x^4-5a^2x^2-15\right)\tanh^{-1}(ax)}{180a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/180*(a^2*x^2 + 3*a^4*x^4 + 2*a*x*(-15 - 5*a^2*x^2 + 6*a^4*x^4)*ArcTanh[a
*x] + 15*(1 - 3*a^4*x^4 + 2*a^6*x^6)*ArcTanh[a*x]^2 - 14*Log[1 - a^2*x^2])/
a^4
```

fricas [A] time = 0.62, size = 109, normalized size = 0.94

$$-\frac{12\,a^4x^4+4\,a^2x^2+15\left(2\,a^6x^6-3\,a^4x^4+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(6\,a^5x^5-5\,a^3x^3-15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-56\,\log\left(\right.}{720\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/720*(12*a^4*x^4 + 4*a^2*x^2 + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*log(-(a*x +
 1)/(a*x - 1))^2 + 4*(6*a^5*x^5 - 5*a^3*x^3 - 15*a*x)*log(-(a*x + 1)/(a*x -
 1)) - 56*log(a^2*x^2 - 1))/a^4
```

giac [B] time = 0.17, size = 522, normalized size = 4.50

$$-\frac{1}{45}\left(\frac{15\left(\frac{3\,(ax+1)^4}{(ax-1)^4}+\frac{2\,(ax+1)^3}{(ax-1)^3}+\frac{3\,(ax+1)^2}{(ax-1)^2}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^6a^5}{(ax-1)^6}-\frac{6\,(ax+1)^5a^5}{(ax-1)^5}+\frac{15\,(ax+1)^4a^5}{(ax-1)^4}-\frac{20\,(ax+1)^3a^5}{(ax-1)^3}+\frac{15\,(ax+1)^2a^5}{(ax-1)^2}-\frac{6\,(ax+1)a^5}{ax-1}+a^5}+\frac{\left(\frac{45\,(ax+1)^3}{(ax-1)^3}-\frac{25\,(ax+}{(ax-1)}\right.}{\frac{(ax+1)^5a^5}{(ax-1)^5}-\frac{5\,(ax+1)^4a^5}{(ax-1)^4}+10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/45*(15*(3*(a*x + 1)^4/(a*x - 1)^4 + 2*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x +
 1)^2/(a*x - 1)^2)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^6*a^5/(a*x - 1)^6
 - 6*(a*x + 1)^5*a^5/(a*x - 1)^5 + 15*(a*x + 1)^4*a^5/(a*x - 1)^4 - 20*(a*x
 + 1)^3*a^5/(a*x - 1)^3 + 15*(a*x + 1)^2*a^5/(a*x - 1)^2 - 6*(a*x + 1)*a^5/
(a*x - 1) + a^5) + (45*(a*x + 1)^3/(a*x - 1)^3 - 25*(a*x + 1)^2/(a*x - 1)^2
 + 35*(a*x + 1)/(a*x - 1) - 7)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^5*a^5/(
a*x - 1)^5 - 5*(a*x + 1)^4*a^5/(a*x - 1)^4 + 10*(a*x + 1)^3*a^5/(a*x - 1)^3
 - 10*(a*x + 1)^2*a^5/(a*x - 1)^2 + 5*(a*x + 1)*a^5/(a*x - 1) - a^5) + (7*(
a*x + 1)^3/(a*x - 1)^3 - 2*(a*x + 1)^2/(a*x - 1)^2 + 7*(a*x + 1)/(a*x - 1))
/((a*x + 1)^4*a^5/(a*x - 1)^4 - 4*(a*x + 1)^3*a^5/(a*x - 1)^3 + 6*(a*x + 1)
^2*a^5/(a*x - 1)^2 - 4*(a*x + 1)*a^5/(a*x - 1) + a^5) + 7*log(-(a*x + 1)/(a
*x - 1) + 1)/a^5 - 7*log(-(a*x + 1)/(a*x - 1))/a^5)*a
```

maple [B] time = 0.06, size = 205, normalized size = 1.77

$$-\frac{a^2x^6\operatorname{arctanh}(ax)^2}{6}+\frac{x^4\operatorname{arctanh}(ax)^2}{4}-\frac{a\,x^5\operatorname{arctanh}(ax)}{15}+\frac{x^3\operatorname{arctanh}(ax)}{18a}+\frac{x\operatorname{arctanh}(ax)}{6a^3}+\frac{\operatorname{arctanh}(ax)\ln}{12a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/6*a^2*x^6*arctanh(a*x)^2+1/4*x^4*arctanh(a*x)^2-1/15*a*x^5*arctanh(a*x)+
1/18*x^3*arctanh(a*x)/a+1/6*x*arctanh(a*x)/a^3+1/12/a^4*arctanh(a*x)*ln(a*x
-1)-1/12/a^4*arctanh(a*x)*ln(a*x+1)+1/48/a^4*ln(a*x-1)^2-1/24/a^4*ln(a*x-1)
*ln(1/2+1/2*a*x)+1/48/a^4*ln(a*x+1)^2+1/24/a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*
a*x)-1/24/a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/60*x^4-1/180*x^2/a^2+7/90/a^4*ln
(a*x-1)+7/90/a^4*ln(a*x+1)
```

maxima [A] time = 0.33, size = 146, normalized size = 1.26

$$-\frac{1}{180}a\left(\frac{2\left(6\,a^4x^5-5\,a^2x^3-15\,x\right)}{a^4}+\frac{15\,\log(ax+1)}{a^5}-\frac{15\,\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)-\frac{1}{12}\left(2\,a^2x^6-3\,x^4\right)\operatorname{artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/180*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*l
og(a*x - 1)/a^5)*arctanh(a*x) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)^2 - 1
/720*(12*a^4*x^4 + 4*a^2*x^2 + 2*(15*log(a*x - 1) - 28)*log(a*x + 1) - 15*l
og(a*x + 1)^2 - 15*log(a*x - 1)^2 - 56*log(a*x - 1))/a^4
```

mupad [B] time = 0.98, size = 101, normalized size = 0.87

$$-\frac{a^2\,x^2-14\,\ln\left(a^2\,x^2-1\right)+3\,a^4\,x^4+15\,\mathrm{atanh}\left(a\,x\right)^2-10\,a^3\,x^3\,\mathrm{atanh}\left(a\,x\right)+12\,a^5\,x^5\,\mathrm{atanh}\left(a\,x\right)-30\,a\,x\,\mathrm{atanh}\left(\right.}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] -(a^2*x^2 - 14*log(a^2*x^2 - 1) + 3*a^4*x^4 + 15*atanh(a*x)^2 - 10*a^3*x^3*
atanh(a*x) + 12*a^5*x^5*atanh(a*x) - 30*a*x*atanh(a*x) - 45*a^4*x^4*atanh(a
*x)^2 + 30*a^6*x^6*atanh(a*x)^2)/(180*a^4)
```

sympy [A] time = 2.14, size = 114, normalized size = 0.98

$$\begin{cases} -\frac{a^2x^6\,\mathrm{atanh}^2\,(ax)}{6}-\frac{ax^5\,\mathrm{atanh}\,(ax)}{15}+\frac{x^4\,\mathrm{atanh}^2\,(ax)}{4}-\frac{x^4}{60}+\frac{x^3\,\mathrm{atanh}\,(ax)}{18a}-\frac{x^2}{180a^2}+\frac{x\,\mathrm{atanh}\,(ax)}{6a^3}+\frac{7\log\left(x-\frac{1}{a}\right)}{45a^4}-\frac{\mathrm{atanh}^2\,(ax)}{12a^4}+\frac{7\,\mathrm{atar}}{45} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*x**6*atanh(a*x)**2/6 - a*x**5*atanh(a*x)/15 + x**4*atanh(a
*x)**2/4 - x**4/60 + x**3*atanh(a*x)/(18*a) - x**2/(180*a**2) + x*atanh(a*x
)/(6*a**3) + 7*log(x - 1/a)/(45*a**4) - atanh(a*x)**2/(12*a**4) + 7*atanh(a
*x)/(45*a**4), Ne(a, 0)), (0, True))
```

3.174 $\int x^2 \left(1-a^2x^2\right) \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=138

$$-\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{15a^3}+\frac{2\tanh^{-1}(ax)^2}{15a^3}-\frac{\tanh^{-1}(ax)}{30a^3}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{x}{30a^2}-\frac{1}{10}ax^4\tan$$

```
[Out] 1/30*x/a^2-1/30*x^3-1/30*arctanh(a*x)/a^3+2/15*x^2*arctanh(a*x)/a-1/10*a*x^
4*arctanh(a*x)+2/15*arctanh(a*x)^2/a^3+1/3*x^3*arctanh(a*x)^2-1/5*a^2*x^5*a
rctanh(a*x)^2-4/15*arctanh(a*x)*ln(2/(-a*x+1))/a^3-2/15*polylog(2,1-2/(-a*x
+1))/a^3
```

Rubi [A] time = 0.41, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6014, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 302}

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{x}{30a^2}+\frac{2\tanh^{-1}(ax)^2}{15a^3}-\frac{\tanh^{-1}(ax)}{30a^3}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{15a^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] x/(30*a^2) - x^3/30 - ArcTanh[a*x]/(30*a^3) + (2*x^2*ArcTanh[a*x])/(15*a) -
 (a*x^4*ArcTanh[a*x])/10 + (2*ArcTanh[a*x]^2)/(15*a^3) + (x^3*ArcTanh[a*x]^
2)/3 - (a^2*x^5*ArcTanh[a*x]^2)/5 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a
^3) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(15*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^2 \left(1-a^2x^2\right) \tanh^{-1}(ax)^2\,dx = -\left(a^2 \int x^4 \tanh^{-1}(ax)^2\,dx\right) + \int x^2 \tanh^{-1}(ax)^2\,dx$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax)^2 - \frac{1}{5}a^2x^5 \tanh^{-1}(ax)^2 - \frac{1}{3}(2a) \int \frac{x^3 \tanh^{-1}(ax)}{1-a^2x^2}\,dx + \frac{1}{5}\left(2a\right.$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax)^2 - \frac{1}{5}a^2x^5 \tanh^{-1}(ax)^2 + \frac{2\int x \tanh^{-1}(ax)\,dx}{3a} - \frac{2\int \frac{x \tanh^{-1}(}{1-a^2x^2}}{3a}$$
$$= \frac{x^2 \tanh^{-1}(ax)}{3a} - \frac{1}{10}ax^4 \tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \tanh^{-1}(ax)^2 - \frac{1}{5}a$$
$$= \frac{x}{3a^2} + \frac{2x^2 \tanh^{-1}(ax)}{15a} - \frac{1}{10}ax^4 \tanh^{-1}(ax) + \frac{2\tanh^{-1}(ax)^2}{15a^3} + \frac{1}{3}x^3 \tanh^{-1}($$
$$= \frac{x}{30a^2} - \frac{x^3}{30} - \frac{\tanh^{-1}(ax)}{3a^3} + \frac{2x^2 \tanh^{-1}(ax)}{15a} - \frac{1}{10}ax^4 \tanh^{-1}(ax) + \frac{2\tanh^{-1}}{15a}$$
$$= \frac{x}{30a^2} - \frac{x^3}{30} - \frac{\tanh^{-1}(ax)}{30a^3} + \frac{2x^2 \tanh^{-1}(ax)}{15a} - \frac{1}{10}ax^4 \tanh^{-1}(ax) + \frac{2\tanh^{-1}}{15a}$$
$$= \frac{x}{30a^2} - \frac{x^3}{30} - \frac{\tanh^{-1}(ax)}{30a^3} + \frac{2x^2 \tanh^{-1}(ax)}{15a} - \frac{1}{10}ax^4 \tanh^{-1}(ax) + \frac{2\tanh^{-1}}{15a}$$

Mathematica [A] time = 0.25, size = 95, normalized size = 0.69

$$-\frac{a^3x^3 + 2\left(3a^5x^5 - 5a^3x^3 + 2\right)\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\left(3a^4x^4 - 4a^2x^2 + 8\log\left(e^{-2\tanh^{-1}(ax)}+1\right)+1\right) - 4\mathrm{Li}}{30a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/30*(-(a*x) + a^3*x^3 + 2*(2 - 5*a^3*x^3 + 3*a^5*x^5)*ArcTanh[a*x]^2 + Ar
cTanh[a*x]*(1 - 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^(-2*ArcTanh[a*x])]) - 4
*PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^3
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^2x^4 - x^2\right)\mathrm{artanh}\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^4 - x^2)*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2x^2 - 1\right)x^2\,\mathrm{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x^2*arctanh(a*x)^2, x)
```

maple [A] time = 0.06, size = 205, normalized size = 1.49

$$-\frac{a^2x^5\operatorname{arctanh}(ax)^2}{5}+\frac{x^3\operatorname{arctanh}(ax)^2}{3}-\frac{a\,x^4\operatorname{arctanh}(ax)}{10}+\frac{2x^2\operatorname{arctanh}(ax)}{15a}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{15a^3}+\frac{2\operatorname{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/5*a^2*x^5*arctanh(a*x)^2+1/3*x^3*arctanh(a*x)^2-1/10*a*x^4*arctanh(a*x)+
2/15*x^2*arctanh(a*x)/a+2/15/a^3*arctanh(a*x)*ln(a*x-1)+2/15/a^3*arctanh(a*
x)*ln(a*x+1)+1/30/a^3*ln(a*x-1)^2-2/15/a^3*dilog(1/2+1/2*a*x)-1/15/a^3*ln(a
*x-1)*ln(1/2+1/2*a*x)-1/30/a^3*ln(a*x+1)^2+1/15/a^3*ln(-1/2*a*x+1/2)*ln(a*x
+1)-1/15/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/30*x^3+1/30*x/a^2+1/60/a^3*
ln(a*x-1)-1/60/a^3*ln(a*x+1)
```

maxima [A] time = 0.32, size = 173, normalized size = 1.25

$$-\frac{1}{60}a^2\left(\frac{2a^3x^3-2ax+2\log(ax+1)^2-4\log(ax+1)\log(ax-1)-2\log(ax-1)^2-\log(ax-1)}{a^5}+\frac{8\left(\log(ax\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/60*a^2*((2*a^3*x^3 - 2*a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x -
 1) - 2*log(a*x - 1)^2 - log(a*x - 1))/a^5 + 8*(log(a*x - 1)*log(1/2*a*x +
1/2) + dilog(-1/2*a*x + 1/2))/a^5 + log(a*x + 1)/a^5) - 1/30*a*((3*a^2*x^4
- 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4)*arctanh(a*x) - 1/15
*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int x^2\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] -int(x^2*atanh(a*x)^2*(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-x^2\operatorname{atanh}^2(ax)\right)dx-\int a^2x^4\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] -Integral(-x**2*atanh(a*x)**2, x) - Integral(a**2*x**4*atanh(a*x)**2, x)
```

3.175 $\int x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=95

$$\frac{1-a^2x^2}{12a^2}+\frac{\log\left(1-a^2x^2\right)}{6a^2}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2}+\frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a}+\frac{x\tanh^{-1}(ax)}{3a}$$

```
[Out] 1/12*(-a^2*x^2+1)/a^2+1/3*x*arctanh(a*x)/a+1/6*x*(-a^2*x^2+1)*arctanh(a*x)/
a-1/4*(-a^2*x^2+1)^2*arctanh(a*x)^2/a^2+1/6*ln(-a^2*x^2+1)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5994, 5942, 5910, 260}

$$\frac{1-a^2x^2}{12a^2}+\frac{\log\left(1-a^2x^2\right)}{6a^2}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2}+\frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a}+\frac{x\tanh^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (1 - a^2*x^2)/(12*a^2) + (x*ArcTanh[a*x])/(3*a) + (x*(1 - a^2*x^2)*ArcTanh[
a*x])/(6*a) - ((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/(4*a^2) + Log[1 - a^2*x^2]/(
6*a^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = -\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2} + \frac{\int \left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx}{2a}$$

$$= \frac{1-a^2x^2}{12a^2} + \frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a} - \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2} + \frac{\int \tanh^{-1}(ax)\,d}{3a}$$

$$= \frac{1-a^2x^2}{12a^2} + \frac{x\tanh^{-1}(ax)}{3a} + \frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a} - \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2}$$

$$= \frac{1-a^2x^2}{12a^2} + \frac{x\tanh^{-1}(ax)}{3a} + \frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a} - \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.69

$$\frac{\left(6ax-2a^3x^3\right)\tanh^{-1}(ax) - a^2x^2 + 2\log\left(1-a^2x^2\right) - 3\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (-(a^2*x^2) + (6*a*x - 2*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTanh
[a*x]^2 + 2*Log[1 - a^2*x^2])/(12*a^2)
```

fricas [A] time = 0.53, size = 91, normalized size = 0.96

$$-\frac{4\,a^2x^2 + 3\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 4\left(a^3x^3 - 3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) - 8\,\log\left(a^2x^2-1\right)}{48\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(4*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
+ 4*(a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 8*log(a^2*x^2 - 1))/a^2
```

giac [B] time = 0.18, size = 305, normalized size = 3.21

$$-\frac{1}{3}a\left(\frac{\left(\frac{3\,(ax+1)}{ax-1}-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^3a^3}{(ax-1)^3} - \frac{3\,(ax+1)^2a^3}{(ax-1)^2} + \frac{3\,(ax+1)a^3}{ax-1} - a^3} + \frac{3\,(ax+1)^2\log\left(-\frac{ax+1}{ax-1}\right)^2}{\left(\frac{(ax+1)^4a^3}{(ax-1)^4} - \frac{4\,(ax+1)^3a^3}{(ax-1)^3} + \frac{6\,(ax+1)^2a^3}{(ax-1)^2} - \frac{4\,(ax+1)a^3}{ax-1} + a^3\right)(ax-1)^2} + \frac{}{\left(\frac{(ax+}{(ax}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/3*a*((3*(a*x + 1)/(a*x - 1) - 1)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^3*
a^3/(a*x - 1)^3 - 3*(a*x + 1)^2*a^3/(a*x - 1)^2 + 3*(a*x + 1)*a^3/(a*x - 1)
 - a^3) + 3*(a*x + 1)^2*log(-(a*x + 1)/(a*x - 1))^2/(((a*x + 1)^4*a^3/(a*x
- 1)^4 - 4*(a*x + 1)^3*a^3/(a*x - 1)^3 + 6*(a*x + 1)^2*a^3/(a*x - 1)^2 - 4*
(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)^2) + (a*x + 1)/(((a*x + 1)^2*a^3/(
a*x - 1)^2 - 2*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)) + log(-(a*x + 1)/(
a*x - 1) + 1)/a^3 - log(-(a*x + 1)/(a*x - 1))/a^3)
```

maple [B] time = 0.06, size = 185, normalized size = 1.95

$$-\frac{a^2\,\mathrm{arctanh}\,(ax)^2\,x^4}{4} + \frac{\mathrm{arctanh}\,(ax)^2\,x^2}{2} - \frac{a\,\mathrm{arctanh}\,(ax)\,x^3}{6} + \frac{x\,\mathrm{arctanh}\,(ax)}{2a} + \frac{\mathrm{arctanh}\,(ax)\ln(ax-1)}{4a^2} - \frac{\mathrm{arctanh}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/4*a^2*arctanh(a*x)^2*x^4+1/2*arctanh(a*x)^2*x^2-1/6*a*arctanh(a*x)*x^3+1
/2*x*arctanh(a*x)/a+1/4/a^2*arctanh(a*x)*ln(a*x-1)-1/4/a^2*arctanh(a*x)*ln(
a*x+1)+1/16/a^2*ln(a*x-1)^2-1/8/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/16/a^2*ln(a
*x+1)^2-1/8/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/8/a^2*ln(-1/2*a*x+1/2)*ln(1/2+
1/2*a*x)-1/12*x^2+1/6/a^2*ln(a*x-1)+1/6/a^2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 74, normalized size = 0.78

$$-\frac{\left(a^{2}x^{2}-1\right)^{2}\operatorname{artanh}\left(ax\right)^{2}}{4\,a^{2}}-\frac{\left(x^{2}-\frac{2\,\log(ax+1)}{a^{2}}-\frac{2\,\log(ax-1)}{a^{2}}\right)a+2\left(a^{2}x^{3}-3\,x\right)\operatorname{artanh}\left(ax\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*x^2 - 1)^2*arctanh(a*x)^2/a^2 - 1/12*((x^2 - 2*log(a*x + 1)/a^2 -
 2*log(a*x - 1)/a^2)*a + 2*(a^2*x^3 - 3*x)*arctanh(a*x))/a
```

mupad [B] time = 0.90, size = 77, normalized size = 0.81

$$\frac{x^{2}\operatorname{atanh}\left(a\,x\right)^{2}}{2}-\frac{\operatorname{atanh}\left(a\,x\right)^{2}}{4\,a^{2}}-\frac{x^{2}}{12}+\frac{\ln\left(a^{2}\,x^{2}-1\right)}{6\,a^{2}}+\frac{x\operatorname{atanh}\left(a\,x\right)}{2\,a}-\frac{a\,x^{3}\operatorname{atanh}\left(a\,x\right)}{6}-\frac{a^{2}\,x^{4}\operatorname{atanh}\left(a\,x\right)^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] (x^2*atanh(a*x)^2)/2 - atanh(a*x)^2/(4*a^2) - x^2/12 + log(a^2*x^2 - 1)/(6*
a^2) + (x*atanh(a*x))/(2*a) - (a*x^3*atanh(a*x))/6 - (a^2*x^4*atanh(a*x)^2)
/4
```

sympy [A] time = 1.28, size = 88, normalized size = 0.93

$$\begin{cases} -\frac{a^{2}x^{4}\operatorname{atanh}^{2}\left(ax\right)}{4}-\frac{ax^{3}\operatorname{atanh}\left(ax\right)}{6}+\frac{x^{2}\operatorname{atanh}^{2}\left(ax\right)}{2}-\frac{x^{2}}{12}+\frac{x\operatorname{atanh}\left(ax\right)}{2a}+\frac{\log\left(x-\frac{1}{a}\right)}{3a^{2}}-\frac{\operatorname{atanh}^{2}\left(ax\right)}{4a^{2}}+\frac{\operatorname{atanh}\left(ax\right)}{3a^{2}} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*x**4*atanh(a*x)**2/4 - a*x**3*atanh(a*x)/6 + x**2*atanh(a*
x)**2/2 - x**2/12 + x*atanh(a*x)/(2*a) + log(x - 1/a)/(3*a**2) - atanh(a*x)
**2/(4*a**2) + atanh(a*x)/(3*a**2), Ne(a, 0)), (0, True))
```

3.176 $\int \left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=115

$$\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}-\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{2\tanh^{-1}(ax)^2}{3a}-\frac{4\log\left(\frac{2}{1-ax}\right.}{}$$

```
[Out] -1/3*x+1/3*(-a^2*x^2+1)*arctanh(a*x)/a+2/3*arctanh(a*x)^2/a+2/3*x*arctanh(a
*x)^2+1/3*x*(-a^2*x^2+1)*arctanh(a*x)^2-4/3*arctanh(a*x)*ln(2/(-a*x+1))/a-2
/3*polylog(2,1-2/(-a*x+1))/a
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5944, 5910, 5984, 5918, 2402, 2315, 8}

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{3a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{2\tanh^{-1}(ax)^2}{3a}-$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -x/3 + ((1 - a^2*x^2)*ArcTanh[a*x])/(3*a) + (2*ArcTanh[a*x]^2)/(3*a) + (2*x
*ArcTanh[a*x]^2)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/3 - (4*ArcTanh[a*x]*L
og[2/(1 - a*x)])/(3*a) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
```

```
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2-\frac{\int 1\,dx}{3}+\frac{2}{3}\int \tanh^{-1}(ax$$
$$=-\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2-$$
$$=-\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x\right.$$
$$=-\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x\right.$$
$$=-\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x\right.$$
$$=-\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x\right.$$

Mathematica [A] time = 0.12, size = 71, normalized size = 0.62

$$-\frac{\tanh^{-1}(ax)\left(a^2x^2+4\log\left(e^{-2\tanh^{-1}(ax)}+1\right)-1\right)-2\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)+ax+(ax-1)^2(ax+2)\tanh^{-1}(ax)^2}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/3*(a*x + (-1 + a*x)^2*(2 + a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-1 + a^2*
x^2 + 4*Log[1 + E^(-2*ArcTanh[a*x])]) - 2*PolyLog[2, -E^(-2*ArcTanh[a*x])])
/a
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

maple [A] time = 0.05, size = 182, normalized size = 1.58

$$-\frac{a^2 \operatorname{arctanh}(ax)^2 x^3}{3} + x \operatorname{arctanh}(ax)^2 - \frac{a \operatorname{arctanh}(ax) x^2}{3} + \frac{2 \operatorname{arctanh}(ax) \ln(ax-1)}{3a} + \frac{2 \operatorname{arctanh}(ax) \ln(ax+1)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/3*a^2*arctanh(a*x)^2*x^3+x*arctanh(a*x)^2-1/3*a*arctanh(a*x)*x^2+2/3/a*a
rctanh(a*x)*ln(a*x-1)+2/3/a*arctanh(a*x)*ln(a*x+1)-1/3*x-1/6/a*ln(a*x-1)+1/
6/a*ln(a*x+1)+1/6/a*ln(a*x-1)^2-2/3/a*dilog(1/2+1/2*a*x)-1/3/a*ln(a*x-1)*ln
(1/2+1/2*a*x)-1/6/a*ln(a*x+1)^2+1/3/a*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/3/a*ln(-
1/2*a*x+1/2)*ln(1/2+1/2*a*x)
```

maxima [A] time = 0.33, size = 144, normalized size = 1.25

$$-\frac{1}{6} a^2 \left(\frac{2\,ax + \log(ax+1)^2 - 2\,\log(ax+1)\log(ax-1) - \log(ax-1)^2 + \log(ax-1)}{a^3} + \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}\,ax\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a^2*((2*a*x + log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x -
 1)^2 + log(a*x - 1))/a^3 + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2
*a*x + 1/2))/a^3 - log(a*x + 1)/a^3) - 1/3*(x^2 - 2*log(a*x + 1)/a^2 - 2*lo
g(a*x - 1)/a^2)*a*arctanh(a*x) - 1/3*(a^2*x^3 - 3*x)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] -int(atanh(a*x)^2*(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int a^2x^2 \operatorname{atanh}^2(ax)\,dx - \int \left(-\operatorname{atanh}^2(ax)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] -Integral(a**2*x**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2, x)
```

3.177 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=146

$$-\frac{1}{2}\log\left(1-a^2x^2\right)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+\frac{1}{2}\text{Li}_3\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{Li}_3\left(\frac{2}{1-ax}-1\right)-\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)+\text{L}$$

```
[Out] -a*x*arctanh(a*x)+1/2*arctanh(a*x)^2-1/2*a^2*x^2*arctanh(a*x)^2-2*arctanh(a
*x)^2*arctanh(-1+2/(-a*x+1))-1/2*ln(-a^2*x^2+1)-arctanh(a*x)*polylog(2,1-2/
(-a*x+1))+arctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*polylog(3,1-2/(-a*x+1))
-1/2*polylog(3,-1+2/(-a*x+1))
```

Rubi [A] time = 0.31, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6014, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 5910, 260}

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\tanh^{-1}(ax)\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

```
[Out] -(a*x*ArcTanh[a*x]) + ArcTanh[a*x]^2/2 - (a^2*x^2*ArcTanh[a*x]^2)/2 + 2*Arc
Tanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - Log[1 - a^2*x^2]/2 - ArcTanh[a*x]*Po
lyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + Pol
yLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x}\,dx = -\left(a^2\int x\tanh^{-1}(ax)^2\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x}\,dx$$

$$= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-(4a)\int \frac{\tanh^{-1}(ax)\,\text{ta}}{1-}$$

$$= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-a\int \tanh^{-1}(ax)\,dx+$$

$$= -ax\tanh^{-1}(ax)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1$$

$$= -ax\tanh^{-1}(ax)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1$$

Mathematica [A] time = 0.04, size = 145, normalized size = 0.99

$$-\frac{1}{2}\log\left(1-a^2x^2\right)-\frac{1}{2}\left(a^2x^2-1\right)\tanh^{-1}(ax)^2-\frac{1}{2}\text{Li}_3\left(\frac{-ax-1}{ax-1}\right)+\frac{1}{2}\text{Li}_3\left(\frac{ax+1}{ax-1}\right)+\text{Li}_2\left(\frac{-ax-1}{ax-1}\right)\tanh^{-1}(ax)-\text{Li}_2$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

```
[Out] -(a*x*ArcTanh[a*x]) - ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/2 + 2*ArcTanh[a*x]^2*
ArcTanh[1 - 2/(1 - a*x)] - Log[1 - a^2*x^2]/2 + ArcTanh[a*x]*PolyLog[2, (-1
 - a*x)/(-1 + a*x)] - ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a*x)] - PolyL
og[3, (-1 - a*x)/(-1 + a*x)]/2 + PolyLog[3, (1 + a*x)/(-1 + a*x)]/2
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

maple [C] time = 1.06, size = 663, normalized size = 4.54

$$-\frac{a^2x^2\,\text{arctanh}\left(ax\right)^2}{2}+\text{arctanh}\left(ax\right)^2\ln\left(ax\right)-\text{arctanh}\left(ax\right)^2\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)+\text{arctanh}\left(ax\right)^2\ln\left(1-\frac{ax+}{\sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x,x)
```

```
[Out] -1/2*a^2*x^2*arctanh(a*x)^2+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1
)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arcta
nh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2
+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/
2))-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*polylog(3,-(a*x+1)^
2/(-a^2*x^2+1))-(a*x+1)*arctanh(a*x)+1/2*arctanh(a*x)^2+ln(1+(a*x+1)^2/(-a^
2*x^2+1))+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x
+1)^2/(-a^2*x^2+1)))^3-1/2*I*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^
2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*
I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x+1)^2/
(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1))
)-1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1
)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}a^2x^2\log(-ax+1)^2+\frac{1}{4}\int -\frac{\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)^2-\left(a^3x^3+2\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+\right.}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*x^2*log(-a*x + 1)^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x + 1
)*log(a*x + 1)^2 - (a^3*x^3 + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1))
*log(-a*x + 1))/(a*x^2 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x,x)
```

```
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\operatorname{atanh}^2(ax)}{x}\right)dx-\int a^2x\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x, x) - Integral(a**2*x*atanh(a*x)**2, x)
```

3.178 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=93

$$-a^2x\tanh^{-1}(ax)^2+a\text{Li}_2\left(1-\frac{2}{1-ax}\right)-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)+2a\log\left(2\right.$$

```
[Out] -arctanh(a*x)^2/x-a^2*x*arctanh(a*x)^2+2*a*arctanh(a*x)*ln(2/(-a*x+1))+2*a*
arctanh(a*x)*ln(2-2/(a*x+1))+a*polylog(2,1-2/(-a*x+1))-a*polylog(2,-1+2/(a*
x+1))
```

Rubi [A] time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6014, 5916, 5988, 5932, 2447, 5910, 5984, 5918, 2402, 2315}

$$a\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-a^2x\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(\frac{2}{1-ax}\right)\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -(ArcTanh[a*x]^2/x) - a^2*x*ArcTanh[a*x]^2 + 2*a*ArcTanh[a*x]*Log[2/(1 - a*
x)] + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + a*PolyLog[2, 1 - 2/(1 - a*x)]
 - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x^2}\,dx = -\left(a^2\int \tanh^{-1}(ax)^2\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + (2a)\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx + \left(2a^3\right)\int \frac{x\tanh^{-1}(a}{1-a^2x^2}$$

$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + (2a)\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx + \left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{1-ax}$$

$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + 2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right) + 2a\tanh^{-1}(ax)\,\mathrm{l}$$

$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + 2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right) + 2a\tanh^{-1}(ax)\,\mathrm{l}$$

$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + 2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right) + 2a\tanh^{-1}(ax)\,\mathrm{l}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 1.10

$$-a\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)+a\left(\tanh^{-1}(ax)\left(-\frac{\tanh^{-1}(ax)}{ax}+\tanh^{-1}(ax)+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-\mathrm{Li}_2\left(e^{-2\tanh^{-1}(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
[Out] -(a*ArcTanh[a*x]*(-ArcTanh[a*x] + a*x*ArcTanh[a*x] - 2*Log[1 + E^(-2*ArcTan
h[a*x])])) - a*PolyLog[2, -E^(-2*ArcTanh[a*x])] + a*(ArcTanh[a*x]*(ArcTanh[
a*x] - ArcTanh[a*x]/(a*x) + 2*Log[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2, E^
(-2*ArcTanh[a*x])])
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

maple [A] time = 0.06, size = 170, normalized size = 1.83

$$-a^2x\,\mathrm{arctanh}\left(ax\right)^2-\frac{\mathrm{arctanh}\left(ax\right)^2}{x}+2a\,\mathrm{arctanh}\left(ax\right)\ln\left(ax\right)-2a\,\mathrm{arctanh}\left(ax\right)\ln\left(ax-1\right)-2a\,\mathrm{arctanh}\left(ax\right)\ln\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x)
[Out] -a^2*x*arctanh(a*x)^2-arctanh(a*x)^2/x+2*a*arctanh(a*x)*ln(a*x)-2*a*arctanh
(a*x)*ln(a*x-1)-2*a*arctanh(a*x)*ln(a*x+1)-a*dilog(a*x)-a*dilog(a*x+1)-a*ln
(a*x)*ln(a*x+1)-1/2*a*ln(a*x-1)^2+2*a*dilog(1/2+1/2*a*x)+a*ln(a*x-1)*ln(1/2
+1/2*a*x)+1/2*a*ln(a*x+1)^2-a*ln(-1/2*a*x+1/2)*ln(a*x+1)+a*ln(-1/2*a*x+1/2)
*ln(1/2+1/2*a*x)
```

maxima [A] time = 0.32, size = 152, normalized size = 1.63

$$\frac{1}{2}a^2\left(\frac{\log\left(ax+1\right)^2-2\,\log\left(ax+1\right)\log\left(ax-1\right)-\log\left(ax-1\right)^2}{a}+\frac{4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a
+ 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 2*(log(a*
x + 1)*log(x) + dilog(-a*x))/a + 2*(log(-a*x + 1)*log(x) + dilog(a*x))/a) -
 2*a*(log(a*x + 1) + log(a*x - 1) - log(x))*arctanh(a*x) - (a^2*x + 1/x)*ar
ctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^2,x)
```

```
[Out] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int a^2\operatorname{atanh}^2(ax)\,dx-\int\left(-\frac{\operatorname{atanh}^2(ax)}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**2,x)
```

```
[Out] -Integral(a**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2/x**2, x)
```

3.179 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{2}a^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)+\frac{1}{2}a^2\text{Li}_3\left(\frac{2}{1-ax}-1\right)+a^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)-a^2\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)-\frac{1}{2}$$

```
[Out] -a*arctanh(a*x)/x+1/2*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+2*a^2*arcta
nh(a*x)^2*arctanh(-1+2/(-a*x+1))+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+a^2*arcta
nh(a*x)*polylog(2,1-2/(-a*x+1))-a^2*arctanh(a*x)*polylog(2,-1+2/(-a*x+1))-1
/2*a^2*polylog(3,1-2/(-a*x+1))+1/2*a^2*polylog(3,-1+2/(-a*x+1))
```

Rubi [A] time = 0.33, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6014, 5916, 5982, 266, 36, 29, 31, 5948, 5914, 6052, 6058, 6610}

$$-\frac{1}{2}a^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)+\frac{1}{2}a^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)+a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a^2\tanh^{-}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) - 2
*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - (a^2*Log[1 - a^
2*x^2])/2 + a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - a^2*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 - a*x)] - (a^2*PolyLog[3, 1 - 2/(1 - a*x)])/2 + (a^2*
PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x^3}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a\int \frac{\tanh^{-1}(ax)}{x^2}\,dx +$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1$$

Mathematica [A] time = 0.08, size = 174, normalized size = 1.01

$$\frac{1}{2}a^2\text{Li}_3\left(\frac{-ax-1}{ax-1}\right)-\frac{1}{2}a^2\text{Li}_3\left(\frac{ax+1}{ax-1}\right)-a^2\text{Li}_2\left(\frac{-ax-1}{ax-1}\right)\tanh^{-1}(ax)+a^2\text{Li}_2\left(\frac{ax+1}{ax-1}\right)\tanh^{-1}(ax)-\frac{1}{2}a^2\log\left(1-a^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) - 2*a^2*Arc
Tanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - (a^2*Log[1 - a^2*x^2])/
2 - a^2*ArcTanh[a*x]*PolyLog[2, (-1 - a*x)/(-1 + a*x)] + a^2*ArcTanh[a*x]*P
olyLog[2, (1 + a*x)/(-1 + a*x)] + (a^2*PolyLog[3, (-1 - a*x)/(-1 + a*x)])/2
 - (a^2*PolyLog[3, (1 + a*x)/(-1 + a*x)])/2
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

maple [C] time = 1.69, size = 741, normalized size = 4.31

$$-a^{2} \operatorname{arctanh}(ax)^{2} \ln(ax) - \frac{\operatorname{arctanh}(ax)^{2}}{2x^{2}} + \frac{a^{2} \operatorname{arctanh}(ax)^{2}}{2} - \frac{i a^{2} \operatorname{arctanh}(ax)^{2} \pi \operatorname{csgn}\left(i\left(\frac{(ax+1)^{2}}{-a^{2}x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{}{1+}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x)
```

```
[Out] -a^2*arctanh(a*x)^2*ln(a*x)-1/2*arctanh(a*x)^2/x^2+1/2*a^2*arctanh(a*x)^2-1
/2*I*a^2*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*
x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))+a^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)-a*arctanh(a*
x)/x+a^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-1/2*I*a^2*arctanh(a*x)^2*Pi*csgn(
I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/2*I*a^2*arctan
h(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+
1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*I*a^2*arctanh(a*x)^2*Pi*csgn(I*((a*
x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))^2+a^2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-a^2*arctanh(a*
x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a^2*arctanh(a*x)*polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))+2*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh
(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a^2*arctanh(a*x)*polylog(2,-(a*x
+1)/(-a^2*x^2+1)^(1/2))+2*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ar
ctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*a^2*polylog(3,-(a*x+1)^2/
(-a^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log(-ax+1)^{2}}{8x^{2}} + \frac{1}{4} \int -\frac{\left(a^{3}x^{3} - a^{2}x^{2} - ax + 1\right) \log(ax+1)^{2} - \left(ax + 2\left(a^{3}x^{3} - a^{2}x^{2} - ax + 1\right) \log(ax+1)\right) \log(}{ax^{4} - x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*log(-a*x + 1)^2/x^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x + 1)*lo
g(a*x + 1)^2 - (a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1))*log(-a*
x + 1))/(a*x^4 - x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)^{2}\,\left(a^{2}\,x^{2}-1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^3,x)
```

```
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\operatorname{atanh}^{2}(ax)}{x^{3}}\right) dx - \int \frac{a^{2} \operatorname{atanh}^{2}(ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**3,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**3, x) - Integral(a**2*atanh(a*x)**2/x, x)
```

3.180 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=116

$$\frac{2}{3}a^3\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-1}(ax)-\frac{4}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{a^2}{3x}+\frac{a^2\tanh^{-1}(ax)}{x}$$

```
[Out] -1/3*a^2/x+1/3*a^3*arctanh(a*x)-1/3*a*arctanh(a*x)/x^2-2/3*a^3*arctanh(a*x)
^2-1/3*arctanh(a*x)^2/x^3+a^2*arctanh(a*x)^2/x-4/3*a^3*arctanh(a*x)*ln(2-2/
(a*x+1))+2/3*a^3*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.31, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6014, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$\frac{2}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{a^2\tanh^{-1}(ax)^2}{x}-\frac{4}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -a^2/(3*x) + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (a^2*ArcTanh[a*x]^2)/x - (4*a^3*Arc
Tanh[a*x]*Log[2 - 2/(1 + a*x)])/3 + (2*a^3*PolyLog[2, -1 + 2/(1 + a*x)])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
```

```
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x^4}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}+\frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx-\left(2a^3\right)\int \frac{\tanh^{-1}(}{x\left(1-a^2\right.}$$

$$= -a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}+\frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3}\,dx+\frac{1}{3}$$

$$= -\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}-2a^3\tanh^{-1}$$

$$= -\frac{a^2}{3x}-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}-\frac{4}{3}a^3\,\mathrm{t}$$

$$= -\frac{a^2}{3x}+\frac{1}{3}a^3\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.31, size = 93, normalized size = 0.80

$$\frac{2a^3x^3\mathrm{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(a^3x^3-4a^3x^3\log\left(1-e^{-2\tanh^{-1}(ax)}\right)-ax\right)-a^2x^2-(ax-1)^2(2ax+1)\,\mathrm{tanh}}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] (-(a^2*x^2) - (-1 + a*x)^2*(1 + 2*a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-(a*x
) + a^3*x^3 - 4*a^3*x^3*Log[1 - E^(-2*ArcTanh[a*x])]) + 2*a^3*x^3*PolyLog[2
, E^(-2*ArcTanh[a*x])])/(3*x^3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

maple [B] time = 0.07, size = 237, normalized size = 2.04

$$\frac{a^2\,\text{arctanh}\left(ax\right)^2}{x}-\frac{\text{arctanh}\left(ax\right)^2}{3x^3}-\frac{a\,\text{arctanh}\left(ax\right)}{3x^2}-\frac{4a^3\,\text{arctanh}\left(ax\right)\ln\left(ax\right)}{3}+\frac{2a^3\,\text{arctanh}\left(ax\right)\ln\left(ax-1\right)}{3}+\frac{2a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x)
```

```
[Out] a^2*arctanh(a*x)^2/x-1/3*arctanh(a*x)^2/x^3-1/3*a*arctanh(a*x)/x^2-4/3*a^3*
arctanh(a*x)*ln(a*x)+2/3*a^3*arctanh(a*x)*ln(a*x-1)+2/3*a^3*arctanh(a*x)*ln
(a*x+1)-1/3*a^2/x-1/6*a^3*ln(a*x-1)+1/6*a^3*ln(a*x+1)+2/3*a^3*dilog(a*x)+2/
3*a^3*dilog(a*x+1)+2/3*a^3*ln(a*x)*ln(a*x+1)+1/6*a^3*ln(a*x-1)^2-2/3*a^3*di
log(1/2+1/2*a*x)-1/3*a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-1/6*a^3*ln(a*x+1)^2-1/3*
a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/3*a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)
```

maxima [A] time = 0.33, size = 188, normalized size = 1.62

$$-\frac{1}{6}\left(4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a-4\left(\log\left(ax+1\right)\log(x)+\text{Li}_2\left(-ax\right)\right)a+4\left(\log\left(-ax\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(lo
g(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*
a - a*log(a*x + 1) + a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a*x +
 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 + 2)/x)*a^2 + 1/3*(2*a^2*log(a^2*x^2
- 1) - 2*a^2*log(x^2) - 1/x^2)*a*arctanh(a*x) + 1/3*(3*a^2*x^2 - 1)*arctanh
(a*x)^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\text{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^4,x)
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\operatorname{atanh}^2(ax)}{x^4}\right)dx-\int\frac{a^2\operatorname{atanh}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**4,x)
[Out] -Integral(-atanh(a*x)**2/x**4, x) - Integral(a**2*atanh(a*x)**2/x**2, x)
```

3.181 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^5}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{3}a^4\log(x)+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{a^2}{12x^2}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^4\log\left(1-a^2x^2\right)-\frac{a\tanh^{-1}(ax)}{6x^3}$$

```
[Out] -1/12*a^2/x^2-1/6*a*arctanh(a*x)/x^3+1/2*a^3*arctanh(a*x)/x-1/4*(-a^2*x^2+1
)^2*arctanh(a*x)^2/x^4-1/3*a^4*ln(x)+1/6*a^4*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6008, 6014, 5916, 266, 44, 36, 29, 31}

$$-\frac{a^2}{12x^2}+\frac{1}{6}a^4\log\left(1-a^2x^2\right)-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\log(x)+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{a\tanh^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] -a^2/(12*x^2) - (a*ArcTanh[a*x])/(6*x^3) + (a^3*ArcTanh[a*x])/(2*x) - ((1 -
 a^2*x^2)^2*ArcTanh[a*x]^2)/(4*x^4) - (a^4*Log[x])/3 + (a^4*Log[1 - a^2*x^2
])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^5}\,dx &= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx\\ &= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int \frac{\tanh^{-1}(ax)}{x^4}\,dx-\frac{1}{2}a^3\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\ &= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^2\int \frac{1}{x^3(1-a^2x^2}\\ &= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\,\mathrm{Subst}\left(\int \frac{}{x^2}\right.\\ &= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\,\mathrm{Subst}\left(\int \left(\frac{1}{x^2}\right.\right.\\ &= -\frac{a^2}{12x^2}-\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 0.92

$$\frac{-4a^4x^4\log(x)+(6a^3x^3-2ax)\tanh^{-1}(ax)-a^2x^2-3(a^2x^2-1)^2\tanh^{-1}(ax)^2+2a^4x^4\log(1-a^2x^2)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] (-(a^2*x^2) + (-2*a*x + 6*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^2 - 4*a^4*x^4*Log[x] + 2*a^4*x^4*Log[1 - a^2*x^2])/(12*x^4)
```

fricas [A] time = 0.87, size = 108, normalized size = 1.21

$$\frac{8\,a^4x^4\log(a^2x^2-1)-16\,a^4x^4\log(x)-4\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(3\,a^3x^3-ax)\log\left(-\frac{ax+1}{ax-1}\right)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(8*a^4*x^4*log(a^2*x^2 - 1) - 16*a^4*x^4*log(x) - 4*a^2*x^2 - 3*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 - a*x)*log(-
(a*x + 1)/(a*x - 1)))/x^4
```

giac [B] time = 0.36, size = 282, normalized size = 3.17

$$-\frac{1}{3}\left(a^3\log\left(-\frac{ax+1}{ax-1}-1\right)-a^3\log\left(-\frac{ax+1}{ax-1}\right)+\frac{3\,(ax+1)^2a^3\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax-1)^2\left(\frac{(ax+1)^4}{(ax-1)^4}+\frac{4\,(ax+1)^3}{(ax-1)^3}+\frac{6\,(ax+1)^2}{(ax-1)^2}+\frac{4\,(ax+1)}{ax-1}+1\right)}-\frac{}{(ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/3*(a^3*log(-(a*x + 1)/(a*x - 1) - 1) - a^3*log(-(a*x + 1)/(a*x - 1)) + 3
*(a*x + 1)^2*a^3*log(-(a*x + 1)/(a*x - 1))^2/((a*x - 1)^2*((a*x + 1)^4/(a*x
 - 1)^4 + 4*(a*x + 1)^3/(a*x - 1)^3 + 6*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x +
1)/(a*x - 1) + 1)) - (a*x + 1)*a^3/((a*x - 1)*((a*x + 1)^2/(a*x - 1)^2 + 2*
(a*x + 1)/(a*x - 1) + 1)) + (3*(a*x + 1)*a^3/(a*x - 1) + a^3)*log(-(a*x + 1
)/(a*x - 1))/((a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2 + 3*(a*x
+ 1)/(a*x - 1) + 1))*a
```

maple [B] time = 0.07, size = 199, normalized size = 2.24

$$\frac{a^2\operatorname{arctanh}(ax)^2}{2x^2}-\frac{\operatorname{arctanh}(ax)^2}{4x^4}+\frac{a^3\operatorname{arctanh}(ax)}{2x}-\frac{a\operatorname{arctanh}(ax)}{6x^3}+\frac{a^4\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{a^4\operatorname{arctanh}(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x)
```

```
[Out] 1/2*a^2*arctanh(a*x)^2/x^2-1/4*arctanh(a*x)^2/x^4+1/2*a^3*arctanh(a*x)/x-1/
6*a*arctanh(a*x)/x^3+1/4*a^4*arctanh(a*x)*ln(a*x-1)-1/4*a^4*arctanh(a*x)*ln
(a*x+1)+1/16*a^4*ln(a*x-1)^2-1/8*a^4*ln(a*x-1)*ln(1/2+1/2*a*x)+1/16*a^4*ln(
a*x+1)^2+1/8*a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/8*a^4*ln(-1/2*a*x+1/2)*
ln(a*x+1)-1/12*a^2/x^2-1/3*a^4*ln(a*x)+1/6*a^4*ln(a*x-1)+1/6*a^4*ln(a*x+1)
```

maxima [B] time = 0.33, size = 164, normalized size = 1.84

$$-\frac{1}{48}\left(16\,a^2\log(x)-\frac{3\,a^2x^2\log(ax+1)^2+3\,a^2x^2\log(ax-1)^2+8\,a^2x^2\log(ax-1)-2\left(3\,a^2x^2\log(ax-1)-4\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/48*(16*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2*log(a*x - 1)^2
 + 8*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 4*a^2*x^2)*log(a*x
+ 1) - 4)/x^2)*a^2 - 1/12*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a
^2*x^2 - 1)/x^3)*a*arctanh(a*x) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/x^4
```

mupad [B] time = 1.36, size = 246, normalized size = 2.76

$$\ln(1-a\,x)^2\left(\frac{\frac{a^2\,x^2}{2}-\frac{1}{4}}{4\,x^4}-\frac{a^4}{16}\right)-\ln(1-a\,x)\left(\ln(a\,x+1)\left(\frac{\frac{a^2\,x^2}{2}-\frac{1}{4}}{2\,x^4}-\frac{a^4}{8}\right)+\frac{3\,a^5\,x-2\,a^4}{24\,a^3\,x^3}-\frac{3\,x\,a^5+2\,a^4}{24\,a^3\,x^3}-\frac{a\,(22}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^5,x)
```

```
[Out] log(1 - a*x)^2*(((a^2*x^2)/2 - 1/4)/(4*x^4) - a^4/16) - log(1 - a*x)*(log(a
*x + 1)*(((a^2*x^2)/2 - 1/4)/(2*x^4) - a^4/8) + (3*a^5*x - 2*a^4)/(24*a^3*x
^3) - (3*a^5*x + 2*a^4)/(24*a^3*x^3) - (a*(6*a*x - 12*a^2*x^2 + 22*a^3*x^3
- 4))/(96*x^3) + (a*(12*a*x + 24*a^2*x^2 + 44*a^3*x^3 + 8))/(192*x^3)) - (a
^4*log(x))/3 + log(a*x + 1)^2*(((a^2*x^2)/8 - 1/16)/x^4 - a^4/16) + (a^4*lo
g(a^2*x^2 - 1))/6 - a^2/(12*x^2) + (a*log(a*x + 1)*((a^2*x^2)/4 - 1/12))/x^
3
```

sympy [A] time = 1.62, size = 102, normalized size = 1.15

$$\begin{cases} -\frac{a^4 \log(x)}{3} + \frac{a^4 \log\left(x-\frac{1}{a}\right)}{3} - \frac{a^4 \operatorname{atanh}^2(ax)}{4} + \frac{a^4 \operatorname{atanh}(ax)}{3} + \frac{a^3 \operatorname{atanh}(ax)}{2x} + \frac{a^2 \operatorname{atanh}^2(ax)}{2x^2} - \frac{a^2}{12x^2} - \frac{a \operatorname{atanh}(ax)}{6x^3} - \frac{\operatorname{atanh}^2(ax)}{4x^4} & \text{fo} \\ 0 & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**5,x)
```

```
[Out] Piecewise((-a**4*log(x)/3 + a**4*log(x - 1/a)/3 - a**4*atanh(a*x)**2/4 + a*
*4*atanh(a*x)/3 + a**3*atanh(a*x)/(2*x) + a**2*atanh(a*x)**2/(2*x**2) - a**
2/(12*x**2) - a*atanh(a*x)/(6*x**3) - atanh(a*x)**2/(4*x**4), Ne(a, 0)), (0
, True))
```

3.182 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^6}\,dx$

Optimal. Leaf size=143

$$\frac{2}{15}a^5\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{1}{30}a^5\tanh^{-1}(ax)-\frac{4}{15}a^5\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)+\frac{a^4}{30x}+\frac{2a^3\tan}{15}$$

```
[Out] -1/30*a^2/x^3+1/30*a^4/x-1/30*a^5*arctanh(a*x)-1/10*a*arctanh(a*x)/x^4+2/15
*a^3*arctanh(a*x)/x^2-2/15*a^5*arctanh(a*x)^2-1/5*arctanh(a*x)^2/x^5+1/3*a^
2*arctanh(a*x)^2/x^3-4/15*a^5*arctanh(a*x)*ln(2-2/(a*x+1))+2/15*a^5*polylog
(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.45, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6014, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$\frac{2}{15}a^5\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{30x^3}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{a^4}{30x}-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{1}{30}a^5\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] -a^2/(30*x^3) + a^4/(30*x) - (a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x])/(10*x
^4) + (2*a^3*ArcTanh[a*x])/(15*x^2) - (2*a^5*ArcTanh[a*x]^2)/15 - ArcTanh[a
*x]^2/(5*x^5) + (a^2*ArcTanh[a*x]^2)/(3*x^3) - (4*a^5*ArcTanh[a*x]*Log[2 -
2/(1 + a*x)])/15 + (2*a^5*PolyLog[2, -1 + 2/(1 + a*x)])/15
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x^6}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5\left(1-a^2x^2\right)}\,dx-\frac{1}{3}\left(2a^3\right)\int \frac{\tanh}{x^3\left(1-\right.}$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5}\,dx+\frac{1}{5}\left(2a^3\right)\int \frac{\tanh^{-1}}{x^3\left(1-\right.}$$

$$= -\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{a^3\tanh^{-1}(ax)}{3x^2}-\frac{1}{3}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2\tanh^{-1}(}{3x^3}$$

$$= -\frac{a^2}{30x^3}+\frac{a^4}{3x}-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax}{5x^5}$$

$$= -\frac{a^2}{30x^3}+\frac{a^4}{30x}-\frac{1}{3}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{2}{15}a^5\tanh^{-}$$

$$= -\frac{a^2}{30x^3}+\frac{a^4}{30x}-\frac{1}{30}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{2}{15}a^5\tanh$$

Mathematica [A] time = 0.62, size = 114, normalized size = 0.80

$$\frac{4a^5x^5\mathrm{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right)+a^2x^2\left(a^2x^2-1\right)-2\left(2a^5x^5-5a^2x^2+3\right)\tanh^{-1}(ax)^2-ax\tanh^{-1}(ax)\left(a^4x^4+8a^4x^4\log\right.}{30x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] (a^2*x^2*(-1 + a^2*x^2) - 2*(3 - 5*a^2*x^2 + 2*a^5*x^5)*ArcTanh[a*x]^2 - a*
x*ArcTanh[a*x]*(3 - 4*a^2*x^2 + a^4*x^4 + 8*a^4*x^4*Log[1 - E^(-2*ArcTanh[a
*x])]) + 4*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(30*x^5)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\operatorname{artanh}(ax)^2}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int-\frac{\left(a^2x^2-1\right)\operatorname{artanh}(ax)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

maple [B] time = 0.07, size = 258, normalized size = 1.80

$$\frac{a^2\operatorname{arctanh}(ax)^2}{3x^3}-\frac{\operatorname{arctanh}(ax)^2}{5x^5}-\frac{a\operatorname{arctanh}(ax)}{10x^4}+\frac{2a^3\operatorname{arctanh}(ax)}{15x^2}-\frac{4a^5\operatorname{arctanh}(ax)\ln(ax)}{15}+\frac{2a^5\operatorname{arctanh}(\ldots}{1\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x)
```

```
[Out] 1/3*a^2*arctanh(a*x)^2/x^3-1/5*arctanh(a*x)^2/x^5-1/10*a*arctanh(a*x)/x^4+2
/15*a^3*arctanh(a*x)/x^2-4/15*a^5*arctanh(a*x)*ln(a*x)+2/15*a^5*arctanh(a*x
)*ln(a*x-1)+2/15*a^5*arctanh(a*x)*ln(a*x+1)-1/30*a^2/x^3+1/30*a^4/x+1/60*a^
5*ln(a*x-1)-1/60*a^5*ln(a*x+1)+2/15*a^5*dilog(a*x)+2/15*a^5*dilog(a*x+1)+2/
15*a^5*ln(a*x)*ln(a*x+1)+1/30*a^5*ln(a*x-1)^2-2/15*a^5*dilog(1/2+1/2*a*x)-1
/15*a^5*ln(a*x-1)*ln(1/2+1/2*a*x)-1/30*a^5*ln(a*x+1)^2+1/15*a^5*ln(-1/2*a*x
+1/2)*ln(a*x+1)-1/15*a^5*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)
```

maxima [A] time = 0.32, size = 228, normalized size = 1.59

$$-\frac{1}{60}\left(8\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^3-8\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)a^3+8\left(\log(-\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 8*
(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 8*(log(-a*x + 1)*log(x) + dilog(a
*x))*a^3 + a^3*log(a*x + 1) - a^3*log(a*x - 1) + 2*(a^3*x^3*log(a*x + 1)^2
- 2*a^3*x^3*log(a*x + 1)*log(a*x - 1) - a^3*x^3*log(a*x - 1)^2 - a^2*x^2 +
1)/x^3)*a^2 + 1/30*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x^2 -
3)/x^4)*a*arctanh(a*x) + 1/15*(5*a^2*x^2 - 3)*arctanh(a*x)^2/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^6,x)
```

```
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\operatorname{atanh}^2(ax)}{x^6}\right)dx-\int \frac{a^2\operatorname{atanh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**6,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**6, x) - Integral(a**2*atanh(a*x)**2/x**4, x)
```

3.183 $\int \left(1-a^2x^2\right) \tanh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=157

$$-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a}+\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{a}-\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(}{a}$$

```
[Out] -x*arctanh(a*x)+1/2*(-a^2*x^2+1)*arctanh(a*x)^2/a+2/3*arctanh(a*x)^3/a+2/3*
x*arctanh(a*x)^3+1/3*x*(-a^2*x^2+1)*arctanh(a*x)^3-2*arctanh(a*x)^2*ln(2/(-
a*x+1))/a-1/2*ln(-a^2*x^2+1)/a-2*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+pol
ylog(3,1-2/(-a*x+1))/a
```

Rubi [A] time = 0.19, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {5944, 5910, 5984, 5918, 5948, 6058, 6610, 260}

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a}-\frac{2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3+\frac{(1-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

```
[Out] -(x*ArcTanh[a*x]) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)/(2*a) + (2*ArcTanh[a*x]^
3)/(3*a) + (2*x*ArcTanh[a*x]^3)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^3)/3 - (2
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - Log[1 - a^2*x^2]/(2*a) - (2*ArcTanh[a
*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/a
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(1-a^2x^2\right)\tanh^{-1}(ax)^3\,dx = \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a} + \frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3 + \frac{2}{3}\int \tanh^{-1}(ax)^3\,dx - \int$$

$$= -x\tanh^{-1}(ax) + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a} + \frac{2}{3}x\tanh^{-1}(ax)^3 + \frac{1}{3}x\left(1-a^2x^2\right)\tanh$$

$$= -x\tanh^{-1}(ax) + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^3 + \frac{1}{3}$$

$$= -x\tanh^{-1}(ax) + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^3 + \frac{1}{3}$$

$$= -x\tanh^{-1}(ax) + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^3 + \frac{1}{3}$$

$$= -x\tanh^{-1}(ax) + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^3 + \frac{1}{3}$$

Mathematica [A] time = 0.30, size = 134, normalized size = 0.85

$$-\frac{2a^3x^3\tanh^{-1}(ax)^3 + 3\log\left(1-a^2x^2\right) + 3a^2x^2\tanh^{-1}(ax)^2 - 12\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) - 6\mathrm{Li}_3\left(-e^{-2\tanh}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

```
[Out] -1/6*(6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2 + 4*
ArcTanh[a*x]^3 - 6*a*x*ArcTanh[a*x]^3 + 2*a^3*x^3*ArcTanh[a*x]^3 + 12*ArcTa
nh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + 3*Log[1 - a^2*x^2] - 12*ArcTanh[a*
x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 6*PolyLog[3, -E^(-2*ArcTanh[a*x])])/a
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

maple [C] time = 1.33, size = 829, normalized size = 5.28

$$-\frac{a^2\,\text{arctanh}\left(ax\right)^3x^3}{3}+x\,\text{arctanh}\left(ax\right)^3-\frac{a\,\text{arctanh}\left(ax\right)^2x^2}{2}+\frac{\text{arctanh}\left(ax\right)^2\ln\left(ax-1\right)}{a}+\frac{\text{arctanh}\left(ax\right)^2\ln\left(ax+\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^3,x)
```

```
[Out] -1/3*a^2*arctanh(a*x)^3*x^3+x*arctanh(a*x)^3-1/2*a*arctanh(a*x)^2*x^2+1/a*a
rctanh(a*x)^2*ln(a*x-1)+1/a*arctanh(a*x)^2*ln(a*x+1)-2/a*arctanh(a*x)^2*ln(
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^3*Pi-I/a*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-1/2*I
/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^
3*Pi-1/2*I/a*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+
1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I/a*arctanh(a*x)^2*csgn(I
/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi-I/a*arctanh(a*x)^2*Pi+1/2*I/a*arctanh(a*x
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^2*Pi-1/2*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/
2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi-I/a*arctanh(a*x)^2*csgn(I*(a*x+1)/(-
a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi+1/2*I/a*arctanh(a*x)^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*Pi+2/3*arctanh(a*x)^3/a-2/a
*arctanh(a*x)^2*ln(2)-x*arctanh(a*x)+1/2*arctanh(a*x)^2/a-arctanh(a*x)/a+1/
a*ln(1+(a*x+1)^2/(-a^2*x^2+1))-2/a*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*
x^2+1))+1/a*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2\,a^3x^3-3\,a^2x^2-12\,ax-6\left(a^3x^3-3\,ax-2\right)\log\left(ax+1\right)\right)\log\left(-ax+1\right)^2}{48\,a}-\frac{\left(\log\left(-ax+1\right)^3-3\,\log\left(-ax+1\right)^2\right.}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(2*a^3*x^3 - 3*a^2*x^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2)*log(a*x + 1)
)*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x
 + 1) - 6)*(a*x - 1)/a + 1/864*(4*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2 +
```

```
6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x + 1)^3 - 6*log(-a*x + 1)^
2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-a*x + 1)^3 - 3*log(-a*x +
1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/8*integrate(-1/3*(3*(a^3*x^3 -
 a^2*x^2 - a*x + 1)*log(a*x + 1)^3 + (2*a^3*x^3 - 3*a^2*x^2 - 9*(a^3*x^3 -
a^2*x^2 - a*x + 1)*log(a*x + 1)^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2)*log(a*
x + 1))*log(-a*x + 1))/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3*(a^2*x^2 - 1),x)
```

```
[Out] -int(atanh(a*x)^3*(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int a^2x^2\operatorname{atanh}^3(ax)\,dx-\int\left(-\operatorname{atanh}^3(ax)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**3,x)
```

```
[Out] -Integral(a**2*x**2*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3, x)
```

3.184 $\int \frac{x \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2} dx$

Optimal. Leaf size=193

$$-\frac{1}{2}\text{Li}_2\left(1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{4}\text{Li}_2\left(\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)+\frac{1}{4}\text{Li}_2\left(1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)+\log\left(\frac{2\sqrt{2}}{x+\sqrt{2}}\right)\tanh^{-1}$$

```
[Out] -1/2*arctanh(1/2*x*2^(1/2))*ln(-4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))+arctanh(1/
2*x*2^(1/2))*ln(2*2^(1/2)/(x+2^(1/2)))-1/2*arctanh(1/2*x*2^(1/2))*ln(4*(1+x
)/(2+2^(1/2))/(x+2^(1/2)))+1/4*polylog(2,1+4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))
-1/2*polylog(2,1-2*2^(1/2)/(x+2^(1/2)))+1/4*polylog(2,1-4*(1+x)/(2+2^(1/2))
/(x+2^(1/2)))
```

Rubi [A] time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5992, 5920, 2402, 2315, 2447}

$$-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)+\frac{1}{4}\text{PolyLog}\left(2,1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```

```
[Out] ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt[2]]*Log
[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/2 - (ArcTanh[x/Sqrt[2]]*Log[(
4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/2 - PolyLog[2, 1 - (2*Sqrt[2])/(
Sqrt[2] + x)]/2 + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]
/4 + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/4
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \frac{x \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = \int \left(-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(-1+x)} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(1+x)}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{-1+x}\,dx\right) - \frac{1}{2}\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1+x}\,dx$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right) - \frac{1}{2}\tanh^{-1}\left(-\right.$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right) - \frac{1}{2}\tanh^{-1}\left(-\right.$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right) - \frac{1}{2}\tanh^{-1}\left(-\right.$$

Mathematica [A] time = 0.28, size = 232, normalized size = 1.20

$$\frac{1}{4}\left(-2\text{Li}_2\left(-e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right) + \text{Li}_2\left(\left(3-2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right) + \text{Li}_2\left(\left(3+2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right) + 4\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\text{lo}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```

```
[Out] (-4*ArcSinh[1]*ArcTanh[x] + 4*ArcTanh[x/Sqrt[2]]*Log[1 + E^(-2*ArcTanh[x/Sq
rt[2]])] + 2*ArcSinh[1]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])]
- 2*ArcTanh[x/Sqrt[2]]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] -
 2*ArcSinh[1]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*ArcTanh
[x/Sqrt[2]]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*PolyLog[2
, -E^(-2*ArcTanh[x/Sqrt[2]])] + PolyLog[2, (3 - 2*Sqrt[2])/E^(2*ArcTanh[x/S
qrt[2]])] + PolyLog[2, (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])])/4
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x\,\text{artanh}\left(\frac{1}{2}\sqrt{2}\,x\right)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x \operatorname{artanh}\left(\frac{1}{2}\sqrt{2}x\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

maple [A] time = 0.06, size = 251, normalized size = 1.30

$$-\frac{\ln\left(x^2-1\right)\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)}{2}-\frac{\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln\left(x^2-1\right)}{4}+\frac{\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{-2+\sqrt{2}}\right)}{4}+\frac{\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}+x\sqrt{2}}{2+\sqrt{2}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x)
```

```
[Out] -1/2*ln(x^2-1)*arctanh(1/2*x*2^(1/2))-1/4*ln(1/2*x*2^(1/2)-1)*ln(x^2-1)+1/4
*ln(1/2*x*2^(1/2)-1)*ln((2^(1/2)-x*2^(1/2))/(-2+2^(1/2)))+1/4*ln(1/2*x*2^(1
/2)-1)*ln((2^(1/2)+x*2^(1/2))/(2+2^(1/2)))+1/4*dilog((2^(1/2)-x*2^(1/2))/(-
2+2^(1/2)))+1/4*dilog((2^(1/2)+x*2^(1/2))/(2+2^(1/2)))+1/4*ln(1/2*x*2^(1/2)
+1)*ln(x^2-1)-1/4*ln(1/2*x*2^(1/2)+1)*ln((2^(1/2)-x*2^(1/2))/(2+2^(1/2)))-1
/4*ln(1/2*x*2^(1/2)+1)*ln((2^(1/2)+x*2^(1/2))/(-2+2^(1/2)))-1/4*dilog((2^(1
/2)-x*2^(1/2))/(2+2^(1/2)))-1/4*dilog((2^(1/2)+x*2^(1/2))/(-2+2^(1/2)))
```

maxima [A] time = 0.43, size = 277, normalized size = 1.44

$$-\frac{1}{2}\operatorname{artanh}\left(\frac{1}{2}\sqrt{2}x\right)\log\left(x^2-1\right)-\frac{1}{4}\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{8}\sqrt{2}\left(\sqrt{2}\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+\sqrt{2}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*arctanh(1/2*sqrt(2)*x)*log(x^2 - 1) - 1/4*log(x^2 - 1)*log((x - sqrt(2
))/(x + sqrt(2))) + 1/8*sqrt(2)*(sqrt(2)*log(x^2 - 1)*log((x - sqrt(2))/(x
+ sqrt(2))) + sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(x^
2 - 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - s
qrt(2))*log((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + s
qrt(2))/(sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) -
1) + 1) - dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2
) + 1)) - dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2
) - 1))))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh((2^(1/2)*x)/2))/(x^2 - 1),x)
```

```
[Out] -int((x*atanh((2^(1/2)*x)/2))/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{2}\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1/2*x*2**(1/2))/(-x**2+1),x)
```

```
[Out] -Integral(x*atanh(sqrt(2)*x/2)/(x**2 - 1), x)
```

3.185 $\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
[Out] Defer[Int][(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)}\,dx = \int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
[Out] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2x^3-x}{\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*x^3 - x)/arctanh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)x}{\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x/arctanh(a*x), x)
```

maple [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a^{2}x^{2}+1\right)}{\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)/arctanh(a*x),x)
[Out] int(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2}x^{2}-1\right)x}{\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*x/arctanh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{x\left(a^{2}\,x^{2}-1\right)}{\operatorname{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a^2*x^2 - 1))/atanh(a*x),x)
[Out] -int((x*(a^2*x^2 - 1))/atanh(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x}{\operatorname{atanh}\left(ax\right)}\right)dx-\int \frac{a^{2}x^{3}}{\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)/atanh(a*x),x)
[Out] -Integral(-x/atanh(a*x), x) - Integral(a**2*x**3/atanh(a*x), x)
```

3.186 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x],x]
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)} \, dx = \int \frac{1-a^2x^2}{\tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x],x]
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x], x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2x^2-1}{\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2x^2-1}{\text{artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/arctanh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{a^2\,x^2-1}{\operatorname{atanh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/atanh(a*x),x)
```

```
[Out] -int((a^2*x^2 - 1)/atanh(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2}{\operatorname{atanh}(ax)}\,dx-\int \left(-\frac{1}{\operatorname{atanh}(ax)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x), x) - Integral(-1/atanh(a*x), x)
```

3.187 $\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1-a^2x^2}{x\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/x/arctanh(a*x),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
[Out] Defer[Int][(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx=\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
[Out] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

fricas [A] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

maple [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{x\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/x/arctanh(a*x),x)
[Out] int((-a^2*x^2+1)/x/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{x\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{a^2\,x^2-1}{x\,\mathrm{atanh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x*atanh(a*x)),x)
[Out] -int((a^2*x^2 - 1)/(x*atanh(a*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{1}{x\,\mathrm{atanh}\,(ax)}\right)dx - \int \frac{a^2x}{\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/x/atanh(a*x),x)
[Out] -Integral(-1/(x*atanh(a*x)), x) - Integral(a**2*x/atanh(a*x), x)
```

3.188 $\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} \, dx = \int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{a^2x^3-x}{\operatorname{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^3 - x)/arctanh(a*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)x}{\operatorname{artanh}(ax)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x/arctanh(a*x)^2, x)
```

maple [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a^{2}x^{2}+1\right)}{\operatorname{arctanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(a^{4}x^{5}-2\,a^{2}x^{3}+x\right)}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}-\int -\frac{2\left(5\,a^{4}x^{4}-6\,a^{2}x^{2}+1\right)}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^5 - 2*a^2*x^3 + x)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrate
(-2*(5*a^4*x^4 - 6*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{x\,\left(a^2\,x^2-1\right)}{\mathrm{atanh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a^2*x^2 - 1))/atanh(a*x)^2,x)
```

```
[Out] -int((x*(a^2*x^2 - 1))/atanh(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x}{\operatorname{atanh}^{2}\left(ax\right)}\right)dx-\int \frac{a^{2}x^{3}}{\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(-x/atanh(a*x)**2, x) - Integral(a**2*x**3/atanh(a*x)**2, x)
```

3.189 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx=\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2x^2-1}{\operatorname{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2x^2-1}{\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(a^4x^4-2\,a^2x^2+1\right)}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}-\int -\frac{8\left(a^3x^3-ax\right)}{\log\left(ax+1\right)-\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrate
(-8*(a^3*x^3 - a*x)/(log(a*x + 1) - log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{a^2\,x^2-1}{\operatorname{atanh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/atanh(a*x)^2,x)
```

```
[Out] -int((a^2*x^2 - 1)/atanh(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2}{\operatorname{atanh}^2(ax)}\,dx-\int \left(-\frac{1}{\operatorname{atanh}^2(ax)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x)**2, x) - Integral(-1/atanh(a*x)**2, x)
```

3.190 $\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1-a^2x^2}{x\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
[Out] Defer[Int][(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx=\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
[Out] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

maple [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{x \operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(a^4x^4-2\,a^2x^2+1\right)}{ax\log\left(ax+1\right)-ax\log\left(-ax+1\right)}-\int -\frac{2\left(3\,a^4x^4-2\,a^2x^2-1\right)}{ax^2\log\left(ax+1\right)-ax^2\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) - integ
rate(-2*(3*a^4*x^4 - 2*a^2*x^2 - 1)/(a*x^2*log(a*x + 1) - a*x^2*log(-a*x +
1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{a^2\,x^2-1}{x\,\mathrm{atanh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x*atanh(a*x)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/(x*atanh(a*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{1}{x\,\mathrm{atanh}^2\left(ax\right)}\right)dx-\int \frac{a^2x}{\mathrm{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/x/atanh(a*x)**2,x)
```

```
[Out] -Integral(-1/(x*atanh(a*x)**2), x) - Integral(a**2*x/atanh(a*x)**2, x)
```

3.191 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} \, dx = \int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2x^2-1}{\text{artanh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2x^2-1}{\text{artanh}(ax)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{\operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(a^4x^4-2\,a^2x^2-2\left(a^5x^5-2\,a^3x^3+ax\right)\log(ax+1)+2\left(a^5x^5-2\,a^3x^3+ax\right)\log(-ax+1)+1\right)}{a\log(ax+1)^2-2\,a\log(ax+1)\log(-ax+1)+a\log(-ax+1)^2}+\int -\frac{4\left(5\,a\right.}{\log(ax\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 - 2*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) + 2*(a
^5*x^5 - 2*a^3*x^3 + a*x)*log(-a*x + 1) + 1)/(a*log(a*x + 1)^2 - 2*a*log(a*
x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2) + integrate(-4*(5*a^4*x^4 - 6*a^2
*x^2 + 1)/(log(a*x + 1) - log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{a^2\,x^2-1}{\operatorname{atanh}(a\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/atanh(a*x)^3,x)
```

```
[Out] -int((a^2*x^2 - 1)/atanh(a*x)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2}{\operatorname{atanh}^3(ax)}\,dx-\int \left(-\frac{1}{\operatorname{atanh}^3(ax)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x)**3, x) - Integral(-1/atanh(a*x)**3, x)
```

3.192 $\int x^4 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=96

$$\frac{1}{9}a^4x^9\tanh^{-1}(ax)+\frac{a^3x^8}{72}+\frac{4x^2}{315a^3}-\frac{2}{7}a^2x^7\tanh^{-1}(ax)+\frac{4\log\left(1-a^2x^2\right)}{315a^5}-\frac{11ax^6}{378}+\frac{1}{5}x^5\tanh^{-1}(ax)+\frac{2x^4}{315a}$$

```
[Out] 4/315*x^2/a^3+2/315*x^4/a-11/378*a*x^6+1/72*a^3*x^8+1/5*x^5*arctanh(a*x)-2/
7*a^2*x^7*arctanh(a*x)+1/9*a^4*x^9*arctanh(a*x)+4/315*ln(-a^2*x^2+1)/a^5
```

Rubi [A] time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6012, 5916, 266, 43}

$$\frac{a^3x^8}{72}+\frac{4x^2}{315a^3}+\frac{4\log\left(1-a^2x^2\right)}{315a^5}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)-\frac{11ax^6}{378}+\frac{2x^4}{315a}+\frac{1}{5}x^5\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5*
ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 + (
4*Log[1 - a^2*x^2])/(315*a^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^4 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx = \int \left(x^4 \tanh^{-1}(ax) - 2a^2x^6 \tanh^{-1}(ax) + a^4x^8 \tanh^{-1}(ax)\right)\,dx$$
$$= -\left(\left(2a^2\right)\int x^6 \tanh^{-1}(ax)\,dx\right) + a^4\int x^8 \tanh^{-1}(ax)\,dx + \int x^4 \tanh^{-1}(ax)\,dx$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9 \tanh^{-1}(ax) - \frac{1}{5}a\int \frac{x^5}{1-a^2x^2}\,dx +$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9 \tanh^{-1}(ax) - \frac{1}{10}a\,\text{Subst}\left(\int \frac{x^2}{1-\right.$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9 \tanh^{-1}(ax) - \frac{1}{10}a\,\text{Subst}\left(\int\left(-\frac{1}{a^4}\right.\right.$$
$$= \frac{4x^2}{315a^3} + \frac{2x^4}{315a} - \frac{11ax^6}{378} + \frac{a^3x^8}{72} + \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9$$

Mathematica [A] time = 0.03, size = 96, normalized size = 1.00

$$\frac{1}{9}a^4x^9 \tanh^{-1}(ax)+\frac{a^3x^8}{72}+\frac{4x^2}{315a^3}-\frac{2}{7}a^2x^7 \tanh^{-1}(ax)+\frac{4\log\left(1-a^2x^2\right)}{315a^5}-\frac{11ax^6}{378}+\frac{1}{5}x^5 \tanh^{-1}(ax)+\frac{2x^4}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5*
ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 + (
4*Log[1 - a^2*x^2])/(315*a^5)
```

fricas [A] time = 0.56, size = 92, normalized size = 0.96

$$\frac{105\,a^8x^8 - 220\,a^6x^6 + 48\,a^4x^4 + 96\,a^2x^2 + 12\left(35\,a^9x^9 - 90\,a^7x^7 + 63\,a^5x^5\right)\log\left(-\frac{ax+1}{ax-1}\right) + 96\,\log\left(a^2x^2-1\right)}{7560\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*x^8 - 220*a^6*x^6 + 48*a^4*x^4 + 96*a^2*x^2 + 12*(35*a^9*x^
9 - 90*a^7*x^7 + 63*a^5*x^5)*log(-(a*x + 1)/(a*x - 1)) + 96*log(a^2*x^2 - 1
))/a^5
```

giac [B] time = 0.20, size = 383, normalized size = 3.99

$$\frac{4}{945}a\left(\frac{6\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6} - \frac{6\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^6} - \frac{\frac{6(ax+1)^7}{(ax-1)^7} - \frac{45(ax+1)^6}{(ax-1)^6} - \frac{274(ax+1)^5}{(ax-1)^5} - \frac{214(ax+1)^4}{(ax-1)^4} - \frac{274(ax+1)^3}{(ax-1)^3} - \frac{45(ax+1)^2}{(ax-1)^2}}{a^6\left(\frac{ax+1}{ax-1}-1\right)^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/945*a*(6*log(abs(-a*x - 1)/abs(a*x - 1))/a^6 - 6*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^6 - (6*(a*x + 1)^7/(a*x - 1)^7 - 45*(a*x + 1)^6/(a*x - 1)^6 -
```

```
274*(a*x + 1)^5/(a*x - 1)^5 - 214*(a*x + 1)^4/(a*x - 1)^4 - 274*(a*x + 1)^3
/(a*x - 1)^3 - 45*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1))/(a^6*((a
*x + 1)/(a*x - 1) - 1)^8) + 6*(210*(a*x + 1)^6/(a*x - 1)^6 + 315*(a*x + 1)^
5/(a*x - 1)^5 + 441*(a*x + 1)^4/(a*x - 1)^4 + 126*(a*x + 1)^3/(a*x - 1)^3 +
 36*(a*x + 1)^2/(a*x - 1)^2 - 9*(a*x + 1)/(a*x - 1) + 1)*log(-(a*((a*x + 1)
/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) +
1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^9))
```

maple [A] time = 0.03, size = 87, normalized size = 0.91

$$\frac{a^4x^9\operatorname{arctanh}(ax)}{9}-\frac{2a^2x^7\operatorname{arctanh}(ax)}{7}+\frac{x^5\operatorname{arctanh}(ax)}{5}+\frac{a^3x^8}{72}-\frac{11x^6a}{378}+\frac{2x^4}{315a}+\frac{4x^2}{315a^3}+\frac{4\ln(ax-1)}{315a^5}+\frac{4\ln(ax}{315a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/9*a^4*x^9*arctanh(a*x)-2/7*a^2*x^7*arctanh(a*x)+1/5*x^5*arctanh(a*x)+1/72
*a^3*x^8-11/378*x^6*a+2/315*x^4/a+4/315*x^2/a^3+4/315/a^5*ln(a*x-1)+4/315/a
^5*ln(a*x+1)
```

maxima [A] time = 0.30, size = 89, normalized size = 0.93

$$\frac{1}{7560}a\left(\frac{105a^6x^8-220a^4x^6+48a^2x^4+96x^2}{a^4}+\frac{96\log(ax+1)}{a^6}+\frac{96\log(ax-1)}{a^6}\right)+\frac{1}{315}\left(35a^4x^9-90a^2x^7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a^2*x^4 + 96*x^2)/a^4 + 96*log(a*
x + 1)/a^6 + 96*log(a*x - 1)/a^6) + 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5
)*arctanh(a*x)
```

mupad [B] time = 1.01, size = 106, normalized size = 1.10

$$\frac{4\ln\left(a^2x^2-1\right)}{315a^5}-\frac{11ax^6}{378}+\ln(ax+1)\left(\frac{a^4x^9}{18}-\frac{a^2x^7}{7}+\frac{x^5}{10}\right)-\ln(1-ax)\left(\frac{a^4x^9}{18}-\frac{a^2x^7}{7}+\frac{x^5}{10}\right)+\frac{2x^4}{315a}+\frac{4x^2}{315a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] (4*log(a^2*x^2 - 1))/(315*a^5) - (11*a*x^6)/378 + log(a*x + 1)*(x^5/10 - (a
^2*x^7)/7 + (a^4*x^9)/18) - log(1 - a*x)*(x^5/10 - (a^2*x^7)/7 + (a^4*x^9)/
18) + (2*x^4)/(315*a) + (4*x^2)/(315*a^3) + (a^3*x^8)/72
```

sympy [A] time = 3.13, size = 100, normalized size = 1.04

$$\begin{cases}\frac{a^4x^9\operatorname{atanh}(ax)}{9}+\frac{a^3x^8}{72}-\frac{2a^2x^7\operatorname{atanh}(ax)}{7}-\frac{11ax^6}{378}+\frac{x^5\operatorname{atanh}(ax)}{5}+\frac{2x^4}{315a}+\frac{4x^2}{315a^3}+\frac{8\log\left(x-\frac{1}{a}\right)}{315a^5}+\frac{8\operatorname{atanh}(ax)}{315a^5} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**9*atanh(a*x)/9 + a**3*x**8/72 - 2*a**2*x**7*atanh(a*x)/7
 - 11*a*x**6/378 + x**5*atanh(a*x)/5 + 2*x**4/(315*a) + 4*x**2/(315*a**3) +
 8*log(x - 1/a)/(315*a**5) + 8*atanh(a*x)/(315*a**5), Ne(a, 0)), (0, True))
```

3.193 $\int x^3 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=87

$$\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{24a^4}+\frac{a^3x^7}{56}+\frac{x}{24a^3}-\frac{1}{3}a^2x^6\tanh^{-1}(ax)-\frac{ax^5}{24}+\frac{1}{4}x^4\tanh^{-1}(ax)+\frac{x^3}{72a}$$

```
[Out] 1/24*x/a^3+1/72*x^3/a-1/24*a*x^5+1/56*a^3*x^7-1/24*arctanh(a*x)/a^4+1/4*x^4
*arctanh(a*x)-1/3*a^2*x^6*arctanh(a*x)+1/8*a^4*x^8*arctanh(a*x)
```

Rubi [A] time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6012, 5916, 302, 206}

$$\frac{a^3x^7}{56}+\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{x}{24a^3}-\frac{\tanh^{-1}(ax)}{24a^4}-\frac{ax^5}{24}+\frac{x^3}{72a}+\frac{1}{4}x^4\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 - ArcTanh[a*x]/(24*a^4)
 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])
/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx=\int\left(x^3\tanh^{-1}(ax)-2a^2x^5\tanh^{-1}(ax)+a^4x^7\tanh^{-1}(ax)\right)dx$$

$$=-\left(\left(2a^2\right)\int x^5\tanh^{-1}(ax)\,dx\right)+a^4\int x^7\tanh^{-1}(ax)\,dx+\int x^3\tanh^{-1}(ax)\,d$$

$$=\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{1}{4}a\int\frac{x^4}{1-a^2x^2}\,d$$

$$=\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{1}{4}a\int\left(-\frac{1}{a^4}-\frac{x^2}{a^2}\right.$$

$$=\frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a^4x^8\,\mathrm{t}$$

$$=\frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}-\frac{\tanh^{-1}(ax)}{24a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}$$

Mathematica [A] time = 0.03, size = 103, normalized size = 1.18

$$\frac{1}{8}a^4x^8\tanh^{-1}(ax)+\frac{\log(1-ax)}{48a^4}-\frac{\log(ax+1)}{48a^4}+\frac{a^3x^7}{56}+\frac{x}{24a^3}-\frac{1}{3}a^2x^6\tanh^{-1}(ax)-\frac{ax^5}{24}+\frac{1}{4}x^4\tanh^{-1}(ax)+\frac{x^3}{72a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 + (x^4*ArcTanh[a*x])/4
- (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])/8 + Log[1 - a*x]/(48*a^
4) - Log[1 + a*x]/(48*a^4)
```

fricas [A] time = 0.51, size = 77, normalized size = 0.89

$$\frac{18\,a^7x^7-42\,a^5x^5+14\,a^3x^3+42\,ax+21\left(3\,a^8x^8-8\,a^6x^6+6\,a^4x^4-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{1008\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/1008*(18*a^7*x^7 - 42*a^5*x^5 + 14*a^3*x^3 + 42*a*x + 21*(3*a^8*x^8 - 8*a
^6*x^6 + 6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^4
```

giac [B] time = 0.20, size = 240, normalized size = 2.76

$$\frac{4}{63}\,a\left(\frac{\frac{28\,(ax+1)^4}{(ax-1)^4}-\frac{7\,(ax+1)^3}{(ax-1)^3}+\frac{21\,(ax+1)^2}{(ax-1)^2}-\frac{7\,(ax+1)}{ax-1}+1}{a^5\left(\frac{ax+1}{ax-1}-1\right)^7}+\frac{84\left(\frac{(ax+1)^5}{(ax-1)^5}+\frac{(ax+1)^4}{(ax-1)^4}+\frac{(ax+1)^3}{(ax-1)^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^5\left(\frac{ax+1}{ax-1}-1\right)^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/63*a*((28*(a*x + 1)^4/(a*x - 1)^4 - 7*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x +
 1)^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x - 1) + 1)/(a^5*((a*x + 1)/(a*x - 1) -
```

```
1)^7) + 84*((a*x + 1)^5/(a*x - 1)^5 + (a*x + 1)^4/(a*x - 1)^4 + (a*x + 1)^3
/(a*x - 1)^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a)
 + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^5*(
(a*x + 1)/(a*x - 1) - 1)^8))
```

maple [A] time = 0.03, size = 85, normalized size = 0.98

$$\frac{a^4x^8\operatorname{arctanh}(ax)}{8}-\frac{a^2x^6\operatorname{arctanh}(ax)}{3}+\frac{x^4\operatorname{arctanh}(ax)}{4}+\frac{a^3x^7}{56}-\frac{a\,x^5}{24}+\frac{x^3}{72a}+\frac{x}{24a^3}+\frac{\ln(ax-1)}{48a^4}-\frac{\ln(ax+1)}{48a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/8*a^4*x^8*arctanh(a*x)-1/3*a^2*x^6*arctanh(a*x)+1/4*x^4*arctanh(a*x)+1/56
*a^3*x^7-1/24*a*x^5+1/72*x^3/a+1/24*x/a^3+1/48/a^4*ln(a*x-1)-1/48/a^4*ln(a*
x+1)
```

maxima [A] time = 0.30, size = 88, normalized size = 1.01

$$\frac{1}{1008}\,a\left(\frac{2\left(9\,a^6x^7-21\,a^4x^5+7\,a^2x^3+21\,x\right)}{a^4}-\frac{21\,\log(ax+1)}{a^5}+\frac{21\,\log(ax-1)}{a^5}\right)+\frac{1}{24}\left(3\,a^4x^8-8\,a^2x^6+6\,x^4\right)\operatorname{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/1008*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x +
1)/a^5 + 21*log(a*x - 1)/a^5) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 + 6*x^4)*arctan
h(a*x)
```

mupad [B] time = 1.21, size = 101, normalized size = 1.16

$$\frac{x}{24\,a^3}-\frac{a\,x^5}{24}+\ln(a\,x+1)\left(\frac{a^4\,x^8}{16}-\frac{a^2\,x^6}{6}+\frac{x^4}{8}\right)-\ln(1-a\,x)\left(\frac{a^4\,x^8}{16}-\frac{a^2\,x^6}{6}+\frac{x^4}{8}\right)+\frac{x^3}{72\,a}+\frac{a^3\,x^7}{56}+\frac{\operatorname{atan}(a\,x\,1\mathrm{i})\,1\mathrm{i}}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] x/(24*a^3) - (a*x^5)/24 + (atan(a*x*1i)*1i)/(24*a^4) + log(a*x + 1)*(x^4/8
- (a^2*x^6)/6 + (a^4*x^8)/16) - log(1 - a*x)*(x^4/8 - (a^2*x^6)/6 + (a^4*x^
8)/16) + x^3/(72*a) + (a^3*x^7)/56
```

sympy [A] time = 2.53, size = 76, normalized size = 0.87

$$\begin{cases}\frac{a^4x^8\operatorname{atanh}(ax)}{8}+\frac{a^3x^7}{56}-\frac{a^2x^6\operatorname{atanh}(ax)}{3}-\frac{ax^5}{24}+\frac{x^4\operatorname{atanh}(ax)}{4}+\frac{x^3}{72a}+\frac{x}{24a^3}-\frac{\operatorname{atanh}(ax)}{24a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**8*atanh(a*x)/8 + a**3*x**7/56 - a**2*x**6*atanh(a*x)/3 -
 a*x**5/24 + x**4*atanh(a*x)/4 + x**3/(72*a) + x/(24*a**3) - atanh(a*x)/(24
*a**4), Ne(a, 0)), (0, True))
```

3.194 $\int x^2 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=86

$$\frac{1}{7}a^4x^7\tanh^{-1}(ax)+\frac{a^3x^6}{42}-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{4\log\left(1-a^2x^2\right)}{105a^3}-\frac{9ax^4}{140}+\frac{1}{3}x^3\tanh^{-1}(ax)+\frac{4x^2}{105a}$$

```
[Out] 4/105*x^2/a-9/140*a*x^4+1/42*a^3*x^6+1/3*x^3*arctanh(a*x)-2/5*a^2*x^5*arcta
nh(a*x)+1/7*a^4*x^7*arctanh(a*x)+4/105*ln(-a^2*x^2+1)/a^3
```

Rubi [A] time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6012, 5916, 266, 43}

$$\frac{a^3x^6}{42}+\frac{4\log\left(1-a^2x^2\right)}{105a^3}+\frac{1}{7}a^4x^7\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)-\frac{9ax^4}{140}+\frac{4x^2}{105a}+\frac{1}{3}x^3\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2*
a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])/(
105*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^2 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx = \int \left(x^2 \tanh^{-1}(ax) - 2a^2x^4 \tanh^{-1}(ax) + a^4x^6 \tanh^{-1}(ax)\right) dx$$
$$= -\left(\left(2a^2\right)\int x^4 \tanh^{-1}(ax)\,dx\right) + a^4 \int x^6 \tanh^{-1}(ax)\,dx + \int x^2 \tanh^{-1}(ax)\,dx$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) - \frac{1}{3}a\int \frac{x^3}{1-a^2x^2}\,dx +$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) - \frac{1}{6}a\,\text{Subst}\left(\int \frac{x}{1-a^2}\right.$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) - \frac{1}{6}a\,\text{Subst}\left(\int\left(-\frac{1}{a^2}\right.\right.$$
$$= \frac{4x^2}{105a} - \frac{9ax^4}{140} + \frac{a^3x^6}{42} + \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(a$$

Mathematica [A] time = 0.02, size = 86, normalized size = 1.00

$$\frac{1}{7}a^4x^7 \tanh^{-1}(ax) + \frac{a^3x^6}{42} - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{4\log\left(1-a^2x^2\right)}{105a^3} - \frac{9ax^4}{140} + \frac{1}{3}x^3 \tanh^{-1}(ax) + \frac{4x^2}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2*
a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])/(
105*a^3)
```

fricas [A] time = 0.78, size = 84, normalized size = 0.98

$$\frac{10\,a^6x^6 - 27\,a^4x^4 + 16\,a^2x^2 + 2\left(15\,a^7x^7 - 42\,a^5x^5 + 35\,a^3x^3\right)\log\left(-\frac{ax+1}{ax-1}\right) + 16\,\log\left(a^2x^2-1\right)}{420\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*x^6 - 27*a^4*x^4 + 16*a^2*x^2 + 2*(15*a^7*x^7 - 42*a^5*x^5 +
35*a^3*x^3)*log(-(a*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a^3
```

giac [B] time = 0.23, size = 319, normalized size = 3.71

$$\frac{4}{105}a\left(\frac{2\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4} - \frac{2\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^4} - \frac{\frac{2(ax+1)^5}{(ax-1)^5} - \frac{11(ax+1)^4}{(ax-1)^4} - \frac{22(ax+1)^3}{(ax-1)^3} - \frac{11(ax+1)^2}{(ax-1)^2} + \frac{2(ax+1)}{ax-1}}{a^4\left(\frac{ax+1}{ax-1}-1\right)^6} + \frac{2\left(\frac{70(ax+1)^4}{(ax-1)^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/105*a*(2*log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - 2*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^4 - (2*(a*x + 1)^5/(a*x - 1)^5 - 11*(a*x + 1)^4/(a*x - 1)^4 -
```

```
22*(a*x + 1)^3/(a*x - 1)^3 - 11*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x
- 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^6) + 2*(70*(a*x + 1)^4/(a*x - 1)^4 + 3
5*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x + 1)^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x -
 1) + 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)
/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x
+ 1)/(a*x - 1) - 1)^7))
```

maple [A] time = 0.03, size = 79, normalized size = 0.92

$$\frac{a^4x^7 \operatorname{arctanh}(ax)}{7}-\frac{2a^2x^5 \operatorname{arctanh}(ax)}{5}+\frac{x^3 \operatorname{arctanh}(ax)}{3}+\frac{x^6a^3}{42}-\frac{9x^4a}{140}+\frac{4x^2}{105a}+\frac{4\ln(ax-1)}{105a^3}+\frac{4\ln(ax+1)}{105a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/7*a^4*x^7*arctanh(a*x)-2/5*a^2*x^5*arctanh(a*x)+1/3*x^3*arctanh(a*x)+1/42
*x^6*a^3-9/140*x^4*a+4/105*x^2/a+4/105/a^3*ln(a*x-1)+4/105/a^3*ln(a*x+1)
```

maxima [A] time = 0.30, size = 81, normalized size = 0.94

$$\frac{1}{420}\,a\left(\frac{10\,a^4x^6-27\,a^2x^4+16\,x^2}{a^2}+\frac{16\,\log(ax+1)}{a^4}+\frac{16\,\log(ax-1)}{a^4}\right)+\frac{1}{105}\left(15\,a^4x^7-42\,a^2x^5+35\,x^3\right)\text{artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a^4 + 16*
log(a*x - 1)/a^4) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^3)*arctanh(a*x)
```

mupad [B] time = 0.95, size = 71, normalized size = 0.83

$$\frac{x^3\operatorname{atanh}(a\,x)}{3}-\frac{9\,a\,x^4}{140}+\frac{4\,\ln\left(a^2\,x^2-1\right)}{105\,a^3}+\frac{4\,x^2}{105\,a}+\frac{a^3\,x^6}{42}-\frac{2\,a^2\,x^5\operatorname{atanh}(a\,x)}{5}+\frac{a^4\,x^7\operatorname{atanh}(a\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] (x^3*atanh(a*x))/3 - (9*a*x^4)/140 + (4*log(a^2*x^2 - 1))/(105*a^3) + (4*x^
2)/(105*a) + (a^3*x^6)/42 - (2*a^2*x^5*atanh(a*x))/5 + (a^4*x^7*atanh(a*x))
/7
```

sympy [A] time = 2.01, size = 90, normalized size = 1.05

$$\begin{cases} \frac{a^4x^7\operatorname{atanh}(ax)}{7}+\frac{a^3x^6}{42}-\frac{2a^2x^5\operatorname{atanh}(ax)}{5}-\frac{9ax^4}{140}+\frac{x^3\operatorname{atanh}(ax)}{3}+\frac{4x^2}{105a}+\frac{8\log\left(x-\frac{1}{a}\right)}{105a^3}+\frac{8\operatorname{atanh}(ax)}{105a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**7*atanh(a*x)/7 + a**3*x**6/42 - 2*a**2*x**5*atanh(a*x)/5
 - 9*a*x**4/140 + x**3*atanh(a*x)/3 + 4*x**2/(105*a) + 8*log(x - 1/a)/(105*
a**3) + 8*atanh(a*x)/(105*a**3), Ne(a, 0)), (0, True))
```

3.195 $\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=50

$$\frac{a^3x^5}{30}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}-\frac{ax^3}{9}+\frac{x}{6a}$$

```
[Out] 1/6*x/a-1/9*a*x^3+1/30*a^3*x^5-1/6*(-a^2*x^2+1)^3*arctanh(a*x)/a^2
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5994, 194}

$$\frac{a^3x^5}{30}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}-\frac{ax^3}{9}+\frac{x}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 - ((1 - a^2*x^2)^3*ArcTanh[a*x])/(6*a^2)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx&=-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}+\frac{\int\left(1-a^2x^2\right)^2\,dx}{6a}\\&=-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}+\frac{\int\left(1-2a^2x^2+a^4x^4\right)dx}{6a}\\&=\frac{x}{6a}-\frac{ax^3}{9}+\frac{a^3x^5}{30}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 93, normalized size = 1.86

$$\frac{1}{6}a^4x^6\tanh^{-1}(ax)+\frac{a^3x^5}{30}-\frac{1}{2}a^2x^4\tanh^{-1}(ax)+\frac{\log(1-ax)}{12a^2}-\frac{\log(ax+1)}{12a^2}-\frac{ax^3}{9}+\frac{1}{2}x^2\tanh^{-1}(ax)+\frac{x}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTan
h[a*x])/2 + (a^4*x^6*ArcTanh[a*x])/6 + Log[1 - a*x]/(12*a^2) - Log[1 + a*x]
/(12*a^2)
```

fricas [A] time = 0.60, size = 68, normalized size = 1.36

$$\frac{6\,a^{5}x^{5}-20\,a^{3}x^{3}+30\,ax+15\left(a^{6}x^{6}-3\,a^{4}x^{4}+3\,a^{2}x^{2}-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{180\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/180*(6*a^5*x^5 - 20*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^
2 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^2
```

giac [B] time = 0.18, size = 176, normalized size = 3.52

$$\frac{8}{45}\,a\left(\frac{\frac{10\,(ax+1)^{2}}{(ax-1)^{2}}-\frac{5\,(ax+1)}{ax-1}+1}{a^{3}\left(\frac{ax+1}{ax-1}-1\right)^{5}}+\frac{30\,(ax+1)^{3}\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^{3}a^{3}\left(\frac{ax+1}{ax-1}-1\right)^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 8/45*a*((10*(a*x + 1)^2/(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)/(a^3*((a*x
 + 1)/(a*x - 1) - 1)^5) + 30*(a*x + 1)^3*log(-(a*((a*x + 1)/(a*x - 1) + 1)/
((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/
(a*x - 1) - a) - 1))/((a*x - 1)^3*a^3*((a*x + 1)/(a*x - 1) - 1)^6))
```

maple [A] time = 0.03, size = 77, normalized size = 1.54

$$\frac{a^{4}\operatorname{arctanh}(ax)\,x^{6}}{6}-\frac{a^{2}\operatorname{arctanh}(ax)\,x^{4}}{2}+\frac{\operatorname{arctanh}(ax)\,x^{2}}{2}+\frac{a^{3}x^{5}}{30}-\frac{x^{3}a}{9}+\frac{x}{6a}+\frac{\ln(ax-1)}{12a^{2}}-\frac{\ln(ax+1)}{12a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/6*a^4*arctanh(a*x)*x^6-1/2*a^2*arctanh(a*x)*x^4+1/2*arctanh(a*x)*x^2+1/30
*a^3*x^5-1/9*x^3*a+1/6*x/a+1/12/a^2*ln(a*x-1)-1/12/a^2*ln(a*x+1)
```

maxima [A] time = 0.30, size = 46, normalized size = 0.92

$$\frac{\left(a^{2}x^{2}-1\right)^{3}\operatorname{artanh}(ax)}{6\,a^{2}}+\frac{3\,a^{4}x^{5}-10\,a^{2}x^{3}+15\,x}{90\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 1)^3*arctanh(a*x)/a^2 + 1/90*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)
/a
```

mupad [B] time = 0.92, size = 64, normalized size = 1.28

$$\frac{x^{2}\operatorname{atanh}(a\,x)}{2}-\frac{\frac{\operatorname{atanh}(a\,x)}{6}-\frac{a\,x}{6}}{a^{2}}-\frac{a\,x^{3}}{9}+\frac{a^{3}\,x^{5}}{30}-\frac{a^{2}\,x^{4}\operatorname{atanh}(a\,x)}{2}+\frac{a^{4}\,x^{6}\operatorname{atanh}(a\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] (x^2*atanh(a*x))/2 - (atanh(a*x)/6 - (a*x)/6)/a^2 - (a*x^3)/9 + (a^3*x^5)/3
0 - (a^2*x^4*atanh(a*x))/2 + (a^4*x^6*atanh(a*x))/6
```

sympy [**A**] time = 1.52, size = 68, normalized size = 1.36

$$\begin{cases} \frac{a^4x^6 \operatorname{atanh}(ax)}{6} + \frac{a^3x^5}{30} - \frac{a^2x^4 \operatorname{atanh}(ax)}{2} - \frac{ax^3}{9} + \frac{x^2 \operatorname{atanh}(ax)}{2} + \frac{x}{6a} - \frac{\operatorname{atanh}(ax)}{6a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**6*atanh(a*x)/6 + a**3*x**5/30 - a**2*x**4*atanh(a*x)/2 -
 a*x**3/9 + x**2*atanh(a*x)/2 + x/(6*a) - atanh(a*x)/(6*a**2), Ne(a, 0)), (
0, True))
```

3.196 $\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=104

$$\frac{\left(1-a^2x^2\right)^2}{20a}+\frac{2\left(1-a^2x^2\right)}{15a}+\frac{4\log\left(1-a^2x^2\right)}{15a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{8}{15}x\tanh$$

```
[Out] 2/15*(-a^2*x^2+1)/a+1/20*(-a^2*x^2+1)^2/a+8/15*x*arctanh(a*x)+4/15*x*(-a^2*
x^2+1)*arctanh(a*x)+1/5*x*(-a^2*x^2+1)^2*arctanh(a*x)+4/15*ln(-a^2*x^2+1)/a
```

Rubi [A] time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5942, 5910, 260}

$$\frac{\left(1-a^2x^2\right)^2}{20a}+\frac{2\left(1-a^2x^2\right)}{15a}+\frac{4\log\left(1-a^2x^2\right)}{15a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{8}{15}x\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (2*(1 - a^2*x^2))/(15*a) + (1 - a^2*x^2)^2/(20*a) + (8*x*ArcTanh[a*x])/15 +
 (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x])/5 +
 (4*Log[1 - a^2*x^2])/(15*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx &= \frac{\left(1-a^2x^2\right)^2}{20a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)+\frac{4}{5}\int \left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx\\
&= \frac{2\left(1-a^2x^2\right)}{15a}+\frac{\left(1-a^2x^2\right)^2}{20a}+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh\\
&= \frac{2\left(1-a^2x^2\right)}{15a}+\frac{\left(1-a^2x^2\right)^2}{20a}+\frac{8}{15}x\tanh^{-1}(ax)+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{1}{5}\\
&= \frac{2\left(1-a^2x^2\right)}{15a}+\frac{\left(1-a^2x^2\right)^2}{20a}+\frac{8}{15}x\tanh^{-1}(ax)+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{1}{5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 0.68

$$\frac{1}{5}a^4x^5\tanh^{-1}(ax)+\frac{a^3x^4}{20}-\frac{2}{3}a^2x^3\tanh^{-1}(ax)+\frac{4\log\left(1-a^2x^2\right)}{15a}-\frac{7ax^2}{30}+x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (-7*a*x^2)/30 + (a^3*x^4)/20 + x*ArcTanh[a*x] - (2*a^2*x^3*ArcTanh[a*x])/3
+ (a^4*x^5*ArcTanh[a*x])/5 + (4*Log[1 - a^2*x^2])/(15*a)
```

fricas [A] time = 0.59, size = 72, normalized size = 0.69

$$\frac{3\,a^4x^4-14\,a^2x^2+2\left(3\,a^5x^5-10\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+16\,\log\left(a^2x^2-1\right)}{60\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*x^4 - 14*a^2*x^2 + 2*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a*
x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a
```

giac [B] time = 0.38, size = 255, normalized size = 2.45

$$\frac{4}{15}\,a\left(\frac{2\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2}-\frac{2\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2}-\frac{\frac{2\,(ax+1)^3}{(ax-1)^3}-\frac{7\,(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}}{a^2\left(\frac{ax+1}{ax-1}-1\right)^4}+\frac{2\left(\frac{10\,(ax+1)^2}{(ax-1)^2}-\frac{5\,(ax+1)}{ax-1}+1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^2\left(\frac{ax+1}{ax-1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/15*a*(2*log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - 2*log(abs(-(a*x + 1)/(a*x -
 1) + 1))/a^2 - (2*(a*x + 1)^3/(a*x - 1)^3 - 7*(a*x + 1)^2/(a*x - 1)^2 + 2*
(a*x + 1)/(a*x - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^4) + 2*(10*(a*x + 1)^2/
(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/
((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/
(a*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^5))
```

maple [A] time = 0.03, size = 68, normalized size = 0.65

$$\frac{a^4\operatorname{arctanh}(ax)\,x^5}{5}-\frac{2a^2\operatorname{arctanh}(ax)\,x^3}{3}+x\operatorname{arctanh}(ax)+\frac{x^4a^3}{20}-\frac{7a\,x^2}{30}+\frac{4\ln(ax-1)}{15a}+\frac{4\ln(ax+1)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/5*a^4*arctanh(a*x)*x^5-2/3*a^2*arctanh(a*x)*x^3+x*arctanh(a*x)+1/20*x^4*a
^3-7/30*a*x^2+4/15/a*ln(a*x-1)+4/15/a*ln(a*x+1)
```

maxima [A] time = 0.30, size = 66, normalized size = 0.63

$$\frac{1}{60}\left(3\,a^2x^4-14\,x^2+\frac{16\,\log(ax+1)}{a^2}+\frac{16\,\log(ax-1)}{a^2}\right)a+\frac{1}{15}\left(3\,a^4x^5-10\,a^2x^3+15\,x\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*(3*a^2*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a + 1
/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)
```

mupad [B] time = 0.91, size = 60, normalized size = 0.58

$$x \operatorname{atanh}(a x) - \frac{7 a x^2}{30} + \frac{4 \ln\left(a^2 x^2 - 1\right)}{15 a} + \frac{a^3 x^4}{20} - \frac{2 a^2 x^3 \operatorname{atanh}(a x)}{3} + \frac{a^4 x^5 \operatorname{atanh}(a x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] x*atanh(a*x) - (7*a*x^2)/30 + (4*log(a^2*x^2 - 1))/(15*a) + (a^3*x^4)/20 -
(2*a^2*x^3*atanh(a*x))/3 + (a^4*x^5*atanh(a*x))/5
```

sympy [A] time = 1.24, size = 75, normalized size = 0.72

$$\begin{cases} \frac{a^4 x^5 \operatorname{atanh}(ax)}{5} + \frac{a^3 x^4}{20} - \frac{2a^2 x^3 \operatorname{atanh}(ax)}{3} - \frac{7ax^2}{30} + x \operatorname{atanh}(ax) + \frac{8 \log\left(x - \frac{1}{a}\right)}{15a} + \frac{8 \operatorname{atanh}(ax)}{15a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**5*atanh(a*x)/5 + a**3*x**4/20 - 2*a**2*x**3*atanh(a*x)/3
 - 7*a*x**2/30 + x*atanh(a*x) + 8*log(x - 1/a)/(15*a) + 8*atanh(a*x)/(15*a)
, Ne(a, 0)), (0, True))
```

3.197 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x} dx$

Optimal. Leaf size=70

$$\frac{1}{4}a^4x^4\tanh^{-1}(ax)+\frac{a^3x^3}{12}-a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}-\frac{3ax}{4}+\frac{3}{4}\tanh^{-1}(ax)$$

```
[Out] -3/4*a*x+1/12*a^3*x^3+3/4*arctanh(a*x)-a^2*x^2*arctanh(a*x)+1/4*a^4*x^4*arc
tanh(a*x)-1/2*polylog(2,-a*x)+1/2*polylog(2,a*x)
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6012, 5912, 5916, 321, 206, 302}

$$-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)+\frac{a^3x^3}{12}+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-a^2x^2\tanh^{-1}(ax)-\frac{3ax}{4}+\frac{3}{4}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

```
[Out] (-3*a*x)/4 + (a^3*x^3)/12 + (3*ArcTanh[a*x])/4 - a^2*x^2*ArcTanh[a*x] + (a^
4*x^4*ArcTanh[a*x])/4 - PolyLog[2, -(a*x)]/2 + PolyLog[2, a*x]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x}\,dx = \int \left(\frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + a^4x^3\tanh^{-1}(ax)\right)dx$$

$$= -\left((2a^2)\int x\tanh^{-1}(ax)\,dx\right) + a^4\int x^3\tanh^{-1}(ax)\,dx + \int \frac{\tanh^{-1}(ax)}{x}\,dx$$

$$= -a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-ax)}{2} + \frac{\text{Li}_2(ax)}{2} + a^3\int \frac{x^2}{1-a^2x^2}$$

$$= -ax - a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-ax)}{2} + \frac{\text{Li}_2(ax)}{2} + a\int \frac{1}{1-}$$

$$= -\frac{3ax}{4} + \frac{a^3x^3}{12} + \tanh^{-1}(ax) - a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-ax}{2}$$

$$= -\frac{3ax}{4} + \frac{a^3x^3}{12} + \frac{3}{4}\tanh^{-1}(ax) - a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-}{2}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 1.04

$$\frac{1}{24}\left(6a^4x^4\tanh^{-1}(ax) + 2a^3x^3 - 24a^2x^2\tanh^{-1}(ax) - 12\text{Li}_2(-ax) + 12\text{Li}_2(ax) - 18ax - 9\log(1-ax) + 9\log(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

```
[Out] (-18*a*x + 2*a^3*x^3 - 24*a^2*x^2*ArcTanh[a*x] + 6*a^4*x^4*ArcTanh[a*x] - 9
*Log[1 - a*x] + 9*Log[1 + a*x] - 12*PolyLog[2, -(a*x)] + 12*PolyLog[2, a*x]
)/24
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a^4x^4 - 2a^2x^2 + 1)\,\text{artanh}(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2x^2 - 1)^2\,\text{artanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x, x)
```

maple [A] time = 0.05, size = 89, normalized size = 1.27

$$\frac{a^4x^4\operatorname{arctanh}(ax)}{4}-a^2x^2\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{dilog}(ax)}{2}-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1)}{2}+\frac{x^3a^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x,x)
```

```
[Out] 1/4*a^4*x^4*arctanh(a*x)-a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*dilo
g(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)+1/12*x^3*a^3-3/4*a*x-3/8*ln(a
*x-1)+3/8*ln(a*x+1)
```

maxima [A] time = 0.31, size = 106, normalized size = 1.51

$$\frac{1}{24}\left(2a^2x^3-18x-\frac{12\left(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax)\right)}{a}+\frac{12\left(\log(-ax+1)\log(x)+\operatorname{Li}_2(ax)\right)}{a}+\frac{9\log(ax+1)}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/24*(2*a^2*x^3 - 18*x - 12*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 12*(log
(-a*x + 1)*log(x) + dilog(a*x))/a + 9*log(a*x + 1)/a - 9*log(a*x - 1)/a)*a
+ 1/4*(a^4*x^4 - 4*a^2*x^2 + 2*log(x^2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x,x)
```

```
[Out] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2(ax+1)^2\operatorname{atanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x, x)
```

3.198 $\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^2} \, dx$

Optimal. Leaf size=64

$$\frac{1}{3}a^4x^3\tanh^{-1}(ax)+\frac{a^3x^2}{6}-\frac{4}{3}a\log\left(1-a^2x^2\right)-2a^2x\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] 1/6*a^3*x^2-arctanh(a*x)/x-2*a^2*x*arctanh(a*x)+1/3*a^4*x^3*arctanh(a*x)+a*
ln(x)-4/3*a*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6012, 5910, 260, 5916, 266, 36, 29, 31, 43}

$$\frac{a^3x^2}{6}-\frac{4}{3}a\log\left(1-a^2x^2\right)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)-2a^2x\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]
```

```
[Out] (a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x]
)/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{x^2}\,dx &= \int \left(-2a^2\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{x^2}+a^4x^2\tanh^{-1}(ax)\right)dx \\
&= -\left(\left(2a^2\right)\int \tanh^{-1}(ax)\,dx\right)+a^4\int x^2\tanh^{-1}(ax)\,dx+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx \\
&= -\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)+a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx+\left(2a\right. \\
&= -\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)-a\log\left(1-a^2x^2\right)+\frac{1}{2}a\,\mathrm{Sub} \\
&= -\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)-a\log\left(1-a^2x^2\right)+\frac{1}{2}a\,\mathrm{Sub} \\
&= \frac{a^3x^2}{6}-\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)+a\log(x)-\frac{4}{3}a\log\left(\right.
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 64, normalized size = 1.00

$$\frac{1}{3}a^4x^3\tanh^{-1}(ax)+\frac{a^3x^2}{6}-\frac{4}{3}a\log\left(1-a^2x^2\right)-2a^2x\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]
```

```
[Out] (a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x]
)/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```

fricas [A] time = 0.53, size = 66, normalized size = 1.03

$$\frac{a^3x^3-8\,ax\log\left(a^2x^2-1\right)+6\,ax\log(x)+\left(a^4x^4-6\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3 - 8*a*x*log(a^2*x^2 - 1) + 6*a*x*log(x) + (a^4*x^4 - 6*a^2*x^2
 - 3)*log(-(a*x + 1)/(a*x - 1)))/x
```

giac [B] time = 0.26, size = 249, normalized size = 3.89

$$\frac{1}{3}\left(\left(\frac{3}{\frac{ax+1}{ax-1}+1} - \frac{\frac{3(ax+1)^2}{(ax-1)^2} - \frac{12(ax+1)}{ax-1} + 5}{\left(\frac{ax+1}{ax-1}-1\right)^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right) + \frac{2(ax+1)}{(ax-1)\left(\frac{ax+1}{ax-1}-1\right)^2} - 8\log\left(\frac{|-ax-1|}{|ax-1|}\right) + 5\,\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*((3/((a*x + 1)/(a*x - 1) + 1) - (3*(a*x + 1)^2/(a*x - 1)^2 - 12*(a*x +
1)/(a*x - 1) + 5)/((a*x + 1)/(a*x - 1) - 1)^3)*log(-(a*((a*x + 1)/(a*x - 1)
 + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x +
 1)*a/(a*x - 1) - a) - 1)) + 2*(a*x + 1)/((a*x - 1)*((a*x + 1)/(a*x - 1) -
1)^2) - 8*log(abs(-a*x - 1)/abs(a*x - 1)) + 5*log(abs(-(a*x + 1)/(a*x - 1)
+ 1)) + 3*log(abs(-(a*x + 1)/(a*x - 1) - 1)))*a
```

maple [A] time = 0.04, size = 65, normalized size = 1.02

$$\frac{a^4x^3\operatorname{arctanh}(ax)}{3} - 2a^2x\operatorname{arctanh}(ax) - \frac{\operatorname{arctanh}(ax)}{x} + \frac{x^2a^3}{6} + a\ln(ax) - \frac{4a\ln(ax-1)}{3} - \frac{4a\ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x)
```

```
[Out] 1/3*a^4*x^3*arctanh(a*x)-2*a^2*x*arctanh(a*x)-arctanh(a*x)/x+1/6*x^2*a^3+a*
ln(a*x)-4/3*a*ln(a*x-1)-4/3*a*ln(a*x+1)
```

maxima [A] time = 0.31, size = 57, normalized size = 0.89

$$\frac{1}{6}\left(a^2x^2 - 8\log(ax+1) - 8\log(ax-1) + 6\log(x)\right)a + \frac{1}{3}\left(a^4x^3 - 6a^2x - \frac{3}{x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 8*log(a*x + 1) - 8*log(a*x - 1) + 6*log(x))*a + 1/3*(a^4*x^3
 - 6*a^2*x - 3/x)*arctanh(a*x)
```

mupad [B] time = 0.91, size = 57, normalized size = 0.89

$$a\ln(x) - \frac{4a\ln\left(a^2x^2-1\right)}{3} - \frac{\operatorname{atanh}(a\,x)}{x} + \frac{a^3x^2}{6} - 2a^2x\operatorname{atanh}(a\,x) + \frac{a^4x^3\operatorname{atanh}(a\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^2,x)
```

```
[Out] a*log(x) - (4*a*log(a^2*x^2 - 1))/3 - atanh(a*x)/x + (a^3*x^2)/6 - 2*a^2*x*
atanh(a*x) + (a^4*x^3*atanh(a*x))/3
```

sympy [A] time = 1.50, size = 68, normalized size = 1.06

$$\begin{cases} \frac{a^4x^3\operatorname{atanh}(ax)}{3} + \frac{a^3x^2}{6} - 2a^2x\operatorname{atanh}(ax) + a\log(x) - \frac{8a\log\left(x-\frac{1}{a}\right)}{3} - \frac{8a\operatorname{atanh}(ax)}{3} - \frac{\operatorname{atanh}(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**2,x)
```

```
[Out] Piecewise((a**4*x**3*atanh(a*x)/3 + a**3*x**2/6 - 2*a**2*x*atanh(a*x) + a*l
og(x) - 8*a*log(x - 1/a)/3 - 8*a*atanh(a*x)/3 - atanh(a*x)/x, Ne(a, 0)), (0
, True))
```

3.199 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=62

$$\frac{1}{2}a^4x^2\tanh^{-1}(ax) + \frac{a^3x}{2} + a^2\text{Li}_2(-ax) - a^2\text{Li}_2(ax) - \frac{\tanh^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

```
[Out] -1/2*a/x+1/2*a^3*x-1/2*arctanh(a*x)/x^2+1/2*a^4*x^2*arctanh(a*x)+a^2*polylo
g(2,-a*x)-a^2*polylog(2,a*x)
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6012, 5916, 325, 206, 5912, 321}

$$a^2\text{PolyLog}(2,-ax) - a^2\text{PolyLog}(2,ax) + \frac{1}{2}a^4x^2\tanh^{-1}(ax) + \frac{a^3x}{2} - \frac{\tanh^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

```
[Out] -a/(2*x) + (a^3*x)/2 - ArcTanh[a*x]/(2*x^2) + (a^4*x^2*ArcTanh[a*x])/2 + a^
2*PolyLog[2, -(a*x)] - a^2*PolyLog[2, a*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{x^3}\,dx &= \int \left(\frac{\tanh^{-1}(ax)}{x^3}-\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)\right)dx\\ &= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x}\,dx\right)+a^4\int x\tanh^{-1}(ax)\,dx+\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)+a^2\mathrm{Li}_2(-ax)-a^2\mathrm{Li}_2(ax)+\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right.}\\ &= -\frac{a}{2x}+\frac{a^3x}{2}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)+a^2\mathrm{Li}_2(-ax)-a^2\mathrm{Li}_2(ax)\end{aligned}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 1.00

$$-\frac{-a^4x^4\tanh^{-1}(ax)-a^3x^3-2a^2x^2\mathrm{Li}_2(-ax)+2a^2x^2\mathrm{Li}_2(ax)+ax+\tanh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*(a*x - a^3*x^3 + ArcTanh[a*x] - a^4*x^4*ArcTanh[a*x] - 2*a^2*x^2*PolyL
og[2, -(a*x)] + 2*a^2*x^2*PolyLog[2, a*x])/x^2
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\left(ax\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2\mathrm{artanh}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^3, x)
```

maple [A] time = 0.05, size = 80, normalized size = 1.29

$$\frac{a^4x^2\,\mathrm{arctanh}\left(ax\right)}{2}-2a^2\,\mathrm{arctanh}\left(ax\right)\ln\left(ax\right)-\frac{\mathrm{arctanh}\left(ax\right)}{2x^2}+a^2\,\mathrm{dilog}\left(ax\right)+a^2\,\mathrm{dilog}\left(ax+1\right)+a^2\ln\left(ax\right)\ln\left(ax+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x)
```

```
[Out] 1/2*a^4*x^2*arctanh(a*x)-2*a^2*arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)/x^2+a^
2*dilog(a*x)+a^2*dilog(a*x+1)+a^2*ln(a*x)*ln(a*x+1)+1/2*a^3*x-1/2*a/x
```

maxima [A] time = 0.31, size = 82, normalized size = 1.32

$$\frac{1}{2}\left(2\left(\log\left(ax+1\right)\log(x)+\mathrm{Li}_2\left(-ax\right)\right)a-2\left(\log\left(-ax+1\right)\log(x)+\mathrm{Li}_2\left(ax\right)\right)a+\frac{a^2x^2-1}{x}\right)a+\frac{1}{2}\left(a^4x^2-2\,a^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + di
log(a*x))*a + (a^2*x^2 - 1)/x)*a + 1/2*(a^4*x^2 - 2*a^2*log(x^2) - 1/x^2)*a
rctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^3,x)
```

```
[Out] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2\mathrm{atanh}\left(ax\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**3, x)
```

3.200 $\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^4} dx$

Optimal. Leaf size=68

$$a^4x\tanh^{-1}(ax)-\frac{5}{3}a^3\log(x)+\frac{2a^2\tanh^{-1}(ax)}{x}+\frac{4}{3}a^3\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{3x^3}-\frac{a}{6x^2}$$

```
[Out] -1/6*a/x^2-1/3*arctanh(a*x)/x^3+2*a^2*arctanh(a*x)/x+a^4*x*arctanh(a*x)-5/3
*a^3*ln(x)+4/3*a^3*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6012, 5910, 260, 5916, 266, 44, 36, 29, 31}

$$\frac{4}{3}a^3\log\left(1-a^2x^2\right)-\frac{5}{3}a^3\log(x)+a^4x\tanh^{-1}(ax)+\frac{2a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh[
a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^4}\,dx &= \int \left(a^4\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{x^4}-\frac{2a^2\tanh^{-1}(ax)}{x^2}\right)dx\\
&= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\right)+a^4\int \tanh^{-1}(ax)\,dx+\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)+\frac{1}{3}a\int \frac{1}{x^3\left(1-a^2x^2\right)}\,dx-(\\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)+\frac{1}{2}a^3\log\left(1-a^2x^2\right)+\frac{1}{6}a\,S\\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)+\frac{1}{2}a^3\log\left(1-a^2x^2\right)+\frac{1}{6}a\,S\\
&= -\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)-\frac{5}{3}a^3\log(x)+\frac{4}{3}a^3\,\mathrm{lo}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.00

$$a^4x\tanh^{-1}(ax)-\frac{5}{3}a^3\log(x)+\frac{2a^2\tanh^{-1}(ax)}{x}+\frac{4}{3}a^3\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{3x^3}-\frac{a}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh[
a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

fricas [A] time = 0.71, size = 72, normalized size = 1.06

$$\frac{8\,a^3x^3\log\left(a^2x^2-1\right)-10\,a^3x^3\log(x)-ax+\left(3\,a^4x^4+6\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(8*a^3*x^3*log(a^2*x^2 - 1) - 10*a^3*x^3*log(x) - a*x + (3*a^4*x^4 + 6*
a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1)))/x^3
```

giac [B] time = 0.23, size = 274, normalized size = 4.03

$$\frac{1}{3}\left(8\,a^2\log\left(\frac{|-ax-1|}{|ax-1|}\right)-3\,a^2\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)-5\,a^2\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)+\left(\frac{3\,a^2}{\frac{ax+1}{ax-1}-1}-\frac{\frac{3\,(ax+1)^2a^2}{(ax-1)^2}+\frac{12\,(ax+1)a^2}{ax-1}}{\left(\frac{ax+1}{ax-1}+1\right)^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(8*a^2*log(abs(-a*x - 1)/abs(a*x - 1)) - 3*a^2*log(abs(-(a*x + 1)/(a*x
- 1) + 1)) - 5*a^2*log(abs(-(a*x + 1)/(a*x - 1) - 1)) + (3*a^2/((a*x + 1)/(
a*x - 1) - 1) - (3*(a*x + 1)^2*a^2/(a*x - 1)^2 + 12*(a*x + 1)*a^2/(a*x - 1)
 + 5*a^2)/((a*x + 1)/(a*x - 1) + 1)^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((
a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a
*x - 1) - a) - 1)) + 2*(a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2
))*a
```

maple [A] time = 0.04, size = 69, normalized size = 1.01

$$a^4x\operatorname{arctanh}(ax)+\frac{2a^2\operatorname{arctanh}(ax)}{x}-\frac{\operatorname{arctanh}(ax)}{3x^3}-\frac{a}{6x^2}-\frac{5a^3\ln(ax)}{3}+\frac{4a^3\ln(ax-1)}{3}+\frac{4a^3\ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x)
```

```
[Out] a^4*x*arctanh(a*x)+2*a^2*arctanh(a*x)/x-1/3*arctanh(a*x)/x^3-1/6*a/x^2-5/3*
a^3*ln(a*x)+4/3*a^3*ln(a*x-1)+4/3*a^3*ln(a*x+1)
```

maxima [A] time = 0.30, size = 66, normalized size = 0.97

$$\frac{1}{6}\left(8\,a^2\log(ax+1)+8\,a^2\log(ax-1)-10\,a^2\log(x)-\frac{1}{x^2}\right)a+\frac{1}{3}\left(3\,a^4x+\frac{6\,a^2x^2-1}{x^3}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(8*a^2*log(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a + 1
/3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)
```

mupad [B] time = 0.88, size = 59, normalized size = 0.87

$$\frac{4\,a^3\,\ln\left(a^2\,x^2-1\right)}{3}-\frac{a}{6\,x^2}-\frac{\operatorname{atanh}(a\,x)}{3\,x^3}-\frac{5\,a^3\,\ln(x)}{3}+a^4\,x\operatorname{atanh}(a\,x)+\frac{2\,a^2\operatorname{atanh}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^4,x)
```

```
[Out] (4*a^3*log(a^2*x^2 - 1))/3 - a/(6*x^2) - atanh(a*x)/(3*x^3) - (5*a^3*log(x)
)/3 + a^4*x*atanh(a*x) + (2*a^2*atanh(a*x))/x
```

sympy [A] time = 1.53, size = 75, normalized size = 1.10

$$\begin{cases} a^4x\,\mathrm{atanh}\,(ax) - \frac{5a^3\log(x)}{3} + \frac{8a^3\log\left(x-\frac{1}{a}\right)}{3} + \frac{8a^3\,\mathrm{atanh}\,(ax)}{3} + \frac{2a^2\,\mathrm{atanh}\,(ax)}{x} - \frac{a}{6x^2} - \frac{\mathrm{atanh}\,(ax)}{3x^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**4,x)
```

```
[Out] Piecewise((a**4*x*atanh(a*x) - 5*a**3*log(x)/3 + 8*a**3*log(x - 1/a)/3 + 8*
a**3*atanh(a*x)/3 + 2*a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3),
 Ne(a, 0)), (0, True))
```

3.201 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^5} dx$

Optimal. Leaf size=77

$$-\frac{1}{2}a^4\text{Li}_2(-ax)+\frac{1}{2}a^4\text{Li}_2(ax)-\frac{3}{4}a^4\tanh^{-1}(ax)+\frac{3a^3}{4x}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{\tanh^{-1}(ax)}{4x^4}-\frac{a}{12x^3}$$

```
[Out] -1/12*a/x^3+3/4*a^3/x-3/4*a^4*arctanh(a*x)-1/4*arctanh(a*x)/x^4+a^2*arctanh
(a*x)/x^2-1/2*a^4*polylog(2,-a*x)+1/2*a^4*polylog(2,a*x)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6012, 5916, 325, 206, 5912}

$$-\frac{1}{2}a^4\text{PolyLog}(2,-ax)+\frac{1}{2}a^4\text{PolyLog}(2,ax)+\frac{a^2\tanh^{-1}(ax)}{x^2}+\frac{3a^3}{4x}-\frac{3}{4}a^4\tanh^{-1}(ax)-\frac{a}{12x^3}-\frac{\tanh^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
```

```
[Out] -a/(12*x^3) + (3*a^3)/(4*x) - (3*a^4*ArcTanh[a*x])/4 - ArcTanh[a*x]/(4*x^4)
 + (a^2*ArcTanh[a*x])/x^2 - (a^4*PolyLog[2, -(a*x)])/2 + (a^4*PolyLog[2, a*
x])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^5}\,dx = \int \left(\frac{\tanh^{-1}(ax)}{x^5}-\frac{2a^2\tanh^{-1}(ax)}{x^3}+\frac{a^4\tanh^{-1}(ax)}{x}\right)dx$$
$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\right)+a^4\int \frac{\tanh^{-1}(ax)}{x}\,dx+\int \frac{\tanh^{-1}(ax)}{x^5}\,dx$$
$$= -\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2(-ax)+\frac{1}{2}a^4\mathrm{Li}_2(ax)+\frac{1}{4}a\int \frac{1}{x^4\left(1-a\right.}$$
$$= -\frac{a}{12x^3}+\frac{a^3}{x}-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2(-ax)+\frac{1}{2}a^4\mathrm{Li}_2(ax)+\frac{1}{4}$$
$$= -\frac{a}{12x^3}+\frac{3a^3}{4x}-a^4\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2(-ax)$$
$$= -\frac{a}{12x^3}+\frac{3a^3}{4x}-\frac{3}{4}a^4\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2(-ax$$

Mathematica [A] time = 0.08, size = 84, normalized size = 1.09

$$\frac{1}{24}\left(-12a^4\mathrm{Li}_2(-ax)+12a^4\mathrm{Li}_2(ax)+9a^4\log(1-ax)-9a^4\log(ax+1)+\frac{18a^3}{x}+\frac{24a^2\tanh^{-1}(ax)}{x^2}-\frac{6\tanh^{-1}(}{x^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
[Out] ((-2*a)/x^3 + (18*a^3)/x - (6*ArcTanh[a*x])/x^4 + (24*a^2*ArcTanh[a*x])/x^2
 + 9*a^4*Log[1 - a*x] - 9*a^4*Log[1 + a*x] - 12*a^4*PolyLog[2, -(a*x)] + 12
*a^4*PolyLog[2, a*x])/24
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\,(ax)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="fricas")
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2\mathrm{artanh}\,(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^5, x)
```

maple [A] time = 0.05, size = 105, normalized size = 1.36

$$a^4\,\mathrm{arctanh}\,(ax)\ln(ax)+\frac{a^2\,\mathrm{arctanh}\,(ax)}{x^2}-\frac{\mathrm{arctanh}\,(ax)}{4x^4}-\frac{a^4\,\mathrm{dilog}\,(ax)}{2}-\frac{a^4\,\mathrm{dilog}\,(ax+1)}{2}-\frac{a^4\ln(ax)\ln(ax+1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x)
```

```
[Out] a^4*arctanh(a*x)*ln(a*x)+a^2*arctanh(a*x)/x^2-1/4*arctanh(a*x)/x^4-1/2*a^4*
dilog(a*x)-1/2*a^4*dilog(a*x+1)-1/2*a^4*ln(a*x)*ln(a*x+1)-1/12*a/x^3+3/4*a^
3/x+3/8*a^4*ln(a*x-1)-3/8*a^4*ln(a*x+1)
```

maxima [A] time = 0.30, size = 112, normalized size = 1.45

$$-\frac{1}{24}\left(12\left(\log\left(ax+1\right)\log(x)+\mathrm{Li}_2\left(-ax\right)\right)a^3-12\left(\log\left(-ax+1\right)\log(x)+\mathrm{Li}_2\left(ax\right)\right)a^3+9\,a^3\log\left(ax+1\right)-9\,a^3\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 - 12*(log(-a*x + 1)*log(x
) + dilog(a*x))*a^3 + 9*a^3*log(a*x + 1) - 9*a^3*log(a*x - 1) - 2*(9*a^2*x^
2 - 1)/x^3)*a + 1/4*(2*a^4*log(x^2) + (4*a^2*x^2 - 1)/x^4)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^5,x)
```

```
[Out] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2\mathrm{atanh}\left(ax\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**5,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**5, x)
```

3.202 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^6} \, dx$

Optimal. Leaf size=83

$$\frac{8}{15}a^5\log(x)-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{7a^3}{30x^2}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{4}{15}a^5\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{5x^5}-\frac{a}{20x^4}$$

```
[Out] -1/20*a/x^4+7/30*a^3/x^2-1/5*arctanh(a*x)/x^5+2/3*a^2*arctanh(a*x)/x^3-a^4*
arctanh(a*x)/x+8/15*a^5*ln(x)-4/15*a^5*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6012, 5916, 266, 44, 36, 29, 31}

$$\frac{7a^3}{30x^2}-\frac{4}{15}a^5\log\left(1-a^2x^2\right)+\frac{2a^2\tanh^{-1}(ax)}{3x^3}+\frac{8}{15}a^5\log(x)-\frac{a^4\tanh^{-1}(ax)}{x}-\frac{a}{20x^4}-\frac{\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

```
[Out] -a/(20*x^4) + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x]
)/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x
^2])/15
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^6}\,dx = \int \left(\frac{\tanh^{-1}(ax)}{x^6} - \frac{2a^2\tanh^{-1}(ax)}{x^4} + \frac{a^4\tanh^{-1}(ax)}{x^2}\right)dx$$

$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)}{x^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^6}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5} + \frac{2a^2\tanh^{-1}(ax)}{3x^3} - \frac{a^4\tanh^{-1}(ax)}{x} + \frac{1}{5}a\int \frac{1}{x^5\left(1-a^2x^2\right)}\,dx - \frac{1}{3}\left(2\right.$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5} + \frac{2a^2\tanh^{-1}(ax)}{3x^3} - \frac{a^4\tanh^{-1}(ax)}{x} + \frac{1}{10}a\,\text{Subst}\left(\int \frac{1}{x^3\left(1-a^2x\right)}\,d\right.$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5} + \frac{2a^2\tanh^{-1}(ax)}{3x^3} - \frac{a^4\tanh^{-1}(ax)}{x} + \frac{1}{10}a\,\text{Subst}\left(\int \left(\frac{1}{x^3} + \frac{a^2}{x^2} + \frac{a^4}{x}\right.\right.$$

$$= -\frac{a}{20x^4} + \frac{7a^3}{30x^2} - \frac{\tanh^{-1}(ax)}{5x^5} + \frac{2a^2\tanh^{-1}(ax)}{3x^3} - \frac{a^4\tanh^{-1}(ax)}{x} + \frac{8}{15}a^5\log(x) -$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.00

$$\frac{8}{15}a^5\log(x) - \frac{a^4\tanh^{-1}(ax)}{x} + \frac{7a^3}{30x^2} + \frac{2a^2\tanh^{-1}(ax)}{3x^3} - \frac{4}{15}a^5\log\left(1-a^2x^2\right) - \frac{\tanh^{-1}(ax)}{5x^5} - \frac{a}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

```
[Out] -1/20*a/x^4 + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x]
)/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x
^2])/15
```

fricas [A] time = 0.55, size = 81, normalized size = 0.98

$$-\frac{16\,a^5x^5\log\left(a^2x^2-1\right) - 32\,a^5x^5\log(x) - 14\,a^3x^3 + 3\,ax + 2\left(15\,a^4x^4 - 10\,a^2x^2 + 3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(16*a^5*x^5*log(a^2*x^2 - 1) - 32*a^5*x^5*log(x) - 14*a^3*x^3 + 3*a*x
 + 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

giac [B] time = 0.18, size = 265, normalized size = 3.19

$$-\frac{4}{15}\left(2\,a^{4}\log\left(\frac{|-ax-1|}{|ax-1|}\right)-2\,a^{4}\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)+\frac{\frac{2\,(ax+1)^{3}a^{4}}{(ax-1)^{3}}+\frac{7\,(ax+1)^{2}a^{4}}{(ax-1)^{2}}+\frac{2\,(ax+1)a^{4}}{ax-1}}{\left(\frac{ax+1}{ax-1}+1\right)^{4}}-\frac{2\left(\frac{10\,(ax+1)^{2}a^{4}}{(ax-1)^{2}}+\frac{5\,(a}{a}\right.}{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] -4/15*(2*a^4*log(abs(-a*x - 1)/abs(a*x - 1)) - 2*a^4*log(abs(-(a*x + 1)/(a*
x - 1) - 1)) + (2*(a*x + 1)^3*a^4/(a*x - 1)^3 + 7*(a*x + 1)^2*a^4/(a*x - 1)
^2 + 2*(a*x + 1)*a^4/(a*x - 1))/((a*x + 1)/(a*x - 1) + 1)^4 - 2*(10*(a*x +
1)^2*a^4/(a*x - 1)^2 + 5*(a*x + 1)*a^4/(a*x - 1) + a^4)*log(-(a*((a*x + 1)/
(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1
)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)/(a*x - 1) + 1)^5)*a
```

maple [A] time = 0.04, size = 80, normalized size = 0.96

$$-\frac{a^{4}\operatorname{arctanh}(ax)}{x}+\frac{2a^{2}\operatorname{arctanh}(ax)}{3x^{3}}-\frac{\operatorname{arctanh}(ax)}{5x^{5}}-\frac{a}{20x^{4}}+\frac{7a^{3}}{30x^{2}}+\frac{8a^{5}\ln(ax)}{15}-\frac{4a^{5}\ln(ax-1)}{15}-\frac{4a^{5}\ln(ax+1)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x)
```

```
[Out] -a^4*arctanh(a*x)/x+2/3*a^2*arctanh(a*x)/x^3-1/5*arctanh(a*x)/x^5-1/20*a/x^
4+7/30*a^3/x^2+8/15*a^5*ln(a*x)-4/15*a^5*ln(a*x-1)-4/15*a^5*ln(a*x+1)
```

maxima [A] time = 0.31, size = 71, normalized size = 0.86

$$-\frac{1}{60}\left(16\,a^{4}\log\left(a^{2}x^{2}-1\right)-16\,a^{4}\log\left(x^{2}\right)-\frac{14\,a^{2}x^{2}-3}{x^{4}}\right)a-\frac{\left(15\,a^{4}x^{4}-10\,a^{2}x^{2}+3\right)\operatorname{artanh}(ax)}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*log(x^2) - (14*a^2*x^2 - 3)/x^4)*a
 - 1/15*(15*a^4*x^4 - 10*a^2*x^2 + 3)*arctanh(a*x)/x^5
```

mupad [B] time = 0.89, size = 70, normalized size = 0.84

$$\frac{8\,a^{5}\,\ln(x)}{15}-\frac{a}{20\,x^{4}}-\frac{\operatorname{atanh}(a\,x)}{5\,x^{5}}-\frac{4\,a^{5}\,\ln\left(a^{2}\,x^{2}-1\right)}{15}+\frac{7\,a^{3}}{30\,x^{2}}+\frac{2\,a^{2}\operatorname{atanh}(a\,x)}{3\,x^{3}}-\frac{a^{4}\operatorname{atanh}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^6,x)
```

```
[Out] (8*a^5*log(x))/15 - a/(20*x^4) - atanh(a*x)/(5*x^5) - (4*a^5*log(a^2*x^2 -
1))/15 + (7*a^3)/(30*x^2) + (2*a^2*atanh(a*x))/(3*x^3) - (a^4*atanh(a*x))/x
```

sympy [A] time = 1.93, size = 88, normalized size = 1.06

$$\begin{cases}\frac{8a^{5}\log(x)}{15}-\frac{8a^{5}\log\left(x-\frac{1}{a}\right)}{15}-\frac{8a^{5}\operatorname{atanh}(ax)}{15}-\frac{a^{4}\operatorname{atanh}(ax)}{x}+\frac{7a^{3}}{30x^{2}}+\frac{2a^{2}\operatorname{atanh}(ax)}{3x^{3}}-\frac{a}{20x^{4}}-\frac{\operatorname{atanh}(ax)}{5x^{5}} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**6,x)
```

```
[Out] Piecewise((8*a**5*log(x)/15 - 8*a**5*log(x - 1/a)/15 - 8*a**5*atanh(a*x)/15
 - a**4*atanh(a*x)/x + 7*a**3/(30*x**2) + 2*a**2*atanh(a*x)/(3*x**3) - a/(2
0*x**4) - atanh(a*x)/(5*x**5), Ne(a, 0)), (0, True))
```

3.203 $\int x^4 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=202

$$-\frac{8\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{315a^5}+\frac{8\tanh^{-1}(ax)^2}{315a^5}-\frac{29\tanh^{-1}(ax)}{3780a^5}-\frac{16\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{315a^5}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2+\frac{29x}{3780a^4}+\frac{1}{36}$$

```
[Out] 29/3780*x/a^4-67/11340*x^3/a^2-23/3780*x^5+1/252*a^2*x^7-29/3780*arctanh(a*
x)/a^5+8/315*x^2*arctanh(a*x)/a^3+4/315*x^4*arctanh(a*x)/a-11/189*a*x^6*arc
tanh(a*x)+1/36*a^3*x^8*arctanh(a*x)+8/315*arctanh(a*x)^2/a^5+1/5*x^5*arctan
h(a*x)^2-2/7*a^2*x^7*arctanh(a*x)^2+1/9*a^4*x^9*arctanh(a*x)^2-16/315*arcta
nh(a*x)*ln(2/(-a*x+1))/a^5-8/315*polylog(2,1-2/(-a*x+1))/a^5
```

Rubi [A] time = 1.02, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 59, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6012, 5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{315a^5}+\frac{a^2x^7}{252}-\frac{67x^3}{11340a^2}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2+\frac{1}{36}a^3x^8\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{8x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (29*x)/(3780*a^4) - (67*x^3)/(11340*a^2) - (23*x^5)/3780 + (a^2*x^7)/252 -
(29*ArcTanh[a*x])/(3780*a^5) + (8*x^2*ArcTanh[a*x])/(315*a^3) + (4*x^4*ArcT
anh[a*x])/(315*a) - (11*a*x^6*ArcTanh[a*x])/189 + (a^3*x^8*ArcTanh[a*x])/36
 + (8*ArcTanh[a*x]^2)/(315*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (2*a^2*x^7*ArcTa
nh[a*x]^2)/7 + (a^4*x^9*ArcTanh[a*x]^2)/9 - (16*ArcTanh[a*x]*Log[2/(1 - a*x
)])/(315*a^5) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(315*a^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^4\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx = \int\left(x^4\tanh^{-1}(ax)^2-2a^2x^6\tanh^{-1}(ax)^2+a^4x^8\tanh^{-1}(ax)^2\right)dx$$

$$= -\left(\left(2a^2\right)\int x^6\tanh^{-1}(ax)^2\,dx\right)+a^4\int x^8\tanh^{-1}(ax)^2\,dx+\int x^4\tanh^{-1}(a$$

$$= \frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2-\frac{1}{5}(2a)\int \frac{x^5}{}$$

$$= \frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2+\frac{2\int x^3\tanh}{5a}$$

$$= \frac{x^4\tanh^{-1}(ax)}{10a}-\frac{2}{21}ax^6\tanh^{-1}(ax)+\frac{1}{36}a^3x^8\tanh^{-1}(ax)+\frac{1}{5}x^5\tanh^{-1}(ax$$

$$= \frac{x^2\tanh^{-1}(ax)}{5a^3}-\frac{3x^4\tanh^{-1}(ax)}{70a}-\frac{11}{189}ax^6\tanh^{-1}(ax)+\frac{1}{36}a^3x^8\tanh^{-1}(a$$

$$= \frac{293x}{1260a^4}+\frac{41x^3}{3780a^2}-\frac{17x^5}{1260}+\frac{a^2x^7}{252}-\frac{3x^2\tanh^{-1}(ax)}{35a^3}+\frac{4x^4\tanh^{-1}(ax)}{315a}-\frac{1}{1}$$

$$= -\frac{601x}{3780a^4}-\frac{277x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}-\frac{293\tanh^{-1}(ax)}{1260a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}-$$

$$= \frac{29x}{3780a^4}-\frac{67x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}+\frac{601\tanh^{-1}(ax)}{3780a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}+$$

$$= \frac{29x}{3780a^4}-\frac{67x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}-\frac{29\tanh^{-1}(ax)}{3780a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}+\frac{4}{}$$

$$= \frac{29x}{3780a^4}-\frac{67x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}-\frac{29\tanh^{-1}(ax)}{3780a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}+\frac{4}{}$$

Mathematica [A] time = 1.99, size = 138, normalized size = 0.68

$$\frac{36\left(35a^9x^9-90a^7x^7+63a^5x^5-8\right)\tanh^{-1}(ax)^2+ax\left(45a^6x^6-69a^4x^4-67a^2x^2+87\right)+3\tanh^{-1}(ax)\left(105a^8\right.}{11340a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(87 - 67*a^2*x^2 - 69*a^4*x^4 + 45*a^6*x^6) + 36*(-8 + 63*a^5*x^5 - 90
*a^7*x^7 + 35*a^9*x^9)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-29 + 96*a^2*x^2 +
48*a^4*x^4 - 220*a^6*x^6 + 105*a^8*x^8 - 192*Log[1 + E^(-2*ArcTanh[a*x])])
+ 288*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(11340*a^5)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4x^8-2\,a^2x^6+x^4\right)\mathrm{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^8 - 2*a^2*x^6 + x^4)*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2x^2-1\right)^2x^4\,\mathrm{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x^4*arctanh(a*x)^2, x)
```

maple [A] time = 0.06, size = 259, normalized size = 1.28

$$\frac{a^4x^9 \operatorname{arctanh}(ax)^2}{9}-\frac{2a^2x^7 \operatorname{arctanh}(ax)^2}{7}+\frac{x^5 \operatorname{arctanh}(ax)^2}{5}+\frac{a^3x^8 \operatorname{arctanh}(ax)}{36}-\frac{11a\,x^6 \operatorname{arctanh}(ax)}{189}+\frac{4x^4 \operatorname{arctan}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/9*a^4*x^9*arctanh(a*x)^2-2/7*a^2*x^7*arctanh(a*x)^2+1/5*x^5*arctanh(a*x)^
2+1/36*a^3*x^8*arctanh(a*x)-11/189*a*x^6*arctanh(a*x)+4/315*x^4*arctanh(a*x
)/a+8/315*x^2*arctanh(a*x)/a^3+8/315/a^5*arctanh(a*x)*ln(a*x-1)+8/315/a^5*a
rctanh(a*x)*ln(a*x+1)+1/252*a^2*x^7-23/3780*x^5-67/11340*x^3/a^2+29/3780*x/
a^4+29/7560/a^5*ln(a*x-1)-29/7560/a^5*ln(a*x+1)+2/315/a^5*ln(a*x-1)^2-8/315
/a^5*dilog(1/2+1/2*a*x)-4/315/a^5*ln(a*x-1)*ln(1/2+1/2*a*x)-2/315/a^5*ln(a*
x+1)^2-4/315/a^5*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+4/315/a^5*ln(-1/2*a*x+1/2
)*ln(a*x+1)
```

maxima [A] time = 0.33, size = 214, normalized size = 1.06

$$\frac{1}{22680}a^2\left(\frac{90\,a^7x^7-138\,a^5x^5-134\,a^3x^3+174\,ax-144\,\log(ax+1)^2+288\,\log(ax+1)\log(ax-1)+144\,\log(}{a^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/22680*a^2*((90*a^7*x^7 - 138*a^5*x^5 - 134*a^3*x^3 + 174*a*x - 144*log(a*
x + 1)^2 + 288*log(a*x + 1)*log(a*x - 1) + 144*log(a*x - 1)^2 + 87*log(a*x
- 1))/a^7 - 576*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a
^7 - 87*log(a*x + 1)/a^7) + 1/3780*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a^2*x
^4 + 96*x^2)/a^4 + 96*log(a*x + 1)/a^6 + 96*log(a*x - 1)/a^6)*arctanh(a*x)
+ 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] int(x^4*atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \left(ax-1\right)^{2} \left(ax+1\right)^{2} \operatorname{atanh}^{2}{\left(ax \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral(x**4*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

3.204 $\int x^3 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=156

$$\frac{1}{8}a^4x^8\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{24a^4}+\frac{1}{28}a^3x^7\tanh^{-1}(ax)+\frac{x\tanh^{-1}(ax)}{12a^3}+\frac{a^2x^6}{168}-\frac{1}{3}a^2x^6\tanh^{-1}(ax)^2-\frac{5x^2}{504a^2}+\frac{2\log}{}$$

```
[Out] -5/504*x^2/a^2-1/84*x^4+1/168*a^2*x^6+1/12*x*arctanh(a*x)/a^3+1/36*x^3*arct
anh(a*x)/a-1/12*a*x^5*arctanh(a*x)+1/28*a^3*x^7*arctanh(a*x)-1/24*arctanh(a
*x)^2/a^4+1/4*x^4*arctanh(a*x)^2-1/3*a^2*x^6*arctanh(a*x)^2+1/8*a^4*x^8*arc
tanh(a*x)^2+2/63*ln(-a^2*x^2+1)/a^4
```

Rubi [A] time = 0.82, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6012, 5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{a^2x^6}{168}-\frac{5x^2}{504a^2}+\frac{2\log\left(1-a^2x^2\right)}{63a^4}+\frac{1}{8}a^4x^8\tanh^{-1}(ax)^2+\frac{1}{28}a^3x^7\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)^2+\frac{x\tanh^{-1}(ax)}{12a^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-5*x^2)/(504*a^2) - x^4/84 + (a^2*x^6)/168 + (x*ArcTanh[a*x])/(12*a^3) + (
x^3*ArcTanh[a*x])/(36*a) - (a*x^5*ArcTanh[a*x])/12 + (a^3*x^7*ArcTanh[a*x])
/28 - ArcTanh[a*x]^2/(24*a^4) + (x^4*ArcTanh[a*x]^2)/4 - (a^2*x^6*ArcTanh[a
*x]^2)/3 + (a^4*x^8*ArcTanh[a*x]^2)/8 + (2*Log[1 - a^2*x^2])/(63*a^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^3 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx = \int \left(x^3 \tanh^{-1}(ax)^2 - 2a^2x^5 \tanh^{-1}(ax)^2 + a^4x^7 \tanh^{-1}(ax)^2\right) dx$$

$$= -\left(\left(2a^2\right)\int x^5 \tanh^{-1}(ax)^2\,dx\right) + a^4\int x^7 \tanh^{-1}(ax)^2\,dx + \int x^3 \tanh^{-1}(ax)^2$$

$$= \frac{1}{4}x^4 \tanh^{-1}(ax)^2 - \frac{1}{3}a^2x^6 \tanh^{-1}(ax)^2 + \frac{1}{8}a^4x^8 \tanh^{-1}(ax)^2 - \frac{1}{2}a\int \frac{x^4 \tanh^{-}}{1-a^2}$$

$$= \frac{1}{4}x^4 \tanh^{-1}(ax)^2 - \frac{1}{3}a^2x^6 \tanh^{-1}(ax)^2 + \frac{1}{8}a^4x^8 \tanh^{-1}(ax)^2 + \frac{\int x^2 \tanh^{-1}(ax}{2a}$$

$$= \frac{x^3 \tanh^{-1}(ax)}{6a} - \frac{2}{15}ax^5 \tanh^{-1}(ax) + \frac{1}{28}a^3x^7 \tanh^{-1}(ax) + \frac{1}{4}x^4 \tanh^{-1}(ax)^2 -$$

$$= \frac{x \tanh^{-1}(ax)}{2a^3} - \frac{x^3 \tanh^{-1}(ax)}{18a} - \frac{1}{12}ax^5 \tanh^{-1}(ax) + \frac{1}{28}a^3x^7 \tanh^{-1}(ax) - \frac{\mathrm{ta}}{}$$

$$= -\frac{x \tanh^{-1}(ax)}{6a^3} + \frac{x^3 \tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5 \tanh^{-1}(ax) + \frac{1}{28}a^3x^7 \tanh^{-1}(ax) +$$

$$= \frac{29x^2}{840a^2} - \frac{41x^4}{1680} + \frac{a^2x^6}{168} + \frac{x \tanh^{-1}(ax)}{12a^3} + \frac{x^3 \tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5 \tanh^{-1}(ax) +$$

$$= -\frac{13x^2}{252a^2} - \frac{x^4}{84} + \frac{a^2x^6}{168} + \frac{x \tanh^{-1}(ax)}{12a^3} + \frac{x^3 \tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5 \tanh^{-1}(ax) +$$

$$= -\frac{5x^2}{504a^2} - \frac{x^4}{84} + \frac{a^2x^6}{168} + \frac{x \tanh^{-1}(ax)}{12a^3} + \frac{x^3 \tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5 \tanh^{-1}(ax) +$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.69

$$\frac{3a^6x^6 - 6a^4x^4 - 5a^2x^2 + 16\log\left(1-a^2x^2\right) + 21\left(a^2x^2-1\right)^3\left(3a^2x^2+1\right)\tanh^{-1}(ax)^2 + 2ax\left(9a^6x^6 - 21a^4x^4 + 7a^2\right.}{504a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-5*a^2*x^2 - 6*a^4*x^4 + 3*a^6*x^6 + 2*a*x*(21 + 7*a^2*x^2 - 21*a^4*x^4 +
9*a^6*x^6)*ArcTanh[a*x] + 21*(-1 + a^2*x^2)^3*(1 + 3*a^2*x^2)*ArcTanh[a*x]^
2 + 16*Log[1 - a^2*x^2])/(504*a^4)
```

fricas [A] time = 0.57, size = 133, normalized size = 0.85

$$\frac{12\,a^6x^6-24\,a^4x^4-20\,a^2x^2+21\left(3\,a^8x^8-8\,a^6x^6+6\,a^4x^4-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(9\,a^7x^7-21\,a^5x^5+7\,a^3x^3+2\right.}{2016\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 + 21*(3*a^8*x^8 - 8*a^6*x^6 +
6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(9*a^7*x^7 - 21*a^5*x^5 + 7*
a^3*x^3 + 21*a*x)*log(-(a*x + 1)/(a*x - 1)) + 64*log(a^2*x^2 - 1))/a^4
```

giac [B] time = 0.23, size = 683, normalized size = 4.38

$$\frac{2}{63}\left(\frac{84\left(\frac{(ax+1)^5}{(ax-1)^5}+\frac{(ax+1)^4}{(ax-1)^4}+\frac{(ax+1)^3}{(ax-1)^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^8a^5}{(ax-1)^8}-\frac{8\,(ax+1)^7a^5}{(ax-1)^7}+\frac{28\,(ax+1)^6a^5}{(ax-1)^6}-\frac{56\,(ax+1)^5a^5}{(ax-1)^5}+\frac{70\,(ax+1)^4a^5}{(ax-1)^4}-\frac{56\,(ax+1)^3a^5}{(ax-1)^3}+\frac{28\,(ax+1)^2a^5}{(ax-1)^2}-\frac{8\,(ax+1)a^5}{ax-1}+a^5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] 2/63*(84*((a*x + 1)^5/(a*x - 1)^5 + (a*x + 1)^4/(a*x - 1)^4 + (a*x + 1)^3/(
a*x - 1)^3)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^8*a^5/(a*x - 1)^8 - 8*(a
*x + 1)^7*a^5/(a*x - 1)^7 + 28*(a*x + 1)^6*a^5/(a*x - 1)^6 - 56*(a*x + 1)^5
*a^5/(a*x - 1)^5 + 70*(a*x + 1)^4*a^5/(a*x - 1)^4 - 56*(a*x + 1)^3*a^5/(a*x
 - 1)^3 + 28*(a*x + 1)^2*a^5/(a*x - 1)^2 - 8*(a*x + 1)*a^5/(a*x - 1) + a^5)
 + 2*(28*(a*x + 1)^4/(a*x - 1)^4 - 7*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x + 1)
^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x - 1) + 1)*log(-(a*x + 1)/(a*x - 1))/((a*x
 + 1)^7*a^5/(a*x - 1)^7 - 7*(a*x + 1)^6*a^5/(a*x - 1)^6 + 21*(a*x + 1)^5*a^
5/(a*x - 1)^5 - 35*(a*x + 1)^4*a^5/(a*x - 1)^4 + 35*(a*x + 1)^3*a^5/(a*x -
1)^3 - 21*(a*x + 1)^2*a^5/(a*x - 1)^2 + 7*(a*x + 1)*a^5/(a*x - 1) - a^5) -
(2*(a*x + 1)^5/(a*x - 1)^5 - 11*(a*x + 1)^4/(a*x - 1)^4 + 6*(a*x + 1)^3/(a*
x - 1)^3 - 11*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x - 1))/((a*x + 1)^6
*a^5/(a*x - 1)^6 - 6*(a*x + 1)^5*a^5/(a*x - 1)^5 + 15*(a*x + 1)^4*a^5/(a*x
- 1)^4 - 20*(a*x + 1)^3*a^5/(a*x - 1)^3 + 15*(a*x + 1)^2*a^5/(a*x - 1)^2 -
6*(a*x + 1)*a^5/(a*x - 1) + a^5) - 2*log(-(a*x + 1)/(a*x - 1) + 1)/a^5 + 2*
log(-(a*x + 1)/(a*x - 1))/a^5)*a
```

maple [A] time = 0.06, size = 239, normalized size = 1.53

$$\frac{a^4x^8\operatorname{arctanh}(ax)^2}{8}-\frac{a^2x^6\operatorname{arctanh}(ax)^2}{3}+\frac{x^4\operatorname{arctanh}(ax)^2}{4}+\frac{a^3x^7\operatorname{arctanh}(ax)}{28}-\frac{a\,x^5\operatorname{arctanh}(ax)}{12}+\frac{x^3\operatorname{arctanh}}{36a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/8*a^4*x^8*arctanh(a*x)^2-1/3*a^2*x^6*arctanh(a*x)^2+1/4*x^4*arctanh(a*x)^
2+1/28*a^3*x^7*arctanh(a*x)-1/12*a*x^5*arctanh(a*x)+1/36*x^3*arctanh(a*x)/a
+1/12*x*arctanh(a*x)/a^3+1/24/a^4*arctanh(a*x)*ln(a*x-1)-1/24/a^4*arctanh(a
*x)*ln(a*x+1)+1/96/a^4*ln(a*x-1)^2-1/48/a^4*ln(a*x-1)*ln(1/2+1/2*a*x)+1/96/
a^4*ln(a*x+1)^2+1/48/a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/48/a^4*ln(-1/2*
```

```
a*x+1/2)*ln(a*x+1)+1/168*a^2*x^6-1/84*x^4-5/504*x^2/a^2+2/63/a^4*ln(a*x-1)+
2/63/a^4*ln(a*x+1)
```

maxima [A] time = 0.32, size = 170, normalized size = 1.09

$$\frac{1}{504}a\left(\frac{2\left(9a^6x^7-21a^4x^5+7a^2x^3+21x\right)}{a^4}-\frac{21\log\left(ax+1\right)}{a^5}+\frac{21\log\left(ax-1\right)}{a^5}\right)\operatorname{artanh}\left(ax\right)+\frac{1}{24}\left(3a^4x^8-8a^2x^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/504*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x + 1
)/a^5 + 21*log(a*x - 1)/a^5)*arctanh(a*x) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 + 6
*x^4)*arctanh(a*x)^2 + 1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 - 2*(21
*log(a*x - 1) - 32)*log(a*x + 1) + 21*log(a*x + 1)^2 + 21*log(a*x - 1)^2 +
64*log(a*x - 1))/a^4
```

mupad [B] time = 1.22, size = 221, normalized size = 1.42

$$\frac{2\ln\left(a^2x^2-1\right)}{63a^4}-\ln\left(1-ax\right)^2\left(\frac{1}{96a^4}-\frac{x^4}{16}+\frac{a^2x^6}{12}-\frac{a^4x^8}{32}\right)-\frac{x^4}{84}-\ln\left(ax+1\right)^2\left(\frac{1}{96a^4}-\frac{x^4}{16}+\frac{a^2x^6}{12}-\frac{a^4x^8}{32}\right)-\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] (2*log(a^2*x^2 - 1))/(63*a^4) - log(1 - a*x)^2*(1/(96*a^4) - x^4/16 + (a^2*
x^6)/12 - (a^4*x^8)/32) - x^4/84 - log(a*x + 1)^2*(1/(96*a^4) - x^4/16 + (a
^2*x^6)/12 - (a^4*x^8)/32) - log(1 - a*x)*(x/(24*a^3) - log(a*x + 1)*(1/(48
*a^4) - x^4/8 + (a^2*x^6)/6 - (a^4*x^8)/16) - (a*x^5)/24 + x^3/(72*a) + (a^
3*x^7)/56) - (5*x^2)/(504*a^2) + (a^2*x^6)/168 + a*log(a*x + 1)*(x/(24*a^4)
 - x^5/24 + x^3/(72*a^2) + (a^2*x^7)/56)
```

sympy [A] time = 3.42, size = 153, normalized size = 0.98

$$\begin{cases}\frac{a^4x^8\operatorname{atanh}^2\left(ax\right)}{8}+\frac{a^3x^7\operatorname{atanh}\left(ax\right)}{28}-\frac{a^2x^6\operatorname{atanh}^2\left(ax\right)}{3}+\frac{a^2x^6}{168}-\frac{ax^5\operatorname{atanh}\left(ax\right)}{12}+\frac{x^4\operatorname{atanh}^2\left(ax\right)}{4}-\frac{x^4}{84}+\frac{x^3\operatorname{atanh}\left(ax\right)}{36a}-\frac{5x^2}{504a^2}+\frac{x\operatorname{atan}}{12} \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*x**8*atanh(a*x)**2/8 + a**3*x**7*atanh(a*x)/28 - a**2*x**6*
atanh(a*x)**2/3 + a**2*x**6/168 - a*x**5*atanh(a*x)/12 + x**4*atanh(a*x)**2
/4 - x**4/84 + x**3*atanh(a*x)/(36*a) - 5*x**2/(504*a**2) + x*atanh(a*x)/(1
2*a**3) + 4*log(x - 1/a)/(63*a**4) - atanh(a*x)**2/(24*a**4) + 4*atanh(a*x)
/(63*a**4), Ne(a, 0)), (0, True))
```

3.205 $\int x^2 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=178

$$\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2-\frac{8\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{105a^3}+\frac{1}{21}a^3x^6\tanh^{-1}(ax)+\frac{8\tanh^{-1}(ax)^2}{105a^3}+\frac{\tanh^{-1}(ax)}{210a^3}-\frac{16\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(a$$

```
[Out] -1/210*x/a^2-17/630*x^3+1/105*a^2*x^5+1/210*arctanh(a*x)/a^3+8/105*x^2*arct
anh(a*x)/a-9/70*a*x^4*arctanh(a*x)+1/21*a^3*x^6*arctanh(a*x)+8/105*arctanh(
a*x)^2/a^3+1/3*x^3*arctanh(a*x)^2-2/5*a^2*x^5*arctanh(a*x)^2+1/7*a^4*x^7*ar
ctanh(a*x)^2-16/105*arctanh(a*x)*ln(2/(-a*x+1))/a^3-8/105*polylog(2,1-2/(-a
*x+1))/a^3
```

Rubi [A] time = 0.78, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6012, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 302}

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{105a^3}+\frac{a^2x^5}{105}+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2+\frac{1}{21}a^3x^6\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2-\frac{x}{210a^2}+\frac{8\tanh}{105}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] -x/(210*a^2) - (17*x^3)/630 + (a^2*x^5)/105 + ArcTanh[a*x]/(210*a^3) + (8*x
^2*ArcTanh[a*x])/(105*a) - (9*a*x^4*ArcTanh[a*x])/70 + (a^3*x^6*ArcTanh[a*x
])/21 + (8*ArcTanh[a*x]^2)/(105*a^3) + (x^3*ArcTanh[a*x]^2)/3 - (2*a^2*x^5*
ArcTanh[a*x]^2)/5 + (a^4*x^7*ArcTanh[a*x]^2)/7 - (16*ArcTanh[a*x]*Log[2/(1
- a*x)])/(105*a^3) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(105*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx = \int\left(x^2\tanh^{-1}(ax)^2-2a^2x^4\tanh^{-1}(ax)^2+a^4x^6\tanh^{-1}(ax)^2\right)dx$$
$$= -\left(\left(2a^2\right)\int x^4\tanh^{-1}(ax)^2\,dx\right)+a^4\int x^6\tanh^{-1}(ax)^2\,dx+\int x^2\tanh^{-1}(a$$
$$= \frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2-\frac{1}{3}(2a)\int\frac{x^3}{}$$
$$= \frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2+\frac{2\int x\tanh^{-}}{3a}$$
$$= \frac{x^2\tanh^{-1}(ax)}{3a}-\frac{1}{5}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3x^6\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)^2}{3a^3}+$$
$$= \frac{x}{3a^2}-\frac{x^2\tanh^{-1}(ax)}{15a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3x^6\tanh^{-1}(ax)-\frac{\tanh^{-1}(}{15a^3}$$
$$= -\frac{23x}{105a^2}-\frac{16x^3}{315}+\frac{a^2x^5}{105}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(a$$
$$= -\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{23\tanh^{-1}(ax)}{105a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-}$$
$$= -\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{\tanh^{-1}(ax)}{210a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(a$$
$$= -\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{\tanh^{-1}(ax)}{210a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(a$$

Mathematica [A] time = 1.26, size = 121, normalized size = 0.68

$$\frac{ax\left(6a^4x^4-17a^2x^2-3\right)+6\left(15a^7x^7-42a^5x^5+35a^3x^3-8\right)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(30a^6x^6-81a^4x^4+48\right.}{630a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(-3 - 17*a^2*x^2 + 6*a^4*x^4) + 6*(-8 + 35*a^3*x^3 - 42*a^5*x^5 + 15*a
^7*x^7)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(3 + 48*a^2*x^2 - 81*a^4*x^4 + 30*a^6
*x^6 - 96*Log[1 + E^(-2*ArcTanh[a*x])]) + 48*PolyLog[2, -E^(-2*ArcTanh[a*x]
)])/(630*a^3)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4x^6-2\,a^2x^4+x^2\right)\mathrm{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^6 - 2*a^2*x^4 + x^2)*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2x^2-1\right)^2x^2\,\mathrm{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x^2*arctanh(a*x)^2, x)
```

maple [A] time = 0.06, size = 239, normalized size = 1.34

$$\frac{a^4x^7 \operatorname{arctanh}(ax)^2}{7} - \frac{2a^2x^5 \operatorname{arctanh}(ax)^2}{5} + \frac{x^3 \operatorname{arctanh}(ax)^2}{3} + \frac{a^3x^6 \operatorname{arctanh}(ax)}{21} - \frac{9a\,x^4 \operatorname{arctanh}(ax)}{70} + \frac{8x^2 \operatorname{arctan}}{105a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/7*a^4*x^7*arctanh(a*x)^2-2/5*a^2*x^5*arctanh(a*x)^2+1/3*x^3*arctanh(a*x)^
2+1/21*a^3*x^6*arctanh(a*x)-9/70*a*x^4*arctanh(a*x)+8/105*x^2*arctanh(a*x)/
a+8/105/a^3*arctanh(a*x)*ln(a*x-1)+8/105/a^3*arctanh(a*x)*ln(a*x+1)+1/105*x
^5*a^2-17/630*x^3-1/210*x/a^2-1/420/a^3*ln(a*x-1)+1/420/a^3*ln(a*x+1)+2/105
/a^3*ln(a*x-1)^2-8/105/a^3*dilog(1/2+1/2*a*x)-4/105/a^3*ln(a*x-1)*ln(1/2+1/
2*a*x)-2/105/a^3*ln(a*x+1)^2-4/105/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+4/1
05/a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)
```

maxima [A] time = 0.33, size = 198, normalized size = 1.11

$$\frac{1}{1260}\,a^2\left(\frac{12\,a^5x^5 - 34\,a^3x^3 - 6\,ax - 24\,\log(ax+1)^2 + 48\,\log(ax+1)\log(ax-1) + 24\,\log(ax-1)^2 - 3\,\log(ax}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/1260*a^2*((12*a^5*x^5 - 34*a^3*x^3 - 6*a*x - 24*log(a*x + 1)^2 + 48*log(a
*x + 1)*log(a*x - 1) + 24*log(a*x - 1)^2 - 3*log(a*x - 1))/a^5 - 96*(log(a*
x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 + 3*log(a*x + 1)/a^5
) + 1/210*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a^4 +
 16*log(a*x - 1)/a^4)*arctanh(a*x) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^
3)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}(a\,x)^2 \left(a^2\,x^2-1\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] int(x^2*atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral(x**2*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

3.206 $\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=138

$$\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{4\log\left(1-a^2x^2\right)}{45a^2}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}+\frac{4x\left(1-a^2x^2\right.}{4}$$

```
[Out] 2/45*(-a^2*x^2+1)/a^2+1/60*(-a^2*x^2+1)^2/a^2+8/45*x*arctanh(a*x)/a+4/45*x*
(-a^2*x^2+1)*arctanh(a*x)/a+1/15*x*(-a^2*x^2+1)^2*arctanh(a*x)/a-1/6*(-a^2*
x^2+1)^3*arctanh(a*x)^2/a^2+4/45*ln(-a^2*x^2+1)/a^2
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5994, 5942, 5910, 260}

$$\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{4\log\left(1-a^2x^2\right)}{45a^2}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}+\frac{4x\left(1-a^2x^2\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (2*(1 - a^2*x^2))/(45*a^2) + (1 - a^2*x^2)^2/(60*a^2) + (8*x*ArcTanh[a*x])/
(45*a) + (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/(45*a) + (x*(1 - a^2*x^2)^2*ArcTa
nh[a*x])/(15*a) - ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(6*a^2) + (4*Log[1 - a^2
*x^2])/(45*a^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx = -\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{\int\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx}{3a}$$
$$=\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{4\int\left(1-a\right.}{}$$
$$=\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}}{15a}$$
$$=\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{8x\tanh^{-1}(ax)}{45a}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}+\frac{x(1}{}$$
$$=\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{8x\tanh^{-1}(ax)}{45a}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}+\frac{x(1}{}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.59

$$\frac{3a^4x^4-14a^2x^2+16\log\left(1-a^2x^2\right)+30\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2+4ax\left(3a^4x^4-10a^2x^2+15\right)\tanh^{-1}(ax)}{180a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-14*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 10*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x]
 + 30*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2 + 16*Log[1 - a^2*x^2])/(180*a^2)
```

fricas [A] time = 0.68, size = 116, normalized size = 0.84

$$\frac{6\,a^4x^4-28\,a^2x^2+15\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(3\,a^5x^5-10\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+32\,\mathrm{l}}{360\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(6*a^4*x^4 - 28*a^2*x^2 + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*lo
g(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a*x +
 1)/(a*x - 1)) + 32*log(a^2*x^2 - 1))/a^2
```

giac [B] time = 0.19, size = 473, normalized size = 3.43

$$\frac{4}{45}a\left(\frac{2\left(\frac{10\,(ax+1)^2}{(ax-1)^2}-\frac{5\,(ax+1)}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^5a^3}{(ax-1)^5}-\frac{5\,(ax+1)^4a^3}{(ax-1)^4}+\frac{10\,(ax+1)^3a^3}{(ax-1)^3}-\frac{10\,(ax+1)^2a^3}{(ax-1)^2}+\frac{5\,(ax+1)a^3}{ax-1}-a^3}+\frac{30\,(ax+1)}{\left(\frac{(ax+1)^6a^3}{(ax-1)^6}-\frac{6\,(ax+1)^5a^3}{(ax-1)^5}+\frac{15\,(ax+1)^4a^3}{(ax-1)^4}-\frac{20\,(ax}{(ax}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] 4/45*a*(2*(10*(a*x + 1)^2/(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)*log(-(a*
x + 1)/(a*x - 1))/((a*x + 1)^5*a^3/(a*x - 1)^5 - 5*(a*x + 1)^4*a^3/(a*x - 1
)^4 + 10*(a*x + 1)^3*a^3/(a*x - 1)^3 - 10*(a*x + 1)^2*a^3/(a*x - 1)^2 + 5*(
a*x + 1)*a^3/(a*x - 1) - a^3) + 30*(a*x + 1)^3*log(-(a*x + 1)/(a*x - 1))^2/
(((a*x + 1)^6*a^3/(a*x - 1)^6 - 6*(a*x + 1)^5*a^3/(a*x - 1)^5 + 15*(a*x + 1
)^4*a^3/(a*x - 1)^4 - 20*(a*x + 1)^3*a^3/(a*x - 1)^3 + 15*(a*x + 1)^2*a^3/(
a*x - 1)^2 - 6*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)^3) - (2*(a*x + 1)^3
```

```
/(a*x - 1)^3 - 7*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x - 1))/((a*x + 1
)^4*a^3/(a*x - 1)^4 - 4*(a*x + 1)^3*a^3/(a*x - 1)^3 + 6*(a*x + 1)^2*a^3/(a*
x - 1)^2 - 4*(a*x + 1)*a^3/(a*x - 1) + a^3) - 2*log(-(a*x + 1)/(a*x - 1) +
1)/a^3 + 2*log(-(a*x + 1)/(a*x - 1))/a^3)
```

maple [A] time = 0.05, size = 219, normalized size = 1.59

$$\frac{a^4 \operatorname{arctanh}(ax)^2 x^6}{6}-\frac{a^2 \operatorname{arctanh}(ax)^2 x^4}{2}+\frac{\operatorname{arctanh}(ax)^2 x^2}{2}+\frac{a^3 \operatorname{arctanh}(ax) x^5}{15}-\frac{2a \operatorname{arctanh}(ax) x^3}{9}+\frac{x \operatorname{arctan}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/6*a^4*arctanh(a*x)^2*x^6-1/2*a^2*arctanh(a*x)^2*x^4+1/2*arctanh(a*x)^2*x^
2+1/15*a^3*arctanh(a*x)*x^5-2/9*a*arctanh(a*x)*x^3+1/3*x*arctanh(a*x)/a+1/6
/a^2*arctanh(a*x)*ln(a*x-1)-1/6/a^2*arctanh(a*x)*ln(a*x+1)+1/24/a^2*ln(a*x-
1)^2-1/12/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/24/a^2*ln(a*x+1)^2+1/12/a^2*ln(-1
/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/12/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/60*a^2*x^
4-7/90*x^2+4/45/a^2*ln(a*x-1)+4/45/a^2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 93, normalized size = 0.67

$$\frac{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^2}{6\,a^2}+\frac{\left(3\,a^2x^4-14\,x^2+\frac{16\,\log(ax+1)}{a^2}+\frac{16\,\log(ax-1)}{a^2}\right)a+4\left(3\,a^4x^5-10\,a^2x^3+15\,x\right)\operatorname{artanh}(a}{180\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 1)^3*arctanh(a*x)^2/a^2 + 1/180*((3*a^2*x^4 - 14*x^2 + 16*lo
g(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a + 4*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)
*arctanh(a*x))/a
```

mupad [B] time = 1.00, size = 111, normalized size = 0.80

$$\frac{x^2 \operatorname{atanh}(a\,x)^2}{2}-\frac{\operatorname{atanh}(a\,x)^2}{6\,a^2}-\frac{7\,x^2}{90}+\frac{4\,\ln\left(a^2\,x^2-1\right)}{45\,a^2}+\frac{a^2\,x^4}{60}+\frac{x\,\operatorname{atanh}(a\,x)}{3\,a}-\frac{2\,a\,x^3\,\operatorname{atanh}(a\,x)}{9}+\frac{a^3\,x^5\,\operatorname{atanh}(a}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] (x^2*atanh(a*x)^2)/2 - atanh(a*x)^2/(6*a^2) - (7*x^2)/90 + (4*log(a^2*x^2 -
 1))/(45*a^2) + (a^2*x^4)/60 + (x*atanh(a*x))/(3*a) - (2*a*x^3*atanh(a*x))/
9 + (a^3*x^5*atanh(a*x))/15 - (a^2*x^4*atanh(a*x)^2)/2 + (a^4*x^6*atanh(a*x
)^2)/6
```

sympy [A] time = 2.11, size = 133, normalized size = 0.96

$$\begin{cases} \frac{a^4x^6 \operatorname{atanh}^2(ax)}{6}+\frac{a^3x^5 \operatorname{atanh}(ax)}{15}-\frac{a^2x^4 \operatorname{atanh}^2(ax)}{2}+\frac{a^2x^4}{60}-\frac{2ax^3 \operatorname{atanh}(ax)}{9}+\frac{x^2 \operatorname{atanh}^2(ax)}{2}-\frac{7x^2}{90}+\frac{x \operatorname{atanh}(ax)}{3a}+\frac{8\log\left(x-\frac{1}{a}\right)}{45a^2} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*x**6*atanh(a*x)**2/6 + a**3*x**5*atanh(a*x)/15 - a**2*x**4*
atanh(a*x)**2/2 + a**2*x**4/60 - 2*a*x**3*atanh(a*x)/9 + x**2*atanh(a*x)**2
/2 - 7*x**2/90 + x*atanh(a*x)/(3*a) + 8*log(x - 1/a)/(45*a**2) - atanh(a*x)
**2/(6*a**2) + 8*atanh(a*x)/(45*a**2), Ne(a, 0)), (0, True))
```

3.207 $\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=171

$$\frac{a^2x^3}{30}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a}$$

```
[Out] -11/30*x+1/30*a^2*x^3+4/15*(-a^2*x^2+1)*arctanh(a*x)/a+1/10*(-a^2*x^2+1)^2*
arctanh(a*x)/a+8/15*arctanh(a*x)^2/a+8/15*x*arctanh(a*x)^2+4/15*x*(-a^2*x^2
+1)*arctanh(a*x)^2+1/5*x*(-a^2*x^2+1)^2*arctanh(a*x)^2-16/15*arctanh(a*x)*l
n(2/(-a*x+1))/a-8/15*polylog(2,1-2/(-a*x+1))/a
```

Rubi [A] time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5944, 5910, 5984, 5918, 2402, 2315, 8}

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a}+\frac{a^2x^3}{30}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-11*x)/30 + (a^2*x^3)/30 + (4*(1 - a^2*x^2)*ArcTanh[a*x])/(15*a) + ((1 - a
^2*x^2)^2*ArcTanh[a*x])/(10*a) + (8*ArcTanh[a*x]^2)/(15*a) + (8*x*ArcTanh[a
*x]^2)/15 + (4*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/15 + (x*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/5 - (16*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a) - (8*PolyLog[2, 1
 - 2/(1 - a*x)])/(15*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx &= \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2-\frac{1}{10}\int\left(1-a^2x^2\right)dx+ \\
&= -\frac{x}{10}+\frac{a^2x^3}{30}+\frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a}+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{4}{15}x\left(1-\right. \\
&= -\frac{11x}{30}+\frac{a^2x^3}{30}+\frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a}+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{8}{15}x\tanh \\
&= -\frac{11x}{30}+\frac{a^2x^3}{30}+\frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a}+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{8\tanh^{-}}{15} \\
&= -\frac{11x}{30}+\frac{a^2x^3}{30}+\frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a}+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{8\tanh^{-}}{15} \\
&= -\frac{11x}{30}+\frac{a^2x^3}{30}+\frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a}+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{8\tanh^{-}}{15} \\
&= -\frac{11x}{30}+\frac{a^2x^3}{30}+\frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a}+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}+\frac{8\tanh^{-}}{15}
\end{aligned}
$$

Mathematica [A] time = 0.69, size = 99, normalized size = 0.58

$$
\frac{ax\left(a^2x^2-11\right)+2\left(3a^2x^2+9ax+8\right)(ax-1)^3\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(3a^4x^4-14a^2x^2-32\log\left(e^{-2\tanh^{-1}(a}\right.\right.}{30a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(-11 + a^2*x^2) + 2*(-1 + a*x)^3*(8 + 9*a*x + 3*a^2*x^2)*ArcTanh[a*x]^
2 + ArcTanh[a*x]*(11 - 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^(-2*ArcTanh[a*
x])]) + 16*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(30*a)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$
\text{integral}\left(\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)^2\,,x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2, x)
```

maple [A] time = 0.05, size = 216, normalized size = 1.26

$$\frac{a^4 \operatorname{arctanh}(ax)^2 x^5}{5}-\frac{2a^2 \operatorname{arctanh}(ax)^2 x^3}{3}+x \operatorname{arctanh}(ax)^2+\frac{a^3 \operatorname{arctanh}(ax)\, x^4}{10}-\frac{7a \operatorname{arctanh}(ax)\, x^2}{15}+\frac{8 \operatorname{arctanh}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/5*a^4*arctanh(a*x)^2*x^5-2/3*a^2*arctanh(a*x)^2*x^3+x*arctanh(a*x)^2+1/10
*a^3*arctanh(a*x)*x^4-7/15*a*arctanh(a*x)*x^2+8/15/a*arctanh(a*x)*ln(a*x-1)
+8/15/a*arctanh(a*x)*ln(a*x+1)+1/30*x^3*a^2-11/30*x-11/60/a*ln(a*x-1)+11/60
/a*ln(a*x+1)+2/15/a*ln(a*x-1)^2-8/15/a*dilog(1/2+1/2*a*x)-4/15/a*ln(a*x-1)*
ln(1/2+1/2*a*x)-2/15/a*ln(a*x+1)^2-4/15/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+
4/15/a*ln(-1/2*a*x+1/2)*ln(a*x+1)
```

maxima [A] time = 0.33, size = 175, normalized size = 1.02

$$\frac{1}{60}\,a^2\left(\frac{2\,a^3x^3-22\,ax-8\,\log(ax+1)^2+16\,\log(ax+1)\log(ax-1)+8\,\log(ax-1)^2-11\,\log(ax-1)}{a^3}-\frac{32\,(\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/60*a^2*((2*a^3*x^3 - 22*a*x - 8*log(a*x + 1)^2 + 16*log(a*x + 1)*log(a*x
- 1) + 8*log(a*x - 1)^2 - 11*log(a*x - 1))/a^3 - 32*(log(a*x - 1)*log(1/2*a
*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 + 11*log(a*x + 1)/a^3) + 1/30*(3*a^2
*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a*arctanh(a*x) +
 1/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] int(atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax-1)^2\,(ax+1)^2 \operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

3.208 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x} \, dx$

Optimal. Leaf size=186

$$\frac{1}{4}a^4x^4 \tanh^{-1}(ax)^2+\frac{1}{6}a^3x^3 \tanh^{-1}(ax)+\frac{a^2x^2}{12}-\frac{2}{3} \log\left(1-a^2x^2\right)-a^2x^2 \tanh^{-1}(ax)^2+\frac{1}{2}\text{Li}_3\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{Li}_3\left(\right.$$

```
[Out] 1/12*a^2*x^2-3/2*a*x*arctanh(a*x)+1/6*a^3*x^3*arctanh(a*x)+3/4*arctanh(a*x)
^2-a^2*x^2*arctanh(a*x)^2+1/4*a^4*x^4*arctanh(a*x)^2-2*arctanh(a*x)^2*arcta
nh(-1+2/(-a*x+1))-2/3*ln(-a^2*x^2+1)-arctanh(a*x)*polylog(2,1-2/(-a*x+1))+a
rctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*polylog(3,1-2/(-a*x+1))-1/2*polylo
g(3,-1+2/(-a*x+1))
```

Rubi [A] time = 0.53, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6012, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 5910, 260, 266, 43}

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\tanh^{-1}(ax)\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

```
[Out] (a^2*x^2)/12 - (3*a*x*ArcTanh[a*x])/2 + (a^3*x^3*ArcTanh[a*x])/6 + (3*ArcTa
nh[a*x]^2)/4 - a^2*x^2*ArcTanh[a*x]^2 + (a^4*x^4*ArcTanh[a*x]^2)/4 + 2*ArcT
anh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 - ArcTanh[a*x]
*PolyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] +
PolyLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
```

```
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + a^4x^3\tanh^{-1}(ax)^2\right)dx$$

$$= -\left(\left(2a^2\right)\int x\tanh^{-1}(ax)^2\,dx\right) + a^4\int x^3\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x}\,dx$$

$$= -a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

$$= -a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

$$= -2ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}$$

$$= -\frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2 +$$

$$= -\frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2 +$$

$$= \frac{a^2x^2}{12} - \frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}($$

Mathematica [A] time = 0.06, size = 191, normalized size = 1.03

$$\frac{1}{4}\left(a^4x^4-1\right)\tanh^{-1}(ax)^2+\frac{a^2x^2}{12}-\frac{2}{3}\log\left(1-a^2x^2\right)+\frac{1}{6}ax\left(a^2x^2+3\right)\tanh^{-1}(ax)-\left(a^2x^2-1\right)\tanh^{-1}(ax)^2-\frac{1}{2}\text{Li}_3\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

```
[Out] (a^2*x^2)/12 - 2*a*x*ArcTanh[a*x] + (a*x*(3 + a^2*x^2)*ArcTanh[a*x])/6 - (-
1 + a^2*x^2)*ArcTanh[a*x]^2 + ((-1 + a^4*x^4)*ArcTanh[a*x]^2)/4 + 2*ArcTanh
[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 + ArcTanh[a*x]*Po
lyLog[2, (-1 - a*x)/(-1 + a*x)] - ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a
*x)] - PolyLog[3, (-1 - a*x)/(-1 + a*x)]/2 + PolyLog[3, (1 + a*x)/(-1 + a*x
)]/2
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2 \text{artanh}\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x, x)
```

maple [C] time = 1.80, size = 728, normalized size = 3.91

$$-a^2x^2 \operatorname{arctanh}(ax)^2 - \frac{i \operatorname{arctanh}(ax)^2 \pi \operatorname{csgn}\left(\frac{i}{1+\frac{(ax+1)^2}{-a^2x^2+1}}\right) \operatorname{csgn}\left(\frac{i\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{1+\frac{(ax+1)^2}{-a^2x^2+1}}\right)^2}{2} - \frac{1}{12} + \frac{3 \operatorname{arctanh}(ax)^2}{4} + \frac{a^4x^4 \operatorname{arctanh}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x)
```

```
[Out] -a^2*x^2*arctanh(a*x)^2-1/2*I*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x
^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/1
2+3/4*arctanh(a*x)^2+1/4*a^4*x^4*arctanh(a*x)^2+1/12*a^2*x^2+1/2*I*arctanh(
a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+
1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+1/2*I*ar
ctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)
))^3-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*arctanh(a*x)^2*Pi*csgn(I
*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^2+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*
x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)
*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-(a*x+1)*arctanh(a*x)-2*polylog(3,-(a*x+
1)/(-a^2*x^2+1)^(1/2))+4/3*ln(1+(a*x+1)^2/(-a^2*x^2+1))+1/2*polylog(3,-(a*x
+1)^2/(-a^2*x^2+1))+1/6*(a^2*x^2-4*a*x+7)*(a*x+1)*arctanh(a*x)+1/2*(a*x-3)*
(a*x+1)*arctanh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(a^4x^4 - 4\,a^2x^2\right)\log\left(-ax+1\right)^2 - \frac{1}{4}\int -\frac{2\left(a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1\right)\log\left(ax+1\right)^2 - \left(a^5x^5 - 4\,a^3x\right.}{2\left(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/16*(a^4*x^4 - 4*a^2*x^2)*log(-a*x + 1)^2 - 1/4*integrate(-1/2*(2*(a^5*x^5
 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (a^5*x^5 - 4
*a^3*x^3 + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x
+ 1))*log(-a*x + 1))/(a*x^2 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2 \operatorname{atanh}^2\left(ax\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x, x)
```

3.209 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=156

$$\frac{1}{3}a^4x^3 \tanh^{-1}(ax)^2+\frac{1}{3}a^3x^2 \tanh^{-1}(ax)+\frac{a^2x}{3}-2a^2x \tanh^{-1}(ax)^2+\frac{5}{3}a\text{Li}_2\left(1-\frac{2}{1-ax}\right)-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{3}a \tanh$$

```
[Out] 1/3*a^2*x-1/3*a*arctanh(a*x)+1/3*a^3*x^2*arctanh(a*x)-2/3*a*arctanh(a*x)^2-
arctanh(a*x)^2/x-2*a^2*x*arctanh(a*x)^2+1/3*a^4*x^3*arctanh(a*x)^2+10/3*a*a
rctanh(a*x)*ln(2/(-a*x+1))+2*a*arctanh(a*x)*ln(2-2/(a*x+1))+5/3*a*polylog(2
,1-2/(-a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.42, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6012, 5910, 5984, 5918, 2402, 2315, 5916, 5988, 5932, 2447, 5980, 321, 206}

$$\frac{5}{3}a\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}a^4x^3 \tanh^{-1}(ax)^2+\frac{1}{3}a^3x^2 \tanh^{-1}(ax)+\frac{a^2x}{3}-2a^2x \tanh^{-}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] (a^2*x)/3 - (a*ArcTanh[a*x])/3 + (a^3*x^2*ArcTanh[a*x])/3 - (2*a*ArcTanh[a*
x]^2)/3 - ArcTanh[a*x]^2/x - 2*a^2*x*ArcTanh[a*x]^2 + (a^4*x^3*ArcTanh[a*x]
^2)/3 + (10*a*ArcTanh[a*x]*Log[2/(1 - a*x)])/3 + 2*a*ArcTanh[a*x]*Log[2 - 2
/(1 + a*x)] + (5*a*PolyLog[2, 1 - 2/(1 - a*x)])/3 - a*PolyLog[2, -1 + 2/(1
+ a*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
```

```
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^2}\,dx = \int \left(-2a^2\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{x^2} + a^4x^2\tanh^{-1}(ax)^2\right)dx$$

$$= -\left(\left(2a^2\right)\int \tanh^{-1}(ax)^2\,dx\right) + a^4\int x^2\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3\tanh^{-1}(ax)^2 + (2a)\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,d$$

$$= -a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3\tanh^{-1}(ax)^2 + (2a)$$

$$= \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3$$

$$= \frac{a^2x}{3} + \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 +$$

$$= \frac{a^2x}{3} - \frac{1}{3}a\tanh^{-1}(ax) + \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a$$

$$= \frac{a^2x}{3} - \frac{1}{3}a\tanh^{-1}(ax) + \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a$$

Mathematica [A] time = 0.06, size = 182, normalized size = 1.17

$$\frac{1}{3}a\left(-\left(1-a^2x^2\right)\left(ax\tanh^{-1}(ax)+1\right)\tanh^{-1}(ax) + \text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + ax + ax\tanh^{-1}(ax)^2 - \tanh^{-1}(ax)^2 - 2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -2*a*ArcTanh[a*x]*(-ArcTanh[a*x] + a*x*ArcTanh[a*x] - 2*Log[1 + E^(-2*ArcTa
nh[a*x])]) - 2*a*PolyLog[2, -E^(-2*ArcTanh[a*x])] + (a*(a*x - ArcTanh[a*x]^
2 + a*x*ArcTanh[a*x]^2 - (1 - a^2*x^2)*ArcTanh[a*x]*(1 + a*x*ArcTanh[a*x])
- 2*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh[a*x])] + PolyLog[2, -E^(-2*ArcTanh[a
*x])]))/3 + a*(ArcTanh[a*x]*(ArcTanh[a*x] - ArcTanh[a*x]/(a*x) + 2*Log[1 -
E^(-2*ArcTanh[a*x])]) - PolyLog[2, E^(-2*ArcTanh[a*x])])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\,(ax)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2\text{artanh}\,(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^2, x)
```

maple [A] time = 0.06, size = 222, normalized size = 1.42

$$\frac{a^4x^3 \operatorname{arctanh}(ax)^2}{3} - 2a^2x \operatorname{arctanh}(ax)^2 - \frac{\operatorname{arctanh}(ax)^2}{x} + \frac{a^3x^2 \operatorname{arctanh}(ax)}{3} + 2a \operatorname{arctanh}(ax) \ln(ax) - \frac{8a \operatorname{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x)
```

```
[Out] 1/3*a^4*x^3*arctanh(a*x)^2-2*a^2*x*arctanh(a*x)^2-arctanh(a*x)^2/x+1/3*a^3*
x^2*arctanh(a*x)+2*a*arctanh(a*x)*ln(a*x)-8/3*a*arctanh(a*x)*ln(a*x-1)-8/3*
a*arctanh(a*x)*ln(a*x+1)+1/3*a^2*x+1/6*a*ln(a*x-1)-1/6*a*ln(a*x+1)-a*dilog(
a*x)-a*dilog(a*x+1)-a*ln(a*x)*ln(a*x+1)-2/3*a*ln(a*x-1)^2+8/3*a*dilog(1/2+1
/2*a*x)+4/3*a*ln(a*x-1)*ln(1/2+1/2*a*x)+2/3*a*ln(a*x+1)^2+4/3*a*ln(-1/2*a*x
+1/2)*ln(1/2+1/2*a*x)-4/3*a*ln(-1/2*a*x+1/2)*ln(a*x+1)
```

maxima [A] time = 0.33, size = 200, normalized size = 1.28

$$\frac{1}{6}a^2\left(\frac{2\left(ax + 2\log(ax+1)^2 - 4\log(ax+1)\log(ax-1) - 2\log(ax-1)^2\right)}{a} + \frac{16\left(\log(ax-1)\log\left(\frac{1}{2}ax + \frac{1}{2}\right)\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*(2*(a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x - 1) - 2*log(a*
x - 1)^2)/a + 16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/
a - 6*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 6*(log(-a*x + 1)*log(x) + dil
og(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) + 1/3*(a^2*x^2 - 8*log(a*x +
1) - 8*log(a*x - 1) + 6*log(x))*a*arctanh(a*x) + 1/3*(a^4*x^3 - 6*a^2*x - 3
/x)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^2,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2\,(ax+1)^2\operatorname{atanh}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**2, x)
```

3.210 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=162

$$\frac{1}{2}a^4x^2 \tanh^{-1}(ax)^2+a^3x \tanh^{-1}(ax)-a^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)+a^2\text{Li}_3\left(\frac{2}{1-ax}-1\right)+2a^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)-2a$$

```
[Out] -a*arctanh(a*x)/x+a^3*x*arctanh(a*x)-1/2*arctanh(a*x)^2/x^2+1/2*a^4*x^2*arc
tanh(a*x)^2+4*a^2*arctanh(a*x)^2*arctanh(-1+2/(-a*x+1))+a^2*ln(x)+2*a^2*arc
tanh(a*x)*polylog(2,1-2/(-a*x+1))-2*a^2*arctanh(a*x)*polylog(2,-1+2/(-a*x+1
))-a^2*polylog(3,1-2/(-a*x+1))+a^2*polylog(3,-1+2/(-a*x+1))
```

Rubi [A] time = 0.46, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {6012, 5916, 5982, 266, 36, 29, 31, 5948, 5914, 6052, 6058, 6610, 5980, 5910, 260}

$$-a^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)+a^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)+2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-2a^2\tanh^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] - ArcTanh[a*x]^2/(2*x^2) + (a^4*
x^2*ArcTanh[a*x]^2)/2 - 4*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2
*Log[x] + 2*a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - 2*a^2*ArcTanh[a*
x]*PolyLog[2, -1 + 2/(1 - a*x)] - a^2*PolyLog[3, 1 - 2/(1 - a*x)] + a^2*Pol
yLog[3, -1 + 2/(1 - a*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
```

```
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^3}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^3} - \frac{2a^2\tanh^{-1}(ax)^2}{x} + a^4x\tanh^{-1}(ax)^2\right)dx$$

$$= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x}\,dx\right) + a^4\int x\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tan$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tan$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tan$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tan$$

Mathematica [A] time = 0.07, size = 183, normalized size = 1.13

$$a^3x\tanh^{-1}(ax)+a^2\text{Li}_3\left(\frac{-ax-1}{ax-1}\right)-a^2\text{Li}_3\left(\frac{ax+1}{ax-1}\right)-2a^2\text{Li}_2\left(\frac{-ax-1}{ax-1}\right)\tanh^{-1}(ax)+2a^2\text{Li}_2\left(\frac{ax+1}{ax-1}\right)\tanh^{-1}(ax)+\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] + (a^2*(-1 + a^2*x^2)*ArcTanh[a*
x]^2)/2 + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) - 4*a^2*ArcTanh[a*x]^2*Ar
cTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - 2*a^2*ArcTanh[a*x]*PolyLog[2, (-1 - a
*x)/(-1 + a*x)] + 2*a^2*ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a*x)] + a^2
*PolyLog[3, (-1 - a*x)/(-1 + a*x)] - a^2*PolyLog[3, (1 + a*x)/(-1 + a*x)]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\left(ax\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^3, x)
```

maple [C] time = 1.88, size = 774, normalized size = 4.78

$$4a^2 \operatorname{polylog}\left(3,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-a^2 \operatorname{polylog}\left(3,-\frac{(ax+1)^2}{-a^2x^2+1}\right)-a^2 \ln\left(1+\frac{(ax+1)^2}{-a^2x^2+1}\right)+a^2 \ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x)
[Out] 4*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)^2/(-a^2
*x^2+1))-a^2*ln(1+(a*x+1)^2/(-a^2*x^2+1))+a^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/
2))+a^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-2*a^2*arctanh(a*x)^2*ln(a*x)+2*a^2
*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-2*a^2*arctanh(a*x)^2*ln(1+(a*x
+1)/(-a^2*x^2+1)^(1/2))-2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2
))-4*a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-4*a^2*arctanh(a
*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x)*polylog(2,-(a
*x+1)^2/(-a^2*x^2+1))+I*a^2*Pi*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-a*arc
tanh(a*x)/x+I*a^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(
-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-I*a^2*arctanh(a
*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+4*
a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-I*a^2*Pi*csgn(I*((a*x+1)^2/(-a^2*
x^2+1)-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^2+a^3*x*arctanh(a*x)+1/2*a^4*
x^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{16}\left(2x^2\log(-ax+1)-a\left(\frac{ax^2+2x}{a^2}+\frac{2\log(ax-1)}{a^3}\right)\right)a^4-\frac{1}{2}a^4\int x\log(ax+1)\log(-ax+1)\,dx+\frac{1}{4}a^3\int a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="maxima")
[Out] -1/16*(2*x^2*log(-a*x + 1) - a*((a*x^2 + 2*x)/a^2 + 2*log(a*x - 1)/a^3))*a^
4 - 1/2*a^4*integrate(x*log(a*x + 1)*log(-a*x + 1), x) + 1/4*a^3*integrate(
a*x*log(a*x + 1)^2, x) + 1/4*a^3*integrate(log(a*x + 1)^2/(a^3*x^3), x) + 1
/4*(a*x - (a*x - 1)*log(-a*x + 1) - 1)*a^2 - 1/2*a^2*integrate(log(a*x + 1)
^2/x, x) + a^2*integrate(log(a*x + 1)*log(-a*x + 1)/x, x) - 1/4*a^2*integra
te(log(-a*x + 1)/x, x) - 1/4*(a*(log(a*x - 1) - log(x)) - log(-a*x + 1)/x)*
a + 1/8*(a^4*x^4 - 1)*log(-a*x + 1)^2/x^2 - 1/2*integrate(log(a*x + 1)*log(
-a*x + 1)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^3,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**3, x)
```

3.211 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^4} \, dx$

Optimal. Leaf size=167

$$a^4x\tanh^{-1}(ax)^2-a^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)+\frac{5}{3}a^3\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-1}(ax)-2a^3\log\left(\frac{2}{1-ax}\right.$$

```
[Out] -1/3*a^2/x+1/3*a^3*arctanh(a*x)-1/3*a*arctanh(a*x)/x^2-2/3*a^3*arctanh(a*x)
^2-1/3*arctanh(a*x)^2/x^3+2*a^2*arctanh(a*x)^2/x+a^4*x*arctanh(a*x)^2-2*a^3
*arctanh(a*x)*ln(2/(-a*x+1))-10/3*a^3*arctanh(a*x)*ln(2-2/(a*x+1))-a^3*poly
log(2,1-2/(-a*x+1))+5/3*a^3*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.43, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6012, 5910, 5984, 5918, 2402, 2315, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$-a^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\frac{5}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}+a^4x\tanh^{-1}(ax)^2-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -a^2/(3*x) + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (2*a^2*ArcTanh[a*x]^2)/x + a^4*x*Ar
cTanh[a*x]^2 - 2*a^3*ArcTanh[a*x]*Log[2/(1 - a*x)] - (10*a^3*ArcTanh[a*x]*L
og[2 - 2/(1 + a*x)])/3 - a^3*PolyLog[2, 1 - 2/(1 - a*x)] + (5*a^3*PolyLog[2
, -1 + 2/(1 + a*x)])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
```

```
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^4}\,dx = \int \left(a^4 \tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{x^4} - \frac{2a^2 \tanh^{-1}(ax)^2}{x^2}\right) dx$$

$$= -\left(\left(2a^2\right) \int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\right) + a^4 \int \tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} + a^4 x \tanh^{-1}(ax)^2 + \frac{1}{3}(2a) \int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right.}$$

$$= -a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} + a^4 x \tanh^{-1}(ax)^2 + \frac{1}{3}(2$$

$$= -\frac{a \tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} + a^4 x \,\mathrm{ta}$$

$$= -\frac{a^2}{3x} - \frac{a \tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} +$$

$$= -\frac{a^2}{3x} + \frac{1}{3}a^3 \tanh^{-1}(ax) - \frac{a \tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} +$$

Mathematica [A] time = 0.07, size = 153, normalized size = 0.92

$$\frac{1}{3}\left(3a^4 x \tanh^{-1}(ax)^2 + 3a^3 \mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 5a^3 \mathrm{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right) - 8a^3 \tanh^{-1}(ax)^2 + a^3 \tanh^{-1}(ax) - 10\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] (-(a^2/x) + a^3*ArcTanh[a*x] - (a*ArcTanh[a*x])/x^2 - 8*a^3*ArcTanh[a*x]^2
- ArcTanh[a*x]^2/x^3 + (6*a^2*ArcTanh[a*x]^2)/x + 3*a^4*x*ArcTanh[a*x]^2 -
10*a^3*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] - 6*a^3*ArcTanh[a*x]*Log[1
 + E^(-2*ArcTanh[a*x])] + 3*a^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 5*a^3*Po
lyLog[2, E^(-2*ArcTanh[a*x])])/3
```

fricas [F] time = 2.10, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\mathrm{artanh}\,(ax)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2 - 1\right)^2 \mathrm{artanh}\,(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^4, x)
```

maple [A] time = 0.07, size = 249, normalized size = 1.49

$$a^4x \operatorname{arctanh}(ax)^2+\frac{2a^2 \operatorname{arctanh}(ax)^2}{x}-\frac{\operatorname{arctanh}(ax)^2}{3x^3}-\frac{a \operatorname{arctanh}(ax)}{3x^2}-\frac{10a^3 \operatorname{arctanh}(ax)\ln(ax)}{3}+\frac{8a^3 \operatorname{arctanh}(a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x)
[Out] a^4*x*arctanh(a*x)^2+2*a^2*arctanh(a*x)^2/x-1/3*arctanh(a*x)^2/x^3-1/3*a*ar
ctanh(a*x)/x^2-10/3*a^3*arctanh(a*x)*ln(a*x)+8/3*a^3*arctanh(a*x)*ln(a*x-1)
+8/3*a^3*arctanh(a*x)*ln(a*x+1)-1/3*a^2/x-1/6*a^3*ln(a*x-1)+1/6*a^3*ln(a*x+
1)+5/3*a^3*dilog(a*x)+5/3*a^3*dilog(a*x+1)+5/3*a^3*ln(a*x)*ln(a*x+1)+2/3*a^
3*ln(a*x-1)^2-8/3*a^3*dilog(1/2+1/2*a*x)-4/3*a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-
2/3*a^3*ln(a*x+1)^2-4/3*a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+4/3*a^3*ln(-1/
2*a*x+1/2)*ln(a*x+1)
```

maxima [A] time = 0.33, size = 203, normalized size = 1.22

$$-\frac{1}{6}\left(16\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-10\left(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax)\right)a+10\left(\log(-ax\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="maxima")
[Out] -1/6*(16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 10*(
log(a*x + 1)*log(x) + dilog(-a*x))*a + 10*(log(-a*x + 1)*log(x) + dilog(a*x
))*a - a*log(a*x + 1) + a*log(a*x - 1) + 2*(2*a*x*log(a*x + 1)^2 - 4*a*x*lo
g(a*x + 1)*log(a*x - 1) - 2*a*x*log(a*x - 1)^2 + 1)/x)*a^2 + 1/3*(8*a^2*log
(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a*arctanh(a*x) + 1/
3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^4,x)
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**4,x)
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**4, x)
```

3.212 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^5} dx$

Optimal. Leaf size=214

$$\frac{1}{2}a^4\text{Li}_3\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}a^4\text{Li}_3\left(\frac{2}{1-ax}-1\right)-a^4\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)+a^4\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)-\frac{4}{3}a$$

```
[Out] -1/12*a^2/x^2-1/6*a*arctanh(a*x)/x^3+3/2*a^3*arctanh(a*x)/x-3/4*a^4*arctanh
(a*x)^2-1/4*arctanh(a*x)^2/x^4+a^2*arctanh(a*x)^2/x^2-2*a^4*arctanh(a*x)^2*
arctanh(-1+2/(-a*x+1))-4/3*a^4*ln(x)+2/3*a^4*ln(-a^2*x^2+1)-a^4*arctanh(a*x
)*polylog(2,1-2/(-a*x+1))+a^4*arctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*a^4
*polylog(3,1-2/(-a*x+1))-1/2*a^4*polylog(3,-1+2/(-a*x+1))
```

Rubi [A] time = 0.55, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6012, 5916, 5982, 266, 44, 36, 29, 31, 5948, 5914, 6052, 6058, 6610}

$$\frac{1}{2}a^4\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}a^4\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-a^4\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+a^4\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] -a^2/(12*x^2) - (a*ArcTanh[a*x])/(6*x^3) + (3*a^3*ArcTanh[a*x])/(2*x) - (3*
a^4*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(4*x^4) + (a^2*ArcTanh[a*x]^2)/x^2 +
 2*a^4*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (4*a^4*Log[x])/3 + (2*a^4*
Log[1 - a^2*x^2])/3 - a^4*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] + a^4*Ar
cTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + (a^4*PolyLog[3, 1 - 2/(1 - a*x)])
/2 - (a^4*PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^5}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^5} - \frac{2a^2\tanh^{-1}(ax)^2}{x^3} + \frac{a^4\tanh^{-1}(ax)^2}{x}\right)dx$$
$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^5}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh^{-1}(ax)^2}{x^2} + 2a^4\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + \frac{1}{2}$$
$$= -\frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh^{-1}(ax)^2}{x^2} + 2a^4\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + \frac{1}{2}$$
$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{2a^3\tanh^{-1}(ax)}{x} - a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh}{x}$$
$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + a^2\tan$$
$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + a^2\tan$$
$$= -\frac{a^2}{12x^2} - \frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4}$$
$$= -\frac{a^2}{12x^2} - \frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4}$$

Mathematica [C] time = 0.37, size = 238, normalized size = 1.11

$$\frac{1}{24}\left(24a^4\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 24a^4\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) + 12a^4\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right) - 12a^4\mathrm{L}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] (2*a^4 + I*a^4*Pi^3 - (2*a^2)/x^2 - (4*a*ArcTanh[a*x])/x^3 + (36*a^3*ArcTan
h[a*x])/x - 18*a^4*ArcTanh[a*x]^2 - (6*ArcTanh[a*x]^2)/x^4 + (24*a^2*ArcTan
h[a*x]^2)/x^2 - 16*a^4*ArcTanh[a*x]^3 - 24*a^4*ArcTanh[a*x]^2*Log[1 + E^(-2
*ArcTanh[a*x])] + 24*a^4*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 32*a^
4*Log[(a*x)/Sqrt[1 - a^2*x^2]] + 24*a^4*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcT
anh[a*x])] + 24*a^4*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 12*a^4*Po
lyLog[3, -E^(-2*ArcTanh[a*x])] - 12*a^4*PolyLog[3, E^(2*ArcTanh[a*x])])/24
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\mathrm{artanh}\,(ax)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2 \mathrm{artanh}\,(ax)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^5, x)
```

maple [C] time = 2.76, size = 927, normalized size = 4.33

$$\frac{3a^3 \operatorname{arctanh}(ax)}{2x}+\frac{a^2 \operatorname{arctanh}(ax)^2}{x^2}-\frac{a \operatorname{arctanh}(ax)}{6x^3}+a^4 \operatorname{arctanh}(ax)^2 \ln(ax)-a^4 \operatorname{arctanh}(ax)^2 \ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x)
```

```
[Out] -3/4*a^4*arctanh(a*x)^2-1/4*arctanh(a*x)^2/x^4+a^2*arctanh(a*x)^2/x^2-2*a^4
*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/6*a*arctanh(a*x)/x^3+3/2*a^3*arcta
nh(a*x)/x-1/2*I*a^4*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*cs
gn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/2*I*a^4*arc
tanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x
^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+4/3*a^4*arctanh(a*x)+1/2*I*a^4*arcta
nh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^
3+a^4*arctanh(a*x)^2*ln(a*x)-a^4*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1
)+a^4*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^4*arctanh(a*x)*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^4*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x
^2+1)^(1/2))+2*a^4*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^4*
arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/12*a^4/(a*x+1-(-a^2*x^2+1
)^(1/2))*(-a^2*x^2+1)^(1/2)+1/12*a^4/((-a^2*x^2+1)^(1/2)+a*x+1)*(-a^2*x^2+1
)^(1/2)-1/24*a^5/((-a^2*x^2+1)^(1/2)-1)*x+1/24*a^5/((-a^2*x^2+1)^(1/2)+1)*x
+1/2*I*a^4*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(
a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))-4/3*a^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a^4*polylog(3,-(a*x+1
)/(-a^2*x^2+1)^(1/2))+1/2*a^4*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-4/3*a^4*ln
((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+1/24*a^4/((-a^2*x^2+1)^(1/2)-1)-1/24*a^4/((-
a^2*x^2+1)^(1/2)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(4\,a^2x^2-1\right)\log(-ax+1)^2}{16\,x^4}-\frac{1}{4}\int -\frac{2\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\log(ax+1)^2-\left(4\,a^3x^3-ax+4\,(a\right.}{2\left(ax^6-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*(4*a^2*x^2 - 1)*log(-a*x + 1)^2/x^4 - 1/4*integrate(-1/2*(2*(a^5*x^5 -
 a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (4*a^3*x^3 - a
*x + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1))*
log(-a*x + 1))/(a*x^6 - x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^5,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**5,x)
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**5, x)
```

3.213 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^6} dx$

Optimal. Leaf size=157

$$-\frac{8}{15}a^5\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{8}{15}a^5\tanh^{-1}(ax)^2-\frac{11}{30}a^5\tanh^{-1}(ax)+\frac{16}{15}a^5\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)+\frac{11a^4}{30x}-\frac{a^4\tanh^{-}}{x}$$

```
[Out] -1/30*a^2/x^3+11/30*a^4/x-11/30*a^5*arctanh(a*x)-1/10*a*arctanh(a*x)/x^4+7/
15*a^3*arctanh(a*x)/x^2+8/15*a^5*arctanh(a*x)^2-1/5*arctanh(a*x)^2/x^5+2/3*
a^2*arctanh(a*x)^2/x^3-a^4*arctanh(a*x)^2/x+16/15*a^5*arctanh(a*x)*ln(2-2/(
a*x+1))-8/15*a^5*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.59, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6012, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$-\frac{8}{15}a^5\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{30x^3}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}+\frac{2a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{11a^4}{30x}+\frac{8}{15}a^5\tanh^{-1}(ax)^2-\frac{11}{30}a^5\tanh^{-}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] -a^2/(30*x^3) + (11*a^4)/(30*x) - (11*a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x
])/(10*x^4) + (7*a^3*ArcTanh[a*x])/(15*x^2) + (8*a^5*ArcTanh[a*x]^2)/15 - A
rcTanh[a*x]^2/(5*x^5) + (2*a^2*ArcTanh[a*x]^2)/(3*x^3) - (a^4*ArcTanh[a*x]^
2)/x + (16*a^5*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/15 - (8*a^5*PolyLog[2, -1
 + 2/(1 + a*x)])/15
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^6}\,dx &= \int \left(\frac{\tanh^{-1}(ax)^2}{x^6}-\frac{2a^2\tanh^{-1}(ax)^2}{x^4}+\frac{a^4\tanh^{-1}(ax)^2}{x^2}\right)dx\\
&= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx\right)+a^4\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx\\
&= -\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{2a^2\tanh^{-1}(ax)^2}{3x^3}-\frac{a^4\tanh^{-1}(ax)^2}{x}+\frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5\left(1-a^2x^2\right)}\\
&= a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{2a^2\tanh^{-1}(ax)^2}{3x^3}-\frac{a^4\tanh^{-1}(ax)^2}{x}+\frac{1}{5}(2a)\\
&= -\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{3x^2}+\frac{1}{3}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{2a^2\text{ta}}{}\\
&= -\frac{a^2}{30x^3}+\frac{2a^4}{3x}-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}+\frac{8}{15}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}}{5}\\
&= -\frac{a^2}{30x^3}+\frac{11a^4}{30x}-\frac{2}{3}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}+\frac{8}{15}a^5\text{ta}\\
&= -\frac{a^2}{30x^3}+\frac{11a^4}{30x}-\frac{11}{30}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}+\frac{8}{15}a^5
\end{aligned}
$$

Mathematica [A] time = 0.82, size = 118, normalized size = 0.75

$$
\frac{-16a^5x^5\text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right)+a^2x^2\left(11a^2x^2-1\right)+2(ax-1)^3\left(8a^2x^2+9ax+3\right)\tanh^{-1}(ax)^2+ax\tanh^{-1}(ax)\left(-\right.}{30x^5}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] (a^2*x^2*(-1 + 11*a^2*x^2) + 2*(-1 + a*x)^3*(3 + 9*a*x + 8*a^2*x^2)*ArcTanh
[a*x]^2 + a*x*ArcTanh[a*x]*(-3 + 14*a^2*x^2 - 11*a^4*x^4 + 32*a^4*x^4*Log[1
 - E^(-2*ArcTanh[a*x])]) - 16*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(30*
x^5)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\,(ax)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2 - 1\right)^2 \text{artanh}\,(ax)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^6, x)
```

maple [A] time = 0.07, size = 272, normalized size = 1.73

$$-\frac{a^4\,\text{arctanh}\,(ax)^2}{x} + \frac{2a^2\,\text{arctanh}\,(ax)^2}{3x^3} - \frac{\text{arctanh}\,(ax)^2}{5x^5} - \frac{a\,\text{arctanh}\,(ax)}{10x^4} + \frac{7a^3\,\text{arctanh}\,(ax)}{15x^2} + \frac{16a^5\,\text{arctanh}\,(ax)\ln (}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x)
```

```
[Out] -a^4*arctanh(a*x)^2/x+2/3*a^2*arctanh(a*x)^2/x^3-1/5*arctanh(a*x)^2/x^5-1/1
0*a*arctanh(a*x)/x^4+7/15*a^3*arctanh(a*x)/x^2+16/15*a^5*arctanh(a*x)*ln(a*
x)-8/15*a^5*arctanh(a*x)*ln(a*x-1)-8/15*a^5*arctanh(a*x)*ln(a*x+1)-1/30*a^2
/x^3+11/30*a^4/x+11/60*a^5*ln(a*x-1)-11/60*a^5*ln(a*x+1)-8/15*a^5*dilog(a*x
)-8/15*a^5*dilog(a*x+1)-8/15*a^5*ln(a*x)*ln(a*x+1)-2/15*a^5*ln(a*x-1)^2+8/1
5*a^5*dilog(1/2+1/2*a*x)+4/15*a^5*ln(a*x-1)*ln(1/2+1/2*a*x)+2/15*a^5*ln(a*x
+1)^2-4/15*a^5*ln(-1/2*a*x+1/2)*ln(a*x+1)+4/15*a^5*ln(-1/2*a*x+1/2)*ln(1/2+
1/2*a*x)
```

maxima [A] time = 0.33, size = 239, normalized size = 1.52

$$\frac{1}{60}\left(32\left(\log\,(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a^3 - 32\left(\log\,(ax+1)\log(x)+\text{Li}_2\,(-ax)\right)a^3 + 32\left(\log\,(-a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/60*(32*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 32
*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 32*(log(-a*x + 1)*log(x) + dilog
(a*x))*a^3 - 11*a^3*log(a*x + 1) + 11*a^3*log(a*x - 1) + 2*(4*a^3*x^3*log(a
```

```
*x + 1)^2 - 8*a^3*x^3*log(a*x + 1)*log(a*x - 1) - 4*a^3*x^3*log(a*x - 1)^2
+ 11*a^2*x^2 - 1)/x^3)*a^2 - 1/30*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*log(x^2
) - (14*a^2*x^2 - 3)/x^4)*a*arctanh(a*x) - 1/15*(15*a^4*x^4 - 10*a^2*x^2 +
3)*arctanh(a*x)^2/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^6,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^2(ax)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**6,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**6, x)
```

3.214 $\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^7}\,dx$

Optimal. Leaf size=113

$$\frac{8}{45}a^6\log(x)-\frac{a^5\tanh^{-1}(ax)}{3x}+\frac{7a^4}{90x^2}+\frac{2a^3\tanh^{-1}(ax)}{9x^3}-\frac{a^2}{60x^4}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6x^6}-\frac{4}{45}a^6\log\left(1-a^2x^2\right)-\frac{a\tan}{1}$$

[Out] -1/60*a^2/x^4+7/90*a^4/x^2-1/15*a*arctanh(a*x)/x^5+2/9*a^3*arctanh(a*x)/x^3 -1/3*a^5*arctanh(a*x)/x-1/6*(-a^2*x^2+1)^3*arctanh(a*x)^2/x^6+8/45*a^6*ln(x)-4/45*a^6*ln(-a^2*x^2+1)

Rubi [A] time = 0.19, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6008, 6012, 5916, 266, 44, 36, 29, 31}

$$\frac{7a^4}{90x^2}-\frac{a^2}{60x^4}-\frac{4}{45}a^6\log\left(1-a^2x^2\right)+\frac{2a^3\tanh^{-1}(ax)}{9x^3}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6x^6}+\frac{8}{45}a^6\log(x)-\frac{a^5\tanh^{-1}(ax)}{3x}-\frac{a\tan}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

```
[Out] -a^2/(60*x^4) + (7*a^4)/(90*x^2) - (a*ArcTanh[a*x])/(15*x^5) + (2*a^3*ArcTa
nh[a*x])/(9*x^3) - (a^5*ArcTanh[a*x])/(3*x) - ((1 - a^2*x^2)^3*ArcTanh[a*x]
^2)/(6*x^6) + (8*a^6*Log[x])/45 - (4*a^6*Log[1 - a^2*x^2])/45
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^7}\,dx = -\frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^6}\,dx$$

$$= -\frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a\int \left(\frac{\tanh^{-1}(ax)}{x^6} - \frac{2a^2\tanh^{-1}(ax)}{x^4} + \frac{a^4\tanh^{-1}(ax)}{x^2}\right.$$

$$= -\frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a\int \frac{\tanh^{-1}(ax)}{x^6}\,dx - \frac{1}{3}\left(2a^3\right)\int \frac{\tanh^{-1}(ax)}{x^4}\,d$$

$$= -\frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6}$$

$$= -\frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6}$$

$$= -\frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6}$$

$$= -\frac{a^2}{60x^4} + \frac{7a^4}{90x^2} - \frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x\right.}{}$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.88

$$\frac{30\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2 + a^2x^2\left(32a^4x^4\log(x)+14a^2x^2-16a^4x^4\log\left(1-a^2x^2\right)-3\right)-4ax\left(15a^4x^4-10a^2\right.}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

```
[Out] (-4*a*x*(3 - 10*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x] + 30*(-1 + a^2*x^2)^3*Ar
cTanh[a*x]^2 + a^2*x^2*(-3 + 14*a^2*x^2 + 32*a^4*x^4*Log[x] - 16*a^4*x^4*Lo
g[1 - a^2*x^2]))/(180*x^6)
```

fricas [A] time = 0.59, size = 132, normalized size = 1.17

$$-\frac{32\,a^6x^6\log\left(a^2x^2-1\right)-64\,a^6x^6\log(x)-28\,a^4x^4+6\,a^2x^2-15\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(15}{360\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/360*(32*a^6*x^6*log(a^2*x^2 - 1) - 64*a^6*x^6*log(x) - 28*a^4*x^4 + 6*a^
2*x^2 - 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))^
2 + 4*(15*a^5*x^5 - 10*a^3*x^3 + 3*a*x)*log(-(a*x + 1)/(a*x - 1)))/x^6
```

giac [B] time = 0.22, size = 440, normalized size = 3.89

$$\frac{4}{45}\left(2\,a^5\log\left(-\frac{ax+1}{ax-1}-1\right)-2\,a^5\log\left(-\frac{ax+1}{ax-1}\right)+\frac{30\,(ax+1)^3a^5\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax-1)^3\left(\frac{(ax+1)^6}{(ax-1)^6}+\frac{6\,(ax+1)^5}{(ax-1)^5}+\frac{15\,(ax+1)^4}{(ax-1)^4}+\frac{20\,(ax+1)^3}{(ax-1)^3}+\frac{15\,(ax+1)^2}{(ax-1)^2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="giac")
```

```
[Out] 4/45*(2*a^5*log(-(a*x + 1)/(a*x - 1) - 1) - 2*a^5*log(-(a*x + 1)/(a*x - 1))
 + 30*(a*x + 1)^3*a^5*log(-(a*x + 1)/(a*x - 1))^2/((a*x - 1)^3*((a*x + 1)^6
/(a*x - 1)^6 + 6*(a*x + 1)^5/(a*x - 1)^5 + 15*(a*x + 1)^4/(a*x - 1)^4 + 20*
(a*x + 1)^3/(a*x - 1)^3 + 15*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1
) + 1)) + 2*(10*(a*x + 1)^2*a^5/(a*x - 1)^2 + 5*(a*x + 1)*a^5/(a*x - 1) + a
^5)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^5/(a*x - 1)^5 + 5*(a*x + 1)^4/(a*x
 - 1)^4 + 10*(a*x + 1)^3/(a*x - 1)^3 + 10*(a*x + 1)^2/(a*x - 1)^2 + 5*(a*x
+ 1)/(a*x - 1) + 1) - (2*(a*x + 1)^3*a^5/(a*x - 1)^3 + 7*(a*x + 1)^2*a^5/(a
*x - 1)^2 + 2*(a*x + 1)*a^5/(a*x - 1))/((a*x + 1)^4/(a*x - 1)^4 + 4*(a*x +
1)^3/(a*x - 1)^3 + 6*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x + 1)/(a*x - 1) + 1))*
a
```

maple [B] time = 0.07, size = 233, normalized size = 2.06

$$-\frac{a^4\operatorname{arctanh}(ax)^2}{2x^2}+\frac{a^2\operatorname{arctanh}(ax)^2}{2x^4}-\frac{\operatorname{arctanh}(ax)^2}{6x^6}-\frac{a\operatorname{arctanh}(ax)}{15x^5}+\frac{2a^3\operatorname{arctanh}(ax)}{9x^3}-\frac{a^5\operatorname{arctanh}(ax)}{3x}-\frac{a^6\operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x)
```

```
[Out] -1/2*a^4*arctanh(a*x)^2/x^2+1/2*a^2*arctanh(a*x)^2/x^4-1/6*arctanh(a*x)^2/x
^6-1/15*a*arctanh(a*x)/x^5+2/9*a^3*arctanh(a*x)/x^3-1/3*a^5*arctanh(a*x)/x-
1/6*a^6*arctanh(a*x)*ln(a*x-1)+1/6*a^6*arctanh(a*x)*ln(a*x+1)-1/24*a^6*ln(a
*x-1)^2+1/12*a^6*ln(a*x-1)*ln(1/2+1/2*a*x)-1/24*a^6*ln(a*x+1)^2-1/12*a^6*ln
(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/12*a^6*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/60*a^2
/x^4+7/90*a^4/x^2+8/45*a^6*ln(a*x)-4/45*a^6*ln(a*x-1)-4/45*a^6*ln(a*x+1)
```

maxima [A] time = 0.33, size = 188, normalized size = 1.66

$$\frac{1}{360}\left(64\,a^4\log(x)-\frac{15\,a^4x^4\log(ax+1)^2+15\,a^4x^4\log(ax-1)^2+32\,a^4x^4\log(ax-1)-28\,a^2x^2-2\left(15\,a^4x^4\log\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/360*(64*a^4*log(x) - (15*a^4*x^4*log(a*x + 1)^2 + 15*a^4*x^4*log(a*x - 1)
^2 + 32*a^4*x^4*log(a*x - 1) - 28*a^2*x^2 - 2*(15*a^4*x^4*log(a*x - 1) - 16
*a^4*x^4)*log(a*x + 1) + 6)/x^4)*a^2 + 1/90*(15*a^5*log(a*x + 1) - 15*a^5*l
og(a*x - 1) - 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)/x^5)*a*arctanh(a*x) - 1/6*(3*
a^4*x^4 - 3*a^2*x^2 + 1)*arctanh(a*x)^2/x^6
```

mupad [B] time = 1.58, size = 335, normalized size = 2.96

$$\frac{8\,a^6\,\ln(x)}{45}-\frac{\frac{3\,a^2}{4}-\frac{7\,a^4\,x^2}{2}}{45\,x^4}-\ln\left(1-a\,x\right)^2\,\left(\frac{\frac{a^4\,x^4}{2}-\frac{a^2\,x^2}{2}+\frac{1}{6}}{4\,x^6}-\frac{a^6}{24}\right)-\ln\left(a\,x+1\right)^2\,\left(\frac{\frac{a^4\,x^4}{8}-\frac{a^2\,x^2}{8}+\frac{1}{24}}{x^6}-\frac{a^6}{24}\right)-\ln\left(1-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^7,x)
```

```
[Out] (8*a^6*log(x))/45 - ((3*a^2)/4 - (7*a^4*x^2)/2)/(45*x^4) - log(1 - a*x)^2*(
((a^4*x^4)/2 - (a^2*x^2)/2 + 1/6)/(4*x^6) - a^6/24) - log(a*x + 1)^2*(((a^4
*x^4)/8 - (a^2*x^2)/8 + 1/24)/x^6 - a^6/24) - log(1 - a*x)*((a*((15*a*x)/2
- 10*a^2*x^2 + 15*a^3*x^3 - 30*a^4*x^4 + (137*a^5*x^5)/2 - 6))/(360*x^5) -
log(a*x + 1)*(((a^4*x^4)/2 - (a^2*x^2)/2 + 1/6)/(2*x^6) - a^6/12) - (a*(15*
a*x + 20*a^2*x^2 + 30*a^3*x^3 + 60*a^4*x^4 + 137*a^5*x^5 + 12))/(720*x^5) +
 (5*a^8*x^2 - (15*a^9*x^3)/2)/(60*a^5*x^5) + (5*a^8*x^2 + (15*a^9*x^3)/2)/(
60*a^5*x^5)) - (4*a^6*log(a^2*x^2 - 1))/45 - (a*log(a*x + 1)*((a^4*x^4)/6 -
 (a^2*x^2)/9 + 1/30))/x^5
```

sympy [A] time = 2.60, size = 148, normalized size = 1.31

$$\begin{cases}\frac{8a^6\log(x)}{45}-\frac{8a^6\log\left(x-\frac{1}{a}\right)}{45}+\frac{a^6\operatorname{atanh}^2(ax)}{6}-\frac{8a^6\operatorname{atanh}(ax)}{45}-\frac{a^5\operatorname{atanh}(ax)}{3x}-\frac{a^4\operatorname{atanh}^2(ax)}{2x^2}+\frac{7a^4}{90x^2}+\frac{2a^3\operatorname{atanh}(ax)}{9x^3}+\frac{a^2\operatorname{atanh}^2(\right. \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**7,x)
```

```
[Out] Piecewise((8*a**6*log(x)/45 - 8*a**6*log(x - 1/a)/45 + a**6*atanh(a*x)**2/6
 - 8*a**6*atanh(a*x)/45 - a**5*atanh(a*x)/(3*x) - a**4*atanh(a*x)**2/(2*x**
2) + 7*a**4/(90*x**2) + 2*a**3*atanh(a*x)/(9*x**3) + a**2*atanh(a*x)**2/(2*
x**4) - a**2/(60*x**4) - a*atanh(a*x)/(15*x**5) - atanh(a*x)**2/(6*x**6), N
e(a, 0)), (0, True))
```

3.215 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^8} dx$

Optimal. Leaf size=183

$$-\frac{8}{105}a^7\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{8}{105}a^7\tanh^{-1}(ax)^2-\frac{1}{210}a^7\tanh^{-1}(ax)+\frac{16}{105}a^7\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)+\frac{a^6}{210x}-\frac{8a^5}{}$$

```
[Out] -1/105*a^2/x^5+17/630*a^4/x^3+1/210*a^6/x-1/210*a^7*arctanh(a*x)-1/21*a*arc
tanh(a*x)/x^6+9/70*a^3*arctanh(a*x)/x^4-8/105*a^5*arctanh(a*x)/x^2+8/105*a^
7*arctanh(a*x)^2-1/7*arctanh(a*x)^2/x^7+2/5*a^2*arctanh(a*x)^2/x^5-1/3*a^4*
arctanh(a*x)^2/x^3+16/105*a^7*arctanh(a*x)*ln(2-2/(a*x+1))-8/105*a^7*polylo
g(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.82, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6012, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$-\frac{8}{105}a^7\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{17a^4}{630x^3}-\frac{a^2}{105x^5}-\frac{8a^5\tanh^{-1}(ax)}{105x^2}-\frac{a^4\tanh^{-1}(ax)^2}{3x^3}+\frac{9a^3\tanh^{-1}(ax)}{70x^4}+\frac{2a^2\tanh^{-1}(}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

```
[Out] -a^2/(105*x^5) + (17*a^4)/(630*x^3) + a^6/(210*x) - (a^7*ArcTanh[a*x])/210
- (a*ArcTanh[a*x])/(21*x^6) + (9*a^3*ArcTanh[a*x])/(70*x^4) - (8*a^5*ArcTan
h[a*x])/(105*x^2) + (8*a^7*ArcTanh[a*x]^2)/105 - ArcTanh[a*x]^2/(7*x^7) + (
2*a^2*ArcTanh[a*x]^2)/(5*x^5) - (a^4*ArcTanh[a*x]^2)/(3*x^3) + (16*a^7*ArcT
anh[a*x]*Log[2 - 2/(1 + a*x)])/105 - (8*a^7*PolyLog[2, -1 + 2/(1 + a*x)])/1
05
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^8}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^8} - \frac{2a^2\tanh^{-1}(ax)^2}{x^6} + \frac{a^4\tanh^{-1}(ax)^2}{x^4}\right)dx$$
$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^8}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{7x^7} + \frac{2a^2\tanh^{-1}(ax)^2}{5x^5} - \frac{a^4\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{7}(2a)\int \frac{\tanh^{-1}(ax)}{x^7\left(1-a^2x^2\right)}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{7x^7} + \frac{2a^2\tanh^{-1}(ax)^2}{5x^5} - \frac{a^4\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{7}(2a)\int \frac{\tanh^{-1}(ax)}{x^7}\,dx +$$
$$= -\frac{a\tanh^{-1}(ax)}{21x^6} + \frac{a^3\tanh^{-1}(ax)}{5x^4} - \frac{a^5\tanh^{-1}(ax)}{3x^2} + \frac{1}{3}a^7\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(a}{7x^7}$$
$$= -\frac{a^2}{105x^5} + \frac{a^4}{15x^3} - \frac{a^6}{3x} - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4} + \frac{a^5\tanh^{-1}(ax)}{15x^2} - \frac{1}{15}a$$
$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{4a^6}{15x} + \frac{1}{3}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4} -$$
$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{a^6}{210x} - \frac{4}{15}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4}$$
$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{a^6}{210x} - \frac{1}{210}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4}$$

Mathematica [A] time = 1.46, size = 140, normalized size = 0.77

$$\frac{-48a^7x^7\text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right) + a^2x^2\left(3a^4x^4+17a^2x^2-6\right) + 6\left(8a^7x^7-35a^4x^4+42a^2x^2-15\right)\tanh^{-1}(ax)^2 + 3ax\,\text{ta}}{630x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

```
[Out] (a^2*x^2*(-6 + 17*a^2*x^2 + 3*a^4*x^4) + 6*(-15 + 42*a^2*x^2 - 35*a^4*x^4 +
 8*a^7*x^7)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh[a*x]*(-10 + 27*a^2*x^2 - 16*a^4*
x^4 - a^6*x^6 + 32*a^6*x^6*Log[1 - E^(-2*ArcTanh[a*x])]) - 48*a^7*x^7*PolyL
og[2, E^(-2*ArcTanh[a*x])])/(630*x^7)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\,(ax)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2 \text{artanh}\,(ax)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^8, x)
```

maple [A] time = 0.07, size = 292, normalized size = 1.60

$$-\frac{a^4 \operatorname{arctanh}(ax)^2}{3x^3}-\frac{\operatorname{arctanh}(ax)^2}{7x^7}+\frac{2a^2 \operatorname{arctanh}(ax)^2}{5x^5}-\frac{a \operatorname{arctanh}(ax)}{21x^6}+\frac{9a^3 \operatorname{arctanh}(ax)}{70x^4}-\frac{8a^5 \operatorname{arctanh}(ax)}{105x^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x)
```

```
[Out] -1/3*a^4*arctanh(a*x)^2/x^3-1/7*arctanh(a*x)^2/x^7+2/5*a^2*arctanh(a*x)^2/x
^5-1/21*a*arctanh(a*x)/x^6+9/70*a^3*arctanh(a*x)/x^4-8/105*a^5*arctanh(a*x)
/x^2+16/105*a^7*arctanh(a*x)*ln(a*x)-8/105*a^7*arctanh(a*x)*ln(a*x-1)-8/105
*a^7*arctanh(a*x)*ln(a*x+1)-8/105*a^7*dilog(a*x)-8/105*a^7*dilog(a*x+1)-8/1
05*a^7*ln(a*x)*ln(a*x+1)-2/105*a^7*ln(a*x-1)^2+8/105*a^7*dilog(1/2+1/2*a*x)
+4/105*a^7*ln(a*x-1)*ln(1/2+1/2*a*x)+2/105*a^7*ln(a*x+1)^2+4/105*a^7*ln(-1/
2*a*x+1/2)*ln(1/2+1/2*a*x)-4/105*a^7*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/210*a^6/x
-1/105*a^2/x^5+17/630*a^4/x^3+1/420*a^7*ln(a*x-1)-1/420*a^7*ln(a*x+1)
```

maxima [A] time = 0.33, size = 254, normalized size = 1.39

$$\frac{1}{1260}\left(96\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^5-96\left(\log\left(ax+1\right)\log(x)+\mathrm{Li}_2\left(-ax\right)\right)a^5+96\left(\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/1260*(96*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^5 -
96*(log(a*x + 1)*log(x) + dilog(-a*x))*a^5 + 96*(log(-a*x + 1)*log(x) + dil
og(a*x))*a^5 - 3*a^5*log(a*x + 1) + 3*a^5*log(a*x - 1) + 2*(12*a^5*x^5*log(
a*x + 1)^2 - 24*a^5*x^5*log(a*x + 1)*log(a*x - 1) - 12*a^5*x^5*log(a*x - 1)
^2 + 3*a^4*x^4 + 17*a^2*x^2 - 6)/x^5)*a^2 - 1/210*(16*a^6*log(a^2*x^2 - 1)
- 16*a^6*log(x^2) + (16*a^4*x^4 - 27*a^2*x^2 + 10)/x^6)*a*arctanh(a*x) - 1/
105*(35*a^4*x^4 - 42*a^2*x^2 + 15)*arctanh(a*x)^2/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}{x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^8,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**8,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**8, x)
```

3.216 $\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^9}\,dx$

Optimal. Leaf size=170

$$\frac{4}{63}a^8\log(x)+\frac{1}{24}a^8\tanh^{-1}(ax)^2-\frac{a^7\tanh^{-1}(ax)}{12x}+\frac{5a^6}{504x^2}-\frac{a^5\tanh^{-1}(ax)}{36x^3}+\frac{a^4}{84x^4}-\frac{a^4\tanh^{-1}(ax)^2}{4x^4}+\frac{a^3\tanh^{-1}(ax)}{12x^5}-$$

```
[Out] -1/168*a^2/x^6+1/84*a^4/x^4+5/504*a^6/x^2-1/28*a*arctanh(a*x)/x^7+1/12*a^3*
arctanh(a*x)/x^5-1/36*a^5*arctanh(a*x)/x^3-1/12*a^7*arctanh(a*x)/x+1/24*a^8
*arctanh(a*x)^2-1/8*arctanh(a*x)^2/x^8+1/3*a^2*arctanh(a*x)^2/x^6-1/4*a^4*a
rctanh(a*x)^2/x^4+4/63*a^8*ln(x)-2/63*a^8*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.84, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 56, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6012, 5916, 5982, 266, 44, 36, 29, 31, 5948}

$$\frac{5a^6}{504x^2}+\frac{a^4}{84x^4}-\frac{a^2}{168x^6}-\frac{2}{63}a^8\log\left(1-a^2x^2\right)-\frac{a^5\tanh^{-1}(ax)}{36x^3}-\frac{a^4\tanh^{-1}(ax)^2}{4x^4}+\frac{a^3\tanh^{-1}(ax)}{12x^5}+\frac{a^2\tanh^{-1}(ax)^2}{3x^6}+\frac{4}{63}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

```
[Out] -a^2/(168*x^6) + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a*ArcTanh[a*x])/(28*x^
7) + (a^3*ArcTanh[a*x])/(12*x^5) - (a^5*ArcTanh[a*x])/(36*x^3) - (a^7*ArcTa
nh[a*x])/(12*x) + (a^8*ArcTanh[a*x]^2)/24 - ArcTanh[a*x]^2/(8*x^8) + (a^2*A
rcTanh[a*x]^2)/(3*x^6) - (a^4*ArcTanh[a*x]^2)/(4*x^4) + (4*a^8*Log[x])/63 -
 (2*a^8*Log[1 - a^2*x^2])/63
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^9}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^9} - \frac{2a^2\tanh^{-1}(ax)^2}{x^7} + \frac{a^4\tanh^{-1}(ax)^2}{x^5}\right)dx$$
$$= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x^7}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x^5}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^9}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2\tanh^{-1}(ax)^2}{3x^6} - \frac{a^4\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{4}a\int \frac{\tanh^{-1}(ax)}{x^8(1-a^2x^2)}\,dx - \frac{1}{3}$$
$$= -\frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2\tanh^{-1}(ax)^2}{3x^6} - \frac{a^4\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{4}a\int \frac{\tanh^{-1}(ax)}{x^8}\,dx + \frac{1}{4}a$$
$$= -\frac{a\tanh^{-1}(ax)}{28x^7} + \frac{2a^3\tanh^{-1}(ax)}{15x^5} - \frac{a^5\tanh^{-1}(ax)}{6x^3} - \frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2\tanh^{-1}(a}{3x^6}$$
$$= -\frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} + \frac{a^5\tanh^{-1}(ax)}{18x^3} - \frac{a^7\tanh^{-1}(ax)}{2x} + \frac{1}{4}a^8\tanh^{-1}$$
$$= -\frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax)}{36x^3} + \frac{a^7\tanh^{-1}(ax)}{6x} - \frac{1}{12}a^8\tanh^{-}$$
$$= -\frac{a^2}{168x^6} + \frac{41a^4}{1680x^4} - \frac{29a^6}{840x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax)}{36x^3} -$$
$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{13a^6}{252x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax)}{36x^3} - a^7$$
$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax)}{36x^3} - a^7$$
$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax)}{36x^3} - a^7$$

<u>Mathematica</u> [A] time = 0.07, size = 124, normalized size = 0.73

$$\frac{21(a^2x^2+3)(a^2x^2-1)^3\tanh^{-1}(ax)^2 + a^2x^2(32a^6x^6\log(x) + 5a^4x^4 + 6a^2x^2 - 16a^6x^6\log(1-a^2x^2) - 3) - 2ax}{504x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

```
[Out] (-2*a*x*(9 - 21*a^2*x^2 + 7*a^4*x^4 + 21*a^6*x^6)*ArcTanh[a*x] + 21*(-1 + a
^2*x^2)^3*(3 + a^2*x^2)*ArcTanh[a*x]^2 + a^2*x^2*(-3 + 6*a^2*x^2 + 5*a^4*x^
4 + 32*a^6*x^6*Log[x] - 16*a^6*x^6*Log[1 - a^2*x^2]))/(504*x^8)
```

<u>fricas</u> [A] time = 0.61, size = 148, normalized size = 0.87

$$-\frac{64a^8x^8\log(a^2x^2-1) - 128a^8x^8\log(x) - 20a^6x^6 - 24a^4x^4 + 12a^2x^2 - 21(a^8x^8 - 6a^4x^4 + 8a^2x^2 - 3)\log\left(-\frac{a}{a}\right.}{2016x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/2016*(64*a^8*x^8*log(a^2*x^2 - 1) - 128*a^8*x^8*log(x) - 20*a^6*x^6 - 24
*a^4*x^4 + 12*a^2*x^2 - 21*(a^8*x^8 - 6*a^4*x^4 + 8*a^2*x^2 - 3)*log(-(a*x
+ 1)/(a*x - 1))^2 + 4*(21*a^7*x^7 + 7*a^5*x^5 - 21*a^3*x^3 + 9*a*x)*log(-(a
*x + 1)/(a*x - 1)))/x^8
```

giac [B] time = 0.24, size = 651, normalized size = 3.83

$$\frac{2}{63}\left(2\,a^7\log\left(-\frac{ax+1}{ax-1}-1\right)-2\,a^7\log\left(-\frac{ax+1}{ax-1}\right)+\frac{84\left(\frac{(ax+1)^5a^7}{(ax-1)^5}-\frac{(ax+1)^4a^7}{(ax-1)^4}+\frac{(ax+1)^3a^7}{(ax-1)^3}\right)\log}{\frac{(ax+1)^8}{(ax-1)^8}+\frac{8\,(ax+1)^7}{(ax-1)^7}+\frac{28\,(ax+1)^6}{(ax-1)^6}+\frac{56\,(ax+1)^5}{(ax-1)^5}+\frac{70\,(ax+1)^4}{(ax-1)^4}+\frac{56\,(ax}{(ax-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="giac")
```

```
[Out] 2/63*(2*a^7*log(-(a*x + 1)/(a*x - 1) - 1) - 2*a^7*log(-(a*x + 1)/(a*x - 1))
 + 84*((a*x + 1)^5*a^7/(a*x - 1)^5 - (a*x + 1)^4*a^7/(a*x - 1)^4 + (a*x + 1
)^3*a^7/(a*x - 1)^3)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^8/(a*x - 1)^8 +
 8*(a*x + 1)^7/(a*x - 1)^7 + 28*(a*x + 1)^6/(a*x - 1)^6 + 56*(a*x + 1)^5/(a
*x - 1)^5 + 70*(a*x + 1)^4/(a*x - 1)^4 + 56*(a*x + 1)^3/(a*x - 1)^3 + 28*(a
*x + 1)^2/(a*x - 1)^2 + 8*(a*x + 1)/(a*x - 1) + 1) + 2*(28*(a*x + 1)^4*a^7/
(a*x - 1)^4 + 7*(a*x + 1)^3*a^7/(a*x - 1)^3 + 21*(a*x + 1)^2*a^7/(a*x - 1)^
2 + 7*(a*x + 1)*a^7/(a*x - 1) + a^7)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^7
/(a*x - 1)^7 + 7*(a*x + 1)^6/(a*x - 1)^6 + 21*(a*x + 1)^5/(a*x - 1)^5 + 35*
(a*x + 1)^4/(a*x - 1)^4 + 35*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x + 1)^2/(a*x
- 1)^2 + 7*(a*x + 1)/(a*x - 1) + 1) - (2*(a*x + 1)^5*a^7/(a*x - 1)^5 + 11*(
a*x + 1)^4*a^7/(a*x - 1)^4 + 6*(a*x + 1)^3*a^7/(a*x - 1)^3 + 11*(a*x + 1)^2
*a^7/(a*x - 1)^2 + 2*(a*x + 1)*a^7/(a*x - 1))/((a*x + 1)^6/(a*x - 1)^6 + 6*
(a*x + 1)^5/(a*x - 1)^5 + 15*(a*x + 1)^4/(a*x - 1)^4 + 20*(a*x + 1)^3/(a*x
- 1)^3 + 15*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1) + 1))*a
```

maple [A] time = 0.07, size = 253, normalized size = 1.49

$$-\frac{\operatorname{arctanh}(ax)^2}{8x^8}-\frac{a^4\operatorname{arctanh}(ax)^2}{4x^4}+\frac{a^2\operatorname{arctanh}(ax)^2}{3x^6}-\frac{a\operatorname{arctanh}(ax)}{28x^7}+\frac{a^3\operatorname{arctanh}(ax)}{12x^5}-\frac{a^5\operatorname{arctanh}(ax)}{36x^3}-\frac{a^7\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x)
```

```
[Out] -1/8*arctanh(a*x)^2/x^8-1/4*a^4*arctanh(a*x)^2/x^4+1/3*a^2*arctanh(a*x)^2/x
^6-1/28*a*arctanh(a*x)/x^7+1/12*a^3*arctanh(a*x)/x^5-1/36*a^5*arctanh(a*x)/
x^3-1/12*a^7*arctanh(a*x)/x-1/24*a^8*arctanh(a*x)*ln(a*x-1)+1/24*a^8*arctan
h(a*x)*ln(a*x+1)-1/96*a^8*ln(a*x-1)^2+1/48*a^8*ln(a*x-1)*ln(1/2+1/2*a*x)-1/
96*a^8*ln(a*x+1)^2-1/48*a^8*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/48*a^8*ln(-1
/2*a*x+1/2)*ln(a*x+1)-1/168*a^2/x^6+1/84*a^4/x^4+5/504*a^6/x^2+4/63*a^8*ln(
a*x)-2/63*a^8*ln(a*x-1)-2/63*a^8*ln(a*x+1)
```

maxima [A] time = 0.32, size = 204, normalized size = 1.20

$$\frac{1}{2016}\left(128\,a^6\log(x)-\frac{21\,a^6x^6\log(ax+1)^2+21\,a^6x^6\log(ax-1)^2+64\,a^6x^6\log(ax-1)-20\,a^4x^4-24\,a^2x^2-}{x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2016*(128*a^6*log(x) - (21*a^6*x^6*log(a*x + 1)^2 + 21*a^6*x^6*log(a*x -
1)^2 + 64*a^6*x^6*log(a*x - 1) - 20*a^4*x^4 - 24*a^2*x^2 - 2*(21*a^6*x^6*lo
g(a*x - 1) - 32*a^6*x^6)*log(a*x + 1) + 12)/x^6)*a^2 + 1/504*(21*a^7*log(a*
x + 1) - 21*a^7*log(a*x - 1) - 2*(21*a^6*x^6 + 7*a^4*x^4 - 21*a^2*x^2 + 9)/
x^7)*a*arctanh(a*x) - 1/24*(6*a^4*x^4 - 8*a^2*x^2 + 3)*arctanh(a*x)^2/x^8
```

mupad [B] time = 2.72, size = 357, normalized size = 2.10

$$\frac{4\,a^8\,\ln(x)}{63}+\frac{a^8\ln(a\,x+1)^2}{96}+\frac{a^8\ln(1-a\,x)^2}{96}-\frac{\ln(a\,x+1)^2}{32\,x^8}-\frac{\ln(1-a\,x)^2}{32\,x^8}-\frac{2\,a^8\,\ln\left(a^2\,x^2-1\right)}{63}-\frac{a^2}{168\,x^6}+\frac{a^4}{84\,x^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^9,x)
```

```
[Out] (4*a^8*log(x))/63 + (a^8*log(a*x + 1)^2)/96 + (a^8*log(1 - a*x)^2)/96 - log
(a*x + 1)^2/(32*x^8) - log(1 - a*x)^2/(32*x^8) - (2*a^8*log(a^2*x^2 - 1))/6
3 - a^2/(168*x^6) + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a^8*log(a*x + 1)*lo
g(1 - a*x))/48 + (log(a*x + 1)*log(1 - a*x))/(16*x^8) + (a^2*log(a*x + 1)^2
)/(12*x^6) - (a^4*log(a*x + 1)^2)/(16*x^4) + (a^2*log(1 - a*x)^2)/(12*x^6)
- (a^4*log(1 - a*x)^2)/(16*x^4) - (a*log(a*x + 1))/(56*x^7) + (a*log(1 - a*
x))/(56*x^7) + (a^3*log(a*x + 1))/(24*x^5) - (a^5*log(a*x + 1))/(72*x^3) -
(a^7*log(a*x + 1))/(24*x) - (a^3*log(1 - a*x))/(24*x^5) + (a^5*log(1 - a*x)
)/(72*x^3) + (a^7*log(1 - a*x))/(24*x) - (a^2*log(a*x + 1)*log(1 - a*x))/(6
*x^6) + (a^4*log(a*x + 1)*log(1 - a*x))/(8*x^4)
```

sympy [A] time = 3.99, size = 168, normalized size = 0.99

$$\begin{cases} \frac{4a^8 \log(x)}{63} - \frac{4a^8 \log\left(x - \frac{1}{a}\right)}{63} + \frac{a^8 \operatorname{atanh}^2(ax)}{24} - \frac{4a^8 \operatorname{atanh}(ax)}{63} - \frac{a^7 \operatorname{atanh}(ax)}{12x} + \frac{5a^6}{504x^2} - \frac{a^5 \operatorname{atanh}(ax)}{36x^3} - \frac{a^4 \operatorname{atanh}^2(ax)}{4x^4} + \frac{a^4}{84x^4} + \frac{a^3 \operatorname{ata}}{1} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**9,x)
```

```
[Out] Piecewise((4*a**8*log(x)/63 - 4*a**8*log(x - 1/a)/63 + a**8*atanh(a*x)**2/2
4 - 4*a**8*atanh(a*x)/63 - a**7*atanh(a*x)/(12*x) + 5*a**6/(504*x**2) - a**
5*atanh(a*x)/(36*x**3) - a**4*atanh(a*x)**2/(4*x**4) + a**4/(84*x**4) + a**
3*atanh(a*x)/(12*x**5) + a**2*atanh(a*x)**2/(3*x**6) - a**2/(168*x**6) - a*
atanh(a*x)/(28*x**7) - atanh(a*x)**2/(8*x**8), Ne(a, 0)), (0, True))
```

3.217 $\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=248

$$-\frac{1-a^2x^2}{20a}-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax$$

```
[Out] 1/20*(a^2*x^2-1)/a-x*arctanh(a*x)-1/10*x*(-a^2*x^2+1)*arctanh(a*x)+2/5*(-a^
2*x^2+1)*arctanh(a*x)^2/a+3/20*(-a^2*x^2+1)^2*arctanh(a*x)^2/a+8/15*arctanh
(a*x)^3/a+8/15*x*arctanh(a*x)^3+4/15*x*(-a^2*x^2+1)*arctanh(a*x)^3+1/5*x*(-
a^2*x^2+1)^2*arctanh(a*x)^3-8/5*arctanh(a*x)^2*ln(2/(-a*x+1))/a-1/2*ln(-a^2
*x^2+1)/a-8/5*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+4/5*polylog(3,1-2/(-a*
x+1))/a
```

Rubi [A] time = 0.25, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5944, 5910, 5984, 5918, 5948, 6058, 6610, 260, 5942}

$$\frac{4\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{5a}-\frac{8\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a}-\frac{1-a^2x^2}{20a}-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

```
[Out] -(1 - a^2*x^2)/(20*a) - x*ArcTanh[a*x] - (x*(1 - a^2*x^2)*ArcTanh[a*x])/10
+ (2*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(5*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x]^
2)/(20*a) + (8*ArcTanh[a*x]^3)/(15*a) + (8*x*ArcTanh[a*x]^3)/15 + (4*x*(1 -
 a^2*x^2)*ArcTanh[a*x]^3)/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/5 - (8*Ar
cTanh[a*x]^2*Log[2/(1 - a*x)])/(5*a) - Log[1 - a^2*x^2]/(2*a) - (8*ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a) + (4*PolyLog[3, 1 - 2/(1 - a*x)])/(
5*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
```

```
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3\,dx = \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{20a} + \frac{1}{5}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3 - \frac{3}{10}\int \left(1-a^2x^2\right) \mathrm{ta}$$

$$= -\frac{1-a^2x^2}{20a} - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a} + \frac{3\left(1-a\right.}{}$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}}{5a}$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}}{5a}$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}}{5a}$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}}{5a}$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}}{5a}$$

Mathematica [A] time = 0.60, size = 183, normalized size = 0.74

$$\frac{12a^5x^5\tanh^{-1}(ax)^3 + 9a^4x^4\tanh^{-1}(ax)^2 - 40a^3x^3\tanh^{-1}(ax)^3 + 6a^3x^3\tanh^{-1}(ax) + 3a^2x^2 - 30\log\left(1-a^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

```
[Out] (-3 + 3*a^2*x^2 - 66*a*x*ArcTanh[a*x] + 6*a^3*x^3*ArcTanh[a*x] + 33*ArcTanh
[a*x]^2 - 42*a^2*x^2*ArcTanh[a*x]^2 + 9*a^4*x^4*ArcTanh[a*x]^2 - 32*ArcTanh
[a*x]^3 + 60*a*x*ArcTanh[a*x]^3 - 40*a^3*x^3*ArcTanh[a*x]^3 + 12*a^5*x^5*Ar
cTanh[a*x]^3 - 96*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 30*Log[1 -
a^2*x^2] + 96*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 48*PolyLog[3,
 -E^(-2*ArcTanh[a*x])])/(60*a)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\mathrm{artanh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2x^2 - 1\right)^2 \mathrm{artanh}\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^3, x)
```

maple [C] time = 2.46, size = 883, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^3,x)
```

```
[Out] -2/3*a^2*arctanh(a*x)^3*x^3-7/10*a*arctanh(a*x)^2*x^2-1/20/a+1/a*ln(1+(a*x+
1)^2/(-a^2*x^2+1))+4/5/a*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+1/20*a*x^2-arct
anh(a*x)/a+4/5/a*arctanh(a*x)^2*ln(a*x-1)+4/5/a*arctanh(a*x)^2*ln(a*x+1)-8/
5/a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+11/20*arctanh(a*x)^2/a+8/
15*arctanh(a*x)^3/a+x*arctanh(a*x)^3-11/10*x*arctanh(a*x)-8/5/a*arctanh(a*x
)^2*ln(2)-8/5/a*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+2/5*I/a*arc
tanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*Pi-4/5*I/a*arcta
nh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-2/5*I/a*arctanh(a*x)^2*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi-2/5*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi+4/5*I/a*arctanh(a*x)^2*csgn(I/(1
+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+2/5*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi-4/5*
I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))*Pi-2/5*I/a*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi-2/5*I/a*arctanh(
a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*P
i+1/5*a^4*arctanh(a*x)^3*x^5+1/10*a^2*arctanh(a*x)*x^3+3/20*a^3*arctanh(a*x
)^2*x^4-4/5*I/a*arctanh(a*x)^2*Pi
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(36\,a^5x^5-45\,a^4x^4-140\,a^3x^3+210\,a^2x^2+480\,ax-60\left(3\,a^5x^5-10\,a^3x^3+15\,ax+8\right)\log\left(ax+1\right)\right)\log\left(-ax+\right.}{2400\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2400*(36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*a^2*x^2 + 480*a*x - 60
*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1))*log(-a*x + 1)^2/a - 1/
8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/a -
 1/1440000*(288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x + 1
) - 6)*(a*x - 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 12*log
(-a*x + 1) - 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2
+ 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*log(-a*x
+ 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3 - 3*log
(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/432*(4*(9*log(-a*x + 1
)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x
 + 1)^3 - 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-
a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a - 1/8*in
tegrate(-1/150*(150*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*l
og(a*x + 1)^3 + (36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*a^2*x^2 - 450*
(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 + 480*
a*x - 60*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1))*log(-a*x + 1))
/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3*(a^2*x^2 - 1)^2,x)
```

```
[Out] int(atanh(a*x)^3*(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax-1)^2 (ax+1)^2 \operatorname{atanh}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3, x)
```

3.218 $\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
[Out] Defer[Int][(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx = \int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
[Out] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^5 - 2\,a^2x^3 + x}{\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
[Out] integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2 x}{\text{artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

maple [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a^{2}x^{2}+1\right)^{2}}{\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2}x^{2}-1\right)^{2}x}{\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(a^2\,x^2-1\right)^2}{\mathrm{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 - 1)^2)/atanh(a*x),x)
```

```
[Out] int((x*(a^2*x^2 - 1)^2)/atanh(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(ax-1\right)^{2}\left(ax+1\right)^{2}}{\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

3.219 $\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^2/arctanh(a*x),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx = \int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

```
[Out] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

fricas [A] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^4-2\,a^2x^2+1}{\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2x^2-1)^2}{\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

maple [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^2}{\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a^2\,x^2-1\right)^2}{\operatorname{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/atanh(a*x),x)
```

```
[Out] int((a^2*x^2 - 1)^2/atanh(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2\,(ax+1)^2}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

3.220 $\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
[Out] Defer[Int][(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx = \int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
[Out] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^4-2a^2x^2+1}{x\,\text{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="fricas")
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2x^2-1)^2}{x\,\text{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

maple [A] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^2}{x \operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
[Out] int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2}{x \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="maxima")
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a^2\,x^2-1\right)^2}{x\,\operatorname{atanh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/(x*atanh(a*x)),x)
[Out] int((a^2*x^2 - 1)^2/(x*atanh(a*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2(ax+1)^2}{x \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/x/atanh(a*x),x)
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)), x)
```

3.221 $\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} dx = \int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^5 - 2\,a^2x^3 + x}{\text{artanh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2 x}{\text{artanh}\,(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x)^2, x)
```

maple [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a^2x^2+1\right)^2}{\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
[Out] int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(a^6x^7-3\,a^4x^5+3\,a^2x^3-x\right)}{a\log(ax+1)-a\log(-ax+1)}+\int -\frac{2\left(7\,a^6x^6-15\,a^4x^4+9\,a^2x^2-1\right)}{a\log(ax+1)-a\log(-ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
[Out] 2*(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)/(a*log(a*x + 1) - a*log(-a*x + 1))
+ integrate(-2*(7*a^6*x^6 - 15*a^4*x^4 + 9*a^2*x^2 - 1)/(a*log(a*x + 1) - a
*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(a^2\,x^2-1\right)^2}{\operatorname{atanh}(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 - 1)^2)/atanh(a*x)^2,x)
[Out] int((x*(a^2*x^2 - 1)^2)/atanh(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(ax-1)^2(ax+1)^2}{\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2/atanh(a*x)**2,x)
[Out] Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

3.222 $\int \frac{\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]
[Out] Defer[Int][(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2}\,dx=\int \frac{\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]
[Out] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^4-2\,a^2x^2+1}{\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2}{\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x)^2, x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{2}}{\operatorname{arctanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(a^{6}x^{6}-3\,a^{4}x^{4}+3\,a^{2}x^{2}-1\right)}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}+\int -\frac{12\left(a^{5}x^{5}-2\,a^{3}x^{3}+ax\right)}{\log\left(ax+1\right)-\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*log(a*x + 1) - a*log(-a*x + 1))
+ integrate(-12*(a^5*x^5 - 2*a^3*x^3 + a*x)/(log(a*x + 1) - log(-a*x + 1)),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a^{2}\,x^{2}-1\right)^{2}}{\operatorname{atanh}\left(a\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/atanh(a*x)^2,x)
```

```
[Out] int((a^2*x^2 - 1)^2/atanh(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^{2}\left(ax+1\right)^{2}}{\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

3.223 $\int \frac{\left(1-a^2x^2\right)^2}{x \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(1-a^2x^2\right)^2}{x \tanh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-a^2x^2\right)^2}{x \tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
[Out] Defer[Int][(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2}{x \tanh^{-1}(ax)^2} \, dx = \int \frac{\left(1-a^2x^2\right)^2}{x \tanh^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\left(1-a^2x^2\right)^2}{x \tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
[Out] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^4 - 2\,a^2x^2 + 1}{x\,\text{artanh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-1\right)^2}{x\,\text{artanh}\,(ax)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)^2), x)
```

maple [A] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{2}}{x \operatorname{arctanh}\left(ax\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(a^{6}x^{6}-3\,a^{4}x^{4}+3\,a^{2}x^{2}-1\right)}{ax\log\left(ax+1\right)-ax\log\left(-ax+1\right)}+\int -\frac{2\left(5\,a^{6}x^{6}-9\,a^{4}x^{4}+3\,a^{2}x^{2}+1\right)}{ax^{2}\log\left(ax+1\right)-ax^{2}\log\left(-ax+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*x*log(a*x + 1) - a*x*log(-a*x +
1)) + integrate(-2*(5*a^6*x^6 - 9*a^4*x^4 + 3*a^2*x^2 + 1)/(a*x^2*log(a*x +
 1) - a*x^2*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a^2\,x^2-1\right)}^2}{x\,{\mathrm{atanh}\left(a\,x\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/(x*atanh(a*x)^2),x)
```

```
[Out] int((a^2*x^2 - 1)^2/(x*atanh(a*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^{2}\left(ax+1\right)^{2}}{x \operatorname{atanh}^{2}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/x/atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)**2), x)
```

3.224 $\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=144

$$\frac{\left(1-a^2x^2\right)^3}{42a}+\frac{3\left(1-a^2x^2\right)^2}{70a}+\frac{4\left(1-a^2x^2\right)}{35a}+\frac{8\log\left(1-a^2x^2\right)}{35a}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(a$$

```
[Out] 4/35*(-a^2*x^2+1)/a+3/70*(-a^2*x^2+1)^2/a+1/42*(-a^2*x^2+1)^3/a+16/35*x*arc
tanh(a*x)+8/35*x*(-a^2*x^2+1)*arctanh(a*x)+6/35*x*(-a^2*x^2+1)^2*arctanh(a*
x)+1/7*x*(-a^2*x^2+1)^3*arctanh(a*x)+8/35*ln(-a^2*x^2+1)/a
```

Rubi [A] time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5942, 5910, 260}

$$\frac{\left(1-a^2x^2\right)^3}{42a}+\frac{3\left(1-a^2x^2\right)^2}{70a}+\frac{4\left(1-a^2x^2\right)}{35a}+\frac{8\log\left(1-a^2x^2\right)}{35a}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (4*(1 - a^2*x^2))/(35*a) + (3*(1 - a^2*x^2)^2)/(70*a) + (1 - a^2*x^2)^3/(42
*a) + (16*x*ArcTanh[a*x])/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x])/35 + (6*x*(
1 - a^2*x^2)^2*ArcTanh[a*x])/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x])/7 + (8*L
og[1 - a^2*x^2])/(35*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{1}{7}x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax) + \frac{6}{7}\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$$
$$= \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{6}{35}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax) + \frac{1}{7}x\left(1-a^2x^2\right)^3 \text{ta}$$
$$= \frac{4\left(1-a^2x^2\right)}{35a} + \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{8}{35}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{6}{35}$$
$$= \frac{4\left(1-a^2x^2\right)}{35a} + \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{16}{35}x\tanh^{-1}(ax) + \frac{8}{35}x\left(1-a^2x^2\right.$$
$$= \frac{4\left(1-a^2x^2\right)}{35a} + \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{16}{35}x\tanh^{-1}(ax) + \frac{8}{35}x\left(1-a^2x^2\right.$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.55

$$\frac{-5a^6x^6 + 24a^4x^4 - 57a^2x^2 + 48\log\left(1-a^2x^2\right) - 6ax\left(5a^6x^6 - 21a^4x^4 + 35a^2x^2 - 35\right)\tanh^{-1}(ax)}{210a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (-57*a^2*x^2 + 24*a^4*x^4 - 5*a^6*x^6 - 6*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x^
4 + 5*a^6*x^6)*ArcTanh[a*x] + 48*Log[1 - a^2*x^2])/(210*a)
```

fricas [A] time = 0.52, size = 88, normalized size = 0.61

$$-\frac{5\,a^6x^6 - 24\,a^4x^4 + 57\,a^2x^2 + 3\left(5\,a^7x^7 - 21\,a^5x^5 + 35\,a^3x^3 - 35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) - 48\,\log\left(a^2x^2-1\right)}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*x^6 - 24*a^4*x^4 + 57*a^2*x^2 + 3*(5*a^7*x^7 - 21*a^5*x^5 + 3
5*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 48*log(a^2*x^2 - 1))/a
```

giac [B] time = 0.19, size = 303, normalized size = 2.10

$$\frac{8}{105}a\left(\frac{6\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2} - \frac{6\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2} - \frac{\frac{6\,(ax+1)^5}{(ax-1)^5} - \frac{33\,(ax+1)^4}{(ax-1)^4} + \frac{74\,(ax+1)^3}{(ax-1)^3} - \frac{33\,(ax+1)^2}{(ax-1)^2} + \frac{6\,(ax+1)}{ax-1}}{a^2\left(\frac{ax+1}{ax-1}-1\right)^6} - \frac{6\left(\frac{35\,(ax+\right.}{(ax-1)}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 8/105*a*(6*log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - 6*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^2 - (6*(a*x + 1)^5/(a*x - 1)^5 - 33*(a*x + 1)^4/(a*x - 1)^4 +
74*(a*x + 1)^3/(a*x - 1)^3 - 33*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x
- 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^6) - 6*(35*(a*x + 1)^3/(a*x - 1)^3 - 2
```

```
1*(a*x + 1)^2/(a*x - 1)^2 + 7*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(
a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)
/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^7))
```

maple [A] time = 0.03, size = 88, normalized size = 0.61

$$-\frac{a^6 \operatorname{arctanh}(ax)\, x^7}{7}+\frac{3a^4 \operatorname{arctanh}(ax)\, x^5}{5}-a^2 \operatorname{arctanh}(ax)\, x^3+x \operatorname{arctanh}(ax)-\frac{a^5x^6}{42}+\frac{4x^4a^3}{35}-\frac{19a\, x^2}{70}+\frac{8\ln(ax-1}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x),x)
```

```
[Out] -1/7*a^6*arctanh(a*x)*x^7+3/5*a^4*arctanh(a*x)*x^5-a^2*arctanh(a*x)*x^3+x*a
rctanh(a*x)-1/42*a^5*x^6+4/35*x^4*a^3-19/70*a*x^2+8/35/a*ln(a*x-1)+8/35/a*l
n(a*x+1)
```

maxima [A] time = 0.31, size = 82, normalized size = 0.57

$$-\frac{1}{210}\left(5\,a^4x^6-24\,a^2x^4+57\,x^2-\frac{48\,\log(ax+1)}{a^2}-\frac{48\,\log(ax-1)}{a^2}\right)a-\frac{1}{35}\left(5\,a^6x^7-21\,a^4x^5+35\,a^2x^3-35\,x\right)\text{art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 - 48*log(a*x
- 1)/a^2)*a - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3 - 35*x)*arctanh(a*x
)
```

mupad [B] time = 0.94, size = 80, normalized size = 0.56

$$x\,\mathrm{atanh}(a\,x)-\frac{19\,a\,x^2}{70}+\frac{8\,\ln\left(a^2\,x^2-1\right)}{35\,a}+\frac{4\,a^3\,x^4}{35}-\frac{a^5\,x^6}{42}-a^2\,x^3\,\mathrm{atanh}(a\,x)+\frac{3\,a^4\,x^5\,\mathrm{atanh}(a\,x)}{5}-\frac{a^6\,x^7\,\mathrm{atanh}(a\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)*(a^2*x^2 - 1)^3,x)
```

```
[Out] x*atanh(a*x) - (19*a*x^2)/70 + (8*log(a^2*x^2 - 1))/(35*a) + (4*a^3*x^4)/35
 - (a^5*x^6)/42 - a^2*x^3*atanh(a*x) + (3*a^4*x^5*atanh(a*x))/5 - (a^6*x^7*
atanh(a*x))/7
```

sympy [A] time = 2.04, size = 97, normalized size = 0.67

$$\begin{cases} -\frac{a^6x^7\operatorname{atanh}(ax)}{7}-\frac{a^5x^6}{42}+\frac{3a^4x^5\operatorname{atanh}(ax)}{5}+\frac{4a^3x^4}{35}-a^2x^3\operatorname{atanh}(ax)-\frac{19ax^2}{70}+x\operatorname{atanh}(ax)+\frac{16\log\left(x-\frac{1}{a}\right)}{35a}+\frac{16\operatorname{atanh}(ax)}{35a} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x),x)
```

```
[Out] Piecewise((-a**6*x**7*atanh(a*x)/7 - a**5*x**6/42 + 3*a**4*x**5*atanh(a*x)/
5 + 4*a**3*x**4/35 - a**2*x**3*atanh(a*x) - 19*a*x**2/70 + x*atanh(a*x) + 1
6*log(x - 1/a)/(35*a) + 16*atanh(a*x)/(35*a), Ne(a, 0)), (0, True))
```

3.225 $\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=227

$$-\frac{1}{105}a^4x^5+\frac{19a^2x^3}{315}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{8}{35}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+$$

```
[Out] -38/105*x+19/315*a^2*x^3-1/105*a^4*x^5+8/35*(-a^2*x^2+1)*arctanh(a*x)/a+3/3
5*(-a^2*x^2+1)^2*arctanh(a*x)/a+1/21*(-a^2*x^2+1)^3*arctanh(a*x)/a+16/35*ar
ctanh(a*x)^2/a+16/35*x*arctanh(a*x)^2+8/35*x*(-a^2*x^2+1)*arctanh(a*x)^2+6/
35*x*(-a^2*x^2+1)^2*arctanh(a*x)^2+1/7*x*(-a^2*x^2+1)^3*arctanh(a*x)^2-32/3
5*arctanh(a*x)*ln(2/(-a*x+1))/a-16/35*polylog(2,1-2/(-a*x+1))/a
```

Rubi [A] time = 0.17, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5944, 5910, 5984, 5918, 2402, 2315, 8, 194}

$$-\frac{16\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a}-\frac{1}{105}a^4x^5+\frac{19a^2x^3}{315}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{8}{35}x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

```
[Out] (-38*x)/105 + (19*a^2*x^3)/315 - (a^4*x^5)/105 + (8*(1 - a^2*x^2)*ArcTanh[a
*x])/(35*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x])/(35*a) + ((1 - a^2*x^2)^3*Ar
cTanh[a*x])/(21*a) + (16*ArcTanh[a*x]^2)/(35*a) + (16*x*ArcTanh[a*x]^2)/35
+ (8*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]
^2)/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2)/7 - (32*ArcTanh[a*x]*Log[2/(1 -
 a*x)])/(35*a) - (16*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2\,dx &= \frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{21a} + \frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2 - \frac{1}{21}\int\left(1-a^2x^2\right)^2\,dx + \cdots \\
&= \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{35a} + \frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{21a} + \frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax \cdots \\
&= -\frac{2x}{15} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{35a} + \cdots \\
&= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{35a} + \cdots \\
&= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{35a} + \cdots \\
&= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{35a} + \cdots \\
&= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{35a} + \cdots \\
&= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{35a} + \cdots
\end{aligned}$$

Mathematica [A] time = 1.22, size = 124, normalized size = 0.55

$$-\frac{3a^5x^5 - 19a^3x^3 + 9(ax-1)^4\left(5a^3x^3+20a^2x^2+29ax+16\right)\tanh^{-1}(ax)^2 + 3\tanh^{-1}(ax)\left(5a^6x^6 - 24a^4x^4 + 57a^2 \cdots\right.}{315a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

```
[Out] -1/315*(114*a*x - 19*a^3*x^3 + 3*a^5*x^5 + 9*(-1 + a*x)^4*(16 + 29*a*x + 20
*a^2*x^2 + 5*a^3*x^3)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-38 + 57*a^2*x^2 - 2
4*a^4*x^4 + 5*a^6*x^6 + 96*Log[1 + E^(-2*ArcTanh[a*x])]) - 144*PolyLog[2, -
E^(-2*ArcTanh[a*x])])/a
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\operatorname{artanh}\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2x^2 - 1\right)^3 \operatorname{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^2, x)
```

maple [A] time = 0.06, size = 250, normalized size = 1.10

$$-\frac{a^6 \operatorname{arctanh}\left(ax\right)^2 x^7}{7}+\frac{3a^4 \operatorname{arctanh}\left(ax\right)^2 x^5}{5}-a^2 \operatorname{arctanh}\left(ax\right)^2 x^3+x \operatorname{arctanh}\left(ax\right)^2-\frac{a^5 \operatorname{arctanh}\left(ax\right) x^6}{21}+\frac{8a^3 \operatorname{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x)^2,x)
```

```
[Out] -1/7*a^6*arctanh(a*x)^2*x^7+3/5*a^4*arctanh(a*x)^2*x^5-a^2*arctanh(a*x)^2*x
^3+x*arctanh(a*x)^2-1/21*a^5*arctanh(a*x)*x^6+8/35*a^3*arctanh(a*x)*x^4-19/
35*a*arctanh(a*x)*x^2+16/35/a*arctanh(a*x)*ln(a*x-1)+16/35/a*arctanh(a*x)*l
n(a*x+1)+4/35/a*ln(a*x-1)^2-16/35/a*dilog(1/2+1/2*a*x)-8/35/a*ln(a*x-1)*ln(
1/2+1/2*a*x)-4/35/a*ln(a*x+1)^2+8/35/a*ln(-1/2*a*x+1/2)*ln(a*x+1)-8/35/a*ln
(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/105*a^4*x^5+19/315*x^3*a^2-38/105*x-19/105
/a*ln(a*x-1)+19/105/a*ln(a*x+1)
```

maxima [A] time = 0.33, size = 199, normalized size = 0.88

$$-\frac{1}{315}a^2\left(\frac{3\,a^5x^5 - 19\,a^3x^3 + 114\,ax + 36\,\log\left(ax+1\right)^2 - 72\,\log\left(ax+1\right)\log\left(ax-1\right) - 36\,\log\left(ax-1\right)^2 + 57\,\mathrm{lo}}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/315*a^2*((3*a^5*x^5 - 19*a^3*x^3 + 114*a*x + 36*log(a*x + 1)^2 - 72*log(
a*x + 1)*log(a*x - 1) - 36*log(a*x - 1)^2 + 57*log(a*x - 1))/a^3 + 144*(log
(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 - 57*log(a*x + 1)
/a^3) - 1/105*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 - 48*l
og(a*x - 1)/a^2)*a*arctanh(a*x) - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3
 - 35*x)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \operatorname{atanh}\left(a\,x\right)^2\left(a^2\,x^2-1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2*(a^2*x^2 - 1)^3,x)
```

```
[Out] -int(atanh(a*x)^2*(a^2*x^2 - 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int 3a^2x^2\operatorname{atanh}^2(ax)\,dx-\int\left(-3a^4x^4\operatorname{atanh}^2(ax)\right)dx-\int a^6x^6\operatorname{atanh}^2(ax)\,dx-\int\left(-\operatorname{atanh}^2(ax)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x)**2,x)
```

```
[Out] -Integral(3*a**2*x**2*atanh(a*x)**2, x) - Integral(-3*a**4*x**4*atanh(a*x)*
*2, x) - Integral(a**6*x**6*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2, x)
```

3.226 $\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3\,dx$

Optimal. Leaf size=338

$$-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{13\left(1-a^2x^2\right)}{210a}-\frac{7\log\left(1-a^2x^2\right)}{15a}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3+\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{14a}+\frac{6}{35}x\left(1\right.$$

```
[Out] -13/210*(-a^2*x^2+1)/a-1/140*(-a^2*x^2+1)^2/a-14/15*x*arctanh(a*x)-13/105*x
*(-a^2*x^2+1)*arctanh(a*x)-1/35*x*(-a^2*x^2+1)^2*arctanh(a*x)+12/35*(-a^2*x
^2+1)*arctanh(a*x)^2/a+9/70*(-a^2*x^2+1)^2*arctanh(a*x)^2/a+1/14*(-a^2*x^2+
1)^3*arctanh(a*x)^2/a+16/35*arctanh(a*x)^3/a+16/35*x*arctanh(a*x)^3+8/35*x*
(-a^2*x^2+1)*arctanh(a*x)^3+6/35*x*(-a^2*x^2+1)^2*arctanh(a*x)^3+1/7*x*(-a^
2*x^2+1)^3*arctanh(a*x)^3-48/35*arctanh(a*x)^2*ln(2/(-a*x+1))/a-7/15*ln(-a^
2*x^2+1)/a-48/35*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+24/35*polylog(3,1-2
/(-a*x+1))/a
```

Rubi [A] time = 0.33, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5944, 5910, 5984, 5918, 5948, 6058, 6610, 260, 5942}

$$\frac{24\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{35a}-\frac{48\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{13\left(1-a^2x^2\right)}{210a}-\frac{7\log\left(1-a^2x^2\right)}{15a}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```

```
[Out] (-13*(1 - a^2*x^2))/(210*a) - (1 - a^2*x^2)^2/(140*a) - (14*x*ArcTanh[a*x])
/15 - (13*x*(1 - a^2*x^2)*ArcTanh[a*x])/105 - (x*(1 - a^2*x^2)^2*ArcTanh[a*
x])/35 + (12*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(35*a) + (9*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/(70*a) + ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(14*a) + (16*ArcTanh[
a*x]^3)/(35*a) + (16*x*ArcTanh[a*x]^3)/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x]
^3)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/35 + (x*(1 - a^2*x^2)^3*ArcTa
nh[a*x]^3)/7 - (48*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(35*a) - (7*Log[1 - a^2
*x^2])/(15*a) - (48*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a) + (24*
PolyLog[3, 1 - 2/(1 - a*x)])/(35*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
```

```
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3\,dx = \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{14a} + \frac{1}{7}x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3 - \frac{1}{7}\int \left(1-a^2x^2\right)^2 \tan$$

$$= -\frac{\left(1-a^2x^2\right)^2}{140a} - \frac{1}{35}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax) + \frac{9\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{70a} + \frac{(1}{}$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax) - \frac{1}{35}x\left(1-a^2x^2\right)^2$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax$$

Mathematica [A] time = 1.10, size = 231, normalized size = 0.68

$$-\frac{60a^7x^7\tanh^{-1}(ax)^3 + 30a^6x^6\tanh^{-1}(ax)^2 - 252a^5x^5\tanh^{-1}(ax)^3 + 12a^5x^5\tanh^{-1}(ax) + 3a^4x^4 - 144a^4x^4\,\mathrm{ta}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```

```
[Out] -1/420*(29 - 32*a^2*x^2 + 3*a^4*x^4 + 456*a*x*ArcTanh[a*x] - 76*a^3*x^3*Arc
Tanh[a*x] + 12*a^5*x^5*ArcTanh[a*x] - 228*ArcTanh[a*x]^2 + 342*a^2*x^2*ArcT
anh[a*x]^2 - 144*a^4*x^4*ArcTanh[a*x]^2 + 30*a^6*x^6*ArcTanh[a*x]^2 + 192*A
rcTanh[a*x]^3 - 420*a*x*ArcTanh[a*x]^3 + 420*a^3*x^3*ArcTanh[a*x]^3 - 252*a
^5*x^5*ArcTanh[a*x]^3 + 60*a^7*x^7*ArcTanh[a*x]^3 + 576*ArcTanh[a*x]^2*Log[
1 + E^(-2*ArcTanh[a*x])] + 196*Log[1 - a^2*x^2] - 576*ArcTanh[a*x]*PolyLog[
2, -E^(-2*ArcTanh[a*x])] - 288*PolyLog[3, -E^(-2*ArcTanh[a*x])])/a
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{artanh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2x^2 - 1\right)^3 \text{artanh}\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^3, x)
```

maple [C] time = 5.40, size = 932, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x)^3,x)
```

```
[Out] -a^2*arctanh(a*x)^3*x^3-57/70*a*arctanh(a*x)^2*x^2-1/35*a^4*arctanh(a*x)*x^
5-29/420/a+14/15/a*ln(1+(a*x+1)^2/(-a^2*x^2+1))+24/35/a*polylog(3,-(a*x+1)^
2/(-a^2*x^2+1))+8/105*a*x^2-14/15*arctanh(a*x)/a+24/35/a*arctanh(a*x)^2*ln(
a*x-1)+24/35/a*arctanh(a*x)^2*ln(a*x+1)-48/35/a*arctanh(a*x)^2*ln((a*x+1)/(
-a^2*x^2+1)^(1/2))-1/140*x^4*a^3+19/35*arctanh(a*x)^2/a+16/35*arctanh(a*x)^
3/a+x*arctanh(a*x)^3-38/35*x*arctanh(a*x)-48/35/a*arctanh(a*x)^2*ln(2)-48/3
5/a*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-12/35*I/a*Pi*csgn(I*(a*
x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2+12/
35*I/a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*
x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-12/35*I/a*arctanh(a*x)^2*Pi*csgn(I/(
1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))^2-24/35*I/a*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))^2-24/35*I/a*Pi*arctanh(a*x)^2+12/35*I/a*Pi*csg
n(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^2+3/5*a^4*arctanh(
a*x)^3*x^5+19/105*a^2*arctanh(a*x)*x^3+12/35*a^3*arctanh(a*x)^2*x^4+24/35*I
/a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-24/35*I/a*Pi*csgn
(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2-12/35*I/a*Pi*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2-12/35*I/a*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2-1/7*a^6*arctanh(a*x)^3*x^7-1/1
4*a^5*arctanh(a*x)^2*x^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/19600*(150*a^7*x^7 - 175*a^6*x^6 - 672*a^5*x^5 + 840*a^4*x^4 + 1330*a^3*x
^3 - 1995*a^2*x^2 - 3360*a*x - 210*(5*a^7*x^7 - 21*a^5*x^5 + 35*a^3*x^3 - 3
5*a*x - 16)*log(a*x + 1))*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*log(
-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/a + 1/691488000*(36000*(343*lo
g(-a*x + 1)^3 - 147*log(-a*x + 1)^2 + 42*log(-a*x + 1) - 6)*(a*x - 1)^7 + 2
401000*(36*log(-a*x + 1)^3 - 18*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 1)*(a*x
 - 1)^6 + 2074464*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x +
 1) - 6)*(a*x - 1)^5 + 13505625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 +
12*log(-a*x + 1) - 3)*(a*x - 1)^4 + 48020000*(9*log(-a*x + 1)^3 - 9*log(-a*
x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 64827000*(4*log(-a*x + 1)^3 -
 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 86436000*(log(-a*x
+ 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a - 1/480000*(
288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x + 1) - 6)*(a*x
- 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 12*log(-a*x + 1) -
 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x
 + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*log(-a*x + 1)^2 + 6*l
og(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2
 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/288*(4*(9*log(-a*x + 1)^3 - 9*log(
-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x + 1)^3 - 6*
log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-a*x + 1)^3 -
 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/8*integrate(-1/1
225*(1225*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x^
```

```
2 - a*x + 1)*log(a*x + 1)^3 + (150*a^7*x^7 - 175*a^6*x^6 - 672*a^5*x^5 + 84
0*a^4*x^4 + 1330*a^3*x^3 - 1995*a^2*x^2 - 3675*(a^7*x^7 - a^6*x^6 - 3*a^5*x
^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x^2 - a*x + 1)*log(a*x + 1)^2 - 3360*a*x
 - 210*(5*a^7*x^7 - 21*a^5*x^5 + 35*a^3*x^3 - 35*a*x - 16)*log(a*x + 1))*lo
g(-a*x + 1))/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3*(a^2*x^2 - 1)^3,x)
```

```
[Out] -int(atanh(a*x)^3*(a^2*x^2 - 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int 3a^2x^2\operatorname{atanh}^3(ax)\,dx-\int\left(-3a^4x^4\operatorname{atanh}^3(ax)\right)dx-\int a^6x^6\operatorname{atanh}^3(ax)\,dx-\int\left(-\operatorname{atanh}^3(ax)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x)**3,x)
```

```
[Out] -Integral(3*a**2*x**2*atanh(a*x)**3, x) - Integral(-3*a**4*x**4*atanh(a*x)*
*3, x) - Integral(a**6*x**6*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3, x)
```

3.227 $\int \frac{x^3 \tanh^{-1}(ax)}{1-a^2x^2} dx$

Optimal. Leaf size=87

$$\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^4}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}-\frac{x}{2a^3}-\frac{x^2\tanh^{-1}(ax)}{2a^2}$$

```
[Out] -1/2*x/a^3+1/2*arctanh(a*x)/a^4-1/2*x^2*arctanh(a*x)/a^2-1/2*arctanh(a*x)^2
/a^4+arctanh(a*x)*ln(2/(-a*x+1))/a^4+1/2*polylog(2,1-2/(-a*x+1))/a^4
```

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5980, 5916, 321, 206, 5984, 5918, 2402, 2315}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{x}{2a^3}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -x/(2*a^3) + ArcTanh[a*x]/(2*a^4) - (x^2*ArcTanh[a*x])/(2*a^2) - ArcTanh[a*
x]^2/(2*a^4) + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4 + PolyLog[2, 1 - 2/(1 -
a*x)]/(2*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3\tanh^{-1}(ax)}{1-a^2x^2}\,dx = -\frac{\int x\tanh^{-1}(ax)\,dx}{a^2} + \frac{\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^3} + \frac{\int \frac{x^2}{1-a^2x^2}\,dx}{2a}$$

$$= -\frac{x}{2a^3} - \frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} + \frac{\int \frac{1}{1-a^2x^2}\,dx}{2a^3} - \frac{\int \frac{\log(\ldots}{1-a\ldots}}{a\ldots}$$

$$= -\frac{x}{2a^3} + \frac{\tanh^{-1}(ax)}{2a^4} - \frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} + \frac{\text{Subst}\ldots}{\ldots}$$

$$= -\frac{x}{2a^3} + \frac{\tanh^{-1}(ax)}{2a^4} - \frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} + \frac{\text{Li}_2\left(1\ldots\right.}{2\ldots}$$

Mathematica [A] time = 0.14, size = 60, normalized size = 0.69

$$\frac{\tanh^{-1}(ax)\left(-a^2x^2+2\log\left(e^{-2\tanh^{-1}(ax)}+1\right)+1\right)-\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-ax+\tanh^{-1}(ax)^2}{2a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] (-(a*x) + ArcTanh[a*x]^2 + ArcTanh[a*x]*(1 - a^2*x^2 + 2*Log[1 + E^(-2*ArcT
anh[a*x])]) - PolyLog[2, -E^(-2*ArcTanh[a*x])])/(2*a^4)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^3\,\text{artanh}\,(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3 \operatorname{artanh}(ax)}{a^2x^2 - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

maple [B] time = 0.06, size = 165, normalized size = 1.90

$$-\frac{x^2 \operatorname{arctanh}(ax)}{2a^2} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2a^4} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2a^4} - \frac{x}{2a^3} - \frac{\ln(ax-1)}{4a^4} + \frac{\ln(ax+1)}{4a^4} - \frac{\ln(ax-1)^2}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1),x)
```

```
[Out] -1/2*x^2*arctanh(a*x)/a^2-1/2/a^4*arctanh(a*x)*ln(a*x-1)-1/2/a^4*arctanh(a*
x)*ln(a*x+1)-1/2*x/a^3-1/4/a^4*ln(a*x-1)+1/4/a^4*ln(a*x+1)-1/8/a^4*ln(a*x-1
)^2+1/2/a^4*dilog(1/2+1/2*a*x)+1/4/a^4*ln(a*x-1)*ln(1/2+1/2*a*x)+1/8/a^4*ln
(a*x+1)^2-1/4/a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/4/a^4*ln(-1/2*a*x+1/2)*ln(1/
2+1/2*a*x)
```

maxima [A] time = 0.32, size = 120, normalized size = 1.38

$$-\frac{1}{8}a\left(\frac{4\,ax - \log(ax+1)^2 + 2\,\log(ax+1)\log(ax-1) + \log(ax-1)^2 + 2\,\log(ax-1)}{a^5} - \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*a*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1
)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2
*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) - 1/2*(x^2/a^2 + log(a^2*x^2 - 1)/a^
4)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3 \operatorname{atanh}(a\,x)}{a^2\,x^2 - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x))/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^3*atanh(a*x))/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3 \operatorname{atanh}(ax)}{a^2x^2 - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**3*atanh(a*x)/(a**2*x**2 - 1), x)
```

3.228 $\int \frac{x^2 \tanh^{-1}(ax)}{1-a^2x^2} \, dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}(ax)^2}{2a^3} - \frac{x\tanh^{-1}(ax)}{a^2} - \frac{\log\left(1-a^2x^2\right)}{2a^3}$$

```
[Out] -x*arctanh(a*x)/a^2+1/2*arctanh(a*x)^2/a^3-1/2*ln(-a^2*x^2+1)/a^3
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5980, 5910, 260, 5948}

$$-\frac{\log\left(1-a^2x^2\right)}{2a^3} + \frac{\tanh^{-1}(ax)^2}{2a^3} - \frac{x\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\tanh^{-1}(ax)}{1-a^2x^2}\,dx &= -\frac{\int \tanh^{-1}(ax)\,dx}{a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{x\tanh^{-1}(ax)}{a^2} + \frac{\tanh^{-1}(ax)^2}{2a^3} + \frac{\int \frac{x}{1-a^2x^2}\,dx}{a}\\
&= -\frac{x\tanh^{-1}(ax)}{a^2} + \frac{\tanh^{-1}(ax)^2}{2a^3} - \frac{\log\left(1-a^2x^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^2}{2a^3}-\frac{x\tanh^{-1}(ax)}{a^2}-\frac{\log\left(1-a^2x^2\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3)
```

fricas [A] time = 0.55, size = 56, normalized size = 1.33

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,\log\left(a^2x^2-1\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(a*x - 1))^2 + 4*log
(a^2*x^2 - 1))/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2\operatorname{artanh}(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)/(a^2*x^2 - 1), x)
```

maple [B] time = 0.05, size = 145, normalized size = 3.45

$$-\frac{x\operatorname{arctanh}(ax)}{a^2}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2a^3}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2a^3}-\frac{\ln(ax-1)^2}{8a^3}+\frac{\ln(ax-1)\ln\left(\frac{1}{2}+\frac{ax}{2}\right)}{4a^3}-\frac{\ln(a}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1),x)
```

```
[Out] -x*arctanh(a*x)/a^2-1/2/a^3*arctanh(a*x)*ln(a*x-1)+1/2/a^3*arctanh(a*x)*ln(
a*x+1)-1/8/a^3*ln(a*x-1)^2+1/4/a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-1/2/a^3*ln(a*x
-1)-1/2/a^3*ln(a*x+1)-1/8/a^3*ln(a*x+1)^2+1/4/a^3*ln(-1/2*a*x+1/2)*ln(a*x+1
)-1/4/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)
```

maxima [B] time = 0.32, size = 85, normalized size = 2.02

$$-\frac{1}{2}\left(\frac{2\,x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)+\frac{2\left(\log(ax-1)-2\right)\log(ax+1)-\log(ax+1)^2-\log(ax-}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x) + 1/8*(2*
(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log(a
*x - 1))/a^3
```

mupad [B] time = 0.96, size = 82, normalized size = 1.95

$$\frac{\ln(a\,x+1)^2}{8\,a^3}-\ln(1-a\,x)\left(\frac{\ln(a\,x+1)}{4\,a^3}-\frac{x}{2\,a^2}\right)+\frac{\ln(1-a\,x)^2}{8\,a^3}-\frac{\ln\left(a^2\,x^2-1\right)}{2\,a^3}-\frac{x\,\ln(a\,x+1)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x))/(a^2*x^2 - 1),x)
```

```
[Out] log(a*x + 1)^2/(8*a^3) - log(1 - a*x)*(log(a*x + 1)/(4*a^3) - x/(2*a^2)) +
log(1 - a*x)^2/(8*a^3) - log(a^2*x^2 - 1)/(2*a^3) - (x*log(a*x + 1))/(2*a^2
)
```

sympy [A] time = 0.99, size = 41, normalized size = 0.98

$$\begin{cases} -\frac{x\,\text{atanh}\,(ax)}{a^2}-\frac{\log\left(x-\frac{1}{a}\right)}{a^3}+\frac{\text{atanh}^2\,(ax)}{2a^3}-\frac{\text{atanh}\,(ax)}{a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((-x*atanh(a*x)/a**2 - log(x - 1/a)/a**3 + atanh(a*x)**2/(2*a**3)
- atanh(a*x)/a**3, Ne(a, 0)), (0, True))
```

3.229 $\int \frac{x \tanh^{-1}(ax)}{1-a^2x^2} \, dx$

Optimal. Leaf size=54

$$\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2}$$

```
[Out] -1/2*arctanh(a*x)^2/a^2+arctanh(a*x)*ln(2/(-a*x+1))/a^2+1/2*polylog(2,1-2/(
-a*x+1))/a^2
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5984, 5918, 2402, 2315}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -ArcTanh[a*x]^2/(2*a^2) + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^2 + PolyLog[2,
1 - 2/(1 - a*x)]/(2*a^2)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx = -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-ax}\right)}{a^2}$$
$$= -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.81

$$-\frac{\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\left(\tanh^{-1}(ax)+2\log\left(e^{-2\tanh^{-1}(ax)}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
[Out] -1/2*(-(ArcTanh[a*x]*(ArcTanh[a*x] + 2*Log[1 + E^(-2*ArcTanh[a*x])])) + Pol
yLog[2, -E^(-2*ArcTanh[a*x])])/a^2
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x\,\text{artanh}\,(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x\,\text{artanh}\,(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

maple [B] time = 0.05, size = 125, normalized size = 2.31

$$-\frac{\text{arctanh}\,(ax)\ln(ax-1)}{2a^2}-\frac{\text{arctanh}\,(ax)\ln(ax+1)}{2a^2}-\frac{\ln(ax-1)^2}{8a^2}+\frac{\text{dilog}\left(\frac{1}{2}+\frac{ax}{2}\right)}{2a^2}+\frac{\ln(ax-1)\ln\left(\frac{1}{2}+\frac{ax}{2}\right)}{4a^2}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1),x)
[Out] -1/2/a^2*arctanh(a*x)*ln(a*x-1)-1/2/a^2*arctanh(a*x)*ln(a*x+1)-1/8/a^2*ln(a
*x-1)^2+1/2/a^2*dilog(1/2+1/2*a*x)+1/4/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/8/a^
```

```
2*ln(a*x+1)^2-1/4/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/4/a^2*ln(-1/2*a*x+1/2)*l
n(1/2+1/2*a*x)
```

maxima [B] time = 0.32, size = 125, normalized size = 2.31

$$-\frac{1}{8}a\left(\frac{\log(ax+1)^2+2\log(ax+1)\log(ax-1)-\log(ax-1)^2}{a^3}-\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right.}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*a*((log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a^3
 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3) + 1/4*(
log(a*x + 1)/a - log(a*x - 1)/a)*log(a^2*x^2 - 1)/a - 1/2*arctanh(a*x)*log(
a^2*x^2 - 1)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x\,\mathrm{atanh}(a\,x)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x))/(a^2*x^2 - 1),x)
```

```
[Out] -int((x*atanh(a*x))/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\,\mathrm{atanh}(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x*atanh(a*x)/(a**2*x**2 - 1), x)
```

3.230 $\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

[Out] 1/2*arctanh(a*x)^2/a

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a*x]/(1 - a^2*x^2),x]

[Out] ArcTanh[a*x]^2/(2*a)

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx = \frac{\tanh^{-1}(ax)^2}{2a}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2),x]

[Out] ArcTanh[a*x]^2/(2*a)

fricas [A] time = 0.63, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")

[Out] 1/8*log(-(a*x + 1)/(a*x - 1))^2/a

giac [A] time = 0.21, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/8*log(-(a*x + 1)/(a*x - 1))^2/a
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$\frac{\operatorname{arctanh}(ax)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1),x)
```

```
[Out] 1/2*arctanh(a*x)^2/a
```

maxima [B] time = 0.32, size = 65, normalized size = 5.00

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax) - \frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) + \log(ax-1)^2}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x) - 1/8*(log(a*x + 1)^2 -
2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)/a
```

mupad [B] time = 0.87, size = 23, normalized size = 1.77

$$\frac{(\ln(a\,x+1) - \ln(1-a\,x))^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(a^2*x^2 - 1),x)
```

```
[Out] (log(a*x + 1) - log(1 - a*x))^2/(8*a)
```

sympy [A] time = 0.90, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

3.231 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

```
[Out] 1/2*arctanh(a*x)^2+arctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5988, 5932, 2447}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1
 + a*x)]/2
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx &= \frac{1}{2}\tanh^{-1}(ax)^2+\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\
&= \frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-a\int \frac{\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx\\
&= \frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{1}{2}\text{Li}_2\left(-1+\frac{2}{1+ax}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 42, normalized size = 0.93

$$\frac{1}{2}\left(\tanh^{-1}(ax)\left(\tanh^{-1}(ax)+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-\text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

```
[Out] (ArcTanh[a*x]*(ArcTanh[a*x] + 2*Log[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2,
E^(-2*ArcTanh[a*x])])/2
```

fricas [F] time = 1.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{artanh}\,(ax)}{(a^2x^2-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x), x)
```

maple [B] time = 0.06, size = 130, normalized size = 2.89

$$\text{arctanh}\,(ax)\ln(ax)-\frac{\text{arctanh}\,(ax)\ln(ax-1)}{2}-\frac{\text{arctanh}\,(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)^2}{8}+\frac{\text{dilog}\left(\frac{1}{2}+\frac{ax}{2}\right)}{2}+\frac{\ln(ax-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1),x)
```

```
[Out] arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)*ln(a*x-1)-1/2*arctanh(a*x)*ln(a*x+1)-
1/8*ln(a*x-1)^2+1/2*dilog(1/2+1/2*a*x)+1/4*ln(a*x-1)*ln(1/2+1/2*a*x)+1/8*ln
(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2+1/2*a*x))*ln(-1/2*a*x+1/2)-1/2*dilog(a*x)-1
/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

maxima [B] time = 0.33, size = 132, normalized size = 2.93

$$\frac{1}{8}a\left(\frac{\log(ax+1)^2-2\log(ax+1)\log(ax-1)-\log(ax-1)^2}{a}+\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*a*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a +
4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*x
+ 1)*log(x) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a) - 1
/2*(log(a^2*x^2 - 1) - log(x^2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\operatorname{atanh}(a\,x)}{x\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x*(a^2*x^2 - 1)),x)
[Out] -int(atanh(a*x)/(x*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(ax)}{a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1),x)
[Out] -Integral(atanh(a*x)/(a**2*x**3 - x), x)
```

3.232 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)+\frac{1}{2}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] -arctanh(a*x)/x+1/2*a*arctanh(a*x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5982, 5916, 266, 36, 29, 31, 5948}

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)+\frac{1}{2}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])/
2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\left(1-a^2x\right)}\,dx,x,x^2\right)\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{1-a^2x}\,dx\right.\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 1.00

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)+\frac{1}{2}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])/
2
```

fricas [A] time = 0.81, size = 63, normalized size = 1.54

$$\frac{ax\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,ax\log\left(a^2x^2-1\right)+8\,ax\log(x)-4\,\log\left(-\frac{ax+1}{ax-1}\right)}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^2 - 4*a*x*log(a^2*x^2 - 1) + 8*a*x*log(x
) - 4*log(-(a*x + 1)/(a*x - 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^2), x)
```

maple [B] time = 0.06, size = 132, normalized size = 3.22

$$-\frac{\operatorname{arctanh}(ax)}{x}-\frac{a\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{a\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{a\ln(ax-1)^2}{8}+\frac{a\ln(ax-1)\ln\left(\frac{1}{2}+\frac{ax}{2}\right)}{4}+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1),x)
```

```
[Out] -arctanh(a*x)/x-1/2*a*arctanh(a*x)*ln(a*x-1)+1/2*a*arctanh(a*x)*ln(a*x+1)-1
/8*a*ln(a*x-1)^2+1/4*a*ln(a*x-1)*ln(1/2+1/2*a*x)+a*ln(a*x)-1/2*a*ln(a*x-1)-
1/2*a*ln(a*x+1)-1/8*a*ln(a*x+1)^2-1/4*a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/
4*a*ln(-1/2*a*x+1/2)*ln(a*x+1)
```

maxima [B] time = 0.32, size = 82, normalized size = 2.00

$$\frac{1}{8}\left(2\left(\log(ax-1)-2\right)\log(ax+1)-\log(ax+1)^2-\log(ax-1)^2-4\,\log(ax-1)+8\,\log(x)\right)a+\frac{1}{2}\left(a\log(ax+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
4*log(a*x - 1) + 8*log(x))*a + 1/2*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*
arctanh(a*x)
```

mupad [B] time = 1.05, size = 80, normalized size = 1.95

$$\frac{a\ln(a\,x+1)^2}{8}+\frac{a\ln(1-a\,x)^2}{8}-\frac{\ln(a\,x+1)}{2\,x}+\frac{\ln(1-a\,x)}{2\,x}-\frac{a\,\ln\left(a^2\,x^2-1\right)}{2}+a\,\ln(x)-\frac{a\,\ln(a\,x+1)\,\ln(1-a\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] (a*log(a*x + 1)^2)/8 + (a*log(1 - a*x)^2)/8 - log(a*x + 1)/(2*x) + log(1 -
a*x)/(2*x) - (a*log(a^2*x^2 - 1))/2 + a*log(x) - (a*log(a*x + 1)*log(1 - a*
x))/4
```

sympy [A] time = 1.42, size = 37, normalized size = 0.90

$$\begin{cases} a\log(x)-a\log\left(x-\frac{1}{a}\right)+\frac{a\operatorname{atanh}^2(ax)}{2}-a\operatorname{atanh}(ax)-\frac{\operatorname{atanh}(ax)}{x} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((a*log(x) - a*log(x - 1/a) + a*atanh(a*x)**2/2 - a*atanh(a*x) - a
tanh(a*x)/x, Ne(a, 0)), (0, True))
```

3.233 $\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{2}a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\tanh^{-1}(ax)^2+\frac{1}{2}a^2\tanh^{-1}(ax)+a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

```
[Out] -1/2*a/x+1/2*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1/2*a^2*arctanh(a*x)^2+a
^2*arctanh(a*x)*ln(2-2/(a*x+1))-1/2*a^2*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5982, 5916, 325, 206, 5988, 5932, 2447}

$$-\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\tanh^{-1}(ax)^2+\frac{1}{2}a^2\tanh^{-1}(ax)+a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -a/(2*x) + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*ArcTanh[a*x]^
2)/2 + a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - (a^2*PolyLog[2, -1 + 2/(1 +
a*x)])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
```

```
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx+\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\tanh^{-1}(ax)^2+\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx+a^2\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\ &= -\frac{a}{2x}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\tanh^{-1}(ax)^2+a^2\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)+\frac{1}{2}a^3\int \frac{1}{1-a^2x^2}\\ &= -\frac{a}{2x}+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\tanh^{-1}(ax)^2+a^2\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)\end{aligned}$$

Mathematica [A] time = 0.27, size = 60, normalized size = 0.71

$$-\frac{1}{2}a^2\left(-\tanh^{-1}(ax)\left(-\frac{1}{a^2x^2}+\tanh^{-1}(ax)+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)+1\right)+\mathrm{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right)+\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
[Out] -1/2*(a^2*(1/(a*x) - ArcTanh[a*x]*(1 - 1/(a^2*x^2) + ArcTanh[a*x] + 2*Log[1
 - E^(-2*ArcTanh[a*x])]) + PolyLog[2, E^(-2*ArcTanh[a*x])]))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\operatorname{artanh}(ax)}{a^2x^5-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-arctanh(a*x)/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^3), x)
```

maple [B] time = 0.07, size = 209, normalized size = 2.49

$$-\frac{\operatorname{arctanh}(ax)}{2x^2}+a^2\operatorname{arctanh}(ax)\ln(ax)-\frac{a^2\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{a^2\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{a}{2x}-\frac{a^2\ln(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1),x)
```

```
[Out] -1/2*arctanh(a*x)/x^2+a^2*arctanh(a*x)*ln(a*x)-1/2*a^2*arctanh(a*x)*ln(a*x-
1)-1/2*a^2*arctanh(a*x)*ln(a*x+1)-1/2*a/x-1/4*a^2*ln(a*x-1)+1/4*a^2*ln(a*x+
1)-1/8*a^2*ln(a*x-1)^2+1/2*a^2*dilog(1/2+1/2*a*x)+1/4*a^2*ln(a*x-1)*ln(1/2+
1/2*a*x)+1/8*a^2*ln(a*x+1)^2+1/4*a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/4*a
^2*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/2*a^2*dilog(a*x)-1/2*a^2*dilog(a*x+1)-1/2*a
^2*ln(a*x)*ln(a*x+1)
```

maxima [B] time = 0.33, size = 162, normalized size = 1.93

$$\frac{1}{8}\left(4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-4\left(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax)\right)a+4\left(\log(-ax+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(log
(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*a
 + 2*a*log(a*x + 1) - 2*a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a*
x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 - 4)/x)*a - 1/2*(a^2*log(a^2*x^2 -
 1) - a^2*log(x^2) + 1/x^2)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)}{x^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)/(x^3*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(ax)}{a^2x^5-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)/(a**2*x**5 - x**3), x)
```

3.234 $\int \frac{x^3 \tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=135

$$-\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^4}+\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^4}-\frac{x\tanh^{-1}(ax)}{a^3}-$$

```
[Out] -x*arctanh(a*x)/a^3+1/2*arctanh(a*x)^2/a^4-1/2*x^2*arctanh(a*x)^2/a^2-1/3*a
rctanh(a*x)^3/a^4+arctanh(a*x)^2*ln(2/(-a*x+1))/a^4-1/2*ln(-a^2*x^2+1)/a^4+
arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^4-1/2*polylog(3,1-2/(-a*x+1))/a^4
```

Rubi [A] time = 0.30, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5980, 5916, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}-\frac{\log\left(1-a^2x^2\right)}{2a^4}-\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^3) + ArcTanh[a*x]^2/(2*a^4) - (x^2*ArcTanh[a*x]^2)/(2*
a^2) - ArcTanh[a*x]^3/(3*a^4) + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - Log
[1 - a^2*x^2]/(2*a^4) + (ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^4 - Po
lyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{1-a^2x^2}\,dx = -\frac{\int x\tanh^{-1}(ax)^2\,dx}{a^2} + \frac{\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x^2\tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^3} + \frac{\int \frac{x^2\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a}$$

$$= -\frac{x^2\tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\int \tanh^{-1}(ax)\,dx}{a^3} + \int \frac{\tan}{1}$$

$$= -\frac{x\tanh^{-1}(ax)}{a^3} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{x^2\tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\tanh^{-1}(ax)^2\log\left(\frac{}{1}\right.}{a^4}$$

$$= -\frac{x\tanh^{-1}(ax)}{a^3} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{x^2\tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\tanh^{-1}(ax)^2\log\left(\frac{}{1}\right.}{a^4}$$

Mathematica [A] time = 0.12, size = 112, normalized size = 0.83

$$-\frac{-\log\left(\frac{1}{\sqrt{1-a^2x^2}}\right) - \frac{1}{2}\left(1-a^2x^2\right)\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + \frac{1}{2}\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right) - \frac{1}{3}\tanh}{a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((a*x*ArcTanh[a*x] - ((1 - a^2*x^2)*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^3/3 -
 ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - Log[1/Sqrt[1 - a^2*x^2]] + A
rcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*
x])]/2)/a^4)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^3\,\text{artanh}\,(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3\,\text{artanh}\,(ax)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

maple [C] time = 0.50, size = 812, normalized size = 6.01

$$-\frac{x^2\,\text{arctanh}\,(ax)^2}{2a^2}-\frac{\text{arctanh}\,(ax)^2\ln\,(ax-1)}{2a^4}-\frac{\text{arctanh}\,(ax)^2\ln\,(ax+1)}{2a^4}+\frac{\text{arctanh}\,(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a^4}+\frac{\text{arctanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] -1/2*x^2*arctanh(a*x)^2/a^2-1/2/a^4*arctanh(a*x)^2*ln(a*x-1)-1/2/a^4*arctan
h(a*x)^2*ln(a*x+1)+1/a^4*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/a^
4*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2/a^4*polylog(3,-(a*x+1
)^2/(-a^2*x^2+1))+1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))+1/2*I/a^4*
arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+
1)^(1/2))+1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+
1)^2/(-a^2*x^2+1)))^3+1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2
*x^2+1)))^3-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I/(1+(a*x+1)^2/(-a
^2*x^2+1)))+1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/2
*I/a^4*arctanh(a*x)^2*Pi-1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-
a^2*x^2+1)))^2-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I/a^4*arctanh(
a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^
2-1/3*arctanh(a*x)^3/a^4+1/a^4*arctanh(a*x)^2*ln(2)-x*arctanh(a*x)/a^3+1/2*
arctanh(a*x)^2/a^4-arctanh(a*x)/a^4+1/a^4*ln(1+(a*x+1)^2/(-a^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\left(a^2x^2+\log\,(ax+1)\right)\log\,(-ax+1)^2+\log\,(-ax+1)^3}{24\,a^4}+\frac{1}{4}\int -\frac{a^3x^3\log\,(ax+1)^2-\left(a^3x^3+a^2x^2+\left(2\,a^3x^3+a\right.\right.}{a^5x^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^4 +
1/4*integrate(-(a^3*x^3*log(a*x + 1)^2 - (a^3*x^3 + a^2*x^2 + (2*a^3*x^3 +
a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^5*x^2 - a^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\mathrm{atanh}\left(a\,x\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\,\mathrm{atanh}^2\left(ax\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**3*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

3.235 $\int \frac{x^2 \tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=75

$$\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^3}{3a^3}-\frac{\tanh^{-1}(ax)^2}{a^3}+\frac{2\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}$$

```
[Out] -arctanh(a*x)^2/a^3-x*arctanh(a*x)^2/a^2+1/3*arctanh(a*x)^3/a^3+2*arctanh(a
*x)*ln(2/(-a*x+1))/a^3+polylog(2,1-2/(-a*x+1))/a^3
```

Rubi [A] time = 0.17, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5980, 5910, 5984, 5918, 2402, 2315, 5948}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^3}{3a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}-\frac{\tanh^{-1}(ax)^2}{a^3}+\frac{2\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -(ArcTanh[a*x]^2/a^3) - (x*ArcTanh[a*x]^2)/a^2 + ArcTanh[a*x]^3/(3*a^3) + (
2*ArcTanh[a*x]*Log[2/(1 - a*x)])/a^3 + PolyLog[2, 1 - 2/(1 - a*x)]/a^3
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
```

```
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{1-a^2x^2}\,dx = -\frac{\int \tanh^{-1}(ax)^2\,dx}{a^2} + \frac{\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x\tanh^{-1}(ax)^2}{a^2} + \frac{\tanh^{-1}(ax)^3}{3a^3} + \frac{2\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^2}{a^3} - \frac{x\tanh^{-1}(ax)^2}{a^2} + \frac{\tanh^{-1}(ax)^3}{3a^3} + \frac{2\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^2}$$

$$= -\frac{\tanh^{-1}(ax)^2}{a^3} - \frac{x\tanh^{-1}(ax)^2}{a^2} + \frac{\tanh^{-1}(ax)^3}{3a^3} + \frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3} - \frac{2\int \frac{\log\left(\frac{2}{1-\ldots}\right)}{1-a^2\ldots}}{a^2}$$

$$= -\frac{\tanh^{-1}(ax)^2}{a^3} - \frac{x\tanh^{-1}(ax)^2}{a^2} + \frac{\tanh^{-1}(ax)^3}{3a^3} + \frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{2\,\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{\tanh^{-1}(ax)^2}{a^3} - \frac{x\tanh^{-1}(ax)^2}{a^2} + \frac{\tanh^{-1}(ax)^3}{3a^3} + \frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{\mathrm{Li}_2\left(1-\ldots\right.}{a^3}$$

Mathematica [A] time = 0.18, size = 59, normalized size = 0.79

$$-\frac{\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) - \frac{1}{3}\tanh^{-1}(ax)\left(-3ax\tanh^{-1}(ax) + \left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax) + 6\log\left(e^{-2\tanh^{-1}(ax)} + \ldots\right.\right.}{a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((-1/3*(ArcTanh[a*x]*(-3*a*x*ArcTanh[a*x] + ArcTanh[a*x]*(3 + ArcTanh[a*x]
) + 6*Log[1 + E^(-2*ArcTanh[a*x])])) + PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^
3)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^2\,\text{artanh}(ax)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2\,\text{artanh}(ax)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

maple [C] time = 0.55, size = 5573, normalized size = 74.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.33, size = 200, normalized size = 2.67

$$-\frac{1}{2}\left(\frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)^2 - \frac{\frac{3\left(\log(ax-1)-2\right)\log(ax+1)^2 - \log(ax+1)^3 + \log(ax-1)^3 - 3\left(\log(ax-1)^2 - 4\log(a\ldots\right.}{a}}{24\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x)^2 - 1/24*
((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 - 3
*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a - 24*
(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a)/a^2 + 1/4*(2*(
log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log(a*
x - 1))*arctanh(a*x)/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2 \operatorname{atanh}(a\,x)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^2*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 \operatorname{atanh}^2(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**2*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

3.236 $\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx$

Optimal. Leaf size=78

$$-\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^2}+\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2}-\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2}$$

```
[Out] -1/3*arctanh(a*x)^3/a^2+arctanh(a*x)^2*ln(2/(-a*x+1))/a^2+arctanh(a*x)*poly
log(2,1-2/(-a*x+1))/a^2-1/2*polylog(3,1-2/(-a*x+1))/a^2
```

Rubi [A] time = 0.16, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5984, 5918, 5948, 6058, 6610}

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2}-\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -ArcTanh[a*x]^3/(3*a^2) + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^2 + (ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^2 - PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^2)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx &= -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a}\\
&= -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{2\int \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}\\
&= -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2}-\frac{\int \frac{\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}\\
&= -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2}-\frac{\mathrm{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.87

$$
-\frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)+\frac{1}{2}\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right)-\frac{1}{3}\tanh^{-1}(ax)^3-\tanh^{-1}(ax)^2\log\left(e^{-2\tanh^{-1}(ax)}+1\right)}{a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
[Out] -((-1/3*ArcTanh[a*x]^3 - ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcT
anh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x])
]/2)/a^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(-\frac{x\,\mathrm{artanh}\,(ax)^2}{a^2x^2-1},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int -\frac{x\,\mathrm{artanh}\,(ax)^2}{a^2x^2-1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

maple [C] time = 0.52, size = 741, normalized size = 9.50

$$
-\frac{\mathrm{arctanh}\,(ax)^2\ln(ax-1)}{2a^2}-\frac{\mathrm{arctanh}\,(ax)^2\ln(ax+1)}{2a^2}+\frac{\mathrm{arctanh}\,(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a^2}-\frac{\mathrm{arctanh}\,(ax)^3}{3a^2}+\frac{\mathrm{arctanh}\,(ax}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] -1/2/a^2*arctanh(a*x)^2*ln(a*x-1)-1/2/a^2*arctanh(a*x)^2*ln(a*x+1)+1/a^2*ar
ctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3/a^2+1/a^2*ar
ctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2/a^2*polylog(3,-(a*x+1)^2/
(-a^2*x^2+1))-1/4*I/a^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I/a^2*arctanh(a
*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2
-1/4*I/a^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1
)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))
+1/2*I/a^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))+1/4*I/a^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*
x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I/a
^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-1/2*I/a^2*arctanh(a*x)
^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*I/a^2*arctanh(a*x)^2*Pi*csgn
(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/4*I/a^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/2*I/a^2*arctanh(a*x)^2*Pi+1/
a^2*arctanh(a*x)^2*ln(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\,\log\left(ax+1\right)\log\left(-ax+1\right)^{2}+\log\left(-ax+1\right)^{3}}{24\,a^{2}}+\frac{1}{4}\int-\frac{ax\log\left(ax+1\right)^{2}-\left(3\,ax+1\right)\log\left(ax+1\right)\log\left(-ax+1\right.}{a^{3}x^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(3*log(a*x + 1)*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^2 + 1/4*integrat
e(-(a*x*log(a*x + 1)^2 - (3*a*x + 1)*log(a*x + 1)*log(-a*x + 1))/(a^3*x^2 -
 a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{x\,\mathrm{atanh}\left(a\,x\right)^{2}}{a^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x\,\mathrm{atanh}^{2}\left(ax\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

3.237 $\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

[Out] 1/3*arctanh(a*x)^3/a

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2),x]

[Out] ArcTanh[a*x]^3/(3*a)

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2} dx = \frac{\tanh^{-1}(ax)^3}{3a}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2),x]

[Out] ArcTanh[a*x]^3/(3*a)

fricas [A] time = 0.54, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")

[Out] 1/24*log(-(a*x + 1)/(a*x - 1))^3/a

giac [A] time = 0.19, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/24*log(-(a*x + 1)/(a*x - 1))^3/a
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$\frac{\operatorname{arctanh}(ax)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] 1/3*arctanh(a*x)^3/a
```

maxima [B] time = 0.33, size = 127, normalized size = 9.77

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2-\frac{\left(\log(ax+1)^2-2\log(ax+1)\log(ax-1)+\log(ax-1)^2\right)\operatorname{arta}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^2 - 1/4*(log(a*x + 1)^2
- 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)*arctanh(a*x)/a + 1/24*(log(
a*x + 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)^2
- log(a*x - 1)^3)/a
```

mupad [B] time = 0.95, size = 68, normalized size = 5.23

$$\frac{\ln(a\,x+1)^3}{24\,a}-\frac{\ln(1-a\,x)^3}{24\,a}+\frac{\ln(a\,x+1)\,\ln(1-a\,x)^2}{8\,a}-\frac{\ln(a\,x+1)^2\,\ln(1-a\,x)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(a^2*x^2 - 1),x)
```

```
[Out] log(a*x + 1)^3/(24*a) - log(1 - a*x)^3/(24*a) + (log(a*x + 1)*log(1 - a*x)^
2)/(8*a) - (log(a*x + 1)^2*log(1 - a*x))/(8*a)
```

sympy [A] time = 0.93, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^{3}(ax)}{3a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

3.238 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)-\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{3}\tanh^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

```
[Out] 1/3*arctanh(a*x)^3+arctanh(a*x)^2*ln(2-2/(a*x+1))-arctanh(a*x)*polylog(2,-1
+2/(a*x+1))-1/2*polylog(3,-1+2/(a*x+1))
```

Rubi [A] time = 0.18, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5988, 5932, 5948, 6056, 6610}

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}\tanh^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyL
og[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx = \frac{1}{3}\tanh^{-1}(ax)^3 + \int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx$$
$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - (2a)\int \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx$$
$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1+ax}\right) + a\int \frac{\text{Li}}{-}$$
$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1+ax}\right) - \frac{1}{2}\text{Li}_3\left(-\right.$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.91

$$\tanh^{-1}(ax)\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)-\frac{1}{2}\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)-\frac{1}{3}\tanh^{-1}(ax)^3+\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

```
[Out] -1/3*ArcTanh[a*x]^3 + ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + ArcTanh[
a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - PolyLog[3, E^(2*ArcTanh[a*x])]/2
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)^2}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{artanh}\,(ax)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x), x)
```

maple [C] time = 0.45, size = 1188, normalized size = 18.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1),x)
```

```
[Out] 1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x+1
)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2
+1)))-1/4*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^
2-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)
```

```
^(1/2))-1/3*arctanh(a*x)^3+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*
x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-
1))^3*arctanh(a*x)^2+1/4*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-1/4*I*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2
*x^2+1)))^2*arctanh(a*x)^2-1/2*I*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^
2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-
1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^
2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+2*arctanh(a*x)*polylog(2,-(
a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1
/2))-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a*x)
^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln(
(a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+a
rctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(2)+1/2*I*P
i*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctan
h(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*arctanh(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^3*arctanh(a*x)^2-1/2*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1))
)^2*arctanh(a*x)^2+1/2*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*
x)^2+1/2*I*Pi*arctanh(a*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\log\left(ax+1\right)\log\left(-ax+1\right)^{2}-\frac{1}{24}\log\left(-ax+1\right)^{3}+\frac{1}{4}\int\frac{\left(a^{2}x^{2}+ax+2\right)\log\left(ax+1\right)\log\left(-ax+1\right)-\log\left(ax+1\right)^{2}}{a^{2}x^{3}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*log(a*x + 1)*log(-a*x + 1)^2 - 1/24*log(-a*x + 1)^3 + 1/4*integrate(((
a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1) - log(a*x + 1)^2)/(a^2*x^3 -
x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int\frac{\mathrm{atanh}\left(a\,x\right)^2}{x\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^2/(x*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{atanh}^{2}\left(ax\right)}{a^{2}x^{3}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a**x)**2/(a**2*x**3 - x), x)
```

3.239 $\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx$

Optimal. Leaf size=66

$$-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{3}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

```
[Out] a*arctanh(a*x)^2-arctanh(a*x)^2/x+1/3*a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln(
2-2/(a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.21, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5982, 5916, 5988, 5932, 2447, 5948}

$$-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a*ArcTanh[a*x]^3)/3 + 2*a*ArcTanh[a*
x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
```

```
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\\
&=-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+(2a)\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\
&=a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+(2a)\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\
&=a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+2a\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\left(2a^2\right)\\
&=a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+2a\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-a\mathrm{Li}_2\left(\right.\end{aligned}$$

Mathematica [A] time = 0.23, size = 61, normalized size = 0.92

$$-a\left(\mathrm{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right)-\frac{1}{3}\tanh^{-1}(ax)\left(\left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)-\frac{3\tanh^{-1}(ax)}{ax}+6\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(a*(-1/3*(ArcTanh[a*x]*((-3*ArcTanh[a*x])/(a*x) + ArcTanh[a*x]*(3 + ArcTan
h[a*x]) + 6*Log[1 - E^(-2*ArcTanh[a*x])])) + PolyLog[2, E^(-2*ArcTanh[a*x])
]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\mathrm{artanh}\left(ax\right)^2}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\mathrm{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^2), x)

maple [C] time = 0.60, size = 4449, normalized size = 67.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x)

[Out] -a*arctanh(a*x)^2-arctanh(a*x)^2/x+1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2+1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2-1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*a*arctanh(a*x)^3-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^

```
2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^
2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-1/4*I*a*Pi*csgn(
I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^
2-1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-a*dilog((a*x+1)/(-a^2*x^2+1)
^(1/2))+a*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2+1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(
I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^
2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1
/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))
+1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^2+1
/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+
1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))
^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I/(1+(a*x+
1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+a*arctanh
(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*
x^2+1)^(1/2))-1/2*a*arctanh(a*x)^2*ln(a*x-1)+1/2*a*arctanh(a*x)^2*ln(a*x+1)
-a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^
2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2
))-1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,-(a*x+1)/(-a^2
*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,(a
*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)/(
-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)*ln(1-(a*x+1
)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*
(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+
1)))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*dilog((a*x+1)/(-a^2*x^2
+1)^(1/2))+1/2*I*a*Pi*arctanh(a*x)^2-1/2*I*a*Pi*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))+1/2*I*a*Pi*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [B] time = 0.33, size = 237, normalized size = 3.59

$$-\frac{1}{24}a^2\left(\frac{3\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)^2-\log\left(ax+1\right)^3+\log\left(ax-1\right)^3-3\left(\log\left(ax-1\right)^2-4\log\left(ax-1\right)\right)\log\left(\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*a^2*((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x
- 1)^3 - 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^
2)/a - 24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 24*
(log(a*x + 1)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilog(a*
x))/a) + 1/4*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x
- 1)^2 - 4*log(a*x - 1) + 8*log(x))*a*arctanh(a*x) + 1/2*(a*log(a*x + 1) -
a*log(a*x - 1) - 2/x)*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x^2*(a^2*x^2 - 1)),x)
[Out] -int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^2x^4-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1),x)
[Out] -Integral(atanh(a*x)**2/(a**2*x**4 - x**2), x)
```

3.240 $\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}\,dx$

Optimal. Leaf size=138

$$-\frac{1}{2}a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)-a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{3}a^2\tanh^{-1}(ax)^3+\frac{1}{2}a^2\tanh$$

```
[Out] -a*arctanh(a*x)/x+1/2*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+1/3*a^2*arc
tanh(a*x)^3+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+a^2*arctanh(a*x)^2*ln(2-2/(a*x
+1))-a^2*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*a^2*polylog(3,-1+2/(a*x+1
))
```

Rubi [A] time = 0.36, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5982, 5916, 266, 36, 29, 31, 5948, 5988, 5932, 6056, 6610}

$$-\frac{1}{2}a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{3}a^2\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) + (
a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log[1 - a^2*x^2])/2 + a^2*ArcTanh
[a*x]^2*Log[2 - 2/(1 + a*x)] - a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)
] - (a^2*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx + a^2\int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) + a\int \frac{\tanh^{-1}(ax)}{x^2}\,dx$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\log(x) - \frac{1}{2}a^2\log$$

Mathematica [C] time = 0.37, size = 133, normalized size = 0.96

$$-a^2\left(-\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right) + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a^2x^2} - \tanh^{-1}(ax)\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) + \frac{1}{2}\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) + \frac{1}{3}\tanh^{-1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -(a^2*((-1/24*I)*Pi^3 + ArcTanh[a*x]/(a*x) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)
/(2*a^2*x^2) + ArcTanh[a*x]^3/3 - ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])
] - Log[(a*x)/Sqrt[1 - a^2*x^2]] - ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x
])] + PolyLog[3, E^(2*ArcTanh[a*x])]/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^2}{a^2x^5-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^3), x)
```

maple [C] time = 0.89, size = 1360, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x)
```

```
[Out] 2*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x
)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-a*arctanh(a*x)/x+a^2*arctanh(a*x)^2
*ln(2)+a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)
^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*a^2*arctanh(a*x)^3+1/2*I*a^2*arctan
h(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)^2-1/4*I*a^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I*a^2*arctanh(a*x)^2*Pi*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+1/4*I*a^2
*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/2*I*a^2*arctanh(a*x)^2*Pi*csgn(I/(1
+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-
a^2*x^2+1)))^2-1/2*I*a^2*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1
))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*a^2*
arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2-a^2*arctanh(a*x)-2*a^2*polylog(3,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-2*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ln
(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a^2*arctanh(a*x)^2*Pi+a^2*ln((a*x+1)/(
-a^2*x^2+1)^(1/2)-1)+a^2*arctanh(a*x)^2*ln(a*x)-1/2*a^2*arctanh(a*x)^2*ln(a
*x-1)-a^2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-1/2*a^2*arctanh(a*x)^
2*ln(a*x+1)+a^2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a^2*arc
tanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+
1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+1/2*I*a^2*arc
tanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x
^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))-1/2
*I*a^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I*a^2*arc
tanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/2*I*a^2*arctanh(a*x)^2*Pi*
csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/4*I*a^2*arctanh(a*x)^2*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/2*I*a^2*arctanh(a*x)^2*P
i*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2x^2\log\left(-ax+1\right)^3+3\left(a^2x^2\log\left(ax+1\right)+1\right)\log\left(-ax+1\right)^2}{24\,x^2}+\frac{1}{4}\int-\frac{\log\left(ax+1\right)^2-\left(a^2x^2+ax+\left(a^4x^4+\cdots\right.\right.}{a^2x^5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(a^2*x^2*log(-a*x + 1)^3 + 3*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1)
^2)/x^2 + 1/4*integrate(-(log(a*x + 1)^2 - (a^2*x^2 + a*x + (a^4*x^4 + a^3*
x^3 + 2)*log(a*x + 1))*log(-a*x + 1))/(a^2*x^5 - x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\mathrm{atanh}\left(a\,x\right)^2}{x^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^2x^5 - x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**2*x**5 - x**3), x)
```

3.241 $\int \frac{x^3 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx$

Optimal. Leaf size=205

$$\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{4a^4}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{2a^4}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{2a^4}-\frac{\tanh^{-1}(ax)^4}{4a^4}+\text{ta}$$

```
[Out] -3/2*arctanh(a*x)^2/a^4-3/2*x*arctanh(a*x)^2/a^3+1/2*arctanh(a*x)^3/a^4-1/2
*x^2*arctanh(a*x)^3/a^2-1/4*arctanh(a*x)^4/a^4+3*arctanh(a*x)*ln(2/(-a*x+1)
)/a^4+arctanh(a*x)^3*ln(2/(-a*x+1))/a^4+3/2*polylog(2,1-2/(-a*x+1))/a^4+3/2
*arctanh(a*x)^2*polylog(2,1-2/(-a*x+1))/a^4-3/2*arctanh(a*x)*polylog(3,1-2/
(-a*x+1))/a^4+3/4*polylog(4,1-2/(-a*x+1))/a^4
```

Rubi [A] time = 0.47, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5980, 5916, 5910, 5984, 5918, 2402, 2315, 5948, 6058, 6062, 6610}

$$\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^4}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{3\tanh^{-1}(ax)\text{PolyLo}}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] (-3*ArcTanh[a*x]^2)/(2*a^4) - (3*x*ArcTanh[a*x]^2)/(2*a^3) + ArcTanh[a*x]^3
/(2*a^4) - (x^2*ArcTanh[a*x]^3)/(2*a^2) - ArcTanh[a*x]^4/(4*a^4) + (3*ArcTa
nh[a*x]*Log[2/(1 - a*x)])/a^4 + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 + (3*
PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(2*a^4) - (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^4) +
 (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx = -\frac{\int x\tanh^{-1}(ax)^3\,dx}{a^2} + \frac{\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a^3} + \frac{3\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a}$$

$$= -\frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{3\int \tanh^{-1}(ax)^2\,dx}{2a^3} + \frac{3}{}$$

$$= -\frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{\tanh^{-1}(ax)^3\log}{a^4}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} +$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} +$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} +$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} +$$

Mathematica [A] time = 0.28, size = 142, normalized size = 0.69

$$-\frac{-2\left(1-a^2x^2\right)\tanh^{-1}(ax)^3 + 6\tanh^{-1}(ax)\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right) + 6\left(\tanh^{-1}(ax)^2+1\right)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 3\mathrm{L}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -1/4*(-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 - 2*(1 - a^2*x^2)*ArcTanh[a*
x]^3 - ArcTanh[a*x]^4 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh[a*x])] - 4*Ar
cTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(1 + ArcTanh[a*x]^2)*PolyLog[
2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])]
+ 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/a^4
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{x^3\,\mathrm{artanh}\,(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3\,\mathrm{artanh}\,(ax)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")

[Out] integrate(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)

maple [A] time = 3.15, size = 248, normalized size = 1.21

$$-\frac{\operatorname{arctanh}(ax)^4}{4a^4}-\frac{x^2\operatorname{arctanh}(ax)^3}{2a^2}-\frac{3x\operatorname{arctanh}(ax)^2}{2a^3}+\frac{\operatorname{arctanh}(ax)^3}{2a^4}-\frac{3\operatorname{arctanh}(ax)^2}{2a^4}+\frac{\operatorname{arctanh}(ax)^3\ln\left(1+\frac{(a}{-a}\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x)

[Out] -1/4*arctanh(a*x)^4/a^4-1/2*x^2*arctanh(a*x)^3/a^2-3/2*x*arctanh(a*x)^2/a^3 +1/2*arctanh(a*x)^3/a^4-3/2*arctanh(a*x)^2/a^4+1/a^4*arctanh(a*x)^3*ln(1+(a *x+1)^2/(-a^2*x^2+1))+3/2/a^4*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2 +1))-3/2/a^4*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4/a^4*polylo g(4,-(a*x+1)^2/(-a^2*x^2+1))+3/a^4*arctanh(a*x)*ln(1+(a*x+1)^2/(-a^2*x^2+1))+3/2/a^4*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\left(a^2x^2+\log(ax+1)\right)\log(-ax+1)^3+\log(-ax+1)^4}{64\,a^4}-\frac{1}{8}\int\frac{2\,a^3x^3\log(ax+1)^3-6\,a^3x^3\log(ax+1)^2\log(-ax}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")

[Out] 1/64*(4*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^4 - 1 /8*integrate(1/2*(2*a^3*x^3*log(a*x + 1)^3 - 6*a^3*x^3*log(a*x + 1)^2*log(- a*x + 1) + 3*(a^3*x^3 + a^2*x^2 + (2*a^3*x^3 + a*x + 1)*log(a*x + 1))*log(- a*x + 1)^2)/(a^5*x^2 - a^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int\frac{x^3\,\mathrm{atanh}\left(a\,x\right)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3*atanh(a*x)^3)/(a^2*x^2 - 1),x)

[Out] -int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3\operatorname{atanh}^3(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1),x)

[Out] -Integral(x**3*atanh(a*x)**3/(a**2*x**2 - 1), x)

3.242 $\int \frac{x^2 \tanh^{-1}(ax)^3}{1-a^2x^2} dx$

Optimal. Leaf size=103

$$-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^3}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3}+\frac{\tanh^{-1}(ax)^4}{4a^3}-\frac{\tanh^{-1}(ax)^3}{a^3}+\frac{3\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}$$

```
[Out] -arctanh(a*x)^3/a^3-x*arctanh(a*x)^3/a^2+1/4*arctanh(a*x)^4/a^3+3*arctanh(a
*x)^2*ln(2/(-a*x+1))/a^3+3*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^3-3/2*pol
ylog(3,1-2/(-a*x+1))/a^3
```

Rubi [A] time = 0.27, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5980, 5910, 5984, 5918, 5948, 6058, 6610}

$$-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^4}{4a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}-\frac{\tanh^{-1}(ax)^3}{a^3}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -(ArcTanh[a*x]^3/a^3) - (x*ArcTanh[a*x]^3)/a^2 + ArcTanh[a*x]^4/(4*a^3) + (
3*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^3 + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(
1 - a*x)])/a^3 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3)
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx &= -\frac{\int \tanh^{-1}(ax)^3\,dx}{a^2}+\frac{\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a}\\
&= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^2}\\
&= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}-\frac{6\int \frac{\tanh^{-1}(a\ldots}{1-\ldots}}{\ldots}\\
&= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{3\tanh^{-1}(a\ldots}{\ldots}\\
&= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{3\tanh^{-1}(a\ldots}{\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 78, normalized size = 0.76

$$
\frac{-12\tanh^{-1}(ax)\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-6\text{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)^2\left(\tanh^{-1}(ax)^2+(4-4ax)\tanh^{-1}(ax)+\ldots\right.}{4a^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*((4 - 4*a*x)*ArcTanh[a*x] + ArcTanh[a*x]^2 + 12*Log[1 + E^(
-2*ArcTanh[a*x])]) - 12*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 6*P
olyLog[3, -E^(-2*ArcTanh[a*x])])/(4*a^3)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$
\text{integral}\left(-\frac{x^2\operatorname{artanh}(ax)^3}{a^2x^2-1}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 \operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

maple [C] time = 0.56, size = 788, normalized size = 7.65

$$-\frac{x \operatorname{arctanh}(ax)^3}{a^2}-\frac{\operatorname{arctanh}(ax)^3 \ln(ax-1)}{2a^3}+\frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{2a^3}-\frac{\operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a^3}+\frac{\operatorname{arctan}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x)
```

```
[Out] -x*arctanh(a*x)^3/a^2-1/2/a^3*arctanh(a*x)^3*ln(a*x-1)+1/2/a^3*arctanh(a*x)
^3*ln(a*x+1)-1/a^3*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*arctan
h(a*x)^4/a^3-arctanh(a*x)^3/a^3-3/2/a^3*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-
1/4*I/a^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*arctanh(a*x)^3+1/2*I/a^3*Pi*arctanh(a*x)^3-1/4*I/a^3*Pi*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/4*I/a^3*arcta
nh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/2*I/a^3*Pi*csgn(I/(1+(a*x+1)
^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/2*I/a^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x
^2+1)))^2*arctanh(a*x)^3+1/4*I/a^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3-1/2*I/a^
3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^
2*x^2-1))^2+1/4*I/a^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arc
tanh(a*x)^3-1/4*I/a^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^
2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3+3/a^3*arctanh(a*
x)^2*ln(1+(a*x+1)^2/(-a^2*x^2+1))+3/a^3*arctanh(a*x)*polylog(2,-(a*x+1)^2/(
-a^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\left(2\,ax-\log(ax+1)-2\right)\log(-ax+1)^3+\log(-ax+1)^4-6\left(4\,(ax+1)\log(ax+1)-\log(ax+1)^2\right)\log(-a}{64\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(4*(2*a*x - log(a*x + 1) - 2)*log(-a*x + 1)^3 + log(-a*x + 1)^4 - 6*(4
*(a*x + 1)*log(a*x + 1) - log(a*x + 1)^2)*log(-a*x + 1)^2)/a^3 + 1/8*integr
ate(-1/2*(2*a^2*x^2*log(a*x + 1)^3 - 3*((2*a^2*x^2 - a*x - 1)*log(a*x + 1)^
2 + 4*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^4*x^2 - a^2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\mathrm{atanh}(a\,x)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x)^3)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^2*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 \operatorname{atanh}^3(ax)}{a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**2*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

3.243 $\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx$

Optimal. Leaf size=108

$$\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{4a^2}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{2a^2}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(a}{a^2}$$

```
[Out] -1/4*arctanh(a*x)^4/a^2+arctanh(a*x)^3*ln(2/(-a*x+1))/a^2+3/2*arctanh(a*x)^
2*polylog(2,1-2/(-a*x+1))/a^2-3/2*arctanh(a*x)*polylog(3,1-2/(-a*x+1))/a^2+
3/4*polylog(4,1-2/(-a*x+1))/a^2
```

Rubi [A] time = 0.21, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5984, 5918, 5948, 6058, 6062, 6610}

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^2}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(a}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -ArcTanh[a*x]^4/(4*a^2) + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^2 + (3*ArcTan
h[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^2) - (3*ArcTanh[a*x]*PolyLog[3,
1 - 2/(1 - a*x)])/(2*a^2) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^2)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
```

```
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx = -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\int \frac{\tanh^{-1}(ax)\text{Li}_2(}{1-a^2x^2}}{a}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}(ax)\text{Li}}{2a^2}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}(ax)\text{Li}}{2a^2}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.81

$$-\frac{6\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right)+3\text{Li}_4\left(-e^{-2\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^4-4\,\text{t}}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -1/4*(-ArcTanh[a*x]^4 - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*A
rcTanh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3,
-E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/a^2
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x\,\text{artanh}\,(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x\,\text{artanh}\,(ax)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

maple [C] time = 0.52, size = 776, normalized size = 7.19

$$-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2a^2}-\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2a^2}+\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a^2}-\frac{\operatorname{arctanh}(ax)^4}{4a^2}+\frac{3\operatorname{arctan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1),x)
```

```
[Out] -1/2/a^2*arctanh(a*x)^3*ln(a*x-1)-1/2/a^2*arctanh(a*x)^3*ln(a*x+1)+1/a^2*ar
ctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4/a^2+3/2/a^2*
arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2/a^2*arctanh(a*x)*poly
log(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4/a^2*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))+1
/2*I/a^2*arctanh(a*x)^3*Pi+1/2*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x
^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi+1/4*I/a^2*arctanh(a*x)^3*csg
n(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(
-a^2*x^2+1)))^2*Pi-1/2*I/a^2*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1
)))^2*Pi+1/2*I/a^2*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-1
/4*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+1/4*I/a^2*arctanh(a*x)^3*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi-1/4*I/a^2*arct
anh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*Pi+1/4*I/a^2*arct
anh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi+1/
4*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi+1/a^2*arctanh(a*x
)^3*ln(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\log(ax+1)\log(-ax+1)^3+\log(-ax+1)^4}{64a^2}-\frac{1}{8}\int\frac{2ax\log(ax+1)^3-6ax\log(ax+1)^2\log(-ax+1)+3(}{2\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(4*log(a*x + 1)*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^2 - 1/8*integrate
(1/2*(2*a*x*log(a*x + 1)^3 - 6*a*x*log(a*x + 1)^2*log(-a*x + 1) + 3*(3*a*x
+ 1)*log(a*x + 1)*log(-a*x + 1)^2)/(a^3*x^2 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{x\operatorname{atanh}(a\,x)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^3)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x\operatorname{atanh}^3(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

3.244 $\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2} dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

```
[Out] 1/4*arctanh(a*x)^4/a
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^4/(4*a)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2} dx = \frac{\tanh^{-1}(ax)^4}{4a}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^4/(4*a)
```

fricas [A] time = 0.75, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

giac [A] time = 0.14, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
[Out] 1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$\frac{\operatorname{arctanh}(ax)^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1),x)
[Out] 1/4*arctanh(a*x)^4/a
```

maxima [B] time = 0.33, size = 209, normalized size = 16.08

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^3 + \frac{1}{64}a\left(\frac{8\left(\log(ax+1)^3 - 3\log(ax+1)^2\log(ax-1) + 3\log(ax+1\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^3 + 1/64*a*(8*(log(a*x +
 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)^2 - log
(a*x - 1)^3)*arctanh(a*x)/a^2 - (log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(a*x
- 1) + 6*log(a*x + 1)^2*log(a*x - 1)^2 - 4*log(a*x + 1)*log(a*x - 1)^3 + lo
g(a*x - 1)^4)/a^2) - 3/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) + lo
g(a*x - 1)^2)*arctanh(a*x)^2/a
```

mupad [B] time = 0.90, size = 90, normalized size = 6.92

$$\frac{\ln(a\,x+1)^4}{64\,a} + \frac{\ln(1-a\,x)^4}{64\,a} - \frac{\ln(a\,x+1)\,\ln(1-a\,x)^3}{16\,a} - \frac{\ln(a\,x+1)^3\,\ln(1-a\,x)}{16\,a} + \frac{3\ln(a\,x+1)^2\ln(1-a\,x)^2}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(a^2*x^2 - 1),x)
[Out] log(a*x + 1)^4/(64*a) + log(1 - a*x)^4/(64*a) - (log(a*x + 1)*log(1 - a*x)^
3)/(16*a) - (log(a*x + 1)^3*log(1 - a*x))/(16*a) + (3*log(a*x + 1)^2*log(1
- a*x)^2)/(32*a)
```

sympy [A] time = 0.96, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1),x)
[Out] Piecewise((atanh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

3.245 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=91

$$-\frac{3}{4}\text{Li}_4\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-\frac{3}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{4}\tanh^{-1}(ax)^4+\log\left(2-\right.$$

```
[Out] 1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2*polylo
g(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*polylog(4,-1
+2/(a*x+1))
```

Rubi [A] time = 0.22, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5988, 5932, 5948, 6056, 6060, 6610}

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*
PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*
x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])/4
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
```

```
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx = \frac{1}{4}\tanh^{-1}(ax)^4 + \int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx$$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - (3a)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx$$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right) + (3a)\int$$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right) - \frac{3}{2}\tanh$$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right) - \frac{3}{2}\tanh$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.91

$$\frac{3}{2}\tanh^{-1}(ax)^2\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)-\frac{3}{2}\tanh^{-1}(ax)\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)+\frac{3}{4}\text{Li}_4\left(e^{2\tanh^{-1}(ax)}\right)-\frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\,\text{l}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

```
[Out] -1/4*ArcTanh[a*x]^4 + ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] + (3*ArcTa
nh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*PolyLog[3, E^
(2*ArcTanh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4
```

fricas [F] time = 1.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}(ax)^3}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{artanh}(ax)^3}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x), x)
```

maple [C] time = 0.57, size = 1245, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1),x)
```

```
[Out] -1/2*arctanh(a*x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a*x)^3*l
n((a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(a*x)-1/4*arctanh(a*x)^4+1/2
*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/2*I*Pi*csgn(I/(
1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3+1/4*I*arctanh(a*x)^3*Pi*csgn(I*
(a*x+1)^2/(a^2*x^2-1))^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^3*arctanh(a*x)^3+1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/
(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-1/4*I*Pi*csgn(I/(1+(a*x+1)^2/
(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^3+1/2*I*arctanh(a*x)^3*Pi*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1
)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+6*polylog(4,-(a*x+1)/(-a^2*
x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylo
g(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1
)+ln(2)*arctanh(a*x)^3+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arct
anh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^3-6*arcta
nh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I/(1+(a*x+1)^2/
(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*a
rctanh(a*x)^3+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*arctanh(a*x)^3+1/2*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctan
h(a*x)^3-1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*
((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/2*I*arctanh(a*x)
^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))^2+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+
1)^(1/2))+3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\log\left(ax+1\right)\log\left(-ax+1\right)^{3}+\frac{1}{64}\log\left(-ax+1\right)^{4}-\frac{1}{8}\int\frac{3\left(a^{2}x^{2}+ax+2\right)\log\left(ax+1\right)\log\left(-ax+1\right)^{2}+2\log}{2\left(a^{2}x^{3}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/16*log(a*x + 1)*log(-a*x + 1)^3 + 1/64*log(-a*x + 1)^4 - 1/8*integrate(1/
2*(3*(a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1)^2 + 2*log(a*x + 1)^3 -
6*log(a*x + 1)^2*log(-a*x + 1))/(a^2*x^3 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\mathrm{atanh}\left(a\,x\right)^{3}}{x\,\left(a^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^3/(x*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^3(ax)}{a^2x^3 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**3 - x), x)
```

3.246 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)}\,dx$

Optimal. Leaf size=90

$$-\frac{3}{2}a\text{Li}_3\left(\frac{2}{ax+1}-1\right)-3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{4}a\tanh^{-1}(ax)^4+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+3a\log\left(2\right.$$

```
[Out] a*arctanh(a*x)^3-arctanh(a*x)^3/x+1/4*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*l
n(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+
2/(a*x+1))
```

Rubi [A] time = 0.27, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5982, 5916, 5988, 5932, 5948, 6056, 6610}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{4}a\tanh^{-1}(ax)^4+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/x + (a*ArcTanh[a*x]^4)/4 + 3*a*ArcTanh[a*
x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] -
 (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
```

```
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)^3}{x}+\frac{1}{4}a\tanh^{-1}(ax)^4+(3a)\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx\\ &= a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{1}{4}a\tanh^{-1}(ax)^4+(3a)\int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx\\ &= a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{1}{4}a\tanh^{-1}(ax)^4+3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-\left(6a^2\right)\\ &= a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{1}{4}a\tanh^{-1}(ax)^4+3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-3a\tan\\ &= a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{1}{4}a\tanh^{-1}(ax)^4+3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-3a\tan\end{aligned}$$

Mathematica [C] time = 0.21, size = 93, normalized size = 1.03

$$-a\left(-3\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)-\frac{1}{4}\tanh^{-1}(ax)^4+\frac{\tanh^{-1}(ax)^3}{ax}+\tanh^{-1}(ax)^3-3\tanh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(a*((-1/8*I)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - ArcTanh[a*x]^4
/4 - 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[
2, E^(2*ArcTanh[a*x])] + (3*PolyLog[3, E^(2*ArcTanh[a*x])])/2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\operatorname{artanh}(ax)^3}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^2), x)
```

maple [C] time = 0.62, size = 826, normalized size = 9.18

$$-\frac{\operatorname{arctanh}(ax)^3}{x}-\frac{a\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}+\frac{a\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}+3a\operatorname{arctanh}(ax)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x)
```

```
[Out] -arctanh(a*x)^3/x-1/2*a*arctanh(a*x)^3*ln(a*x-1)+1/2*a*arctanh(a*x)^3*ln(a*
x+1)+3*a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-a*arc
tanh(a*x)^3+1/4*a*arctanh(a*x)^4+1/2*I*a*Pi*arctanh(a*x)^3+1/2*I*a*Pi*csgn(
I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/4*I*a*Pi*csgn(I/(1+(a*x+1)
^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^
2*arctanh(a*x)^3-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,-(
a*x+1)/(-a^2*x^2+1)^(1/2))-a*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-
1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arcta
nh(a*x)^3-1/4*I*a*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/4*I*a
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^2*arctanh(a*x)^3+1/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1
)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))*arctanh(a*x)^3-1/2*I*a*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-1/2*I*a*Pi*csgn(I/(1+(a*x+
1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3-1/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ax\log(-ax+1)^4-4\left(ax\log(ax+1)+2\,ax-2\right)\log(-ax+1)^3+6\left(ax\log(ax+1)^2-4\,(ax+1)\log(ax+1)\right.}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(a*x*log(-a*x + 1)^4 - 4*(a*x*log(a*x + 1) + 2*a*x - 2)*log(-a*x + 1)^
3 + 6*(a*x*log(a*x + 1)^2 - 4*(a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/x -
1/8*integrate(1/2*(2*log(a*x + 1)^3 + 3*((a^3*x^3 + a^2*x^2 - 2)*log(a*x +
1)^2 - 4*(a^3*x^3 + 2*a^2*x^2 + a*x)*log(a*x + 1))*log(-a*x + 1))/(a^2*x^4
- x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^3(ax)}{a^2x^4 - x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**4 - x**2), x)
```

3.247 $\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=200

$$-\frac{3}{2}a^2\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{3}{4}a^2\mathrm{Li}_4\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-\frac{3}{2}a^2\mathrm{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(a$$

```
[Out] 3/2*a^2*arctanh(a*x)^2-3/2*a*arctanh(a*x)^2/x+1/2*a^2*arctanh(a*x)^3-1/2*ar
ctanh(a*x)^3/x^2+1/4*a^2*arctanh(a*x)^4+3*a^2*arctanh(a*x)*ln(2-2/(a*x+1))+
a^2*arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+2/(a*x+1))-3/2*a^2*
arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))-3/2*a^2*arctanh(a*x)*polylog(3,-1+2/
(a*x+1))-3/4*a^2*polylog(4,-1+2/(a*x+1))
```

Rubi [A] time = 0.50, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5982, 5916, 5988, 5932, 2447, 5948, 6056, 6060, 6610}

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{4}a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\ \mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3
)/2 - ArcTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^4)/4 + 3*a^2*ArcTanh[a*x]*
Log[2 - 2/(1 + a*x)] + a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*a^2*Pol
yLog[2, -1 + 2/(1 + a*x)])/2 - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 +
 a*x)])/2 - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*a^2*Po
lyLog[4, -1 + 2/(1 + a*x)])/4
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx + a^2\int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) + \frac{1}{2}(3a)\int \frac{\tanh^{-1}}{x^2}$$
$$= -\frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\tanh^{-1}(ax)$$
$$= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)$$
$$= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)$$
$$= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)$$

Mathematica [A] time = 0.46, size = 165, normalized size = 0.82

$$-\frac{1}{64}a^2\left(\frac{32(1-a^2x^2)\tanh^{-1}(ax)^3}{a^2x^2} - 96\tanh^{-1}(ax)^2\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) + 96\tanh^{-1}(ax)\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) + 96\text{Li}_2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -1/64*(a^2*(-Pi^4 - 96*ArcTanh[a*x]^2 + (96*ArcTanh[a*x]^2)/(a*x) + (32*(1
- a^2*x^2)*ArcTanh[a*x]^3)/(a^2*x^2) + 16*ArcTanh[a*x]^4 - 192*ArcTanh[a*x]
*Log[1 - E^(-2*ArcTanh[a*x])] - 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x]
)] + 96*PolyLog[2, E^(-2*ArcTanh[a*x])] - 96*ArcTanh[a*x]^2*PolyLog[2, E^(2
*ArcTanh[a*x])] + 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] - 48*PolyL
og[4, E^(2*ArcTanh[a*x])]))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}(ax)^3}{a^2x^5 - x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{artanh}(ax)^3}{(a^2x^2-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^3), x)
```

maple [B] time = 4.73, size = 406, normalized size = 2.03

$$-\frac{a^2 \operatorname{arctanh}(ax)^4}{4}+\frac{a^2 \operatorname{arctanh}(ax)^3}{2}-\frac{3a^2 \operatorname{arctanh}(ax)^2}{2}-\frac{3a \operatorname{arctanh}(ax)^2}{2x}-\frac{\operatorname{arctanh}(ax)^3}{2x^2}+a^2 \operatorname{arctanh}(ax)^3 \ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x)
```

```
[Out] -1/4*a^2*arctanh(a*x)^4+1/2*a^2*arctanh(a*x)^3-3/2*a^2*arctanh(a*x)^2-3/2*a
*arctanh(a*x)^2/x-1/2*arctanh(a*x)^3/x^2+a^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-
a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2
))-6*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2*polylog(
4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)
^(1/2))+3*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2*ar
ctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2*polylog(4,(a*x+1)/(-
a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a
^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2x^2 \log(-ax+1)^4 + 4\left(a^2x^2 \log(ax+1)+1\right)\log(-ax+1)^3}{64\,x^2}-\frac{1}{8}\int \frac{2\,\log(ax+1)^3 - 6\,\log(ax+1)^2 \log(-ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(a^2*x^2*log(-a*x + 1)^4 + 4*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1)^
3)/x^2 - 1/8*integrate(1/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x +
1) + 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 + 2)*log(a*x + 1))*log(-a*x + 1)
^2)/(a^2*x^5 - x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\mathrm{atanh}\left(a\,x\right)^3}{x^3\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^{3}{\left(ax \right)}}{a^{2} x^{5} - x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**5 - x**3), x)
```

3.248 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{1-a^2x^2} dx$

Optimal. Leaf size=15

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

```
[Out] 2/3*arctanh(a*x)^(3/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5948}

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (2*ArcTanh[a*x]^(3/2))/(3*a)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{1-a^2x^2} dx = \frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (2*ArcTanh[a*x]^(3/2))/(3*a)
```

fricas [B] time = 0.65, size = 25, normalized size = 1.67

$$\frac{\sqrt{2}\log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*log(-(a*x + 1)/(a*x - 1))^(3/2)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\operatorname{artanh}(ax)}}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(arctanh(a*x))/(a^2*x^2 - 1), x)
```

maple [A] time = 0.04, size = 12, normalized size = 0.80

$$\frac{2\,\mathrm{arctanh}\,(ax)^{\frac{3}{2}}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x)
```

```
[Out] 2/3*arctanh(a*x)^(3/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\mathrm{artanh}\,(ax)}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1), x)
```

mupad [B] time = 0.87, size = 11, normalized size = 0.73

$$\frac{2\,\mathrm{atanh}\,(a\,x)^{3/2}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1),x)
```

```
[Out] (2*atanh(a*x)^(3/2))/(3*a)
```

sympy [A] time = 1.22, size = 14, normalized size = 0.93

$$\begin{cases} \frac{2\,\mathrm{atanh}^{\frac{3}{2}}\,(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((2*atanh(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True))
```

3.249 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
[Out] Defer[Int][x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx = \int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{x}{(a^2x^2-1)\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{(a^2x^2-1)\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-a^{2}x^{2}+1\right)\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(a^{2}x^{2}-1\right)\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)\,\left(a^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)*(a^2*x^2 - 1)),x)
```

```
[Out] -int(x/(atanh(a*x)*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{2}x^{2}\operatorname{atanh}\left(ax\right)-\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x) - atanh(a*x)), x)
```

3.250 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

[Out] ln(arctanh(a*x))/a

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5946}

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]

[Out] Log[ArcTanh[a*x]]/a

Rule 5946

```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx = \frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.03, size = 9, normalized size = 1.00

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]

[Out] Log[ArcTanh[a*x]]/a

fricas [B] time = 0.97, size = 20, normalized size = 2.22

$$\frac{\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")

[Out] log(log(-(a*x + 1)/(a*x - 1)))/a

giac [B] time = 0.22, size = 21, normalized size = 2.33

$$\frac{\log\left(\left|\log\left(-\frac{ax+1}{ax-1}\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] log(abs(log(-(a*x + 1)/(a*x - 1))))/a
```

maple [A] time = 0.02, size = 10, normalized size = 1.11

$$\frac{\ln\left(\operatorname{arctanh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] ln(arctanh(a*x))/a
```

maxima [B] time = 0.31, size = 21, normalized size = 2.33

$$\frac{\log\left(-\log\left(ax+1\right)+\log\left(-ax+1\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] log(-log(a*x + 1) + log(-a*x + 1))/a
```

mupad [B] time = 0.82, size = 9, normalized size = 1.00

$$\frac{\ln\left(\operatorname{atanh}\left(a\,x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)*(a^2*x^2 - 1)),x)
```

```
[Out] log(atanh(a*x))/a
```

sympy [A] time = 0.64, size = 7, normalized size = 0.78

$$\frac{\log\left(\operatorname{atanh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] log(atanh(a*x))/a
```

3.251 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
[Out] Defer[Int][1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx=\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

fricas [A] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^2x^3-x\right)\text{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)x\,\text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2}x^{2}+1\right)\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^{2}x^{2}-1\right)x\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate(1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\,\left(a^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)*(a^2*x^2 - 1)),x)
[Out] -int(1/(x*atanh(a*x)*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2}x^{3}\operatorname{atanh}\left(ax\right)-x\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x),x)
[Out] -Integral(1/(a**2*x**3*atanh(a*x) - x*atanh(a*x)), x)
```

3.252 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=26

$$\frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a}-\frac{x}{a\tanh^{-1}(ax)}$$

```
[Out] -x/a/arctanh(a*x)+Unintegrable(1/arctanh(a*x),x)/a
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
[Out] -(x/(a*ArcTanh[a*x])) + Defer[Int][ArcTanh[a*x]^(-1), x]/a
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2} dx = -\frac{x}{a\tanh^{-1}(ax)}+\frac{\int \frac{1}{\tanh^{-1}(ax)} dx}{a}$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x}{(a^2x^2-1)\,\text{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{(a^2x^2-1)\,\text{artanh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-a^{2} x^{2}+1\right) \operatorname{arctanh}\left(a x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x}{a \log\left(a x+1\right)-a \log\left(-a x+1\right)}-2 \int -\frac{1}{a \log\left(a x+1\right)-a \log\left(-a x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x/(a*log(a*x + 1) - a*log(-a*x + 1)) - 2*integrate(-1/(a*log(a*x + 1) -
a*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^{2}\,\left(a^{2}\,x^{2}-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(x/(atanh(a*x)^2*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{2} x^{2} \operatorname{atanh}^{2}\left(a x\right)-\operatorname{atanh}^{2}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

3.253 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{a\tanh^{-1}(ax)}$$

```
[Out] -1/a/arctanh(a*x)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$-\frac{1}{a\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*ArcTanh[a*x]))
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\tanh^{-1}(ax)}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{1}{a\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*ArcTanh[a*x]))
```

fricas [A] time = 0.50, size = 22, normalized size = 2.00

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1)))
```

giac [A] time = 0.19, size = 22, normalized size = 2.00

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1)))
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$-\frac{1}{a \operatorname{arctanh}(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] -1/a/arctanh(a*x)
```

maxima [B] time = 0.33, size = 23, normalized size = 2.09

$$-\frac{2}{a \log(ax+1) - a \log(-ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/(a*log(a*x + 1) - a*log(-a*x + 1))
```

mupad [B] time = 0.78, size = 23, normalized size = 2.09

$$-\frac{2}{a\,\left(\ln\left(a\,x+1\right)-\ln\left(1-a\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^2*(a^2*x^2 - 1)),x)
```

```
[Out] -2/(a*(log(a*x + 1) - log(1 - a*x)))
```

sympy [A] time = 0.93, size = 8, normalized size = 0.73

$$-\frac{1}{a \operatorname{atanh}(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -1/(a*atanh(a*x))
```

3.254 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{\text{Int}\left(\frac{1}{x^2\tanh^{-1}(ax)},x\right)}{a}-\frac{1}{ax\tanh^{-1}(ax)}$$

```
[Out] -1/a/x/arctanh(a*x)-Unintegrable(1/x^2/arctanh(a*x),x)/a
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
[Out] -(1/(a*x*ArcTanh[a*x])) - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx=-\frac{1}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)}\,dx}{a}$$

Mathematica [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{(a^2x^3-x)\,\text{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2x^2-1)x\,\text{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^2), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)\operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{ax\log\left(ax+1\right)-ax\log\left(-ax+1\right)}+2\int -\frac{1}{ax^2\log\left(ax+1\right)-ax^2\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) + 2*integrate(-1/(a*x^2*log(a*x +
 1) - a*x^2*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^3\operatorname{atanh}^2\left(ax\right)-x\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**2*x**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

3.255 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=31

$$\frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)^2},x\right)}{2a}-\frac{x}{2a\tanh^{-1}(ax)^2}$$

```
[Out] -1/2*x/a/arctanh(a*x)^2+1/2*Unintegrable(1/arctanh(a*x)^2,x)/a
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] -x/(2*a*ArcTanh[a*x]^2) + Defer[Int][ArcTanh[a*x]^(-2), x]/(2*a)
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx=-\frac{x}{2a\tanh^{-1}(ax)^2}+\frac{\int \frac{1}{\tanh^{-1}(ax)^2}\,dx}{2a}$$

Mathematica [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-a^{2}x^{2}+1\right)\operatorname{arctanh}\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,ax-\left(a^{2}x^{2}-1\right)\log\left(ax+1\right)+\left(a^{2}x^{2}-1\right)\log\left(-ax+1\right)}{a^{2}\log\left(ax+1\right)^{2}-2\,a^{2}\log\left(ax+1\right)\log\left(-ax+1\right)+a^{2}\log\left(-ax+1\right)^{2}}+2\int-\frac{x}{\log\left(ax+1\right)-\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*lo
g(a*x + 1)^2 - 2*a^2*log(a*x + 1)*log(-a*x + 1) + a^2*log(-a*x + 1)^2) + 2*
integrate(-x/(log(a*x + 1) - log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^{3}\,\left(a^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(x/(atanh(a*x)^3*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{2}x^{2}\operatorname{atanh}^{3}\left(ax\right)-\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

3.256 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

[Out] -1/2/a/arctanh(a*x)^2

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]

[Out] -1/(2*a*ArcTanh[a*x]^2)

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\tanh^{-1}(ax)^2}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]

[Out] -1/2*1/(a*ArcTanh[a*x]^2)

fricas [A] time = 0.42, size = 22, normalized size = 1.69

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")

[Out] -2/(a*log(-(a*x + 1)/(a*x - 1))^2)

giac [A] time = 0.15, size = 22, normalized size = 1.69

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1))^2)
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$-\frac{1}{2a\operatorname{arctanh}(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x)^3,x)
[Out] -1/2/a/arctanh(a*x)^2
```

maxima [B] time = 0.33, size = 42, normalized size = 3.23

$$-\frac{2}{a\log(ax+1)^2 - 2\,a\log(ax+1)\log(-ax+1) + a\log(-ax+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
[Out] -2/(a*log(a*x + 1)^2 - 2*a*log(a*x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2)
```

mupad [B] time = 0.80, size = 23, normalized size = 1.77

$$-\frac{2}{a\,(\ln(a\,x+1) - \ln(1-a\,x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^3*(a^2*x^2 - 1)),x)
[Out] -2/(a*(log(a*x + 1) - log(1 - a*x))^2)
```

sympy [A] time = 1.12, size = 12, normalized size = 0.92

$$-\frac{1}{2a\operatorname{atanh}^2(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x)**3,x)
[Out] -1/(2*a*atanh(a*x)**2)
```

3.257 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=37

$$-\frac{\text{Int}\left(\frac{1}{x^2\tanh^{-1}(ax)^2},x\right)}{2a}-\frac{1}{2ax\tanh^{-1}(ax)^2}$$

```
[Out] -1/2/a/x/arctanh(a*x)^2-1/2*Unintegrable(1/x^2/arctanh(a*x)^2,x)/a
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - Defer[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx=-\frac{1}{2ax\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)^2}\,dx}{2a}$$

Mathematica [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{(a^2x^3-x)\,\text{artanh}(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2x^2-1)x\,\text{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^3), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)\operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,ax+\left(a^2x^2-1\right)\log(ax+1)-\left(a^2x^2-1\right)\log(-ax+1)}{a^2x^2\log(ax+1)^2-2\,a^2x^2\log(ax+1)\log(-ax+1)+a^2x^2\log(-ax+1)^2}-2\int -\frac{1}{a^2x^3\log(ax+1)-a^2x^3\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*x^
2*log(a*x + 1)^2 - 2*a^2*x^2*log(a*x + 1)*log(-a*x + 1) + a^2*x^2*log(-a*x
+ 1)^2) - 2*integrate(-1/(a^2*x^3*log(a*x + 1) - a^2*x^3*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{x\operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^3\operatorname{atanh}^3(ax)-x\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**2*x**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

3.258 $\int \frac{\tanh^{-1}(ax)^p}{1-a^2x^2}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^{-1}(ax)^{p+1}}{a(p+1)}$$

```
[Out] arctanh(a*x)^(1+p)/a/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^{p+1}}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^p/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^(1 + p)/(a*(1 + p))
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^p}{1-a^2x^2}\,dx = \frac{\tanh^{-1}(ax)^{1+p}}{a(1+p)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^{p+1}}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^p/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^(1 + p)/(a*(1 + p))
```

fricas [B] time = 0.51, size = 84, normalized size = 4.94

$$\frac{\cosh\left(p\log\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)+\log\left(-\frac{ax+1}{ax-1}\right)\sinh\left(p\log\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(p*log(1/2*log(-(a*x + 1)/(a*x - 1))))*log(-(a*x + 1)/(a*x - 1)) +
 log(-(a*x + 1)/(a*x - 1))*sinh(p*log(1/2*log(-(a*x + 1)/(a*x - 1)))))/(a*p
 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-arctanh(a*x)^p/(a^2*x^2 - 1), x)
```

maple [A] time = 0.02, size = 18, normalized size = 1.06

$$\frac{\operatorname{arctanh}(ax)^{1+p}}{a(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^p/(-a^2*x^2+1),x)
[Out] arctanh(a*x)^(1+p)/a/(1+p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{artanh}(ax)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="maxima")
[Out] -integrate(arctanh(a*x)^p/(a^2*x^2 - 1), x)
```

mupad [B] time = 1.05, size = 33, normalized size = 1.94

$$\begin{cases} \frac{\ln(\operatorname{atanh}(a\,x))}{a} & \text{if } p=-1 \\ \frac{\operatorname{atanh}(a\,x)^{p+1}}{a\,(p+1)} & \text{if } p\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^p/(a^2*x^2 - 1),x)
[Out] piecewise(p == -1, log(atanh(a*x))/a, p ~= -1, atanh(a*x)^(p + 1)/(a*(p + 1
)))
```

sympy [A] time = 1.53, size = 26, normalized size = 1.53

$$\begin{cases} \frac{\begin{cases} \frac{\operatorname{atanh}^{p+1}(ax)}{p+1} & \text{for } p\neq -1 \\ \log(\operatorname{atanh}(ax)) & \text{otherwise} \end{cases}}{a} & \text{for } a\neq 0 \\ 0^p x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**p/(-a**2*x**2+1),x)
[Out] Piecewise((Piecewise((atanh(a*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(atanh(a
*x)), True))/a, Ne(a, 0)), (0**p*x, True))
```

3.259 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=109

$$-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^4}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{\tanh^{-1}(ax)}{4a^4}-\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)}-\frac{x}{4a^3\left(1-a^2x^2\right)}$$

```
[Out] -1/4*x/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)/a^4+1/2*arctanh(a*x)/a^4/(-a^2*x^2
+1)+1/2*arctanh(a*x)^2/a^4-arctanh(a*x)*ln(2/(-a*x+1))/a^4-1/2*polylog(2,1-
2/(-a*x+1))/a^4
```

Rubi [A] time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6028, 5984, 5918, 2402, 2315, 5994, 199, 206}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{x}{4a^3\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{\tanh^{-1}(ax)}{4a^4}-\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -x/(4*a^3*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^4) + ArcTanh[a*x]/(2*a^4*(1 -
a^2*x^2)) + ArcTanh[a*x]^2/(2*a^4) - (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4 -
PolyLog[2, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

```
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx &= \frac{\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}\\
&= \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx}{2a^3} - \frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^3}\\
&= -\frac{x}{4a^3\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\int \frac{1}{1-a^2x^2}\,dx}{4a^3} +\\
&= -\frac{x}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} -\\
&= -\frac{x}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} -
\end{aligned}$$

Mathematica [A] time = 0.17, size = 64, normalized size = 0.59

$$-\frac{-4\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 4\tanh^{-1}(ax)^2 + \sinh\left(2\tanh^{-1}(ax)\right) - 2\tanh^{-1}(ax)\left(\cosh\left(2\tanh^{-1}(ax)\right) - 4\log\left(e^{-2}\right.\right.}{8a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/8*(4*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 4*Log[1 + E
^(-2*ArcTanh[a*x])]) - 4*PolyLog[2, -E^(-2*ArcTanh[a*x])] + Sinh[2*ArcTanh[
a*x]])/a^4
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 \operatorname{artanh}(ax)}{a^4x^4 - 2a^2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{artanh}(ax)}{\left(a^2x^2 - 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

maple [B] time = 0.06, size = 203, normalized size = 1.86

$$-\frac{\operatorname{arctanh}(ax)}{4a^4(ax-1)} + \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2a^4} + \frac{\operatorname{arctanh}(ax)}{4a^4(ax+1)} + \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2a^4} + \frac{\ln(ax-1)^2}{8a^4} - \frac{\operatorname{dilog}\left(\frac{1}{2} + \frac{a}{2}\right.}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4/a^4*arctanh(a*x)/(a*x-1)+1/2/a^4*arctanh(a*x)*ln(a*x-1)+1/4/a^4*arctan
h(a*x)/(a*x+1)+1/2/a^4*arctanh(a*x)*ln(a*x+1)+1/8/a^4*ln(a*x-1)^2-1/2/a^4*d
ilog(1/2+1/2*a*x)-1/4/a^4*ln(a*x-1)*ln(1/2+1/2*a*x)-1/8/a^4*ln(a*x+1)^2+1/4
/a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/4/a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/
8/a^4/(a*x-1)+1/8/a^4*ln(a*x-1)+1/8/a^4/(a*x+1)-1/8/a^4*ln(a*x+1)
```

maxima [A] time = 0.33, size = 177, normalized size = 1.62

$$-\frac{1}{8}a\left(\frac{\left(a^2x^2-1\right)\log(ax+1)^2 - 2\left(a^2x^2-1\right)\log(ax+1)\log(ax-1) - \left(a^2x^2-1\right)\log(ax-1)^2 - 2\,ax - \left(a^2x^2\right.}{a^7x^2 - a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*
x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2*a*x - (a^2*x^2 - 1)*log(a*x - 1))
/(a^7*x^2 - a^5) + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/
2))/a^5 + log(a*x + 1)/a^5) - 1/2*(1/(a^6*x^2 - a^4) - log(a^2*x^2 - 1)/a^4
)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atanh}(a\,x)}{\left(a^2\,x^2 - 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

```
[Out] int((x^3*atanh(a*x))/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.260 $\int \frac{x^2 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2} \, dx$

Optimal. Leaf size=57

$$-\frac{\tanh^{-1}(ax)^2}{4a^3}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{1}{4a^3\left(1-a^2x^2\right)}$$

```
[Out] -1/4/a^3/(-a^2*x^2+1)+1/2*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-1/4*arctanh(a*x)^
2/a^3
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5998, 5948}

$$-\frac{1}{4a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/(4*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*a^2*(1 - a^2*x^2)) - ArcTanh
[a*x]^2/(4*a^3)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx &= -\frac{1}{4a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{2a^2}\\
&= -\frac{1}{4a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 0.79

$$\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2-2ax\tanh^{-1}(ax)+1}{4a^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (1 - a^2*x^2)*ArcTanh[a*x]^2)/(4*a^3*(-1 + a^2*x^
2))
```

fricas [A] time = 0.72, size = 65, normalized size = 1.14

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
- 1))^2 - 4)/(a^5*x^2 - a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

maple [B] time = 0.06, size = 169, normalized size = 2.96

$$-\frac{\operatorname{arctanh}(ax)}{4a^3(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4a^3}-\frac{\operatorname{arctanh}(ax)}{4a^3(ax+1)}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4a^3}+\frac{\ln(ax+1)^2}{16a^3}+\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4/a^3*arctanh(a*x)/(a*x-1)+1/4/a^3*arctanh(a*x)*ln(a*x-1)-1/4/a^3*arctan
h(a*x)/(a*x+1)-1/4/a^3*arctanh(a*x)*ln(a*x+1)+1/16/a^3*ln(a*x+1)^2+1/8/a^3*
ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/8/a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/16/a^
3*ln(a*x-1)^2-1/8/a^3*ln(a*x-1)*ln(1/2+1/2*a*x)+1/8/a^3/(a*x-1)-1/8/a^3/(a*
x+1)
```

maxima [B] time = 0.32, size = 126, normalized size = 2.21

$$-\frac{1}{4}\left(\frac{2\,x}{a^4x^2-a^2}+\frac{\log(ax+1)}{a^3}-\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)+\frac{\left(\left(a^2x^2-1\right)\log(ax+1)^2-2\left(a^2x^2-1\right)\log(ax+1)\log\right.}{16\left(a^6x^2-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x) + 1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(
a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a/(a^6*x^2 - a^4)
```

mupad [B] time = 0.96, size = 110, normalized size = 1.93

$$\ln(1-a\,x)\left(\frac{\ln(a\,x+1)}{8\,a^3}+\frac{x}{2\,a^2\left(2\,a^2\,x^2-2\right)}\right)-\frac{\ln(a\,x+1)^2}{16\,a^3}-\frac{\ln(1-a\,x)^2}{16\,a^3}-\frac{1}{2\,a^2\left(2\,a-2\,a^3\,x^2\right)}-\frac{x\,\ln(a\,x+1)}{4\,a^3\left(a\,x^2-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(1 - a*x)*(log(a*x + 1)/(8*a^3) + x/(2*a^2*(2*a^2*x^2 - 2))) - log(a*x +
 1)^2/(16*a^3) - log(1 - a*x)^2/(16*a^3) - 1/(2*a^2*(2*a - 2*a^3*x^2)) - (x
*log(a*x + 1))/(4*a^3*(a*x^2 - 1/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}(ax)}{(ax-1)^2 (ax+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.261 $\int \frac{x \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2} \, dx$

Optimal. Leaf size=55

$$-\frac{x}{4a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{4a^2}$$

```
[Out] -1/4*x/a/(-a^2*x^2+1)-1/4*arctanh(a*x)/a^2+1/2*arctanh(a*x)/a^2/(-a^2*x^2+1
)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5994, 199, 206}

$$-\frac{x}{4a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -x/(4*a*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^2) + ArcTanh[a*x]/(2*a^2*(1 - a^
2*x^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx = \frac{\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx}{2a}$$
$$= -\frac{x}{4a\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)} - \frac{\int \frac{1}{1-a^2x^2}\,dx}{4a}$$
$$= -\frac{x}{4a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{4a^2} + \frac{\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 1.20

$$\frac{-a^2x^2\log(ax+1)+\left(a^2x^2-1\right)\log(1-ax)+2ax+\log(ax+1)-4\tanh^{-1}(ax)}{8a^2\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] (2*a*x - 4*ArcTanh[a*x] + (-1 + a^2*x^2)*Log[1 - a*x] + Log[1 + a*x] - a^2*
x^2*Log[1 + a*x])/(8*a^2*(-1 + a^2*x^2))
```

fricas [A] time = 0.40, size = 48, normalized size = 0.87

$$\frac{2\,ax-\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{8\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

giac [B] time = 0.18, size = 154, normalized size = 2.80

$$-\frac{1}{16}\left(\left(\frac{ax+1}{(ax-1)a^3}+\frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)-\frac{ax+1}{(ax-1)a^3}+\frac{ax-1}{(ax+1)a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/16*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*((a*
x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x -
 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) - (a*x + 1)/((a*x - 1)*a^3) + (a
*x - 1)/((a*x + 1)*a^3))*a
```

maple [A] time = 0.04, size = 68, normalized size = 1.24

$$-\frac{\operatorname{arctanh}(ax)}{2a^2\left(a^2x^2-1\right)}+\frac{1}{8a^2(ax-1)}+\frac{\ln(ax-1)}{8a^2}+\frac{1}{8a^2(ax+1)}-\frac{\ln(ax+1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2/a^2/(a^2*x^2-1)*arctanh(a*x)+1/8/a^2/(a*x-1)+1/8/a^2*ln(a*x-1)+1/8/a^2
/(a*x+1)-1/8/a^2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 62, normalized size = 1.13

$$\frac{\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}}{8a} - \frac{\operatorname{artanh}(ax)}{2\left(a^2x^2-1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)/a - 1/2*arctanh(a
*x)/((a^2*x^2 - 1)*a^2)
```

mupad [B] time = 0.91, size = 37, normalized size = 0.67

$$-\frac{\operatorname{atanh}(a\,x)}{4\,a^2} - \frac{\frac{\operatorname{atanh}(a\,x)}{2} - \frac{a\,x}{4}}{a^2\,\left(a^2\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

```
[Out] - atanh(a*x)/(4*a^2) - (atanh(a*x)/2 - (a*x)/4)/(a^2*(a^2*x^2 - 1))
```

sympy [A] time = 1.31, size = 61, normalized size = 1.11

$$\begin{cases} -\frac{a^2x^2\operatorname{atanh}(ax)}{4a^4x^2-4a^2} + \frac{ax}{4a^4x^2-4a^2} - \frac{\operatorname{atanh}(ax)}{4a^4x^2-4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Piecewise((-a**2*x**2*atanh(a*x)/(4*a**4*x**2 - 4*a**2) + a*x/(4*a**4*x**2
- 4*a**2) - atanh(a*x)/(4*a**4*x**2 - 4*a**2), Ne(a, 0)), (0, True))
```

3.262 $\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{4a}$$

```
[Out] -1/4/a/(-a^2*x^2+1)+1/2*x*arctanh(a*x)/(-a^2*x^2+1)+1/4*arctanh(a*x)^2/a
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5956, 261}

$$-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/(4*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^
2/(4*a)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx &= \frac{x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{4a}-\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx\\ &= -\frac{1}{4a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{4a}\end{aligned}$$

Mathematica [A] time = 0.07, size = 44, normalized size = 0.81

$$\frac{\left(a^2x^2-1\right)\tanh^{-1}(ax)^2-2ax\tanh^{-1}(ax)+1}{4a\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (-1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a*(-1 + a^2*x^2
))
```

fricas [A] time = 0.56, size = 64, normalized size = 1.19

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
- 1))^2 - 4)/(a^3*x^2 - a)
```

giac [B] time = 2.01, size = 255, normalized size = 4.72

$$\frac{1}{8}\,a^2\left(\frac{(ax-1)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}-1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}+1}\right)}{(ax+1)a^4}+\frac{ax-1}{(ax+1)a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*((a*x - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(
a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)
/((a*x + 1)*a/(a*x - 1) - a) - 1)) - 1)/(a*((a*x + 1)/(a*x - 1) + 1)/(a - a
*(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1
)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + 1))/((a*x + 1)*a^4) +
(a*x - 1)/((a*x + 1)*a^4))
```

maple [B] time = 0.06, size = 169, normalized size = 3.13

$$-\frac{\operatorname{arctanh}(ax)}{4a(ax-1)}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4a}-\frac{\operatorname{arctanh}(ax)}{4a(ax+1)}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4a}-\frac{\ln(ax-1)^2}{16a}+\frac{\ln(ax-1)\ln\left(\frac{1}{2}\right.}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4/a*arctanh(a*x)/(a*x-1)-1/4/a*arctanh(a*x)*ln(a*x-1)-1/4/a*arctanh(a*x)
/(a*x+1)+1/4/a*arctanh(a*x)*ln(a*x+1)-1/16/a*ln(a*x-1)^2+1/8/a*ln(a*x-1)*ln
(1/2+1/2*a*x)-1/16/a*ln(a*x+1)^2+1/8/a*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/8/a*ln(
-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/8/a/(a*x-1)-1/8/a/(a*x+1)
```

maxima [B] time = 0.31, size = 122, normalized size = 2.26

$$-\frac{1}{4}\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)-\frac{\left((a^2x^2-1)\log(ax+1)^2-2\left(a^2x^2-1\right)\log(ax+1)\right.}{16\left(a^4x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x) - 1
/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x -
1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a/(a^4*x^2 - a^2)
```

mupad [B] time = 0.97, size = 106, normalized size = 1.96

$$\frac{\ln(a\,x+1)^2}{16\,a}-\ln(1-a\,x)\left(\frac{\ln(a\,x+1)}{8\,a}-\frac{x}{2\,\left(2\,a^2\,x^2-2\right)}\right)+\frac{\ln(1-a\,x)^2}{16\,a}+\frac{1}{2\,a\,\left(2\,a^2\,x^2-2\right)}-\frac{x\,\ln(a\,x+1)}{4\,a\,\left(a\,x^2-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(a*x + 1)^2/(16*a) - log(1 - a*x)*(log(a*x + 1)/(8*a) - x/(2*(2*a^2*x^2
- 2))) + log(1 - a*x)^2/(16*a) + 1/(2*a*(2*a^2*x^2 - 2)) - (x*log(a*x + 1))
/(4*a*(a*x^2 - 1/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.263 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^2} dx$

Optimal. Leaf size=91

$$-\frac{ax}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}-\frac{1}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{4}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

```
[Out] -1/4*a*x/(-a^2*x^2+1)-1/4*arctanh(a*x)+1/2*arctanh(a*x)/(-a^2*x^2+1)+1/2*ar
ctanh(a*x)^2+arctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6030, 5988, 5932, 2447, 5994, 199, 206}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{ax}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{4}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] -(a*x)/(4*(1 - a^2*x^2)) - ArcTanh[a*x]/4 + ArcTanh[a*x]/(2*(1 - a^2*x^2))
+ ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/
(1 + a*x)]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
```

```
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx$$

$$= \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 - \frac{1}{2}a\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx$$

$$= -\frac{ax}{4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{4}a\int \frac{}{1}$$

$$= -\frac{ax}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{}{1}\right.$$

Mathematica [A] time = 0.18, size = 63, normalized size = 0.69

$$\frac{1}{8}\left(-4\text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right) + 4\tanh^{-1}(ax)^2 - \sinh\left(2\tanh^{-1}(ax)\right) + 2\tanh^{-1}(ax)\left(4\log\left(1-e^{-2\tanh^{-1}(ax)}\right) + \cosh\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (4*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] + 4*Log[1 - E^(-2*
ArcTanh[a*x])]) - 4*PolyLog[2, E^(-2*ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]])
/8
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(ax)}{a^4x^5 - 2a^2x^3 + x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x), x)
```

maple [B] time = 0.07, size = 190, normalized size = 2.09

$$\operatorname{arctanh}(ax)\ln(ax) - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} + \frac{\operatorname{arctanh}(ax)}{4ax+4} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} - \frac{\operatorname{dilog}(ax)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^2,x)
```

```
[Out] arctanh(a*x)*ln(a*x)-1/4*arctanh(a*x)/(a*x-1)-1/2*arctanh(a*x)*ln(a*x-1)+1/
4*arctanh(a*x)/(a*x+1)-1/2*arctanh(a*x)*ln(a*x+1)-1/2*dilog(a*x)-1/2*dilog(
a*x+1)-1/2*ln(a*x)*ln(a*x+1)-1/8*ln(a*x-1)^2+1/2*dilog(1/2+1/2*a*x)+1/4*ln(
a*x-1)*ln(1/2+1/2*a*x)+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2+1/2*a*x))*ln(-
1/2*a*x+1/2)+1/8/(a*x-1)+1/8*ln(a*x-1)+1/8/(a*x+1)-1/8*ln(a*x+1)
```

maxima [B] time = 0.34, size = 206, normalized size = 2.26

$$\frac{1}{8}a\left(\frac{\left(a^2x^2-1\right)\log(ax+1)^2 - 2\left(a^2x^2-1\right)\log(ax+1)\log(ax-1) - \left(a^2x^2-1\right)\log(ax-1)^2 + 2ax}{a^3x^2-a} + \frac{4\left(\log(ax\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
 - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 + 2*a*x)/(a^3*x^2 - a) + 4*(log(a*x -
1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*x + 1)*log(x) +
 dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a - log(a*x + 1)/a
+ log(a*x - 1)/a) - 1/2*(1/(a^2*x^2 - 1) + log(a^2*x^2 - 1) - log(x^2))*arc
tanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)/(x*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.264 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^2} dx$

Optimal. Leaf size=82

$$-\frac{a}{4(1-a^2x^2)}-\frac{1}{2}a\log(1-a^2x^2)+\frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)}+a\log(x)+\frac{3}{4}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] -1/4*a/(-a^2*x^2+1)-arctanh(a*x)/x+1/2*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+3/4*
a*arctanh(a*x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.15, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6030, 5982, 5916, 266, 36, 29, 31, 5948, 5956, 261}

$$-\frac{a}{4(1-a^2x^2)}-\frac{1}{2}a\log(1-a^2x^2)+\frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)}+a\log(x)+\frac{3}{4}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] -a/(4*(1 - a^2*x^2)) - ArcTanh[a*x]/x + (a^2*x*ArcTanh[a*x])/(2*(1 - a^2*x^
2)) + (3*a*ArcTanh[a*x]^2)/4 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx\\
&= \frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{4}a\tanh^{-1}(ax)^2+a^2\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx-\frac{1}{2}a^3\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tan
\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\left(1-a\right.
\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,\right.
\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-\right.
\end{aligned}$$

Mathematica [A] time = 0.14, size = 77, normalized size = 0.94

$$\frac{1}{4}\left(a\left(\frac{1}{a^2x^2-1}-2\log\left(1-a^2x^2\right)+4\log(ax)\right)-\frac{2\left(3a^2x^2-2\right)\tanh^{-1}(ax)}{x\left(a^2x^2-1\right)}+3a\tanh^{-1}(ax)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] ((-2*(-2 + 3*a^2*x^2)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)) + 3*a*ArcTanh[a*x]^2
 + a*((-1 + a^2*x^2)^(-1) + 4*Log[a*x] - 2*Log[1 - a^2*x^2]))/4
```

fricas [A] time = 0.90, size = 118, normalized size = 1.44

$$\frac{3\left(a^{3}x^{3}-ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}+4\,ax-8\left(a^{3}x^{3}-ax\right)\log\left(a^{2}x^{2}-1\right)+16\left(a^{3}x^{3}-ax\right)\log(x)-4\left(3\,a^{2}x^{2}-2\right)\log\left(\cdots\right.}{16\left(a^{2}x^{3}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3*x^3 - a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x - 8*(a^3*x^3 -
a*x)*log(a^2*x^2 - 1) + 16*(a^3*x^3 - a*x)*log(x) - 4*(3*a^2*x^2 - 2)*log(-
(a*x + 1)/(a*x - 1)))/(a^2*x^3 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\left(a^{2}x^{2}-1\right)^{2}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^2), x)
```

maple [B] time = 0.06, size = 180, normalized size = 2.20

$$-\frac{\operatorname{arctanh}(ax)}{x}-\frac{a\operatorname{arctanh}(ax)}{4(ax-1)}-\frac{3a\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{a\operatorname{arctanh}(ax)}{4(ax+1)}+\frac{3a\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{3a\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -arctanh(a*x)/x-1/4*a*arctanh(a*x)/(a*x-1)-3/4*a*arctanh(a*x)*ln(a*x-1)-1/4
*a*arctanh(a*x)/(a*x+1)+3/4*a*arctanh(a*x)*ln(a*x+1)-3/16*a*ln(a*x-1)^2+3/8
*a*ln(a*x-1)*ln(1/2+1/2*a*x)-3/16*a*ln(a*x+1)^2-3/8*a*ln(-1/2*a*x+1/2)*ln(1
/2+1/2*a*x)+3/8*a*ln(-1/2*a*x+1/2)*ln(a*x+1)+a*ln(a*x)-1/2*a*ln(a*x-1)+1/8*
a/(a*x-1)-1/2*a*ln(a*x+1)-1/8*a/(a*x+1)
```

maxima [B] time = 0.33, size = 150, normalized size = 1.83

$$-\frac{1}{16}a\left(\frac{3\left(a^{2}x^{2}-1\right)\log(ax+1)^{2}-6\left(a^{2}x^{2}-1\right)\log(ax+1)\log(ax-1)+3\left(a^{2}x^{2}-1\right)\log(ax-1)^{2}-4}{a^{2}x^{2}-1}+8\,\mathrm{lo}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/16*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)*log(a*x + 1)*log
(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2 - 1) + 8*log(a*x +
 1) + 8*log(a*x - 1) - 16*log(x)) + 1/4*(3*a*log(a*x + 1) - 3*a*log(a*x - 1
) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arctanh(a*x)
```

mupad [B] time = 1.15, size = 132, normalized size = 1.61

$$\frac{3\,a\ln\left(a\,x+1\right)^2}{16}+\frac{3\,a\ln\left(1-a\,x\right)^2}{16}+\frac{a}{2\,\left(2\,a^2\,x^2-2\right)}-\frac{a\,\ln\left(a^2\,x^2-1\right)}{2}+a\,\ln(x)-\ln\left(1-a\,x\right)\left(\frac{\frac{3\,a^2\,x^2}{2}-1}{2\,x-2\,a^2\,x^3}+\frac{3\,a\,\ln(\ldots}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] (3*a*log(a*x + 1)^2)/16 + (3*a*log(1 - a*x)^2)/16 + a/(2*(2*a^2*x^2 - 2)) -
 (a*log(a^2*x^2 - 1))/2 + a*log(x) - log(1 - a*x)*(((3*a^2*x^2)/2 - 1)/(2*x
 - 2*a^2*x^3) + (3*a*log(a*x + 1))/8) + (log(a*x + 1)*((3*a*x^2)/4 - 1/(2*a
)))/(x/a - a*x^3)
```

sympy [A] time = 2.72, size = 253, normalized size = 3.09

$$\begin{cases}\frac{4a^3x^3\log(x)}{4a^2x^3-4x}-\frac{4a^3x^3\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x}+\frac{3a^3x^3\operatorname{atanh}^2(ax)}{4a^2x^3-4x}-\frac{4a^3x^3\operatorname{atanh}(ax)}{4a^2x^3-4x}-\frac{6a^2x^2\operatorname{atanh}(ax)}{4a^2x^3-4x}-\frac{4ax\log(x)}{4a^2x^3-4x}+\frac{4ax\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x}-\frac{3ax\operatorname{atanh}^2(a\ldots}{4a^2x^3-4x} \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Piecewise((4*a**3*x**3*log(x)/(4*a**2*x**3 - 4*x) - 4*a**3*x**3*log(x - 1/a
)/(4*a**2*x**3 - 4*x) + 3*a**3*x**3*atanh(a*x)**2/(4*a**2*x**3 - 4*x) - 4*a
**3*x**3*atanh(a*x)/(4*a**2*x**3 - 4*x) - 6*a**2*x**2*atanh(a*x)/(4*a**2*x*
*3 - 4*x) - 4*a*x*log(x)/(4*a**2*x**3 - 4*x) + 4*a*x*log(x - 1/a)/(4*a**2*x
**3 - 4*x) - 3*a*x*atanh(a*x)**2/(4*a**2*x**3 - 4*x) + 4*a*x*atanh(a*x)/(4*
a**2*x**3 - 4*x) + a*x/(4*a**2*x**3 - 4*x) + 4*atanh(a*x)/(4*a**2*x**3 - 4*
x), Ne(a, 0)), (0, True))
```

3.265 $\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=123

$$-a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+a^2\tanh^{-1}(ax)^2+\frac{1}{4}a^2\tanh^{-1}(ax)+2a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{}{4\left(1\right.}$$

```
[Out] -1/2*a/x-1/4*a^3*x/(-a^2*x^2+1)+1/4*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1
/2*a^2*arctanh(a*x)/(-a^2*x^2+1)+a^2*arctanh(a*x)^2+2*a^2*arctanh(a*x)*ln(2
-2/(a*x+1))-a^2*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.36, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6030, 5982, 5916, 325, 206, 5988, 5932, 2447, 5994, 199}

$$-a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^3x}{4\left(1-a^2x^2\right)}+\frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+a^2\tanh^{-1}(ax)^2+\frac{1}{4}a^2\tanh^{-1}(ax)+2a^2\log\left(2-\frac{2}{ax\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] -a/(2*x) - (a^3*x)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x])/4 - ArcTanh[a*x]/
(2*x^2) + (a^2*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + a^2*ArcTanh[a*x]^2 + 2*a^2
*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a^2*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx$$

$$= 2\left(a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\right) + a^4\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx + 2\left(\frac{1}{2}a^2\tanh^{-1}(ax)^2 + a^2\int \right.$$

$$= -\frac{a}{2x} - \frac{a^3x}{4\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{1}{4}a^3\int \frac{1}{1-a^2x^2}\,dx + \frac{1}{2}a^3\int \frac{}{1-}$$

$$= -\frac{a}{2x} - \frac{a^3x}{4\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + 2\left(\frac{1}{2}a^2\tanh^{-1}\right.$$

Mathematica [A] time = 0.40, size = 83, normalized size = 0.67

$$\frac{1}{8}a^2\left(2\tanh^{-1}(ax)\left(-\frac{2}{a^2x^2} + 8\log\left(1-e^{-2\tanh^{-1}(ax)}\right) + \cosh\left(2\tanh^{-1}(ax)\right) + 2\right) - 8\text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right) - \frac{4}{ax} + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*(-4/(a*x) + 8*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(2 - 2/(a^2*x^2) + Cosh[
2*ArcTanh[a*x]] + 8*Log[1 - E^(-2*ArcTanh[a*x])]) - 8*PolyLog[2, E^(-2*ArcT
anh[a*x])] - Sinh[2*ArcTanh[a*x]]))/8
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(ax)}{a^4x^7 - 2a^2x^5 + x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}(ax)}{\left(a^2x^2-1\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^3), x)
```

maple [B] time = 0.07, size = 265, normalized size = 2.15

$$-\frac{\text{arctanh}(ax)}{2x^2} + 2a^2\,\text{arctanh}(ax)\ln(ax) - \frac{a^2\,\text{arctanh}(ax)}{4(ax-1)} - a^2\,\text{arctanh}(ax)\ln(ax-1) + \frac{a^2\,\text{arctanh}(ax)}{4ax+4} - a^2\,\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2*arctanh(a*x)/x^2+2*a^2*arctanh(a*x)*ln(a*x)-1/4*a^2*arctanh(a*x)/(a*x-
1)-a^2*arctanh(a*x)*ln(a*x-1)+1/4*a^2*arctanh(a*x)/(a*x+1)-a^2*arctanh(a*x)
*ln(a*x+1)-a^2*dilog(a*x)-a^2*dilog(a*x+1)-a^2*ln(a*x)*ln(a*x+1)-1/4*a^2*ln
(a*x-1)^2+a^2*dilog(1/2+1/2*a*x)+1/2*a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/4*a^2*
ln(a*x+1)^2+1/2*a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/2*a^2*ln(-1/2*a*x+1/
2)*ln(a*x+1)-1/2*a/x+1/8*a^2/(a*x-1)-1/8*a^2*ln(a*x-1)+1/8*a^2/(a*x+1)+1/8*
a^2*ln(a*x+1)
```

maxima [B] time = 0.33, size = 233, normalized size = 1.89

$$\frac{1}{8}\left(8\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a-8\left(\log\left(ax+1\right)\log(x)+\mathrm{Li}_2\left(-ax\right)\right)a+8\left(\log\left(-ax+1\right)\mathrm{l}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 8*(log
(a*x + 1)*log(x) + dilog(-a*x))*a + 8*(log(-a*x + 1)*log(x) + dilog(a*x))*a
 + a*log(a*x + 1) - a*log(a*x - 1) - 2*(a^2*x^2 - (a^3*x^3 - a*x)*log(a*x +
 1)^2 + 2*(a^3*x^3 - a*x)*log(a*x + 1)*log(a*x - 1) + (a^3*x^3 - a*x)*log(a
*x - 1)^2 - 2)/(a^2*x^3 - x))*a - 1/2*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*log(x
^2) + (2*a^2*x^2 - 1)/(a^2*x^4 - x^2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(a\,x\right)}{x^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)/(x^3*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}{\left(ax \right)}}{x^{3} \left(ax - 1\right)^{2} \left(ax + 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.266 $\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2} \, dx$

Optimal. Leaf size=161

$$\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^4}-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)^3}{3a^4}-\frac{\tanh^{-1}(ax)^2}{4a^4}-\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^4}+\frac{1}{4a^4\left(1-a^2x^2\right.}$$

```
[Out] 1/4/a^4/(-a^2*x^2+1)-1/2*x*arctanh(a*x)/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)^2
/a^4+1/2*arctanh(a*x)^2/a^4/(-a^2*x^2+1)+1/3*arctanh(a*x)^3/a^4-arctanh(a*x
)^2*ln(2/(-a*x+1))/a^4-arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^4+1/2*polylog
(3,1-2/(-a*x+1))/a^4
```

Rubi [A] time = 0.29, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6028, 5984, 5918, 5948, 6058, 6610, 5994, 5956, 261}

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}+\frac{1}{4a^4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)}+\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] 1/(4*a^4*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a^3*(1 - a^2*x^2)) - ArcTanh[
a*x]^2/(4*a^4) + ArcTanh[a*x]^2/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(3*a
^4) - (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - (ArcTanh[a*x]*PolyLog[2, 1 -
2/(1 - a*x)])/a^4 + PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a^3} - \frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^3}$$

$$= -\frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right.}{a^4}$$

$$= \frac{1}{4a^4\left(1-a^2x^2\right)} - \frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-}}{}$$

$$= \frac{1}{4a^4\left(1-a^2x^2\right)} - \frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-}}{}$$

Mathematica [A] time = 0.18, size = 103, normalized size = 0.64

$$\frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)+\frac{1}{2}\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right)-\frac{1}{3}\tanh^{-1}(ax)^3-\tanh^{-1}(ax)^2\log\left(e^{-2\tanh^{-1}(ax)}+1\right)-}{a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-1/3*ArcTanh[a*x]^3 + ((1 + 2*ArcTanh[a*x]^2)*Cosh[2*ArcTanh[a*x]])/8 - Ar
cTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcTanh[a*x]*PolyLog[2, -E^(-2*
ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x])]/2 - (ArcTanh[a*x]*Sinh[2*
ArcTanh[a*x]])/4)/a^4
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^3\,\mathrm{artanh}\,(ax)^2}{a^4x^4-2\,a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\,\mathrm{artanh}\,(ax)^2}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

maple [C] time = 0.70, size = 907, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2*I/a^4*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))+1/4*I/a^4*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/4*I/a^4*Pi
*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/4*I/a^4*Pi*arctanh(a*x)^2*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2+1/2/a^4*arctanh(a*x)
^2*ln(a*x-1)+1/4/a^4*arctanh(a*x)^2/(a*x+1)+1/2/a^4*arctanh(a*x)^2*ln(a*x+1
)-1/a^4*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/a^4*arctanh(a*x)*po
lylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/a^4*arctanh(a*x)^2*ln(2)-1/4/a^4*arctanh
(a*x)^2/(a*x-1)-1/16/a^3*x/(a*x-1)-1/16/a^3/(a*x+1)*x-1/4*I/a^4*Pi*arctanh(
a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-1/2*I/a^4*Pi*arctanh(a*x)^2*csgn(I/(
1+(a*x+1)^2/(-a^2*x^2+1)))^3-1/4*I/a^4*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/2*I/a^4*Pi*arctanh(a*x)^2*csgn(I
/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/3*arctanh(a*x)^3/a^4+1/8/a^4*arctanh(a*x)/
(a*x-1)+1/8/a^4*arctanh(a*x)/(a*x+1)-1/16/a^4/(a*x-1)+1/16/a^4/(a*x+1)+1/2/
a^4*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-1/4*arctanh(a*x)^2/a^4+1/4*I/a^4*Pi*
arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+
```

```
1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))-1/2*I/a^4*Pi*
arctanh(a*x)^2+1/8/a^3*arctanh(a*x)/(a*x-1)*x-1/8/a^3*arctanh(a*x)/(a*x+1)*
x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{4}a^3\int\frac{x^3\log(ax+1)\log(-ax+1)}{a^7x^4-2a^5x^2+a^3}\,dx-\frac{1}{4}a^2\int\frac{x^2\log(ax+1)\log(-ax+1)}{a^7x^4-2a^5x^2+a^3}\,dx-\frac{1}{32}\left(a\left(\frac{2}{a^7x-a^6}-\frac{\log(ax+1}{a^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -3/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^
3), x) - 1/4*a^2*integrate(x^2*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*
x^2 + a^3), x) - 1/32*(a*(2/(a^7*x - a^6) - log(a*x + 1)/a^6 + log(a*x - 1)
/a^6) + 4*log(-a*x + 1)/(a^7*x^2 - a^5))*a + 1/4*a*integrate(x*log(a*x + 1)
*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x) + 1/24*((a^2*x^2 - 1)*log(-a
*x + 1)^3 + 3*((a^2*x^2 - 1)*log(a*x + 1) - 1)*log(-a*x + 1)^2)/(a^6*x^2 -
a^4) + 1/4*integrate(a^3*x^3*log(a*x + 1)^2/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
 + 1/4*integrate(log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
 + 1/4*integrate(log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3\,\mathrm{atanh}\left(a\,x\right)^2}{\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

```
[Out] int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\,\mathrm{atanh}^2\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.267 $\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=94

$$-\frac{\tanh^{-1}(ax)^3}{6a^3}+\frac{\tanh^{-1}(ax)}{4a^3}+\frac{x}{4a^2\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)}$$

```
[Out] 1/4*x/a^2/(-a^2*x^2+1)+1/4*arctanh(a*x)/a^3-1/2*arctanh(a*x)/a^3/(-a^2*x^2+
1)+1/2*x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)-1/6*arctanh(a*x)^3/a^3
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6000, 5994, 199, 206}

$$\frac{x}{4a^2\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^3}{6a^3}+\frac{\tanh^{-1}(ax)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] x/(4*a^2*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a^3) - ArcTanh[a*x]/(2*a^3*(1 - a
^2*x^2)) + (x*ArcTanh[a*x]^2)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(6*a^3
)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6000

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> -Simp[(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^
2, x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2} dx = \frac{x \tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} - \frac{\int \frac{x \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2} dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x \tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^2} dx}{2a^2}$$

$$= \frac{x}{4a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x \tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{1-a^2x^2} dx}{4a^2}$$

$$= \frac{x}{4a^2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{4a^3} - \frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x \tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3}$$

Mathematica [A] time = 0.13, size = 93, normalized size = 0.99

$$\frac{-3\left(\left(a^2x^2-1\right)\log(1-ax)+\left(1-a^2x^2\right)\log(ax+1)+2ax\right)+\left(4-4a^2x^2\right)\tanh^{-1}(ax)^3-12ax\tanh^{-1}(ax)^2+12\text{ ta}}{24a^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + (4 - 4*a^2*x^2)*ArcTanh[a*x]^3 -
 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(24*
a^3*(-1 + a^2*x^2))
```

fricas [A] time = 0.57, size = 96, normalized size = 1.02

$$-\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*
x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^2 -
a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{artanh}(ax)^2}{\left(a^2x^2-1\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

maple [C] time = 0.81, size = 1722, normalized size = 18.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(
-a^2*x^2+1)^(1/2))*arctanh(a*x)^2*Pi+1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+
1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*arctanh(a*x)^2*Pi+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1
)^2/(a^2*x^2-1))^3*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2-1/4*I/a/(a*x-1)/(a*x+1
)*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+1/4*I/a/(a*x-1
)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-1/8*I/
a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+
1)^(1/2))^2*arctanh(a*x)^2*Pi-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*
arctanh(a*x)^2*Pi+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^
2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1
)))*x^2+1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2-1/8*I/a/(a*x-1)/(a*x+1)*arcta
nh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+
(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2
*x^2+1)))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2+1/8*I/a^3/(a*x-1)/(a*x+1)*
csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(
a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^2*Pi+1/8*I/a/(a
*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)/(-a^2*x^2+1)^(1/2))^2*x^2+1/4*I/a^3/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi+1/4
/a^3*arctanh(a*x)^2*ln(a*x-1)-1/4/a^3*arctanh(a*x)^2/(a*x+1)-1/4/a^3*arctan
h(a*x)^2*ln(a*x+1)+1/2/a^3*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/
4/a^3*arctanh(a*x)^2/(a*x-1)-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2*Pi+1/4*I/a^3/(a*x-1)/
(a*x+1)*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2*Pi-1/4*I/a^3/(a
*x-1)/(a*x+1)*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2*Pi-1/4*I/
a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*x^2-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a
*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*Pi-1/6/a/(a*x-1)/(a*x+1)*arctanh(a*x)
^3*x^2+1/4/a/(a*x-1)/(a*x+1)*arctanh(a*x)*x^2-1/4/a^2/(a*x-1)/(a*x+1)*x+1/6
/a^3/(a*x-1)/(a*x+1)*arctanh(a*x)^3+1/4/a^3/(a*x-1)/(a*x+1)*arctanh(a*x)
```

maxima [B] time = 0.34, size = 273, normalized size = 2.90

$$-\frac{1}{4}\left(\frac{2\,x}{a^4x^2-a^2}+\frac{\log\left(ax+1\right)}{a^3}-\frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\left(\left(a^2x^2-1\right)\log\left(ax+1\right)^3-3\left(a^2x^2-1\right)\log\left(ax+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x)^2 - 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*
log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12*a*x - 3*(2*a^2*x^2 - (a^2*
x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(a^2*x^2 - 1)*log(a*x - 1))*a
^2/(a^7*x^2 - a^5) + 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*lo
g(a*x + 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh(a*x)/
(a^6*x^2 - a^4)
```

mupad [B] time = 1.76, size = 231, normalized size = 2.46

$$\frac{\ln\left(1-a\,x\right)}{4\,a^3-4\,a^5\,x^2}-\frac{\ln\left(a\,x+1\right)^3}{48\,a^3}+\frac{\ln\left(1-a\,x\right)^3}{48\,a^3}+\frac{x}{4\,a^2-4\,a^4\,x^2}-\frac{\ln\left(a\,x+1\right)}{4\,\left(a^3-a^5\,x^2\right)}+\frac{x\ln\left(1-a\,x\right)^2}{8\,a^2-8\,a^4\,x^2}-\frac{\ln\left(a\,x+1\right)\,\ln\left(1-a\right.}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(1 - a*x)/(4*a^3 - 4*a^5*x^2) - log(a*x + 1)^3/(48*a^3) + log(1 - a*x)^3
/(48*a^3) + x/(4*a^2 - 4*a^4*x^2) - (atan(a*x*1i)*1i)/(4*a^3) - log(a*x + 1
)/(4*(a^3 - a^5*x^2)) + (x*log(1 - a*x)^2)/(8*a^2 - 8*a^4*x^2) - (log(a*x +
 1)*log(1 - a*x)^2)/(16*a^3) + (log(a*x + 1)^2*log(1 - a*x))/(16*a^3) + (x*
log(a*x + 1)^2)/(8*(a^2 - a^4*x^2)) - (x*log(a*x + 1)*log(1 - a*x))/(4*a^2
- 4*a^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.268 $\int \frac{x \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=82

$$\frac{1}{4a^2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^2}$$

```
[Out] 1/4/a^2/(-a^2*x^2+1)-1/2*x*arctanh(a*x)/a/(-a^2*x^2+1)-1/4*arctanh(a*x)^2/a
^2+1/2*arctanh(a*x)^2/a^2/(-a^2*x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5994, 5956, 261}

$$\frac{1}{4a^2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] 1/(4*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^2/(4*a^2) + ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx = \frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a}$$
$$= -\frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^2} + \frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} + \frac{1}{2}\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx$$
$$= \frac{1}{4a^2\left(1-a^2x^2\right)} - \frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^2} + \frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.52

$$\frac{\left(a^2x^2+1\right)\tanh^{-1}(ax)^2 - 2ax\tanh^{-1}(ax) + 1}{4a^2 - 4a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a^2 - 4*a^4*x^2)
```

fricas [A] time = 0.62, size = 66, normalized size = 0.80

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right) - \left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 4}{16\left(a^4x^2 - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1))^2 - 4)/(a^4*x^2 - a^2)
```

giac [A] time = 0.17, size = 140, normalized size = 1.71

$$-\frac{1}{32}\left(\left(\frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 2\left(\frac{ax+1}{(ax-1)a^3} - \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{2\,(ax+1)}{(ax-1)a^3} + \frac{2\,(a}{(ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/32*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*x +
1)/(a*x - 1))^2 - 2*((a*x + 1)/((a*x - 1)*a^3) - (a*x - 1)/((a*x + 1)*a^3))
*log(-(a*x + 1)/(a*x - 1)) + 2*(a*x + 1)/((a*x - 1)*a^3) + 2*(a*x - 1)/((a*
x + 1)*a^3))*a
```

maple [B] time = 0.06, size = 191, normalized size = 2.33

$$-\frac{\operatorname{arctanh}(ax)^2}{2a^2\left(a^2x^2-1\right)} + \frac{\operatorname{arctanh}(ax)}{4a^2(ax-1)} + \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4a^2} + \frac{\operatorname{arctanh}(ax)}{4a^2(ax+1)} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4a^2} + \frac{\ln(ax-1)^2}{16a^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2/a^2/(a^2*x^2-1)*arctanh(a*x)^2+1/4/a^2*arctanh(a*x)/(a*x-1)+1/4/a^2*ar
ctanh(a*x)*ln(a*x-1)+1/4/a^2*arctanh(a*x)/(a*x+1)-1/4/a^2*arctanh(a*x)*ln(a
*x+1)+1/16/a^2*ln(a*x-1)^2-1/8/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/16/a^2*ln(a*
x+1)^2+1/8/a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/8/a^2*ln(-1/2*a*x+1/2)*ln
(a*x+1)-1/8/a^2/(a*x-1)+1/8/a^2/(a*x+1)
```

maxima [B] time = 0.33, size = 146, normalized size = 1.78

$$\frac{\left(\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)}{4a} + \frac{\left(a^2x^2-1\right)\log(ax+1)^2 - 2\left(a^2x^2-1\right)\log(ax+1)\log(ax-1) + (}{16\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)/a +
1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x -
 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^4*x^2 - a^2) - 1/2*arctanh(a*x)^
2/((a^2*x^2 - 1)*a^2)
```

mupad [B] time = 1.16, size = 198, normalized size = 2.41

$$\ln(1-a\,x)\left(\frac{\frac{x}{2}-\frac{1}{2\,a}}{4\,a-4\,a^3\,x^2} + \frac{\frac{x}{2}+\frac{1}{2\,a}}{4\,a-4\,a^3\,x^2} + \ln(a\,x+1)\left(\frac{1}{8\,a^2} + \frac{1}{2\,a^2\,\left(2\,a^2\,x^2-2\right)}\right)\right) - \ln(1-a\,x)^2\left(\frac{1}{16\,a^2} + \frac{}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(1 - a*x)*((x/2 - 1/(2*a))/(4*a - 4*a^3*x^2) + (x/2 + 1/(2*a))/(4*a - 4*
a^3*x^2) + log(a*x + 1)*(1/(8*a^2) + 1/(2*a^2*(2*a^2*x^2 - 2)))) - log(1 -
a*x)^2*(1/(16*a^2) + 1/(2*a^2*(4*a^2*x^2 - 4))) - 1/(2*a^2*(2*a^2*x^2 - 2))
 - log(a*x + 1)^2*(1/(8*a^3*(a*x^2 - 1/a)) + 1/(16*a^2)) + (x*log(a*x + 1))
/(4*a^2*(a*x^2 - 1/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{atanh}^2(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.269 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=88

$$\frac{x}{4\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{\tanh^{-1}(ax)}{4a}$$

```
[Out] 1/4*x/(-a^2*x^2+1)+1/4*arctanh(a*x)/a-1/2*arctanh(a*x)/a/(-a^2*x^2+1)+1/2*x
*arctanh(a*x)^2/(-a^2*x^2+1)+1/6*arctanh(a*x)^3/a
```

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5956, 5994, 199, 206}

$$\frac{x}{4\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{\tanh^{-1}(ax)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

```
[Out] x/(4*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a) - ArcTanh[a*x]/(2*a*(1 - a^2*x^2))
 + (x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx &= \frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}-a\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{1}{2}\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx\\ &= \frac{x}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{1}{4}\int \frac{1}{1-a^2x^2}\,dx\\ &= \frac{x}{4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4a}-\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}\end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 1.06

$$\frac{-3\left(\left(a^2x^2-1\right)\log(1-ax)+\left(1-a^2x^2\right)\log(ax+1)+2ax\right)+4\left(a^2x^2-1\right)\tanh^{-1}(ax)^3-12ax\tanh^{-1}(ax)^2+1}{24a\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

```
[Out] (12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^3
- 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(24
*a*(-1 + a^2*x^2))
```

fricas [A] time = 0.54, size = 95, normalized size = 1.08

$$-\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 - 1)*log(-(a*x + 1)/(a*
x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^3*x^2 -
a)
```

giac [A] time = 2.08, size = 88, normalized size = 1.00

$$\frac{1}{16}\,a^2\left(\frac{(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax+1)a^4}+\frac{2\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)}{(ax+1)a^4}+\frac{2\,(ax-1)}{(ax+1)a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*a^2*((a*x - 1)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)*a^4) + 2*(a*x -
1)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)*a^4) + 2*(a*x - 1)/((a*x + 1)*a^4))
```

maple [C] time = 0.81, size = 1695, normalized size = 19.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+
(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*cs
gn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2-1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)
^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2-1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a
*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2+1/8*I/a/(a*x-1)/(a*x+1)*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a
*x)^2*Pi+1/4*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*
x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2*Pi+1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^
2*x^2+1)))*arctanh(a*x)^2*Pi-1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(
a*x)^2*Pi+1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-1/8*I
*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2-1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2-1/8*
I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1))
)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(
a*x)^2*Pi-1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2-
1/6/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3+1/4/a/(a*x-1)/(a*x+1)*arctanh(a*x)+1/8
*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I/(1+(a*x+1)^2/(-a^
2*x^2+1)))*x^2-1/4/a*arctanh(a*x)^2*ln(a*x-1)+1/4/a*arctanh(a*x)^2*ln(a*x+1
)-1/2/a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I/a/(a*x-1)/(a*x+
1)*arctanh(a*x)^2*Pi+1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2*Pi-1/4*I/a/(a*x-1)/(a*x+1)*csgn
(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2*Pi+1/8*I/a/(a*x-1)/(a*x+1)*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*Pi+1/4*I/a/(a*x-1)/(a*x+1)*c
sgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2*Pi+1/4*I*a/(a*x-1)/(a*x+
1)*arctanh(a*x)^2*Pi*x^2-1/4/(a*x-1)/(a*x+1)*x+1/6*a/(a*x-1)/(a*x+1)*arctan
h(a*x)^3*x^2+1/4*a/(a*x-1)/(a*x+1)*arctanh(a*x)*x^2-1/4/a*arctanh(a*x)^2/(a
*x-1)-1/4/a*arctanh(a*x)^2/(a*x+1)
```

maxima **[B]** time = 0.34, size = 268, normalized size = 3.05

$$-\frac{1}{4}\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2+\frac{\left(\left(a^2x^2-1\right)\log(ax+1)^3-3\left(a^2x^2-1\right)\log(ax+1)^2\,l\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2 +
 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*log(a*
x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x^2 -
1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a^2/(a^
5*x^2 - a^3) - 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x
+ 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/(a^4*x
^2 - a^2)
```

mupad **[B]** time = 1.50, size = 213, normalized size = 2.42

$$\frac{\ln(a\,x+1)^3}{48\,a}-\frac{\ln(a\,x+1)}{4\,\left(a-a^3\,x^2\right)}-\frac{\ln(1-a\,x)^3}{48\,a}-\frac{x}{4\,a^2\,x^2-4}+\frac{\ln(1-a\,x)}{4\,a-4\,a^3\,x^2}+\frac{\ln(a\,x+1)\,\ln(1-a\,x)^2}{16\,a}-\frac{\ln(a\,x+1)^2\,\ln(1\cdots}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(a*x + 1)^3/(48*a) - log(a*x + 1)/(4*(a - a^3*x^2)) - log(1 - a*x)^3/(48
*a) - x/(4*a^2*x^2 - 4) - (atan(a*x*1i)*1i)/(4*a) + log(1 - a*x)/(4*a - 4*a
^3*x^2) + (log(a*x + 1)*log(1 - a*x)^2)/(16*a) - (log(a*x + 1)^2*log(1 - a*
x))/(16*a) - (x*log(a*x + 1)^2)/(8*(a^2*x^2 - 1)) - (x*log(1 - a*x)^2)/(2*(
4*a^2*x^2 - 4)) + (x*log(a*x + 1)*log(1 - a*x))/(4*a^2*x^2 - 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.270 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=136

$$\frac{1}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{ax\tanh^{-1}(ax)}{2(1-a^2x^2)}-\frac{1}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)-\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{3}\tanh^{-1}(ax)^3-\frac{1}{4}\text{t}$$

```
[Out] 1/4/(-a^2*x^2+1)-1/2*a*x*arctanh(a*x)/(-a^2*x^2+1)-1/4*arctanh(a*x)^2+1/2*a
rctanh(a*x)^2/(-a^2*x^2+1)+1/3*arctanh(a*x)^3+arctanh(a*x)^2*ln(2-2/(a*x+1)
)-arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*polylog(3,-1+2/(a*x+1))
```

Rubi [A] time = 0.29, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6030, 5988, 5932, 5948, 6056, 6610, 5994, 5956, 261}

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{ax\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] 1/(4*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) - ArcTanh[a*x]^2
/4 + ArcTanh[a*x]^2/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*L
og[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3
, -1 + 2/(1 + a*x)]/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
```

```
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx\\
&= \frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{3}\tanh^{-1}(ax)^3-a\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx\\
&= -\frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)^2+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{3}\tanh^{-1}(ax)^3+\tanh^{-1}(ax)^2\log\Bigl(2\\
&= \frac{1}{4\left(1-a^2x^2\right)}-\frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)^2+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{3}\tanh^{-1}(ax)^3+\tanh^{-}\\
&= \frac{1}{4\left(1-a^2x^2\right)}-\frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)^2+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{3}\tanh^{-1}(ax)^3+\tanh^{-}\end{aligned}$$

Mathematica [C] time = 0.18, size = 106, normalized size = 0.78

$$\frac{1}{24}\Bigl(24\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)-12\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)-8\tanh^{-1}(ax)^3+24\tanh^{-1}(ax)^2\log\Bigl(1-e^{2\tanh^{-1}(ax}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (I*Pi^3 - 8*ArcTanh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*Cosh
[2*ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 24*ArcTa
nh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - 12*PolyLog[3, E^(2*ArcTanh[a*x])]
- 6*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/24
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)^2}{a^4x^5 - 2a^2x^3 + x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2 - 1\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x), x)
```

maple [C] time = 0.73, size = 1290, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4*arctanh(a*x)^2/(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x-1)+1/4*arctanh(a*x)^2
/(a*x+1)-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1
)^(1/2))-1/16*(a*x+1)/(a*x-1)-1/16*(a*x-1)/(a*x+1)-2*polylog(3,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3
-1/4*arctanh(a*x)^2+1/2*I*Pi*arctanh(a*x)^2-1/8*arctanh(a*x)*(a*x-1)/(a*x+1
)+1/8*(a*x+1)*arctanh(a*x)/(a*x-1)+arctanh(a*x)^2*ln(2)+arctanh(a*x)^2*ln(a
*x)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)
/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arct
anh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a
*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x
^2+1)-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)
-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))-1/4*I*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2
/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))-1/4*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I*arc
tanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x
^2-1))-1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*I*arctanh(a*x)^2
*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-1/2*I*a
rctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2
*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*I*arctanh(a*x)^2*Pi*csgn(I*(a*
x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/4*I*arctanh(a*x)
^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a
*x+1)^2/(-a^2*x^2+1)))^2+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^3*arctanh(a*x)^2-1/2*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1))
```

```
)^2*arctanh(a*x)^2+1/2*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*
x)^2+1/4*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}a^4\int \frac{x^4\log(ax+1)\log(-ax+1)}{a^4x^5-2a^2x^3+x}\,dx+\frac{1}{4}a^3\int \frac{x^3\log(ax+1)\log(-ax+1)}{a^4x^5-2a^2x^3+x}\,dx-\frac{1}{32}\left(a\left(\frac{2}{a^4x-a^3}-\frac{\log(ax+}{a^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*integrate(x^4*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x),
 x) + 1/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3
 + x), x) - 1/32*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)
 + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^2 - 1/4*a^2*integrate(x^2*log(a*x + 1
)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/4*a*integrate(x*log(a*x +
 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) + 1/4*a*integrate(x*log(-a*
x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/24*((a^2*x^2 - 1)*log(-a*x + 1)^3
+ 3*((a^2*x^2 - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^2*x^2 - 1) + 1/4*i
ntegrate(log(a*x + 1)^2/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/2*integrate(log(a
*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2}{x\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atanh}^2(ax)}{x(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.271 $\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{a^2x}{4\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}-\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+$$

```
[Out] 1/4*a^2*x/(-a^2*x^2+1)+1/4*a*arctanh(a*x)-1/2*a*arctanh(a*x)/(-a^2*x^2+1)+a
*arctanh(a*x)^2-arctanh(a*x)^2/x+1/2*a^2*x*arctanh(a*x)^2/(-a^2*x^2+1)+1/2*
a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln(2-2/(a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.32, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6030, 5982, 5916, 5988, 5932, 2447, 5948, 5956, 5994, 199, 206}

$$-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{a^2x}{4\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}-\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*x)/(4*(1 - a^2*x^2)) + (a*ArcTanh[a*x])/4 - (a*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2)/(2*
(1 - a^2*x^2)) + (a*ArcTanh[a*x]^3)/2 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x
)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx$$

$$= \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{6}a\tanh^{-1}(ax)^3 + a^2\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx - a^3\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int$$

$$= -\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{2}a\tanh^{-1}(ax)^3 + (2a)\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}$$

$$= \frac{a^2x}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{2}a\tanh$$

$$= \frac{a^2x}{4\left(1-a^2x^2\right)} + \frac{1}{4}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh}{2\left(1-\right.}$$

$$= \frac{a^2x}{4\left(1-a^2x^2\right)} + \frac{1}{4}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh}{2\left(1-\right.}$$

Mathematica [A] time = 0.32, size = 97, normalized size = 0.68

$$\frac{-8ax\text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right) + 4ax\tanh^{-1}(ax)^3 + 2\tanh^{-1}(ax)^2\left(4ax + ax\sinh\left(2\tanh^{-1}(ax)\right) - 4\right) + ax\sinh\left(2\tanh\right.}{8x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (4*a*x*ArcTanh[a*x]^3 - 2*a*x*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 8*Log[1
- E^(-2*ArcTanh[a*x])]) - 8*a*x*PolyLog[2, E^(-2*ArcTanh[a*x])] + a*x*Sinh[
2*ArcTanh[a*x]] + 2*ArcTanh[a*x]^2*(-4 + 4*a*x + a*x*Sinh[2*ArcTanh[a*x]]))
/(8*x)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(ax\right)^2}{a^4x^6 - 2\,a^2x^4 + x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^2), x)
```

maple [C] time = 0.87, size = 4589, normalized size = 32.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -a*arctanh(a*x)^2+1/8*a*arctanh(a*x)/(a*x-1)-1/8*a*arctanh(a*x)/(a*x+1)-3/8
*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+
1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh
(a*x)^2/x+3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1
)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a*x+1)/(-a^2*x
^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)
^2/(a^2*x^2-1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)*ln(1-(a*x+1)/(-
a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2+3/8*I*a*Pi*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*d
ilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,(a*x+
1)/(-a^2*x^2+1)^(1/2))-1/16*a^2*x/(a*x-1)+1/16/(a*x+1)*a^2*x-1/4*a*arctanh(
a*x)^2/(a*x-1)-3/4*a*arctanh(a*x)^2*ln(a*x-1)-1/4*a*arctanh(a*x)^2/(a*x+1)+
3/4*a*arctanh(a*x)^2*ln(a*x+1)-3/2*a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)
^(1/2))+2*a*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*arctanh(a*x)*ln
(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*d
ilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3
*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2-3/4*I*a*Pi*csgn(I/(1+(a*x+
1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*arctan
h(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3
/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylo
g(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)
))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+
3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))-3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,(a*x+1
)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog
((a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^
3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a*arctanh(a*x)^3+a*polylog(2,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-a*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+a*polylog(2,(a
*x+1)/(-a^2*x^2+1)^(1/2))+a*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*
csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^
2/(-a^2*x^2+1)))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,-(a*x+1)/
(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(
a^2*x^2-1))*arctanh(a*x)^2-3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(
I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))
^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
+3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1
)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/
2))+3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2
```

```
-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4
*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^
2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x
)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(
1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2)
)-3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2-1/16*a/(a*x-1)-1/16*a/(a*x+1
)+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))+3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog((a*x+1)/(-a^
2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2
/(-a^2*x^2+1)))^3*arctanh(a*x)^2-3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1
)))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I/(1+(a*x+1)^2
/(-a^2*x^2+1)))^2*arctanh(a*x)^2-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3
*arctanh(a*x)^2-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*
a*Pi*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*polylog(2,-(a*x+1)/(-a^2*
x^2+1)^(1/2))+3/4*I*a*Pi*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csg
n(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*polylog(2,(a*x+1)/(-a^2*x^2+1)^
(1/2))+1/8*arctanh(a*x)/(a*x-1)*a^2*x+1/8*arctanh(a*x)/(a*x+1)*a^2*x+3/4*I*
a*Pi*arctanh(a*x)^2+3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)
))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2
*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctan
h(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*
x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arc
tanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(
-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1
+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^2-3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a
^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(
a*x+1)^2/(-a^2*x^2+1)))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)*l
n(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)
))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/
(-a^2*x^2+1)))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

<u>maxima</u> [B] time = 0.35, size = 406, normalized size = 2.86

$$\frac{1}{16}a^2\left(\frac{\left(a^2x^2-1\right)\log\left(ax+1\right)^3-\left(a^2x^2-1\right)\log\left(ax-1\right)^3+\left(4\,a^2x^2-3\left(a^2x^2-1\right)\log\left(ax-1\right)-4\right)\log\left(ax+1\right)^2-}{a^3x^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/16*a^2*(((a^2*x^2 - 1)*log(a*x + 1)^3 - (a^2*x^2 - 1)*log(a*x - 1)^3 + (4
*a^2*x^2 - 3*(a^2*x^2 - 1)*log(a*x - 1) - 4)*log(a*x + 1)^2 - 4*(a^2*x^2 -
1)*log(a*x - 1)^2 - 4*a*x + (3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 8*(a^2*x^2 -
1)*log(a*x - 1))*log(a*x + 1))/(a^3*x^2 - a) + 16*(log(a*x - 1)*log(1/2*a*x
 + 1/2) + dilog(-1/2*a*x + 1/2))/a - 16*(log(a*x + 1)*log(x) + dilog(-a*x))
/a + 16*(log(-a*x + 1)*log(x) + dilog(a*x))/a + 2*log(a*x + 1)/a - 2*log(a*
x - 1)/a) - 1/8*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)*log(a*
x + 1)*log(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2 - 1) + 8
*log(a*x + 1) + 8*log(a*x - 1) - 16*log(x))*arctanh(a*x) + 1/4*(3*a*log(a*x
 + 1) - 3*a*log(a*x - 1) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arctanh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^2),x)
[Out] int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^2\,(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**2,x)
[Out] Integral(atanh(a*x)**2/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.272 $\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=205

$$-a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)-2a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{a^2}{4\left(1-a^2x^2\right)}-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+\frac{a^2\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+a^2\log($$

```
[Out] 1/4*a^2/(-a^2*x^2+1)-a*arctanh(a*x)/x-1/2*a^3*x*arctanh(a*x)/(-a^2*x^2+1)+1
/4*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+1/2*a^2*arctanh(a*x)^2/(-a^2*x
^2+1)+2/3*a^2*arctanh(a*x)^3+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+2*a^2*arctanh
(a*x)^2*ln(2-2/(a*x+1))-2*a^2*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-a^2*poly
log(3,-1+2/(a*x+1))
```

Rubi [A] time = 0.70, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {6030, 5982, 5916, 266, 36, 29, 31, 5948, 5988, 5932, 6056, 6610, 5994, 5956, 261}

$$-a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{a^2}{4\left(1-a^2x^2\right)}-\frac{1}{2}a^2\log\left(1-a^2x^2\right)-\frac{a^3x\tanh}{2\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] a^2/(4*(1 - a^2*x^2)) - (a*ArcTanh[a*x])/x - (a^3*x*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + (a^2*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(2*x^2) + (a^2*ArcTanh[a
*x]^2)/(2*(1 - a^2*x^2)) + (2*a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log
[1 - a^2*x^2])/2 + 2*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a^2*ArcTan
h[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - a^2*PolyLog[3, -1 + 2/(1 + a*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}\,dx$$

$$= 2\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx\right) + a^4\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + a\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx + 2\left(\frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\int \frac{\tan}{x(}\right.$$

$$= -\frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + a\int \frac{\tanh^{-1}(ax)}{x^2}\,d$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tan}{2(1}$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tan}{2(1}$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tan}{2(1}$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tan}{2(1}$$

Mathematica [C] time = 1.02, size = 146, normalized size = 0.71

$$a^2\left(2\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) + \frac{1}{24}\left(24\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right) + 6\tanh^{-1}(ax)^2\left(-\frac{2}{a^2x^2} + 8\log\left(1-e^{2\tanh^{-1}(ax)}\right) + \right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] a^2*(2*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((2*I)*Pi^3 - 16*ArcTa
nh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*(2 - 2/(a^2*x^2) + Co
sh[2*ArcTanh[a*x]] + 8*Log[1 - E^(2*ArcTanh[a*x])]) + 24*Log[(a*x)/Sqrt[1 -
 a^2*x^2]] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] - (6*ArcTanh[a*x]*(4 + a*x*S
inh[2*ArcTanh[a*x]]))/(a*x))/24)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\,(ax)^2}{a^4x^7-2\,a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
[Out] integral(arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\,(ax)^2}{\left(a^2x^2-1\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^3), x)
```

maple [C] time = 2.26, size = 3040, normalized size = 14.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x)
[Out] I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi-I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^
2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1))*Pi+1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi+1/2*I*
a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*Pi-I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^
2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I
*((a*x+1)^2/(-a^2*x^2+1)-1))*Pi+1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^
2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1
)^2/(-a^2*x^2+1)))^2*Pi-I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/(1+
(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2*Pi-1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*Pi+4*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2*arctanh
(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ln((a*x+1)/(-a^2*x^2+1)^(1/
2)-1)-1/16*a^4/(a*x-1)/(a*x+1)*x^2+2/3*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^3-1
/4*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2+a^2/(a*x-1)/(a*x+1)*arctanh(a*x)-1/2*
arctanh(a*x)^2/x^2-4*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-4*a^2*polyl
og(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*c
sgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-1/2*I*a^2
/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+1/2*I*a^2/(a*x-1)/(a*
x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1))
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi-1/2*I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)
^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi-I*
a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^2*Pi+1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*
csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi+1/2*I*a^4*x^2
/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi-I*a^4*x^
```

```
2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I*
a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(
I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I*a^2/(a*x-1)
/(a*x+1)*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*Pi+I*a^4*x^2/(a*x-1)/(a*x+1
)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi+I*a^4*x^2/(a*x-1)/
(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2
/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)
-1))*Pi-1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-
a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*Pi-2/3*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^3+1/4*
a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2-a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)
-1/2*a^3*x/(a*x-1)/(a*x+1)*arctanh(a*x)+a/x/(a*x-1)/(a*x+1)*arctanh(a*x)-2*
a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*ln(2)-1/2*I*a^2/(a*x-1)/(a*x+1)*arctanh(
a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-I*a^2/
(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)
^2/(-a^2*x^2+1)))^3*Pi-1/2*I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x
+1)^2/(a^2*x^2-1))^3*Pi+a^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+I*a^2/(a*x-1)/
(a*x+1)*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi+I*a^4*x^2/(a
*x-1)/(a*x+1)*arctanh(a*x)^2*Pi-I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I
/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi-3/16*a^2/(a*x-1)/(a*x+1)+2*a^4*x^2/(a*x-1
)/(a*x+1)*arctanh(a*x)^2*ln(2)-I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi+2*a^
2*arctanh(a*x)^2*ln(a*x)-1/4*a^2*arctanh(a*x)^2/(a*x-1)-a^2*arctanh(a*x)^2*
ln(a*x-1)-2*a^2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/4*a^2*arctanh
(a*x)^2/(a*x+1)-a^2*arctanh(a*x)^2*ln(a*x+1)+2*a^2*arctanh(a*x)^2*ln((a*x+1
)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))
+2*a^2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^6\int \frac{x^6\log(ax+1)\log(-ax+1)}{a^4x^7-2\,a^2x^5+x^3}\,dx+\frac{1}{2}a^5\int \frac{x^5\log(ax+1)\log(-ax+1)}{a^4x^7-2\,a^2x^5+x^3}\,dx-\frac{1}{16}\left(a\left(\frac{2}{a^4x-a^3}-\frac{\log(ax+1)}{a^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^6*integrate(x^6*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3
), x) + 1/2*a^5*integrate(x^5*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x
^5 + x^3), x) - 1/16*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x - 1)/
a^3) + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^4 - 1/2*a^4*integrate(x^4*log(a*x
 + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/2*a^3*integrate(x^3
*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) + 1/2*a^3*integ
rate(x^3*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a^2*integrate(
x^2*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a*integrate(x*log(-
a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/24*(2*(a^4*x^4 - a^2*x^2)*log(
-a*x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1) - 1)*log(-a
*x + 1)^2)/(a^2*x^4 - x^2) + 1/4*integrate(log(a*x + 1)^2/(a^4*x^7 - 2*a^2*
x^5 + x^3), x) - 1/2*integrate(log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*
x^5 + x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atanh}\left(a\,x\right)}^2}{x^3\,{\left(a^2\,x^2-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^3(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**2,x)
[Out] Integral(atanh(a*x)**2/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.273 $\int \frac{x^3 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=227

$$-\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{4a^4}-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{2a^4}+\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{2a^4}+\frac{\tanh^{-1}(ax)^4}{4a^4}-\frac{\tanh^{-1}(ax)^3}{4a^4}-\frac{3\tanh}{8a}$$

```
[Out] -3/8*x/a^3/(-a^2*x^2+1)-3/8*arctanh(a*x)/a^4+3/4*arctanh(a*x)/a^4/(-a^2*x^2
+1)-3/4*x*arctanh(a*x)^2/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)^3/a^4+1/2*arctan
h(a*x)^3/a^4/(-a^2*x^2+1)+1/4*arctanh(a*x)^4/a^4-arctanh(a*x)^3*ln(2/(-a*x+
1))/a^4-3/2*arctanh(a*x)^2*polylog(2,1-2/(-a*x+1))/a^4+3/2*arctanh(a*x)*pol
ylog(3,1-2/(-a*x+1))/a^4-3/4*polylog(4,1-2/(-a*x+1))/a^4
```

Rubi [A] time = 0.40, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6028, 5984, 5918, 5948, 6058, 6062, 6610, 5994, 5956, 199, 206}

$$-\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^4}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{3x}{8a^3\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^4) + (3*ArcTanh[a*x])/
(4*a^4*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) - ArcTan
h[a*x]^3/(4*a^4) + ArcTanh[a*x]^3/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(4
*a^4) - (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 - (3*ArcTanh[a*x]^2*PolyLog[2
, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/
(2*a^4) - (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}dx}{a^3} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}dx}{2a^3}$$

$$= -\frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-a\ldots}\right.}{a^4}$$

$$= \frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \frac{\tanh\ldots}{}$$

$$= -\frac{3x}{8a^3\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \ldots$$

$$= -\frac{3x}{8a^3\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)}{8a^4} + \frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\ldots}{2a\ldots}$$

Mathematica [A] time = 0.19, size = 139, normalized size = 0.61

$$\frac{24\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)\text{Li}_3\left(-e^{-2\tanh^{-1}(ax)}\right) + 12\text{Li}_4\left(-e^{-2\tanh^{-1}(ax)}\right) - 4\tanh^{-1}(ax)^4 - \ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-4*ArcTanh[a*x]^4 + 6*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^3
*Cosh[2*ArcTanh[a*x]] - 16*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 24
*ArcTanh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 24*ArcTanh[a*x]*PolyLog[
3, -E^(-2*ArcTanh[a*x])] + 12*PolyLog[4, -E^(-2*ArcTanh[a*x])] - 3*Sinh[2*A
rcTanh[a*x]] - 6*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/(16*a^4)
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\,\text{artanh}\,(ax)^3}{a^4x^4 - 2\,a^2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,\text{artanh}\,(ax)^3}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

maple [C] time = 0.70, size = 1015, normalized size = 4.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))^3+1/2*I/a^4*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))
^2-3/16/a^3*arctanh(a*x)^2/(a*x+1)*x-3/16/a^3*arctanh(a*x)/(a*x+1)*x+3/16/a
^3*arctanh(a*x)^2/(a*x-1)*x-3/16/a^3*arctanh(a*x)/(a*x-1)*x-1/2*I/a^4*arcta
nh(a*x)^3*Pi-3/32/a^3/(a*x+1)*x+1/4/a^4*arctanh(a*x)^3/(a*x+1)+1/2/a^4*arct
anh(a*x)^3*ln(a*x+1)+3/2/a^4*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1)
)-1/a^4*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/2/a^4*arctanh(a*x)^
2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/a^4*arctanh(a*x)^3*ln(2)-1/4*I/a^4*a
rctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+3/32/a^3*x/(a*x-1)+3/16/a^
4*arctanh(a*x)^2/(a*x+1)+3/16/a^4*arctanh(a*x)^2/(a*x-1)-1/4/a^4*arctanh(a*
x)^3/(a*x-1)+1/2/a^4*arctanh(a*x)^3*ln(a*x-1)-1/4*arctanh(a*x)^3/a^4+1/4*ar
ctanh(a*x)^4/a^4-1/2*I/a^4*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+
1)))^3-1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I/a^4*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2
*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/4*I/a^4*arctanh(a*x)^3*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2-1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1))
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-3/16/a^4*arcta
nh(a*x)/(a*x-1)+3/16/a^4*arctanh(a*x)/(a*x+1)-3/4/a^4*polylog(4,-(a*x+1)^2/
(-a^2*x^2+1))+3/32/a^4/(a*x-1)+3/32/a^4/(a*x+1)+1/4*I/a^4*arctanh(a*x)^3*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(a^2x^2-1\right)\log\left(-ax+1\right)^4+4\left(\left(a^2x^2-1\right)\log\left(ax+1\right)-1\right)\log\left(-ax+1\right)^3}{64\left(a^6x^2-a^4\right)}+\frac{1}{8}\int\frac{2\,a^3x^3\log\left(ax+1\right)^3-6\,a^3x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) - 1)*l
og(-a*x + 1)^3)/(a^6*x^2 - a^4) + 1/8*integrate(1/2*(2*a^3*x^3*log(a*x + 1)
^3 - 6*a^3*x^3*log(a*x + 1)^2*log(-a*x + 1) - 3*(a*x - (3*a^3*x^3 + a^2*x^2
 - a*x - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^7*x^4 - 2*a^5*x^2 + a^3),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^3\,\mathrm{atanh}\left(a\,x\right)^3}{\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

```
[Out] int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}^3(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
[Out] Integral(x**3*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.274 $\int \frac{x^2 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=121

$$-\frac{\tanh^{-1}(ax)^4}{8a^3}+\frac{3\tanh^{-1}(ax)^2}{8a^3}+\frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{3}{8a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)}$$

```
[Out] -3/8/a^3/(-a^2*x^2+1)+3/4*x*arctanh(a*x)/a^2/(-a^2*x^2+1)+3/8*arctanh(a*x)^
2/a^3-3/4*arctanh(a*x)^2/a^3/(-a^2*x^2+1)+1/2*x*arctanh(a*x)^3/a^2/(-a^2*x^
2+1)-1/8*arctanh(a*x)^4/a^3
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6000, 5994, 5956, 261}

$$-\frac{3}{8a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^4}{8a^3}+\frac{3\tanh^{-1}(ax)^2}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] -3/(8*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*a^2*(1 - a^2*x^2)) + (3*Ar
cTanh[a*x]^2)/(8*a^3) - (3*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) + (x*ArcTa
nh[a*x]^3)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(8*a^3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6000

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> -Simp[(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^
2, x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} - \frac{3\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{2a}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} + \frac{3\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{2a^2}$$

$$= \frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a^3} - \frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} - \frac{3\int}{}$$

$$= -\frac{3}{8a^3\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a^3} - \frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\mathrm{t}}{}$$

Mathematica [A] time = 0.09, size = 72, normalized size = 0.60

$$\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^4 + 3\left(a^2x^2+1\right)\tanh^{-1}(ax)^2 - 4ax\tanh^{-1}(ax)^3 - 6ax\tanh^{-1}(ax) + 3}{8a^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a*
x]^3 + (1 - a^2*x^2)*ArcTanh[a*x]^4)/(8*a^3*(-1 + a^2*x^2))
```

fricas [A] time = 1.03, size = 114, normalized size = 0.94

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3 + \left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 48\,ax\log\left(-\frac{ax+1}{ax-1}\right) - 12\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 48}{128\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*x
+ 1)/(a*x - 1))^2 - 48)/(a^5*x^2 - a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\mathrm{artanh}\,(ax)^3}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

maple [C] time = 0.82, size = 1771, normalized size = 14.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] 3/16/a/(a*x-1)/(a*x+1)*x^2+1/8/a^3/(a*x-1)/(a*x+1)*arctanh(a*x)^4+3/8/a^3/(
a*x-1)/(a*x+1)*arctanh(a*x)^2-1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3*Pi+1/8*I/a^3
/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*Pi-1/8*I/a^3/(a*x-1)/(a*x+1)*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(
a*x)^3*Pi-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)
^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^3*Pi+1/
8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2
+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+
(a*x+1)^2/(-a^2*x^2+1)))^3*x^2-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*cs
gn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)
^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2+3/16/a^3/(a*x-1)/(a*x+1)-1/4
/a^3*arctanh(a*x)^3/(a*x-1)+1/4/a^3*arctanh(a*x)^3*ln(a*x-1)-1/4/a^3*arctan
h(a*x)^3/(a*x+1)-1/4/a^3*arctanh(a*x)^3*ln(a*x+1)+1/2/a^3*arctanh(a*x)^3*ln
((a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x
^2+1)))^2*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a
^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2+1/8*I/a^3/(a*x-1)/(a*x+
1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^3*Pi+1/8*I/a
/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^
2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2+1/4*I/a/(a*x-1)/(
a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))*x^2-1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*
csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2+1/4*I/a^3/(a*x-1)/(a*x+1)*arctanh(a*
x)^3*Pi+1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arct
anh(a*x)^3*Pi-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*x^2-1/8*I/a^3/(a*x-
1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3*Pi-1/8*I/a^3/(a*x
-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arct
anh(a*x)^3*Pi-1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^
2*arctanh(a*x)^3*Pi-1/8/a/(a*x-1)/(a*x+1)*arctanh(a*x)^4*x^2+3/8/a/(a*x-1)/
(a*x+1)*arctanh(a*x)^2*x^2-3/4/a^2/(a*x-1)/(a*x+1)*arctanh(a*x)*x
```

maxima [B] time = 0.35, size = 465, normalized size = 3.84

$$-\frac{1}{4}\left(\frac{2x}{a^4x^2-a^2}+\frac{\log(ax+1)}{a^3}-\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)^3+\frac{3\left(\left(a^2x^2-1\right)\log(ax+1)^2-2\left(a^2x^2-1\right)\log(ax\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x)^3 + 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*lo
g(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh(a*x)^2/(a^6*x^2 -
a^4) + 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x + 1)^
3*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 - 6*(2*a^2*x^2 - (a^2*x^2 - 1
)*log(a*x - 1)^2 - 2)*log(a*x + 1)^2 - 12*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4*
((a^2*x^2 - 1)*log(a*x - 1)^3 - 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1)
+ 48)*a^2/(a^8*x^2 - a^6) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 -
1)*log(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12*a*x - 3*
(2*a^2*x^2 - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(a^2*x^2 -
1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^2 - a^5))*a
```

mupad [B] time = 1.71, size = 410, normalized size = 3.39

$$\frac{3\ln(a\,x+1)^2}{32\,a^3}-\frac{3\ln(a\,x+1)^2}{16\,\left(a^3-a^5\,x^2\right)}+\frac{3\ln(1-a\,x)^2}{32\,a^3}-\frac{\ln(a\,x+1)^4}{128\,a^3}-\frac{\ln(1-a\,x)^4}{128\,a^3}-\frac{3\ln(1-a\,x)^2}{16\,a^3-16\,a^5\,x^2}-\frac{3}{2\,\left(4\,a^3-4\,a^5\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (3*log(a*x + 1)^2)/(32*a^3) - (3*log(a*x + 1)^2)/(16*(a^3 - a^5*x^2)) + (3*
log(1 - a*x)^2)/(32*a^3) - log(a*x + 1)^4/(128*a^3) - log(1 - a*x)^4/(128*a
^3) - (3*log(1 - a*x)^2)/(16*a^3 - 16*a^5*x^2) - 3/(2*(4*a^3 - 4*a^5*x^2))
- (x*log(1 - a*x)^3)/(2*(8*a^2 - 8*a^4*x^2)) - (3*log(a*x + 1)*log(1 - a*x)
)/(16*a^3) + (3*log(a*x + 1)*log(1 - a*x))/(8*a^3 - 8*a^5*x^2) + (3*x*log(a
*x + 1))/(8*(a^2 - a^4*x^2)) + (log(a*x + 1)*log(1 - a*x)^3)/(32*a^3) + (lo
g(a*x + 1)^3*log(1 - a*x))/(32*a^3) - (6*x*log(1 - a*x))/(16*a^2 - 16*a^4*x
^2) + (x*log(a*x + 1)^3)/(16*(a^2 - a^4*x^2)) - (3*log(a*x + 1)^2*log(1 - a
*x)^2)/(64*a^3) + (6*x*log(a*x + 1)*log(1 - a*x)^2)/(32*a^2 - 32*a^4*x^2) -
 (6*x*log(a*x + 1)^2*log(1 - a*x))/(32*a^2 - 32*a^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.275 $\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{3x}{8a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^3}{4a^2}-\frac{3\tanh^{-1}(ax)}{8a^2}$$

```
[Out] -3/8*x/a/(-a^2*x^2+1)-3/8*arctanh(a*x)/a^2+3/4*arctanh(a*x)/a^2/(-a^2*x^2+1
)-3/4*x*arctanh(a*x)^2/a/(-a^2*x^2+1)-1/4*arctanh(a*x)^3/a^2+1/2*arctanh(a*
x)^3/a^2/(-a^2*x^2+1)
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5994, 5956, 199, 206}

$$-\frac{3x}{8a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^3}{4a^2}-\frac{3\tanh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^2) + (3*ArcTanh[a*x])/(4
*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^3/(4*a^2) + ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}dx}{2a}$$
$$= -\frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} + \frac{3}{2}\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2}dx$$
$$= \frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\int \frac{1}{(1-a^2x^2)^2}dx}{4a}$$
$$= -\frac{3x}{8a(1-a^2x^2)} + \frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\int}{}$$
$$= -\frac{3x}{8a(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)}{8a^2} + \frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}}{2a^2(}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.76

$$\frac{3(a^2x^2-1)\log(1-ax) - 3(a^2x^2-1)\log(ax+1) - 4(a^2x^2+1)\tanh^{-1}(ax)^3 + 6ax + 12ax\tanh^{-1}(ax)^2 - 12\tan}{16a^2(a^2x^2-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (6*a*x - 12*ArcTanh[a*x] + 12*a*x*ArcTanh[a*x]^2 - 4*(1 + a^2*x^2)*ArcTanh[
a*x]^3 + 3*(-1 + a^2*x^2)*Log[1 - a*x] - 3*(-1 + a^2*x^2)*Log[1 + a*x])/(16
*a^2*(-1 + a^2*x^2))
```

fricas [A] time = 0.46, size = 97, normalized size = 0.82

$$\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2 - (a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 12\,ax - 6(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{32(a^4x^2-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a
^2)
```

giac [A] time = 0.19, size = 192, normalized size = 1.61

$$-\frac{1}{64}\left(\left(\frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 - 3\left(\frac{ax+1}{(ax-1)a^3} - \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 6\left(\frac{ax+1}{(ax-1)a^3} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/64*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*x +
1)/(a*x - 1))^3 - 3*((a*x + 1)/((a*x - 1)*a^3) - (a*x - 1)/((a*x + 1)*a^3))
*log(-(a*x + 1)/(a*x - 1))^2 + 6*((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a
```

```
*x + 1)*a^3))*log(-(a*x + 1)/(a*x - 1)) - 6*(a*x + 1)/((a*x - 1)*a^3) + 6*(
a*x - 1)/((a*x + 1)*a^3))*a
```

maple [C] time = 0.81, size = 1708, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] 3/8*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csg
n(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2+3/16*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*P
i*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2+3/
16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^
2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*Pi-3/16*I/a^2/(a*x-1)
/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/
(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^2*Pi-3/8*I/a^2/(a*x-1)/(a*x+1)*csg
n(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x
)^2*Pi-3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)^2*Pi-3/16*I/(a*x-1)/(a*x+1)*arctanh(
a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*
x+1)^2/(-a^2*x^2+1)))^2*x^2+3/16*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-
a^2*x^2+1)))*x^2-1/2/a^2/(a^2*x^2-1)*arctanh(a*x)^3+3/8/a^2*arctanh(a*x)^2/
(a*x-1)+3/8/a^2*arctanh(a*x)^2*ln(a*x-1)+3/8/a^2*arctanh(a*x)^2/(a*x+1)-3/8
/a^2*arctanh(a*x)^2*ln(a*x+1)+3/4/a^2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1
)^(1/2))+3/8/a/(a*x-1)/(a*x+1)*x+1/4/a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^3-3/8
/a^2/(a*x-1)/(a*x+1)*arctanh(a*x)+3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*cs
gn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^2*Pi-3/16*I/(a*x-1)/(a*x+1)*a
rctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2-1/4/(a*
x-1)/(a*x+1)*arctanh(a*x)^3*x^2-3/8/(a*x-1)/(a*x+1)*arctanh(a*x)*x^2+3/8*I/
a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi-3/8*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*
Pi*x^2-3/8*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^
2+1)))^3*x^2-3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*
x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2*Pi-3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*Pi+3/8*I/a^2/(a*x-1)/(a*x+1)*csgn(
I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2*Pi-3/8*I/a^2/(a*x-1)/(a*x+1)
*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2*Pi+3/8*I/(a*x-1)/(a*x+
1)*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2+3/16*I/(a*x-1
)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2
*x^2+1)))^3*x^2+3/16*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(
a^2*x^2-1))^3*x^2
```

maxima [B] time = 0.34, size = 298, normalized size = 2.50

$$\frac{3\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2}{8a}-\frac{\left((a^2x^2-1)\log(ax+1)^3-3(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3-12a\right.}{a^5x^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 3/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2/a
- 1/32*(((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*log(
a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x^2
- 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a^2/(
a^5*x^2 - a^3) - 6*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x
```

```
+ 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/(a^4*x
^2 - a^2))/a - 1/2*arctanh(a*x)^3/((a^2*x^2 - 1)*a^2)
```

mupad [B] time = 1.76, size = 239, normalized size = 2.01

$$-\frac{6\ \ln\left(1-a\,x\right)-6\ \ln\left(a\,x+1\right)+12\,a\,x-\ln\left(a\,x+1\right)^3+\ln\left(1-a\,x\right)^3-3\ \ln\left(a\,x+1\right)\ \ln\left(1-a\,x\right)^2+3\ln\left(a\,x+1\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

```
[Out] -(6*log(1 - a*x) - 6*log(a*x + 1) + 12*a*x - log(a*x + 1)^3 + log(1 - a*x)^
3 - 3*log(a*x + 1)*log(1 - a*x)^2 + 3*log(a*x + 1)^2*log(1 - a*x) - a^2*x^2
*(6*log(a*x + 1) - 6*log(1 - a*x)) - a^2*x^2*log(a*x + 1)^3 + a^2*x^2*log(1
 - a*x)^3 + 6*a*x*log(a*x + 1)^2 + 6*a*x*log(1 - a*x)^2 - 12*a*x*log(a*x +
1)*log(1 - a*x) - 3*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 + 3*a^2*x^2*log(a*x
 + 1)^2*log(1 - a*x))/(32*a^2 - 32*a^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{atanh}^3\left(ax\right)}{\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.276 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{3}{8a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^4}{8a}+\frac{3\tanh^{-1}(ax)^2}{8a}$$

```
[Out] -3/8/a/(-a^2*x^2+1)+3/4*x*arctanh(a*x)/(-a^2*x^2+1)+3/8*arctanh(a*x)^2/a-3/
4*arctanh(a*x)^2/a/(-a^2*x^2+1)+1/2*x*arctanh(a*x)^3/(-a^2*x^2+1)+1/8*arcta
nh(a*x)^4/a
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5956, 5994, 261}

$$-\frac{3}{8a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^4}{8a}+\frac{3\tanh^{-1}(ax)^2}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

```
[Out] -3/(8*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*ArcTanh[
a*x]^2)/(8*a) - (3*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)
/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{8a} - \frac{1}{2}(3a)\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{8a} + \frac{3}{2}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{3x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a} - \frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{8a} - \frac{1}{4}(3a)$$

$$= -\frac{3}{8a\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a} - \frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh}{}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.62

$$\frac{\left(a^2x^2-1\right)\tanh^{-1}(ax)^4 + 3\left(a^2x^2+1\right)\tanh^{-1}(ax)^2 - 4ax\tanh^{-1}(ax)^3 - 6ax\tanh^{-1}(ax) + 3}{8a\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

```
[Out] (3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a*
x]^3 + (-1 + a^2*x^2)*ArcTanh[a*x]^4)/(8*a*(-1 + a^2*x^2))
```

fricas [A] time = 1.15, size = 113, normalized size = 0.98

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3 - \left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 48\,ax\log\left(-\frac{ax+1}{ax-1}\right) - 12\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 48}{128\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 - (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*x
+ 1)/(a*x - 1))^2 - 48)/(a^3*x^2 - a)
```

giac [A] time = 2.75, size = 122, normalized size = 1.06

$$\frac{1}{32}a^2\left(\frac{(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^3}{(ax+1)a^4} + \frac{3\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax+1)a^4} + \frac{6\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)}{(ax+1)a^4} + \frac{6\,(ax-1)}{(ax+1)a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/32*a^2*((a*x - 1)*log(-(a*x + 1)/(a*x - 1))^3/((a*x + 1)*a^4) + 3*(a*x -
1)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)*a^4) + 6*(a*x - 1)*log(-(a*x + 1)
/(a*x - 1))/((a*x + 1)*a^4) + 6*(a*x - 1)/((a*x + 1)*a^4))
```

maple [C] time = 0.82, size = 1742, normalized size = 15.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*
x^2-1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi
*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2-1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a
*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2+1/8*I/a/(a*x-1)/(a*x+1)*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)
^2/(-a^2*x^2+1)))*arctanh(a*x)^3*Pi+1/4*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^
2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3*Pi-1/8*I
/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))
^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*Pi+1/8*I/a/(a*x-1)/(a*x+1)*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(
a*x)^3*Pi-1/4/a*arctanh(a*x)^3/(a*x-1)-1/4/a*arctanh(a*x)^3*ln(a*x-1)-1/4/a
*arctanh(a*x)^3/(a*x+1)+1/4/a*arctanh(a*x)^3*ln(a*x+1)-1/2/a*arctanh(a*x)^3
*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/
(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*x)^3*Pi-1/8*I*a/(a*x-1)/(a*x+1)*arcta
nh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2
))^2*x^2-1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2-1
/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2+1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^2*x^2+1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))
*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2+3/16/(a*x-1)/(a*x+1)/a-1/8/a/(a*x-1
)/(a*x+1)*arctanh(a*x)^4+3/8/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2-3/4/(a*x-1)/(
a*x+1)*arctanh(a*x)*x-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi+3/16/(a*x-1
)/(a*x+1)*a*x^2+1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arc
tanh(a*x)^3*Pi+1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*x^2-1/4*I/a/(a*x-1
)/(a*x+1)*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3*Pi+1/4*I/a/(a
*x-1)/(a*x+1)*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3*Pi+1/8*I/
a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^
3*arctanh(a*x)^3*Pi+1/8*a/(a*x-1)/(a*x+1)*arctanh(a*x)^4*x^2+3/8*a/(a*x-1)/
(a*x+1)*arctanh(a*x)^2*x^2
```

maxima [B] time = 0.35, size = 459, normalized size = 3.99

$$-\frac{1}{4}\left(\frac{2\,x}{a^2x^2-1}-\frac{\log\left(ax+1\right)}{a}+\frac{\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^3-\frac{3\left(\left(a^2x^2-1\right)\log\left(ax+1\right)^2-2\left(a^2x^2-1\right)\log\left(ax+\right.\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^3 -
 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
- 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)^2/(a^4*x^2 - a^2) -
 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x + 1)^3*log(
a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 + 6*(2*a^2*x^2 + (a^2*x^2 - 1)*log(
a*x - 1)^2 - 2)*log(a*x + 1)^2 + 12*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4*((a^2*
x^2 - 1)*log(a*x - 1)^3 + 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1) - 48)*
a^2/(a^6*x^2 - a^4) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log
(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2
*x^2 + (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log
(a*x - 1))*a*arctanh(a*x)/(a^5*x^2 - a^3))*a
```

mupad [B] time = 1.72, size = 378, normalized size = 3.29

$$\frac{3\ln\left(a\,x+1\right)^2}{32\,a}-\frac{3}{2\,\left(4\,a-4\,a^3\,x^2\right)}-\frac{3\ln\left(1-a\,x\right)^2}{16\,a-16\,a^3\,x^2}+\frac{3\ln\left(1-a\,x\right)^2}{32\,a}+\frac{\ln\left(a\,x+1\right)^4}{128\,a}+\frac{\ln\left(1-a\,x\right)^4}{128\,a}-\frac{3\ln\left(a\,x+1\right)^2}{16\,\left(a-a^3\,x^2\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(a^2*x^2 - 1)^2,x)
```

```
[Out] (3*log(a*x + 1)^2)/(32*a) - 3/(2*(4*a - 4*a^3*x^2)) - (3*log(1 - a*x)^2)/(1
6*a - 16*a^3*x^2) + (3*log(1 - a*x)^2)/(32*a) + log(a*x + 1)^4/(128*a) + lo
g(1 - a*x)^4/(128*a) - (3*log(a*x + 1)^2)/(16*(a - a^3*x^2)) - (3*log(a*x +
 1)*log(1 - a*x))/(16*a) - (log(a*x + 1)*log(1 - a*x)^3)/(32*a) - (log(a*x
+ 1)^3*log(1 - a*x))/(32*a) - (3*x*log(a*x + 1))/(8*(a^2*x^2 - 1)) + (6*x*l
og(1 - a*x))/(16*a^2*x^2 - 16) + (3*log(a*x + 1)*log(1 - a*x))/(8*a - 8*a^3
*x^2) + (3*log(a*x + 1)^2*log(1 - a*x)^2)/(64*a) - (x*log(a*x + 1)^3)/(16*(
a^2*x^2 - 1)) + (x*log(1 - a*x)^3)/(2*(8*a^2*x^2 - 8)) - (6*x*log(a*x + 1)*
log(1 - a*x)^2)/(32*a^2*x^2 - 32) + (6*x*log(a*x + 1)^2*log(1 - a*x))/(32*a
^2*x^2 - 32)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.277 $\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=193

$$-\frac{3ax}{8\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}-\frac{3}{4}\text{Li}_4\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(\ldots$$

```
[Out] -3/8*a*x/(-a^2*x^2+1)-3/8*arctanh(a*x)+3/4*arctanh(a*x)/(-a^2*x^2+1)-3/4*a*
x*arctanh(a*x)^2/(-a^2*x^2+1)-1/4*arctanh(a*x)^3+1/2*arctanh(a*x)^3/(-a^2*x
^2+1)+1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2*
polylog(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*polylo
g(4,-1+2/(a*x+1))
```

Rubi [A] time = 0.39, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6030, 5988, 5932, 5948, 6056, 6060, 6610, 5994, 5956, 199, 206}

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a*x)/(8*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/8 + (3*ArcTanh[a*x])/(4*(1 -
a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) - ArcTanh[a*x]^3/4 + A
rcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 -
 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcT
anh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])
/4
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 - \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx$$

$$= -\frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log($$

$$= \frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 + \tan$$

$$= -\frac{3ax}{8\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\,t$$

$$= -\frac{3ax}{8\left(1-a^2x^2\right)} - \frac{3}{8}\tanh^{-1}(ax) + \frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tan}{2(}$$

Mathematica [A] time = 0.20, size = 135, normalized size = 0.70

$$\frac{1}{64}\left(96\tanh^{-1}(ax)^2\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) + 48\mathrm{Li}_4\left(e^{2\tanh^{-1}(ax)}\right) - 16\tanh^{-1}(ax)^4 +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (Pi^4 - 16*ArcTanh[a*x]^4 + 24*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 16*ArcTa
nh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x
])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*Po
lyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])] - 12*Sinh[
2*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/64
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\operatorname{artanh}(ax)^3}{a^4x^5 - 2\,a^2x^3 + x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(a^2x^2-1\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x), x)
```

maple [C] time = 0.74, size = 1387, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x)
```

```
[Out] 3/32*(a*x+1)/(a*x-1)+1/4*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3+3/16*(a*x+1
)*arctanh(a*x)^2/(a*x-1)-3/16*arctanh(a*x)*(a*x-1)/(a*x+1)-3/16*arctanh(a*x
)^2*(a*x-1)/(a*x+1)+1/2*I*Pi*arctanh(a*x)^3-1/4*arctanh(a*x)^4-1/4*arctanh(
a*x)^3-3/16*(a*x+1)*arctanh(a*x)/(a*x-1)-1/4*arctanh(a*x)^3/(a*x-1)+1/4*arc
tanh(a*x)^3/(a*x+1)+arctanh(a*x)^3*ln(a*x)+6*polylog(4,-(a*x+1)/(-a^2*x^2+1
)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*arctanh(a*x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3
*ln(a*x+1)+arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/(1
+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^3-1/2*I*Pi*csgn(I/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2*arctanh(a*x)^3+1/4*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*
x^2-1))^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))
^3*arctanh(a*x)^3+1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)
/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1
)^(1/2))-1/4*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*
x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*arctanh(a*
x)^3+1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(
a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh
(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^3*ln((a*x+1)^2/
(-a^2*x^2+1)-1)+ln(2)*arctanh(a*x)^3+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/32*(a*x-1)/(a*x
+1)-1/2*I*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x
+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I*Pi*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3+1/2*I*a
rctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x
^2-1))^2-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^3-1/2*I*arctanh(a*x)^3*Pi*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)
^2/(-a^2*x^2+1)))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(a^2x^2-1\right)\log\left(-ax+1\right)^4+4\left(\left(a^2x^2-1\right)\log\left(ax+1\right)+1\right)\log\left(-ax+1\right)^3}{64\left(a^2x^2-1\right)}-\frac{1}{8}\int-\frac{2\log\left(ax+1\right)^3-6\log\left(ax+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) + 1)*lo
g(-a*x + 1)^3)/(a^2*x^2 - 1) - 1/8*integrate(-1/2*(2*log(a*x + 1)^3 - 6*log
(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 - a^2*x^2
 - a*x - 2)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atanh}\left(a\,x\right)}^3}{x\,{\left(a^2\,x^2-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^2), x)
```

sympy **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{x\,(ax-1)^{2}\,(ax+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.278 $\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=191

$$-\frac{3a}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}+\frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}-\frac{3}{2}a\mathrm{Li}_3\left(\frac{2}{ax+1}-1\right)-3a\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)\tan$$

```
[Out] -3/8*a/(-a^2*x^2+1)+3/4*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+3/8*a*arctanh(a*x)^
2-3/4*a*arctanh(a*x)^2/(-a^2*x^2+1)+a*arctanh(a*x)^3-arctanh(a*x)^3/x+1/2*a
^2*x*arctanh(a*x)^3/(-a^2*x^2+1)+3/8*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*ln
(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+2
/(a*x+1))
```

Rubi [A] time = 0.44, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6030, 5982, 5916, 5988, 5932, 5948, 6056, 6610, 5956, 5994, 261}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3a}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}}{4\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a)/(8*(1 - a^2*x^2)) + (3*a^2*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*a*
ArcTanh[a*x]^2)/8 - (3*a*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + a*ArcTanh[a*x]
^3 - ArcTanh[a*x]^3/x + (a^2*x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (3*a*Arc
Tanh[a*x]^4)/8 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)}\,dx$$

$$= \frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{8}a\tanh^{-1}(ax)^4 + a^2\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx - \frac{1}{2}\left(3a^3\right)\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{x} + \frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^4 + (3a)\int \frac{\tanh^{-1}(a}{x\left(1-a^2\right.}$$

$$= \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{a}{}$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 -$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 -$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 -$$

Mathematica [C] time = 0.35, size = 144, normalized size = 0.75

$$\frac{1}{16}a\left(48\tanh^{-1}(ax)\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) - 24\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) + 6\tanh^{-1}(ax)^4 - \frac{16\tanh^{-1}(ax)^3}{ax} - 16\tanh^{-1}(ax)^3 +\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (a*((2*I)*Pi^3 - 16*ArcTanh[a*x]^3 - (16*ArcTanh[a*x]^3)/(a*x) + 6*ArcTanh[
a*x]^4 - 3*Cosh[2*ArcTanh[a*x]] - 6*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]] + 4
8*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 48*ArcTanh[a*x]*PolyLog[2, E
^(2*ArcTanh[a*x])] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*Sin
h[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^3*Sinh[2*ArcTanh[a*x]]))/16
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(ax\right)^3}{a^4x^6 - 2\,a^2x^4 + x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(ax\right)^3}{\left(a^2x^2-1\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^2), x)
```

maple [B] time = 3.03, size = 442, normalized size = 2.31

$$\frac{3a^2x}{32(ax-1)}+\frac{3a^2x}{32(ax+1)}+\frac{3a}{32(ax-1)}-\frac{3a}{32(ax+1)}-\frac{\operatorname{arctanh}(ax)^3}{x}-\frac{3a\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{3a\operatorname{arctanh}(ax)}{16(ax+1)}+6a\operatorname{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -3/16*a*arctanh(a*x)/(a*x-1)-3/16*a*arctanh(a*x)/(a*x+1)-6*a*polylog(3,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^3/x+6*a*arctanh(a*x)*polylog(2,-(a*x+
1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/32*a^2*x/(a*x-1)+3/32/(a*x+1
)*a^2*x+3/16*a*arctanh(a*x)^2/(a*x-1)-3/16*a*arctanh(a*x)^2/(a*x+1)-a*arcta
nh(a*x)^3+3/8*a*arctanh(a*x)^4+3/32*a/(a*x-1)-3/32*a/(a*x+1)-3/16*arctanh(a
*x)/(a*x-1)*a^2*x+3/16*arctanh(a*x)/(a*x+1)*a^2*x-1/8/(a*x-1)*arctanh(a*x)^
3*x*a^2+3/16/(a*x-1)*arctanh(a*x)^2*x*a^2+1/8/(a*x+1)*arctanh(a*x)^3*x*a^2+
3/16/(a*x+1)*arctanh(a*x)^2*x*a^2+3*a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2
+1)^(1/2))+3*a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a/(a*x-1
)*arctanh(a*x)^3-1/8*a/(a*x+1)*arctanh(a*x)^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^2(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.279 $\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=302

$$-\frac{3}{2}a^2\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\mathrm{Li}_4\left(\frac{2}{ax+1}-1\right)-3a^2\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-3a^2\mathrm{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+$$

```
[Out] -3/8*a^3*x/(-a^2*x^2+1)-3/8*a^2*arctanh(a*x)+3/4*a^2*arctanh(a*x)/(-a^2*x^2
+1)+3/2*a^2*arctanh(a*x)^2-3/2*a*arctanh(a*x)^2/x-3/4*a^3*x*arctanh(a*x)^2/
(-a^2*x^2+1)+1/4*a^2*arctanh(a*x)^3-1/2*arctanh(a*x)^3/x^2+1/2*a^2*arctanh(
a*x)^3/(-a^2*x^2+1)+1/2*a^2*arctanh(a*x)^4+3*a^2*arctanh(a*x)*ln(2-2/(a*x+1
))+2*a^2*arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+2/(a*x+1))-3*a
^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))-3*a^2*arctanh(a*x)*polylog(3,-1+2
/(a*x+1))-3/2*a^2*polylog(4,-1+2/(a*x+1))
```

Rubi [A] time = 0.96, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {6030, 5982, 5916, 5988, 5932, 2447, 5948, 6056, 6060, 6610, 5994, 5956, 199, 206}

$$-\frac{3}{2}a^2\mathrm{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\mathrm{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-3a^2\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-3a^2\tanh$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a^3*x)/(8*(1 - a^2*x^2)) - (3*a^2*ArcTanh[a*x])/8 + (3*a^2*ArcTanh[a*x]
)/(4*(1 - a^2*x^2)) + (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x)
 - (3*a^3*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x]^3)/4 - Ar
cTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (a^2*ArcTan
h[a*x]^4)/2 + 3*a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + 2*a^2*ArcTanh[a*x]^
3*Log[2 - 2/(1 + a*x)] - (3*a^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - 3*a^2*Arc
Tanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)] - 3*a^2*ArcTanh[a*x]*PolyLog[3, -1
 + 2/(1 + a*x)] - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)}\,dx$$

$$= 2\left(a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx\right)+a^4\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^3}{2x^2}+\frac{a^2\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx+2\left(\frac{1}{4}a^2\tanh^{-1}(ax)^4+a^2\int$$

$$= -\frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}-\frac{1}{4}a^2\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{2x^2}+\frac{a^2\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{2}(3a)\int \frac{\tanh}{}$$

$$= \frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}(ax)^2}{2x}-\frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}+\frac{1}{4}a^2\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{2x^2}$$

$$= -\frac{3a^3x}{8\left(1-a^2x^2\right)}+\frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{2x}-\frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}$$

$$= -\frac{3a^3x}{8\left(1-a^2x^2\right)}-\frac{3}{8}a^2\tanh^{-1}(ax)+\frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{2x}-$$

$$= -\frac{3a^3x}{8\left(1-a^2x^2\right)}-\frac{3}{8}a^2\tanh^{-1}(ax)+\frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{2x}-$$

Mathematica [A]   time = 0.81, size = 215, normalized size = 0.71

$$\frac{1}{32}a^2\left(-\frac{16\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}{a^2x^2}+96\tanh^{-1}(ax)^2\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)-96\tanh^{-1}(ax)\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right)-48\text{Li}_2\left(e\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```

[Out] (a^2*(Pi^4 + 48*ArcTanh[a*x]^2 - (48*ArcTanh[a*x]^2)/(a*x) - (16*(1 - a^2*x^2)*ArcTanh[a*x]^3)/(a^2*x^2) - 16*ArcTanh[a*x]^4 + 12*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 8*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 96*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] - 48*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])] - 6*Sinh[2*ArcTanh[a*x]] - 12*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]]))/32

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)^3}{a^4x^7 - 2a^2x^5 + x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")

[Out] integral(arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2 - 1)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")

[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^3), x)

maple [B] time = 6.42, size = 672, normalized size = 2.23

$$\frac{a^2\operatorname{arctanh}(ax)^3}{8ax+8} - \frac{a^3\operatorname{arctanh}(ax)^3 x}{8(ax-1)} + \frac{3a^3\operatorname{arctanh}(ax)^2 x}{16(ax-1)} - \frac{3a^3\operatorname{arctanh}(ax)\,x}{16(ax-1)} - \frac{a^3\operatorname{arctanh}(ax)^3 x}{8(ax+1)} - \frac{3a^3\operatorname{arcta}}{16(a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x)

[Out] -3/16*a^3/(a*x-1)*arctanh(a*x)*x-1/8*a^3/(a*x+1)*arctanh(a*x)^3*x-3/16*a^3/(a*x+1)*arctanh(a*x)^2*x-3/16*a^3/(a*x+1)*arctanh(a*x)*x-1/8*a^3/(a*x-1)*arctanh(a*x)^3*x+3/16*a^3/(a*x-1)*arctanh(a*x)^2*x+6*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a^2/(a*x-1)*arctanh(a*x)^3+3/16*a^2/(a*x-1)*arctanh(a*x)^2-3/16*a^2/(a*x-1)*arctanh(a*x)+1/8*a^2/(a*x+1)*arctanh(a*x)^3+3/16*a^2/(a*x+1)*arctanh(a*x)^2+3/16*a^2/(a*x+1)*arctanh(a*x)+3/32*a^3*x/(a*x-1)-3/32*a^3/(a*x+1)*x+1/2*a^2*arctanh(a*x)^3-3/2*a*arctanh(a*x)^2/x-1/2*arctanh(a*x)^3/x^2-1/2*a^2*arctanh(a*x)^4+3/32*a^2/(a*x+1)+3/32*a^2/(a*x-1)-3/2*a^2*arctanh(a*x)^2+12*a^2*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a^4x^4 - a^2x^2)\log(-ax+1)^4 + 2\left(2a^2x^2 + 2(a^4x^4 - a^2x^2)\log(ax+1) - 1\right)\log(-ax+1)^3}{32(a^2x^4 - x^2)} - \frac{1}{8}\int -\frac{2\log(ax+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/32*((a^4*x^4 - a^2*x^2)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^4 - a^2
*x^2)*log(a*x + 1) - 1)*log(-a*x + 1)^3)/(a^2*x^4 - x^2) - 1/8*integrate(-1
/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x + 1) - 3*(2*a^4*x^4 + 2*a^
3*x^3 - a^2*x^2 - a*x + 2*(a^6*x^6 + a^5*x^5 - a^4*x^4 - a^3*x^3 - 1)*log(a
*x + 1))*log(-a*x + 1)^2)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^3\,(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

3.280 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}$$

```
[Out] 1/3*arctanh(a*x)^(3/2)/a+1/32*erf(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)/a-1/32*erfi(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+1/2*x*arctan
h(a*x)^(1/2)/(-a^2*x^2+1)
```

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5956, 6034, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

```
[Out] (x*Sqrt[ArcTanh[a*x]])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^(3/2)/(3*a) + (Sqrt
[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*S
qrt[ArcTanh[a*x]]])/(16*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^2}\,dx = \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{1}{4}a\int \frac{x}{\left(1-a^2x^2\right)^2\sqrt{\tanh^{-1}(ax)}}\,dx$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{4a}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{4a}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{8a}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}+\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{16a}-\frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx,x,\right.}{16a}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}+\frac{\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\tanh^{-1}(ax)}\right)}{8a}-\frac{\text{Subst}\left(\int e^{2x^2}\,d\right.}{}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}+\frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(a\right.}}{16a}$$

Mathematica [A] time = 0.24, size = 87, normalized size = 0.84

$$\sqrt{\tanh^{-1}(ax)}\left(\frac{\tanh^{-1}(ax)}{3a}-\frac{x}{2\left(a^2x^2-1\right)}\right)-\frac{\sqrt{\frac{\pi}{2}}\left(\text{erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)-\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

```
[Out] Sqrt[ArcTanh[a*x]]*(-1/2*x/(-1 + a^2*x^2) + ArcTanh[a*x]/(3*a)) - (Sqrt[Pi/
2]*(-Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]]))/(
16*a)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{\left(-a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{atanh}(a\,x)}}{\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^2,x)
```

```
[Out] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}(ax)}}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**2,x)
[Out] Integral(sqrt(atanh(a*x))/((a*x - 1)**2*(a*x + 1)**2), x)
```

3.281 $\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=40

$$\frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a^4}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}-\frac{3\log\left(\tanh^{-1}(ax)\right)}{2a^5}$$

```
[Out] 1/2*Chi(2*arctanh(a*x))/a^5-3/2*ln(arctanh(a*x))/a^5+Unintegrable(1/arctanh
(a*x),x)/a^4
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
[Out] Defer[Int][x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx=\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 4.93, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
[Out] Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
[Out] integral(x^4/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a^2x^2-1\right)^2 \text{artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

maple [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-a^2x^2+1\right)^2 \operatorname{arctanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x**4/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

3.282 $\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=43

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4} - \frac{\text{Int}\left(\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}, x\right)}{a^2}$$

```
[Out] 1/2*Shi(2*arctanh(a*x))/a^4-Unintegrable(x/(-a^2*x^2+1)/arctanh(a*x),x)/a^2
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
[Out] Defer[Int][x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx = \int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2x^2 - 1\right)^2 \text{artanh}\left(ax\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-a^2x^2+1\right)^2 \operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
[Out] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x),x)
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

3.283 $\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

[Out] 1/2*Chi(2*arctanh(a*x))/a^3-1/2*ln(arctanh(a*x))/a^3

Rubi [A] time = 0.10, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 3312, 3301}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]

[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} dx, x, \tanh^{-1}(ax)\right)}{2a^3}\\
&= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 27, normalized size = 1.00

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)
```

fricas [B] time = 0.52, size = 58, normalized size = 2.15

$$-\frac{2\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)-\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
[Out] -1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a*x + 1)/(a*x - 1))
 - log_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(a^2x^2-1\right)^2\text{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

maple [A] time = 0.20, size = 24, normalized size = 0.89

$$\frac{\text{X}(2\,\text{arctanh}(ax))}{2a^3}-\frac{\ln(\text{arctanh}(ax))}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x)
[Out] 1/2*Chi(2*arctanh(a*x))/a^3-1/2*ln(arctanh(a*x))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(a^2x^2-1\right)^2\text{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{x^2}{\text{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
[Out] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x),x)
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

3.284 $\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

```
[Out] 1/2*Shi(2*arctanh(a*x))/a^2
```

Rubi [A] time = 0.07, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^2}$$
$$= \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A] time = 0.07, size = 14, normalized size = 1.00

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

fricas [B] time = 0.46, size = 38, normalized size = 2.71

$$\frac{\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(log_integral(-(a*x + 1)/(a*x - 1)) - log_integral(-(a*x - 1)/(a*x + 1)
))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

maple [A] time = 0.20, size = 13, normalized size = 0.93

$$\frac{\text{Shi}\left(2\operatorname{arctanh}(ax)\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] 1/2*Shi(2*arctanh(a*x))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

3.285 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\log\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/2*ln(arctanh(a*x))/a
```

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5968, 3312, 3301}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\log\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + Log[ArcTanh[a*x]]/(2*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\log\left(\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a}\\
&= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\log\left(\tanh^{-1}(ax)\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 27, normalized size = 1.00

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\log\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + Log[ArcTanh[a*x]]/(2*a)
```

fricas [B] time = 1.02, size = 54, normalized size = 2.00

$$\frac{2\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a*x + 1)/(a*x - 1))
+ log_integral(-(a*x - 1)/(a*x + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

maple [A] time = 0.18, size = 24, normalized size = 0.89

$$\frac{\mathrm{X}\left(2\operatorname{arctanh}(ax)\right)}{2a}+\frac{\ln\left(\operatorname{arctanh}(ax)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/2*ln(arctanh(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{1}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^{2}\left(ax+1\right)^{2}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

3.286 $\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)},x\right)+\frac{1}{2}\text{Shi}\left(2\tanh^{-1}(ax)\right)$$

[Out] 1/2*Shi(2*arctanh(a*x))+Unintegrable(1/x/(-a^2*x^2+1)/arctanh(a*x),x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]

[Out] Defer[Int][1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]), x]

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx=\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]

[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]), x]

fricas [A] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^4x^5-2\,a^2x^3+x\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")

[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^2x\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

maple [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2}x^{2}+1\right)^{2}\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}x^{2}-1\right)^{2}x\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\,\left(a^{2}\,x^{2}-1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^2),x)
[Out] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax-1\right)^{2}\left(ax+1\right)^{2}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x),x)
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

3.287 $\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=61

$$-\frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a^3}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^4}+\frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] x/a^3/arctanh(a*x)-x/a^3/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))/a^4-
Unintegrable(1/arctanh(a*x),x)/a^3
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] x/(a^3*ArcTanh[a*x]) - x/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*
ArcTanh[a*x]]/a^4 - Defer[Int][ArcTanh[a*x]^(-1), x]/a^3
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx &= \frac{\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx}{a^2}-\frac{\int \frac{x}{\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} dx}{a^2}\\
&= \frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\int \frac{1}{\tanh^{-1}(ax)} dx}{a^3}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx}{a^3}\\
&= \frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x} dx,x,\tanh^{-1}(ax)\right)}{a^4}\\
&= \frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right) dx,x,\tanh^{-1}(ax)\right)}{a^4}\\
&= \frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}+2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} dx,x,\tanh^{-1}(ax)\right)}{2a^4}\\
&= \frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^4}-\frac{\int \frac{1}{\tanh^{-1}(ax)} dx}{a^3}
\end{aligned}$$

Mathematica [A] time = 3.40, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2x^2 - 1\right)^2 \text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

maple [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-a^2x^2 + 1\right)^2 \text{arctanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x^3}{\left(a^3x^2 - a\right)\log\left(ax + 1\right) - \left(a^3x^2 - a\right)\log\left(-ax + 1\right)} + \int -\frac{2\left(a^2x^4 - 3\,x^2\right)}{\left(a^5x^4 - 2\,a^3x^2 + a\right)\log\left(ax + 1\right) - \left(a^5x^4 - 2\,a^3x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
[Out] 2*x^3/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) + integrat
e(-2*(a^2*x^4 - 3*x^2)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 -
 2*a^3*x^2 + a)*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\text{atanh}\,(a\,x)^2\left(a^2\,x^2 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
[Out] int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

3.288 $\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=38

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -x^2/a/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a^3
```

Rubi [A] time = 0.13, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6006, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(x^2/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx = -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx}{a}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}$$

Mathematica [A] time = 0.15, size = 36, normalized size = 0.95

$$\frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3} + \frac{x^2}{a\left(a^2x^2-1\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] x^2/(a*(-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

fricas [B] time = 1.37, size = 111, normalized size = 2.92

$$\frac{4\,a^2x^2 + \left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^5x^2-a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2 + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x
^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^
5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

maple [A] time = 0.20, size = 36, normalized size = 0.95

$$\frac{\frac{1}{2\,\mathrm{arctanh}(ax)} - \frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)} + \mathrm{Shi}\left(2\,\mathrm{arctanh}\left(ax\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh
(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x^2}{\left(a^3x^2-a\right)\log\left(ax+1\right)-\left(a^3x^2-a\right)\log\left(-ax+1\right)}-4\int -\frac{x}{\left(a^5x^4-2\,a^3x^2+a\right)\log\left(ax+1\right)-\left(a^5x^4-2\,a^3x^2+a\right)\log\left(-ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x^2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - 4*integr
ate(-x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

3.289 $\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=36

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -x/a/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))/a^2
```

Rubi [A] time = 0.21, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6032, 6034, 3312, 3301, 5968}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/a^2
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx = -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx}{a} + a\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{\text{Su}}{}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} dx, x, \tanh^{-1}(ax)\right)}{2a^2}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 0.89

$$\frac{\frac{ax}{\left(a^2x^2-1\right)\tanh^{-1}(ax)} + \text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] ((a*x)/((-1 + a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]])/a^2
```

fricas [B] time = 0.59, size = 106, normalized size = 2.94

$$\frac{4\,ax + \left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + \left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^4x^2-a^2\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 -
 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^4*x^
2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

maple [A] time = 0.20, size = 28, normalized size = 0.78

$$\frac{-\frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)} + \text{X}\left(2\operatorname{arctanh}(ax)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] 1/a^2*(-1/2*sinh(2*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2x}{\left(a^3x^2-a\right)\log\left(ax+1\right)-\left(a^3x^2-a\right)\log\left(-ax+1\right)}-\int-\frac{2\left(a^2x^2+1\right)}{\left(a^5x^4-2a^3x^2+a\right)\log\left(ax+1\right)-\left(a^5x^4-2a^3x^2+a\right)\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - integrate(
-2*(a^2*x^2 + 1)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3
*x^2 + a)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x}{\mathrm{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

3.290 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=35

$$\frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -1/a/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a
```

Rubi [A] time = 0.09, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5966, 6034, 5448, 12, 3298}

$$\frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + (2a)\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx$$
$$= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.09, size = 30, normalized size = 0.86

$$\frac{\frac{1}{(a^2x^2-1)\tanh^{-1}(ax)} + \text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
[Out] (1/((-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]])/a
```

fricas [B] time = 0.59, size = 102, normalized size = 2.91

$$\frac{\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right) + 4}{2\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
[Out] 1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)) + 4)/((a^3*x^2 -
a)*log(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

maple [A] time = 0.18, size = 36, normalized size = 1.03

$$\frac{-\frac{1}{2\,\text{arctanh}(ax)} - \frac{\cosh(2\,\text{arctanh}(ax))}{2\,\text{arctanh}(ax)} + \text{Shi}\left(2\,\text{arctanh}(ax)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(-1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh(
a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,a\int -\frac{x}{\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)-\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-ax+1\right)}\,dx+\frac{2}{\left(a^3x^2-a\right)\log\left(ax+1\right)-\left(a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -4*a*integrate(-x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^
2*x^2 + 1)*log(-a*x + 1)), x) + 2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 -
a)*log(-a*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

3.291 $\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=62

$$-\frac{\text{Int}\left(\frac{1}{x^2\tanh^{-1}(ax)},x\right)}{a}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\text{Chi}\left(2\tanh^{-1}(ax)\right)-\frac{1}{ax\tanh^{-1}(ax)}$$

```
[Out] -1/a/x/arctanh(a*x)-a*x/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))-Unint
egrable(1/x^2/arctanh(a*x),x)/a
```

Rubi [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral
[2*ArcTanh[a*x]] - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)^2} \, dx = a^2\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^2} \, dx + \int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2} \, dx$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} \, dx}{a}+a\int \frac{}{(1-a^2x^2)}$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} \, dx}{a}+\text{Subst}\left(\int \frac{\cosh}{x}\right.$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} \, dx}{a}-\text{Subst}\left(\int \left(\frac{1}{2x}\right.\right.$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+2\left(\frac{1}{2}\text{Subst}\left(\int \frac{\cosh(2x)}{x} \, dx,x,\tan\right.\right.$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\text{Chi}\left(2\tanh^{-1}(ax)\right)-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)}}{a}$$

Mathematica [A] time = 4.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^4x^5 - 2\,a^2x^3 + x\right)\text{artanh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2 - 1\right)^2 x\,\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^2), x)
```

maple [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2 + 1\right)^2 \text{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{\left(a^3x^3 - ax\right)\log\left(ax + 1\right) - \left(a^3x^3 - ax\right)\log\left(-ax + 1\right)} - \int -\frac{2\left(3\,a^2x^2 - 1\right)}{\left(a^5x^6 - 2\,a^3x^4 + ax^2\right)\log\left(ax + 1\right) - \left(a^5x^6 - 2\,a^3x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
[Out] 2/((a^3*x^3 - a*x)*log(a*x + 1) - (a^3*x^3 - a*x)*log(-a*x + 1)) - integrat
e(-2*(3*a^2*x^2 - 1)/((a^5*x^6 - 2*a^3*x^4 + a*x^2)*log(a*x + 1) - (a^5*x^6
 - 2*a^3*x^4 + a*x^2)*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\text{atanh}\left(a\,x\right)^2 \left(a^2\,x^2 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
[Out] int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax - 1\right)^2 \left(ax + 1\right)^2 \text{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

3.292 $\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=102

$$-\frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)^2}, x\right)}{2a^3}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^4}+\frac{x}{2a^3\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}}$$

```
[Out] 1/2*x/a^3/arctanh(a*x)^2-1/2*x/a^3/(-a^2*x^2+1)/arctanh(a*x)^2+1/2*(-a^2*x^
2-1)/a^4/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a^4-1/2*Unintegrable
(1/arctanh(a*x)^2,x)/a^3
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] x/(2*a^3*ArcTanh[a*x]^2) - x/(2*a^3*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^
2*x^2)/(2*a^4*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^
4 - Defer[Int][ArcTanh[a*x]^(-2), x]/(2*a^3)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx &= \frac{\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx}{a^2}-\frac{\int \frac{x}{\left(1-a^2x^2\right) \tanh^{-1}(ax)^3} dx}{a^2}\\
&= \frac{x}{2a^3\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\\
&= \frac{x}{2a^3\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\\
&= \frac{x}{2a^3\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\\
&= \frac{x}{2a^3\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\\
&= \frac{x}{2a^3\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}+
\end{aligned}$$

Mathematica [A] time = 9.43, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2x^2-1\right)^2\operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

maple [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-a^2x^2+1\right)^2\operatorname{arctanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,ax^3-\left(a^2x^4-3\,x^2\right)\log\left(ax+1\right)+\left(a^2x^4-3\,x^2\right)\log\left(-ax+1\right)}{\left(a^4x^2-a^2\right)\log\left(ax+1\right)^2-2\left(a^4x^2-a^2\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^4x^2-a^2\right)\log\left(-ax+1\right)^2}-\int -\frac{}{\left(a^6x^4-2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
[Out] (2*a*x^3 - (a^2*x^4 - 3*x^2)*log(a*x + 1) + (a^2*x^4 - 3*x^2)*log(-a*x + 1)
)/((a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x
 + 1) + (a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-2*(a^4*x^5 - 2*a^2*x^
3 + 3*x)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1) - (a^6*x^4 - 2*a^4*x^2 +
 a^2)*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{atanh}\left(a\,x\right)^3\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

3.293 $\int \frac{x^2}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=64

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -1/2*x^2/a/(-a^2*x^2+1)/arctanh(a*x)^2-x/a^2/(-a^2*x^2+1)/arctanh(a*x)+Chi(
2*arctanh(a*x))/a^3
```

Rubi [A] time = 0.27, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6006, 6032, 6034, 3312, 3301, 5968}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -x^2/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(a^2*(1 - a^2*x^2)*ArcTanh[a*x]
) + CoshIntegral[2*ArcTanh[a*x]]/a^3
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
```

```
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}\,dx = -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}+\frac{\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx}{a}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,d\right.}{a^3}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\right.\right.}{}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}+2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\right.}{2}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^3}$$

Mathematica [A] time = 0.12, size = 47, normalized size = 0.73

$$\frac{\frac{ax\left(ax+2\tanh^{-1}(ax)\right)}{\left(a^2x^2-1\right)\tanh^{-1}(ax)^2}+2\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] ((a*x*(a*x + 2*ArcTanh[a*x]))/((-1 + a^2*x^2)*ArcTanh[a*x]^2) + 2*CoshInteg
ral[2*ArcTanh[a*x]])/(2*a^3)
```

fricas [B] time = 0.45, size = 131, normalized size = 2.05

$$\frac{4a^2x^2+4ax\log\left(-\frac{ax+1}{ax-1}\right)+\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^5x^2-a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2 + 4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_integ
ral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)
))*log(-(a*x + 1)/(a*x - 1))^2)/((a^5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1))^
2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

maple [A] time = 0.20, size = 51, normalized size = 0.80

$$\frac{\frac{1}{4\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)^2}-\frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\mathrm{X}\left(2\operatorname{arctanh}(ax)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2*sinh(
2*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(ax^2+x\log(ax+1)-x\log(-ax+1)\right)}{\left(a^4x^2-a^2\right)\log(ax+1)^2-2\left(a^4x^2-a^2\right)\log(ax+1)\log(-ax+1)+\left(a^4x^2-a^2\right)\log(-ax+1)^2}-\int-\frac{}{\left(a^6x^4-2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(a*x^2 + x*log(a*x + 1) - x*log(-a*x + 1))/((a^4*x^2 - a^2)*log(a*x + 1)^
2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^4*x^2 - a^2)*log(-a*x
 + 1)^2) - integrate(-2*(a^2*x^2 + 1)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x
+ 1) - (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax-1)^2(ax+1)^2\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

3.294 $\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=72

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -1/2*x/a/(-a^2*x^2+1)/arctanh(a*x)^2+1/2*(-a^2*x^2-1)/a^2/(-a^2*x^2+1)/arct
anh(a*x)+Shi(2*arctanh(a*x))/a^2
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5996, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2*x^2)*
ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 2\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh}{x}\right.}{}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,d\right.}{a^2}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx,\right.}{a^2}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.92

$$\frac{2\left(a^2x^2-1\right)\tanh^{-1}(ax)^2\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2+1\right)\tanh^{-1}(ax)+ax}{2a^2\left(a^2x^2-1\right)\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] (a*x + (1 + a^2*x^2)*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*SinhInt
egral[2*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

fricas [B] time = 0.54, size = 135, normalized size = 1.88

$$\frac{\left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,ax+2\left(a^2x^2+1\right)\log\left(-\frac{a}{a}\right.}{2\left(a^4x^2-a^2\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x + 2*(a^
2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/((a^4*x^2 - a^2)*log(-(a*x + 1)/(a*x
- 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

maple [A] time = 0.20, size = 43, normalized size = 0.60

$$\frac{-\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] 1/a^2*(-1/4*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/2/arctanh(a*x)*cosh(2*arc
tanh(a*x))+Shi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,ax+\left(a^2x^2+1\right)\log\left(ax+1\right)-\left(a^2x^2+1\right)\log\left(-ax+1\right)}{\left(a^4x^2-a^2\right)\log\left(ax+1\right)^2-2\left(a^4x^2-a^2\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^4x^2-a^2\right)\log\left(-ax+1\right)^2}-4\int-\frac{}{\left(a^4x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (a^2*x^2 + 1)*log(a*x + 1) - (a^2*x^2 + 1)*log(-a*x + 1))/((a^4*x^
2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a
^4*x^2 - a^2)*log(-a*x + 1)^2) - 4*integrate(-x/((a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^3\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

3.295 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=58

$$-\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)} - \frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)/arctanh(a*x)^2-x/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctan
h(a*x))/a
```

Rubi [A] time = 0.23, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5966, 6032, 6034, 3312, 3301, 5968}

$$-\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)} - \frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/((1 - a^2*x^2)*ArcTanh[a*x]) + Co
shIntegral[2*ArcTanh[a*x]]/a
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
```

```
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} + a\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + a^2\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh \cdots}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, \cdots\right)}{a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2 \cdots}{2x} \cdots\right)\right)}{\cdots}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx \cdots\right)}{2a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.08, size = 58, normalized size = 1.00

$$\frac{2\left(a^2x^2-1\right)\tanh^{-1}(ax)^2\text{Chi}\left(2\tanh^{-1}(ax)\right)+2ax\tanh^{-1}(ax)+1}{2a\left(a^2x^2-1\right)\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] (1 + 2*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*CoshIntegral[2*Ar
cTanh[a*x]])/(2*a*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

fricas [B] time = 0.52, size = 122, normalized size = 2.10

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4}{2\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_integral(-(a*x +
1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x
 + 1)/(a*x - 1))^2 + 4)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

maple [A] time = 0.18, size = 51, normalized size = 0.88

$$\frac{-\frac{1}{4\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)^2}-\frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\mathrm{X}(2\operatorname{arctanh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] 1/a*(-1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2*sinh(2
*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(ax\log(ax+1)-ax\log(-ax+1)+1\right)}{\left(a^3x^2-a\right)\log(ax+1)^2-2\left(a^3x^2-a\right)\log(ax+1)\log(-ax+1)+\left(a^3x^2-a\right)\log(-ax+1)^2}-\int -\frac{}{\left(a^4x^4-2\,a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(a*x*log(a*x + 1) - a*x*log(-a*x + 1) + 1)/((a^3*x^2 - a)*log(a*x + 1)^2
- 2*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1) + (a^3*x^2 - a)*log(-a*x + 1)^
2) - integrate(-2*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (
a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^3(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

3.296 $\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=99

$$-\frac{\mathrm{Int}\left(\frac{1}{x^2 \tanh^{-1}(ax)^2}, x\right)}{2a} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{a^2x^2+1}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \mathrm{Shi}\left(2\tanh^{-1}(ax)\right) - \frac{1}{2ax\tanh^{-1}(ax}$$

```
[Out] -1/2/a/x/arctanh(a*x)^2-1/2*a*x/(-a^2*x^2+1)/arctanh(a*x)^2+1/2*(-a^2*x^2-1
)/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))-1/2*Unintegrable(1/x^2/arct
anh(a*x)^2,x)/a
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a
^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]] - Def
er[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx = a^2 \int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx + \int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3} dx$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)} -$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)} +$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)} +$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)} -$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)} +$$

Mathematica [A] time = 3.83, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\left(a^4x^5 - 2\,a^2x^3 + x\right)\mathrm{artanh}\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2 - 1\right)^2 x\,\mathrm{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^3), x)
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2 + 1\right)^2 \mathrm{arctanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,ax + \left(3\,a^2x^2 - 1\right)\log\left(ax + 1\right) - \left(3\,a^2x^2 - 1\right)\log\left(-ax + 1\right)}{\left(a^4x^4 - a^2x^2\right)\log\left(ax + 1\right)^2 - 2\left(a^4x^4 - a^2x^2\right)\log\left(ax + 1\right)\log\left(-ax + 1\right) + \left(a^4x^4 - a^2x^2\right)\log\left(-ax + 1\right)^2} - \int -\frac{}{\left(a^6x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
[Out] (2*a*x + (3*a^2*x^2 - 1)*log(a*x + 1) - (3*a^2*x^2 - 1)*log(-a*x + 1))/((a^
4*x^4 - a^2*x^2)*log(a*x + 1)^2 - 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1)*log(-a
*x + 1) + (a^4*x^4 - a^2*x^2)*log(-a*x + 1)^2) - integrate(-2*(3*a^4*x^4 -
2*a^2*x^2 + 1)/((a^6*x^7 - 2*a^4*x^5 + a^2*x^3)*log(a*x + 1) - (a^6*x^7 - 2
*a^4*x^5 + a^2*x^3)*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^3\left(a^2\,x^2 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
[Out] int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(ax-1)^2\,(ax+1)^2\,\text{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

3.297 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}+\frac{2\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{3a}$$

```
[Out] -1/3/a/(-a^2*x^2+1)/arctanh(a*x)^3-1/3*x/(-a^2*x^2+1)/arctanh(a*x)^2+1/3*(-
a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)+2/3*Shi(2*arctanh(a*x))/a
```

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5966, 5996, 6034, 5448, 12, 3298}

$$-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}+\frac{2\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

```
[Out] -1/(3*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(3*(1 - a^2*x^2)*ArcTanh[a*x]^2)
- (1 + a^2*x^2)/(3*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*SinhIntegral[2*ArcTan
h[a*x]])/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
```

```
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4}\,dx = -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3} + \frac{1}{3}(2a)\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3} - \frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3} - \frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3} - \frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3} - \frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3} - \frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh}$$

Mathematica [A] time = 0.14, size = 73, normalized size = 0.75

$$\frac{2\left(a^2x^2-1\right)\tanh^{-1}(ax)^3\text{Shi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+ax\tanh^{-1}(ax)+1}{3a\left(a^2x^2-1\right)\tanh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

```
[Out] (1 + a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*Arc
Tanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]])/(3*a*(-1 + a^2*x^2)*ArcTanh[a*x]^
3)
```

fricas [A] time = 0.92, size = 151, normalized size = 1.56

$$\frac{\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+2\left(a^2\right.}{3\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^3 + 4*a*x*log(-(a
```

```
*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 8)/((a^3
*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^4), x)
```

maple [A] time = 0.18, size = 68, normalized size = 0.70

$$\frac{-\frac{1}{6\operatorname{arctanh}(ax)^3}-\frac{\cosh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^3}-\frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{3\operatorname{arctanh}(ax)}+\frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x)
```

```
[Out] 1/a*(-1/6/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6*sinh(2
*arctanh(a*x))/arctanh(a*x)^2-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*Shi
(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-8\,a\int -\frac{x}{3\left(\left(a^4x^4-2\,a^2x^2+1\right)\log(ax+1)-\left(a^4x^4-2\,a^2x^2+1\right)\log(-ax+1)\right)}\,dx+\frac{2\left(2\,ax\log(ax+1)+\right.}{3\left(\left(a^3x^2-a\right)\log(ax+1)^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -8*a*integrate(-1/3*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 -
2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/3*(2*a*x*log(a*x + 1) + (a^2*x^2 + 1)
*log(a*x + 1)^2 + (a^2*x^2 + 1)*log(-a*x + 1)^2 - 2*(a*x + (a^2*x^2 + 1)*lo
g(a*x + 1))*log(-a*x + 1) + 4)/((a^3*x^2 - a)*log(a*x + 1)^3 - 3*(a^3*x^2 -
 a)*log(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x +
1)^2 - (a^3*x^2 - a)*log(-a*x + 1)^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}(a\,x)^4\,\left(a^2\,x^2-1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2\,(ax+1)^2\operatorname{atanh}^4(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**4,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**4), x)
```

3.298 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^5} dx$

Optimal. Leaf size=120

$$-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{a^2x^2+1}{12a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4}+$$

```
[Out] -1/4/a/(-a^2*x^2+1)/arctanh(a*x)^4-1/6*x/(-a^2*x^2+1)/arctanh(a*x)^3+1/12*(
-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^2-1/3*x/(-a^2*x^2+1)/arctanh(a*x)+1
/3*Chi(2*arctanh(a*x))/a
```

Rubi [A] time = 0.29, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5966, 5996, 6032, 6034, 3312, 3301, 5968}

$$-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{a^2x^2+1}{12a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
```

```
[Out] -1/(4*a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(6*(1 - a^2*x^2)*ArcTanh[a*x]^3)
- (1 + a^2*x^2)/(12*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(3*(1 - a^2*x^2)*Ar
cTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
```

```
2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5}\,dx &= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}+\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4}\,dx \\
&= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.70

$$
\frac{4\left(a^2x^2-1\right)\tanh^{-1}(ax)^4\text{Chi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+4ax\tanh^{-1}(ax)^3+2ax\tanh^{-1}(ax)+3}{12a\left(a^2x^2-1\right)\tanh^{-1}(ax)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
```

[Out] (3 + 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh[a*x]^3 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^4*CoshIntegral[2*ArcTanh[a*x]])/(12*a*(-1 + a^2*x^2)*ArcTanh[a*x]^4)

fricas [A] time = 0.70, size = 171, normalized size = 1.42

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3+\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+8\,ax}{6\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="fricas")

[Out] 1/6*(4*a*x*log(-(a*x + 1)/(a*x - 1))^3 + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^4 + 8*a*x*log(-(a*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 24)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^4)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="giac")

[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^5), x)

maple [A] time = 0.19, size = 83, normalized size = 0.69

$$\frac{-\frac{1}{8\operatorname{arctanh}(ax)^4}-\frac{\cosh(2\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)^4}-\frac{\sinh(2\operatorname{arctanh}(ax))}{12\operatorname{arctanh}(ax)^3}-\frac{\cosh(2\operatorname{arctanh}(ax))}{12\operatorname{arctanh}(ax)^2}-\frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)}+\frac{\mathrm{X}(2\operatorname{arctanh}(ax))}{3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x)

[Out] 1/a*(-1/8/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12/arctanh(a*x)^3*sinh(2*arctanh(a*x))-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/6*sinh(2*arctanh(a*x))/arctanh(a*x)+1/3*Chi(2*arctanh(a*x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,ax\log(ax+1)^3-2\,ax\log(-ax+1)^3+4\,ax\log(ax+1)+\left(a^2x^2+1\right)\log(ax+1)^2+\left(a^2x^2+6\,ax\log(ax+\right.}{3\left(\left(a^3x^2-a\right)\log(ax+1)^4-4\left(a^3x^2-a\right)\log(ax+1)^3\log(-ax+1)+6\left(a^3x^2-a\right)\log(ax+1)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="maxima")

[Out] 1/3*(2*a*x*log(a*x + 1)^3 - 2*a*x*log(-a*x + 1)^3 + 4*a*x*log(a*x + 1) + (a^2*x^2 + 1)*log(a*x + 1)^2 + (a^2*x^2 + 6*a*x*log(a*x + 1) + 1)*log(-a*x + 1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*a*x + (a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 12)/((a^3*x^2 - a)*log(a*x + 1)^4 - 4*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1) + 6*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^3 + (a^3*x^2 - a)*log(-a*x + 1)^4) - integrate(-2/3*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}(a\,x)^5\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^2),x)
[Out] int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^5(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**5,x)
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**5), x)
```

3.299 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^6} dx$

Optimal. Leaf size=154

$$-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{a^2x^2+1}{15a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{a^2x^2+1}{30a\left(1-a^2x^2\right)\tanh^{-1}(ax$$

```
[Out] -1/5/a/(-a^2*x^2+1)/arctanh(a*x)^5-1/10*x/(-a^2*x^2+1)/arctanh(a*x)^4+1/30*
(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^3-1/15*x/(-a^2*x^2+1)/arctanh(a*x)
^2+1/15*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)+2/15*Shi(2*arctanh(a*x))/a
```

Rubi [A] time = 0.20, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5966, 5996, 6034, 5448, 12, 3298}

$$-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{a^2x^2+1}{15a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{a^2x^2+1}{30a\left(1-a^2x^2\right)\tanh^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```

```
[Out] -1/(5*a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(10*(1 - a^2*x^2)*ArcTanh[a*x]^4)
 - (1 + a^2*x^2)/(30*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(15*(1 - a^2*x^2)*
ArcTanh[a*x]^2) - (1 + a^2*x^2)/(15*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*Sinh
Integral[2*ArcTanh[a*x]])/(15*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
```

```
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^6}\,dx = -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} + \frac{1}{5}(2a)\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5}\,dx$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 0.66

$$\frac{4\left(a^2x^2-1\right)\tanh^{-1}(ax)^5\text{Shi}\left(2\tanh^{-1}(ax)\right)+2\left(a^2x^2+1\right)\tanh^{-1}(ax)^4+\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+2ax\tanh^{-1}(a}{30a\left(a^2x^2-1\right)\tanh^{-1}(ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```

```
[Out] (6 + 3*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*a*x*ArcTanh[a*x]
^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^5*SinhI
ntegral[2*ArcTanh[a*x]])/(30*a*(-1 + a^2*x^2)*ArcTanh[a*x]^5)
```

fricas [A] time = 0.79, size = 200, normalized size = 1.30

$$\frac{\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^5+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3+2\left(a^2x^2\right.}{15\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log
_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^5 + 4*a*x*log(-(
a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 24*a*
x*log(-(a*x + 1)/(a*x - 1)) + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 +
 96)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^6), x)
```

maple [A] time = 0.18, size = 98, normalized size = 0.64

$$\frac{-\frac{1}{10\operatorname{arctanh}(ax)^5}-\frac{\cosh(2\operatorname{arctanh}(ax))}{10\operatorname{arctanh}(ax)^5}-\frac{\sinh(2\operatorname{arctanh}(ax))}{20\operatorname{arctanh}(ax)^4}-\frac{\cosh(2\operatorname{arctanh}(ax))}{30\operatorname{arctanh}(ax)^3}-\frac{\sinh(2\operatorname{arctanh}(ax))}{30\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{15\operatorname{arctanh}(ax)}+2S}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x)
```

```
[Out] 1/a*(-1/10/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/arc
tanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1
/30*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/15/arctanh(a*x)*cosh(2*arctanh(a*
x))+2/15*Shi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-8\,a\int -\frac{x}{15\left(\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)-\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-ax+1\right)\right)}\,dx+\frac{2\left(2\,ax\log\left(ax+1\right)^3+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="maxima")
```

```
[Out] -8*a*integrate(-1/15*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 -
 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/15*(2*a*x*log(a*x + 1)^3 + (a^2*x^2
+ 1)*log(a*x + 1)^4 + (a^2*x^2 + 1)*log(-a*x + 1)^4 - 2*(a*x + 2*(a^2*x^2 +
 1)*log(a*x + 1))*log(-a*x + 1)^3 + 12*a*x*log(a*x + 1) + 2*(a^2*x^2 + 1)*l
og(a*x + 1)^2 + 2*(a^2*x^2 + 3*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*log(a*x +
 1)^2 + 1)*log(-a*x + 1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*(a^2*x^2 + 1)*log(
a*x + 1)^3 + 6*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 48)/((a^
3*x^2 - a)*log(a*x + 1)^5 - 5*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x + 1) +
10*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^3*x^2 - a)*log(a*x
+ 1)^2*log(-a*x + 1)^3 + 5*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^4 - (a^
3*x^2 - a)*log(-a*x + 1)^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^6\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^{2}\left(ax+1\right)^{2}\operatorname{atanh}^{6}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**6,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**6), x)
```

3.300 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^7}\,dx$

Optimal. Leaf size=177

$$-\frac{2x}{45\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{x}{45\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{a^2x^2+1}{90a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -1/6/a/(-a^2*x^2+1)/arctanh(a*x)^6-1/15*x/(-a^2*x^2+1)/arctanh(a*x)^5+1/60*
(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^4-1/45*x/(-a^2*x^2+1)/arctanh(a*x)
^3+1/90*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^2-2/45*x/(-a^2*x^2+1)/arct
anh(a*x)+2/45*Chi(2*arctanh(a*x))/a
```

Rubi [A] time = 0.34, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 5996, 6032, 6034, 3312, 3301, 5968}

$$-\frac{2x}{45\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{x}{45\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{a^2x^2+1}{90a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```

```
[Out] -1/(6*a*(1 - a^2*x^2)*ArcTanh[a*x]^6) - x/(15*(1 - a^2*x^2)*ArcTanh[a*x]^5)
 - (1 + a^2*x^2)/(60*a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(45*(1 - a^2*x^2)*
ArcTanh[a*x]^3) - (1 + a^2*x^2)/(90*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*x)
/(45*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*CoshIntegral[2*ArcTanh[a*x]])/(45*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
```

```
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^7}\,dx &= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}+\frac{1}{3}a\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^6}\,dx \\
&= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-} \\
&= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-} \\
&= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-} \\
&= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-} \\
&= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-} \\
&= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-} \\
&= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.63

$$
\frac{8\left(a^2x^2-1\right)\tanh^{-1}(ax)^6\text{Chi}\left(2\tanh^{-1}(ax)\right)+2\left(a^2x^2+1\right)\tanh^{-1}(ax)^4+3\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+8ax\tanh^{-1}}{180a\left(a^2x^2-1\right)\tanh^{-1}(ax)^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```

```
[Out] (30 + 12*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh[
a*x]^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 8*a*x*ArcTanh[a*x]^5 + 8*(-1 + a^
2*x^2)*ArcTanh[a*x]^6*CoshIntegral[2*ArcTanh[a*x]])/(180*a*(-1 + a^2*x^2)*A
rcTanh[a*x]^6)
```

fricas [A] time = 0.58, size = 220, normalized size = 1.24

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{5}+\left(\left(a^{2}x^{2}-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^{2}x^{2}-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{6}+8\,ax}{45\left(a^{3}x^{2}-a\right)\log\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="fricas")
```

```
[Out] 1/45*(4*a*x*log(-(a*x + 1)/(a*x - 1))^5 + ((a^2*x^2 - 1)*log_integral(-(a*x
 + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(
a*x + 1)/(a*x - 1))^6 + 8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 + 1)
*log(-(a*x + 1)/(a*x - 1))^4 + 96*a*x*log(-(a*x + 1)/(a*x - 1)) + 12*(a^2*x
^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 480)/((a^3*x^2 - a)*log(-(a*x + 1)/(a
*x - 1))^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}x^{2}-1\right)^{2}\operatorname{artanh}\left(ax\right)^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^7), x)
```

maple [A] time = 0.18, size = 113, normalized size = 0.64

$$\frac{-\frac{1}{12\operatorname{arctanh}(ax)^{6}}-\frac{\cosh(2\operatorname{arctanh}(ax))}{12\operatorname{arctanh}(ax)^{6}}-\frac{\sinh(2\operatorname{arctanh}(ax))}{30\operatorname{arctanh}(ax)^{5}}-\frac{\cosh(2\operatorname{arctanh}(ax))}{60\operatorname{arctanh}(ax)^{4}}-\frac{\sinh(2\operatorname{arctanh}(ax))}{90\operatorname{arctanh}(ax)^{3}}-\frac{\cosh(2\operatorname{arctanh}(ax))}{90\operatorname{arctanh}(ax)^{2}}-\text{si}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x)
```

```
[Out] 1/a*(-1/12/arctanh(a*x)^6-1/12/arctanh(a*x)^6*cosh(2*arctanh(a*x))-1/30/arc
tanh(a*x)^5*sinh(2*arctanh(a*x))-1/60/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1
/90/arctanh(a*x)^3*sinh(2*arctanh(a*x))-1/90/arctanh(a*x)^2*cosh(2*arctanh(
a*x))-1/45*sinh(2*arctanh(a*x))/arctanh(a*x)+2/45*Chi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(2\,ax\log\left(ax+1\right)^{5}-2\,ax\log\left(-ax+1\right)^{5}+4\,ax\log\left(ax+1\right)^{3}+\left(a^{2}x^{2}+1\right)\log\left(ax+1\right)^{4}+\left(a^{2}x^{2}+10\,ax\log\left(\right.\right.\right.}{45\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="maxima")
```

```
[Out] 2/45*(2*a*x*log(a*x + 1)^5 - 2*a*x*log(-a*x + 1)^5 + 4*a*x*log(a*x + 1)^3 +
 (a^2*x^2 + 1)*log(a*x + 1)^4 + (a^2*x^2 + 10*a*x*log(a*x + 1) + 1)*log(-a*
x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + a*x + (a^2*x^2 + 1)*log(a*x + 1))*log(
```

```
-a*x + 1)^3 + 48*a*x*log(a*x + 1) + 6*(a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(10*
a*x*log(a*x + 1)^3 + 3*a^2*x^2 + 6*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*log(a
*x + 1)^2 + 3)*log(-a*x + 1)^2 - 2*(5*a*x*log(a*x + 1)^4 + 6*a*x*log(a*x +
1)^2 + 2*(a^2*x^2 + 1)*log(a*x + 1)^3 + 24*a*x + 6*(a^2*x^2 + 1)*log(a*x +
1))*log(-a*x + 1) + 240)/((a^3*x^2 - a)*log(a*x + 1)^6 - 6*(a^3*x^2 - a)*lo
g(a*x + 1)^5*log(-a*x + 1) + 15*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x + 1)^
2 - 20*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^3 + 15*(a^3*x^2 - a)*log(
a*x + 1)^2*log(-a*x + 1)^4 - 6*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^5 +
 (a^3*x^2 - a)*log(-a*x + 1)^6) - integrate(-4/45*(a^2*x^2 + 1)/((a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)^7\,{\left(a^2\,x^2-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^7*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^7*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax - 1\right)^{2} \left(ax + 1\right)^{2} \operatorname{atanh}^{7}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**7,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**7), x)
```

3.301 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^8}\,dx$

Optimal. Leaf size=211

$$-\frac{2x}{315\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{105\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{2\left(a^2x^2+1\right)}{315a\left(1-a^2x^2\right)\tanh^{-1}}$$

```
[Out] -1/7/a/(-a^2*x^2+1)/arctanh(a*x)^7-1/21*x/(-a^2*x^2+1)/arctanh(a*x)^6+1/105
*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^5-1/105*x/(-a^2*x^2+1)/arctanh(a*
x)^4+1/315*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^3-2/315*x/(-a^2*x^2+1)/
arctanh(a*x)^2-2/315*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)+4/315*Shi(2*ar
ctanh(a*x))/a
```

Rubi [A] time = 0.26, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5966, 5996, 6034, 5448, 12, 3298}

$$-\frac{2x}{315\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{105\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{2\left(a^2x^2+1\right)}{315a\left(1-a^2x^2\right)\tanh^{-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

```
[Out] -1/(7*a*(1 - a^2*x^2)*ArcTanh[a*x]^7) - x/(21*(1 - a^2*x^2)*ArcTanh[a*x]^6)
 - (1 + a^2*x^2)/(105*a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(105*(1 - a^2*x^2
)*ArcTanh[a*x]^4) - (1 + a^2*x^2)/(315*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - (2
*x)/(315*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*(1 + a^2*x^2))/(315*a*(1 - a^2*
x^2)*ArcTanh[a*x]) + (4*SinhIntegral[2*ArcTanh[a*x]])/(315*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^8}\,dx = -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}+\frac{1}{7}(2a)\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^7}\,dx$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh}$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh}$$

Mathematica [A] time = 0.14, size = 128, normalized size = 0.61

$$\frac{4\left(a^2x^2-1\right)\tanh^{-1}(ax)^7\text{Shi}\left(2\tanh^{-1}(ax)\right)+2\left(a^2x^2+1\right)\tanh^{-1}(ax)^6+\left(a^2x^2+1\right)\tanh^{-1}(ax)^4+3\left(a^2x^2+1\right)\text{t}}{315a\left(a^2x^2-1\right)\tanh^{-1}(ax)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

```
[Out] (45 + 15*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh[
a*x]^3 + (1 + a^2*x^2)*ArcTanh[a*x]^4 + 2*a*x*ArcTanh[a*x]^5 + 2*(1 + a^2*x
^2)*ArcTanh[a*x]^6 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^7*SinhIntegral[2*ArcTanh
[a*x]])/(315*a*(-1 + a^2*x^2)*ArcTanh[a*x]^7)
```

fricas [A] time = 1.08, size = 249, normalized size = 1.18

$$\frac{2\left(\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^7+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^5+2\cdots\right.}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="fricas")
```

```
[Out] 2/315*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*lo
g_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^7 + 4*a*x*log(-
(a*x + 1)/(a*x - 1))^5 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^6 + 24*a
*x*log(-(a*x + 1)/(a*x - 1))^3 + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^
4 + 480*a*x*log(-(a*x + 1)/(a*x - 1)) + 48*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*
x - 1))^2 + 2880)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^2\operatorname{artanh}\left(ax\right)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^8), x)
```

maple [A] time = 0.18, size = 128, normalized size = 0.61

$$\frac{-\frac{1}{14\operatorname{arctanh}(ax)^7}-\frac{\cosh(2\operatorname{arctanh}(ax))}{14\operatorname{arctanh}(ax)^7}-\frac{\sinh(2\operatorname{arctanh}(ax))}{42\operatorname{arctanh}(ax)^6}-\frac{\cosh(2\operatorname{arctanh}(ax))}{105\operatorname{arctanh}(ax)^5}-\frac{\sinh(2\operatorname{arctanh}(ax))}{210\operatorname{arctanh}(ax)^4}-\frac{\cosh(2\operatorname{arctanh}(ax))}{315\operatorname{arctanh}(ax)^3}-\frac{\sin\cdots}{3\cdots}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x)
```

```
[Out] 1/a*(-1/14/arctanh(a*x)^7-1/14/arctanh(a*x)^7*cosh(2*arctanh(a*x))-1/42/arc
tanh(a*x)^6*sinh(2*arctanh(a*x))-1/105/arctanh(a*x)^5*cosh(2*arctanh(a*x))-
1/210/arctanh(a*x)^4*sinh(2*arctanh(a*x))-1/315/arctanh(a*x)^3*cosh(2*arcta
nh(a*x))-1/315*sinh(2*arctanh(a*x))/arctanh(a*x)^2-2/315/arctanh(a*x)*cosh(
2*arctanh(a*x))+4/315*Shi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-16\,a\int -\frac{x}{315\left(\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)-\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-ax+1\right)\right)}\,dx+\frac{4\left(2\,ax\log\left(ax+1\right)^5\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="maxima")
```

```
[Out] -16*a*integrate(-1/315*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4
 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 4/315*(2*a*x*log(a*x + 1)^5 + (a^2*x
^2 + 1)*log(a*x + 1)^6 + (a^2*x^2 + 1)*log(-a*x + 1)^6 - 2*(a*x + 3*(a^2*x^
2 + 1)*log(a*x + 1))*log(-a*x + 1)^5 + 12*a*x*log(a*x + 1)^3 + 2*(a^2*x^2 +
 1)*log(a*x + 1)^4 + (2*a^2*x^2 + 10*a*x*log(a*x + 1) + 15*(a^2*x^2 + 1)*lo
g(a*x + 1)^2 + 2)*log(-a*x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + 5*(a^2*x^2 +
1)*log(a*x + 1)^3 + 3*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1)^3 +
```

```
 240*a*x*log(a*x + 1) + 24*(a^2*x^2 + 1)*log(a*x + 1)^2 + (20*a*x*log(a*x +
 1)^3 + 15*(a^2*x^2 + 1)*log(a*x + 1)^4 + 24*a^2*x^2 + 36*a*x*log(a*x + 1)
+ 12*(a^2*x^2 + 1)*log(a*x + 1)^2 + 24)*log(-a*x + 1)^2 - 2*(5*a*x*log(a*x
+ 1)^4 + 3*(a^2*x^2 + 1)*log(a*x + 1)^5 + 18*a*x*log(a*x + 1)^2 + 4*(a^2*x^
2 + 1)*log(a*x + 1)^3 + 120*a*x + 24*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x +
 1) + 1440)/((a^3*x^2 - a)*log(a*x + 1)^7 - 7*(a^3*x^2 - a)*log(a*x + 1)^6*
log(-a*x + 1) + 21*(a^3*x^2 - a)*log(a*x + 1)^5*log(-a*x + 1)^2 - 35*(a^3*x
^2 - a)*log(a*x + 1)^4*log(-a*x + 1)^3 + 35*(a^3*x^2 - a)*log(a*x + 1)^3*lo
g(-a*x + 1)^4 - 21*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x + 1)^5 + 7*(a^3*x^
2 - a)*log(a*x + 1)*log(-a*x + 1)^6 - (a^3*x^2 - a)*log(-a*x + 1)^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\operatorname{atanh}(a\,x)^8\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^8*(a^2*x^2 - 1)^2),x)
[Out] int(1/(atanh(a*x)^8*(a^2*x^2 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^8(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**8,x)
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**8), x)
```

3.302 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=77

$$-\frac{3\tanh^{-1}(ax)}{32a^4}+\frac{x^4\tanh^{-1}(ax)}{4(1-a^2x^2)^2}-\frac{x^3}{16a(1-a^2x^2)^2}+\frac{3x}{32a^3(1-a^2x^2)}$$

```
[Out] -1/16*x^3/a/(-a^2*x^2+1)^2+3/32*x/a^3/(-a^2*x^2+1)-3/32*arctanh(a*x)/a^4+1/
4*x^4*arctanh(a*x)/(-a^2*x^2+1)^2
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6008, 288, 206}

$$-\frac{x^3}{16a(1-a^2x^2)^2}+\frac{3x}{32a^3(1-a^2x^2)}+\frac{x^4\tanh^{-1}(ax)}{4(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -x^3/(16*a*(1 - a^2*x^2)^2) + (3*x)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x
])/(32*a^4) + (x^4*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx = \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} - \frac{1}{4}a\int \frac{x^4}{\left(1-a^2x^2\right)^3}\,dx$$

$$= -\frac{x^3}{16a\left(1-a^2x^2\right)^2} + \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3\int \frac{x^2}{\left(1-a^2x^2\right)^2}\,dx}{16a}$$

$$= -\frac{x^3}{16a\left(1-a^2x^2\right)^2} + \frac{3x}{32a^3\left(1-a^2x^2\right)} + \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} - \frac{3\int \frac{1}{1-a^2x^2}\,dx}{32a^3}$$

$$= -\frac{x^3}{16a\left(1-a^2x^2\right)^2} + \frac{3x}{32a^3\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)}{32a^4} + \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 1.27

$$-\frac{5\log(1-ax)}{64a^4} + \frac{5\log(ax+1)}{64a^4} + \frac{\left(2a^2x^2-1\right)\tanh^{-1}(ax)}{4a^4\left(a^2x^2-1\right)^2} - \frac{5x}{32a^3\left(a^2x^2-1\right)} - \frac{x}{16a^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*x/(a^3*(-1 + a^2*x^2)^2) - (5*x)/(32*a^3*(-1 + a^2*x^2)) + ((-1 + 2*a
^2*x^2)*ArcTanh[a*x])/(4*a^4*(-1 + a^2*x^2)^2) - (5*Log[1 - a*x])/(64*a^4)
+ (5*Log[1 + a*x])/(64*a^4)
```

fricas [A] time = 0.80, size = 71, normalized size = 0.92

$$-\frac{10\,a^3x^3 - 6\,ax - \left(5\,a^4x^4 + 6\,a^2x^2 - 3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{64\left(a^8x^4 - 2\,a^6x^2 + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*a^3*x^3 - 6*a*x - (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x
 - 1)))/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

giac [B] time = 0.19, size = 239, normalized size = 3.10

$$\frac{1}{256}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5} + \frac{\frac{(ax+1)^2a^5}{(ax-1)^2} + \frac{4(ax+1)a^5}{ax-1}}{a^{10}}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right) + \frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5} - \frac{\frac{(ax+1)^2a^5}{(ax-1)^2}+\cdots}{a\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/256*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + ((a*x
 + 1)^2*a^5/(a*x - 1)^2 + 4*(a*x + 1)*a^5/(a*x - 1))/a^10)*log(-(a*((a*x +
1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1)
+ 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + (a*x - 1)^2*(8*(a*x + 1)/(a*x - 1)
 + 1)/((a*x + 1)^2*a^5) - ((a*x + 1)^2*a^5/(a*x - 1)^2 + 8*(a*x + 1)*a^5/(a
*x - 1))/a^10)*a
```

maple [A] time = 0.04, size = 136, normalized size = 1.77

$$\frac{\operatorname{arctanh}(ax)}{16a^4(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{16a^4(ax-1)}+\frac{\operatorname{arctanh}(ax)}{16a^4(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16a^4(ax+1)}-\frac{1}{64a^4(ax-1)^2}-\frac{5}{64a^4(ax-1)}-\frac{5\ln(ax-1)}{64a^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^4*arctanh(a*x)/(a*x-1)^2+3/16/a^4*arctanh(a*x)/(a*x-1)+1/16/a^4*arct
anh(a*x)/(a*x+1)^2-3/16/a^4*arctanh(a*x)/(a*x+1)-1/64/a^4/(a*x-1)^2-5/64/a^
4/(a*x-1)-5/64/a^4*ln(a*x-1)+1/64/a^4/(a*x+1)^2-5/64/a^4/(a*x+1)+5/64/a^4*l
n(a*x+1)
```

maxima [A] time = 0.31, size = 99, normalized size = 1.29

$$-\frac{1}{64}a\left(\frac{2\left(5a^2x^3-3x\right)}{a^8x^4-2a^6x^2+a^4}-\frac{5\log(ax+1)}{a^5}+\frac{5\log(ax-1)}{a^5}\right)+\frac{\left(2a^2x^2-1\right)\operatorname{artanh}(ax)}{4\left(a^8x^4-2a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/(a^8*x^4 - 2*a^
6*x^2 + a^4)
```

mupad [B] time = 1.39, size = 83, normalized size = 1.08

$$\frac{5\operatorname{atanh}(a\,x)}{32\,a^4}+\frac{\frac{\ln(1-a\,x)}{8}-\frac{\ln(a\,x+1)}{8}+\frac{3\,a\,x}{32}+x^2\left(\frac{a^2\,\ln(a\,x+1)}{4}-\frac{a^2\,\ln(1-a\,x)}{4}\right)-\frac{5\,a^3\,x^3}{32}}{a^4\left(a^2\,x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

```
[Out] (5*atanh(a*x))/(32*a^4) + (log(1 - a*x)/8 - log(a*x + 1)/8 + (3*a*x)/32 + x
^2*((a^2*log(a*x + 1))/4 - (a^2*log(1 - a*x))/4) - (5*a^3*x^3)/32)/(a^4*(a^
2*x^2 - 1)^2)
```

sympy [A] time = 2.41, size = 158, normalized size = 2.05

$$\begin{cases}\frac{5a^4x^4\operatorname{atanh}(ax)}{32a^8x^4-64a^6x^2+32a^4}-\frac{5a^3x^3}{32a^8x^4-64a^6x^2+32a^4}+\frac{6a^2x^2\operatorname{atanh}(ax)}{32a^8x^4-64a^6x^2+32a^4}+\frac{3ax}{32a^8x^4-64a^6x^2+32a^4}-\frac{3\operatorname{atanh}(ax)}{32a^8x^4-64a^6x^2+32a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((5*a**4*x**4*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) -
 5*a**3*x**3/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 6*a**2*x**2*atanh(a*
x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 3*a*x/(32*a**8*x**4 - 64*a**6*
x**2 + 32*a**4) - 3*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4), Ne(
a, 0)), (0, True))
```

3.303 $\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=100

$$-\frac{\tanh^{-1}(ax)^2}{16a^3}-\frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}+\frac{1}{16a^3(1-a^2x^2)}-\frac{1}{16a^3(1-a^2x^2)^2}$$

```
[Out] -1/16/a^3/(-a^2*x^2+1)^2+1/16/a^3/(-a^2*x^2+1)+1/4*x*arctanh(a*x)/a^2/(-a^2
*x^2+1)^2-1/8*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-1/16*arctanh(a*x)^2/a^3
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5998, 5956, 261}

$$\frac{1}{16a^3(1-a^2x^2)}-\frac{1}{16a^3(1-a^2x^2)^2}-\frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\tanh^{-1}(ax)^2}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/(16*a^3*(1 - a^2*x^2)^2) + 1/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(
4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x])/(8*a^2*(1 - a^2*x^2)) - ArcTanh[a
*x]^2/(16*a^3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx = -\frac{1}{16a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{4a^2}$$

$$= -\frac{1}{16a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)}{8a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{16a^3} + \frac{\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx}{8a}$$

$$= -\frac{1}{16a^3\left(1-a^2x^2\right)^2} + \frac{1}{16a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)}{8a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{16a^3}$$

Mathematica [A] time = 0.09, size = 61, normalized size = 0.61

$$-\frac{-2\left(a^3x^3+ax\right)\tanh^{-1}(ax)+a^2x^2+\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2}{16a^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*(a^2*x^2 - 2*(a*x + a^3*x^3)*ArcTanh[a*x] + (-1 + a^2*x^2)^2*ArcTanh[
a*x]^2)/(a^3*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.51, size = 95, normalized size = 0.95

$$-\frac{4\,a^2x^2+\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\left(a^3x^3+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{64\left(a^7x^4-2\,a^5x^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*a^2*x^2 + (a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 -
4*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2\,\mathrm{artanh}\,(ax)}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

maple [B] time = 0.06, size = 225, normalized size = 2.25

$$\frac{\mathrm{arctanh}\,(ax)}{16a^3\,(ax-1)^2}+\frac{\mathrm{arctanh}\,(ax)}{16a^3\,(ax-1)}+\frac{\mathrm{arctanh}\,(ax)\ln\,(ax-1)}{16a^3}-\frac{\mathrm{arctanh}\,(ax)}{16a^3\,(ax+1)^2}+\frac{\mathrm{arctanh}\,(ax)}{16a^3\,(ax+1)}-\frac{\mathrm{arctanh}\,(ax)\ln\,(ax+}{16a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^3*arctanh(a*x)/(a*x-1)^2+1/16/a^3*arctanh(a*x)/(a*x-1)+1/16/a^3*arct
anh(a*x)*ln(a*x-1)-1/16/a^3*arctanh(a*x)/(a*x+1)^2+1/16/a^3*arctanh(a*x)/(a
*x+1)-1/16/a^3*arctanh(a*x)*ln(a*x+1)+1/64/a^3*ln(a*x+1)^2+1/32/a^3*ln(-1/2
*a*x+1/2)*ln(1/2+1/2*a*x)-1/32/a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/64/a^3*ln(a
*x-1)^2-1/32/a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-1/64/a^3/(a*x-1)^2-1/64/a^3/(a*x
-1)-1/64/a^3/(a*x+1)^2+1/64/a^3/(a*x+1)
```

maxima [B] time = 0.32, size = 179, normalized size = 1.79

$$\frac{1}{16}\left(\frac{2\left(a^2x^3+x\right)}{a^6x^4-2\,a^4x^2+a^2}-\frac{\log\left(ax+1\right)}{a^3}+\frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right)-\frac{\left(4\,a^2x^2-\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x) - 1/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*lo
g(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

mupad [B] time = 1.19, size = 150, normalized size = 1.50

$$\ln\left(1-a\,x\right)\left(\frac{\ln\left(a\,x+1\right)}{32\,a^3}-\frac{\frac{x}{8\,a^2}+\frac{x^3}{8}}{2\,a^4\,x^4-4\,a^2\,x^2+2}\right)-\frac{\ln\left(a\,x+1\right)^2}{64\,a^3}-\frac{\ln\left(1-a\,x\right)^2}{64\,a^3}-\frac{x^2}{2\,\left(8\,a^5\,x^4-16\,a^3\,x^2+8\,a\right)}+\frac{\ln\left(a\,x\right.}{\frac{1}{a}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(1 - a*x)*(log(a*x + 1)/(32*a^3) - (x/(8*a^2) + x^3/8)/(2*a^4*x^4 - 4*a^
2*x^2 + 2)) - log(a*x + 1)^2/(64*a^3) - log(1 - a*x)^2/(64*a^3) - x^2/(2*(8
*a - 16*a^3*x^2 + 8*a^5*x^4)) + (log(a*x + 1)*(x/(16*a^3) + x^3/(16*a)))/(1
/a - 2*a*x^2 + a^3*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2\operatorname{atanh}\left(ax\right)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

3.304 $\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx$

Optimal. Leaf size=75

$$-\frac{3x}{32a\left(1-a^2x^2\right)}-\frac{x}{16a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{32a^2}$$

```
[Out] -1/16*x/a/(-a^2*x^2+1)^2-3/32*x/a/(-a^2*x^2+1)-3/32*arctanh(a*x)/a^2+1/4*ar
ctanh(a*x)/a^2/(-a^2*x^2+1)^2
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5994, 199, 206}

$$-\frac{3x}{32a\left(1-a^2x^2\right)}-\frac{x}{16a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -x/(16*a*(1 - a^2*x^2)^2) - (3*x)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(
32*a^2) + ArcTanh[a*x]/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx = \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{\int \frac{1}{(1-a^2x^2)^3}\,dx}{4a}$$
$$= -\frac{x}{16a(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{1}{(1-a^2x^2)^2}\,dx}{16a}$$
$$= -\frac{x}{16a(1-a^2x^2)^2} - \frac{3x}{32a(1-a^2x^2)} + \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{1}{1-a^2x^2}\,dx}{32a}$$
$$= -\frac{x}{16a(1-a^2x^2)^2} - \frac{3x}{32a(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)}{32a^2} + \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 1.17

$$\frac{3x}{32a(a^2x^2-1)} - \frac{x}{16a(a^2x^2-1)^2} + \frac{\tanh^{-1}(ax)}{4a^2(a^2x^2-1)^2} + \frac{3\log(1-ax)}{64a^2} - \frac{3\log(ax+1)}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*x/(a*(-1 + a^2*x^2)^2) + (3*x)/(32*a*(-1 + a^2*x^2)) + ArcTanh[a*x]/(
4*a^2*(-1 + a^2*x^2)^2) + (3*Log[1 - a*x])/(64*a^2) - (3*Log[1 + a*x])/(64*
a^2)
```

fricas [A] time = 0.56, size = 71, normalized size = 0.95

$$\frac{6\,a^3x^3 - 10\,ax - (3\,a^4x^4 - 6\,a^2x^2 - 5)\log\left(-\frac{ax+1}{ax-1}\right)}{64\,(a^6x^4 - 2\,a^4x^2 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(6*a^3*x^3 - 10*a*x - (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x
- 1)))/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

giac [B] time = 0.25, size = 239, normalized size = 3.19

$$-\frac{1}{256}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3} - \frac{\frac{(ax+1)^2a^3}{(ax-1)^2} - \frac{4(ax+1)a^3}{ax-1}}{a^6}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right) + \frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3} + \frac{\frac{(ax+1)^2a^3}{(ax-1)^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/256*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - ((a*
x + 1)^2*a^3/(a*x - 1)^2 - 4*(a*x + 1)*a^3/(a*x - 1))/a^6)*log(-(a*((a*x +
1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1)
+ 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + (a*x - 1)^2*(8*(a*x + 1)/(a*x - 1)
```

```
 - 1)/((a*x + 1)^2*a^3) + ((a*x + 1)^2*a^3/(a*x - 1)^2 - 8*(a*x + 1)*a^3/(a
*x - 1))/a^6)*a
```

maple [A] time = 0.04, size = 92, normalized size = 1.23

$$\frac{\operatorname{arctanh}(ax)}{4a^2\left(a^2x^2-1\right)^2}-\frac{1}{64a^2\left(ax-1\right)^2}+\frac{3}{64a^2\left(ax-1\right)}+\frac{3\ln\left(ax-1\right)}{64a^2}+\frac{1}{64a^2\left(ax+1\right)^2}+\frac{3}{64a^2\left(ax+1\right)}-\frac{3\ln\left(ax+1\right)}{64a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/4/a^2/(a^2*x^2-1)^2*arctanh(a*x)-1/64/a^2/(a*x-1)^2+3/64/a^2/(a*x-1)+3/64
/a^2*ln(a*x-1)+1/64/a^2/(a*x+1)^2+3/64/a^2/(a*x+1)-3/64/a^2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 82, normalized size = 1.09

$$\frac{\frac{2\left(3a^2x^3-5x\right)}{a^4x^4-2a^2x^2+1}-\frac{3\log(ax+1)}{a}+\frac{3\log(ax-1)}{a}}{64a}+\frac{\operatorname{artanh}(ax)}{4\left(a^2x^2-1\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)/a + 1/4*arctanh(a*x)/((a^2*x^2 - 1)^2*a^2)
```

mupad [B] time = 1.08, size = 105, normalized size = 1.40

$$\frac{\frac{3\ln(a\,x-1)}{64}-\frac{3\ln(a\,x+1)}{64}}{a^2}+\frac{\frac{\operatorname{atanh}(a\,x)}{4}-x^2\left(a^2\left(\frac{3\ln(a\,x-1)}{32}-\frac{3\ln(a\,x+1)}{32}\right)-2\,a^2\left(\frac{3\ln(a\,x-1)}{64}-\frac{3\ln(a\,x+1)}{64}\right)\right)-\frac{5\,a\,x}{32}+\frac{3\,a^3\,x^3}{32}}{a^2\left(a^2\,x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

```
[Out] ((3*log(a*x - 1))/64 - (3*log(a*x + 1))/64)/a^2 + (atanh(a*x)/4 - x^2*(a^2*
((3*log(a*x - 1))/32 - (3*log(a*x + 1))/32) - 2*a^2*((3*log(a*x - 1))/64 -
(3*log(a*x + 1))/64)) - (5*a*x)/32 + (3*a^3*x^3)/32)/(a^2*(a^2*x^2 - 1)^2)
```

sympy [A] time = 2.41, size = 158, normalized size = 2.11

$$\begin{cases} -\frac{3a^4x^4\operatorname{atanh}(ax)}{32a^6x^4-64a^4x^2+32a^2}+\frac{3a^3x^3}{32a^6x^4-64a^4x^2+32a^2}+\frac{6a^2x^2\operatorname{atanh}(ax)}{32a^6x^4-64a^4x^2+32a^2}-\frac{5ax}{32a^6x^4-64a^4x^2+32a^2}+\frac{5\operatorname{atanh}(ax)}{32a^6x^4-64a^4x^2+32a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((-3*a**4*x**4*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2)
+ 3*a**3*x**3/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) + 6*a**2*x**2*atanh(a
*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) - 5*a*x/(32*a**6*x**4 - 64*a**4
*x**2 + 32*a**2) + 5*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2), Ne
(a, 0)), (0, True))
```

3.305 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=94

$$-\frac{3}{16a\left(1-a^2x^2\right)}-\frac{1}{16a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3\tanh^{-1}(ax)^2}{16a}$$

```
[Out] -1/16/a/(-a^2*x^2+1)^2-3/16/a/(-a^2*x^2+1)+1/4*x*arctanh(a*x)/(-a^2*x^2+1)^
2+3/8*x*arctanh(a*x)/(-a^2*x^2+1)+3/16*arctanh(a*x)^2/a
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5960, 5956, 261}

$$-\frac{3}{16a\left(1-a^2x^2\right)}-\frac{1}{16a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3\tanh^{-1}(ax)^2}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/(16*a*(1 - a^2*x^2)^2) - 3/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(4*(1
 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^2)/
(16*a)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx = -\frac{1}{16a\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3}{4}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx$$
$$= -\frac{1}{16a\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{16a} - \frac{1}{8}(3a)\int \frac{x}{\left(1-a^2x^2\right.}$$
$$= -\frac{1}{16a\left(1-a^2x^2\right)^2} - \frac{3}{16a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{16a}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.69

$$\frac{\left(10ax-6a^3x^3\right)\tanh^{-1}(ax)+3a^2x^2+3\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2-4}{16a\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```

```
[Out] (-4 + 3*a^2*x^2 + (10*a*x - 6*a^3*x^3)*ArcTanh[a*x] + 3*(-1 + a^2*x^2)^2*Ar
cTanh[a*x]^2)/(16*a*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.81, size = 97, normalized size = 1.03

$$\frac{12\,a^2x^2+3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\left(3\,a^3x^3-5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-16}{64\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
 - 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^5*x^4 - 2*a^3*x^
2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

maple [B] time = 0.06, size = 225, normalized size = 2.39

$$\frac{\operatorname{arctanh}(ax)}{16a\,(ax-1)^2} - \frac{3\operatorname{arctanh}(ax)}{16a\,(ax-1)} - \frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{16a} - \frac{\operatorname{arctanh}(ax)}{16a\,(ax+1)^2} - \frac{3\operatorname{arctanh}(ax)}{16a\,(ax+1)} + \frac{3\operatorname{arctanh}(ax)\ln(a}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a*arctanh(a*x)/(a*x-1)^2-3/16/a*arctanh(a*x)/(a*x-1)-3/16/a*arctanh(a*
x)*ln(a*x-1)-1/16/a*arctanh(a*x)/(a*x+1)^2-3/16/a*arctanh(a*x)/(a*x+1)+3/16
/a*arctanh(a*x)*ln(a*x+1)-3/64/a*ln(a*x-1)^2+3/32/a*ln(a*x-1)*ln(1/2+1/2*a*
x)-3/64/a*ln(a*x+1)^2-3/32/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+3/32/a*ln(-1/
2*a*x+1/2)*ln(a*x+1)-1/64/a/(a*x-1)^2+7/64/a/(a*x-1)-1/64/a/(a*x+1)^2-7/64/
a/(a*x+1)
```

maxima [B] time = 0.32, size = 182, normalized size = 1.94

$$-\frac{1}{16}\left(\frac{2\left(3\,a^2x^3-5\,x\right)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log\left(ax+1\right)}{a}+\frac{3\,\log\left(ax-1\right)}{a}\right)\mathrm{artanh}\left(ax\right)+\frac{\left(12\,a^2x^2-3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x) + 1/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 +
 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1)
- 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a/(a^6*x^4 - 2*a^4*x^2 +
 a^2)
```

mupad [B] time = 1.37, size = 154, normalized size = 1.64

$$\frac{\frac{3\,a\,x^2}{2}-\frac{2}{a}}{8\,a^4\,x^4-16\,a^2\,x^2+8}-\ln\left(1-a\,x\right)\left(\frac{3\,\ln\left(a\,x+1\right)}{32\,a}+\frac{\frac{5\,x}{8}-\frac{3\,a^2\,x^3}{8}}{2\,a^4\,x^4-4\,a^2\,x^2+2}\right)+\frac{3\,\ln\left(a\,x+1\right)^2}{64\,a}+\frac{3\,\ln\left(1-a\,x\right)^2}{64\,a}+\frac{\ln\left(a\,x\right.}{\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(a^2*x^2 - 1)^3,x)
```

```
[Out] ((3*a*x^2)/2 - 2/a)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 - a*x)*((3*log(a*x
 + 1))/(32*a) + ((5*x)/8 - (3*a^2*x^3)/8)/(2*a^4*x^4 - 4*a^2*x^2 + 2)) + (3
*log(a*x + 1)^2)/(64*a) + (3*log(1 - a*x)^2)/(64*a) + (log(a*x + 1)*((5*x)/
(16*a) - (3*a*x^3)/16))/(1/a - 2*a*x^2 + a^3*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\mathrm{atanh}\left(ax\right)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

3.306 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{11ax}{32\left(1-a^2x^2\right)}-\frac{ax}{16\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}-\frac{1}{2}\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{11}{32}\tanh^{-1}(a$$

```
[Out] -1/16*a*x/(-a^2*x^2+1)^2-11/32*a*x/(-a^2*x^2+1)-11/32*arctanh(a*x)+1/4*arct
anh(a*x)/(-a^2*x^2+1)^2+1/2*arctanh(a*x)/(-a^2*x^2+1)+1/2*arctanh(a*x)^2+ar
ctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.25, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6030, 5988, 5932, 2447, 5994, 199, 206}

$$-\frac{1}{2}\mathrm{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{11ax}{32\left(1-a^2x^2\right)}-\frac{ax}{16\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{11}{32}\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*x)/(16*(1 - a^2*x^2)^2) - (11*a*x)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x
])/32 + ArcTanh[a*x]/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(2*(1 - a^2*x^2)) +
 ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(
1 + a*x)]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} - \frac{1}{4}a\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx + a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx$$

$$= -\frac{ax}{16\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 - \frac{1}{16}(3a)\int \frac{1}{\left(1-a^2x^2\right)^2}\,d$$

$$= -\frac{ax}{16\left(1-a^2x^2\right)^2} - \frac{11ax}{32\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}$$

$$= -\frac{ax}{16\left(1-a^2x^2\right)^2} - \frac{11ax}{32\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh$$

Mathematica [A] time = 0.21, size = 81, normalized size = 0.63

$$\frac{1}{128}\left(-64\text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right) + 64\tanh^{-1}(ax)^2 - 24\sinh\left(2\tanh^{-1}(ax)\right) - \sinh\left(4\tanh^{-1}(ax)\right) + 4\tanh^{-1}(ax)\left(32\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (64*ArcTanh[a*x]^2 + 4*ArcTanh[a*x]*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcTa
nh[a*x]] + 32*Log[1 - E^(-2*ArcTanh[a*x])]) - 64*PolyLog[2, E^(-2*ArcTanh[a
*x])] - 24*Sinh[2*ArcTanh[a*x]] - Sinh[4*ArcTanh[a*x]])/128
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)}{a^6x^7 - 3\,a^4x^5 + 3\,a^2x^3 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x), x)
```

maple [B] time = 0.07, size = 234, normalized size = 1.81

$$\operatorname{arctanh}(ax)\ln(ax)+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{5\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}+\frac{5\operatorname{arctanh}(ax)}{16(ax+1)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^3,x)
```

```
[Out] arctanh(a*x)*ln(a*x)+1/16*arctanh(a*x)/(a*x-1)^2-5/16*arctanh(a*x)/(a*x-1)-
1/2*arctanh(a*x)*ln(a*x-1)+1/16*arctanh(a*x)/(a*x+1)^2+5/16*arctanh(a*x)/(a
*x+1)-1/2*arctanh(a*x)*ln(a*x+1)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x
)*ln(a*x+1)-1/8*ln(a*x-1)^2+1/2*dilog(1/2+1/2*a*x)+1/4*ln(a*x-1)*ln(1/2+1/2
*a*x)+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2+1/2*a*x))*ln(-1/2*a*x+1/2)-1/64
/(a*x-1)^2+11/64/(a*x-1)+11/64*ln(a*x-1)+1/64/(a*x+1)^2+11/64/(a*x+1)-11/64
*ln(a*x+1)
```

maxima [B] time = 0.33, size = 268, normalized size = 2.08

$$\frac{1}{64}a\left(\frac{2\left(11a^3x^3+4\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)^2-8\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)\log(ax-1)-4\left(a^4x^4\right.\right.}{a^5x^4-2a^3x^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a*(2*(11*a^3*x^3 + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 - 8*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 4*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x - 1)^2 - 13*a*x)/(a^5*x^4 - 2*a^3*x^2 + a) + 32*(log(a*x - 1)*lo
g(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 32*(log(a*x + 1)*log(x) + dil
og(-a*x))/a + 32*(log(-a*x + 1)*log(x) + dilog(a*x))/a - 11*log(a*x + 1)/a
+ 11*log(a*x - 1)/a) - 1/4*((2*a^2*x^2 - 3)/(a^4*x^4 - 2*a^2*x^2 + 1) + 2*l
og(a^2*x^2 - 1) - 2*log(x^2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)}{x\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)/(x*(a^2*x^2 - 1)^3), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(ax)}{a^{6}x^{7}-3a^{4}x^{5}+3a^{2}x^{3}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

3.307 $\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+a\log(x)+\frac{15}{16}a\tanh^{-1}(a$$

```
[Out] -1/16*a/(-a^2*x^2+1)^2-7/16*a/(-a^2*x^2+1)-arctanh(a*x)/x+1/4*a^2*x*arctanh
(a*x)/(-a^2*x^2+1)^2+7/8*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+15/16*a*arctanh(a*
x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {6030, 5982, 5916, 266, 36, 29, 31, 5948, 5956, 261, 5960}

$$-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+a\log(x)+\frac{15}{16}a\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -a/(16*(1 - a^2*x^2)^2) - (7*a)/(16*(1 - a^2*x^2)) - ArcTanh[a*x]/x + (a^2*
x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x])/(8*(1 - a^2*x^
2)) + (15*a*ArcTanh[a*x]^2)/16 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^3}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx + \int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^2}\,dx$$

$$= -\frac{a}{16(1-a^2x^2)^2} + \frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{1}{4}(3a^2)\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx + a^2\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx \cdot$$

$$= -\frac{a}{16(1-a^2x^2)^2} + \frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8(1-a^2x^2)} + \frac{7}{16}a\tanh^{-1}(ax)^2 + a^2\int \frac{\tanh}{1-}$$

$$= -\frac{a}{16(1-a^2x^2)^2} - \frac{7a}{16(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8(1-a^2x^2)} +$$

$$= -\frac{a}{16(1-a^2x^2)^2} - \frac{7a}{16(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8(1-a^2x^2)} +$$

$$= -\frac{a}{16(1-a^2x^2)^2} - \frac{7a}{16(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8(1-a^2x^2)} +$$

$$= -\frac{a}{16(1-a^2x^2)^2} - \frac{7a}{16(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8(1-a^2x^2)} +$$

Mathematica [A] time = 0.17, size = 94, normalized size = 0.76

$$\frac{1}{16}\left(a\left(\frac{7a^2x^2-8}{(a^2x^2-1)^2} - 8\log(1-a^2x^2) + 16\log(x)\right) - \frac{2(15a^4x^4-25a^2x^2+8)\tanh^{-1}(ax)}{x(a^2x^2-1)^2} + 15a\tanh^{-1}(ax)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] ((-2*(8 - 25*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)^2) + 15*
a*ArcTanh[a*x]^2 + a*((-8 + 7*a^2*x^2)/(-1 + a^2*x^2)^2 + 16*Log[x] - 8*Log
[1 - a^2*x^2]))/16
```

fricas [A] time = 0.87, size = 161, normalized size = 1.31

$$\frac{28\,a^3x^3 + 15\left(a^5x^5 - 2\,a^3x^3 + ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 32\,ax - 32\left(a^5x^5 - 2\,a^3x^3 + ax\right)\log\left(a^2x^2-1\right) + 64\left(a^5x^5 - 2\right.}{64\left(a^4x^5 - 2\,a^2x^3 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(28*a^3*x^3 + 15*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))
^2 - 32*a*x - 32*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a^2*x^2 - 1) + 64*(a^5*x^5
 - 2*a^3*x^3 + a*x)*log(x) - 4*(15*a^4*x^4 - 25*a^2*x^2 + 8)*log(-(a*x + 1)
/(a*x - 1)))/(a^4*x^5 - 2*a^2*x^3 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x^2), x)
```

maple [B] time = 0.07, size = 228, normalized size = 1.85

$$-\frac{\operatorname{arctanh}(ax)}{x}+\frac{a\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{7a\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{15a\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{a\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{7a\operatorname{arctanh}(ax)}{16(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x)
```

```
[Out] -arctanh(a*x)/x+1/16*a*arctanh(a*x)/(a*x-1)^2-7/16*a*arctanh(a*x)/(a*x-1)-1
5/16*a*arctanh(a*x)*ln(a*x-1)-1/16*a*arctanh(a*x)/(a*x+1)^2-7/16*a*arctanh(
a*x)/(a*x+1)+15/16*a*arctanh(a*x)*ln(a*x+1)-15/64*a*ln(a*x-1)^2+15/32*a*ln(
a*x-1)*ln(1/2+1/2*a*x)-15/64*a*ln(a*x+1)^2+15/32*a*ln(-1/2*a*x+1/2)*ln(a*x+
1)-15/32*a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+a*ln(a*x)-1/64*a/(a*x-1)^2+15/6
4*a/(a*x-1)-1/2*a*ln(a*x-1)-1/64*a/(a*x+1)^2-15/64*a/(a*x+1)-1/2*a*ln(a*x+1
)
```

maxima [A] time = 0.33, size = 204, normalized size = 1.66

$$\frac{1}{64}a\left(\frac{28a^2x^2-15\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)^2+30\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)\log(ax-1)-15\left(a^4x^4-\right.}{a^4x^4-2a^2x^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a*((28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2 +
 1)*log(a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 32*l
og(a*x - 1) + 64*log(x)) + 1/16*(15*a*log(a*x + 1) - 15*a*log(a*x - 1) - 2*
(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*arctanh(a*x)
```

mupad [B] time = 1.39, size = 183, normalized size = 1.49

$$\frac{15a\ln(ax+1)^2}{64}-\frac{4a-\frac{7a^3x^2}{2}}{8a^4x^4-16a^2x^2+8}+\frac{15a\ln(1-ax)^2}{64}-\frac{a\ln\left(a^2x^2-1\right)}{2}+a\ln(x)+\ln(1-ax)\left(\frac{\frac{15a^4x^4}{8}-\frac{25a^2}{8}}{2a^4x^5-4a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] (15*a*log(a*x + 1)^2)/64 - (4*a - (7*a^3*x^2)/2)/(8*a^4*x^4 - 16*a^2*x^2 +
8) + (15*a*log(1 - a*x)^2)/64 - (a*log(a^2*x^2 - 1))/2 + a*log(x) + log(1 -
 a*x)*(((15*a^4*x^4)/8 - (25*a^2*x^2)/8 + 1)/(2*x - 4*a^2*x^3 + 2*a^4*x^5)
- (15*a*log(a*x + 1))/32) - (log(a*x + 1)*(1/(2*a) - (25*a*x^2)/16 + (15*a^
3*x^4)/16))/(x/a - 2*a*x^3 + a^3*x^5)
```

sympy [A] time = 4.90, size = 549, normalized size = 4.46

$$\begin{cases}\frac{16a^5x^5\log(x)}{16a^4x^5-32a^2x^3+16x}-\frac{16a^5x^5\log\left(x-\frac{1}{a}\right)}{16a^4x^5-32a^2x^3+16x}+\frac{15a^5x^5\operatorname{atanh}^2(ax)}{16a^4x^5-32a^2x^3+16x}-\frac{16a^5x^5\operatorname{atanh}(ax)}{16a^4x^5-32a^2x^3+16x}-\frac{30a^4x^4\operatorname{atanh}(ax)}{16a^4x^5-32a^2x^3+16x}-\frac{32a^3x^3\log(x)}{16a^4x^5-32a^2x^3+16x}+ \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((16*a**5*x**5*log(x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a*
*5*x**5*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 15*a**5*x**5*at
anh(a*x)**2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a**5*x**5*atanh(a*x)/
(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 30*a**4*x**4*atanh(a*x)/(16*a**4*x**
5 - 32*a**2*x**3 + 16*x) - 32*a**3*x**3*log(x)/(16*a**4*x**5 - 32*a**2*x**3
 + 16*x) + 32*a**3*x**3*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) -
 30*a**3*x**3*atanh(a*x)**2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 32*a**3*
x**3*atanh(a*x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 7*a**3*x**3/(16*a**4
*x**5 - 32*a**2*x**3 + 16*x) + 50*a**2*x**2*atanh(a*x)/(16*a**4*x**5 - 32*a
**2*x**3 + 16*x) + 16*a*x*log(x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*
a*x*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 15*a*x*atanh(a*x)**
2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a*x*atanh(a*x)/(16*a**4*x**5 -
32*a**2*x**3 + 16*x) - 8*a*x/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*atan
h(a*x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x), Ne(a, 0)), (0, True))
```

3.308 $\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx$

Optimal. Leaf size=127

$$-\frac{3\tanh^{-1}(ax)^2}{32a^4}+\frac{x^4}{32\left(1-a^2x^2\right)^2}+\frac{x^4\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{x^3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3}{32a^4\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)}$$

```
[Out] 1/32*x^4/(-a^2*x^2+1)^2-3/32/a^4/(-a^2*x^2+1)-1/8*x^3*arctanh(a*x)/a/(-a^2*
x^2+1)^2+3/16*x*arctanh(a*x)/a^3/(-a^2*x^2+1)-3/32*arctanh(a*x)^2/a^4+1/4*x
^4*arctanh(a*x)^2/(-a^2*x^2+1)^2
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6008, 6002, 5998, 5948}

$$\frac{x^4}{32\left(1-a^2x^2\right)^2}-\frac{3}{32a^4\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{x^3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] x^4/(32*(1 - a^2*x^2)^2) - 3/(32*a^4*(1 - a^2*x^2)) - (x^3*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(16*a^3*(1 - a^2*x^2)) - (3*ArcTan
h[a*x]^2)/(32*a^4) + (x^4*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rule 6002

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(q_), x_Symbol] :> -Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x]
 + (-Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a
 + b*ArcTanh[c*x]), x], x] + Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTanh[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2
*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
```

```
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx = \frac{x^4 \tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} - \frac{1}{2}a\int \frac{x^4 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx$$

$$= \frac{x^4}{32\left(1-a^2x^2\right)^2} - \frac{x^3 \tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} + \frac{x^4 \tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{3\int \frac{x^2 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{8a}$$

$$= \frac{x^4}{32\left(1-a^2x^2\right)^2} - \frac{3}{32a^4\left(1-a^2x^2\right)} - \frac{x^3 \tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} + \frac{3x \tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)} + \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}$$

$$= \frac{x^4}{32\left(1-a^2x^2\right)^2} - \frac{3}{32a^4\left(1-a^2x^2\right)} - \frac{x^3 \tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} + \frac{3x \tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)} - \frac{3 \tanh^{-1}(ax)^2}{32a^4}$$

Mathematica [A] time = 0.10, size = 71, normalized size = 0.56

$$\frac{\left(6ax-10a^3x^3\right)\tanh^{-1}(ax)+5a^2x^2+\left(5a^4x^4+6a^2x^2-3\right)\tanh^{-1}(ax)^2-4}{32a^4\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-4 + 5*a^2*x^2 + (6*a*x - 10*a^3*x^3)*ArcTanh[a*x] + (-3 + 6*a^2*x^2 + 5*a
^4*x^4)*ArcTanh[a*x]^2)/(32*a^4*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.70, size = 99, normalized size = 0.78

$$\frac{20\,a^2x^2+\left(5\,a^4x^4+6\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\left(5\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-16}{128\left(a^8x^4-2\,a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*a^2*x^2 + (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1))^2
 - 4*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^8*x^4 - 2*a^6*x
^2 + a^4)
```

giac [B] time = 0.17, size = 250, normalized size = 1.97

$$\frac{1}{512}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{(ax+1)^2}{(ax-1)^2a^5}+\frac{4(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+2\left(\frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}-\frac{(ax}{(ax}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/512*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (a*x
+ 1)^2/((a*x - 1)^2*a^5) + 4*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/(a*x
```

```
 - 1))^2 + 2*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) - (
a*x + 1)^2/((a*x - 1)^2*a^5) - 8*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/
(a*x - 1)) + (a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (
a*x + 1)^2/((a*x - 1)^2*a^5) + 16*(a*x + 1)/((a*x - 1)*a^5))*a
```

maple [B] time = 0.07, size = 297, normalized size = 2.34

$$\frac{\operatorname{arctanh}(ax)^2}{16a^4(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)^2}{16a^4(ax-1)}+\frac{\operatorname{arctanh}(ax)^2}{16a^4(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)^2}{16a^4(ax+1)}-\frac{\operatorname{arctanh}(ax)}{32a^4(ax-1)^2}-\frac{5\operatorname{arctanh}(ax)}{32a^4(ax-1)}-\frac{5\operatorname{arctanh}(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^4*arctanh(a*x)^2/(a*x-1)^2+3/16/a^4*arctanh(a*x)^2/(a*x-1)+1/16/a^4*
arctanh(a*x)^2/(a*x+1)^2-3/16/a^4*arctanh(a*x)^2/(a*x+1)-1/32/a^4*arctanh(a
*x)/(a*x-1)^2-5/32/a^4*arctanh(a*x)/(a*x-1)-5/32/a^4*arctanh(a*x)*ln(a*x-1)
+1/32/a^4*arctanh(a*x)/(a*x+1)^2-5/32/a^4*arctanh(a*x)/(a*x+1)+5/32/a^4*arc
tanh(a*x)*ln(a*x+1)-5/128/a^4*ln(a*x-1)^2+5/64/a^4*ln(a*x-1)*ln(1/2+1/2*a*x
)-5/128/a^4*ln(a*x+1)^2-5/64/a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+5/64/a^4*
ln(-1/2*a*x+1/2)*ln(a*x+1)+1/128/a^4/(a*x-1)^2+9/128/a^4/(a*x-1)+1/128/a^4/
(a*x+1)^2-9/128/a^4/(a*x+1)
```

maxima [B] time = 0.33, size = 226, normalized size = 1.78

$$-\frac{1}{32}a\left(\frac{2\left(5a^2x^3-3x\right)}{a^8x^4-2a^6x^2+a^4}-\frac{5\log(ax+1)}{a^5}+\frac{5\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)+\frac{\left(20a^2x^2-5\left(a^4x^4-2a^2x^2+1\right)\log(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/32*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x) + 1/128*(20*a^2*x^2 - 5*(a^4*x^4 - 2*
a^2*x^2 + 1)*log(a*x + 1)^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log
(a*x - 1) - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a^2/(a^10*x^4
- 2*a^8*x^2 + a^6) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/(a^8*x^4 - 2*a^6*x^
2 + a^4)
```

mupad [B] time = 1.53, size = 372, normalized size = 2.93

$$\ln(a\,x+1)^2\left(\frac{5}{128\,a^4}-\frac{\frac{1}{16\,a^5}-\frac{x^2}{8\,a^3}}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}\right)-\ln(1-a\,x)\left(\frac{\frac{3\,x}{8}+a\,x^2-\frac{3}{4\,a}-\frac{5\,a^2\,x^3}{8}}{8\,a^7\,x^4-16\,a^5\,x^2+8\,a^3}+\frac{\frac{3\,x}{8}-a\,x^2+\frac{3}{4\,a}-\frac{5\,a^2\,x^3}{8}}{8\,a^7\,x^4-16\,a^5\,x^2+8\,a^3}-\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(a*x + 1)^2*(5/(128*a^4) - (1/(16*a^5) - x^2/(8*a^3))/(1/a - 2*a*x^2 + a
^3*x^4)) - log(1 - a*x)*(((3*x)/8 + a*x^2 - 3/(4*a) - (5*a^2*x^3)/8)/(8*a^3
 - 16*a^5*x^2 + 8*a^7*x^4) + ((3*x)/8 - a*x^2 + 3/(4*a) - (5*a^2*x^3)/8)/(8
*a^3 - 16*a^5*x^2 + 8*a^7*x^4) - log(a*x + 1)*((1/(4*a^4) - x^2/(2*a^2))/(2
*a^4*x^4 - 4*a^2*x^2 + 2) - (5*(a^4*x^4 - 2*a^2*x^2 + 1))/(32*a^4*(2*a^4*x^
4 - 4*a^2*x^2 + 2)))) - log(1 - a*x)^2*((1/(4*a^4) - x^2/(2*a^2))/(4*a^4*x^
4 - 8*a^2*x^2 + 4) - 5/(128*a^4)) - (2/a^2 - (5*x^2)/2)/(16*a^2 - 32*a^4*x^
2 + 16*a^6*x^4) + (log(a*x + 1)*((3*x)/(32*a^4) - (5*x^3)/(32*a^2)))/(1/a -
 2*a*x^2 + a^3*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3\operatorname{atanh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**3*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

3.309 $\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3} \, dx$

Optimal. Leaf size=163

$$-\frac{\tanh^{-1}(ax)^3}{24a^3}-\frac{\tanh^{-1}(ax)}{64a^3}-\frac{x}{64a^2\left(1-a^2x^2\right)}+\frac{x}{32a^2\left(1-a^2x^2\right)^2}-\frac{x\tanh^{-1}(ax)^2}{8a^2\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)}$$

```
[Out] 1/32*x/a^2/(-a^2*x^2+1)^2-1/64*x/a^2/(-a^2*x^2+1)-1/64*arctanh(a*x)/a^3-1/8
*arctanh(a*x)/a^3/(-a^2*x^2+1)^2+1/8*arctanh(a*x)/a^3/(-a^2*x^2+1)+1/4*x*ar
ctanh(a*x)^2/a^2/(-a^2*x^2+1)^2-1/8*x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)-1/24*
arctanh(a*x)^3/a^3
```

Rubi [A] time = 0.25, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6028, 5956, 5994, 199, 206, 5964}

$$-\frac{x}{64a^2\left(1-a^2x^2\right)}+\frac{x}{32a^2\left(1-a^2x^2\right)^2}-\frac{x\tanh^{-1}(ax)^2}{8a^2\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(a}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] x/(32*a^2*(1 - a^2*x^2)^2) - x/(64*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]/(64*a^
3) - ArcTanh[a*x]/(8*a^3*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(8*a^3*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x]^2)/(8*a^
2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(24*a^3)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
```

```
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{a^2}$$

$$= -\frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx}{8a^2}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)}{8a^2\left(1-a^2x\right.}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{13x}{64a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(a}{4a^2\left(1-a^2x\right.}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{x}{64a^2\left(1-a^2x^2\right)} - \frac{13\tanh^{-1}(ax)}{64a^3} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x\right.}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{x}{64a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{64a^3} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.12, size = 121, normalized size = 0.74

$$\frac{48\left(a^3x^3+ax\right)\tanh^{-1}(ax)^2 + 6ax\left(a^2x^2+1\right) + 3\left(a^2x^2-1\right)^2\log(1-ax) - 3\left(a^2x^2-1\right)^2\log(ax+1) - 16\left(a^2x^2\right.}{384a^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (6*a*x*(1 + a^2*x^2) - 48*a^2*x^2*ArcTanh[a*x] + 48*(a*x + a^3*x^3)*ArcTanh
[a*x]^2 - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^3 + 3*(-1 + a^2*x^2)^2*Log[1 - a
*x] - 3*(-1 + a^2*x^2)^2*Log[1 + a*x])/(384*a^3*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.62, size = 136, normalized size = 0.83

$$\frac{6\,a^{3}x^{3}-2\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}+12\left(a^{3}x^{3}+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}+6\,ax-3\left(a^{4}x^{4}+6\,a^{2}x^{2}+1\right)\log\left(-\frac{ax}{ax}\right.}{384\left(a^{7}x^{4}-2\,a^{5}x^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^3
+ 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 6*a*x - 3*(a^4*x^4 + 6*a
^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^{2}\operatorname{artanh}\left(ax\right)^{2}}{\left(a^{2}x^{2}-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

maple [C] time = 0.89, size = 2571, normalized size = 15.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

```
[Out] -1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2+1/32*I/a^3/(a*x-1)^2/(a*x
+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)
^2/(a^2*x^2-1))+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+
(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^
2+1)))^2-1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/3
2*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a
*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^4+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^4+1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctan
h(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^4+1/8*I/a/(a*x-1)^2/(a*x
+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^2-1/16*I/a/(
a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2
/(-a^2*x^2+1)))^3*Pi*x^2-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^2-1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2
*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^2-1/16*I*a/(a*x-1)^2/(a*x+1)^2*a
rctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^4+1/16*I/a^3/(a*x-1
)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(
a*x+1)^2/(a^2*x^2-1))^2+1/64/(a*x-1)^2/(a*x+1)^2*x^3+1/16*I/a/(a*x-1)^2/(a*
x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^2
-1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+
1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*Pi*x^4+1/8/a^3*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2
))+1/16/a^3*arctanh(a*x)^2/(a*x-1)^2+1/16/a^3*arctanh(a*x)^2/(a*x-1)+1/16/a
^3*arctanh(a*x)^2*ln(a*x-1)-1/16/a^3*arctanh(a*x)^2/(a*x+1)^2+1/16/a^3*arct
anh(a*x)^2/(a*x+1)-1/16/a^3*arctanh(a*x)^2*ln(a*x+1)+1/64/a^2/(a*x-1)^2/(a*
x+1)^2*x-1/24/a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3-1/64/a^3/(a*x-1)^2/(a*
x+1)^2*arctanh(a*x)+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x
```

```
+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^4-1/16*I/a/(a*
x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^2+1/16*I/a/(a*x-1)^
2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^2-1/8*I/a/(a*x-1)^2/(a*x+1)^2
*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))*Pi*x^2-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2
/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^2-1/32*I/a^3/(a*x-1
)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(
a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1
)))+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x
^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^4-1
/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+
(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^4+1/16*I*a/(a
*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*
x+1)/(-a^2*x^2+1)^(1/2))*Pi*x^4-1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2
*Pi*x^4+1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^2+1/32*I/a^3/(a*x-1
)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(
-a^2*x^2+1)))^3+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^3+1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn
(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(
a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-1/24*a/(a*x-1)^2/(a*x+1)^2*arct
anh(a*x)^3*x^4-1/64*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^4+1/12/a/(a*x-1)^2
/(a*x+1)^2*arctanh(a*x)^3*x^2-3/32/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^2
```

maxima [B] time = 0.33, size = 388, normalized size = 2.38

$$\frac{1}{16}\left(\frac{2\left(a^2x^3+x\right)}{a^6x^4-2\,a^4x^2+a^2}-\frac{\log\left(ax+1\right)}{a^3}+\frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right)^2+\frac{\left(6\,a^3x^3-2\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x)^2 + 1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1)
 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*a^2*
x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 1)*log(a*x + 1) + 3*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^9*x^4 - 2*a^7*x^2 + a^5) - 1/32*
(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)
^2)*a*arctanh(a*x)/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

mupad [B] time = 1.97, size = 350, normalized size = 2.15

$$\ln\left(1-a\,x\right)\left(\frac{\frac{3\,a\,x^3}{2}-\frac{x}{2\,a}+x^2}{32\,a^5\,x^4-64\,a^3\,x^2+32\,a}+\frac{\frac{x}{2\,a}-\frac{3\,a\,x^3}{2}+x^2}{32\,a^5\,x^4-64\,a^3\,x^2+32\,a}+\frac{\ln\left(a\,x+1\right)^2}{64\,a^3}-\frac{\ln\left(a\,x+1\right)\,\left(2\,a^2\,x^3+2\,x\right.}{32\,a^6\,x^4-64\,a^4\,x^2+32\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(1 - a*x)*(((3*a*x^3)/2 - x/(2*a) + x^2)/(32*a - 64*a^3*x^2 + 32*a^5*x^4
) + (x/(2*a) - (3*a*x^3)/2 + x^2)/(32*a - 64*a^3*x^2 + 32*a^5*x^4) + log(a*
x + 1)^2/(64*a^3) - (log(a*x + 1)*(2*x + 2*a^2*x^3))/(32*a^2 - 64*a^4*x^2 +
 32*a^6*x^4)) + (x/(8*a^2) + x^3/8)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 -
a*x)^2*(log(a*x + 1)/(64*a^3) - (x/(8*a^2) + x^3/8)/(4*a^4*x^4 - 8*a^2*x^2
+ 4)) - log(a*x + 1)^3/(192*a^3) + log(1 - a*x)^3/(192*a^3) + (atan(a*x*1i)
```

```
*1i)/(64*a^3) + (log(a*x + 1)^2*(x/(32*a^3) + x^3/(32*a)))/(1/a - 2*a*x^2 +
 a^3*x^4) - (x^2*log(a*x + 1))/(16*a^2*(1/a - 2*a*x^2 + a^3*x^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 \operatorname{atanh}^2(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

3.310 $\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=125

$$\frac{3}{32a^2\left(1-a^2x^2\right)}+\frac{1}{32a^2\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2}-\frac{3x\tanh^{-1}(ax)}{16a\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)^2}{32a^2}$$

```
[Out] 1/32/a^2/(-a^2*x^2+1)^2+3/32/a^2/(-a^2*x^2+1)-1/8*x*arctanh(a*x)/a/(-a^2*x^
2+1)^2-3/16*x*arctanh(a*x)/a/(-a^2*x^2+1)-3/32*arctanh(a*x)^2/a^2+1/4*arcta
nh(a*x)^2/a^2/(-a^2*x^2+1)^2
```

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5994, 5960, 5956, 261}

$$\frac{3}{32a^2\left(1-a^2x^2\right)}+\frac{1}{32a^2\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2}-\frac{3x\tanh^{-1}(ax)}{16a\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)^2}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] 1/(32*a^2*(1 - a^2*x^2)^2) + 3/(32*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(16*a*(1 - a^2*x^2)) - (3*ArcTanh[
a*x]^2)/(32*a^2) + ArcTanh[a*x]^2/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2} - \frac{\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx}{2a}$$

$$= \frac{1}{32a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx}{8a}$$

$$= \frac{1}{32a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)}{16a(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)^2}{32a^2} + \frac{\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2} +$$

$$= \frac{1}{32a^2(1-a^2x^2)^2} + \frac{3}{32a^2(1-a^2x^2)} - \frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)}{16a(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)^2}{32a^2} +$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.57

$$\frac{-3a^2x^2 + 2ax(3a^2x^2-5)\tanh^{-1}(ax) + (-3a^4x^4+6a^2x^2+5)\tanh^{-1}(ax)^2 + 4}{32a^2(a^2x^2-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (4 - 3*a^2*x^2 + 2*a*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x] + (5 + 6*a^2*x^2 - 3*a
^4*x^4)*ArcTanh[a*x]^2)/(32*a^2*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.49, size = 99, normalized size = 0.79

$$-\frac{12\,a^2x^2 + (3\,a^4x^4 - 6\,a^2x^2 - 5)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 4\,(3\,a^3x^3 - 5\,ax)\log\left(-\frac{ax+1}{ax-1}\right) - 16}{128\,(a^6x^4 - 2\,a^4x^2 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*a^2*x^2 + (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1))^
2 - 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^6*x^4 - 2*a^4*
x^2 + a^2)
```

giac [B] time = 0.25, size = 251, normalized size = 2.01

$$-\frac{1}{512}\left(2\left(\frac{(ax-1)^2\left(\frac{4\,(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3} - \frac{(ax+1)^2}{(ax-1)^2a^3} + \frac{4\,(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 2\left(\frac{(ax-1)^2\left(\frac{8\,(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3} + \frac{(ax+}{(ax-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/512*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - (a*x
 + 1)^2/((a*x - 1)^2*a^3) + 4*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)/(a*
x - 1))^2 + 2*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) +
(a*x + 1)^2/((a*x - 1)^2*a^3) - 8*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)
/(a*x - 1)) + (a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) -
(a*x + 1)^2/((a*x - 1)^2*a^3) + 16*(a*x + 1)/((a*x - 1)*a^3))*a
```

maple [B] time = 0.07, size = 247, normalized size = 1.98

$$\frac{\operatorname{arctanh}(ax)^2}{4a^2(a^2x^2-1)^2}-\frac{\operatorname{arctanh}(ax)}{32a^2(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{32a^2(ax-1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{32a^2}+\frac{\operatorname{arctanh}(ax)}{32a^2(ax+1)^2}+\frac{3\operatorname{arctanh}(ax)}{32a^2(ax+1)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/4/a^2/(a^2*x^2-1)^2*arctanh(a*x)^2-1/32/a^2*arctanh(a*x)/(a*x-1)^2+3/32/a
^2*arctanh(a*x)/(a*x-1)+3/32/a^2*arctanh(a*x)*ln(a*x-1)+1/32/a^2*arctanh(a*
x)/(a*x+1)^2+3/32/a^2*arctanh(a*x)/(a*x+1)-3/32/a^2*arctanh(a*x)*ln(a*x+1)+
3/128/a^2*ln(a*x-1)^2-3/64/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+3/128/a^2*ln(a*x+1
)^2+3/64/a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-3/64/a^2*ln(-1/2*a*x+1/2)*ln(
a*x+1)+1/128/a^2/(a*x-1)^2-7/128/a^2/(a*x-1)+1/128/a^2/(a*x+1)^2+7/128/a^2/
(a*x+1)
```

maxima [A] time = 0.33, size = 206, normalized size = 1.65

$$\frac{\left(\frac{2\left(3\,a^2x^3-5\,x\right)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log(ax+1)}{a}+\frac{3\,\log(ax-1)}{a}\right)\operatorname{artanh}(ax)}{32\,a}-\frac{12\,a^2x^2-3\left(a^4x^4-2\,a^2x^2+1\right)\log(ax+1)^2+6\left(a^4x^4-\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)*arctanh(a*x)/a - 1/128*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1
) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)/(a^6*x^4 - 2*a^4*x^2 +
 a^2) + 1/4*arctanh(a*x)^2/((a^2*x^2 - 1)^2*a^2)
```

mupad [B] time = 1.48, size = 319, normalized size = 2.55

$$\ln(a\,x+1)^2\left(\frac{1}{16\,a^3\,\left(\frac{1}{a}-2\,a\,x^2+a^3\,x^4\right)}-\frac{3}{128\,a^2}\right)-\ln(1-a\,x)^2\left(\frac{3}{128\,a^2}-\frac{1}{4\,a^2\,\left(4\,a^4\,x^4-8\,a^2\,x^2+4\right)}\right)-\ln(1-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(a*x + 1)^2*(1/(16*a^3*(1/a - 2*a*x^2 + a^3*x^4)) - 3/(128*a^2)) - log(1
 - a*x)^2*(3/(128*a^2) - 1/(4*a^2*(4*a^4*x^4 - 8*a^2*x^2 + 4))) - log(1 - a
*x)*((1/(4*a) - (5*x)/8 + (3*a^2*x^3)/8)/(8*a - 16*a^3*x^2 + 8*a^5*x^4) - (
(5*x)/8 + 1/(4*a) - (3*a^2*x^3)/8)/(8*a - 16*a^3*x^2 + 8*a^5*x^4) + log(a*x
 + 1)*(1/(4*a^2*(2*a^4*x^4 - 4*a^2*x^2 + 2)) - (3*(a^4*x^4 - 2*a^2*x^2 + 1)
)/(32*a^2*(2*a^4*x^4 - 4*a^2*x^2 + 2)))) + (2/a^2 - (3*x^2)/2)/(16*a^4*x^4
- 32*a^2*x^2 + 16) - (log(a*x + 1)*((5*x)/(32*a^2) - (3*x^3)/32))/(1/a - 2*
a*x^2 + a^3*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x\operatorname{atanh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

3.311 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx$

Optimal. Leaf size=151

$$\frac{15x}{64\left(1-a^2x^2\right)}+\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{8a}+\frac{15}{}$$

```
[Out] 1/32*x/(-a^2*x^2+1)^2+15/64*x/(-a^2*x^2+1)+15/64*arctanh(a*x)/a-1/8*arctanh
(a*x)/a/(-a^2*x^2+1)^2-3/8*arctanh(a*x)/a/(-a^2*x^2+1)+1/4*x*arctanh(a*x)^2
/(-a^2*x^2+1)^2+3/8*x*arctanh(a*x)^2/(-a^2*x^2+1)+1/8*arctanh(a*x)^3/a
```

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 199, 206}

$$\frac{15x}{64\left(1-a^2x^2\right)}+\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{8a}+\frac{15}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```

```
[Out] x/(32*(1 - a^2*x^2)^2) + (15*x)/(64*(1 - a^2*x^2)) + (15*ArcTanh[a*x])/(64*
a) - ArcTanh[a*x]/(8*a*(1 - a^2*x^2)^2) - (3*ArcTanh[a*x])/(8*a*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x]^2)/(8*(1 -
 a^2*x^2)) + ArcTanh[a*x]^3/(8*a)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
```

```
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx = -\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{1}{8}\int\frac{1}{\left(1-a^2x^2\right)^3}\,dx+\frac{3}{4}\int\frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{8a}+\frac{3}{32}$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{3x}{64\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{3x\,\mathrm{t}}{8(}$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{15x}{64\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{64a}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}+\frac{x\,\mathrm{tan}}{4(1}$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{15x}{64\left(1-a^2x^2\right)}+\frac{15\tanh^{-1}(ax)}{64a}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}+\frac{x\,\mathrm{ta}}{4(1}$$

Mathematica [A] time = 0.11, size = 127, normalized size = 0.84

$$\frac{1}{128}\left(-\frac{30x}{a^2x^2-1}+\frac{4x}{\left(a^2x^2-1\right)^2}-\frac{16x\left(3a^2x^2-5\right)\tanh^{-1}(ax)^2}{\left(a^2x^2-1\right)^2}+\frac{16\left(3a^2x^2-4\right)\tanh^{-1}(ax)}{a\left(a^2x^2-1\right)^2}-\frac{15\log(1-ax)}{a}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```

```
[Out] ((4*x)/(-1 + a^2*x^2)^2 - (30*x)/(-1 + a^2*x^2) + (16*(-4 + 3*a^2*x^2)*ArcT
anh[a*x])/(a*(-1 + a^2*x^2)^2) - (16*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x]^2)/(-1
 + a^2*x^2)^2 + (16*ArcTanh[a*x]^3)/a - (15*Log[1 - a*x])/a + (15*Log[1 + a
*x])/a)/128
```

fricas [A] time = 0.54, size = 137, normalized size = 0.91

$$-\frac{30\,a^3x^3-2\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+4\left(3\,a^3x^3-5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-34\,ax-\left(15\,a^4x^4-6\,a^2x^2-1\right.}{128\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^
3 + 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 34*a*x - (15*a^4*x^
4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^4 - 2*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")

[Out] integrate(-arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)

maple [C] time = 0.87, size = 2571, normalized size = 17.03

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^3,x)

[Out] -3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^2+3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^4+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^2+3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^2+3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^4-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^4-3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^4+3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^2-3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^2+3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi*x^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^2-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^4-3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^4-3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^2-3/16/a*arctanh(a*x)^2*ln(a*x-1)+3/16/a*arctanh(a*x)^2*ln(a*x+1)-3/8/a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^2-3/32*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^2+1/8*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^4+15/64*a

```
^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^4+3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arc
tanh(a*x)^2+17/64/(a*x-1)^2/(a*x+1)^2*x+1/16/a*arctanh(a*x)^2/(a*x-1)^2-3/1
6/a*arctanh(a*x)^2/(a*x-1)-1/16/a*arctanh(a*x)^2/(a*x+1)^2-3/16/a*arctanh(a
*x)^2/(a*x+1)-3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^2+3/16*I/a/(a
*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-3/
32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^
3-3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*
x^2+1)))^2+3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^4-3/32*I/a/(a
*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)
^2/(-a^2*x^2+1)))^3-15/64*a^2/(a*x-1)^2/(a*x+1)^2*x^3+1/8/a/(a*x-1)^2/(a*x+
1)^2*arctanh(a*x)^3-17/64/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)
```

maxima [**B**] time = 0.34, size = 392, normalized size = 2.60

$$-\frac{1}{16}\left(\frac{2\left(3\,a^2x^3-5\,x\right)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log\left(ax+1\right)}{a}+\frac{3\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\left(30\,a^3x^3-2\left(a^4x^4-2\,a^2x^2+1\right)\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^
2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x
- 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4 -
 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1)
+ 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x^2 + a^3
) + 1/32*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

mupad [**B**] time = 2.06, size = 358, normalized size = 2.37

$$\frac{\frac{17\,x}{8}-\frac{15\,a^2\,x^3}{8}}{8\,a^4\,x^4-16\,a^2\,x^2+8}-\ln\left(1-a\,x\right)\left(\frac{3\ln\left(a\,x+1\right)^2}{64\,a}-\frac{\frac{7\,x}{2}-3\,a\,x^2+\frac{4}{a}-\frac{5\,a^2\,x^3}{2}}{32\,a^4\,x^4-64\,a^2\,x^2+32}+\frac{\frac{7\,x}{2}+3\,a\,x^2-\frac{4}{a}-\frac{5\,a^2\,x^3}{2}}{32\,a^4\,x^4-64\,a^2\,x^2+32}+\frac{\ln\left(\right.}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(a^2*x^2 - 1)^3,x)
```

```
[Out] ((17*x)/8 - (15*a^2*x^3)/8)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 - a*x)*((3
*log(a*x + 1)^2)/(64*a) - ((7*x)/2 - 3*a*x^2 + 4/a - (5*a^2*x^3)/2)/(32*a^4
*x^4 - 64*a^2*x^2 + 32) + ((7*x)/2 + 3*a*x^2 - 4/a - (5*a^2*x^3)/2)/(32*a^4
*x^4 - 64*a^2*x^2 + 32) + (log(a*x + 1)*(10*x - 6*a^2*x^3))/(32*a^4*x^4 - 6
4*a^2*x^2 + 32)) + log(1 - a*x)^2*((3*log(a*x + 1))/(64*a) + ((5*x)/8 - (3*
a^2*x^3)/8)/(4*a^4*x^4 - 8*a^2*x^2 + 4)) + log(a*x + 1)^3/(64*a) - log(1 -
a*x)^3/(64*a) - (atan(a*x*1i)*15i)/(64*a) - (log(a*x + 1)*(1/(4*a^2) - (3*x
^2)/16))/(1/a - 2*a*x^2 + a^3*x^4) + (log(a*x + 1)^2*((5*x)/(32*a) - (3*a*x
^3)/32))/(1/a - 2*a*x^2 + a^3*x^4)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2\left(ax\right)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

3.312 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=196

$$\frac{11}{32\left(1-a^2x^2\right)}+\frac{1}{32\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)}-\frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{1}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)$$

```
[Out] 1/32/(-a^2*x^2+1)^2+11/32/(-a^2*x^2+1)-1/8*a*x*arctanh(a*x)/(-a^2*x^2+1)^2-
11/16*a*x*arctanh(a*x)/(-a^2*x^2+1)-11/32*arctanh(a*x)^2+1/4*arctanh(a*x)^2
/(-a^2*x^2+1)^2+1/2*arctanh(a*x)^2/(-a^2*x^2+1)+1/3*arctanh(a*x)^3+arctanh(
a*x)^2*ln(2-2/(a*x+1))-arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*polylog(3,-
1+2/(a*x+1))
```

Rubi [A] time = 0.45, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6030, 5988, 5932, 5948, 6056, 6610, 5994, 5956, 261, 5960}

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{11}{32\left(1-a^2x^2\right)}+\frac{1}{32\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] 1/(32*(1 - a^2*x^2)^2) + 11/(32*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(8*(1 -
 a^2*x^2)^2) - (11*a*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^
2)/32 + ArcTanh[a*x]^2/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^2/(2*(1 - a^2*x^2
)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*
PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} - \frac{1}{2}a\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx + a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{3}\tanh^{-1}(ax)^3 - \frac{1}{8}(3a)\int$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{\text{ta}}{2}$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} + \frac{11}{32\left(1-a^2x^2\right)} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}(ax)^2 + \frac{\text{ta}}{4}$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} + \frac{11}{32\left(1-a^2x^2\right)} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}(ax)^2 + \frac{\text{ta}}{4}$$

Mathematica [C] time = 0.41, size = 129, normalized size = 0.66

$$\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right)+\frac{1}{768}\left(-384\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) - 256\tanh^{-1}(ax)^3 - 12\tanh^{-1}(ax)\left(24\sinh\left(2\tanh^{-1}(a\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((32*I)*Pi^3 - 256*ArcTanh[a*
x]^3 + 144*Cosh[2*ArcTanh[a*x]] + 3*Cosh[4*ArcTanh[a*x]] + 24*ArcTanh[a*x]^
2*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcTanh[a*x]] + 32*Log[1 - E^(2*ArcTanh
[a*x])]) - 384*PolyLog[3, E^(2*ArcTanh[a*x])] - 12*ArcTanh[a*x]*(24*Sinh[2*
ArcTanh[a*x]] + Sinh[4*ArcTanh[a*x]]))/768
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\mathrm{artanh}\left(ax\right)^2}{a^6x^7 - 3\,a^4x^5 + 3\,a^2x^3 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\mathrm{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x), x)
```

maple [C] time = 0.84, size = 1392, normalized size = 7.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x)
```

```
[Out] -3/32*(a*x+1)/(a*x-1)-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3-11/32*arctanh(a*x)^2+1/512*(
a*x+1)^2/(a*x-1)^2-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x+1
)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/512*(a*x-1)^2/(a*x+1)^2+a
rctanh(a*x)^2*ln(2)-3/32*(a*x-1)/(a*x+1)+1/16*arctanh(a*x)^2/(a*x-1)^2-5/16
*arctanh(a*x)^2/(a*x-1)+1/16*arctanh(a*x)^2/(a*x+1)^2+5/16*arctanh(a*x)^2/(
a*x+1)+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+a
rctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^2-3/16*arctanh(a*x)*(a*x-1)/(a*x+
1)+1/128*arctanh(a*x)*(a*x-1)^2/(a*x+1)^2-1/128*arctanh(a*x)*(a*x+1)^2/(a*x
-1)^2+3/16*(a*x+1)*arctanh(a*x)/(a*x-1)+2*arctanh(a*x)*polylog(2,-(a*x+1)/(
-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2
*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x+1)^2
/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)
))-1/4*I*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+1
/2*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2+1/4*I*Pi*arctan
h(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/4*I*Pi*arctanh(a*x)^2*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-1/2*I*arctanh(a*x)^2*Pi*cs
gn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+
1)^2/(-a^2*x^2+1)))^2-1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I
*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a
^2*x^2-1))+1/4*I*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*I*Pi*arctanh(a*x)
^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-1/4*I
*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/2*I*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1))
)^2*arctanh(a*x)^2+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1
)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^6\int\frac{x^6\log(ax+1)\log(-ax+1)}{2\left(a^6x^7-3a^4x^5+3a^2x^3-x\right)}\,dx+\frac{1}{2}a^5\int\frac{x^5\log(ax+1)\log(-ax+1)}{2\left(a^6x^7-3a^4x^5+3a^2x^3-x\right)}\,dx-\frac{1}{256}\left(a\left(\frac{2\left(5a^2x^2+3ax\right.}{a^8x^3-a^7x^2-a^6x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^6*integrate(1/2*x^6*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 +
 3*a^2*x^3 - x), x) + 1/2*a^5*integrate(1/2*x^5*log(a*x + 1)*log(-a*x + 1)/
(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/256*(a*(2*(5*a^2*x^2 + 3*a*x
- 6)/(a^8*x^3 - a^7*x^2 - a^6*x + a^5) - 5*log(a*x + 1)/a^5 + 5*log(a*x - 1
)/a^5) + 16*(2*a^2*x^2 - 1)*log(-a*x + 1)/(a^8*x^4 - 2*a^6*x^2 + a^4))*a^4
- a^4*integrate(1/2*x^4*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3
*a^2*x^3 - x), x) - a^3*integrate(1/2*x^3*log(a*x + 1)*log(-a*x + 1)/(a^6*x
^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + 1/2*a^3*integrate(1/2*x^3*log(-a*x +
1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 3/512*(a*(2*(3*a^2*x^2 - 3*a
*x - 2)/(a^6*x^3 - a^5*x^2 - a^4*x + a^3) - 3*log(a*x + 1)/a^3 + 3*log(a*x
- 1)/a^3) - 16*log(-a*x + 1)/(a^6*x^4 - 2*a^4*x^2 + a^2))*a^2 + 1/2*a^2*int
egrate(1/2*x^2*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3
```

```
- x), x) + 1/2*a*integrate(1/2*x*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^
4*x^5 + 3*a^2*x^3 - x), x) - 3/4*a*integrate(1/2*x*log(-a*x + 1)/(a^6*x^7 -
 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/48*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*
x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - 3)*log
(-a*x + 1)^2)/(a^4*x^4 - 2*a^2*x^2 + 1) - 1/2*integrate(1/2*log(a*x + 1)^2/
(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + integrate(1/2*log(a*x + 1)*log(
-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)^2}{x\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^6x^7-3a^4x^5+3a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

3.313 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=209

$$\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-a\text{Li}_2\left(\frac{2}{ax+\cdots}\right.$$

```
[Out] 1/32*a^2*x/(-a^2*x^2+1)^2+31/64*a^2*x/(-a^2*x^2+1)+31/64*a*arctanh(a*x)-1/8
*a*arctanh(a*x)/(-a^2*x^2+1)^2-7/8*a*arctanh(a*x)/(-a^2*x^2+1)+a*arctanh(a*
x)^2-arctanh(a*x)^2/x+1/4*a^2*x*arctanh(a*x)^2/(-a^2*x^2+1)^2+7/8*a^2*x*arc
tanh(a*x)^2/(-a^2*x^2+1)+5/8*a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln(2-2/(a*x+
1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A] time = 0.49, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6030, 5982, 5916, 5988, 5932, 2447, 5948, 5956, 5994, 199, 206, 5964}

$$-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(a\cdots}{8\left(1-a^2x^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] (a^2*x)/(32*(1 - a^2*x^2)^2) + (31*a^2*x)/(64*(1 - a^2*x^2)) + (31*a*ArcTan
h[a*x])/64 - (a*ArcTanh[a*x])/(8*(1 - a^2*x^2)^2) - (7*a*ArcTanh[a*x])/(8*(
1 - a^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2
)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^2)/(8*(1 - a^2*x^2)) + (5*a*A
rcTanh[a*x]^3)/8 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1
+ 2/(1 + a*x)]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{1}{8}a^2\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx + \frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,d$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)} + \frac{7}{24}a\tanh^{-1}(a$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{3a^2x}{64\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x}{4(}$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{31a^2x}{64\left(1-a^2x^2\right)} + \frac{3}{64}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + a$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{31a^2x}{64\left(1-a^2x^2\right)} + \frac{31}{64}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + a$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{31a^2x}{64\left(1-a^2x^2\right)} + \frac{31}{64}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + a$$

Mathematica [A] time = 0.59, size = 127, normalized size = 0.61

$$-a\left(\tanh^{-1}(ax)^2\left(\frac{ax}{a^2x^2-1} + \frac{1}{ax} - \frac{1}{32}\sinh\left(4\tanh^{-1}(ax)\right) - 1\right) + \text{Li}_2\left(e^{-2\tanh^{-1}(ax)}\right) - \frac{5}{8}\tanh^{-1}(ax)^3 - \frac{1}{4}\sinh$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*((-5*ArcTanh[a*x]^3)/8 + (ArcTanh[a*x]*(32*Cosh[2*ArcTanh[a*x]] + Cosh[
4*ArcTanh[a*x]] - 128*Log[1 - E^(-2*ArcTanh[a*x])]))/64 + PolyLog[2, E^(-2*
ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]]/4 + ArcTanh[a*x]^2*(-1 + 1/(a*x) + (a
*x)/(-1 + a^2*x^2) - Sinh[4*ArcTanh[a*x]]/32) - Sinh[4*ArcTanh[a*x]]/256))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^2}{a^6x^8 - 3a^4x^6 + 3a^2x^4 - x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2-1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x^2), x)
```

maple [C] time = 0.95, size = 4797, normalized size = 22.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x)
[Out] -a*arctanh(a*x)^2+1/4*a*arctanh(a*x)/(a*x-1)-1/4*a*arctanh(a*x)/(a*x+1)+1/5
12*a/(a*x-1)^2-15/32*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*
arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2/x+15/16*I*a*Pi
*arctanh(a*x)^2+15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)
))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog((a*x+1)/(-a^
2*x^2+1)^(1/2))-15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog((a*
x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1
)^(1/2))-15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog
((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(
a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*
Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*
I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-
1/8*a^2*x/(a*x-1)+1/8/(a*x+1)*a^2*x-7/16*a*arctanh(a*x)^2/(a*x-1)-7/16*a*ar
ctanh(a*x)^2/(a*x+1)+5/8*a*arctanh(a*x)^3-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^2-1/512*a/(a*x+1)^2+15/
32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*
x+1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2
*x^2+1)))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*csgn(I/(1+(a
*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/1
6*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a
^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog(1+(a*x+1
)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog
(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1
+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1
5/32*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*
Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+
1)^2/(-a^2*x^2+1)))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*cs
gn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/
(-a^2*x^2+1)))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2+15/32*I*a*Pi*csgn(I*(a*x+1)/(-
a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a*x+1)/(-a^2*x^2+1
)^(1/2))-15/32*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2)
```

```
)-1/8*a/(a*x-1)-1/8*a/(a*x+1)+a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a*di
log((a*x+1)/(-a^2*x^2+1)^(1/2))+a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a*d
ilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1
))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,-(a*x+1)/
(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh
(a*x)^2-15/32*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2+15/32*I*a*Pi*csgn(I*
(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+
1)))^2*arctanh(a*x)^2-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*polylog(2,-(a*x+1)/(-a^2*x
^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1
5/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2-15/16*I*a*Pi*csg
n(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/
16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,-(a*x+1)/(-a^2*x^2
+1)^(1/2))-15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*arctanh(a*x)^2
+15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*dilog(1+(a*x+1)/(-a^2*x^
2+1)^(1/2))+15/16*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*arctanh(a*x)^
2-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*d
ilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/512
/(a*x-1)^2*a^3*x^2-15/8*a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+2*a
*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*arctanh(a*x)*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))+1/4*arctanh(a*x)/(a*x-1)*a^2*x+1/4*arctanh(a*x)/(a*x+1)
*a^2*x+15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^
2*x^2-1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn
(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-
a^2*x^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*c
sgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*dilog(1+(a*x+1)/(-a^2*x^2+1)^
(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(
a^2*x^2-1))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x
^2+1)))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I
/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1
/2))-15/32*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*
x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*dilog((a*x
+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))*arctanh(a*x)^2-15/32*I*a*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*polylog(2,(a*x+
1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+
15/16*I*a*Pi*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*dilog((a*x+1)
/(-a^2*x^2+1)^(1/2))-1/128*arctanh(a*x)/(a*x+1)^2*a^3*x^2+1/64*arctanh(a*x)
/(a*x+1)^2*a^2*x-1/128*arctanh(a*x)/(a*x-1)^2*a^3*x^2-1/64*arctanh(a*x)/(a*
x-1)^2*a^2*x+1/256/(a*x-1)^2*a^2*x-1/512/(a*x+1)^2*a^3*x^2+1/256/(a*x+1)^2*
a^2*x+1/16*a*arctanh(a*x)^2/(a*x-1)^2-1/16*a*arctanh(a*x)^2/(a*x+1)^2-15/16
*a*arctanh(a*x)^2*ln(a*x-1)+15/16*a*arctanh(a*x)^2*ln(a*x+1)-1/128*a*arctan
h(a*x)/(a*x-1)^2-1/128*a*arctanh(a*x)/(a*x+1)^2
```

maxima [B] time = 0.35, size = 534, normalized size = 2.56

$$-\frac{1}{128}a^2\left(\frac{2\left(31a^3x^3-5\left(a^4x^4-2a^2x^2+1\right)\log\left(ax+1\right)^3+5\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)^3-\left(16a^4x^4-32a\right.\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")

[Out] -1/128*a^2*(2*(31*a^3*x^3 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - (16*a^4*x^4 - 32*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1) + 16)*log(a*x + 1)^2 + 16*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 33*a*x - (15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 32*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))/(a^5*x^4 - 2*a^3*x^2 + a) - 128*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 128*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 128*(log(-a*x + 1)*log(x) + dilog(a*x))/a - 31*log(a*x + 1)/a + 31*log(a*x - 1)/a) + 1/32*a*((28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 32*log(a*x - 1) + 64*log(x))*arctanh(a*x) + 1/16*(15*a*log(a*x + 1) - 15*a*log(a*x - 1) - 2*(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*arctanh(a*x)^2

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^3),x)

[Out] -int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^6x^8-3a^4x^6+3a^2x^4-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**3,x)

[Out] -Integral(atanh(a*x)**2/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)

3.314 $\int \frac{x^3 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3} \, dx$

Optimal. Leaf size=192

$$-\frac{3\tanh^{-1}(ax)^3}{32a^4}+\frac{27\tanh^{-1}(ax)}{256a^4}+\frac{x^4\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3x^4\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3x^3}{128a\left(1-a^2x^2\right)^2}-\frac{3x^3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}-\frac{9\,\text{ta}}{32a^4}$$

```
[Out] -3/128*x^3/a/(-a^2*x^2+1)^2+45/256*x/a^3/(-a^2*x^2+1)+27/256*arctanh(a*x)/a
^4+3/32*x^4*arctanh(a*x)/(-a^2*x^2+1)^2-9/32*arctanh(a*x)/a^4/(-a^2*x^2+1)-
3/16*x^3*arctanh(a*x)^2/a/(-a^2*x^2+1)^2+9/32*x*arctanh(a*x)^2/a^3/(-a^2*x^
2+1)-3/32*arctanh(a*x)^3/a^4+1/4*x^4*arctanh(a*x)^3/(-a^2*x^2+1)^2
```

Rubi [A] time = 0.27, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6008, 6004, 6000, 5994, 199, 206, 288}

$$-\frac{3x^3}{128a\left(1-a^2x^2\right)^2}+\frac{45x}{256a^3\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3x^4\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3x^3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{9x\tanh^{-1}(ax)^2}{32a^3\left(1-a^2x^2\right)}-\frac{}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*x^3)/(128*a*(1 - a^2*x^2)^2) + (45*x)/(256*a^3*(1 - a^2*x^2)) + (27*Arc
Tanh[a*x])/(256*a^4) + (3*x^4*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) - (9*ArcTa
nh[a*x])/(32*a^4*(1 - a^2*x^2)) - (3*x^3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2
)^2) + (9*x*ArcTanh[a*x]^2)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32
*a^4) + (x^4*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6000

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> -Simp[(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^
2, x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6004

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> -Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*
ArcTanh[c*x])^(p - 1))/(c*d*m^2), x] + (-Dist[(f^2*(m - 1))/(c^2*d*m), Int[
(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^
2*p*(p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x
], x] + Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(
c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && Eq
Q[m + 2*q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = \frac{x^4 \tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} - \frac{1}{4}(3a)\int \frac{x^4 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx$$

$$= \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3x^3 \tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} + \frac{x^4 \tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{9\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{16a} - \frac{1}{32}(3a)\int$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3x^3 \tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} + \frac{9x \tanh^{-1}(ax)^2}{32a^3\left(1-a^2x^2\right)} - \frac{3 \tanh^{-1}(a}{32a^4}$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{9x}{256a^3\left(1-a^2x^2\right)} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{9 \tanh^{-1}(ax)}{32a^4\left(1-a^2x^2\right)} - \frac{3x^3 \tanh^{-1}}{16a\left(1-a\right.}$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{45x}{256a^3\left(1-a^2x^2\right)} - \frac{9 \tanh^{-1}(ax)}{256a^4} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{9 \tanh^{-1}(ax}{32a^4\left(1-a^2x\right.}$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{45x}{256a^3\left(1-a^2x^2\right)} + \frac{27 \tanh^{-1}(ax)}{256a^4} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{9 \tanh^{-1}(a}{32a^4\left(1-a^2\right.}$$

Mathematica [A] time = 0.13, size = 135, normalized size = 0.70

$$\frac{-48ax\left(5a^2x^2-3\right)\tanh^{-1}(ax)^2+48\left(5a^2x^2-4\right)\tanh^{-1}(ax)+16\left(5a^4x^4+6a^2x^2-3\right)\tanh^{-1}(ax)^3+3\left(-34a^3\right.}{512a^4\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (48*(-4 + 5*a^2*x^2)*ArcTanh[a*x] - 48*a*x*(-3 + 5*a^2*x^2)*ArcTanh[a*x]^2
+ 16*(-3 + 6*a^2*x^2 + 5*a^4*x^4)*ArcTanh[a*x]^3 + 3*(30*a*x - 34*a^3*x^3 -
 17*(-1 + a^2*x^2)^2*Log[1 - a*x] + 17*(-1 + a^2*x^2)^2*Log[1 + a*x]))/(512
*a^4*(-1 + a^2*x^2)^2)
```

fricas [A] time = 1.52, size = 140, normalized size = 0.73

$$-\frac{102\,a^3x^3-2\left(5\,a^4x^4+6\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\left(5\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-90\,ax-3\left(17\,a^4x^4+6\,a^2\right.}{512\left(a^8x^4-2\,a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/512*(102*a^3*x^3 - 2*(5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1
))^3 + 12*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 90*a*x - 3*(17*
a^4*x^4 + 6*a^2*x^2 - 15)*log(-(a*x + 1)/(a*x - 1)))/(a^8*x^4 - 2*a^6*x^2 +
 a^4)
```

giac [B] time = 0.21, size = 341, normalized size = 1.78

$$\frac{1}{2048}\left(4\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{(ax+1)^2}{(ax-1)^2a^5}+\frac{4(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+6\left(\frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}-\frac{(a}{(ax}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(4*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (a*x
 + 1)^2/((a*x - 1)^2*a^5) + 4*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/(a*
x - 1))^3 + 6*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) -
(a*x + 1)^2/((a*x - 1)^2*a^5) - 8*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)
/(a*x - 1))^2 + 6*((a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^
5) + (a*x + 1)^2/((a*x - 1)^2*a^5) + 16*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*
x + 1)/(a*x - 1)) + 3*(a*x - 1)^2*(32*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2
*a^5) - 3*(a*x + 1)^2/((a*x - 1)^2*a^5) - 96*(a*x + 1)/((a*x - 1)*a^5))*a
```

maple [C] time = 0.90, size = 2634, normalized size = 13.72

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] 15/64*I/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+15/64*I/a^2/(a*x-1)^2/(a*x+1)^2*arcta
nh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2+15/32*I/a^2/(a*x-1)^2/(a*x
+1)^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-15/128*I/(
```

```
a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^4-15/128*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^
2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*
x+1)^2/(-a^2*x^2+1)))^2*x^4+15/128*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2
/(a^2*x^2-1))*x^4+15/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2
+1)))^2-15/64*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-
a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-15/128*I/a^4/(a*x-1)^2/(a
*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+
1)^2/(a^2*x^2-1))-15/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I
/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))^2-15/32*I/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a
*x+1)^2/(-a^2*x^2+1)))^3*x^2+3/16/a^4*arctanh(a*x)^3/(a*x-1)-3/16/a^4*arcta
nh(a*x)^3/(a*x+1)+1/16/a^4*arctanh(a*x)^3/(a*x-1)^2+1/16/a^4*arctanh(a*x)^3
/(a*x+1)^2-15/64/a^4*arctanh(a*x)^2*ln(a*x-1)-15/64*I/(a*x-1)^2/(a*x+1)^2*a
rctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*x^4+15/64*I/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2+15/64*I/a^2/(a*x
-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-15/64*I/a^2/(a*x-1
)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(
-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^2+15/128*I/(a*x-1)^2/(a*x+1
)^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^4+15
/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*
x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x
+1)^2/(-a^2*x^2+1)))+15/32*I/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn
(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2-3/64/a^4
*arctanh(a*x)^2/(a*x-1)^2-15/64/a^4*arctanh(a*x)^2/(a*x-1)+3/64/a^4*arctanh
(a*x)^2/(a*x+1)^2-15/64/a^4*arctanh(a*x)^2/(a*x+1)-15/64*I/a^2/(a*x-1)^2/(a
*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^
2+15/64/a^4*arctanh(a*x)^2*ln(a*x+1)-15/32/a^4*arctanh(a*x)^2*ln((a*x+1)/(-
a^2*x^2+1)^(1/2))+15/64*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2+15/64*I
/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^4-15/128*I/a^4/(a*x-1)^2/(a*x+1)^2
*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-15/64*I/a^4/(a*x-1)^2/(a
*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-15/128*I/(a*
x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^4-15/6
4*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1))
)^2*x^4+15/64*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)
^2/(-a^2*x^2+1)))^3-15/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-15/32*I/a^2/(a*x-1)^
2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^2+15/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^
2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^4-15/128*I/(a*x-1)^2/(a*x+1)^2*
arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^
3*x^4-5/16/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^2+9/128/a^2/(a*x-1)^2/(
a*x+1)^2*arctanh(a*x)*x^2-51/256/a/(a*x-1)^2/(a*x+1)^2*x^3+45/256/a^3/(a*x-
1)^2/(a*x+1)^2*x+5/32/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^4+51/256/(a*x-1)
^2/(a*x+1)^2*arctanh(a*x)*x^4+5/32/a^4/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3-4
5/256/a^4/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)
```

maxima [B] time = 0.34, size = 437, normalized size = 2.28

$$-\frac{3}{64}\,a\left(\frac{2\left(5\,a^{2}x^{3}-3\,x\right)}{a^{8}x^{4}-2\,a^{6}x^{2}+a^{4}}-\frac{5\,\log\left(ax+1\right)}{a^{5}}+\frac{5\,\log\left(ax-1\right)}{a^{5}}\right)\operatorname{artanh}\left(ax\right)^{2}+\frac{\left(2\,a^{2}x^{2}-1\right)\operatorname{artanh}\left(ax\right)^{3}}{4\left(a^{8}x^{4}-2\,a^{6}x^{2}+a^{4}\right)}-\frac{1}{512}\left(\frac{\left(102\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -3/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x)^2 + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^
3/(a^8*x^4 - 2*a^6*x^2 + a^4) - 1/512*((102*a^3*x^3 - 10*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x + 1)^3 + 30*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*
x - 1) + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 90*a*x - 3*(17*a^4*x
^4 - 34*a^2*x^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 17)*log(a*x
 + 1) + 51*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^11*x^4 - 2*a^9*x^
2 + a^7) - 12*(20*a^2*x^2 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 10
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 5*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^10*x^4 - 2*a^8*x^2 + a^6))*a
```

mupad [B] time = 3.09, size = 414, normalized size = 2.16

$$48\ \ln(1-a\,x)-48\ \ln(a\,x+1)+51\,\mathrm{atanh}(a\,x)+45\,a\,x-3\ln(a\,x+1)^3+3\ln(1-a\,x)^3-9\ \ln(a\,x+1)\ \ln(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

```
[Out] (48*log(1 - a*x) - 48*log(a*x + 1) + 51*atanh(a*x) + 45*a*x - 3*log(a*x + 1
)^3 + 3*log(1 - a*x)^3 - 9*log(a*x + 1)*log(1 - a*x)^2 + 9*log(a*x + 1)^2*l
og(1 - a*x) - 51*a^3*x^3 + 6*a^2*x^2*log(a*x + 1)^3 - 6*a^2*x^2*log(1 - a*x
)^3 - 30*a^3*x^3*log(a*x + 1)^2 - 30*a^3*x^3*log(1 - a*x)^2 + 5*a^4*x^4*log
(a*x + 1)^3 - 5*a^4*x^4*log(1 - a*x)^3 - 102*a^2*x^2*atanh(a*x) + 51*a^4*x^
4*atanh(a*x) + 18*a*x*log(a*x + 1)^2 + 18*a*x*log(1 - a*x)^2 + 60*a^2*x^2*l
og(a*x + 1) - 60*a^2*x^2*log(1 - a*x) - 36*a*x*log(a*x + 1)*log(1 - a*x) +
18*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 - 18*a^2*x^2*log(a*x + 1)^2*log(1 -
a*x) + 15*a^4*x^4*log(a*x + 1)*log(1 - a*x)^2 - 15*a^4*x^4*log(a*x + 1)^2*l
og(1 - a*x) + 60*a^3*x^3*log(a*x + 1)*log(1 - a*x))/(256*a^4*(a^2*x^2 - 1)^
2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\,\mathrm{atanh}^3\,(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**3*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

3.315 $\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=215

$$-\frac{\tanh^{-1}(ax)^4}{32a^3}-\frac{3\tanh^{-1}(ax)^2}{128a^3}-\frac{x\tanh^{-1}(ax)^3}{8a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3x\tanh^{-1}(ax)}{64a^2(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2}+\frac{3}{128a^3(1-a}$$

```
[Out] -3/128/a^3/(-a^2*x^2+1)^2+3/128/a^3/(-a^2*x^2+1)+3/32*x*arctanh(a*x)/a^2/(-
a^2*x^2+1)^2-3/64*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-3/128*arctanh(a*x)^2/a^3-
3/16*arctanh(a*x)^2/a^3/(-a^2*x^2+1)^2+3/16*arctanh(a*x)^2/a^3/(-a^2*x^2+1)
+1/4*x*arctanh(a*x)^3/a^2/(-a^2*x^2+1)^2-1/8*x*arctanh(a*x)^3/a^2/(-a^2*x^2
+1)-1/32*arctanh(a*x)^4/a^3
```

Rubi [A] time = 0.36, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6028, 5956, 5994, 261, 5964, 5960}

$$\frac{3}{128a^3(1-a^2x^2)}-\frac{3}{128a^3(1-a^2x^2)^2}-\frac{x\tanh^{-1}(ax)^3}{8a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{4a^2(1-a^2x^2)^2}+\frac{3\tanh^{-1}(ax)^2}{16a^3(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{16a^3(1-a^2x^2)^2}-\frac{3x\tan}{64a^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] -3/(128*a^3*(1 - a^2*x^2)^2) + 3/(128*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x
])/(32*a^2*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(64*a^2*(1 - a^2*x^2)) - (
3*ArcTanh[a*x]^2)/(128*a^3) - (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)^2) +
 (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*a^2*(1 -
 a^2*x^2)^2) - (x*ArcTanh[a*x]^3)/(8*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(3
2*a^3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
```

```
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx}{a^2}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} + \cdots$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)^2} + \frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \cdots$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{39x\tanh^{-1}(ax)}{64a^2\left(1-a^2x^2\right)} - \frac{39\tanh^{-1}(ax)^2}{128a^3} - \cdots$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{39}{128a^3\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)}{64a^2\left(1-a^2x^2\right)} - \cdots$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{3}{128a^3\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)}{64a^2\left(1-a^2x^2\right)} - \cdots$$

Mathematica [A] time = 0.12, size = 107, normalized size = 0.50

$$\frac{16\left(a^3x^3+ax\right)\tanh^{-1}(ax)^3 + 6\left(a^3x^3+ax\right)\tanh^{-1}(ax) - 3a^2x^2 - 4\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^4 - 3\left(a^4x^4+6a^2x^2 \cdots\right.}{128a^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*a^2*x^2 + 6*(a*x + a^3*x^3)*ArcTanh[a*x] - 3*(1 + 6*a^2*x^2 + a^4*x^4)*
ArcTanh[a*x]^2 + 16*(a*x + a^3*x^3)*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2*Arc
Tanh[a*x]^4)/(128*a^3*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.52, size = 161, normalized size = 0.75

$$-\frac{\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}+12\,a^{2}x^{2}-8\left(a^{3}x^{3}+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}+3\left(a^{4}x^{4}+6\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-}{512\left(a^{7}x^{4}-2\,a^{5}x^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/512*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 12*a^2*x^2
- 8*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(a^4*x^4 + 6*a^2*x^2 +
1)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1
)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^{2}\operatorname{artanh}\left(ax\right)^{3}}{\left(a^{2}x^{2}-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

maple [C] time = 0.85, size = 2646, normalized size = 12.31

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] -1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x
^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+
1)^2/(-a^2*x^2+1)))*x^4+1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn
(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2+1/32*I/a^3/(a*x-1)^2/(a*x+1)
^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctan
h(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^4+1/32*I*a/(a*x-1)^2/(a*
x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^4+1/32*I*a/(a*x-
1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/
(-a^2*x^2+1)))^3*x^4+1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/
(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4+1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^
3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2-1/16*I/a/(a*x-1)^2/(a*x+1)^2*
arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2-1/16*I/a/(a*x-1)^2/(a
*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x
^2+1)))^3*x^2-1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+
1)^2/(-a^2*x^2+1)))^2*x^2-1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+
1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+1/16*I/a^3/(a*x-1)^2
/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x
+1)^2/(a^2*x^2-1))^2-3/1024/a^3/(a*x-1)^2/(a*x+1)^2+1/16/a^3*arctanh(a*x)^3
/(a*x-1)^2+1/16/a^3*arctanh(a*x)^3/(a*x-1)+1/16/a^3*arctanh(a*x)^3*ln(a*x-1
```

```
)-1/16/a^3*arctanh(a*x)^3/(a*x+1)^2+1/16/a^3*arctanh(a*x)^3/(a*x+1)-1/16/a^
3*arctanh(a*x)^3*ln(a*x+1)+1/8/a^3*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(
1/2))-1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3+1/16*I*a/(a*x-1)^2/(
a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(
-a^2*x^2+1)^(1/2))*x^4-1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^
2+1)))^2*x^4+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^4-1/16*I/a/(a*x-1)^2/
(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+
1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-1/8*I/a/(a*x-1)^2/(a*x+1
)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))*x^2+1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1))
)^2*x^2-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2-1/32*I/a^3/(a*x-1)^2/(a*
x+1)^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+1/3
2*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1
)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4-9/64/a/(
a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*x^2+3/64/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(
a*x)*x-1/32*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^4*x^4-1/16*I/a^3/(a*x-1)^2/(
a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-1/16*I*a/(a
*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*x^4+1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh
(a*x)^3*Pi*x^2+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x
+1)^2/(a^2*x^2-1))^3+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+1/16*I/a^3/(a*x-1)^2/
(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+3/64/(a*x-
1)^2/(a*x+1)^2*arctanh(a*x)*x^3-1/32/a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^4
-3/128/a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2-3/1024*a/(a*x-1)^2/(a*x+1)^2*
x^4-9/512/a/(a*x-1)^2/(a*x+1)^2*x^2-3/128*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x
)^2*x^4+1/16/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^4*x^2
```

maxima [B] time = 0.35, size = 657, normalized size = 3.06

$$\frac{1}{16}\left(\frac{2\left(a^2x^3+x\right)}{a^6x^4-2\,a^4x^2+a^2}-\frac{\log\left(ax+1\right)}{a^3}+\frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right)^3-\frac{3\left(4\,a^2x^2-\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x)^3 - 3/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a*arctanh(a*x)^2/(a^8*x^4 - 2*a^6*x
^2 + a^4) + 1/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^4 - 4*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log
(a*x - 1)^4 - 12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2
+ 1)*log(a*x - 1)^2 + 1)*log(a*x + 1)^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a
*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 3*(a^4*x^4 - 2*
a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))*a^2/(a^10*x^4 - 2*a^8*x^2 + a^6) +
 4*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2
*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log
(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x - 1)^2 + 1)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)
)*a*arctanh(a*x)/(a^9*x^4 - 2*a^7*x^2 + a^5))*a
```

mupad [B] time = 3.19, size = 831, normalized size = 3.87

$$\frac{3\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)}{4\,\left(64\,a^7\,x^4-128\,a^5\,x^2+64\,a^3\right)}-\frac{3\ln\left(1-a\,x\right)^2}{512\,a^3}-\frac{\ln\left(a\,x+1\right)^4}{512\,a^3}-\frac{\ln\left(1-a\,x\right)^4}{512\,a^3}-\frac{3\,x^2}{2\,\left(64\,a^5\,x^4-128\,a^3\,x^2+64\,a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

```
[Out] (3*log(a*x + 1)*log(1 - a*x))/(4*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4)) - (3*
log(1 - a*x)^2)/(512*a^3) - log(a*x + 1)^4/(512*a^3) - log(1 - a*x)^4/(512*
a^3) - (3*x^2)/(2*(64*a - 128*a^3*x^2 + 64*a^5*x^4)) - (x*log(1 - a*x)^3)/(
8*(8*a^2 - 16*a^4*x^2 + 8*a^6*x^4)) - (6*x^2*log(1 - a*x)^2)/(128*a - 256*a
^3*x^2 + 128*a^5*x^4) - (3*log(a*x + 1)^2)/(512*a^3) + (x^3*log(a*x + 1)^3)
/(64*(a^4*x^4 - 2*a^2*x^2 + 1)) - (x^3*log(1 - a*x)^3)/(8*(8*a^4*x^4 - 16*a
^2*x^2 + 8)) + (3*x*log(a*x + 1))/(128*(a^2 - 2*a^4*x^2 + a^6*x^4)) + (log(
a*x + 1)*log(1 - a*x)^3)/(128*a^3) + (log(a*x + 1)^3*log(1 - a*x))/(128*a^3
) - (3*x*log(1 - a*x))/(128*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (3*x^2*log(a
*x + 1)^2)/(64*(a - 2*a^3*x^2 + a^5*x^4)) + (x*log(a*x + 1)^3)/(64*(a^2 - 2
*a^4*x^2 + a^6*x^4)) - (3*log(a*x + 1)^2*log(1 - a*x)^2)/(256*a^3) + (3*x^3
*log(a*x + 1))/(128*(a^4*x^4 - 2*a^2*x^2 + 1)) - (3*a*x^3*log(1 - a*x))/(12
8*a - 256*a^3*x^2 + 128*a^5*x^4) + (6*x*log(a*x + 1)*log(1 - a*x)^2)/(128*a
^2 - 256*a^4*x^2 + 128*a^6*x^4) - (6*x*log(a*x + 1)^2*log(1 - a*x))/(128*a^
2 - 256*a^4*x^2 + 128*a^6*x^4) + (6*x^2*log(a*x + 1)*log(1 - a*x))/(64*a -
128*a^3*x^2 + 64*a^5*x^4) + (6*a^2*x^3*log(a*x + 1)*log(1 - a*x)^2)/(128*a^
2 - 256*a^4*x^2 + 128*a^6*x^4) - (6*a^2*x^3*log(a*x + 1)^2*log(1 - a*x))/(1
28*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (3*a^2*x^2*log(a*x + 1)*log(1 - a*x))
/(2*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4)) + (3*a^4*x^4*log(a*x + 1)*log(1 -
a*x))/(4*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 \operatorname{atanh}^3(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

3.316 $\int \frac{x \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3} \, dx$

Optimal. Leaf size=188

$$-\frac{45x}{256a\left(1-a^2x^2\right)}-\frac{3x}{128a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{9x\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)}+\frac{3}{32a}$$

```
[Out] -3/128*x/a/(-a^2*x^2+1)^2-45/256*x/a/(-a^2*x^2+1)-45/256*arctanh(a*x)/a^2+3
/32*arctanh(a*x)/a^2/(-a^2*x^2+1)^2+9/32*arctanh(a*x)/a^2/(-a^2*x^2+1)-3/16
*x*arctanh(a*x)^2/a/(-a^2*x^2+1)^2-9/32*x*arctanh(a*x)^2/a/(-a^2*x^2+1)-3/3
2*arctanh(a*x)^3/a^2+1/4*arctanh(a*x)^3/a^2/(-a^2*x^2+1)^2
```

Rubi [A] time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5994, 5964, 5956, 199, 206}

$$-\frac{45x}{256a\left(1-a^2x^2\right)}-\frac{3x}{128a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{9x\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)}+\frac{3}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*x)/(128*a*(1 - a^2*x^2)^2) - (45*x)/(256*a*(1 - a^2*x^2)) - (45*ArcTanh
[a*x])/(256*a^2) + (3*ArcTanh[a*x])/(32*a^2*(1 - a^2*x^2)^2) + (9*ArcTanh[a
*x])/(32*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) -
 (9*x*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32*a^2) +
ArcTanh[a*x]^3/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
```

```
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = \frac{\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2} - \frac{3\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx}{32a} - \frac{9\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,d}{16a}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} - \frac{9x\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax}{32a^2}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} - \frac{9x}{256a\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} + \frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(}{16a\left(1-a^2x\right.}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} - \frac{45x}{256a\left(1-a^2x^2\right)} - \frac{9\tanh^{-1}(ax)}{256a^2} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} + \frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right.}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} - \frac{45x}{256a\left(1-a^2x^2\right)} - \frac{45\tanh^{-1}(ax)}{256a^2} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} + \frac{9\tanh^{-1}(ax}{32a^2\left(1-a^2x\right.}$$

Mathematica [A] time = 0.09, size = 148, normalized size = 0.79

$$\frac{-45a^4x^4\log(ax+1)+90a^3x^3+90a^2x^2\log(ax+1)+45\left(a^2x^2-1\right)^2\log(1-ax)+48ax\left(3a^2x^2-5\right)\tanh^{-1}(ax)^2}{512a^2\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-102*a*x + 90*a^3*x^3 - 48*(-4 + 3*a^2*x^2)*ArcTanh[a*x] + 48*a*x*(-5 + 3*
a^2*x^2)*ArcTanh[a*x]^2 + (80 + 96*a^2*x^2 - 48*a^4*x^4)*ArcTanh[a*x]^3 + 4
5*(-1 + a^2*x^2)^2*Log[1 - a*x] - 45*Log[1 + a*x] + 90*a^2*x^2*Log[1 + a*x]
 - 45*a^4*x^4*Log[1 + a*x])/(512*a^2*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.46, size = 140, normalized size = 0.74

$$\frac{90\,a^3x^3-2\left(3\,a^4x^4-6\,a^2x^2-5\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\left(3\,a^3x^3-5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-102\,ax-3\left(15\,a^4x^4-6\,a^2x^2-\right.}{512\left(a^6x^4-2\,a^4x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(90*a^3*x^3 - 2*(3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1))
^3 + 12*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 102*a*x - 3*(15*a
^4*x^4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^6*x^4 - 2*a^4*x^2 +
a^2)
```

giac [B] time = 0.29, size = 342, normalized size = 1.82

$$-\frac{1}{2048}\left(4\left(\frac{(ax-1)^2\left(\frac{4\,(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{(ax+1)^2}{(ax-1)^2a^3}+\frac{4\,(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+6\left(\frac{(ax-1)^2\left(\frac{8\,(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}+\frac{(}{(a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(4*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - (a*
x + 1)^2/((a*x - 1)^2*a^3) + 4*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)/(a
*x - 1))^3 + 6*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) +
 (a*x + 1)^2/((a*x - 1)^2*a^3) - 8*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1
)/(a*x - 1))^2 + 6*((a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a
^3) - (a*x + 1)^2/((a*x - 1)^2*a^3) + 16*(a*x + 1)/((a*x - 1)*a^3))*log(-(a
*x + 1)/(a*x - 1)) + 3*(a*x - 1)^2*(32*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^
2*a^3) + 3*(a*x + 1)^2/((a*x - 1)^2*a^3) - 96*(a*x + 1)/((a*x - 1)*a^3))*a
```

maple [C] time = 0.72, size = 2582, normalized size = 13.73

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] -9/32*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi*x^2-9/64*I/(a*x-1)^2/(a*x+1)^2*arctan
h(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2
*Pi*x^2-9/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2
*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^2
-9/64*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^
2+1)))^3*Pi*x^4+9/128*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+
1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^4+9/128*I*a^2/(a*x-1)^2
/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^4+9/64*I*a^2
/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi
*x^4+9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2
*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+9/128*I/a^2/(a*x-1)^2/(a*x+1
)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi
*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))^2+9/64*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x
)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+9/64
*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+
1)^2/(-a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^2-9/128*I*a^2/(a*x
-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-
a^2*x^2+1)))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^4+9/64*I*a^2/(a*x-1)^2/(a
*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2
*x^2+1)^(1/2))*Pi*x^4+9/128*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^4+9/64*I/
(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*Pi*x^2-9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(
```

```
a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+9/128*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh
(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+
(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^4-9/128*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a
*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a
*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^4-3/32*a^2/(a*x-1
)^2/(a*x+1)^2*arctanh(a*x)^3*x^4-45/256*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x
)*x^4+1/4/a^2/(a^2*x^2-1)^2*arctanh(a*x)^3-3/64/a^2*arctanh(a*x)^2/(a*x-1)^
2+9/64/a^2*arctanh(a*x)^2/(a*x-1)+9/64/a^2*arctanh(a*x)^2*ln(a*x-1)+3/64/a^
2*arctanh(a*x)^2/(a*x+1)^2+9/64/a^2*arctanh(a*x)^2/(a*x+1)-9/64/a^2*arctanh
(a*x)^2*ln(a*x+1)+9/32/a^2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-9/
64*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2+9/32*I/(a*x-1)^2/(a*x+1)^2*a
rctanh(a*x)^2*Pi*x^2+9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn
(I*(a*x+1)^2/(a^2*x^2-1))^3+9/64*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^
2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-9/64*I*a^2/(a*x-1)^2/(a*x+1)^2*arcta
nh(a*x)^2*Pi*x^4-9/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x^2-9/64*I/(a*x-1)^2/(a*x+1)^
2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^2-9/32*I/(a*x-1)^2/(a
*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*Pi*x^2+9/32*I/(
a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*Pi*x
^2-9/64*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a
^2*x^2+1)))^3+9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+3/16/(a*x-1)^2/(a*x+1)^2*ar
ctanh(a*x)^3*x^2+9/128/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^2-3/32/a^2/(a*x-1
)^2/(a*x+1)^2*arctanh(a*x)^3+51/256/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)+45
/256*a/(a*x-1)^2/(a*x+1)^2*x^3-51/256/a/(a*x-1)^2/(a*x+1)^2*x
```

maxima [B] time = 0.34, size = 422, normalized size = 2.24

$$\frac{3\left(\frac{2\left(3a^2x^3-5x\right)}{a^4x^4-2a^2x^2+1}-\frac{3\log(ax+1)}{a}+\frac{3\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2}{64a}+\frac{3\left(\left(30a^3x^3-2\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)^3+6\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)*arctanh(a*x)^2/a + 3/512*((30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*
x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4
 - 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1
) + 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x^2 + a
^3) - 4*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)
*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2))/a + 1/4*a
rctanh(a*x)^3/((a^2*x^2 - 1)^2*a^2)
```

mupad [B] time = 2.69, size = 414, normalized size = 2.20

$$-\frac{48\ln(1-ax)-48\ln(ax+1)+45\operatorname{atanh}(ax)+51ax-5\ln(ax+1)^3+5\ln(1-ax)^3-15\ln(ax+1)\ln(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

```
[Out] -(48*log(1 - a*x) - 48*log(a*x + 1) + 45*atanh(a*x) + 51*a*x - 5*log(a*x +
1)^3 + 5*log(1 - a*x)^3 - 15*log(a*x + 1)*log(1 - a*x)^2 + 15*log(a*x + 1)^
2*log(1 - a*x) - 45*a^3*x^3 - 6*a^2*x^2*log(a*x + 1)^3 + 6*a^2*x^2*log(1 -
```

```
a*x)^3 - 18*a^3*x^3*log(a*x + 1)^2 - 18*a^3*x^3*log(1 - a*x)^2 + 3*a^4*x^4*
log(a*x + 1)^3 - 3*a^4*x^4*log(1 - a*x)^3 - 90*a^2*x^2*atanh(a*x) + 45*a^4*
x^4*atanh(a*x) + 30*a*x*log(a*x + 1)^2 + 30*a*x*log(1 - a*x)^2 + 36*a^2*x^2
*log(a*x + 1) - 36*a^2*x^2*log(1 - a*x) - 60*a*x*log(a*x + 1)*log(1 - a*x)
- 18*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 + 18*a^2*x^2*log(a*x + 1)^2*log(1
- a*x) + 9*a^4*x^4*log(a*x + 1)*log(1 - a*x)^2 - 9*a^4*x^4*log(a*x + 1)^2*l
og(1 - a*x) + 36*a^3*x^3*log(a*x + 1)*log(1 - a*x))/(256*a^2*(a^2*x^2 - 1)^
2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x \operatorname{atanh}^3(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

3.317 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=203

$$-\frac{45}{128a\left(1-a^2x^2\right)}-\frac{3}{128a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)^3}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{9\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{45x\tanh}{64(1-}$$

```
[Out] -3/128/a/(-a^2*x^2+1)^2-45/128/a/(-a^2*x^2+1)+3/32*x*arctanh(a*x)/(-a^2*x^2
+1)^2+45/64*x*arctanh(a*x)/(-a^2*x^2+1)+45/128*arctanh(a*x)^2/a-3/16*arctan
h(a*x)^2/a/(-a^2*x^2+1)^2-9/16*arctanh(a*x)^2/a/(-a^2*x^2+1)+1/4*x*arctanh(
a*x)^3/(-a^2*x^2+1)^2+3/8*x*arctanh(a*x)^3/(-a^2*x^2+1)+3/32*arctanh(a*x)^4
/a
```

Rubi [A] time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 261, 5960}

$$-\frac{45}{128a\left(1-a^2x^2\right)}-\frac{3}{128a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)^3}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{9\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{45x\tanh}{64(1-}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```

```
[Out] -3/(128*a*(1 - a^2*x^2)^2) - 45/(128*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/
(32*(1 - a^2*x^2)^2) + (45*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) + (45*ArcTanh
[a*x]^2)/(128*a) - (3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) - (9*ArcTanh[a
*x]^2)/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (3*x
*ArcTanh[a*x]^3)/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^4)/(32*a)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
```

```
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = -\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3}{8}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx+\frac{3}{4}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx$$
$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}$$
$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{9x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{9\tanh^{-1}(ax)^2}{128a}-\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}$$
$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}-\frac{9}{128a\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{45x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{45\tanh^{-1}(a}{128a}$$
$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}-\frac{45}{128a\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{45x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{45\tanh^{-1}(a}{128a}$$

Mathematica [A] time = 0.16, size = 111, normalized size = 0.55

$$\frac{\left(80ax-48a^3x^3\right)\tanh^{-1}(ax)^3+\left(102ax-90a^3x^3\right)\tanh^{-1}(ax)+45a^2x^2+12\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^4+3\left(15a^4\right.}{128a\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```

```
[Out] (-48 + 45*a^2*x^2 + (102*a*x - 90*a^3*x^3)*ArcTanh[a*x] + 3*(-17 - 6*a^2*x^
2 + 15*a^4*x^4)*ArcTanh[a*x]^2 + (80*a*x - 48*a^3*x^3)*ArcTanh[a*x]^3 + 12*
(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)/(128*a*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.57, size = 166, normalized size = 0.82

$$\frac{3\left(a^4x^4-2a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+180a^2x^2-8\left(3a^3x^3-5ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+3\left(15a^4x^4-6a^2x^2-17\right)\log}{512\left(a^5x^4-2a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 180*a^2*x^
2 - 8*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(15*a^4*x^4 - 6*a
^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^3*x^3 - 17*a*x)*log(-(a
*x + 1)/(a*x - 1)) - 192)/(a^5*x^4 - 2*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^3}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

maple [C] time = 0.92, size = 2646, normalized size = 13.03

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] -3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^4+3/32*I*a^3/(a*x-1)^2/(a*x+1)^
2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctan
h(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*x^4+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/
(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x
^2+3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-
1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2-3/16*I*a/(a*x-1)^2/(a*x+1)^2*a
rctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2+3/32*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x
)^4*x^4+45/128*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*x^4-195/1024/a/(a*x-1
)^2/(a*x+1)^2+1/16/a*arctanh(a*x)^3/(a*x-1)^2-1/16/a*arctanh(a*x)^3/(a*x+1)
^2+3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a
^2*x^2+1)))^3*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(
a*x+1)^2/(a^2*x^2-1))^3*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*P
i*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^4-3/16*I*a^3
/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2
*x^4-3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^
2*x^2+1)))^3*x^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1))^3*x^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+3/8*I*a/(a*x-1)
^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-3/3
2*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1
)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-3/16*I/a/(a*
x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^2+3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^
2*x^2+1)))^2-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/
(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-3/16/a*arctanh(a*x)^3/(
a*x-1)-3/16/a*arctanh(a*x)^3*ln(a*x-1)-3/16/a*arctanh(a*x)^3/(a*x+1)+3/16/a
*arctanh(a*x)^3*ln(a*x+1)-3/8/a*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2
))+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2-3/32*I*a^3/(a*x-1)^2/(a*x+1)^
2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4+3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*a
rctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^4-3/16*I*
a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))*x^2+3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a
```

```
*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(
a*x)^3+3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(
-a^2*x^2+1)))^3-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+
1)^2/(a^2*x^2-1))^3-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-3/16*I/a/(a*x-1)^2/(a*x+
1)^2*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+3/16*I*a^3/(a*x
-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*x^4-3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a
*x)^3*Pi*x^2+189/1024*a^3/(a*x-1)^2/(a*x+1)^2*x^4-9/512*a/(a*x-1)^2/(a*x+1)
^2*x^2+51/64/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x+3/32/a/(a*x-1)^2/(a*x+1)^2*
arctanh(a*x)^4-51/128/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2-3/16*a/(a*x-1)^2
/(a*x+1)^2*arctanh(a*x)^4*x^2-45/64*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^
3-9/64*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*x^2
```

maxima [B] time = 0.35, size = 663, normalized size = 3.27

$$-\frac{1}{16}\left(\frac{2\left(3\,a^2x^3-5\,x\right)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log\left(ax+1\right)}{a}+\frac{3\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^3+\frac{3\left(12\,a^2x^2-3\left(a^4x^4-2\,a^2x^2+1\right)\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)^3 + 3/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1
) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)^2/(a^6*
x^4 - 2*a^4*x^2 + a^2) - 3/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^4 -
 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 - 2*a^2
*x^2 + 1)*log(a*x - 1)^4 - 60*a^2*x^2 + 3*(5*a^4*x^4 - 10*a^2*x^2 + 2*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1)^2 + 15*(a^4*x^4 - 2*a
^2*x^2 + 1)*log(a*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3
+ 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1) + 64)*a^2/(a^8*x^
4 - 2*a^6*x^2 + a^4) + 4*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x
+ 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1) + 2*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4 - 10*a^2*x^2 + 2
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1) + 15*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^4 - 2*a^5*x^2 + a^3))*a
```

mupad [B] time = 2.29, size = 736, normalized size = 3.63

$$\frac{\frac{45\,a\,x^2}{2}-\frac{24}{a}}{64\,a^4\,x^4-128\,a^2\,x^2+64}-\ln\left(a\,x+1\right)^2\left(\frac{\frac{3}{16\,a^2}-\frac{9\,x^2}{64}}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}-\frac{45}{512\,a}\right)-\ln\left(1-a\,x\right)^3\left(\frac{3\,\ln\left(a\,x+1\right)}{128\,a}+\frac{\frac{5\,x}{8}-}{8\,a^4\,x^4-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(a^2*x^2 - 1)^3,x)
```

```
[Out] ((45*a*x^2)/2 - 24/a)/(64*a^4*x^4 - 128*a^2*x^2 + 64) - log(a*x + 1)^2*((3/
(16*a^2) - (9*x^2)/64)/(1/a - 2*a*x^2 + a^3*x^4) - 45/(512*a)) - log(1 - a*
x)^3*((3*log(a*x + 1))/(128*a) + ((5*x)/8 - (3*a^2*x^3)/8)/(8*a^4*x^4 - 16*
a^2*x^2 + 8)) - log(1 - a*x)*((3*log(a*x + 1)^3)/(128*a) + log(a*x + 1)*(((
21*x)/2 + 9*a*x^2 - 12/a - (15*a^2*x^3)/2)/(64*a^4*x^4 - 128*a^2*x^2 + 64)
- ((21*x)/2 - 9*a*x^2 + 12/a - (15*a^2*x^3)/2)/(64*a^4*x^4 - 128*a^2*x^2 +
64) + (45*(a^4*x^4 - 2*a^2*x^2 + 1))/(4*a*(64*a^4*x^4 - 128*a^2*x^2 + 64)))
 + ((9*x)/2 + (3*a*x^2)/2 - 3/(2*a) - (9*a^2*x^3)/2)/(128*a^4*x^4 - 256*a^2
*x^2 + 128) + (21*x + (33*a*x^2)/2 - 39/(2*a) - 18*a^2*x^3)/(128*a^4*x^4 -
256*a^2*x^2 + 128) + ((51*x)/2 - 18*a*x^2 + 21/a - (45*a^2*x^3)/2)/(128*a^4
*x^4 - 256*a^2*x^2 + 128) + (log(a*x + 1)^2*(30*x - 18*a^2*x^3))/(128*a^4*x
```

```
^4 - 256*a^2*x^2 + 128)) + (3*log(a*x + 1)^4)/(512*a) + (3*log(1 - a*x)^4)/
(512*a) + log(1 - a*x)^2*((9*log(a*x + 1)^2)/(256*a) + 45/(512*a) - ((21*x)
/2 - 9*a*x^2 + 12/a - (15*a^2*x^3)/2)/(128*a^4*x^4 - 256*a^2*x^2 + 128) + (
(21*x)/2 + 9*a*x^2 - 12/a - (15*a^2*x^3)/2)/(128*a^4*x^4 - 256*a^2*x^2 + 12
8) + (log(a*x + 1)*(30*x - 18*a^2*x^3))/(128*a^4*x^4 - 256*a^2*x^2 + 128))
+ (log(a*x + 1)*((51*x)/(128*a) - (45*a*x^3)/128))/(1/a - 2*a*x^2 + a^3*x^4
) + (log(a*x + 1)^3*((5*x)/(64*a) - (3*a*x^3)/64))/(1/a - 2*a*x^2 + a^3*x^4
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^3(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

3.318 $\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^3}\,dx$

Optimal. Leaf size=277

$$-\frac{141ax}{256\left(1-a^2x^2\right)}-\frac{3ax}{128\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{33ax\tanh^{-1}(ax)^2}{32\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}+\frac{33\tanh}{32\left(1-\right.}$$

```
[Out] -3/128*a*x/(-a^2*x^2+1)^2-141/256*a*x/(-a^2*x^2+1)-141/256*arctanh(a*x)+3/3
2*arctanh(a*x)/(-a^2*x^2+1)^2+33/32*arctanh(a*x)/(-a^2*x^2+1)-3/16*a*x*arct
anh(a*x)^2/(-a^2*x^2+1)^2-33/32*a*x*arctanh(a*x)^2/(-a^2*x^2+1)-11/32*arcta
nh(a*x)^3+1/4*arctanh(a*x)^3/(-a^2*x^2+1)^2+1/2*arctanh(a*x)^3/(-a^2*x^2+1)
+1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2*polyl
og(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*polylog(4,-
1+2/(a*x+1))
```

Rubi [A] time = 0.63, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6030, 5988, 5932, 5948, 6056, 6060, 6610, 5994, 5956, 199, 206, 5964}

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (-3*a*x)/(128*(1 - a^2*x^2)^2) - (141*a*x)/(256*(1 - a^2*x^2)) - (141*ArcTa
nh[a*x])/256 + (3*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) + (33*ArcTanh[a*x])/(3
2*(1 - a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (33*a*x*Ar
cTanh[a*x]^2)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^3)/32 + ArcTanh[a*x]^3/
(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 +
 ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/
(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLo
g[4, -1 + 2/(1 + a*x)])/4
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
```

```
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} - \frac{1}{4}(3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx + a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx$$

$$= \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 - \frac{1}{32}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} - \frac{33ax\tanh^{-1}(ax)^2}{32\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} - \frac{9ax}{256\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)^2}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} - \frac{141ax}{256\left(1-a^2x^2\right)} - \frac{9}{256}\tanh^{-1}(ax) + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} - \frac{141ax}{256\left(1-a^2x^2\right)} - \frac{141}{256}\tanh^{-1}(ax) + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.30, size = 189, normalized size = 0.68

$$1536\tanh^{-1}(ax)^2\text{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) - 1536\tanh^{-1}(ax)\text{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) + 768\text{Li}_4\left(e^{2\tanh^{-1}(ax)}\right) - 256\tanh^{-1}(ax$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (16*Pi^4 - 256*ArcTanh[a*x]^4 + 576*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 384
*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 12*ArcTanh[a*x]*Cosh[4*ArcTanh[a*x]]
 + 32*ArcTanh[a*x]^3*Cosh[4*ArcTanh[a*x]] + 1024*ArcTanh[a*x]^3*Log[1 - E^(
2*ArcTanh[a*x])] + 1536*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 153
6*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 768*PolyLog[4, E^(2*ArcTanh
[a*x])] - 288*Sinh[2*ArcTanh[a*x]] - 576*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]
] - 3*Sinh[4*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[4*ArcTanh[a*x]])/1024
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)^3}{a^6x^7 - 3\,a^4x^5 + 3\,a^2x^3 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x), x)
```

maple [C] time = 0.86, size = 1533, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x)
```

```
[Out] 9/64*(a*x+1)/(a*x-1)-1/4*arctanh(a*x)^4-11/32*arctanh(a*x)^3+6*polylog(4,-(
a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2048*(
a*x+1)^2/(a*x-1)^2-9/64*(a*x-1)/(a*x+1)-6*arctanh(a*x)*polylog(3,-(a*x+1)/(
-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2
))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-1/2*I*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1
)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2
+1)))^2-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*arctanh(a*
x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a*x)^3*ln((a*x+1)/(-a^2
*x^2+1)^(1/2))+3/512*arctanh(a*x)*(a*x+1)^2/(a*x-1)^2-9/32*(a*x+1)*arctanh(
a*x)/(a*x-1)+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh
(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*
x^2+1)^(1/2))-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+ln(2)*arctanh(a*x
)^3+3/2048*(a*x-1)^2/(a*x+1)^2+1/16*arctanh(a*x)^3/(a*x-1)^2-5/16*arctanh(a
*x)^3/(a*x-1)+1/16*arctanh(a*x)^3/(a*x+1)^2+5/16*arctanh(a*x)^3/(a*x+1)+arc
tanh(a*x)^3*ln(a*x)+3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
+1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/(1+(a*x+
1)^2/(-a^2*x^2+1)))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^
2+1)))-1/4*I*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)
))+1/2*I*Pi*arctanh(a*x)^3-9/32*arctanh(a*x)*(a*x-1)/(a*x+1)+3/512*arctanh(
a*x)*(a*x-1)^2/(a*x+1)^2+1/2*I*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*
x^2+1)))^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1))
)^3*arctanh(a*x)^3+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-
1/2*I*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/2*I*Pi*arcta
nh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3-9
/32*arctanh(a*x)^2*(a*x-1)/(a*x+1)+3/256*arctanh(a*x)^2*(a*x-1)^2/(a*x+1)^2
-3/256*arctanh(a*x)^2*(a*x+1)^2/(a*x-1)^2+9/32*(a*x+1)*arctanh(a*x)^2/(a*x-
1)-1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+
1)^2/(-a^2*x^2+1)-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-1/4*I*Pi*arctanh(a*x)^3*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a
^2*x^2+1)))^2+1/4*I*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+1/4*I*Pi*arctanh(a
*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-ax+1\right)^4+2\left(2\,a^2x^2+2\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)-3\right)\log\left(-ax+1\right)^3}{64\left(a^4x^4-2\,a^2x^2+1\right)}-\frac{1}{8}\int\frac{4\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^4
 - 2*a^2*x^2 + 1)*log(a*x + 1) - 3)*log(-a*x + 1)^3)/(a^4*x^4 - 2*a^2*x^2 +
 1) - 1/8*integrate(1/4*(4*log(a*x + 1)^3 - 12*log(a*x + 1)^2*log(-a*x + 1)
 + 3*(2*a^4*x^4 + 2*a^3*x^3 - 3*a^2*x^2 - 3*a*x + 2*(a^6*x^6 + a^5*x^5 - 2*
a^4*x^4 - 2*a^3*x^3 + a^2*x^2 + a*x + 2)*log(a*x + 1))*log(-a*x + 1)^2)/(a^
6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\operatorname{atanh}(a\,x)^3}{x\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^3(ax)}{a^6x^7-3a^4x^5+3a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

3.319 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=281

$$-\frac{93a}{128\left(1-a^2x^2\right)}-\frac{3a}{128\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^3}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{21a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}+\frac{93a^2}{64}$$

```
[Out] -3/128*a/(-a^2*x^2+1)^2-93/128*a/(-a^2*x^2+1)+3/32*a^2*x*arctanh(a*x)/(-a^2
*x^2+1)^2+93/64*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+93/128*a*arctanh(a*x)^2-3/1
6*a*arctanh(a*x)^2/(-a^2*x^2+1)^2-21/16*a*arctanh(a*x)^2/(-a^2*x^2+1)+a*arc
tanh(a*x)^3-arctanh(a*x)^3/x+1/4*a^2*x*arctanh(a*x)^3/(-a^2*x^2+1)^2+7/8*a^
2*x*arctanh(a*x)^3/(-a^2*x^2+1)+15/32*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*l
n(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+
2/(a*x+1))
```

Rubi [A] time = 0.69, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6030, 5982, 5916, 5988, 5932, 5948, 6056, 6610, 5956, 5994, 261, 5964, 5960}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{93a}{128\left(1-a^2x^2\right)}-\frac{3a}{128\left(1-a^2x^2\right)^2}+\frac{7a^2x\tan}{8\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] (-3*a)/(128*(1 - a^2*x^2)^2) - (93*a)/(128*(1 - a^2*x^2)) + (3*a^2*x*ArcTan
h[a*x])/(32*(1 - a^2*x^2)^2) + (93*a^2*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) +
 (93*a*ArcTanh[a*x]^2)/128 - (3*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (2
1*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/
x + (a^2*x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^3)/(
8*(1 - a^2*x^2)) + (15*a*ArcTanh[a*x]^4)/32 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/
(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3
, -1 + 2/(1 + a*x)])/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
```

```
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{1}{8}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx + \frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{\cdots}}$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}\cdots}{8\left(1-a^2x\cdots\right.}$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{9a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{9}{128}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}\cdots}{16\left(1-a^{\cdots}\right.}$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{9a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\tanh^{\cdots}$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{93a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\tanh^{\cdots}$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{93a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\tanh^{\cdots}$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{93a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\tanh^{\cdots}$$

Mathematica [C] time = 0.70, size = 218, normalized size = 0.78

$$-a\left(-\frac{ax\tanh^{-1}(ax)^3}{1-a^2x^2} - 3\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{2\tanh^{-1}(ax)}\right) + \frac{3}{2}\mathrm{Li}_3\left(e^{2\tanh^{-1}(ax)}\right) - \frac{15}{32}\tanh^{-1}(ax)^4 + \frac{\tanh^{-1}(ax)^3}{ax} + \mathrm{ta}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*((-1/8*I)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - (a*x*ArcTanh[a
*x]^3)/(1 - a^2*x^2) - (15*ArcTanh[a*x]^4)/32 + (3*Cosh[2*ArcTanh[a*x]])/8
+ (3*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]])/4 + (3*Cosh[4*ArcTanh[a*x]])/1024
 + (3*ArcTanh[a*x]^2*Cosh[4*ArcTanh[a*x]])/128 - 3*ArcTanh[a*x]^2*Log[1 - E
```

```
^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + (3*Pol
yLog[3, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/4 -
(3*ArcTanh[a*x]*Sinh[4*ArcTanh[a*x]])/256 - (ArcTanh[a*x]^3*Sinh[4*ArcTanh[
a*x]])/32))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^3}{a^6x^8 - 3a^4x^6 + 3a^2x^4 - x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(ax)^3}{\left(a^2x^2 - 1\right)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x^2), x)
```

maple [B] time = 3.04, size = 842, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x)
```

```
[Out] -3/8*a*arctanh(a*x)/(a*x-1)-3/8*a*arctanh(a*x)/(a*x+1)-6*a*polylog(3,-(a*x+
1)/(-a^2*x^2+1)^(1/2))-3/2048*a/(a*x-1)^2-arctanh(a*x)^3/x+6*a*arctanh(a*x)
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/16*a^2*x/(a
*x-1)+3/16/(a*x+1)*a^2*x+3/8*a*arctanh(a*x)^2/(a*x-1)-3/8*a*arctanh(a*x)^2/
(a*x+1)-a*arctanh(a*x)^3+15/32*a*arctanh(a*x)^4+1/32/(a*x-1)^2*arctanh(a*x)
^3*x*a^2-3/128/(a*x-1)^2*arctanh(a*x)^2*x*a^2-1/64/(a*x+1)^2*arctanh(a*x)^3
*x^2*a^3-3/256/(a*x+1)^2*arctanh(a*x)^2*x^2*a^3+1/32/(a*x+1)^2*arctanh(a*x)
^3*x*a^2+3/128/(a*x+1)^2*arctanh(a*x)^2*x*a^2+1/64/(a*x-1)^2*arctanh(a*x)^3
*x^2*a^3-3/256/(a*x-1)^2*arctanh(a*x)^2*x^2*a^3-3/512*arctanh(a*x)/(a*x+1)^
2*a^3*x^2+3/256*arctanh(a*x)/(a*x+1)^2*a^2*x+3/512*arctanh(a*x)/(a*x-1)^2*a
^3*x^2+3/256*arctanh(a*x)/(a*x-1)^2*a^2*x+3/16*a/(a*x-1)-3/16*a/(a*x+1)+1/6
4*a/(a*x-1)^2*arctanh(a*x)^3-1/64*a/(a*x+1)^2*arctanh(a*x)^3-3/256*a*arctan
h(a*x)^2/(a*x+1)^2-3/8*arctanh(a*x)/(a*x-1)*a^2*x+3/8*arctanh(a*x)/(a*x+1)*
a^2*x+3/512*a*arctanh(a*x)/(a*x-1)^2-3/512*a*arctanh(a*x)/(a*x+1)^2-3/2048/
(a*x-1)^2*a^3*x^2-3/1024/(a*x-1)^2*a^2*x-3/2048/(a*x+1)^2*a^3*x^2+3/1024/(a
*x+1)^2*a^2*x-3/256*a*arctanh(a*x)^2/(a*x-1)^2-3/2048*a/(a*x+1)^2-1/4/(a*x-
1)*arctanh(a*x)^3*x*a^2+3/8/(a*x-1)*arctanh(a*x)^2*x*a^2+1/4/(a*x+1)*arctan
h(a*x)^3*x*a^2+3/8/(a*x+1)*arctanh(a*x)^2*x*a^2+3*a*arctanh(a*x)^2*ln(1+(a*
x+1)/(-a^2*x^2+1)^(1/2))+3*a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2)
)-1/4*a/(a*x-1)*arctanh(a*x)^3-1/4*a/(a*x+1)*arctanh(a*x)^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^{3}{\left(ax \right)}}{a^{6}x^{8} - 3a^{4}x^{6} + 3a^{2}x^{4} - x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)
```

3.320 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)\right.}}{16a}$$

```
[Out] 1/4*arctanh(a*x)^(3/2)/a+1/32*erf(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)/a-1/32*erfi(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+1/256*erf(2*
arctanh(a*x)^(1/2))*Pi^(1/2)/a-1/256*erfi(2*arctanh(a*x)^(1/2))*Pi^(1/2)/a+
1/4*sinh(2*arctanh(a*x))*arctanh(a*x)^(1/2)/a+1/32*sinh(4*arctanh(a*x))*arc
tanh(a*x)^(1/2)/a
```

Rubi [A] time = 0.20, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5968, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(a\right.}}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```

```
[Out] ArcTanh[a*x]^(3/2)/(4*a) + (Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(256*a) + (
Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi]*Erfi[2*Sqrt[
ArcTanh[a*x]]])/(256*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16
*a) + (Sqrt[ArcTanh[a*x]]*Sinh[2*ArcTanh[a*x]])/(4*a) + (Sqrt[ArcTanh[a*x]]
*Sinh[4*ArcTanh[a*x]])/(32*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \sqrt{x}\cosh^4(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8}+\frac{1}{2}\sqrt{x}\cosh(2x)+\frac{1}{8}\sqrt{x}\cosh(4x)\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\text{Subst}\left(\int \sqrt{x}\cosh(4x)\,dx, x, \tanh^{-1}(ax)\right)}{8a} + \frac{\text{Subst}\left(\int \sqrt{x}\cosh(2x)\,dx, x\right.}{2a}$$

$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{32a}$$

$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{32a}$$

$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{32a}$$

$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a} - \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\text{ta}}\right.}{256a}$$

Mathematica [A] time = 0.50, size = 152, normalized size = 0.90

$$\frac{\frac{32\sqrt{\tanh^{-1}(ax)}\left(-3a^3x^3+2\left(a^2x^2-1\right)^2\tanh^{-1}(ax)+5ax\right)}{\left(a^2x^2-1\right)^2}+\frac{\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\tanh^{-1}(ax)\right)}{\sqrt{-\tanh^{-1}(ax)}}+\frac{8\sqrt{2}\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\tanh^{-1}(ax)\right)}{\sqrt{-\tanh^{-1}(ax)}}-8\sqrt{2}\,\Gamma\left(\right.}{256a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```

```
[Out] ((32*Sqrt[ArcTanh[a*x]]*(5*a*x - 3*a^3*x^3 + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x
]))/(-1 + a^2*x^2)^2 + (Sqrt[ArcTanh[a*x]]*Gamma[1/2, -4*ArcTanh[a*x]])/Sqr
t[-ArcTanh[a*x]] + (8*Sqrt[2]*Sqrt[ArcTanh[a*x]]*Gamma[1/2, -2*ArcTanh[a*x]
])/Sqrt[-ArcTanh[a*x]] - 8*Sqrt[2]*Gamma[1/2, 2*ArcTanh[a*x]] - Gamma[1/2,
4*ArcTanh[a*x]])/(256*a)
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{\left(-a^2x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{\operatorname{atanh}(a\,x)}}{\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1)^3,x)
```

```
[Out] int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1)^3, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\operatorname{atanh}(ax)}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(sqrt(atanh(a*x))/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

3.321 $\int \frac{x^6}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^6}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Defer[Int][x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^6}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx = \int \frac{x^6}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 7.86, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^6}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-x^6/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^6}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-a^2x^2+1\right)^3 \operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^6}{\operatorname{atanh}\left(a\,x\right)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^6/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{a^6x^6 \operatorname{atanh}\left(ax\right) - 3a^4x^4 \operatorname{atanh}\left(ax\right) + 3a^2x^2 \operatorname{atanh}\left(ax\right) - \operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**6/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

3.322 $\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Defer[Int][x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx = \int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 15.25, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^5}{(a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1)\,\text{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^5}{(a^2x^2 - 1)^3 \,\text{artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-a^2x^2+1\right)^3 \operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^5}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^5}{\operatorname{atanh}\left(a\,x\right)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^5}{a^6x^6 \operatorname{atanh}\left(ax\right) - 3a^4x^4 \operatorname{atanh}\left(ax\right) + 3a^2x^2 \operatorname{atanh}\left(ax\right) - \operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**5/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

3.323 $\int \frac{x^4}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

```
[Out] -1/2*Chi(2*arctanh(a*x))/a^5+1/8*Chi(4*arctanh(a*x))/a^5+3/8*ln(arctanh(a*x
))/a^5
```

Rubi [A] time = 0.12, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 3312, 3301}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(2*a^5) + CoshIntegral[4*ArcTanh[a*x]]/(8*a^5
) + (3*Log[ArcTanh[a*x]])/(8*a^5)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}-\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^5}\\
&= \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^5}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, \right.}{2a^5}\\
&= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 31, normalized size = 0.76

$$\frac{-4\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)+\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)+3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] (-4*CoshIntegral[2*ArcTanh[a*x]] + CoshIntegral[4*ArcTanh[a*x]] + 3*Log[Arc
Tanh[a*x]])/(8*a^5)
```

fricas [B] time = 0.59, size = 118, normalized size = 2.88

$$\frac{6\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\mathrm{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)+\mathrm{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)-4\ \mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-4\ \mathrm{l}}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a
*x + 1)) - 4*log_integral(-(a*x + 1)/(a*x - 1)) - 4*log_integral(-(a*x - 1)
/(a*x + 1)))/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

maple [A] time = 0.26, size = 36, normalized size = 0.88

$$-\frac{\mathrm{X}(2\operatorname{arctanh}(ax))}{2a^5}+\frac{\mathrm{X}(4\operatorname{arctanh}(ax))}{8a^5}+\frac{3\ln(\operatorname{arctanh}(ax))}{8a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] -1/2*Chi(2*arctanh(a*x))/a^5+1/8*Chi(4*arctanh(a*x))/a^5+3/8*ln(arctanh(a*x
))/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^4}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{a^6x^6\operatorname{atanh}(ax)-3a^4x^4\operatorname{atanh}(ax)+3a^2x^2\operatorname{atanh}(ax)-\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

3.324 $\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4} - \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^4}$$

```
[Out] -1/4*Shi(2*arctanh(a*x))/a^4+1/8*Shi(4*arctanh(a*x))/a^4
```

Rubi [A] time = 0.12, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3298}

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4} - \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -SinhIntegral[2*ArcTanh[a*x]]/(4*a^4) + SinhIntegral[4*ArcTanh[a*x]]/(8*a^4
)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x} dx, x, \tanh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x} dx, x, \tanh^{-1}(ax)\right)}{4a^4} \\
&= -\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^4} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 24, normalized size = 0.83

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)-2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] (-2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^4)
```

fricas [B] time = 0.50, size = 102, normalized size = 3.52

$$\frac{\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)-2\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+2\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
[Out] 1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_integ
ral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*log_integral(-(a*x + 1
)/(a*x - 1)) + 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

maple [A] time = 0.18, size = 24, normalized size = 0.83

$$\frac{\frac{\text{Shi}(4\,\text{arctanh}(ax))}{8}-\frac{\text{Shi}(2\,\text{arctanh}(ax))}{4}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x)
[Out] 1/a^4*(1/8*Shi(4*arctanh(a*x))-1/4*Shi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate(x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int\frac{x^3}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

3.325 $\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

```
[Out] 1/8*Chi(4*arctanh(a*x))/a^3-1/8*ln(arctanh(a*x))/a^3
```

Rubi [A] time = 0.11, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6034, 5448, 3301}

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[4*ArcTanh[a*x]]/(8*a^3) - Log[ArcTanh[a*x]]/(8*a^3)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{8x} + \frac{\cosh(4x)}{8x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^3} \\
&= -\frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{8a^3} \\
&= \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 22, normalized size = 0.81

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)-\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] (CoshIntegral[4*ArcTanh[a*x]] - Log[ArcTanh[a*x]])/(8*a^3)
```

fricas [B] time = 0.61, size = 88, normalized size = 3.26

$$-\frac{2\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)-\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/16*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral((a^2*x^2 + 2*a*x + 1
)/(a^2*x^2 - 2*a*x + 1)) - log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*
a*x + 1)))/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(a^2x^2-1\right)^3\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

maple [A] time = 0.19, size = 24, normalized size = 0.89

$$\frac{\text{X}\,(4\,\text{arctanh}\,(ax))}{8a^3}-\frac{\ln\,(\text{arctanh}\,(ax))}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] 1/8*Chi(4*arctanh(a*x))/a^3-1/8*ln(arctanh(a*x))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(a^2x^2-1\right)^3\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^2}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

3.326 $\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

```
[Out] 1/4*Shi(2*arctanh(a*x))/a^2+1/8*Shi(4*arctanh(a*x))/a^2
```

Rubi [A] time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6034, 5448, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(4*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8*a^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx &= \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{4a^2}\\
&= \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 24, normalized size = 0.83

$$\frac{2\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)+\mathrm{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] (2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^2)
```

fricas [B] time = 0.62, size = 102, normalized size = 3.52

$$\frac{\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+2\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-2\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_integ
ral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 2*log_integral(-(a*x + 1
)/(a*x - 1)) - 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

maple [A] time = 0.19, size = 24, normalized size = 0.83

$$\frac{\frac{\mathrm{Shi}(4\operatorname{arctanh}(ax))}{8}+\frac{\mathrm{Shi}(2\operatorname{arctanh}(ax))}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] 1/a^2*(1/8*Shi(4*arctanh(a*x))+1/4*Shi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^6x^6\,\mathrm{atanh}\,(ax) - 3a^4x^4\,\mathrm{atanh}\,(ax) + 3a^2x^2\,\mathrm{atanh}\,(ax) - \mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*at
anh(a*x) - atanh(a*x)), x)
```

3.327 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/8*Chi(4*arctanh(a*x))/a+3/8*ln(arctanh(a*x))/a
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5968, 3312, 3301}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + CoshIntegral[4*ArcTanh[a*x]]/(8*a) + (
3*Log[ArcTanh[a*x]])/(8*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx\right.}{2a}\\
&= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 33, normalized size = 0.80

$$-\frac{-4\operatorname{Chi}\left(2\tanh^{-1}(ax)\right)-\operatorname{Chi}\left(4\tanh^{-1}(ax)\right)-3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -1/8*(-4*CoshIntegral[2*ArcTanh[a*x]] - CoshIntegral[4*ArcTanh[a*x]] - 3*Lo
g[ArcTanh[a*x]])/a
```

fricas [B] time = 0.51, size = 118, normalized size = 2.88

$$\frac{6\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\mathrm{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)+\mathrm{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+4\ \mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+4\ \mathrm{l}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a
*x + 1)) + 4*log_integral(-(a*x + 1)/(a*x - 1)) + 4*log_integral(-(a*x - 1)
/(a*x + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

maple [A] time = 0.18, size = 36, normalized size = 0.88

$$\frac{\mathrm{X}\left(2\operatorname{arctanh}(ax)\right)}{2a}+\frac{\mathrm{X}\left(4\operatorname{arctanh}(ax)\right)}{8a}+\frac{3\ln\left(\operatorname{arctanh}(ax)\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/8*Chi(4*arctanh(a*x))/a+3/8*ln(arctanh(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{6}x^{6}\operatorname{atanh}\left(ax\right)-3a^{4}x^{4}\operatorname{atanh}\left(ax\right)+3a^{2}x^{2}\operatorname{atanh}\left(ax\right)-\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*at
anh(a*x) - atanh(a*x)), x)
```

3.328 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=49

$$-\text{Int}\left(\frac{1}{x\left(a^2x^2-1\right)\tanh^{-1}(ax)},x\right)+\frac{3}{4}\text{Shi}\left(2\tanh^{-1}(ax)\right)+\frac{1}{8}\text{Shi}\left(4\tanh^{-1}(ax)\right)$$

```
[Out] 3/4*Shi(2*arctanh(a*x))+1/8*Shi(4*arctanh(a*x))-Unintegrable(1/x/(a^2*x^2-1
)/arctanh(a*x),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx = \int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^6x^7-3\,a^4x^5+3\,a^2x^3-x\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)^3 x\,\text{artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2}x^{2}+1\right)^{3}\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^{2}x^{2}-1\right)^{3}x\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\,\left(a^{2}\,x^{2}-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{6}x^{7}\operatorname{atanh}\left(ax\right)-3a^{4}x^{5}\operatorname{atanh}\left(ax\right)+3a^{2}x^{3}\operatorname{atanh}\left(ax\right)-x\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x) - 3*a**4*x**5*atanh(a*x) + 3*a**2*x**3*at
anh(a*x) - x*atanh(a*x)), x)
```

3.329 $\int \frac{x^5}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=102

$$\frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a^5}-\frac{3\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^6}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^6}-\frac{x}{a^5\tanh^{-1}(ax)}+\frac{2x}{a^5\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{}{a^5 (1}$$

```
[Out] -x/a^5/arctanh(a*x)-x/a^5/(-a^2*x^2+1)^2/arctanh(a*x)+2*x/a^5/(-a^2*x^2+1)/
arctanh(a*x)-3/2*Chi(2*arctanh(a*x))/a^6+1/2*Chi(4*arctanh(a*x))/a^6+Uninte
grable(1/arctanh(a*x),x)/a^5
```

Rubi [A] time = 0.90, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a^5*ArcTanh[a*x])) - x/(a^5*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (2*x)/(a^5
*(1 - a^2*x^2)*ArcTanh[a*x]) - (3*CoshIntegral[2*ArcTanh[a*x]])/(2*a^6) + C
oshIntegral[4*ArcTanh[a*x]]/(2*a^6) + Defer[Int][ArcTanh[a*x]^(-1), x]/a^5
```

Rubi steps

$$\int \frac{x^5}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx = \frac{\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx}{a^2}-\frac{\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$=\frac{\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx}{a^4}-2\frac{\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx}{a^4}+\frac{\int \frac{x}{\left(1-a^2x^2\right) \tanh^{-1}(ax)^2}\,dx}{a^4}$$

$$=-\frac{x}{a^5\tanh^{-1}(ax)}-\frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{\int \frac{1}{\tanh^{-1}(ax)}\,dx}{a^5}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^3 \text{ta}}}{a^5}$$

$$=-\frac{x}{a^5\tanh^{-1}(ax)}-\frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx,x,\tanh^{-}\right.}{a^6}$$

$$=-\frac{x}{a^5\tanh^{-1}(ax)}-\frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-2\left(-\frac{x}{a^5\left(1-a^2x^2\right)\tanh^{-1}(ax)}\right.$$

$$=-\frac{x}{a^5\tanh^{-1}(ax)}-\frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx,x,\tanh^{-}\right.}{8a^6}$$

$$=-\frac{x}{a^5\tanh^{-1}(ax)}-\frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^6}-2\left(-\frac{}{a^5}\right.$$

Mathematica [A] time = 12.13, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
[Out] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

fricas [A] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^5}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\operatorname{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^5}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-a^2x^2+1\right)^3\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
[Out] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^5}{\left(a^5x^4-2\,a^3x^2+a\right)\log(ax+1)-\left(a^5x^4-2\,a^3x^2+a\right)\log(-ax+1)}-\int -\frac{2(}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log(ax+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
[Out] -2*x^5/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) - integrate(-2*(a^2*x^6 - 5*x^4)/((a^7*x^6 - 3*a^5*x^4 + 3*a
^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^5}{\operatorname{atanh}(a\,x)^2\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^5/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^5}{a^6x^6\operatorname{atanh}^2(ax) - 3a^4x^4\operatorname{atanh}^2(ax) + 3a^2x^2\operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**5/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

3.330 $\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=53

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}-\frac{x^4}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -x^4/a/(-a^2*x^2+1)^2/arctanh(a*x)-Shi(2*arctanh(a*x))/a^5+1/2*Shi(4*arctan
h(a*x))/a^5
```

Rubi [A] time = 0.19, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6006, 6034, 5448, 3298}

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}-\frac{x^4}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x^4/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) - SinhIntegral[2*ArcTanh[a*x]]/a^5
+ SinhIntegral[4*ArcTanh[a*x]]/(2*a^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx = -\frac{x^4}{a(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{4\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)}\,dx}{a}$$

$$= -\frac{x^4}{a(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{x^4}{a(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right) dx, x, \tanh^{-1}(a$$

$$= -\frac{x^4}{a(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^5} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x^4}{a(1-a^2x^2)^2 \tanh^{-1}(ax)} - \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Mathematica [A] time = 0.20, size = 49, normalized size = 0.92

$$\frac{-\frac{2a^4x^4}{(a^2x^2-1)^2 \tanh^{-1}(ax)} - 2\text{Shi}\left(2\tanh^{-1}(ax)\right) + \text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] ((-2*a^4*x^4)/((-1 + a^2*x^2)^2*ArcTanh[a*x]) - 2*SinhIntegral[2*ArcTanh[a*
x]] + SinhIntegral[4*ArcTanh[a*x]])/(2*a^5)
```

fricas [B] time = 0.69, size = 232, normalized size = 4.38

$$-\frac{8\,a^4x^4 - \left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - \left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - 2}{4\left(a^9x^4 - 2\,a^7x^2 + a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^4*x^4 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_in
tegral(-(a*x + 1)/(a*x - 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^9*x^4 - 2*a^7*x^2 + a^5)
*log(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

maple [A] time = 0.25, size = 62, normalized size = 1.17

$$\frac{-\frac{3}{8\operatorname{arctanh}(ax)}+\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}-\operatorname{Shi}\left(2\operatorname{arctanh}\left(ax\right)\right)-\frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^5*(-3/8/arctanh(a*x)+1/2/arctanh(a*x)*cosh(2*arctanh(a*x))-Shi(2*arctan
h(a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^4}{\left(a^5x^4-2\,a^3x^2+a\right)\log\left(ax+1\right)-\left(a^5x^4-2\,a^3x^2+a\right)\log\left(-ax+1\right)}+8\int-\frac{}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^4/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + 8*integrate(-x^3/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*lo
g(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^4}{\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^4/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{a^6x^6\operatorname{atanh}^2\left(ax\right)-3a^4x^4\operatorname{atanh}^2\left(ax\right)+3a^2x^2\operatorname{atanh}^2\left(ax\right)-\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

3.331 $\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=55

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4}-\frac{x^3}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -x^3/a/(-a^2*x^2+1)^2/arctanh(a*x)-1/2*Chi(2*arctanh(a*x))/a^4+1/2*Chi(4*ar
ctanh(a*x))/a^4
```

Rubi [A] time = 0.52, antiderivative size = 76, normalized size of antiderivative = 1.38, number of steps used = 20, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6028, 6032, 6034, 3312, 3301, 5968, 5448}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4}+\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x])) + x/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]
) - CoshIntegral[2*ArcTanh[a*x]]/(2*a^4) + CoshIntegral[4*ArcTanh[a*x]]/(2*
a^4)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
```

```
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx = \frac{\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx}{a^2} - \frac{\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx}{a^3}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x\right.}{a^4}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right.\right.}{a}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x\right.}{8a^4}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4} +$$

Mathematica [A] time = 0.12, size = 80, normalized size = 1.45

$$\frac{-2a^3x^3 - \left(a^2x^2-1\right)^2\tanh^{-1}(ax)\text{Chi}\left(2\tanh^{-1}(ax)\right) + \left(a^2x^2-1\right)^2\tanh^{-1}(ax)\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4\left(a^2x^2-1\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a^3*x^3 - (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] +
(-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^4*(-1 + a^
2*x^2)^2*ArcTanh[a*x])
```

fricas [B] time = 0.71, size = 231, normalized size = 4.20

$$-\frac{8\,a^3x^3 - \left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + \left(a^4x^4 - 2\,a^2x^2 + 1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - \left(a \right.\right.}{4\left(a^8x^4 - 2\,a^6x^2 + a^4\right)\mathrm{l}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*x^3 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_inte
gral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x -
 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4)*log
(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

maple [A] time = 0.18, size = 54, normalized size = 0.98

$$\frac{-\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} + \frac{\mathrm{X}(4\operatorname{arctanh}(ax))}{2} + \frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)} - \frac{\mathrm{X}(2\operatorname{arctanh}(ax))}{2}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^4*(-1/8/arctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))+1/4*s
inh(2*arctanh(a*x))/arctanh(a*x)-1/2*Chi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^3}{\left(a^5x^4 - 2\,a^3x^2 + a\right)\log\left(ax+1\right) - \left(a^5x^4 - 2\,a^3x^2 + a\right)\log\left(-ax+1\right)} + \int -\frac{}{\left(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a\right)\log\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + integrate(-2*(a^2*x^4 + 3*x^2)/((a^7*x^6 - 3*a^5*x^4 + 3*a
^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^3}{\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{a^6x^6\operatorname{atanh}^2(ax) - 3a^4x^4\operatorname{atanh}^2(ax) + 3a^2x^2\operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

3.332 $\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=41

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^3}-\frac{x^2}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -x^2/a/(-a^2*x^2+1)^2/arctanh(a*x)+1/2*Shi(4*arctanh(a*x))/a^3
```

Rubi [A] time = 0.29, antiderivative size = 60, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6028, 5966, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^3}+\frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x])) + 1/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]
) + SinhIntegral[4*ArcTanh[a*x]]/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx = \frac{\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx}{a^2} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{2\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,d}{a}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(}{x}\right.}{a^3}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx,\right.}{a^3}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx,x\right.}{2a^3}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^3}$$

Mathematica [A] time = 0.21, size = 56, normalized size = 1.37

$$\frac{\left(a^2x^2-1\right)^2\tanh^{-1}(ax)\text{Shi}\left(4\tanh^{-1}(ax)\right)-2a^2x^2}{2a^3\left(a^2x^2-1\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a^2*x^2 + (-1 + a^2*x^2)^2*ArcTanh[a*x]*SinhIntegral[4*ArcTanh[a*x]])/(
2*a^3*(-1 + a^2*x^2)^2*ArcTanh[a*x])
```

fricas [B] time = 0.62, size = 164, normalized size = 4.00

$$-\frac{8\,a^2x^2 - \left(\left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - \left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)\right)\log\left(-\right.}{4\left(a^7x^4-2\,a^5x^2+a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
```

```
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^7*x^
4 - 2*a^5*x^2 + a^3)*log(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

maple [A] time = 0.16, size = 38, normalized size = 0.93

$$\frac{\frac{1}{8\operatorname{arctanh}(ax)}-\frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/8/arctanh(a*x)-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arc
tanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^2}{\left(a^5x^4-2\,a^3x^2+a\right)\log(ax+1)-\left(a^5x^4-2\,a^3x^2+a\right)\log(-ax+1)}+\int -\frac{}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + integrate(-4*(a^2*x^3 + x)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x
^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2}{\operatorname{atanh}(a\,x)^2\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{a^6x^6\operatorname{atanh}^2(ax)-3a^4x^4\operatorname{atanh}^2(ax)+3a^2x^2\operatorname{atanh}^2(ax)-\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

3.333 $\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=53

$$\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -x/a/(-a^2*x^2+1)^2/arctanh(a*x)+1/2*Chi(2*arctanh(a*x))/a^2+1/2*Chi(4*arct
anh(a*x))/a^2
```

Rubi [A] time = 0.25, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6032, 6034, 5448, 3301, 5968, 3312}

$$\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/(2*a^2
) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
```

```
&& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx = -\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx}{a} + (3a)\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tan$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{3\,\text{Subst}\,(}{}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \tanh^{-1}\right.}{a^2}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^2} + \frac{3\,\text{Subst}\,(}{}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^2} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 1.42

$$\frac{\left(a^2x^2-1\right)^2\tanh^{-1}(ax)\text{Chi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2-1\right)^2\tanh^{-1}(ax)\text{Chi}\left(4\tanh^{-1}(ax)\right)-2ax}{2a^2\left(a^2x^2-1\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a*x + (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] + (-1
+ a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^
2)^2*ArcTanh[a*x])
```

fricas [B] time = 0.54, size = 225, normalized size = 4.25

$$-\frac{8\,ax-\left(\left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\left(a^4\right.\right.}{4\left(a^6x^4-2\,a^4x^2+a^2\right)\text{lo}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a*x - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 -
```

```
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral
(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/
(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(-(a
*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

maple [A] time = 0.19, size = 54, normalized size = 1.02

$$\frac{-\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{\mathrm{X}(4\operatorname{arctanh}(ax))}{2}-\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\mathrm{X}(2\operatorname{arctanh}(ax))}{2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^2*(-1/8/arctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-1/4*s
inh(2*arctanh(a*x))/arctanh(a*x)+1/2*Chi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x}{\left(a^5x^4-2\,a^3x^2+a\right)\log\left(ax+1\right)-\left(a^5x^4-2\,a^3x^2+a\right)\log\left(-ax+1\right)}+\int -\frac{2}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(ax+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*lo
g(-a*x + 1)) + integrate(-2*(3*a^2*x^2 + 1)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x
^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^6x^6\operatorname{atanh}^2\left(ax\right)-3a^4x^4\operatorname{atanh}^2\left(ax\right)+3a^2x^2\operatorname{atanh}^2\left(ax\right)-\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*x
**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

3.334 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] -1/a/(-a^2*x^2+1)^2/arctanh(a*x)+Shi(2*arctanh(a*x))/a+1/2*Shi(4*arctanh(a*
x))/a
```

Rubi [A] time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a + Si
nhIntegral[4*ArcTanh[a*x]]/(2*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx = -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + (4a)\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{\sin\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

Mathematica [A] time = 0.15, size = 43, normalized size = 0.88

$$\frac{-\frac{2}{(a^2x^2-1)^2\tanh^{-1}(ax)} + 2\text{Shi}\left(2\tanh^{-1}(ax)\right) + \text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2/((-1 + a^2*x^2)^2*ArcTanh[a*x]) + 2*SinhIntegral[2*ArcTanh[a*x]] + Sinh
Integral[4*ArcTanh[a*x]])/(2*a)
```

fricas [B] time = 0.55, size = 222, normalized size = 4.53

$$\frac{\left((a^4x^4 - 2a^2x^2 + 1)\,\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right) - (a^4x^4 - 2a^2x^2 + 1)\,\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right) + 2\,(a^4x^4 - 2a^2x\cdots\right.}{4\,(a^5x^4 - 2a^3x^2 + a)\log\left(-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2
 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x +
1)/(a^2*x^2 + 2*a*x + 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1)) - 8)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x
+ 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2x^2-1)^3\,\text{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

maple [A] time = 0.23, size = 60, normalized size = 1.22

$$\frac{-\frac{3}{8\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}\left(2\,\mathrm{arctanh}\left(ax\right)\right)-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(-3/8/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh(
a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$8\,a\int-\frac{x}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax+1\right)-\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-ax+1\right)}\,dx-\frac{}{\left(a^5x^4-2\,a^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 8*a*integrate(-x/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x) - 2/((a^5*x^4 - 2*a^3*
x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int\frac{1}{\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{a^6x^6\,\mathrm{atanh}^2\left(ax\right)-3a^4x^4\,\mathrm{atanh}^2\left(ax\right)+3a^2x^2\,\mathrm{atanh}^2\left(ax\right)-\mathrm{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*x
**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

3.335 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=99

$$-\frac{\text{Int}\left(\frac{1}{x^2 \tanh^{-1}(ax)}, x\right)}{a}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2 \tanh^{-1}(ax)}+\frac{3}{2}\text{Chi}\left(2\tanh^{-1}(ax)\right)+\frac{1}{2}\text{Chi}\left(4\tanh^{-1}(ax\right.$$

```
[Out] -1/a/x/arctanh(a*x)-a*x/(-a^2*x^2+1)^2/arctanh(a*x)-a*x/(-a^2*x^2+1)/arctan
h(a*x)+3/2*Chi(2*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-Unintegrable(1/x^2/a
rctanh(a*x),x)/a
```

Rubi [A] time = 0.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)^2*ArcTanh[a*x]) - (a*x)/((1
- a^2*x^2)*ArcTanh[a*x]) + (3*CoshIntegral[2*ArcTanh[a*x]])/2 + CoshIntegra
l[4*ArcTanh[a*x]]/2 - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx &= a^2 \int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx + \int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^2} \, dx \\
&= -\frac{ax}{(1-a^2x^2)^2 \tanh^{-1}(ax)} + a \int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx + a^2 \int \frac{}{(1-a^2x^2)^2} \\
&= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)^2 \tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)} + 3\,\text{Subst}\Big(\\
&= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)^2 \tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)} + 3\,\text{Subst} \\
&= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)^2 \tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)} + \frac{1}{8}\,\text{Subst} \\
&= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)^2 \tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)} + \frac{1}{2}\text{Chi}\big(2 \\
&= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)^2 \tanh^{-1}(ax)} - \frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)} + \frac{3}{2}\text{Chi}\big(2
\end{aligned}$$

Mathematica [A] time = 4.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^{6}x^{7}-3\,a^{4}x^{5}+3\,a^{2}x^{3}-x\right)\operatorname{artanh}\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^{2}x^{2}-1\right)^{3}x\operatorname{artanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^2), x)
```

maple [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2}x^{2}+1\right)^{3}\operatorname{arctanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{\left(a^{5}x^{5}-2\,a^{3}x^{3}+ax\right)\log\left(ax+1\right)-\left(a^{5}x^{5}-2\,a^{3}x^{3}+ax\right)\log\left(-ax+1\right)}+\int -\frac{[illegible]}{\left(a^{7}x^{8}-3\,a^{5}x^{6}+3\,a^{3}x^{4}-ax^{2}\right)\mathrm{lo}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/((a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) - (a^5*x^5 - 2*a^3*x^3 + a*x)*
log(-a*x + 1)) + integrate(-2*(5*a^2*x^2 - 1)/((a^7*x^8 - 3*a^5*x^6 + 3*a^3
*x^4 - a*x^2)*log(a*x + 1) - (a^7*x^8 - 3*a^5*x^6 + 3*a^3*x^4 - a*x^2)*log(
-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x\,\operatorname{atanh}\left(a\,x\right)^{2}\,\left(a^{2}\,x^{2}-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^7 \operatorname{atanh}^2(ax) - 3a^4x^5 \operatorname{atanh}^2(ax) + 3a^2x^3 \operatorname{atanh}^2(ax) - x \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x)**2 - 3*a**4*x**5*atanh(a*x)**2 + 3*a**2*x
**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

3.336 $\int \frac{x^4}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=100

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^5}-\frac{x^4}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{2x}{a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{}{a^4\left(1-a^2x\right.}$$

```
[Out] -1/2*x^4/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2*x/a^4/(-a^2*x^2+1)^2/arctanh(a*x
)+2*x/a^4/(-a^2*x^2+1)/arctanh(a*x)-Chi(2*arctanh(a*x))/a^5+Chi(4*arctanh(a
*x))/a^5
```

Rubi [A] time = 0.59, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6006, 6028, 6032, 6034, 3312, 3301, 5968, 5448}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^5}-\frac{x^4}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{2x}{a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{}{a^4\left(1-a^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -x^4/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/(a^4*(1 - a^2*x^2)^2*ArcT
anh[a*x]) + (2*x)/(a^4*(1 - a^2*x^2)*ArcTanh[a*x]) - CoshIntegral[2*ArcTanh
[a*x]]/a^5 + CoshIntegral[4*ArcTanh[a*x]]/a^5
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
```

```
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^3\tanh^{-1}(ax)^3}\,dx = -\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2} + \frac{2\int \frac{x^3}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}dx}{a}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2} + \frac{2\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}dx}{a^3} - \frac{2\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}\cdots}}{a^3}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)\tan\cdots}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)\tan\cdots}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)\tan\cdots}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)\tan\cdots}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)\tan\cdots}$$

Mathematica [A] time = 0.22, size = 60, normalized size = 0.60

$$-\frac{\frac{a^3x^3(ax+4\tanh^{-1}(ax))}{(a^2x^2-1)^2\tanh^{-1}(ax)^2} + 2\mathrm{Chi}\left(2\tanh^{-1}(ax)\right) - 2\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*((a^3*x^3*(a*x + 4*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTanh[a*x]^2) +
2*CoshIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]])/a^5
```

fricas [B] time = 0.67, size = 256, normalized size = 2.56

$$-\frac{4a^4x^4 + 8a^3x^3\log\left(-\frac{ax+1}{ax-1}\right) - \left((a^4x^4 - 2a^2x^2 + 1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right) + (a^4x^4 - 2a^2x^2 + 1)\,\mathrm{log\_inte}\cdots\right.}{2(a^9x^4 - 2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4*x^4 + 8*a^3*x^3*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*x
^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^
4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)
) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4
 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x -
 1))^2)/((a^9*x^4 - 2*a^7*x^2 + a^5)*log(-(a*x + 1)/(a*x - 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

maple [A] time = 0.25, size = 90, normalized size = 0.90

$$\frac{-\frac{3}{16\operatorname{arctanh}(ax)^2}+\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)^2}+\frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}-\mathrm{X}\left(2\operatorname{arctanh}(ax)\right)-\frac{\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\sinh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\mathrm{X}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a^5*(-3/16/arctanh(a*x)^2+1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))+1/2*sin
h(2*arctanh(a*x))/arctanh(a*x)-Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*cosh
(4*arctanh(a*x))-1/4/arctanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(ax^4+2\,x^3\log(ax+1)-2\,x^3\log(-ax+1)\right)}{\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)^2-2\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)\log(-ax+1)+\left(a^6x^4-2\,a^4x^2+a^2\right)\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a*x^4 + 2*x^3*log(a*x + 1) - 2*x^3*log(-a*x + 1))/((a^6*x^4 - 2*a^4*x^2
 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)*log(-a*
x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + integrate(-4*(a^2*x
^4 + 3*x^2)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1) - (a^8*x^
6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^4}{\operatorname{atanh}(a\,x)^3\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^4/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{a^6x^6\operatorname{atanh}^3(ax)-3a^4x^4\operatorname{atanh}^3(ax)+3a^2x^2\operatorname{atanh}^3(ax)-\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

3.337 $\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=107

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^4}-\frac{3x^2}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x^4}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{}{2a\left(1-a^2x^2\right.}$$

```
[Out] -1/2*x^3/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-3/2*x^2/a^2/(-a^2*x^2+1)^2/arctanh
(a*x)-1/2*x^4/(-a^2*x^2+1)^2/arctanh(a*x)-1/2*Shi(2*arctanh(a*x))/a^4+Shi(4
*arctanh(a*x))/a^4
```

Rubi [A] time = 0.65, antiderivative size = 160, normalized size of antiderivative = 1.50, number of steps used = 25, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6028, 5996, 6034, 5448, 12, 3298, 6032, 5966}

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^4}+\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{}{2a^4\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + x/(2*a^3*(1 - a^2*x^2)*ArcTanh[
a*x]^2) - 2/(a^4*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 3/(2*a^4*(1 - a^2*x^2)*Arc
Tanh[a*x]) + (1 + a^2*x^2)/(2*a^4*(1 - a^2*x^2)*ArcTanh[a*x]) - SinhIntegra
l[2*ArcTanh[a*x]]/(2*a^4) + SinhIntegral[4*ArcTanh[a*x]]/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
```

```
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx = \frac{\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx}{a^2} - \frac{\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}\,dx}{a^2}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} + \frac{1+a}{2a^4\left(1-a^2x^2\right)}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{1}{2a^4\left(1-a^2x^2\right)}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2}$$

Mathematica [A] time = 0.26, size = 66, normalized size = 0.62

$$-\frac{\frac{a^2x^2\left(\left(a^2x^2+3\right)\tanh^{-1}(ax)+ax\right)}{\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2} + \mathrm{Shi}\left(2\tanh^{-1}(ax)\right) - 2\mathrm{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*((a^2*x^2*(a*x + (3 + a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTan
h[a*x]^2) + SinhIntegral[2*ArcTanh[a*x]] - 2*SinhIntegral[4*ArcTanh[a*x]])/
a^4
```

fricas [B] time = 0.60, size = 267, normalized size = 2.50

$$-\frac{8\,a^3x^3 - \left(2\left(a^4x^4 - 2\,a^2x^2 + 1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - 2\left(a^4x^4 - 2\,a^2x^2 + 1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)\right.}{4\left(a^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*x^3 - (2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*
x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a
```

```
^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_
integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*(a^4*x^4 + 3*a^2*x^2)*l
og(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4)*log(-(a*x + 1)/(a*x
- 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

maple [A] time = 0.18, size = 82, normalized size = 0.77

$$\frac{\frac{\sinh(2\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)^2}+\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}-\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2}-\frac{\sinh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\cosh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\operatorname{Shi}(4\operatorname{arctanh}(ax))}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a^4*(1/8*sinh(2*arctanh(a*x))/arctanh(a*x)^2+1/4/arctanh(a*x)*cosh(2*arct
anh(a*x))-1/2*Shi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*x))-
1/4/arctanh(a*x)*cosh(4*arctanh(a*x))+Shi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,ax^3+\left(a^2x^4+3\,x^2\right)\log(ax+1)-\left(a^2x^4+3\,x^2\right)\log(-ax+1)}{\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)^2-2\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)\log(-ax+1)+\left(a^6x^4-2\,a^4x^2+a^2\right)\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x^3 + (a^2*x^4 + 3*x^2)*log(a*x + 1) - (a^2*x^4 + 3*x^2)*log(-a*x + 1
))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a
^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^
2) + integrate(-2*(5*a^2*x^3 + 3*x)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2
)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3}{\operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{a^6x^6\operatorname{atanh}^3(ax)-3a^4x^4\operatorname{atanh}^3(ax)+3a^2x^2\operatorname{atanh}^3(ax)-\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

3.338 $\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=86

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{2a\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2}+\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2x}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -1/2*x^2/a/(a^2*x^2-1)^2/arctanh(a*x)^2-2*x/a^2/(-a^2*x^2+1)^2/arctanh(a*x)
+x/a^2/(-a^2*x^2+1)/arctanh(a*x)+Chi(4*arctanh(a*x))/a^3
```

Rubi [A] time = 0.59, antiderivative size = 109, normalized size of antiderivative = 1.27, number of steps used = 22, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6028, 5966, 6032, 6034, 3312, 3301, 5968, 5448}

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}+\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2x}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{1}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{}{2a^3\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 1/(2*a^3*(1 - a^2*x^2)*ArcTanh[
a*x]^2) - (2*x)/(a^2*(1 - a^2*x^2)^2*ArcTanh[a*x]) + x/(a^2*(1 - a^2*x^2)*A
rcTanh[a*x]) + CoshIntegral[4*ArcTanh[a*x]]/a^3
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
```

```
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx = \frac{\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx}{a^2} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}\,dx}{a^2}$$

$$= -\frac{1}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh}}{a}$$

$$= -\frac{1}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2x}{a^2\left(1-a^2x^2\right)^2}$$

$$= -\frac{1}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2x}{a^2\left(1-a^2x^2\right)^2}$$

$$= -\frac{1}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2x}{a^2\left(1-a^2x^2\right)^2}$$

$$= -\frac{1}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2x}{a^2\left(1-a^2x^2\right)^2}$$

$$= -\frac{1}{2a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3\left(1-a^2x^2\right) \tanh^{-1}(ax)^2} - \frac{2x}{a^2\left(1-a^2x^2\right)^2}$$

Mathematica [A] time = 0.17, size = 56, normalized size = 0.65

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}-\frac{x\left(2\left(a^2x^2+1\right)\tanh^{-1}(ax)+ax\right)}{2a^2\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*(x*(a*x + 2*(1 + a^2*x^2)*ArcTanh[a*x]))/(a^2*(-1 + a^2*x^2)^2*ArcTanh
[a*x]^2) + CoshIntegral[4*ArcTanh[a*x]]/a^3
```

fricas [B] time = 0.53, size = 193, normalized size = 2.24

$$-\frac{4\,a^2x^2-\left(\left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)\right)\log\left(-\right.}{2\left(a^7x^4-2\,a^5x^2+a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*(a^
3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/((a^7*x^4 - 2*a^5*x^2 + a^3)*log(-(
a*x + 1)/(a*x - 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

maple [A] time = 0.16, size = 51, normalized size = 0.59

$$\frac{\frac{1}{16\operatorname{arctanh}(ax)^2}-\frac{\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\sinh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\mathrm{X}(4\operatorname{arctanh}(ax))}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/16/arctanh(a*x)^2-1/16/arctanh(a*x)^2*cosh(4*arctanh(a*x))-1/4/arc
tanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(ax^2+\left(a^2x^3+x\right)\log(ax+1)-\left(a^2x^3+x\right)\log(-ax+1)\right)}{\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)^2-2\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)\log(-ax+1)+\left(a^6x^4-2\,a^4x^2+a^2\right)\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a*x^2 + (a^2*x^3 + x)*log(a*x + 1) - (a^2*x^3 + x)*log(-a*x + 1))/((a^6
*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*log(
a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + int
egrate(-2*(a^4*x^4 + 6*a^2*x^2 + 1)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2
)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\operatorname{atanh}(a\,x)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{a^6x^6\operatorname{atanh}^3(ax)-3a^4x^4\operatorname{atanh}^3(ax)+3a^2x^2\operatorname{atanh}^3(ax)-\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

3.339 $\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=100

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{3}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2}{a^2\left(1-a^2x^2\right)^2}$$

```
[Out] -1/2*x/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2/a^2/(-a^2*x^2+1)^2/arctanh(a*x)+3/
2/a^2/(-a^2*x^2+1)/arctanh(a*x)+1/2*Shi(2*arctanh(a*x))/a^2+Shi(4*arctanh(a
*x))/a^2
```

Rubi [A] time = 0.46, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6032, 6028, 5966, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{3}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2}{a^2\left(1-a^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - 2/(a^2*(1 - a^2*x^2)^2*ArcTanh[a*
x]) + 3/(2*a^2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/(
2*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
```

```
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx}{2a}+\frac{1}{2}(3a)\int \frac{}{\left(1-a^2x^2\right.}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{1}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+2\int \frac{}{\left(1-a^2x^2\right)^3}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{2}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{3}{2a^2\left(1-a^2x^2\right)\text{ta}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{2}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{3}{2a^2\left(1-a^2x^2\right)\text{ta}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{2}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{3}{2a^2\left(1-a^2x^2\right)\text{ta}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{2}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{3}{2a^2\left(1-a^2x^2\right)\text{ta}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{2}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{3}{2a^2\left(1-a^2x^2\right)\text{ta}}$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.96

$$-\frac{-\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2\text{Shi}\left(2\tanh^{-1}(ax)\right)-2\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2\text{Shi}\left(4\tanh^{-1}(ax)\right)+3a^2x^2\tanh^{-1}(}{2a^2\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

[Out] -1/2*(a*x + ArcTanh[a*x] + 3*a^2*x^2*ArcTanh[a*x] - (-1 + a^2*x^2)^2*ArcTan
h[a*x]^2*SinhIntegral[2*ArcTanh[a*x]] - 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*S
inhIntegral[4*ArcTanh[a*x]])/(a^2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)

fricas [B] time = 0.56, size = 256, normalized size = 2.56

$$\frac{\left(2\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-2\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+\left(a^4x^4-2a^2\right.\right.}{4\left(a^6x^4-2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")

[Out] 1/4*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1))^2 - 8*a*x - 4*(3*a^2*x^2 + 1)*log(-(a*x +
1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(-(a*x + 1)/(a*x - 1))^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")

[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)

maple [A] time = 0.18, size = 82, normalized size = 0.82

$$\frac{-\frac{\sinh(2\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2}-\frac{\sinh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\cosh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\operatorname{Shi}\left(4\operatorname{arctanh}\left(ax\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)

[Out] 1/a^2*(-1/8*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/4/arctanh(a*x)*cosh(2*arc
tanh(a*x))+1/2*Shi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*x))
-1/4/arctanh(a*x)*cosh(4*arctanh(a*x))+Shi(4*arctanh(a*x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,ax+\left(3\,a^2x^2+1\right)\log\left(ax+1\right)-\left(3\,a^2x^2+1\right)\log\left(-ax+1\right)}{\left(a^6x^4-2\,a^4x^2+a^2\right)\log\left(ax+1\right)^2-2\left(a^6x^4-2\,a^4x^2+a^2\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^6x^4-2\,a^4x^2+a^2\right)\log}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")

[Out] -(2*a*x + (3*a^2*x^2 + 1)*log(a*x + 1) - (3*a^2*x^2 + 1)*log(-a*x + 1))/((a
^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*lo
g(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + i
ntegrate(-2*(3*a^2*x^3 + 5*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*
x + 1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{6}x^{6}\operatorname{atanh}^{3}\left(ax\right)-3a^{4}x^{4}\operatorname{atanh}^{3}\left(ax\right)+3a^{2}x^{2}\operatorname{atanh}^{3}\left(ax\right)-\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*x
**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

3.340 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=69

$$-\frac{2x}{(1-a^2x^2)^2 \tanh^{-1}(ax)}-\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2}+\frac{\operatorname{Chi}(2\tanh^{-1}(ax))}{a}+\frac{\operatorname{Chi}(4\tanh^{-1}(ax))}{a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2*x/(-a^2*x^2+1)^2/arctanh(a*x)+Chi(2*
arctanh(a*x))/a+Chi(4*arctanh(a*x))/a
```

Rubi [A] time = 0.27, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{2x}{(1-a^2x^2)^2 \tanh^{-1}(ax)}-\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2}+\frac{\operatorname{Chi}(2\tanh^{-1}(ax))}{a}+\frac{\operatorname{Chi}(4\tanh^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/((1 - a^2*x^2)^2*ArcTanh[a*
x]) + CoshIntegral[2*ArcTanh[a*x]]/a + CoshIntegral[4*ArcTanh[a*x]]/a
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + (2a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + 2\int \frac{1}{\left(1-a^2x^2\right)^3 \tan$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh^4(x)}{x}\right.}{a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \left(\frac{3}{8x} + \right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx\right.}{4a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.20, size = 86, normalized size = 1.25

$$\frac{2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2 \mathrm{Chi}\left(2\tanh^{-1}(ax)\right) + 2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2 \mathrm{Chi}\left(4\tanh^{-1}(ax)\right) - 4ax\tanh^{-1}(ax}{2a\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] (-1 - 4*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[2
*ArcTanh[a*x]] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[4*ArcTanh[a
*x]])/(2*a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)
```

fricas [B] time = 0.98, size = 241, normalized size = 3.49

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\left(\left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2\,ax-}{a^2x^2+2\,ax-}\right.\right.}{2\left(a^5x^4-2\,a^3x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a*x*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_inte
gral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 +
1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2
*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1
)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4)/((a^
5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

maple [A] time = 0.24, size = 88, normalized size = 1.28

$$\frac{-\frac{3}{16\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)^2}-\frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\mathrm{X}\left(2\operatorname{arctanh}\left(ax\right)\right)-\frac{\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\sinh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\mathrm{X}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a*(-3/16/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2*sinh(
2*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*cosh(4
*arctanh(a*x))-1/4/arctanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(2\,ax\log\left(ax+1\right)-2\,ax\log\left(-ax+1\right)+1\right)}{\left(a^5x^4-2\,a^3x^2+a\right)\log\left(ax+1\right)^2-2\left(a^5x^4-2\,a^3x^2+a\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^5x^4-2\,a^3x^2+a\right)\log\left(-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(2*a*x*log(a*x + 1) - 2*a*x*log(-a*x + 1) + 1)/((a^5*x^4 - 2*a^3*x^2 + a
)*log(a*x + 1)^2 - 2*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1) +
 (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^2) + integrate(-4*(3*a^2*x^2 + 1)/
((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6*x^6 - 3*a^4*x^4
+ 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6\operatorname{atanh}^3(ax) - 3a^4x^4\operatorname{atanh}^3(ax) + 3a^2x^2\operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*x
**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

3.341 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=176

$$-\frac{\text{Int}\left(\frac{1}{x^2 \tanh^{-1}(ax)^2}, x\right)}{2a} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{a^2x^2+1}{2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{}{2(1-a^2x}$$

```
[Out] -1/2/a/x/arctanh(a*x)^2-1/2*a*x/(-a^2*x^2+1)^2/arctanh(a*x)^2-1/2*a*x/(-a^2
*x^2+1)/arctanh(a*x)^2-2/(-a^2*x^2+1)^2/arctanh(a*x)+3/2/(-a^2*x^2+1)/arcta
nh(a*x)+1/2*(-a^2*x^2-1)/(-a^2*x^2+1)/arctanh(a*x)+3/2*Shi(2*arctanh(a*x))+
Shi(4*arctanh(a*x))-1/2*Unintegrable(1/x^2/arctanh(a*x)^2,x)/a
```

Rubi [A] time = 0.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (a*x
)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - 2/((1 - a^2*x^2)^2*ArcTanh[a*x]) + 3/(
2*(1 - a^2*x^2)*ArcTanh[a*x]) - (1 + a^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*x]
) + (3*SinhIntegral[2*ArcTanh[a*x]])/2 + SinhIntegral[4*ArcTanh[a*x]] - Def
er[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx = a^2\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx + \int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{1}{2}a\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx + a^2\int \frac{}{(1}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}$$

Mathematica [A] time = 5.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^6x^7-3\,a^4x^5+3\,a^2x^3-x\right)\operatorname{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)^3 x\operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^3), x)
```

maple [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^3 \operatorname{arctanh}(ax)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,ax + \left(5\,a^2x^2-1\right)\log(ax+1) - \left(5\,a^2x^2-1\right)\log(-ax+1)}{\left(a^6x^6-2\,a^4x^4+a^2x^2\right)\log(ax+1)^2 - 2\left(a^6x^6-2\,a^4x^4+a^2x^2\right)\log(ax+1)\log(-ax+1) + \left(a^6x^6-2\,a^4x^4+a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (5*a^2*x^2 - 1)*log(a*x + 1) - (5*a^2*x^2 - 1)*log(-a*x + 1))/((a
^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(a*x + 1)^2 - 2*(a^6*x^6 - 2*a^4*x^4 + a^2
*x^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(-a*x
 + 1)^2) + integrate(-2*(10*a^4*x^4 - 3*a^2*x^2 + 1)/((a^8*x^9 - 3*a^6*x^7
+ 3*a^4*x^5 - a^2*x^3)*log(a*x + 1) - (a^8*x^9 - 3*a^6*x^7 + 3*a^4*x^5 - a^
2*x^3)*log(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x\,\operatorname{atanh}(a\,x)^3\left(a^2\,x^2-1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^7\operatorname{atanh}^3(ax) - 3a^4x^5\operatorname{atanh}^3(ax) + 3a^2x^3\operatorname{atanh}^3(ax) - x\operatorname{atanh}^3(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x)**3 - 3*a**4*x**5*atanh(a*x)**3 + 3*a**2*x
**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

3.342 $\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=125

$$-\frac{2x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}$$

```
[Out] -1/3/a/(-a^2*x^2+1)^2/arctanh(a*x)^3-2/3*x/(-a^2*x^2+1)^2/arctanh(a*x)^2-8/
3/a/(-a^2*x^2+1)^2/arctanh(a*x)+2/a/(-a^2*x^2+1)/arctanh(a*x)+2/3*Shi(2*arc
tanh(a*x))/a+4/3*Shi(4*arctanh(a*x))/a
```

Rubi [A] time = 0.50, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 6032, 6028, 6034, 5448, 12, 3298}

$$-\frac{2x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```

```
[Out] -1/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) - (2*x)/(3*(1 - a^2*x^2)^2*ArcTanh[
a*x]^2) - 8/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 2/(a*(1 - a^2*x^2)*ArcTanh
[a*x]) + (2*SinhIntegral[2*ArcTanh[a*x]])/(3*a) + (4*SinhIntegral[4*ArcTanh
[a*x]])/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
```

```
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^4}\,dx = -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} + \frac{1}{3}(4a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{2}{3}\int \frac{1}{\left(1-a^2x^2\right)^3 \tan}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2}{3a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{8}{3a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{8}{3a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{8}{3a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{8}{3a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{8}{3a\left(1-a^2x^2\right)^2 \tanh}$$

Mathematica [A] time = 0.26, size = 108, normalized size = 0.86

$$-\frac{-2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^3 \text{Shi}\left(2\tanh^{-1}(ax)\right) - 4\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^3 \text{Shi}\left(4\tanh^{-1}(ax)\right) + 6a^2x^2 \tanh^{-1}(a}{3a\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```

```
[Out] -1/3*(1 + 2*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^2
- 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]] - 4*(-1 +
a^2*x^2)^2*ArcTanh[a*x]^3*SinhIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2
*ArcTanh[a*x]^3)
```

fricas [B] time = 0.51, size = 272, normalized size = 2.18

$$\left(2\left(a^4x^4 - 2a^2x^2 + 1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right) - 2\left(a^4x^4 - 2a^2x^2 + 1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right) + \left(a^4x^4 - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1))^3 - 8*a*x*log(-(a*x + 1)/(a*x - 1)) - 4*(3
*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 - 8)/((a^5*x^4 - 2*a^3*x^2 + a)*l
og(-(a*x + 1)/(a*x - 1))^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2 - 1\right)^3 \operatorname{artanh}(ax)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^4), x)
```

maple [A] time = 0.23, size = 122, normalized size = 0.98

$$\frac{-\frac{1}{8\operatorname{arctanh}(ax)^3} - \frac{\cosh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^3} - \frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^2} - \frac{\cosh(2\operatorname{arctanh}(ax))}{3\operatorname{arctanh}(ax)} + \frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{3} - \frac{\cosh(4\operatorname{arctanh}(ax))}{24\operatorname{arctanh}(ax)^3} - \frac{\sin}{1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x)
```

```
[Out] 1/a*(-1/8/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6*sinh(2
*arctanh(a*x))/arctanh(a*x)^2-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*Shi
(2*arctanh(a*x))-1/24/arctanh(a*x)^3*cosh(4*arctanh(a*x))-1/12/arctanh(a*x)
^2*sinh(4*arctanh(a*x))-1/3/arctanh(a*x)*cosh(4*arctanh(a*x))+4/3*Shi(4*arc
tanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\left(2\,ax\log(ax+1) + \left(3a^2x^2+1\right)\log(ax+1)^2 + \left(3a^2x^2+1\right)\log(-ax+1)^2 - 2\left(ax + \right.\right.}{3\left(\left(a^5x^4 - 2a^3x^2 + a\right)\log(ax+1)^3 - 3\left(a^5x^4 - 2a^3x^2 + a\right)\log(ax+1)^2\log(-ax+1) + 3\left(a^5x^4 - 2a^3x^2 + a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -4/3*(2*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 + 1)
*log(-a*x + 1)^2 - 2*(a*x + (3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 2
)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^3 - 3*(a^5*x^4 - 2*a^3*x^2 + a)*l
og(a*x + 1)^2*log(-a*x + 1) + 3*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(
-a*x + 1)^2 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^3) + integrate(-8/3*(
3*a^3*x^3 + 5*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a
^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\operatorname{atanh}(a\,x)^4\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^4*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6\operatorname{atanh}^4(ax)-3a^4x^4\operatorname{atanh}^4(ax)+3a^2x^2\operatorname{atanh}^4(ax)-\operatorname{atanh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**4,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**4 - 3*a**4*x**4*atanh(a*x)**4 + 3*a**2*x
**2*atanh(a*x)**4 - atanh(a*x)**4), x)
```

3.343 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^5}\,dx$

Optimal. Leaf size=170

$$\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}+\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{\quad}{3a\,(}$$

```
[Out] -1/4/a/(-a^2*x^2+1)^2/arctanh(a*x)^4-1/3*x/(-a^2*x^2+1)^2/arctanh(a*x)^3-2/
3/a/(-a^2*x^2+1)^2/arctanh(a*x)^2+1/2/a/(-a^2*x^2+1)/arctanh(a*x)^2-8/3*x/(
-a^2*x^2+1)^2/arctanh(a*x)+x/(-a^2*x^2+1)/arctanh(a*x)+1/3*Chi(2*arctanh(a*
x))/a+4/3*Chi(4*arctanh(a*x))/a
```

Rubi [A] time = 0.96, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5966, 6032, 6028, 6034, 3312, 3301, 5968, 5448}

$$\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}+\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{\quad}{3a\,(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```

```
[Out] -1/(4*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^4) - x/(3*(1 - a^2*x^2)^2*ArcTanh[a*x]
^3) - 2/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 1/(2*a*(1 - a^2*x^2)*ArcTanh
[a*x]^2) - (8*x)/(3*(1 - a^2*x^2)^2*ArcTanh[a*x]) + x/((1 - a^2*x^2)*ArcTan
h[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a) + (4*CoshIntegral[4*ArcTanh[a*
x]])/(3*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^5}\,dx = -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} + a\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^4}\,dx$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} + \frac{1}{3}\int \frac{}{\left(1-a^2x^2\right)^3}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{1}{6a\left(1-a^2x^2\right)^2 \mathrm{ta}}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)^2 \mathrm{ta}}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)^2 \mathrm{ta}}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)^2 \mathrm{ta}}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)^2 \mathrm{ta}}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)^2 \mathrm{ta}}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)^2 \mathrm{ta}}$$

Mathematica [A] time = 0.18, size = 132, normalized size = 0.78

$$-\frac{12a^3x^3\tanh^{-1}(ax)^3 - 4\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^4\mathrm{Chi}\left(2\tanh^{-1}(ax)\right) - 16\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^4\mathrm{Chi}\left(4\tanh\right.}{12a\left(a^2x^2-1\right)^2\tanh^{-1}(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```

```
[Out] -1/12*(3 + 4*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^2
 + 20*a*x*ArcTanh[a*x]^3 + 12*a^3*x^3*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2*A
rcTanh[a*x]^4*CoshIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh[a*
x]^4*CoshIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)
```

fricas [B] time = 0.59, size = 303, normalized size = 1.78

$$\frac{\left(4\left(a^4x^4-2a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)+4\left(a^4x^4-2a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+\left(a^4x^4-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="fricas")
```

```
[Out] 1/6*((4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
```

```
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x + 1)/(
a*x - 1))^3 - 16*a*x*log(-(a*x + 1)/(a*x - 1)) - 4*(3*a^2*x^2 + 1)*log(-(a*
x + 1)/(a*x - 1))^2 - 24)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x -
1))^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^5), x)
```

maple [A] time = 0.23, size = 152, normalized size = 0.89

$$\frac{-\frac{3}{32\operatorname{arctanh}(ax)^4}-\frac{\cosh(2\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)^4}-\frac{\sinh(2\operatorname{arctanh}(ax))}{12\operatorname{arctanh}(ax)^3}-\frac{\cosh(2\operatorname{arctanh}(ax))}{12\operatorname{arctanh}(ax)^2}-\frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)}+\frac{X(2\operatorname{arctanh}(ax))}{3}-\frac{\cosh(4\operatorname{ar}}{32\operatorname{arcta}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x)
```

```
[Out] 1/a*(-3/32/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12/arct
anh(a*x)^3*sinh(2*arctanh(a*x))-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/
6*sinh(2*arctanh(a*x))/arctanh(a*x)+1/3*Chi(2*arctanh(a*x))-1/32/arctanh(a*
x)^4*cosh(4*arctanh(a*x))-1/24/arctanh(a*x)^3*sinh(4*arctanh(a*x))-1/12/arc
tanh(a*x)^2*cosh(4*arctanh(a*x))-1/3/arctanh(a*x)*sinh(4*arctanh(a*x))+4/3*
Chi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\left(3a^3x^3+5ax\right)\log(ax+1)^3-\left(3a^3x^3+5ax\right)\log(-ax+1)^3+4ax\log(ax+1)+\left(3a^2x^2+1\right)\log(ax+1)^2}{3\left(\left(a^5x^4-2a^3x^2+a\right)\log(ax+1)^4-4\left(a^5x^4-2a^3x^2+a\right)\log(ax+1)^3\log(-ax+1)+6\left(a^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="maxima")
```

```
[Out] -2/3*((3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - (3*a^3*x^3 + 5*a*x)*log(-a*x + 1
)^3 + 4*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 + 3*
(3*a^3*x^3 + 5*a*x)*log(a*x + 1) + 1)*log(-a*x + 1)^2 - (3*(3*a^3*x^3 + 5*a
*x)*log(a*x + 1)^2 + 4*a*x + 2*(3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1)
+ 6)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4 - 4*(a^5*x^4 - 2*a^3*x^2 + a
)*log(a*x + 1)^3*log(-a*x + 1) + 6*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^2
*log(-a*x + 1)^2 - 4*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1)^3
 + (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^4) + integrate(-2/3*(3*a^4*x^4 +
 24*a^2*x^2 + 5)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\operatorname{atanh}(a\,x)^5\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^5*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6 \operatorname{atanh}^5(ax) - 3a^4x^4 \operatorname{atanh}^5(ax) + 3a^2x^2 \operatorname{atanh}^5(ax) - \operatorname{atanh}^5(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**5,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**5 - 3*a**4*x**4*atanh(a*x)**5 + 3*a**2*x
**2*atanh(a*x)**5 - atanh(a*x)**5), x)
```

3.344 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^6} dx$

Optimal. Leaf size=257

$$\frac{x}{5(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{8x}{15(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{x}{5(1-a^2x^2)^2\tanh^{-1}(ax)^4} + \frac{a^2x^2+1}{5a(1-a^2x^2)\tanh^{-1}(ax)} + \frac{}{5a}$$

```
[Out] -1/5/a/(-a^2*x^2+1)^2/arctanh(a*x)^5-1/5*x/(-a^2*x^2+1)^2/arctanh(a*x)^4-4/
15/a/(-a^2*x^2+1)^2/arctanh(a*x)^3+1/5/a/(-a^2*x^2+1)/arctanh(a*x)^3-8/15*x
/(-a^2*x^2+1)^2/arctanh(a*x)^2+1/5*x/(-a^2*x^2+1)/arctanh(a*x)^2-32/15/a/(-
a^2*x^2+1)^2/arctanh(a*x)+8/5/a/(-a^2*x^2+1)/arctanh(a*x)+1/5*(a^2*x^2+1)/a
/(-a^2*x^2+1)/arctanh(a*x)+2/15*Shi(2*arctanh(a*x))/a+16/15*Shi(4*arctanh(a
*x))/a
```

Rubi [A] time = 1.35, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5966, 6032, 6028, 5996, 6034, 5448, 12, 3298}

$$\frac{x}{5(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{8x}{15(1-a^2x^2)^2\tanh^{-1}(ax)^2} - \frac{x}{5(1-a^2x^2)^2\tanh^{-1}(ax)^4} + \frac{a^2x^2+1}{5a(1-a^2x^2)\tanh^{-1}(ax)} + \frac{}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

```
[Out] -1/(5*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^5) - x/(5*(1 - a^2*x^2)^2*ArcTanh[a*x]
^4) - 4/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) + 1/(5*a*(1 - a^2*x^2)*ArcTan
h[a*x]^3) - (8*x)/(15*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + x/(5*(1 - a^2*x^2)*
ArcTanh[a*x]^2) - 32/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 8/(5*a*(1 - a^2*
x^2)*ArcTanh[a*x]) + (1 + a^2*x^2)/(5*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*Si
nhIntegral[2*ArcTanh[a*x]])/(15*a) + (16*SinhIntegral[4*ArcTanh[a*x]])/(15*
a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
```

```
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^6}\,dx = -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} + \frac{1}{5}(4a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^5}\,dx$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} + \frac{1}{5}\int \frac{1}{\left(1-a^2x^2\right)^3 \tan}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{1}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh}$$

Mathematica [A] time = 0.33, size = 166, normalized size = 0.65

$$-\frac{3a^4x^4 \tanh^{-1}(ax)^4 + 3a^3x^3 \tanh^{-1}(ax)^3 - 2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^5 \mathrm{Shi}\left(2\tanh^{-1}(ax)\right) - 16\left(a^2x^2-1\right)^2 \tanh^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

```
[Out] -1/15*(3 + 3*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2 +
 5*a*x*ArcTanh[a*x]^3 + 3*a^3*x^3*ArcTanh[a*x]^3 + 5*ArcTanh[a*x]^4 + 24*a^
2*x^2*ArcTanh[a*x]^4 + 3*a^4*x^4*ArcTanh[a*x]^4 - 2*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^5*SinhIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^5*
SinhIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^5)
```

fricas [A] time = 0.52, size = 341, normalized size = 1.33

$$\frac{\left(8\left(a^4x^4-2a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right) - 8\left(a^4x^4-2a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right) + \left(a^4x^4-2a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*((8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) - 8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a
*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
 + 1)))*log(-(a*x + 1)/(a*x - 1))^5 - 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(-(
a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x + 1)/(a*x - 1))^3 -
 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 8*(3*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1))^2 - 96)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^6), x)
```

maple [A] time = 0.23, size = 182, normalized size = 0.71

$$-\frac{3}{40\,\mathrm{arctanh}(ax)^5} - \frac{\cosh(2\,\mathrm{arctanh}(ax))}{10\,\mathrm{arctanh}(ax)^5} - \frac{\sinh(2\,\mathrm{arctanh}(ax))}{20\,\mathrm{arctanh}(ax)^4} - \frac{\cosh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^3} - \frac{\sinh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^2} - \frac{\cosh(2\,\mathrm{arctanh}(ax))}{15\,\mathrm{arctanh}(ax)} + 2S$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x)
```

```
[Out] 1/a*(-3/40/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/arc
tanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1
/30*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/15/arctanh(a*x)*cosh(2*arctanh(a*
x))+2/15*Shi(2*arctanh(a*x))-1/40/arctanh(a*x)^5*cosh(4*arctanh(a*x))-1/40/
arctanh(a*x)^4*sinh(4*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(4*arctanh(a*x)
)-1/15/arctanh(a*x)^2*sinh(4*arctanh(a*x))-4/15/arctanh(a*x)*cosh(4*arctanh
(a*x))+16/15*Shi(4*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\left(3\,a^4x^4+24\,a^2x^2+5\right)\log(ax+1)^4+\left(3\,a^4x^4+24\,a^2x^2+5\right)\log(-ax+1)^4+2\left(3\,a^3x^3+5\,ax\right)\log(ax+\right.}{15\left(\left(a^5x^4-2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="maxima")
```

```
[Out] -2/15*((3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^4 + (3*a^4*x^4 + 24*a^2*x^
2 + 5)*log(-a*x + 1)^4 + 2*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - 2*(3*a^3*x^
3 + 5*a*x + 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1))*log(-a*x + 1)^3 +
24*a*x*log(a*x + 1) + 4*(3*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(6*a^2*x^2 + 3*(
3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^2 + 3*(3*a^3*x^3 + 5*a*x)*log(a*x
+ 1) + 2)*log(-a*x + 1)^2 - 2*(2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^
3 + 3*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^2 + 12*a*x + 4*(3*a^2*x^2 + 1)*log(a
*x + 1))*log(-a*x + 1) + 48)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^5 - 5*
(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4*log(-a*x + 1) + 10*(a^5*x^4 - 2*a^
3*x^2 + a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^5*x^4 - 2*a^3*x^2 + a)*lo
g(a*x + 1)^2*log(-a*x + 1)^3 + 5*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log
```

```
(-a*x + 1)^4 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^5) + integrate(-8/15
*(15*a^3*x^3 + 17*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)
- (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{1}{\operatorname{atanh}(a\,x)^6\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^6*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6\operatorname{atanh}^6(ax) - 3a^4x^4\operatorname{atanh}^6(ax) + 3a^2x^2\operatorname{atanh}^6(ax) - \operatorname{atanh}^6(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**6,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**6 - 3*a**4*x**4*atanh(a*x)**6 + 3*a**2*x
**2*atanh(a*x)**6 - atanh(a*x)**6), x)
```

3.345 $\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^4}\,dx$

Optimal. Leaf size=134

$$-\frac{5}{32a\left(1-a^2x^2\right)}-\frac{5}{96a\left(1-a^2x^2\right)^2}-\frac{1}{36a\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)}{6\left(1-a^2x^2\right)^3}+\frac{5\tanh^{-1}(}{32a}$$

```
[Out] -1/36/a/(-a^2*x^2+1)^3-5/96/a/(-a^2*x^2+1)^2-5/32/a/(-a^2*x^2+1)+1/6*x*arct
anh(a*x)/(-a^2*x^2+1)^3+5/24*x*arctanh(a*x)/(-a^2*x^2+1)^2+5/16*x*arctanh(a
*x)/(-a^2*x^2+1)+5/32*arctanh(a*x)^2/a
```

Rubi [A] time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5960, 5956, 261}

$$-\frac{5}{32a\left(1-a^2x^2\right)}-\frac{5}{96a\left(1-a^2x^2\right)^2}-\frac{1}{36a\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)}{6\left(1-a^2x^2\right)^3}+\frac{5\tanh^{-1}(}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]
```

```
[Out] -1/(36*a*(1 - a^2*x^2)^3) - 5/(96*a*(1 - a^2*x^2)^2) - 5/(32*a*(1 - a^2*x^2
)) + (x*ArcTanh[a*x])/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x])/(24*(1 - a^2
*x^2)^2) + (5*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^2)/(32*a
)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^4}\,dx = -\frac{1}{36a\left(1-a^2x^2\right)^3} + \frac{x\tanh^{-1}(ax)}{6\left(1-a^2x^2\right)^3} + \frac{5}{6}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx$$

$$= -\frac{1}{36a\left(1-a^2x^2\right)^3} - \frac{5}{96a\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{6\left(1-a^2x^2\right)^3} + \frac{5x\tanh^{-1}(ax)}{24\left(1-a^2x^2\right)^2} + \frac{5}{8}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{1}{36a\left(1-a^2x^2\right)^3} - \frac{5}{96a\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{6\left(1-a^2x^2\right)^3} + \frac{5x\tanh^{-1}(ax)}{24\left(1-a^2x^2\right)^2} + \frac{5x\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)} + \frac{5\,t}{}$$

$$= -\frac{1}{36a\left(1-a^2x^2\right)^3} - \frac{5}{96a\left(1-a^2x^2\right)^2} - \frac{5}{32a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)}{6\left(1-a^2x^2\right)^3} + \frac{5x\tanh^{-1}(ax)}{24\left(1-a^2x^2\right)^2} + \frac{5x}{16}$$

Mathematica [A] time = 0.18, size = 81, normalized size = 0.60

$$\frac{45a^4x^4 - 105a^2x^2 + 45\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2 - 6ax\left(15a^4x^4 - 40a^2x^2 + 33\right)\tanh^{-1}(ax) + 68}{288a\left(a^2x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]
```

```
[Out] (68 - 105*a^2*x^2 + 45*a^4*x^4 - 6*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTa
nh[a*x] + 45*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2)/(288*a*(-1 + a^2*x^2)^3)
```

fricas [A] time = 0.47, size = 131, normalized size = 0.98

$$\frac{180\,a^4x^4 - 420\,a^2x^2 + 45\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 12\left(15\,a^5x^5 - 40\,a^3x^3 + 33\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{1152\left(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] 1/1152*(180*a^4*x^4 - 420*a^2*x^2 + 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1
)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*log(-
(a*x + 1)/(a*x - 1)) + 272)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(a^2*x^2 - 1)^4, x)
```

maple [B] time = 0.07, size = 281, normalized size = 2.10

$$-\frac{\operatorname{arctanh}(ax)}{48a\,(ax-1)^3}+\frac{\operatorname{arctanh}(ax)}{16a\,(ax-1)^2}-\frac{5\operatorname{arctanh}(ax)}{32a\,(ax-1)}-\frac{5\operatorname{arctanh}(ax)\ln(ax-1)}{32a}-\frac{\operatorname{arctanh}(ax)}{48a\,(ax+1)^3}-\frac{\operatorname{arctanh}(ax)}{16a\,(ax+1)^2}-\frac{5\operatorname{arcta}}{32a\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^4,x)
```

```
[Out] -1/48/a*arctanh(a*x)/(a*x-1)^3+1/16/a*arctanh(a*x)/(a*x-1)^2-5/32/a*arctanh
(a*x)/(a*x-1)-5/32/a*arctanh(a*x)*ln(a*x-1)-1/48/a*arctanh(a*x)/(a*x+1)^3-1
/16/a*arctanh(a*x)/(a*x+1)^2-5/32/a*arctanh(a*x)/(a*x+1)+5/32/a*arctanh(a*x
)*ln(a*x+1)-5/128/a*ln(a*x-1)^2+5/64/a*ln(a*x-1)*ln(1/2+1/2*a*x)-5/128/a*ln
(a*x+1)^2-5/64/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+5/64/a*ln(-1/2*a*x+1/2)*l
n(a*x+1)+1/288/a/(a*x-1)^3-7/384/a/(a*x-1)^2+37/384/a/(a*x-1)-1/288/a/(a*x+
1)^3-7/384/a/(a*x+1)^2-37/384/a/(a*x+1)
```

maxima [B] time = 0.32, size = 240, normalized size = 1.79

$$-\frac{1}{96}\left(\frac{2\left(15\,a^4x^5-40\,a^2x^3+33\,x\right)}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}-\frac{15\,\log\left(ax+1\right)}{a}+\frac{15\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)+\frac{\left(180\,a^4x^4-420\,a^2x^2-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x) + 1/1152*(180*a^
4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)
^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1) - 4
5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a/(a^8*x^6 -
3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

mupad [B] time = 1.30, size = 206, normalized size = 1.54

$$\frac{\frac{34}{3\,a}-\frac{35\,a\,x^2}{2}+\frac{15\,a^3\,x^4}{2}}{48\,a^6\,x^6-144\,a^4\,x^4+144\,a^2\,x^2-48}-\ln\left(1-a\,x\right)\left(\frac{5\,\ln\left(a\,x+1\right)}{64\,a}-\frac{\frac{5\,a^4\,x^5}{16}-\frac{5\,a^2\,x^3}{6}+\frac{11\,x}{16}}{2\,a^6\,x^6-6\,a^4\,x^4+6\,a^2\,x^2-2}\right)+\frac{5\ln\left(a\,x+\right.}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(a^2*x^2 - 1)^4,x)
```

```
[Out] (34/(3*a) - (35*a*x^2)/2 + (15*a^3*x^4)/2)/(144*a^2*x^2 - 144*a^4*x^4 + 48*
a^6*x^6 - 48) - log(1 - a*x)*((5*log(a*x + 1))/(64*a) - ((11*x)/16 - (5*a^2
*x^3)/6 + (5*a^4*x^5)/16)/(6*a^2*x^2 - 6*a^4*x^4 + 2*a^6*x^6 - 2)) + (5*log
(a*x + 1)^2)/(128*a) + (5*log(1 - a*x)^2)/(128*a) - (log(a*x + 1)*((11*x)/(
32*a) - (5*a*x^3)/12 + (5*a^3*x^5)/32))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^
6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(ax\right)}{\left(ax-1\right)^4\left(ax+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)/((a*x - 1)**4*(a*x + 1)**4), x)
```

3.346 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^4}\,dx$

Optimal. Leaf size=214

$$\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^2}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}}{16a\left(1-\right.}$$

```
[Out] 1/108*x/(-a^2*x^2+1)^3+65/1728*x/(-a^2*x^2+1)^2+245/1152*x/(-a^2*x^2+1)+245
/1152*arctanh(a*x)/a-1/18*arctanh(a*x)/a/(-a^2*x^2+1)^3-5/48*arctanh(a*x)/a
/(-a^2*x^2+1)^2-5/16*arctanh(a*x)/a/(-a^2*x^2+1)+1/6*x*arctanh(a*x)^2/(-a^2
*x^2+1)^3+5/24*x*arctanh(a*x)^2/(-a^2*x^2+1)^2+5/16*x*arctanh(a*x)^2/(-a^2*
x^2+1)+5/48*arctanh(a*x)^3/a
```

Rubi [A] time = 0.17, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 199, 206}

$$\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^2}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}}{16a\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

```
[Out] x/(108*(1 - a^2*x^2)^3) + (65*x)/(1728*(1 - a^2*x^2)^2) + (245*x)/(1152*(1
- a^2*x^2)) + (245*ArcTanh[a*x])/(1152*a) - ArcTanh[a*x]/(18*a*(1 - a^2*x^2
)^3) - (5*ArcTanh[a*x])/(48*a*(1 - a^2*x^2)^2) - (5*ArcTanh[a*x])/(16*a*(1
- a^2*x^2)) + (x*ArcTanh[a*x]^2)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^2)
/(24*(1 - a^2*x^2)^2) + (5*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + (5*ArcTan
h[a*x]^3)/(48*a)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
```

```
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^4}\,dx = -\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}+\frac{1}{18}\int\frac{1}{\left(1-a^2x^2\right)^4}\,dx+\frac{5}{6}\int\frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx$$

$$=\frac{x}{108\left(1-a^2x^2\right)^3}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^2}{24\left(1-a^2x^2\right)^2}$$

$$=\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}$$

$$=\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{65x}{1152\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)}$$

$$=\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{65\tanh^{-1}(ax)}{1152a}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)}$$

$$=\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{245\tanh^{-1}(ax)}{1152a}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x\right.}$$

Mathematica [A] time = 0.26, size = 157, normalized size = 0.73

$$\frac{-\frac{1470x}{a^2x^2-1}+\frac{260x}{\left(a^2x^2-1\right)^2}-\frac{64x}{\left(a^2x^2-1\right)^3}-\frac{144x\left(15a^4x^4-40a^2x^2+33\right)\tanh^{-1}(ax)^2}{\left(a^2x^2-1\right)^3}+\frac{48\left(45a^4x^4-105a^2x^2+68\right)\tanh^{-1}(ax)}{a\left(a^2x^2-1\right)^3}-\frac{735\log(1-ax)}{a}+}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

```
[Out] ((-64*x)/(-1 + a^2*x^2)^3 + (260*x)/(-1 + a^2*x^2)^2 - (1470*x)/(-1 + a^2*x
^2) + (48*(68 - 105*a^2*x^2 + 45*a^4*x^4)*ArcTanh[a*x])/(a*(-1 + a^2*x^2)^3
) - (144*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^2)/(-1 + a^2*x^2)^3
+ (720*ArcTanh[a*x]^3)/a - (735*Log[1 - a*x])/a + (735*Log[1 + a*x])/a)/691
2
```

fricas [A] time = 0.53, size = 179, normalized size = 0.84

$$-\frac{1470a^5x^5-3200a^3x^3-90\left(a^6x^6-3a^4x^4+3a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+36\left(15a^5x^5-40a^3x^3+33ax\right)\log\left(-\right.}{6912\left(a^7x^6-3a^5x^4+3a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] -1/6912*(1470*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1)*log(-(a*x + 1)/(a*x - 1))^3 + 36*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*lo
g(-(a*x + 1)/(a*x - 1))^2 + 1794*a*x - 3*(245*a^6*x^6 - 375*a^4*x^4 - 105*a
^2*x^2 + 299)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 -
 a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(a^2*x^2 - 1)^4, x)
```

maple [C] time = 0.86, size = 3447, normalized size = 16.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^4,x)
```

```
[Out] -15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-
1))^3*x^2-15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+15/32*I*a/(a*x-1)^3/(a*x+1)^3
*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2-15/32*I*a/(a*x-
1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-5
/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1
))^3*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^6+5/32*I*a^5/(a*x-1)^3/(a*x+1)^
3*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^6-5/32*I*a^5/(a*
x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^6
+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^3*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^4+5/32*I/a/(a*x-1)^3/(a*x+1
)^3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(
a^2*x^2-1))^2-5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)
^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+
5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^
(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arct
anh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
(1+(a*x+1)^2/(-a^2*x^2+1)))^2-15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^
2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^4+15/32*I*a^3/(a*x-1)^3/(a*x+1)
^3*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4-245/1152*a^4/
(a*x-1)^3/(a*x+1)^3*x^5+25/54*a^2/(a*x-1)^3/(a*x+1)^3*x^3-5/48/a/(a*x-1)^3/
(a*x+1)^3*arctanh(a*x)^3+299/1152/a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)-15/64*
I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1))
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-5/32*I*a^5
/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn
(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^6+5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a
*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x
+1)^2/(-a^2*x^2+1)))^2*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^6-5/6
4*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2
+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^6+15/32*
I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2
```

```
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^4-15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arc
tanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4-5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*
x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))+15/64*I*a^3/(a*x-1)^3/(
a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))^2*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*cs
gn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/
(-a^2*x^2+1)))^2*x^4-15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2+15/64*I*a/
(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-15/64*I*a/(a*x-1)^3
/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(
-a^2*x^2+1)^(1/2))^2*x^2-15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^4+5/64*I*a^5/(a*x-1)^3/(a*
x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/(1+(a*x+1)^2/
(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^6
+15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-
1))*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x
+1)^2/(-a^2*x^2+1)))*x^2-5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2-299
/1152/(a*x-1)^3/(a*x+1)^3*x-1/48/a*arctanh(a*x)^2/(a*x-1)^3+1/16/a*arctanh(
a*x)^2/(a*x-1)^2-5/32/a*arctanh(a*x)^2/(a*x-1)-1/48/a*arctanh(a*x)^2/(a*x+1
)^3-1/16/a*arctanh(a*x)^2/(a*x+1)^2-5/32/a*arctanh(a*x)^2/(a*x+1)-5/32/a*ar
ctanh(a*x)^2*ln(a*x-1)+5/32/a*arctanh(a*x)^2*ln(a*x+1)-5/16/a*arctanh(a*x)^
2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*
x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*a
rctanh(a*x)^2*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+5/64*I/a/(a*x-1)^3/(a*x+
1)^3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+5/64*I/a/(a*x-1)^3/(
a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*
x^2+1)))^3+5/32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*x^6-15/32*I*a^3
/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*x^4+15/32*I*a/(a*x-1)^3/(a*x+1)^3*ar
ctanh(a*x)^2*Pi*x^2+5/16*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*x^2-35/384*a/
(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x^2+5/48*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a
*x)^3*x^6+245/1152*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x^6-5/16*a^3/(a*x-1
)^3/(a*x+1)^3*arctanh(a*x)^3*x^4-125/384*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*
x)*x^4
```

maxima [B] time = 0.34, size = 516, normalized size = 2.41

$$-\frac{1}{96}\left(\frac{2\left(15\,a^4x^5-40\,a^2x^3+33\,x\right)}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}-\frac{15\,\log\left(ax+1\right)}{a}+\frac{15\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\left(1470\,a^5x^5-3200\,a^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/6912*(1470
*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
+ 1)^3 + 270*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a*x -
 1) + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a*x -
15*(49*a^6*x^6 - 147*a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^
2*x^2 - 1)*log(a*x - 1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*x^4 + 3
*a^2*x^2 - 1)*log(a*x - 1))*a^2/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x^2 - a^3) + 1
/576*(180*a^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*
log(a*x + 1)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(
a*x - 1) - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a
*arctanh(a*x)/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

mupad [B] time = 2.49, size = 493, normalized size = 2.30

$$\ln(1-a\,x)^2\left(\frac{5\,\ln(a\,x+1)}{128\,a}-\frac{\frac{5\,a^4\,x^5}{16}-\frac{5\,a^2\,x^3}{6}+\frac{11\,x}{16}}{4\,a^6\,x^6-12\,a^4\,x^4+12\,a^2\,x^2-4}\right)-\frac{\frac{245\,a^4\,x^5}{8}-\frac{200\,a^2\,x^3}{3}+\frac{299\,x}{8}}{144\,a^6\,x^6-432\,a^4\,x^4+432\,a^2\,x^2-144}-\ln(1-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(a^2*x^2 - 1)^4,x)
```

```
[Out] log(1 - a*x)^2*((5*log(a*x + 1))/(128*a) - ((11*x)/16 - (5*a^2*x^3)/6 + (5*
a^4*x^5)/16)/(12*a^2*x^2 - 12*a^4*x^4 + 4*a^6*x^6 - 4)) - ((299*x)/8 - (200
*a^2*x^3)/3 + (245*a^4*x^5)/8)/(432*a^2*x^2 - 432*a^4*x^4 + 144*a^6*x^6 - 1
44) - log(1 - a*x)*((5*log(a*x + 1)^2)/(128*a) + ((37*x)/2 - 35*a*x^2 + 68/
(3*a) - (82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(576*a^2*x^2 - 576*a^
4*x^4 + 192*a^6*x^6 - 192) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*a^2*x^3)
/3 - 15*a^3*x^4 + (23*a^4*x^5)/2)/(576*a^2*x^2 - 576*a^4*x^4 + 192*a^6*x^6
- 192) - (log(a*x + 1)*(22*x - (80*a^2*x^3)/3 + 10*a^4*x^5))/(192*a^2*x^2 -
 192*a^4*x^4 + 64*a^6*x^6 - 64)) + (5*log(a*x + 1)^3)/(384*a) - (5*log(1 -
a*x)^3)/(384*a) - (atan(a*x*1i)*245i)/(1152*a) + (log(a*x + 1)*(17/(72*a^2)
 - (35*x^2)/96 + (5*a^2*x^4)/32))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6) - (
log(a*x + 1)^2*((11*x)/(64*a) - (5*a*x^3)/24 + (5*a^3*x^5)/64))/(3*a*x^2 -
1/a - 3*a^3*x^4 + a^5*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{(ax-1)^4\,(ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)**2/((a*x - 1)**4*(a*x + 1)**4), x)
```

3.347 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^4}\,dx$

Optimal. Leaf size=291

$$-\frac{245}{768a\left(1-a^2x^2\right)}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{1}{216a\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^3}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^3}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}-\cdots$$

```
[Out] -1/216/a/(-a^2*x^2+1)^3-65/2304/a/(-a^2*x^2+1)^2-245/768/a/(-a^2*x^2+1)+1/3
6*x*arctanh(a*x)/(-a^2*x^2+1)^3+65/576*x*arctanh(a*x)/(-a^2*x^2+1)^2+245/38
4*x*arctanh(a*x)/(-a^2*x^2+1)+245/768*arctanh(a*x)^2/a-1/12*arctanh(a*x)^2/
a/(-a^2*x^2+1)^3-5/32*arctanh(a*x)^2/a/(-a^2*x^2+1)^2-15/32*arctanh(a*x)^2/
a/(-a^2*x^2+1)+1/6*x*arctanh(a*x)^3/(-a^2*x^2+1)^3+5/24*x*arctanh(a*x)^3/(-
a^2*x^2+1)^2+5/16*x*arctanh(a*x)^3/(-a^2*x^2+1)+5/64*arctanh(a*x)^4/a
```

Rubi [A] time = 0.33, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 261, 5960}

$$-\frac{245}{768a\left(1-a^2x^2\right)}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{1}{216a\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^3}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^3}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

```
[Out] -1/(216*a*(1 - a^2*x^2)^3) - 65/(2304*a*(1 - a^2*x^2)^2) - 245/(768*a*(1 -
a^2*x^2)) + (x*ArcTanh[a*x])/(36*(1 - a^2*x^2)^3) + (65*x*ArcTanh[a*x])/(57
6*(1 - a^2*x^2)^2) + (245*x*ArcTanh[a*x])/(384*(1 - a^2*x^2)) + (245*ArcTan
h[a*x]^2)/(768*a) - ArcTanh[a*x]^2/(12*a*(1 - a^2*x^2)^3) - (5*ArcTanh[a*x]
^2)/(32*a*(1 - a^2*x^2)^2) - (15*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) + (x*
ArcTanh[a*x]^3)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^3)/(24*(1 - a^2*x^2
)^2) + (5*x*ArcTanh[a*x]^3)/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^4)/(64*a)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^4}\,dx = -\frac{\tanh^{-1}(ax)^2}{12a\left(1-a^2x^2\right)^3}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}+\frac{1}{6}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^4}\,dx+\frac{5}{6}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}-\frac{\tanh^{-1}(ax)^2}{12a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}+$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)^2}{12a\left(1-a^2x^2\right.}$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}+\frac{65x\tanh^{-1}(a}{384\left(1-a^2x\right.}$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{65}{768a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(a}{576\left(1-a^2x^2\right.}$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{245}{768a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(a}{576\left(1-a^2x^2\right.}$$

Mathematica [A] time = 0.15, size = 143, normalized size = 0.49

$$\frac{2205a^4x^4-4605a^2x^2+540\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^4-144ax\left(15a^4x^4-40a^2x^2+33\right)\tanh^{-1}(ax)^3-6ax\left(735a^4x\right.}{6912a\left(a^2x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

```
[Out] (2432 - 4605*a^2*x^2 + 2205*a^4*x^4 - 6*a*x*(897 - 1600*a^2*x^2 + 735*a^4*x
^4)*ArcTanh[a*x] + 9*(299 - 105*a^2*x^2 - 375*a^4*x^4 + 245*a^6*x^6)*ArcTan
h[a*x]^2 - 144*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^3 + 540*(-1
+ a^2*x^2)^3*ArcTanh[a*x]^4)/(6912*a*(-1 + a^2*x^2)^3)
```

fricas [A] time = 0.71, size = 216, normalized size = 0.74

$$\frac{8820\,a^4x^4+135\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4-18420\,a^2x^2-72\left(15\,a^5x^5-40\,a^3x^3+33\,ax\right)\log\left(-\frac{a}{a}\right.}{27648\left(a^7x^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] 1/27648*(8820*a^4*x^4 + 135*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x
 + 1)/(a*x - 1))^4 - 18420*a^2*x^2 - 72*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*
log(-(a*x + 1)/(a*x - 1))^3 + 9*(245*a^6*x^6 - 375*a^4*x^4 - 105*a^2*x^2 +
299)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(735*a^5*x^5 - 1600*a^3*x^3 + 897*a*x
)*log(-(a*x + 1)/(a*x - 1)) + 9728)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(a^2x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a^2*x^2 - 1)^4, x)
```

maple [C] time = 0.97, size = 3550, normalized size = 12.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^4,x)
```

```
[Out] 5/32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*
x^2+1)))^3*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^6-5/32*I*a^5/(a*x-1)^3/(a
*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^6+15/64*I*
a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x
^4-15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-
a^2*x^2+1)))^3*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^4+15/32*I*a^3/(a*x-1
)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4-15
/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))
^3*x^2+15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/
(-a^2*x^2+1)))^3*x^2-15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3*x^2+5/64*I/a/(a*x-1)^3
/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a
*x+1)^2/(a^2*x^2-1))+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I/
(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2
*x^2+1)))^2+5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(
-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-5/64*I/a/(a*x-1)^3/(a*x+
1)^3*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2-15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(
a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-5/64*I*a^5/(a*x-1)^3/(a*
x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^6+5/64*I*a^5/(a*
x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^
2+1)))*x^6+15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1
)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^2-15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(
a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*
csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))*x^4+9485/55296*a^5
/(a*x-1)^3/(a*x+1)^3*x^6-3605/18432*a^3/(a*x-1)^3/(a*x+1)^3*x^4-2795/18432*
a/(a*x-1)^3/(a*x+1)^3*x^2-299/384/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x-5/64/a
/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^4+299/768/a/(a*x-1)^3/(a*x+1)^3*arctanh(a
```

```
*x)^2+15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^4-5/64*I/a/(a*x-1)^3/(a*x
+1)^3*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2
/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))-15/6
4*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1
)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-15/64*I*
a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^4+15/64*I*a^3/(a*
x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*
x+1)/(-a^2*x^2+1)^(1/2))^2*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)
^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a
*x+1)^2/(-a^2*x^2+1)))^2*x^4-15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*P
i*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2+15
/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^2-15/64*I*a/(
a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2-5/32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x
)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x
^6+5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^6-5/64*
I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arct
anh(a*x)^3*Pi*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^2*x^6-5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctan
h(a*x)^3+5/32/a*arctanh(a*x)^3*ln(a*x+1)-5/16/a*arctanh(a*x)^3*ln((a*x+1)/(
-a^2*x^2+1)^(1/2))+15/64*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^4*x^2+25/18*a^2
/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x^3+9971/55296/a/(a*x-1)^3/(a*x+1)^3-1/48
/a*arctanh(a*x)^3/(a*x-1)^3+1/16/a*arctanh(a*x)^3/(a*x-1)^2-5/32/a*arctanh(
a*x)^3/(a*x-1)-5/32/a*arctanh(a*x)^3*ln(a*x-1)-1/48/a*arctanh(a*x)^3/(a*x+1
)^3-1/16/a*arctanh(a*x)^3/(a*x+1)^2-35/256*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*
x)^2*x^2+5/64*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^4*x^6+245/768*a^5/(a*x-1
)^3/(a*x+1)^3*arctanh(a*x)^2*x^6-15/64*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)
^4*x^4-245/384*a^4/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x^5-125/256*a^3/(a*x-1)
^3/(a*x+1)^3*arctanh(a*x)^2*x^4-5/32/a*arctanh(a*x)^3/(a*x+1)-15/32*I*a^3/(
a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*x^4+15/32*I*a/(a*x-1)^3/(a*x+1)^3*arct
anh(a*x)^3*Pi*x^2+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^3-5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I
/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)
^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(1+(a*x+1)^2/(-a^2*x^2+1)))^3+5/32*I/a/(a*x
-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I/(1+(a*x+1)^2/(-a^2*x^2+1)))^2+5/32
*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*x^6
```

maxima [B] time = 0.37, size = 871, normalized size = 2.99

$$-\frac{1}{96}\left(\frac{2\left(15\,a^4x^5-40\,a^2x^3+33\,x\right)}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}-\frac{15\,\log\left(ax+1\right)}{a}+\frac{15\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^3+\frac{\left(180\,a^4x^4-420\,a^2x^2-45\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^3 + 1/384*(180*a
^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1
)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1) -
45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a*arctanh(a*
x)^2/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2) + 1/27648*((8820*a^4*x^4 - 135
*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^4 + 540*(a^6*x^6 - 3*a^
4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3*log(a*x - 1) - 135*(a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^4 - 18420*a^2*x^2 - 45*(49*a^6*x^6 - 147*a
```

```
^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1
)^2 - 49)*log(a*x + 1)^2 - 2205*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a
*x - 1)^2 + 90*(6*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 49
*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1) + 9728)*a
^2/(a^10*x^6 - 3*a^8*x^4 + 3*a^6*x^2 - a^4) - 12*(1470*a^5*x^5 - 3200*a^3*x
^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3 + 270*(a^6*x^6
 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a*x - 1) + 90*(a^6*x^6 - 3
*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a*x - 15*(49*a^6*x^6 - 147*
a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x -
1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
 - 1))*a*arctanh(a*x)/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x^2 - a^3))*a
```

mupad [B] time = 2.97, size = 1041, normalized size = 3.58

$$\frac{\frac{1216}{3a}-\frac{1535\,a\,x^2}{2}+\frac{735\,a^3\,x^4}{2}}{1152\,a^6\,x^6-3456\,a^4\,x^4+3456\,a^2\,x^2-1152}-\ln\left(1-a\,x\right)^3\left(\frac{5\,\ln\left(a\,x+1\right)}{256\,a}-\frac{\frac{5\,a^4\,x^5}{16}-\frac{5\,a^2\,x^3}{6}+\frac{11\,x}{16}}{8\,a^6\,x^6-24\,a^4\,x^4+24\,a^2\,x^2-8}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(a^2*x^2 - 1)^4,x)
```

```
[Out] (1216/(3*a) - (1535*a*x^2)/2 + (735*a^3*x^4)/2)/(3456*a^2*x^2 - 3456*a^4*x^
4 + 1152*a^6*x^6 - 1152) - log(1 - a*x)^3*((5*log(a*x + 1))/(256*a) - ((11*
x)/16 - (5*a^2*x^3)/6 + (5*a^4*x^5)/16)/(24*a^2*x^2 - 24*a^4*x^4 + 8*a^6*x^
6 - 8)) + (5*log(a*x + 1)^4)/(1024*a) + (5*log(1 - a*x)^4)/(1024*a) + log(1
 - a*x)^2*((15*log(a*x + 1)^2)/(512*a) + 245/(3072*a) + ((37*x)/2 - 35*a*x^
2 + 68/(3*a) - (82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(768*a^2*x^2 -
 768*a^4*x^4 + 256*a^6*x^6 - 256) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*a
^2*x^3)/3 - 15*a^3*x^4 + (23*a^4*x^5)/2)/(768*a^2*x^2 - 768*a^4*x^4 + 256*a
^6*x^6 - 256) - (log(a*x + 1)*(66*x - 80*a^2*x^3 + 30*a^4*x^5))/(768*a^2*x^
2 - 768*a^4*x^4 + 256*a^6*x^6 - 256)) + log(a*x + 1)^2*((17/(96*a^2) - (35*
x^2)/128 + (15*a^2*x^4)/128)/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6) + 245/(3
072*a)) + log(1 - a*x)*((36*x + 22*a*x^2 - 23/(2*a) - 67*a^2*x^3 - (21*a^3*
x^4)/2 + 31*a^4*x^5)/(2304*a^2*x^2 - 2304*a^4*x^4 + 768*a^6*x^6 - 768) - (5
*log(a*x + 1)^3)/(256*a) - log(a*x + 1)*(((37*x)/2 - 35*a*x^2 + 68/(3*a) -
(82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(384*a^2*x^2 - 384*a^4*x^4 +
128*a^6*x^6 - 128) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*a^2*x^3)/3 - 15*
a^3*x^4 + (23*a^4*x^5)/2)/(384*a^2*x^2 - 384*a^4*x^4 + 128*a^6*x^6 - 128) +
 (245*(3*a^2*x^2 - 3*a^4*x^4 + a^6*x^6 - 1))/(12*a*(384*a^2*x^2 - 384*a^4*x
^4 + 128*a^6*x^6 - 128))) + ((227*x)/2 + 173*a*x^2 - 593/(6*a) - (599*a^2*x
^3)/3 - (159*a^3*x^4)/2 + (183*a^4*x^5)/2)/(2304*a^2*x^2 - 2304*a^4*x^4 + 7
68*a^6*x^6 - 768) + ((299*x)/2 - 195*a*x^2 + 331/(3*a) - (800*a^2*x^3)/3 +
90*a^3*x^4 + (245*a^4*x^5)/2)/(2304*a^2*x^2 - 2304*a^4*x^4 + 768*a^6*x^6 -
768) + (log(a*x + 1)^2*(66*x - 80*a^2*x^3 + 30*a^4*x^5))/(768*a^2*x^2 - 768
*a^4*x^4 + 256*a^6*x^6 - 256)) - (log(a*x + 1)*((299*x)/(768*a) - (25*a*x^3
)/36 + (245*a^3*x^5)/768))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6) - (log(a*x
 + 1)^3*((11*x)/(128*a) - (5*a*x^3)/48 + (5*a^3*x^5)/128))/(3*a*x^2 - 1/a -
 3*a^3*x^4 + a^5*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{(ax-1)^4\,(ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)**3/((a*x - 1)**4*(a*x + 1)**4), x)
```

3.348 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^4}\,dx$

Optimal. Leaf size=252

$$\frac{3\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\,\mathrm{erf}\left(\sqrt{6}\sqrt{\tanh^{-1}(ax)}\right)}{768a}-\frac{3\sqrt{\pi}\,\mathrm{erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}$$

```
[Out] 5/24*arctanh(a*x)^(3/2)/a+1/4608*erf(6^(1/2)*arctanh(a*x)^(1/2))*6^(1/2)*Pi
^(1/2)/a-1/4608*erfi(6^(1/2)*arctanh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a+15/512*
erf(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a-15/512*erfi(2^(1/2)*arct
anh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+3/512*erf(2*arctanh(a*x)^(1/2))*Pi^(1/2)
/a-3/512*erfi(2*arctanh(a*x)^(1/2))*Pi^(1/2)/a+15/64*sinh(2*arctanh(a*x))*a
rctanh(a*x)^(1/2)/a+3/64*sinh(4*arctanh(a*x))*arctanh(a*x)^(1/2)/a+1/192*si
nh(6*arctanh(a*x))*arctanh(a*x)^(1/2)/a
```

Rubi [A] time = 0.30, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5968, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\,\mathrm{Erf}\left(\sqrt{6}\sqrt{\tanh^{-1}(ax)}\right)}{768a}-\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```

```
[Out] (5*ArcTanh[a*x]^(3/2))/(24*a) + (3*Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(512
*a) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a) + (Sqrt[Pi/6]
*Erf[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) - (3*Sqrt[Pi]*Erfi[2*Sqrt[ArcTanh
[a*x]]])/(512*a) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a)
 - (Sqrt[Pi/6]*Erfi[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) + (15*Sqrt[ArcTanh
[a*x]]*Sinh[2*ArcTanh[a*x]])/(64*a) + (3*Sqrt[ArcTanh[a*x]]*Sinh[4*ArcTanh[
a*x]])/(64*a) + (Sqrt[ArcTanh[a*x]]*Sinh[6*ArcTanh[a*x]])/(192*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^4}\,dx = \frac{\text{Subst}\left(\int \sqrt{x}\cosh^6(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5\sqrt{x}}{16}+\frac{15}{32}\sqrt{x}\cosh(2x)+\frac{3}{16}\sqrt{x}\cosh(4x)+\frac{1}{32}\sqrt{x}\cosh(6x)\right)dx, x, \tanh^{-1}(a\right.}{a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{\text{Subst}\left(\int \sqrt{x}\cosh(6x)\,dx, x, \tanh^{-1}(ax)\right)}{32a}+\frac{3\,\text{Subst}\left(\int \sqrt{x}\cosh(4x\right.}{16a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{15\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{64a}+\frac{3\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh\right.}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{15\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{64a}+\frac{3\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh\right.}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{15\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{64a}+\frac{3\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh\right.}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{3\sqrt{\pi}\,\text{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\,\text{er}}{}$$

Mathematica [A] time = 0.75, size = 257, normalized size = 1.02

$$-\frac{3168x\sqrt{\tanh^{-1}(ax)}}{\left(a^2x^2-1\right)^3}+\frac{3840a^2x^3\sqrt{\tanh^{-1}(ax)}}{\left(a^2x^2-1\right)^3}-\frac{1440a^4x^5\sqrt{\tanh^{-1}(ax)}}{\left(a^2x^2-1\right)^3}+\frac{960\tanh^{-1}(ax)^{3/2}}{a}+\frac{\sqrt{6}\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-6\tanh^{-1}(ax)\right)}{a\sqrt{-\tanh^{-1}(ax)}}+\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```

```
[Out] ((-3168*x*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^2)^3 + (3840*a^2*x^3*Sqrt[ArcTanh
[a*x]])/(-1 + a^2*x^2)^3 - (1440*a^4*x^5*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^2)
```

```
^3 + (960*ArcTanh[a*x]^(3/2))/a + (Sqrt[6]*Sqrt[ArcTanh[a*x]]*Gamma[1/2, -6
*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (27*Sqrt[ArcTanh[a*x]]*Gamma[1/2,
 -4*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (135*Sqrt[2]*Sqrt[ArcTanh[a*x]
]*Gamma[1/2, -2*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) - (135*Sqrt[2]*Gamma
[1/2, 2*ArcTanh[a*x]])/a - (27*Gamma[1/2, 4*ArcTanh[a*x]])/a - (Sqrt[6]*Gam
ma[1/2, 6*ArcTanh[a*x]])/a)/4608
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{\left(-a^2x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\operatorname{atanh}(a\,x)}}{\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^4,x)
```

```
[Out] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}(ax)}}{(ax-1)^4(ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**4,x)
[Out] Integral(sqrt(atanh(a*x))/((a*x - 1)**4*(a*x + 1)**4), x)
```

3.349 $\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Defer[Int][x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx = \int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 8.52, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^8}{(a^8x^8 - 4a^6x^6 + 6a^4x^4 - 4a^2x^2 + 1) \operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
[Out] integral(x^8/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*x
)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a^2x^2 - 1)^4 \operatorname{artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-a^2x^2+1\right)^4 \operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
[Out] int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^8}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x^8/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(ax-1)^4\,(ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(x**8/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.350 $\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^7}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^7}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Defer[Int][x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^7}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx = \int \frac{x^7}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 33.24, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\left(a^8x^8 - 4\,a^6x^6 + 6\,a^4x^4 - 4\,a^2x^2 + 1\right) \text{artanh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
[Out] integral(x^7/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*x
)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a^2x^2-1\right)^4 \text{artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(-a^{2} x^{2}+1\right)^{4} \operatorname{arctanh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
[Out] int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{{\left(a^{2} x^{2}-1\right)}^{4} \operatorname{artanh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^7}{\mathrm{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x^7/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(a x - 1\right)^{4} \left(a x + 1\right)^{4} \operatorname{atanh}{\left (a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(x**7/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.351 $\int \frac{x^6}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=55

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7}-\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7}-\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7}$$

```
[Out] 15/32*Chi(2*arctanh(a*x))/a^7-3/16*Chi(4*arctanh(a*x))/a^7+1/32*Chi(6*arcta
nh(a*x))/a^7-5/16*ln(arctanh(a*x))/a^7
```

Rubi [A] time = 0.15, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 3312, 3301}

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7}-\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7}-\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a^7) - (3*CoshIntegral[4*ArcTanh[a*x]
])/(16*a^7) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^7) - (5*Log[ArcTanh[a*x]])
/(16*a^7)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^6(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^7}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{5}{16x}-\frac{15\cosh(2x)}{32x}+\frac{3\cosh(4x)}{16x}-\frac{\cosh(6x)}{32x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^7}$$
$$= -\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^7}-\frac{3\,\text{Subst}\left(\int \frac{\cosh}{x}\right.}{}$$
$$= \frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7}-\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7}-\frac{5\log(}{}$$

Mathematica [A] time = 0.17, size = 40, normalized size = 0.73

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)-6\text{Chi}\left(4\tanh^{-1}(ax)\right)+\text{Chi}\left(6\tanh^{-1}(ax)\right)-10\log\left(\tanh^{-1}(ax)\right)}{32a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]] - 6*CoshIntegral[4*ArcTanh[a*x]] + CoshInt
egral[6*ArcTanh[a*x]] - 10*Log[ArcTanh[a*x]])/(32*a^7)
```

fricas [B] time = 0.94, size = 220, normalized size = 4.00

$$-\frac{20\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)-\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+6\,\text{log\_integ}}{64\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*x
^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_integr
al((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x^2 -
 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 15*log_integral(-(a*x + 1)/(a*x - 1))
- 15*log_integral(-(a*x - 1)/(a*x + 1)))/a^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a^2x^2-1\right)^4\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.30, size = 48, normalized size = 0.87

$$\frac{15\text{X}\,(2\,\text{arctanh}\,(ax))}{32a^7}-\frac{3\text{X}\,(4\,\text{arctanh}\,(ax))}{16a^7}+\frac{\text{X}\,(6\,\text{arctanh}\,(ax))}{32a^7}-\frac{5\ln\,(\text{arctanh}\,(ax))}{16a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 15/32*Chi(2*arctanh(a*x))/a^7-3/16*Chi(4*arctanh(a*x))/a^7+1/32*Chi(6*arcta
nh(a*x))/a^7-5/16*ln(arctanh(a*x))/a^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(ax-1)^4\,(ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**6/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.352 $\int \frac{x^5}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^6}-\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^6}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

```
[Out] 5/32*Shi(2*arctanh(a*x))/a^6-1/8*Shi(4*arctanh(a*x))/a^6+1/32*Shi(6*arctanh
(a*x))/a^6
```

Rubi [A] time = 0.14, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3298}

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^6}-\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^6}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^6) - SinhIntegral[4*ArcTanh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^6)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} - \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x} dx, x, \tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x} dx, x, \tanh^{-1}(ax)\right)}{8a^6} + \frac{5\,\text{Su}}{}$$
$$= \frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^6} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Mathematica [A] time = 0.17, size = 33, normalized size = 0.77

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right) - 4\text{Shi}\left(4\tanh^{-1}(ax)\right) + \text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]] - 4*SinhIntegral[4*ArcTanh[a*x]] + SinhInte
gral[6*ArcTanh[a*x]])/(32*a^6)
```

fricas [B] time = 0.51, size = 200, normalized size = 4.65

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right) - 4\,\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right) + 4\,\text{log\_int}}{64\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) - 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2
*a*x + 1)) + 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) +
5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x + 1))
)/a^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a^2x^2-1)^4 \operatorname{artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.23, size = 33, normalized size = 0.77

$$\frac{-\frac{\text{Shi}(4\operatorname{arctanh}(ax))}{8} + \frac{\text{Shi}(6\operatorname{arctanh}(ax))}{32} + \frac{5\,\text{Shi}(2\operatorname{arctanh}(ax))}{32}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 1/a^6*(-1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(a*x))+5/32*Shi(2*arctanh
(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(ax-1)^4\,(ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**5/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.353 $\int \frac{x^4}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=55

$$-\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5}-\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5}+\frac{\mathrm{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5}+\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5}$$

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^5-1/16*Chi(4*arctanh(a*x))/a^5+1/32*Chi(6*arcta
nh(a*x))/a^5+1/16*ln(arctanh(a*x))/a^5
```

Rubi **[A]** time = 0.15, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3301}

$$-\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5}-\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5}+\frac{\mathrm{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5}+\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(32*a^5) - CoshIntegral[4*ArcTanh[a*x]]/(16*a
^5) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^5) + Log[ArcTanh[a*x]]/(16*a^5)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16x} - \frac{\cosh(2x)}{32x} - \frac{\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,d\right.}{32a}$$
$$= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5} - \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5} + \frac{\log\left(\tan\right.}{16}$$

Mathematica [A] time = 0.15, size = 40, normalized size = 0.73

$$\frac{-\text{Chi}\left(2\tanh^{-1}(ax)\right) - 2\text{Chi}\left(4\tanh^{-1}(ax)\right) + \text{Chi}\left(6\tanh^{-1}(ax)\right) + 2\log\left(\tanh^{-1}(ax)\right)}{32a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (-CoshIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]] + CoshInteg
ral[6*ArcTanh[a*x]] + 2*Log[ArcTanh[a*x]])/(32*a^5)
```

fricas [B] time = 0.46, size = 216, normalized size = 3.93

$$\frac{4\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right) + \text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right) - 2\,\text{log\_integra}}{64\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x^2
 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^3 -
 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_integral
((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^2 - 2
*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - log_integral(-(a*x + 1)/(a*x - 1)) - log
_integral(-(a*x - 1)/(a*x + 1)))/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.26, size = 48, normalized size = 0.87

$$-\frac{\text{X}(2\operatorname{arctanh}(ax))}{32a^5} - \frac{\text{X}(4\operatorname{arctanh}(ax))}{16a^5} + \frac{\text{X}(6\operatorname{arctanh}(ax))}{32a^5} + \frac{\ln(\operatorname{arctanh}(ax))}{16a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^5-1/16*Chi(4*arctanh(a*x))/a^5+1/32*Chi(6*arcta
nh(a*x))/a^5+1/16*ln(arctanh(a*x))/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a^2x^2-1\right)^4 \operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\operatorname{atanh}\left(a\,x\right)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax-1\right)^4\left(ax+1\right)^4 \operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**4/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.354 $\int \frac{x^3}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}-\frac{3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}$$

```
[Out] -3/32*Shi(2*arctanh(a*x))/a^4+1/32*Shi(6*arctanh(a*x))/a^4
```

Rubi [A] time = 0.13, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3298}

$$\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}-\frac{3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[2*ArcTanh[a*x]])/(32*a^4) + SinhIntegral[6*ArcTanh[a*x]]/(
32*a^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^3(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(2x)}{32x}+\frac{\sinh(6x)}{32x}\right)dx,x,\tanh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{32a^4}-\frac{3\,\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{32a^4}\\
&= -\frac{3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 24, normalized size = 0.83

$$\frac{\mathrm{Shi}\left(6\tanh^{-1}(ax)\right)-3\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] (-3*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[6*ArcTanh[a*x]])/(32*a^4)
```

fricas [B] time = 0.47, size = 136, normalized size = 4.69

$$\frac{\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)-3\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+3\,\mathrm{log\_integra}}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) - 3*log_integral(-(a*x + 1)/(a*x - 1)) + 3*log_inte
gral(-(a*x - 1)/(a*x + 1)))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.19, size = 24, normalized size = 0.83

$$\frac{\frac{\mathrm{Shi}(6\operatorname{arctanh}(ax))}{32}-\frac{3\,\mathrm{Shi}(2\operatorname{arctanh}(ax))}{32}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x)
[Out] 1/a^4*(1/32*Shi(6*arctanh(a*x))-3/32*Shi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\mathrm{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(x**3/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.355 $\int \frac{x^2}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=55

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^3+1/16*Chi(4*arctanh(a*x))/a^3+1/32*Chi(6*arcta
nh(a*x))/a^3-1/16*ln(arctanh(a*x))/a^3
```

Rubi [A] time = 0.14, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6034, 5448, 3301}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(32*a^3) + CoshIntegral[4*ArcTanh[a*x]]/(16*a
^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16x} - \frac{\cosh(2x)}{32x} + \frac{\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\right.}{32}$$

$$= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3} - \frac{\log\left(\tan\right.}{16}$$

Mathematica [A] time = 0.17, size = 55, normalized size = 1.00

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -1/32*CoshIntegral[2*ArcTanh[a*x]]/a^3 + CoshIntegral[4*ArcTanh[a*x]]/(16*a
^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

fricas [B] time = 0.44, size = 216, normalized size = 3.93

$$-\frac{4\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right) - 2\ \text{log\_integr}}{64\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*x^
2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x^3
- 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_integra
l((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^2 -
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + log_integral(-(a*x + 1)/(a*x - 1)) + lo
g_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.26, size = 48, normalized size = 0.87

$$-\frac{\mathrm{X}(2\operatorname{arctanh}(ax))}{32a^3} + \frac{\mathrm{X}(4\operatorname{arctanh}(ax))}{16a^3} + \frac{\mathrm{X}(6\operatorname{arctanh}(ax))}{32a^3} - \frac{\ln(\operatorname{arctanh}(ax))}{16a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^3+1/16*Chi(4*arctanh(a*x))/a^3+1/32*Chi(6*arcta
nh(a*x))/a^3-1/16*ln(arctanh(a*x))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**2/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.356 $\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

```
[Out] 5/32*Shi(2*arctanh(a*x))/a^2+1/8*Shi(4*arctanh(a*x))/a^2+1/32*Shi(6*arctanh
(a*x))/a^2
```

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6034, 5448, 3298}

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8
*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x}+\frac{\sinh(4x)}{8x}+\frac{\sinh(6x)}{32x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^2}+\frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^2}+\cdots\\
&= \frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 43, normalized size = 1.00

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8
*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

fricas [B] time = 0.57, size = 200, normalized size = 4.65

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+4\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-4\,\text{log\_int}}{64\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) + 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2
*a*x + 1)) - 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) +
5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x + 1))
)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(a^2x^2-1\right)^4\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.23, size = 33, normalized size = 0.77

$$\frac{\frac{\text{Shi}(4\,\text{arctanh}(ax))}{8}+\frac{\text{Shi}(6\,\text{arctanh}(ax))}{32}+\frac{5\,\text{Shi}(2\,\text{arctanh}(ax))}{32}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 1/a^2*(1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(a*x))+5/32*Shi(2*arctanh(
a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(a^2x^2-1\right)^4\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax-1)^4\,(ax+1)^4\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.357 $\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a}+\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] 15/32*Chi(2*arctanh(a*x))/a+3/16*Chi(4*arctanh(a*x))/a+1/32*Chi(6*arctanh(a
*x))/a+5/16*ln(arctanh(a*x))/a
```

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5968, 3312, 3301}

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a}+\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a) + (3*CoshIntegral[4*ArcTanh[a*x]])
/(16*a) + CoshIntegral[6*ArcTanh[a*x]]/(32*a) + (5*Log[ArcTanh[a*x]])/(16*a
)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Subst}\left(\int \left(\frac{5}{16x}+\frac{15\cosh(2x)}{32x}+\frac{3\cosh(4x)}{16x}+\frac{\cosh(6x)}{32x}\right)dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= \frac{5\log\left(\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{32a}+\frac{3\,\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,d\right.}{16\right.}\\ &= \frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a}+\frac{5\log\left(\tan\right.}{16}\end{aligned}$$

Mathematica [A] time = 0.18, size = 40, normalized size = 0.73

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)+6\text{Chi}\left(4\tanh^{-1}(ax)\right)+\text{Chi}\left(6\tanh^{-1}(ax)\right)+10\log\left(\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]] + 6*CoshIntegral[4*ArcTanh[a*x]] + CoshInt
egral[6*ArcTanh[a*x]] + 10*Log[ArcTanh[a*x]])/(32*a)
```

fricas [B] time = 0.67, size = 216, normalized size = 3.93

$$\frac{20\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right)+\text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right)+6\text{ log\_integr}}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x^
2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^3
- 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_integra
l((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x^2 -
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 15*log_integral(-(a*x + 1)/(a*x - 1)) +
 15*log_integral(-(a*x - 1)/(a*x + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

maple [A] time = 0.28, size = 48, normalized size = 0.87

$$\frac{15\text{X}(2\operatorname{arctanh}(ax))}{32a}+\frac{3\text{X}(4\operatorname{arctanh}(ax))}{16a}+\frac{\text{X}(6\operatorname{arctanh}(ax))}{32a}+\frac{5\ln(\operatorname{arctanh}(ax))}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 15/32*Chi(2*arctanh(a*x))/a+3/16*Chi(4*arctanh(a*x))/a+1/32*Chi(6*arctanh(a
*x))/a+5/16*ln(arctanh(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(1/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^4\,(ax+1)^4\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.358 $\int \frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx = \int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^8x^9-4\,a^6x^7+6\,a^4x^5-4\,a^2x^3+x\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
[Out] integral(1/((a^8*x^9 - 4*a^6*x^7 + 6*a^4*x^5 - 4*a^2*x^3 + x)*arctanh(a*x))
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^4 x\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

maple [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2}x^{2}+1\right)^{4}\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
[Out] int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}x^{2}-1\right)^{4}x\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\left(a^{2}\,x^{2}-1\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax-1\right)^{4}\left(ax+1\right)^{4}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(1/(x*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.359 $\int \frac{1}{x^2(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^2 \left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}, x\right)$$

[Out] Unintegrable(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]

[Out] Defer[Int][1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]

Rubi steps

$$\int \frac{1}{x^2 \left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx = \int \frac{1}{x^2 \left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 3.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]

[Out] Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^8x^{10} - 4\,a^6x^8 + 6\,a^4x^6 - 4\,a^2x^4 + x^2\right)\text{artanh}\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")

[Out] integral(1/((a^8*x^10 - 4*a^6*x^8 + 6*a^4*x^6 - 4*a^2*x^4 + x^2)*arctanh(a*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^4 x^2 \,\text{artanh}\left(ax\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-a^{2} x^{2}+1\right)^{4} \operatorname{arctanh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
[Out] int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(a^{2} x^{2} - 1\right)}^{4} x^{2} \operatorname{artanh}\left(a x\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\,\mathrm{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(1/(x^2*atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a x - 1\right)^{4} \left(a x + 1\right)^{4} \operatorname{atanh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(1/(x**2*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

3.360 $\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=67

$$\frac{5\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}+\frac{3\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}$$

```
[Out] -x/a/(-a^2*x^2+1)^3/arctanh(a*x)+5/16*Chi(2*arctanh(a*x))/a^2+1/2*Chi(4*arc
tanh(a*x))/a^2+3/16*Chi(6*arctanh(a*x))/a^2
```

Rubi [A] time = 0.31, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6032, 6034, 5448, 3301, 5968, 3312}

$$\frac{5\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}+\frac{3\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (5*CoshIntegral[2*ArcTanh[a*x]])/(1
6*a^2) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2) + (3*CoshIntegral[6*ArcTanh[a
*x]])/(16*a^2)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
```

```
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^2}\,dx = -\frac{x}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx}{a} + (5a)\int \frac{x^2}{\left(1-a^2x^2\right)^4 \tanh^{-1}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{5\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{5}{16x} + \frac{15\cosh(2x)}{32x} + \frac{3\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right) d\right.}{a^2}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^2} - \frac{5\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{5\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a^2} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2} + \frac{3\text{Ch}}{}$$

Mathematica [A] time = 0.20, size = 56, normalized size = 0.84

$$\frac{\frac{16ax}{\left(a^2x^2-1\right)^3 \tanh^{-1}(ax)} + 5\text{Chi}\left(2\tanh^{-1}(ax)\right) + 8\text{Chi}\left(4\tanh^{-1}(ax)\right) + 3\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] ((16*a*x)/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + 5*CoshIntegral[2*ArcTanh[a*x]]
+ 8*CoshIntegral[4*ArcTanh[a*x]] + 3*CoshIntegral[6*ArcTanh[a*x]])/(16*a^2)
```

fricas [B] time = 0.59, size = 418, normalized size = 6.24

$$\frac{64\,ax + \left(3\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) + 3\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{log\_int}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(64*a*x + (3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*
x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 3*(a^6*x^
6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
```

```
1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 8*(a^6*x^
6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2
+ 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*
x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^4 + 3*a^4
*x^2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2x^2-1\right)^4 \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)
```

maple [A] time = 0.23, size = 78, normalized size = 1.16

$$\frac{-\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{X(4\operatorname{arctanh}(ax))}{2}-\frac{\sinh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{3X(6\operatorname{arctanh}(ax))}{16}-\frac{5\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{5X(2\operatorname{arctanh}(ax))}{16}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x)
```

```
[Out] 1/a^2*(-1/8/arctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-1/32/
arctanh(a*x)*sinh(6*arctanh(a*x))+3/16*Chi(6*arctanh(a*x))-5/32*sinh(2*arct
anh(a*x))/arctanh(a*x)+5/16*Chi(2*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(ax+1\right)-\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(-ax+1\right)}-\int -\frac{}{\left(a^9x^8-4\,a^7x^6+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*
x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)) - integrate(-2*(5*a^2*x^2 + 1)/((a^9*x^
8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(a*x + 1) - (a^9*x^8 - 4*a^7*
x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax-1\right)^4\left(ax+1\right)^4\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)
```

3.361 $\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=66

$$-\frac{1}{a(1-a^2x^2)^3 \tanh^{-1}(ax)}+\frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] -1/a/(-a^2*x^2+1)^3/arctanh(a*x)+15/16*Shi(2*arctanh(a*x))/a+3/4*Shi(4*arct
anh(a*x))/a+3/16*Shi(6*arctanh(a*x))/a
```

Rubi [A] time = 0.14, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a(1-a^2x^2)^3 \tanh^{-1}(ax)}+\frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (15*SinhIntegral[2*ArcTanh[a*x]])/(
16*a) + (3*SinhIntegral[4*ArcTanh[a*x]])/(4*a) + (3*SinhIntegral[6*ArcTanh[
a*x]])/(16*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + (6a)\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{6\,\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{6\,\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} + \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right) dx, x, \text{tar}\right.}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a} + \frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a} + \frac{3\text{S}}{}$$

Mathematica [A] time = 0.30, size = 56, normalized size = 0.85

$$\frac{\frac{1}{\left(a^2x^2-1\right)^3 \tanh^{-1}(ax)} + \frac{15}{16}\text{Shi}\left(2\tanh^{-1}(ax)\right) + \frac{3}{4}\text{Shi}\left(4\tanh^{-1}(ax)\right) + \frac{3}{16}\text{Shi}\left(6\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] (1/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + (15*SinhIntegral[2*ArcTanh[a*x]])/16 +
 (3*SinhIntegral[4*ArcTanh[a*x]])/4 + (3*SinhIntegral[6*ArcTanh[a*x]])/16)/
a
```

fricas [B] time = 0.49, size = 413, normalized size = 6.26

$$\frac{3\left(\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) - \left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{a}{a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(3*((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - (a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^
3 + 3*a^2*x^2 + 3*a*x + 1)) + 4*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_i
ntegral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1
)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x -
 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x
 + 1)))*log(-(a*x + 1)/(a*x - 1)) + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 -
 a)*log(-(a*x + 1)/(a*x - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")

[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)

maple [A] time = 0.28, size = 86, normalized size = 1.30

$$\frac{-\frac{5}{16\,\mathrm{arctanh}(ax)}-\frac{15\cosh(2\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}+\frac{15\,\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{16}-\frac{3\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)}+\frac{3\,\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{4}-\frac{\cosh(6\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x)

[Out] 1/a*(-5/16/arctanh(a*x)-15/32/arctanh(a*x)*cosh(2*arctanh(a*x))+15/16*Shi(2*arctanh(a*x))-3/16/arctanh(a*x)*cosh(4*arctanh(a*x))+3/4*Shi(4*arctanh(a*x))-1/32/arctanh(a*x)*cosh(6*arctanh(a*x))+3/16*Shi(6*arctanh(a*x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-12\,a\int -\frac{x}{\left(a^8x^8-4\,a^6x^6+6\,a^4x^4-4\,a^2x^2+1\right)\log\left(ax+1\right)-\left(a^8x^8-4\,a^6x^6+6\,a^4x^4-4\,a^2x^2+1\right)\log\left(-ax+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")

[Out] -12*a*integrate(-x/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^4),x)

[Out] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^4), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^4\left(ax+1\right)^4\mathrm{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**2,x)

[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)

3.362 $\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^3}\, dx$

Optimal. Leaf size=114

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}+\frac{9\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}+\frac{5}{2a^2\left(1-a^2x^2\right)^2 \tan}$$

```
[Out] -1/2*x/a/(-a^2*x^2+1)^3/arctanh(a*x)^2-3/a^2/(-a^2*x^2+1)^3/arctanh(a*x)+5/
2/a^2/(-a^2*x^2+1)^2/arctanh(a*x)+5/16*Shi(2*arctanh(a*x))/a^2+Shi(4*arctan
h(a*x))/a^2+9/16*Shi(6*arctanh(a*x))/a^2
```

Rubi [A] time = 0.59, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6032, 6028, 5966, 6034, 5448, 3298}

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}+\frac{9\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}+\frac{5}{2a^2\left(1-a^2x^2\right)^2 \tan}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - 3/(a^2*(1 - a^2*x^2)^3*ArcTanh[a*
x]) + 5/(2*a^2*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (5*SinhIntegral[2*ArcTanh[a*
x]])/(16*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2 + (9*SinhIntegral[6*ArcTan
h[a*x]])/(16*a^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)^2}\,dx}{2a} + \frac{1}{2}(5a)\int \frac{}{\left(1-a^2x^2\right.}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2} - \frac{1}{2a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + 3\int \frac{}{\left(1-a^2x^2\right)^{}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right)^2\text{t}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right)^2\text{t}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right)^2\text{t}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right)^2\text{t}}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right)^2\text{t}}$$

Mathematica [A] time = 0.38, size = 73, normalized size = 0.64

$$\frac{\frac{8\left(\left(5a^2x^2+1\right)\tanh^{-1}(ax)+ax\right)}{\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2} + 5\text{Shi}\left(2\tanh^{-1}(ax)\right) + 16\text{Shi}\left(4\tanh^{-1}(ax)\right) + 9\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] ((8*(a*x + (1 + 5*a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^3*ArcTanh[a*x]^2)
 + 5*SinhIntegral[2*ArcTanh[a*x]] + 16*SinhIntegral[4*ArcTanh[a*x]] + 9*Sin
hIntegral[6*ArcTanh[a*x]])/(16*a^2)
```

fricas [B] time = 0.51, size = 447, normalized size = 3.92

$$\frac{\left(9\left(a^6x^6-3a^4x^4+3a^2x^2-1\right)\text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right)-9\left(a^6x^6-3a^4x^4+3a^2x^2-1\right)\text{log\_integral}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((9*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - 9*(a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*
x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 16*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*lo
g_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 16*(a^6*x^6 - 3*a
^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x
 + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a
*x - 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x - 1)/
(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 64*a*x + 32*(5*a^2*x^2 + 1)*log(-
(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-(a*x +
1)/(a*x - 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

maple [A] time = 0.24, size = 121, normalized size = 1.06

$$\frac{-\frac{5\sinh(2\operatorname{arctanh}(ax))}{64\operatorname{arctanh}(ax)^2}-\frac{5\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{5\operatorname{Shi}(2\operatorname{arctanh}(ax))}{16}-\frac{\sinh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\cosh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\operatorname{Shi}(4\operatorname{arctanh}(a}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x)
```

```
[Out] 1/a^2*(-5/64*sinh(2*arctanh(a*x))/arctanh(a*x)^2-5/32/arctanh(a*x)*cosh(2*a
rctanh(a*x))+5/16*Shi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*
x))-1/4/arctanh(a*x)*cosh(4*arctanh(a*x))+Shi(4*arctanh(a*x))-1/64/arctanh(
a*x)^2*sinh(6*arctanh(a*x))-3/32/arctanh(a*x)*cosh(6*arctanh(a*x))+9/16*Shi
(6*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2ax+\left(5a^2x^2+1\right)\log(ax+1)-\left(5a^2x^2+1\right)\log(-ax+1)}{\left(a^8x^6-3a^6x^4+3a^4x^2-a^2\right)\log(ax+1)^2-2\left(a^8x^6-3a^6x^4+3a^4x^2-a^2\right)\log(ax+1)\log(-ax+1)+\left(a^8x^6-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (5*a^2*x^2 + 1)*log(a*x + 1) - (5*a^2*x^2 + 1)*log(-a*x + 1))/((a^
8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^8*x^6 - 3*a^6*x^
4 + 3*a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^8*x^6 - 3*a^6*x^4 + 3*
```

```
a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-4*(5*a^2*x^3 + 4*x)/((a^8*x^8
- 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^
6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax-1\right)^{4}\left(ax+1\right)^{4}\operatorname{atanh}^{3}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

3.363 $\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=89

$$-\frac{3x}{(1-a^2x^2)^3 \tanh^{-1}(ax)}-\frac{1}{2a(1-a^2x^2)^3 \tanh^{-1}(ax)^2}+\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a}+\frac{9\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^3/arctanh(a*x)^2-3*x/(-a^2*x^2+1)^3/arctanh(a*x)+15/16*
Chi(2*arctanh(a*x))/a+3/2*Chi(4*arctanh(a*x))/a+9/16*Chi(6*arctanh(a*x))/a
```

Rubi [A] time = 0.39, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{3x}{(1-a^2x^2)^3 \tanh^{-1}(ax)}-\frac{1}{2a(1-a^2x^2)^3 \tanh^{-1}(ax)^2}+\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a}+\frac{9\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - (3*x)/((1 - a^2*x^2)^3*ArcTanh[a*
x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/(16*a) + (3*CoshIntegral[4*ArcTanh[
a*x]])/(2*a) + (9*CoshIntegral[6*ArcTanh[a*x]])/(16*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
```

```
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} + (3a)\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + 3\int \frac{1}{\left(1-a^2x^2\right)^4 \tan}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh^6(x)}{x}\right.}{a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \left(\frac{5}{16x} + \right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(6x)}{x}\right.}{32}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{15\mathrm{Chi}\left(2\tanh^{-1}(a\right.}{16a}$$

Mathematica [A] time = 0.20, size = 83, normalized size = 0.93

$$\frac{1}{16}\left(\frac{48x}{\left(a^2x^2-1\right)^3 \tanh^{-1}(ax)} + \frac{8}{a\left(a^2x^2-1\right)^3 \tanh^{-1}(ax)^2} + \frac{15\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{a} + \frac{24\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{a} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] (8/(a*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2) + (48*x)/((-1 + a^2*x^2)^3*ArcTanh[a
*x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/a + (24*CoshIntegral[4*ArcTanh[a*x
]])/a + (9*CoshIntegral[6*ArcTanh[a*x]])/a)/16
```

fricas [B] time = 0.71, size = 435, normalized size = 4.89

$$\frac{192\,ax\log\left(-\frac{ax+1}{ax-1}\right)+3\left(3\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+3\left(a^6x^6-3\,a^4x^4+3\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(192*a*x*log(-(a*x + 1)/(a*x - 1)) + 3*(3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2
*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)) + 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(
a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8*(a^
6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a
^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2
*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a
^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2
 + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-1\right)^4\operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

maple [A] time = 0.28, size = 131, normalized size = 1.47

$$\frac{-\frac{5}{32\operatorname{arctanh}(ax)^2}-\frac{15\cosh(2\operatorname{arctanh}(ax))}{64\operatorname{arctanh}(ax)^2}-\frac{15\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{15\mathrm{X}(2\operatorname{arctanh}(ax))}{16}-\frac{3\cosh(4\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)^2}-\frac{3\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x)
```

```
[Out] 1/a*(-5/32/arctanh(a*x)^2-15/64/arctanh(a*x)^2*cosh(2*arctanh(a*x))-15/32*s
inh(2*arctanh(a*x))/arctanh(a*x)+15/16*Chi(2*arctanh(a*x))-3/32/arctanh(a*x
)^2*cosh(4*arctanh(a*x))-3/8/arctanh(a*x)*sinh(4*arctanh(a*x))+3/2*Chi(4*ar
ctanh(a*x))-1/64/arctanh(a*x)^2*cosh(6*arctanh(a*x))-3/32/arctanh(a*x)*sinh
(6*arctanh(a*x))+9/16*Chi(6*arctanh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(3\,ax\log\left(ax+1\right)-3\,ax\log\left(-ax+1\right)+1\right)}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(ax+1\right)^2-2\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^7x^6-3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(3*a*x*log(a*x + 1) - 3*a*x*log(-a*x + 1) + 1)/((a^7*x^6 - 3*a^5*x^4 + 3*
a^3*x^2 - a)*log(a*x + 1)^2 - 2*(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a
*x + 1)*log(-a*x + 1) + (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)
^2) - integrate(-6*(5*a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^
```

```
2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)
*log(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^4),x)
[Out] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^4\,(ax+1)^4\,\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

3.364 $\int \frac{x^5 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=139

$$\frac{89 \sin^{-1}(ax)}{120a^6} - \frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2} - \frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6} - \frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4} - \frac{x^3\sqrt{}}{}$$

```
[Out] 89/120*arcsin(a*x)/a^6-5/24*x*(-a^2*x^2+1)^(1/2)/a^5-1/20*x^3*(-a^2*x^2+1)^
(1/2)/a^3-8/15*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^6-4/15*x^2*arctanh(a*x)*(-
a^2*x^2+1)^(1/2)/a^4-1/5*x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.23, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6016, 321, 216, 5994}

$$-\frac{x^3\sqrt{1-a^2x^2}}{20a^3} - \frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2} - \frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4} - \frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6} + \frac{89}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-5*x*Sqrt[1 - a^2*x^2])/(24*a^5) - (x^3*Sqrt[1 - a^2*x^2])/(20*a^3) + (89*
ArcSin[a*x])/(120*a^6) - (8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^6) - (4*x
^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^4*Sqrt[1 - a^2*x^2]*ArcTan
h[a*x])/(5*a^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int \frac{x^5 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2} + \frac{4\int \frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2} + \frac{\int \frac{x^4}{\sqrt{1-a^2x^2}}dx}{5a}$$
$$= -\frac{x^3\sqrt{1-a^2x^2}}{20a^3} - \frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4} - \frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2} + \frac{8\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}}{15a^4}$$
$$= -\frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{x^3\sqrt{1-a^2x^2}}{20a^3} - \frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6} - \frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}$$
$$= -\frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{x^3\sqrt{1-a^2x^2}}{20a^3} + \frac{89\sin^{-1}(ax)}{120a^6} - \frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6} - \frac{4x^2\sqrt{1-a^2}}{}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 0.57

$$-\frac{ax\sqrt{1-a^2x^2}\left(6a^2x^2+25\right)+8\sqrt{1-a^2x^2}\left(3a^4x^4+4a^2x^2+8\right)\tanh^{-1}(ax)-89\sin^{-1}(ax)}{120a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/120*(a*x*Sqrt[1 - a^2*x^2]*(25 + 6*a^2*x^2) - 89*ArcSin[a*x] + 8*Sqrt[1
- a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/a^6
```

fricas [A] time = 0.60, size = 91, normalized size = 0.65

$$-\frac{\left(6\,a^3x^3+25\,ax+4\left(3\,a^4x^4+4\,a^2x^2+8\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+178\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*((6*a^3*x^3 + 25*a*x + 4*(3*a^4*x^4 + 4*a^2*x^2 + 8)*log(-(a*x + 1)/
(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 178*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))
)/a^6
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.44, size = 120, normalized size = 0.86

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(24a^4x^4\operatorname{arctanh}(ax)+6x^3a^3+32a^2x^2\operatorname{arctanh}(ax)+25ax+64\operatorname{arctanh}(ax)\right)}{120a^6}+\frac{89i\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/120/a^6*(-(a*x-1)*(a*x+1))^(1/2)*(24*a^4*x^4*arctanh(a*x)+6*x^3*a^3+32*a
^2*x^2*arctanh(a*x)+25*a*x+64*arctanh(a*x))+89/120*I*ln((a*x+1)/(-a^2*x^2+1
)^(1/2)+I)/a^6-89/120*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^6
```

maxima [A] time = 0.41, size = 163, normalized size = 1.17

$$-\frac{1}{120}a\left(\frac{3\left(\frac{2\sqrt{-a^2x^2+1}\,x^3}{a^2}+\frac{3\sqrt{-a^2x^2+1}\,x}{a^4}-\frac{3\arcsin(ax)}{a^5}\right)}{a^2}+\frac{16\left(\frac{\sqrt{-a^2x^2+1}\,x}{a^2}-\frac{\arcsin(ax)}{a^3}\right)}{a^4}-\frac{64\arcsin(ax)}{a^7}\right)-\frac{1}{15}\left(\frac{3\sqrt{-a^2x}}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/120*a*(3*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2 + 1)*x/a^4 - 3*
arcsin(a*x)/a^5)/a^2 + 16*(sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)/a^4
- 64*arcsin(a*x)/a^7) - 1/15*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\mathrm{atanh}\left(a\,x\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^5*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\,\mathrm{atanh}\,(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.365 $\int \frac{x^4 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=197

$$-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}-\frac{3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a^5}-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a^5}-\frac{5\sqrt{}}{}$$

```
[Out] 1/12*(-a^2*x^2+1)^(3/2)/a^5-3/4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctan
h(a*x)/a^5-3/8*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+3/8*I*polyl
og(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5-5/8*(-a^2*x^2+1)^(1/2)/a^5-3/8*x*a
rctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/4*x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/
a^2
```

Rubi [A] time = 0.22, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6016, 266, 43, 261, 5950}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a^5}-\frac{5\sqrt{1-a^2x^2}}{8a^5}-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2}-\frac{3x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-5*Sqrt[1 - a^2*x^2])/(8*a^5) + (1 - a^2*x^2)^(3/2)/(12*a^5) - (3*x*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/(8*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4
*a^2) - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^5) - (((3
*I)/8)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (((3*I)/8)*Pol
yLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int \frac{x^4 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2} + \frac{3\int \frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{4a^2} + \frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}\,dx}{4a}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2} + \frac{3\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^4} + \frac{3\int \frac{x}{\sqrt{1-a^2x^2}}\,d}{8a^3}$$

$$= -\frac{3\sqrt{1-a^2x^2}}{8a^5} - \frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2} - \frac{3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\,\mathrm{ta}}{4a^5}$$

$$= -\frac{5\sqrt{1-a^2x^2}}{8a^5} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a^5} - \frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2} - \frac{3}{}$$

Mathematica [A] time = 0.51, size = 160, normalized size = 0.81

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{9i\left(\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}} - 2a^2x^2 - 6ax\left(a^2x^2-1\right)\tanh^{-1}(ax) - \frac{9i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)\right.}{\sqrt{1-a^2x^2}}\right.}{24a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-13 - 2*a^2*x^2 - 15*a*x*ArcTanh[a*x] - 6*a*x*(-1 + a^2
*x^2)*ArcTanh[a*x] - ((9*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1
 + I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((9*I)*(PolyLog[2, (-I)/E^ArcTan
h[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^5)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^4\,\mathrm{artanh}\,(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\mathrm{artanh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.47, size = 175, normalized size = 0.89

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(6a^3x^3\,\mathrm{arctanh}\,(ax)+2a^2x^2+9ax\,\mathrm{arctanh}\,(ax)+13\right)}{24a^5}-\frac{3i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\mathrm{arctanh}\,(a}{8a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/24/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2+9*a*x*
arctanh(a*x)+13)-3/8*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5+
3/8*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5-3/8*I*dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+3/8*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\mathrm{artanh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{atanh}\,(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\mathrm{atanh}\,(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.366 $\int \frac{x^3 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{5\sin^{-1}(ax)}{6a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^4}-\frac{x\sqrt{1-a^2x^2}}{6a^3}$$

```
[Out] 5/6*arcsin(a*x)/a^4-1/6*x*(-a^2*x^2+1)^(1/2)/a^3-2/3*arctanh(a*x)*(-a^2*x^2
+1)^(1/2)/a^4-1/3*x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{6016, 321, 216, 5994\}$

$$-\frac{x\sqrt{1-a^2x^2}}{6a^3}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^4}+\frac{5\sin^{-1}(ax)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x*Sqrt[1 - a^2*x^2])/(6*a^3) + (5*ArcSin[a*x])/(6*a^4) - (2*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{3a^2} + \frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{3a}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{6a^3} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{6a^3} +$$
$$= -\frac{x\sqrt{1-a^2x^2}}{6a^3} + \frac{5\sin^{-1}(ax)}{6a^4} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^2}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.69

$$-\frac{ax\sqrt{1-a^2x^2} + 2\sqrt{1-a^2x^2}\left(a^2x^2+2\right)\tanh^{-1}(ax) - 5\sin^{-1}(ax)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/6*(a*x*Sqrt[1 - a^2*x^2] - 5*ArcSin[a*x] + 2*Sqrt[1 - a^2*x^2]*(2 + a^2*
x^2)*ArcTanh[a*x])/a^4
```

fricas [A] time = 0.49, size = 72, normalized size = 0.83

$$-\frac{\sqrt{-a^2x^2+1}\left(ax + \left(a^2x^2+2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right) + 10\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1))) +
10*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.38, size = 99, normalized size = 1.14

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(2a^2x^2\operatorname{arctanh}(ax) + ax + 4\operatorname{arctanh}(ax)\right)}{6a^4} + \frac{5i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}} + i\right)}{6a^4} - \frac{5i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}} - i\right)}{6a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/6/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x+4*arctanh(a*x
))+5/6*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^4-5/6*I*ln((a*x+1)/(-a^2*x^2+1)
^(1/2)-I)/a^4
```

maxima [A] time = 0.41, size = 88, normalized size = 1.01

$$-\frac{1}{6}\,a\left(\frac{\frac{\sqrt{-a^2x^2+1}\,x}{a^2}-\frac{\arcsin(ax)}{a^3}}{a^2}-\frac{4\,\arcsin\left(ax\right)}{a^5}\right)-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*((sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)/a^2 - 4*arcsin(a*x)/a^
5) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arctanh(a*
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atanh}\left(a\,x\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,\operatorname{atanh}\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.367 $\int \frac{x^2 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=146

$$-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}$$

```
[Out] -arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-1/2*I*polylog(2,-I*(
-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a^3-1/2*(-a^2*x^2+1)^(1/2)/a^3-1/2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/
a^2
```

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6016, 261, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}-\frac{\sqrt{1-a^2x^2}}{2a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) - (
ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - ((I/2)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{\int \frac{x}{\sqrt{1-a^2x^2}}\,dx}{2a}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2a^2} - \frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a^3} + \frac{i}{}$$

Mathematica [A] time = 0.23, size = 125, normalized size = 0.86

$$-\frac{\sqrt{1-a^2x^2} + ax\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) + i\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right) - i\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right) + i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right.}{2a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/2*(Sqrt[1 - a^2*x^2] + a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + I*ArcTanh[a*
x]*Log[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + I
*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh[a*x]])/a^3
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^2\,\text{artanh}\,(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.41, size = 154, normalized size = 1.05

$$-\frac{(ax\,\text{arctanh}\,(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2a^3} - \frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\text{arctanh}\,(ax)}{2a^3} + \frac{i\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\text{arctanh}\,(ax)}{2a^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(a*x*arctanh(a*x)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3-1/2*I*ln(1+I*(a*x+1)
/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*arctanh(a*x)/a^3-1/2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/2*I*
dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.368 $\int \frac{x \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\, dx$

Optimal. Leaf size=32

$$\frac{\sin^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^2}$$

```
[Out] arcsin(a*x)/a^2-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5994, 216}

$$\frac{\sin^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
[Out] ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\, dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\, dx}{a} \\ &= \frac{\sin^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.91

$$\frac{\sin^{-1}(ax) - \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
[Out] (ArcSin[a*x] - Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

fricas [A] time = 0.47, size = 58, normalized size = 1.81

$$-\frac{\sqrt{-a^2x^2+1}\,\log\left(-\frac{ax+1}{ax-1}\right) + 4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)) + 4*arctan((sqrt(-a^2*x^
2 + 1) - 1)/(a*x)))/a^2
```

giac [A] time = 0.22, size = 47, normalized size = 1.47

$$\frac{\arcsin\left(ax\right)\operatorname{sgn}(a)}{a|a|} - \frac{\sqrt{-a^2x^2+1}\,\log\left(-\frac{ax+1}{ax-1}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/2*sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1))/a^2
```

maple [C] time = 0.31, size = 81, normalized size = 2.53

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\;\operatorname{arctanh}\left(ax\right)}{a^2} + \frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{a^2} - \frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-i\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/a^2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)+I*ln((a*x+1)/(-a^2*x^2+1)^(1/2
)+I)/a^2-I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^2
```

maxima [A] time = 0.40, size = 30, normalized size = 0.94

$$-\frac{\sqrt{-a^2x^2+1}\;\operatorname{artanh}\left(ax\right)}{a^2} + \frac{\arcsin\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*arctanh(a*x)/a^2 + arcsin(a*x)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\operatorname{atanh}\left(a\,x\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{atanh}\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.369 $\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

```
[Out] -2*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-I*polylog(2,-I*(-a*x
+1)^(1/2)/(a*x+1)^(1/2))/a+I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx=-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.08, size = 76, normalized size = 0.80

$$-\frac{i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]])
 + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.45, size = 126, normalized size = 1.33

$$\frac{i\arctanh(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{i\arctanh(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{i\operatorname{dilog}\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{a}+\frac{i\operatorname{dilog}\left(1-\frac{i(ax+}{\sqrt{-a^2x}}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-I/a*arctanh(a*x)*ln(1+I
*(a*x+1)/(-a^2*x^2+1)^(1/2))-I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+I/a*
dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.370 $\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=75

$$\mathrm{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\mathrm{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

```
[Out] -2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(
1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6018}

$$\mathrm{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\mathrm{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1
- a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx=-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\mathrm{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\mathrm{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

Mathematica [A] time = 0.10, size = 57, normalized size = 0.76

$$\mathrm{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-\mathrm{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\log\left(e^{-\tanh^{-1}(ax)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ArcTanh[a*x]*(Log[1 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])]) + Po
lyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\ \mathrm{artanh}(ax)}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.42, size = 99, normalized size = 1.32

$$-\operatorname{arctanh}(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\operatorname{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\operatorname{polylo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x)
[Out] -arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-polylog(2,-(a*x+1)/(-a^2*x^2
+1)^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.371 $\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6008, 266, 63, 208}

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}$$
$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.05, size = 48, normalized size = 1.14

$$-a\log\left(\sqrt{1-a^2x^2}+1\right) - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + a*Log[x] - a*Log[1 + Sqrt[1 - a^2*x
^2]]
```

fricas [A] time = 0.53, size = 58, normalized size = 1.38

$$\frac{2\,ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}\,\log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1)*log(-(a*x +
 1)/(a*x - 1)))/x
```

giac [B] time = 0.27, size = 111, normalized size = 2.64

$$-\frac{1}{2}a\left(\log\left(\sqrt{-a^2x^2+1}+1\right) - \log\left(-\sqrt{-a^2x^2+1}+1\right)\right) + \frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}\,|a|+a}{x|a|}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*(log(sqrt(-a^2*x^2 + 1) + 1) - log(-sqrt(-a^2*x^2 + 1) + 1)) + 1/4*(
a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(x*abs(a)))*log(-(a*x + 1)/(a*x - 1))
```

maple [A] time = 0.40, size = 72, normalized size = 1.71

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{arctanh}(ax)}{x} - a\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right) + a\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)/x-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

maxima [A] time = 0.41, size = 51, normalized size = 1.21

$$-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*arctanh
(a*x)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.372 $\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=137

$$\frac{1}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+a^2\left(-\tanh^{-1}(ax)\right)\tanh^{-1}\left(\frac{\sqrt{1}}{\sqrt{a}}\right.$$

```
[Out] -a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/2*a^2*polylog(2,-
(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2
))-1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6026, 264, 6018}

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+a^2\left(-\tanh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 - (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}-a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\frac{1}{2}a^2\text{Li}_2\left(-\right.\end{aligned}$$

Mathematica [A] time = 0.74, size = 126, normalized size = 0.92

$$\frac{1}{8}a^2\left(4\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-4\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)+4\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-4\,\text{t}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 + 4*Arc
Tanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a
*x])] + 4*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*PolyLog[2, E^(-ArcTanh[a*x])]
- ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{a^2x^5-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.45, size = 141, normalized size = 1.03

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,(ax+\text{artanh}\,(ax))}{2x^2}-\frac{a^2\,\text{artanh}\,(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{a^2\,\text{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{a^2\,\text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x+arctanh(a*x))/x^2-1/2*a^2*arctanh(a*x)*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/2*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.373 $\int \frac{x^3 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=205

$$-\frac{5i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}+\frac{5i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}-\frac{10\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2}-\frac{\sqrt{1-a^2x^2}}{3a^4}-\frac{2\sqrt{1-}$$

```
[Out] -10/3*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^4-5/3*I*polylog(2
,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4+5/3*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x
+1)^(1/2))/a^4-1/3*(-a^2*x^2+1)^(1/2)/a^4-1/3*x*arctanh(a*x)*(-a^2*x^2+1)^(
1/2)/a^3-2/3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arctanh(a*x)^2*(
-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.31, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6016, 261, 5950, 5994}

$$-\frac{5i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}+\frac{5i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}-\frac{\sqrt{1-a^2x^2}}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*a^4) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^3) - (
10*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^4) - (2*Sqrt[1 -
a^2*x^2]*ArcTanh[a*x]^2)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(
3*a^2) - (((5*I)/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (
((5*I)/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
```

```
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{3a^2} + \frac{2\int \frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{3a}$$

$$= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^3} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^2} + \int$$

$$= -\frac{\sqrt{1-a^2x^2}}{3a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^3} - \frac{10\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{3}$$

Mathematica [A] time = 0.43, size = 160, normalized size = 0.78

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{5i\left(\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}} + \left(1-a^2x^2\right)\tanh^{-1}(ax)^2 - \frac{5i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-}\right.\right.}{\sqrt{1-a^2x^2}}\right.}{3a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1 - a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + (1 - a^2*x^2
)*ArcTanh[a*x]^2 - ((5*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
 I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((5*I)*(PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(3*a^4)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^3\,\mathrm{artanh}\,(ax)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.39, size = 175, normalized size = 0.85

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(a^2x^2\,\mathrm{arctanh}\,(ax)^2 + ax\,\mathrm{arctanh}\,(ax) + 2\,\mathrm{arctanh}\,(ax)^2 + 1\right)}{3a^4} - \frac{5i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\mathrm{arc}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)+
2*arctanh(a*x)^2+1)-5/3*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a
^4+5/3*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4-5/3*I*dilog(1+
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5/3*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \operatorname{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.374 $\int \frac{x^2 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=161

$$-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{\sin^{-1}(a}{a^3}$$

```
[Out] arcsin(a*x)/a^3+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^3-I*arc
tanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+I*arctanh(a*x)*polyl
og(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^
(1/2))/a^3-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-arctanh(a*x)*(-a^2
*x^2+1)^(1/2)/a^3-1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.25, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6016, 5994, 216, 5952, 4180, 2531, 2282, 6589}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{iI}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ArcSin[a*x]/a^3 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^3 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^2) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^3 -
(I*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (I*ArcTanh[a*x]*Poly
Log[2, I*E^ArcTanh[a*x]])/a^3 + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - (
I*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(\right.}{2a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh}{a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh}{a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh}{a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh}{a^3}$$

Mathematica [A] time = 0.82, size = 188, normalized size = 1.17

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(2\tanh^{-1}(ax)\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+2\text{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)-2\text{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)^2\log\left(\right.\right.}{\sqrt{1-a^2x^2}}}{2a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2*ArcTanh[a*x] - a*x*ArcTanh[a*x]^2 - (I*((4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a^
3)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^2\,\text{artanh}\,(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{arctanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.375 $\int \frac{x \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}+\frac{2i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}-\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^2}$$

```
[Out] -4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^2-2*I*polylog(2,-I*(
-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2+2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a^2-arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5994, 5950}

$$-\frac{2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}+\frac{2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}-\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^2 - (Sqrt[1 - a^2*x
^2]*ArcTanh[a*x]^2)/a^2 - ((2*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a^2 + ((2*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])]])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}+\frac{2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a}\\ &= -\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}-\frac{2i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^2}+\frac{2i\text{Li}_2\left(\frac{i\sqrt{}}{\sqrt{}}\right.}{a^2}\end{aligned}$$

Mathematica [A] time = 0.24, size = 104, normalized size = 0.87

$$-\frac{\tanh^{-1}(ax)\left(\sqrt{1-a^2x^2}\tanh^{-1}(ax)+2i\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)+2i\text{Li}_2\left(-ie^{-\tanh^{-1}(\right.}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + (2*I)*(Log[1 - I/E^ArcTan
h[a*x]] - Log[1 + I/E^ArcTanh[a*x]])) + (2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x
]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]])/a^2)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x\,\mathrm{artanh}\,(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\mathrm{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.33, size = 151, normalized size = 1.26

$$-\frac{\sqrt{-(ax-1)(ax+1)}\;\mathrm{arctanh}\,(ax)^2}{a^2}-\frac{2i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\mathrm{arctanh}\,(ax)}{a^2}+\frac{2i\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\mathrm{arctanh}\,(ax)}{a^2}-\frac{2i\,\mathrm{d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/a^2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2-2*I*ln(1+I*(a*x+1)/(-a^2*x^2
+1)^(1/2))*arctanh(a*x)/a^2+2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(
a*x)/a^2-2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+2*I*dilog(1-I*(a*x+1
)/(-a^2*x^2+1)^(1/2))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\mathrm{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atanh}\,(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.376 $\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\mathrm{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{2i\mathrm{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2\tan\ldots}{\ldots}$$

```
[Out] 2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-2*I*arctanh(a*x)*poly
log(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+2*I*arctanh(a*x)*polylog(2,I*(a*x+1)
/(-a^2*x^2+1)^(1/2))/a+2*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-2*I*p
olylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5952, 4180, 2531, 2282, 6589}

$$-\frac{2i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\mathrm{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - ((2*I)*ArcTanh[a*x]*PolyLog[2
, (-I)*E^ArcTanh[a*x]])/a + ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]
])/a + ((2*I)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((2*I)*PolyLog[3, I*E^Ar
cTanh[a*x]])/a
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

```
&& GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{(2i)\,\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \tanh^{-1}(ax)\right)}{a} + \frac{(2i)}{}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(i\right.}{a}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(i\right.}{a}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(i\right.}{a}$$

Mathematica [A] time = 0.10, size = 119, normalized size = 1.16

$$\frac{i\left(-2\tanh^{-1}(ax)\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)\right)-2\left(\text{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right)\right)-\left(\tan\right.}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (I*(-(ArcTanh[a*x]^2*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]
)) - 2*ArcTanh[a*x]*(PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTa
nh[a*x]]) - 2*(PolyLog[3, (-I)/E^ArcTanh[a*x]] - PolyLog[3, I/E^ArcTanh[a*x
]])))/a
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)^{2}}{\sqrt{-a^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
[Out] int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^{2}}{\sqrt{-a^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^{2}}{\sqrt{1-a^{2}\,x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.377 $\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=68

$$-2\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+2\mathrm{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-2\mathrm{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-2\tanh^{-1}\left(\right.$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh(a*x)*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a
^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6020, 4182, 2531, 2282, 6589}

$$-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-2\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*PolyLog[2, -E^Ar
cTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 2*PolyLog[3, -E^A
rcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \tanh^{-1}(ax)\right) + 2\,\text{Subs}$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh}\right.$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh}\right.$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh}\right.$$

Mathematica [A] time = 0.09, size = 100, normalized size = 1.47

$$2\tanh^{-1}(ax)\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+2\text{Li}_3\left(-e^{-\tanh^{-1}(ax)}\right)-2\text{Li}_3\left(e^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(a$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]^2*Log[1 + E^(-ArcT
anh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 2*ArcTanh[a*x]
*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*PolyL
og[3, E^(-ArcTanh[a*x])]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^2}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.44, size = 158, normalized size = 2.32

$$-\text{arctanh}\left(ax\right)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-2\,\text{arctanh}\left(ax\right)\text{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+2\,\text{polylog}\left(3,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-
(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctan
h(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.378 $\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}+2a\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-4a\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)$$

```
[Out] -4*a*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*a*polylog(2,-(-a*
x+1)^(1/2)/(a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-arcta
nh(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6008, 6018}

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-4a\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a
*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}+(2a)\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-4a\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+2a\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-2a\text{Li}_2\end{aligned}$$

Mathematica [A] time = 0.52, size = 89, normalized size = 0.85

$$-\frac{\tanh^{-1}(ax)\left(\sqrt{1-a^2x^2}\tanh^{-1}(ax)+2ax\left(\log\left(e^{-\tanh^{-1}(ax)}+1\right)-\log\left(1-e^{-\tanh^{-1}(ax)}\right)\right)\right)}{x}+2a\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2*a*x*(-Log[1 - E^(-ArcTa
nh[a*x])] + Log[1 + E^(-ArcTanh[a*x])])))/x) + 2*a*PolyLog[2, -E^(-ArcTanh[
a*x])] - 2*a*PolyLog[2, E^(-ArcTanh[a*x])]
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\ \mathrm{artanh}\left(ax\right)^2}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.41, size = 131, normalized size = 1.25

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \mathrm{arctanh}\left(ax\right)^2}{x}-2a\,\mathrm{arctanh}\left(ax\right)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-2a\,\mathrm{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2/x-2*a*arctanh(a*x)*ln(1+(a*x+1)/(-
a^2*x^2+1)^(1/2))-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*arctanh(a*
x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(a\,x\right)^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.379 $\int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=152

$$-a^2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+a^2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+a^2\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-a^2\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-a^2\tan$$

```
[Out] -a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-a^2*x
^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ar
ctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-a*arctanh(a*x)*(
-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.42, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6026, 6008, 266, 63, 208, 6020, 4182, 2531, 2282, 6589}

$$-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+a^2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-a$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] - a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] + a^2*PolyLog[3, -E^ArcTanh[a*x]] - a^2*Poly
Log[3, E^ArcTanh[a*x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} + a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int x^2\mathrm{csch}(x)\,dx, x, \mathrm{ta}\right.$$

$$= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)$$

$$= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)$$

$$= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)$$

$$= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)$$

Mathematica [A] time = 1.27, size = 188, normalized size = 1.24

$$\frac{1}{8}a^2\left(8\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) - 8\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{-\tanh^{-1}(ax)}\right) + 8\mathrm{Li}_3\left(-e^{-\tanh^{-1}(ax)}\right) - 8\mathrm{Li}_3\left(e^{-\tanh^{-1}(ax)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
[Out] (a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*
x]/2]^2 + 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]^2*Lo
g[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 8*ArcTanh[a*x]*Pol
yLog[2, -E^(-ArcTanh[a*x])] - 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])]
+ 8*PolyLog[3, -E^(-ArcTanh[a*x])] - 8*PolyLog[3, E^(-ArcTanh[a*x])] - ArcT
anh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\mathrm{artanh}(ax)^2}{a^2x^5-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}(ax)^2}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.46, size = 231, normalized size = 1.52

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\mathrm{arctanh}(ax)(2ax+\mathrm{arctanh}(ax))}{2x^2} - \frac{a^2\,\mathrm{arctanh}(ax)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2} - a^2\,\mathrm{arctanh}(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2-1/2*a^2
*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/
2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*poly
log(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
-2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.380 $\int \frac{x^3 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=219

$$-\frac{5i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{5i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{\text{si}}{}$$

```
[Out] arcsin(a*x)/a^4+5*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^4-5*I
*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5*I*arctanh(a*x)
*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5*I*polylog(3,-I*(a*x+1)/(-a^2
*x^2+1)^(1/2))/a^4-5*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-arctanh(
a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^3-2/3
*arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arctanh(a*x)^3*(-a^2*x^2+1)^
(1/2)/a^2
```

Rubi [A] time = 0.57, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6016, 5994, 216, 5952, 4180, 2531, 2282, 6589}

$$-\frac{5i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ArcSin[a*x]/a^4 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^3) + (5*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^4
- (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x]^3)/(3*a^2) - ((5*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])
/a^4 + ((5*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 + ((5*I)*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a^4 - ((5*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2} + \frac{\int \frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}$$

$$= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^2} + \cdots$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^4} - \frac{x^2\sqrt{\cdots}}{\cdots}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\cdots}{a^4}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\cdots}{a^4}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\cdots}{a^4}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\cdots}{a^4}$$

Mathematica [A] time = 1.01, size = 215, normalized size = 0.98

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{3i\left(10\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-10\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+10\mathrm{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)-10\mathrm{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right)+5\tanh^{-1}(a\cdots\right)}{\sqrt{1-a^2x^2}}\cdots\right)}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-3*a*x*ArcTanh[a*x]^2 + 2*(1 - a^2*x^2)*ArcTanh[a*x]^3
- 6*ArcTanh[a*x]*(1 + ArcTanh[a*x]^2) - ((3*I)*((4*I)*ArcTan[Tanh[ArcTanh[a
*x]/2]] + 5*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 5*ArcTanh[a*x]^2*Log
[1 + I/E^ArcTanh[a*x]] + 10*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] -
10*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 10*PolyLog[3, (-I)/E^ArcTanh
[a*x]] - 10*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(6*a^4)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^3\,\mathrm{artanh}\,(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
[Out] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.381 $\int \frac{x^2 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=305

$$-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}+\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{2a^3}+\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{2a^3}+\frac{3i\tanh}{}$$

```
[Out] -6*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3+arctan((a*x+1)/(-a
^2*x^2+1)^(1/2))*arctanh(a*x)^3/a^3-3/2*I*arctanh(a*x)^2*polylog(2,-I*(a*x+
1)/(-a^2*x^2+1)^(1/2))/a^3+3/2*I*arctanh(a*x)^2*polylog(2,I*(a*x+1)/(-a^2*x
^2+1)^(1/2))/a^3-3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+3*I*pol
ylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+3*I*arctanh(a*x)*polylog(3,-I*(a
*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^
2+1)^(1/2))/a^3-3*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+3*I*polylo
g(4,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
/a^3-1/2*x*arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.34, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6016, 5994, 5950, 5952, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{2a^3}+\frac{3i\tanh^{-1}(ax)^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - (3*Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^2)/(2*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(2*a^2
) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 - (((3*I)/2)*ArcTanh[a*x]^2
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2
, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 +
a*x]])/a^3 + ((3*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((3*
I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]*
PolyLog[3, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])
/a^3 + ((3*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{3\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{2a}$$

$$= -\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2} + \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \tan\right.}{2a^3}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2}$$

Mathematica [A] time = 5.23, size = 570, normalized size = 1.87

$$-\frac{i\left(-64iax\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3 - 192i\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2 + 192\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right) + 192i\pi\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (1
92*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 - (64*I)*
a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + 384*ArcTanh[a*x]
*Log[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] - 384*Arc
Tanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcT
anh[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh
[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTan
h[a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*L
og[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8
*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(Pi^2 - (4*I)*Pi*ArcTan
h[a*x] - 4*(2 + ArcTanh[a*x]^2))*PolyLog[2, (-I)/E^ArcTanh[a*x]] - 384*Poly
Log[2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x
]] - 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog
[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*Ar
cTanh[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-
I)*E^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[
4, (-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a^3
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^2\,\text{artanh}\,(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)

[Out] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)

[Out] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)

[Out] Integral(x**2*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)

3.382 $\int \frac{x \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=128

$$-\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^2}-\frac{6i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^2}-\cdots$$

```
[Out] 6*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^2-6*I*arctanh(a*x)*po
lylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+6*I*arctanh(a*x)*polylog(2,I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^2+6*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
^2-6*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2-arctanh(a*x)^3*(-a^2*x^2
+1)^(1/2)/a^2
```

Rubi [A] time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5994, 5952, 4180, 2531, 2282, 6589}

$$-\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^2}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (6*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/a^2 - ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^2 + ((
6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^2 + ((6*I)*PolyLog[3, (-I
)*E^ArcTanh[a*x]])/a^2 - ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

```
&& GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2}+\frac{3\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a} \\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2}+\frac{3\,\mathrm{Subst}\left(\int x^2\mathrm{sech}(x)\,dx,x,\tanh^{-1}(ax)\right)}{a^2} \\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2}-\frac{(6i)\,\mathrm{Subst}\left(\int x\log\left(1-ie^x\right)\right.}{a^2} \\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2}-\frac{6i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\tanh^{-1}(a\right.}}{a^2} \\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2}-\frac{6i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\tanh^{-1}(a\right.}}{a^2} \\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2}-\frac{6i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\tanh^{-1}(a\right.}}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 157, normalized size = 1.23

$$-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3+6i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-6i\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+6i\mathrm{Li}_3\left(-ie^{-\tanh^{-1}\right.}}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 + (3*I)*ArcTanh[a*x]^2*Log[1 - I/E^ArcT
anh[a*x]] - (3*I)*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*ArcTanh[
a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*PolyLog[2, I/E^Ar
cTanh[a*x]] + (6*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (6*I)*PolyLog[3, I/E^
ArcTanh[a*x]])/a^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x\,\mathrm{artanh}\left(ax\right)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
[Out] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.383 $\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=153

$$-\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{6i\tanh^{-1}(ax)\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{6i\tanh^{-1}(a$$

```
[Out] 2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a-3*I*arctanh(a*x)^2*po
lylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+3*I*arctanh(a*x)^2*polylog(2,I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a+6*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a^2*x^2+
1)^(1/2))/a-6*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-6*I*
polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+6*I*polylog(4,I*(a*x+1)/(-a^2*x^
2+1)^(1/2))/a
```

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5952, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(3,-\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - ((3*I)*ArcTanh[a*x]^2*PolyLog
[2, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[
a*x]])/a + ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((6*I)*
ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a - ((6*I)*PolyLog[4, (-I)*E^Arc
Tanh[a*x]])/a + ((6*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{(3i)\,\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \tanh^{-1}(ax)\right)}{a} + \frac{(3i}{}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2}{a}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2}{a}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2}{a}$$
$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2}{a}$$

Mathematica [B] time = 0.44, size = 451, normalized size = 2.95

$$-\frac{i\left(192\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right) + 192i\pi\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right) + 384\tanh^{-1}(ax)\text{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-1/64*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (32
*I)*Pi*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[
a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] - (96*I)*Pi*ArcTanh[
a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*
x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] + (96*I)*Pi*ArcTanh[a*
x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + 64*
ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*A
rcTanh[a*x])/4]] - 48*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E^ArcTanh
```

```
[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - 48*Pi^2*PolyL
og[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*
x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcTanh[a*x]*PolyLog
[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]]
 - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)/E^ArcTanh[
a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
[Out] int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.384 $\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=102

$$-3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\text{Li}_3\left(\right.$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-3*arctanh(a*x)^2*poly
log(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arct
anh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.17, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6020, 4182, 2531, 6609, 2282, 6589}

$$-3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{\tanh}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^
ArcTanh[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x
]*PolyLog[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] -
 6*PolyLog[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\,\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \tanh^{-1}(ax)\right) + 3\,\text{S}$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2$$

Mathematica [A] time = 0.14, size = 146, normalized size = 1.43

$$\frac{1}{8}\left(24\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right) + 48\tanh^{-1}(ax)\text{Li}_3\left(-e^{-\tanh^{-1}(ax)}\right) - 48$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Pi^4 - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*
ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -E^(-
ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[
a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[
a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[a*x]])/
8
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^3}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.46, size = 215, normalized size = 2.11

$$-\operatorname{arctanh}(ax)^3 \ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-3\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+6\operatorname{arctanh}(ax)\operatorname{polylog}\left(3,-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1
)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(1-(a*x+
1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)
/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.385 $\int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=98

$$-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x}-6a\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+6a\mathrm{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-$$

```
[Out] -6*a*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-6*a*arctanh(a*x)*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(
a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.25, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6008, 6020, 4182, 2531, 2282, 6589}

$$-6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6a\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x]*P
olyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLog[3
, E^ArcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
```

```
&& NeQ[m, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} + (3a)\,\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right) \\
&= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - (6a)\,\text{Subst}\left(\int x\log\left(1-e\right.\right. \\
&= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^-}\right. \\
&= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^-}\right. \\
&= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^-}\right.
\end{aligned}$$

Mathematica [A] time = 0.39, size = 131, normalized size = 1.34

$$a\left(-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{ax} + 6\tanh^{-1}(ax)\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) - 6\tanh^{-1}(ax)\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right) + 6\text{Li}_3\left(-e^{-\tanh^{-1}(a\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(a*x)) + 3*ArcTanh[a*x]^2*Log[1 - E
^(-ArcTanh[a*x])] - 3*ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 6*ArcTanh
[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*PolyLog[2, E^(-ArcTan
h[a*x])] + 6*PolyLog[3, -E^(-ArcTanh[a*x])] - 6*PolyLog[3, E^(-ArcTanh[a*x]
)])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)^3}{a^2x^4-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.43, size = 190, normalized size = 1.94

$$-\frac{\sqrt{-(ax-1)(ax+1)}\ \operatorname{arctanh}(ax)^3}{x}-3a\operatorname{arctanh}(ax)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-6a\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,-\frac{}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^3/x-3*a*arctanh(a*x)^2*ln(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))-6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a*arctanh(a*x)^2*ln(1-(a*x+1)
/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.386 $\int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=267

$$3a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+3a^2$$

```
[Out] -a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-6*a^2*arctanh(a*x)*
arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*polylog(2,-(a*
x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+3*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3*a^2*polylog(2,(
-a*x+1)^(1/2)/(a*x+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^
2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*
polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(4,(a*x+1)/(-a^2*x^2+1)
^(1/2))-3/2*a*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^3*(-a^2*
x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.45, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6026, 6008, 6018, 6020, 4182, 2531, 6609, 2282, 6589}

$$3a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+\frac{3}{2}a^2\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a^2*ArcTan
h[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*ArcTanh[a*x]^2*PolyLog
[2, -E^ArcTanh[a*x]])/2 + (3*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]])
/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*PolyLog[2, Sq
rt[1 - a*x]/Sqrt[1 + a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcTanh[a*x]]
 - 3*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 3*a^2*PolyLog[4, -E^ArcT
anh[a*x]] + 3*a^2*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}+\frac{1}{2}a^2\,\text{Subst}\left(\int x^3\text{csch}(x)\,dx,x,\tan$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3$$

Mathematica [A] time = 8.97, size = 416, normalized size = 1.56

$$\frac{1}{16}a^2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\left(-\frac{4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{ax}-\frac{ax\tanh^{-1}(ax)^3\text{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}+24\tanh^{-1}(ax)^2\text{L}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a^2*(12*ArcTanh[a*x]^2 - (4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(a*x) + Pi^4
*Coth[ArcTanh[a*x]/2] - 2*ArcTanh[a*x]^4*Coth[ArcTanh[a*x]/2] - 12*ArcTanh[
a*x]^2*Coth[ArcTanh[a*x]/2]^2 - (a*x*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^4)
/Sqrt[1 - a^2*x^2] + 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*Log[1 - E^(-ArcTa
nh[a*x])] - 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*Log[1 + E^(-ArcTanh[a*x])]
 - 8*ArcTanh[a*x]^3*Coth[ArcTanh[a*x]/2]*Log[1 + E^(-ArcTanh[a*x])] + 8*Arc
Tanh[a*x]^3*Coth[ArcTanh[a*x]/2]*Log[1 - E^ArcTanh[a*x]] + 24*(2 + ArcTanh[
a*x]^2)*Coth[ArcTanh[a*x]/2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 48*Coth[ArcTa
nh[a*x]/2]*PolyLog[2, E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*Coth[ArcTanh[a
*x]/2]*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*Po
lyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*PolyLog
[3, E^ArcTanh[a*x]] + 48*Coth[ArcTanh[a*x]/2]*PolyLog[4, -E^(-ArcTanh[a*x])
] + 48*Coth[ArcTanh[a*x]/2]*PolyLog[4, E^ArcTanh[a*x]])*Tanh[ArcTanh[a*x]/2
])/16
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)^3}{a^2x^5-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.50, size = 386, normalized size = 1.45

$$-\frac{\sqrt{-(ax-1)(ax+1)}\ \operatorname{arctanh}(ax)^2(3ax+\operatorname{arctanh}(ax))}{2x^2}-\frac{a^2\operatorname{arctanh}(ax)^3\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{3a^2\operatorname{arctanh}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2*(3*a*x+arctanh(a*x))/x^2-1/2*a
^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(3,-(a*x+1)
/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*a
rctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2
*x^2+1)^(1/2))+3*a^2*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*
x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(
1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,(
a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.387 $\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$$

Mathematica [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
[Out] Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

fricas [A] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m\,\text{artanh}\,(ax)}{a^4x^4-2\,a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\,\text{artanh}\,(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arctanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
[Out] int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atanh}(a\,x)}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^m*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atanh}(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(x**m*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.388 $\int \frac{x^3 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=74

$$-\frac{\sin^{-1}(ax)}{a^4}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}-\frac{x}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] -arcsin(a*x)/a^4-x/a^3/(-a^2*x^2+1)^(1/2)+arctanh(a*x)/a^4/(-a^2*x^2+1)^(1/
2)+arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.17, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6028, 5994, 216, 191}

$$-\frac{x}{a^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(x/(a^3*Sqrt[1 - a^2*x^2])) - ArcSin[a*x]/a^4 + ArcTanh[a*x]/(a^4*Sqrt[1 -
 a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= \frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3}$$
$$= -\frac{x}{a^3\sqrt{1-a^2x^2}} - \frac{\sin^{-1}(ax)}{a^4} + \frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 1.03

$$\frac{ax\sqrt{1-a^2x^2}+\left(1-a^2x^2\right)\sin^{-1}(ax)+\sqrt{1-a^2x^2}\left(a^2x^2-2\right)\tanh^{-1}(ax)}{a^4\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2] + (1 - a^2*x^2)*ArcSin[a*x] + Sqrt[1 - a^2*x^2]*(-2
+ a^2*x^2)*ArcTanh[a*x])/(a^4*(-1 + a^2*x^2))
```

fricas [A] time = 0.62, size = 94, normalized size = 1.27

$$\frac{4\left(a^2x^2-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\left(2\,ax+\left(a^2x^2-2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^2*x^2 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(2*a*x + (a^2*x^2 - 2)*log(-(a*x + 1)/(a*x - 1))))/(a^6*x^2 - a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.42, size = 144, normalized size = 1.95

$$-\frac{(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2a^4(ax-1)}+\frac{(\operatorname{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2a^4(ax+1)}+\frac{\operatorname{arctanh}(ax)\sqrt{-(ax-1)}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x-1)+1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x+1)+arctanh(a*x)*(-(a*x-1)*(a*x+1))^(
1/2)/a^4+I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^4-I*ln((a*x+1)/(-a^2*x^2+1)^(
1/2)+I)/a^4
```

maxima [A] time = 0.41, size = 96, normalized size = 1.30

$$a\left(\frac{\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin(ax)}{a^3}}{a^2}-\frac{2\,x}{\sqrt{-a^2x^2+1}\,a^4}\right)-\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*((x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3)/a^2 - 2*x/(sqrt(-a^2*x^2
+ 1)*a^4)) - (x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4))*ar
ctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atanh}\left(a\,x\right)}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\operatorname{atanh}{\left(ax \right)}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.389 $\int \frac{x^2 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}+\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{1}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] 2*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3+I*polylog(2,-I*(-a*
x+1)^(1/2)/(a*x+1)^(1/2))/a^3-I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
^3-1/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5998, 5950}

$$\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{1}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(1/(a^3*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) + (
2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 + (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 - (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqr
t[1 + a*x]])/a^3
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx=-\frac{1}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$=-\frac{1}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3}+\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3}-\frac{i\text{Li}_2}{}$$

Mathematica [A] time = 0.25, size = 121, normalized size = 0.88

$$\frac{i\left(\frac{i}{\sqrt{1-a^2x^2}}-\frac{iax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(a}{a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (I*(I/Sqrt[1 - a^2*x^2] - (I*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[
a*x]*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + P
olyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a^3
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^2\,\text{artanh}\,(ax)}{a^4x^4-2\,a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,\text{artanh}\,(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.39, size = 190, normalized size = 1.39

$$-\frac{(\text{arctanh}\,(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2a^3(ax-1)}-\frac{(\text{arctanh}\,(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2a^3(ax+1)}+\frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\text{arctanh}\,(}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x-1)-1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x+1)+I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*arctanh(a*x)/a^3-I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+
I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-I*dilog(1-I*(a*x+1)/(-a^2*x^2+1
)^(1/2))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,\text{artanh}\,(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{atanh}(a\,x)}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(x**2*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.390 $\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}$$

```
[Out] -x/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5994, 191}

$$\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 0.63

$$\frac{\tanh^{-1}(ax)-ax}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-(a*x) + ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.53, size = 51, normalized size = 1.19

$$\frac{\sqrt{-a^{2} x^{2}+1}\left(2 a x-\log\left(-\frac{a x+1}{a x-1}\right)\right)}{2\left(a^{4} x^{2}-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(2*a*x - log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

giac [A] time = 0.29, size = 61, normalized size = 1.42

$$\frac{\sqrt{-a^{2} x^{2}+1}\, x}{\left(a^{2} x^{2}-1\right) a}+\frac{\log\left(-\frac{a x+1}{a x-1}\right)}{2 \sqrt{-a^{2} x^{2}+1}\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*a) + 1/2*log(-(a*x + 1)/(a*x - 1))/(sqr
t(-a^2*x^2 + 1)*a^2)
```

maple [A] time = 0.32, size = 66, normalized size = 1.53

$$-\frac{(\operatorname{arctanh}(a x)-1) \sqrt{-(a x-1)(a x+1)}}{2 a^{2}(a x-1)}+\frac{(\operatorname{arctanh}(a x)+1) \sqrt{-(a x-1)(a x+1)}}{2 a^{2}(a x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)+1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)
```

maxima [A] time = 0.31, size = 39, normalized size = 0.91

$$-\frac{x}{\sqrt{-a^{2} x^{2}+1}\, a}+\frac{\operatorname{artanh}(a x)}{\sqrt{-a^{2} x^{2}+1}\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \operatorname{atanh}(a\, x)}{\left(1-a^{2} x^{2}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}(a x)}{(-(a x-1)(a x+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.391 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}$$

```
[Out] -1/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5958}

$$\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
[Out] -(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx=-\frac{1}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 0.68

$$\frac{ax\tanh^{-1}(ax)-1}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
[Out] (-1 + a*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.51, size = 47, normalized size = 1.18

$$-\frac{\sqrt{-a^2x^2+1}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)-2\right)}{2\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*x^2 - a)
```

giac [A] time = 0.33, size = 59, normalized size = 1.48

$$-\frac{\sqrt{-a^2x^2+1}\,x\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^2x^2-1\right)}-\frac{1}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1) - 1/(sqrt
(-a^2*x^2 + 1)*a)
```

maple [A] time = 0.27, size = 38, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+1}\,\left(ax\operatorname{arctanh}(ax)-1\right)}{a\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(a*x*arctanh(a*x)-1)/(a^2*x^2-1)
```

maxima [A] time = 0.30, size = 36, normalized size = 0.90

$$\frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] x*arctanh(a*x)/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(3/2),x)
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.392 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

```
[Out] -2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(
1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-a*x/(-a^2*x^2+1
)^(1/2)+arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6030, 6018, 5994, 191}

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] - 2*ArcTanh[a*x
]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
+ a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - a\int$$
$$= -\frac{ax}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{Li}_2\Bigg($$

Mathematica [A] time = 0.19, size = 97, normalized size = 0.87

$$-\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\,\text{lo}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]*
Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + Poly
Log[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\ \text{artanh}(ax)}{a^4x^5-2\,a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

maple [A] time = 0.43, size = 157, normalized size = 1.40

$$-\frac{(\text{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{(\text{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2ax+2}-\text{arctanh}(ax)\ln\left(1+\frac{ax+}{\sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)+1)
*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1
/2))-polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x*(1 - a^2*x^2)^(3/2)),x)
[Out] int(atanh(a*x)/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.393 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-a/(-a^2*x^2+1)^(1/2)+a^2*x*arctanh(a*x)/(-a^
2*x^2+1)^(1/2)-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.17, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6030, 6008, 266, 63, 208, 5958}

$$-\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -(a/Sqrt[1 - a^2*x^2]) + (a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x])/x - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,d\right.$$

$$= -\frac{a}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-\right.}{a}$$

$$= -\frac{a}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.12, size = 89, normalized size = 1.09

$$\frac{ax\left(\sqrt{1-a^2x^2}\,\log(x)-\sqrt{1-a^2x^2}\,\log\left(\sqrt{1-a^2x^2}+1\right)-1\right)+\left(2a^2x^2-1\right)\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] ((-1 + 2*a^2*x^2)*ArcTanh[a*x] + a*x*(-1 + Sqrt[1 - a^2*x^2]*Log[x] - Sqrt[
1 - a^2*x^2]*Log[1 + Sqrt[1 - a^2*x^2]]))/(x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.43, size = 107, normalized size = 1.30

$$-\frac{2\,a^3x^3-2\,ax-2\left(a^3x^3-ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\left(2\,ax-\left(2\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*x^3 - 2*a*x - 2*(a^3*x^3 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x)
 - sqrt(-a^2*x^2 + 1)*(2*a*x - (2*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))))/
(a^2*x^3 - x)
```

giac [B] time = 0.31, size = 155, normalized size = 1.89

$$-\frac{1}{2}\,a\log\left(\sqrt{-a^2x^2+1}+1\right)+\frac{1}{2}\,a\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{2\sqrt{-a^2x^2+1}\,a^2x}{a^2x^2-1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(sqrt(-a^2*x^2 + 1) + 1) + 1/2*a*log(-sqrt(-a^2*x^2 + 1) + 1) + 1
/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 2*sqrt(-a^2*x^2 + 1)*a
^2*x/(a^2*x^2 - 1) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a)))*log(-(a*x
+ 1)/(a*x - 1)) - a/sqrt(-a^2*x^2 + 1)
```

maple [A] time = 0.44, size = 132, normalized size = 1.61

$$-\frac{a\left(\operatorname{arctanh}\left(ax\right)-1\right)\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2\left(ax-1\right)}-\frac{\left(\operatorname{arctanh}\left(ax\right)+1\right)a\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2\left(ax+1\right)}-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \text{arcta}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)-1/2*(arctanh(a*x)+
1)*a*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)
/x+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.31, size = 84, normalized size = 1.02

$$-a\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}+\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)\right)+\left(\frac{2a^{2}x}{\sqrt{-a^{2}x^{2}+1}}-\frac{1}{\sqrt{-a^{2}x^{2}+1}\,x}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(1/sqrt(-a^2*x^2 + 1) + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))) + (
2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(a\,x\right)}{x^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}{\left(ax \right)}}{x^{2} \left(- \left(ax - 1\right) \left(ax + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.394 $\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=179

$$\frac{3}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}-3a^2\tanh^{-1}(ax$$

```
[Out] -3*a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/2*a^2*polylog(2
,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1
/2))-a^3*x/(-a^2*x^2+1)^(1/2)+a^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-1/2*a*(-a
^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.40, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6030, 6026, 264, 6018, 5994, 191}

$$\frac{3}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^3x}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a^3*x)/Sqrt[1 - a^2*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2*x) + (a^2*ArcTanh[a
*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - 3*a^2*A
rcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (3*a^2*PolyLog[2, -(Sqrt
[1 - a*x]/Sqrt[1 + a*x])])/2 - (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx + a^2\int \frac{\tanh}{x\sqrt{1}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{2x} + \frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} - 3a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-}}{\sqrt{1+}}\right.$$

$$= -\frac{a^3x}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}}{2x} + \frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} - 3a^2\tanh^{-1}(ax)$$

Mathematica [A] time = 1.63, size = 182, normalized size = 1.02

$$\frac{1}{8}a^2\left(-\frac{8ax}{\sqrt{1-a^2x^2}} + \frac{8\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{ax\operatorname{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} + 12\mathrm{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) - 12\mathrm{Li}_2\left(e^{-\tanh^{-1}(ax)}\right) + 2\tan\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*((-8*a*x)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a*
x*Csch[ArcTanh[a*x]/2]^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]*Csch[ArcTanh[a*x
]/2]^2 + 12*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]*Log[1
 + E^(-ArcTanh[a*x])] + 12*PolyLog[2, -E^(-ArcTanh[a*x])] - 12*PolyLog[2, E
^(-ArcTanh[a*x])] - ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*
x]/2]))/8
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{a^4x^7-2\,a^2x^5+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)

maple [A] time = 0.45, size = 205, normalized size = 1.15

$$-\frac{a^2(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{(\operatorname{arctanh}(ax)+1)a^2\sqrt{-(ax-1)(ax+1)}}{2ax+2}-\frac{\sqrt{-(ax-1)(ax+1)}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x)

[Out] -1/2*a^2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)+1)*a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x+arctanh(a*x))/x^2-3/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x^3\,{(1-a^2\,x^2)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(3/2)),x)

[Out] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x^3(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(3/2),x)

[Out] Integral(atanh(a*x)/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)

3.395 $\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx = \int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$$

Mathematica [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\, x^m \operatorname{artanh}(ax)^2}{a^4x^4 - 2\,a^2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atanh}(a\,x)^2}{(1-a^2\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^m*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**m*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.396 $\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=186

$$\frac{2i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}-\frac{2i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}+\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^4}+\frac{2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^4}+\frac{\tanh^{-1}(\ldots}{a^4\sqrt{1-\ldots}}$$

```
[Out] 4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^4+2*I*polylog(2,-I*(-
a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4-2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2
))/a^4+2/a^4/(-a^2*x^2+1)^(1/2)-2*x*arctanh(a*x)/a^3/(-a^2*x^2+1)^(1/2)+arc
tanh(a*x)^2/a^4/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.31, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6028, 5994, 5950, 5958}

$$\frac{2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}-\frac{2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}+\frac{2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^4}+\frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}-\frac{2x\tanh\ldots}{a^3\sqrt{1-\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 2/(a^4*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) + (4
*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^4 + ArcTanh[a*x]^2/(a^
4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^4 + ((2*I)*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 - ((2*I)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
```

```
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx=\frac{\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2}-\frac{\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$=\frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{a^4}-\frac{2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^3}-\frac{2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^3}$$

$$=\frac{2}{a^4\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}}+\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}+\sqrt{1-$$

Mathematica [A] time = 0.40, size = 165, normalized size = 0.89

$$\frac{\frac{-2i\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\left(2-a^2x^2\right)\tanh^{-1}(ax)^2-2\tanh^{-1}(ax)\left(-i\sqrt{1-a^2x^2}\,\log\left(1-ie^{-\tanh^{-1}(ax)}\right)+i\sqrt{1-a^2x^2}\,\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+ax\right)+2}{\sqrt{1-a^2x^2}}}{a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (2 + (2 - a^2*x^2)*ArcTanh[a*x]^2
- 2*ArcTanh[a*x]*(a*x - I*Sqrt[1 - a^2*x^2]*Log[1 - I/E^ArcTanh[a*x]] + I*S
qrt[1 - a^2*x^2]*Log[1 + I/E^ArcTanh[a*x]]) - (2*I)*Sqrt[1 - a^2*x^2]*PolyL
og[2, I/E^ArcTanh[a*x]])/Sqrt[1 - a^2*x^2])/a^4
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^3\,\mathrm{artanh}\,(ax)^2}{a^4x^4-2\,a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.43, size = 230, normalized size = 1.24

$$-\frac{\left(\mathrm{arctanh}\,(ax)^2-2\,\mathrm{arctanh}\,(ax)+2\right)\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2a^4\left(ax-1\right)}+\frac{\left(\mathrm{arctanh}\,(ax)^2+2\,\mathrm{arctanh}\,(ax)+2\right)\sqrt{-\left(ax-1\right.}}{2a^4\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x+1)
+arctanh(a*x)^2*(-(a*x-1)*(a*x+1))^(1/2)/a^4+2*I*ln(1+I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*arctanh(a*x)/a^4-2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*
x)/a^4+2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-2*I*dilog(1-I*(a*x+1)/
(-a^2*x^2+1)^(1/2))/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atanh}(a\,x)^2}{\left(1-a^2\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.397 $\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{2i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\mathrm{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{2i\mathrm{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2\,\mathrm{ta}}{}$$

```
[Out] -2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^3+2*I*arctanh(a*x)*p
olylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-2*I*arctanh(a*x)*polylog(2,I*(a
*x+1)/(-a^2*x^2+1)^(1/2))/a^3-2*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^3+2*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+2*x/a^2/(-a^2*x^2+1)^(1
/2)-2*arctanh(a*x)/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)
^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6028, 5952, 4180, 2531, 2282, 6589, 5962, 191}

$$\frac{2i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\mathrm{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*x)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) +
(x*ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcT
anh[a*x]^2)/a^3 + ((2*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3
- ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^3 - ((2*I)*PolyLog[3,
 (-I)*E^ArcTanh[a*x]])/a^3 + ((2*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a^3} + \frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}}{a^2}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{(2i)}{}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{2i\,\text{ta}}{}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{2i\,\text{ta}}{}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{2i\,\text{ta}}{}$$

Mathematica [A] time = 0.35, size = 193, normalized size = 1.13

$$i\left(-\frac{2iax}{\sqrt{1-a^2x^2}}-\frac{iax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{2i\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+2\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (I*(((-2*I)*a*x)/Sqrt[1 - a^2*x^2] + ((2*I)*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
 - (I*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - I/E^Ar
cTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*Pol
yLog[2, (-I)/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]]
+ 2*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/a^3
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^2\,\text{artanh}(ax)^2}{a^4x^4-2\,a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{arctanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{atanh}(a x)^2}{\left(1 - a^2 x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.398 $\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2}{a^2\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

```
[Out] 2/a^2/(-a^2*x^2+1)^(1/2)-2*x*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)
^2/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5994, 5958}

$$\frac{2}{a^2\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 2/(a^2*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + ArcT
anh[a*x]^2/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a}\\ &= \frac{2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.50

$$\frac{\tanh^{-1}(ax)^2-2ax\tanh^{-1}(ax)+2}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2 - 2*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.51, size = 69, normalized size = 1.01

$$\frac{\sqrt{-a^2x^2+1}\left(4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\log\left(-\frac{ax+1}{ax-1}\right)^2-8\right)}{4\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(a
*x - 1))^2 - 8)/(a^4*x^2 - a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.32, size = 82, normalized size = 1.21

$$-\frac{\left(\operatorname{arctanh}(ax)^2-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2a^2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^2+2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(a}}{2a^2(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)
```

maxima [A] time = 0.31, size = 62, normalized size = 0.91

$$-\frac{2\,x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,a}+\frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}\,a^2}+\frac{2}{\sqrt{-a^2x^2+1}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*x*arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^2/(sqrt(-a^2*x^2 +
1)*a^2) + 2/(sqrt(-a^2*x^2 + 1)*a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\operatorname{atanh}(a\,x)^2}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.399 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2x}{\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

```
[Out] 2*x/(-a^2*x^2+1)^(1/2)-2*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^2
/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5962, 191}

$$\frac{2x}{\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTa
nh[a*x]^2)/Sqrt[1 - a^2*x^2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx &= -\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx\\ &= \frac{2x}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.60

$$\frac{2ax+ax\tanh^{-1}(ax)^2-2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*a*x - 2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.51, size = 69, normalized size = 1.10

$$-\frac{\sqrt{-a^2x^2+1}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)^2+8\,ax-4\,\log\left(-\frac{ax+1}{ax-1}\right)\right)}{4\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1))^2 + 8*a*x - 4*log(-(
a*x + 1)/(a*x - 1)))/(a^3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.27, size = 49, normalized size = 0.78

$$-\frac{\sqrt{-a^2x^2+1}\,\left(\operatorname{arctanh}(ax)^2\,ax+2ax-2\operatorname{arctanh}(ax)\right)}{a\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^2*a*x+2*a*x-2*arctanh(a*x))/(a^2*x^2-
1)
```

maxima [A] time = 0.31, size = 57, normalized size = 0.90

$$\frac{x\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}+\frac{2\,x}{\sqrt{-a^2x^2+1}}-\frac{2\,\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1) + 2*x/sqrt(-a^2*x^2 + 1) - 2*arctanh(a*
x)/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\operatorname{atanh}(a\,x)^2}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.400 $\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+2\text{Li}_3$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh(a*x)*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a
^2*x^2+1)^(1/2))+2/(-a^2*x^2+1)^(1/2)-2*a*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
+arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6030, 6020, 4182, 2531, 2282, 6589, 5994, 5958}

$$-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-2\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] 2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
^2/Sqrt[1 - a^2*x^2] - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh
[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x
]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-(2a)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx+\text{Subst}\left(\int x^2\text{csch}(x)\,dx,x,\tanh^{-1}(ax)\right)\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-2\,\text{Sub}\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-2\tanh\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-2\tanh\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-2\tanh
\end{aligned}$$

Mathematica [A] time = 0.26, size = 159, normalized size = 1.25

$$\frac{2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+2\tanh^{-1}(ax)\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+2\text{Li}_3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] 2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
^2/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[
a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[
a*x])] - 2*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-A
rcTanh[a*x])] - 2*PolyLog[3, E^(-ArcTanh[a*x])]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)^2}{a^4x^5-2\,a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

maple [A] time = 0.45, size = 232, normalized size = 1.83

$$-\frac{\left(\operatorname{arctanh}(ax)^2-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^2+2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1}}{2ax+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2
*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-arctanh
(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-(a*x+1)/
(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2
*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^
2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.401 $\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{2a^2x}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}+2a\mathrm{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\mathrm{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

```
[Out] -4*a*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*a*polylog(2,-(-a*
x+1)^(1/2)/(a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*a^2
*x/(-a^2*x^2+1)^(1/2)-2*a*arctanh(a*x)/(-a^2*x^2+1)^(1/2)+a^2*x*arctanh(a*x
)^2/(-a^2*x^2+1)^(1/2)-arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6030, 6008, 6018, 5962, 191}

$$2a\mathrm{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\mathrm{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{2a^2x}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*x)/Sqrt[1 - a^2*x^2] - (2*a*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (a^2*x
*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x -
 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(S
qrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x} + (2a)\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{2a^2x}{\sqrt{1-a^2x^2}} - \frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x} - 4a\tanh^{-1}(a$$

Mathematica [A] time = 1.15, size = 215, normalized size = 1.26

$$\frac{a\left(4\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) - 4\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(e^{-\tanh^{-1}(ax)}\right) + 4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right) -\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a*(4*a*x - 4*ArcTanh[a*x] + 2*a*x*ArcTanh[a*x]^2 - (a*x*ArcTanh[a*x]^2*Csc
h[ArcTanh[a*x]/2]^2)/2 + 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-ArcTa
nh[a*x])] - 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + 4
*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*Sqrt[1 - a^2*x^2]*Pol
yLog[2, E^(-ArcTanh[a*x])] - (2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*Sinh[ArcTanh[
a*x]/2]^2)/(a*x)))/(2*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{a^4x^6-2\,a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)

maple [A] time = 0.46, size = 207, normalized size = 1.21

$$-\frac{a\left(\operatorname{arctanh}(ax)^2-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}-\frac{\left(\operatorname{arctanh}(ax)^2+2\operatorname{arctanh}(ax)+2\right)a\sqrt{-(ax}}{2(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x)

[Out] -1/2*a*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)-1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*a*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2/x-2*a*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(3/2)),x)

[Out] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(3/2),x)

[Out] Integral(atanh(a*x)**2/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)

3.402 $\int \frac{\tanh^{-1}(ax)^2}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=221

$$-3a^2\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+3a^2\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+3a^2\mathrm{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-3a^2\mathrm{Li}_3\left(e^{\tanh^{-1}(ax)}\right)+\frac{}{\sqrt{1}}$$

```
[Out] -3*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-a^2
*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*
a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(3,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2/
(-a^2*x^2+1)^(1/2)-2*a^3*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)+a^2*arctanh(a*x)
^2/(-a^2*x^2+1)^(1/2)-a*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^
2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.78, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6030, 6026, 6008, 266, 63, 208, 6020, 4182, 2531, 2282, 6589, 5994, 5958}

$$-3a^2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3a^2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+3a^2\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (2*a^2)/Sqrt[1 - a^2*x^2] - (2*a^3*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a*S
qrt[1 - a^2*x^2]*ArcTanh[a*x])/x + (a^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] -
 (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]
*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1 - a^2*x^2]] - 3*a^2*ArcTanh[a*x]*PolyL
og[2, -E^ArcTanh[a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 3*
a^2*PolyLog[3, -E^ArcTanh[a*x]] - 3*a^2*PolyLog[3, E^ArcTanh[a*x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} + a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx + a^2\int \frac{\tanh}{x\sqrt{1}}$$

$$= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int x\right.$$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2}$$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2}$$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2}$$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2}$$

Mathematica [A] time = 2.85, size = 266, normalized size = 1.20

$$\frac{1}{8}a^2\left(\frac{16}{\sqrt{1-a^2x^2}} + \frac{8\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{16ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{2ax\tanh^{-1}(ax)\operatorname{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} + 24\tanh^{-1}(ax)\mathrm{Li}_2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*(16/Sqrt[1 - a^2*x^2] - (16*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (8*A
rcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]
^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*x]/2]^2 + 12*ArcTanh[
a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a
*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 24*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTa
nh[a*x])] - 24*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 24*PolyLog[3, -
E^(-ArcTanh[a*x])] - 24*PolyLog[3, E^(-ArcTanh[a*x])] - ArcTanh[a*x]^2*Sech
[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}\left(ax\right)^2}{a^4x^7-2\,a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

maple [A] time = 0.48, size = 313, normalized size = 1.42

$$-\frac{a^2\left(\operatorname{arctanh}\left(ax\right)^2-2\operatorname{arctanh}\left(ax\right)+2\right)\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2\left(ax-1\right)}+\frac{\left(\operatorname{arctanh}\left(ax\right)^2+2\operatorname{arctanh}\left(ax\right)+2\right)a^2\sqrt{-(a}}{2ax+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x)
[Out] -1/2*a^2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)
-1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2-2*a^2*a
rctanh((a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^
2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3
*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*ln(1-(a*
x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^
(1/2))-3*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(a\,x\right)^2}{x^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)**2/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.403 $\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}, x\right)$$

[Out] Unintegrable(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$$

Verification is Not applicable to the result.

[In] Int[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]

[Out] Defer[Int][(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx = \int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$$

Mathematica [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]

[Out] Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\, x^m \operatorname{artanh}(ax)^3}{a^4x^4 - 2\,a^2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arctanh}\left(ax\right)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{artanh}\left(ax\right)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{m}\,\mathrm{atanh}\left(a\,x\right)^{3}}{{\left(1-a^{2}\,x^{2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^m*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{atanh}^{3}\left(ax\right)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**m*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.404 $\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=220

$$\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{6i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6\,\text{ta}}{}$$

```
[Out] -6*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^4+6*I*arctanh(a*x)*p
olylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-6*I*arctanh(a*x)*polylog(2,I*(a
*x+1)/(-a^2*x^2+1)^(1/2))/a^4-6*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^4+6*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-6*x/a^3/(-a^2*x^2+1)^(1
/2)+6*arctanh(a*x)/a^4/(-a^2*x^2+1)^(1/2)-3*x*arctanh(a*x)^2/a^3/(-a^2*x^2+
1)^(1/2)+arctanh(a*x)^3/a^4/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^3*(-a^2*x^2+1)^
(1/2)/a^4
```

Rubi [A] time = 0.41, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6028, 5994, 5952, 4180, 2531, 2282, 6589, 5962, 191}

$$\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6*x)/(a^3*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^4*Sqrt[1 - a^2*x^2]) -
 (3*x*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) - (6*ArcTan[E^ArcTanh[a*x]]*A
rcTanh[a*x]^2)/a^4 + ArcTanh[a*x]^3/(a^4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^3)/a^4 + ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a
*x]])/a^4 - ((6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 - ((6*I)*
PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^4 + ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])
/a^4
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^4} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{a^3} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^3}$$

$$= \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^4} - \frac{3\,\mathrm{Subst}(}{}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} +$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} +$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} +$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} +$$

Mathematica [A] time = 0.47, size = 249, normalized size = 1.13

$$\frac{6i\sqrt{1-a^2x^2}\,\mathrm{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right) - 6i\sqrt{1-a^2x^2}\,\mathrm{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right) - a^2x^2\tanh^{-1}(ax)^3 + 3i\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right) - 3i\sqrt{1-a^2x^2}\,\mathrm{tanh}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*Po
lyLog[2, I/E^ArcTanh[a*x]] + (-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x]^
2 + 2*ArcTanh[a*x]^3 - a^2*x^2*ArcTanh[a*x]^3 + (3*I)*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - (3*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, (-I)/E^A
rcTanh[a*x]] - (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, I/E^ArcTanh[a*x]])/Sqrt[1
 - a^2*x^2])/a^4
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^3\,\mathrm{artanh}\,(ax)^3}{a^4x^4-2\,a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \operatorname{atanh}(a\,x)^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.405 $\int \frac{x^2 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=246

$$\frac{3i\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\mathrm{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{6i\tanh^{-1}(ax)\mathrm{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{6i\tanh^{-1}(}{}$$

```
[Out] -2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a^3+3*I*arctanh(a*x)^2
*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*arctanh(a*x)^2*polylog(2,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-6*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a
^2*x^2+1)^(1/2))/a^3+6*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a^3+6*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-6*I*polylog(4,I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^3-6/a^3/(-a^2*x^2+1)^(1/2)+6*x*arctanh(a*x)/a^2/
(-a^2*x^2+1)^(1/2)-3*arctanh(a*x)^2/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^3
/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6028, 5952, 4180, 2531, 6609, 2282, 6589, 5962, 5958}

$$\frac{3i\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{6i\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,-\right.}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -6/(a^3*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(a^2*Sqrt[1
- a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 + ((3*I)*ArcTan
h[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]^2*PolyL
og[2, I*E^ArcTanh[a*x]])/a^3 - ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTan
h[a*x]])/a^3 + ((6*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a^3 + ((6*
I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a^3 - ((6*I)*PolyLog[4, I*E^ArcTanh[a*x
]])/a^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a^3} + \frac{6\int \frac{\tanh^{-1}}{(1-a^2}}{a^{}}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(a}\right.}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(a}\right.}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(a}\right.}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(a}\right.}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(a}\right.}{a^3}$$

Mathematica [B] time = 1.07, size = 541, normalized size = 2.20

$$-\frac{384}{\sqrt{1-a^2x^2}} + \frac{64ax\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{192\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{384ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + 192i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right) - 192\pi\tanh^{-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((7*I)*Pi^4 - 384/Sqrt[1 - a^2*x^2] - 8*Pi^3*ArcTanh[a*x] + (384*a*x*ArcTan
h[a*x])/Sqrt[1 - a^2*x^2] + (24*I)*Pi^2*ArcTanh[a*x]^2 - (192*ArcTanh[a*x]^
2)/Sqrt[1 - a^2*x^2] + 32*Pi*ArcTanh[a*x]^3 + (64*a*x*ArcTanh[a*x]^3)/Sqrt[
1 - a^2*x^2] - (16*I)*ArcTanh[a*x]^4 - 8*Pi^3*Log[1 + I/E^ArcTanh[a*x]] + (
48*I)*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 96*Pi*ArcTanh[a*x]^2*Lo
g[1 + I/E^ArcTanh[a*x]] - (64*I)*ArcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] -
 (48*I)*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] - 96*Pi*ArcTanh[a*x]^2*
Log[1 - I*E^ArcTanh[a*x]] + 8*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + (64*I)*ArcTa
nh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] - 8*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*
x])/4]] - (48*I)*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E^ArcTanh[a*x]
] + (192*I)*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (48*I)*Pi^2*Po
lyLog[2, I*E^ArcTanh[a*x]] - 192*Pi*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x
]] - 192*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + (384*I)*ArcTanh[a*x]*PolyLog[
3, (-I)/E^ArcTanh[a*x]] - (384*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*
x]] + 192*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + (384*I)*PolyLog[4, (-I)/E^ArcTa
nh[a*x]] + (384*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/(64*a^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^2\,\text{artanh}\,(ax)^3}{a^4x^4-2\,a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atanh}\left(a\,x\right)^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.406 $\int \frac{x \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=94

$$-\frac{6x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] -6*x/a/(-a^2*x^2+1)^(1/2)+6*arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)-3*x*arctanh
(a*x)^2/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^3/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5994, 5962, 191}

$$-\frac{6x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6*x)/(a*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^
2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a}$$

$$= \frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{6\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a}$$

$$= -\frac{6x}{a\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 0.48

$$\frac{-6ax + \tanh^{-1}(ax)^3 - 3ax\tanh^{-1}(ax)^2 + 6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x]^2 + ArcTanh[a*x]^3)/(a^2*Sqrt
[1 - a^2*x^2])
```

fricas [A] time = 0.42, size = 91, normalized size = 0.97

$$\frac{\sqrt{-a^2x^2+1}\left(6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2 - \log\left(-\frac{ax+1}{ax-1}\right)^3 + 48\,ax - 24\,\log\left(-\frac{ax+1}{ax-1}\right)\right)}{8\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-a^2*x^2 + 1)*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - log(-(a*x + 1)/
(a*x - 1))^3 + 48*a*x - 24*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\mathrm{artanh}\left(ax\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.33, size = 98, normalized size = 1.04

$$-\frac{\left(\mathrm{arctanh}\left(ax\right)^3 - 3\,\mathrm{arctanh}\left(ax\right)^2 + 6\,\mathrm{arctanh}\left(ax\right) - 6\right)\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2a^2\left(ax-1\right)} + \frac{\left(\mathrm{arctanh}\left(ax\right)^3 + 3\,\mathrm{arctanh}\left(ax\right)^2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1))^
(1/2)/a^2/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*(-
(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)
```

maxima [A] time = 0.32, size = 88, normalized size = 0.94

$$-\frac{3\,x\,\operatorname{artanh}\left(ax\right)^{2}}{\sqrt{-a^{2}x^{2}+1}\,a}+\frac{\operatorname{artanh}\left(ax\right)^{3}}{\sqrt{-a^{2}x^{2}+1}\,a^{2}}-\frac{6\left(\frac{x}{\sqrt{-a^{2}x^{2}+1}}-\frac{\operatorname{artanh}(ax)}{\sqrt{-a^{2}x^{2}+1}\,a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*x*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^3/(sqrt(-a^2*x^2
+ 1)*a^2) - 6*(x/sqrt(-a^2*x^2 + 1) - arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a))/
a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\operatorname{atanh}\left(a\,x\right)^{3}}{\left(1-a^{2}\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{atanh}^{3}\left(ax\right)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.407 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -6/a/(-a^2*x^2+1)^(1/2)+6*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*arctanh(a*x)^
2/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5962, 5958}

$$-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -6/(a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*ArcTan
h[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx&=-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}+6\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx\\&=-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.51

$$\frac{ax\tanh^{-1}(ax)^3-3\tanh^{-1}(ax)^2+6ax\tanh^{-1}(ax)-6}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6 + 6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + a*x*ArcTanh[a*x]^3)/(a*Sqrt[1
 - a^2*x^2])
```

fricas [A] time = 0.47, size = 87, normalized size = 0.99

$$-\frac{\left(ax\log\left(-\frac{ax+1}{ax-1}\right)^3+24\,ax\log\left(-\frac{ax+1}{ax-1}\right)-6\,\log\left(-\frac{ax+1}{ax-1}\right)^2-48\right)\sqrt{-a^2x^2+1}}{8\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^3 + 24*a*x*log(-(a*x + 1)/(a*x - 1)) -
6*log(-(a*x + 1)/(a*x - 1))^2 - 48)*sqrt(-a^2*x^2 + 1)/(a^3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{artanh}\left(ax\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.27, size = 56, normalized size = 0.64

$$-\frac{\sqrt{-a^2x^2+1}\,\left(\operatorname{arctanh}\left(ax\right)^3ax+6ax\operatorname{arctanh}\left(ax\right)-3\operatorname{arctanh}\left(ax\right)^2-6\right)}{a\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^3*a*x+6*a*x*arctanh(a*x)-3*arctanh(a*
x)^2-6)/(a^2*x^2-1)
```

maxima [A] time = 0.32, size = 86, normalized size = 0.98

$$\frac{x\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}}+6\,a\left(\frac{x\operatorname{artanh}\left(ax\right)}{\sqrt{-a^2x^2+1}\,a}-\frac{1}{\sqrt{-a^2x^2+1}\,a^2}\right)-\frac{3\,\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1) + 6*a*(x*arctanh(a*x)/(sqrt(-a^2*x^2 +
1)*a) - 1/(sqrt(-a^2*x^2 + 1)*a^2)) - 3*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*
a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{atanh}\left(a\,x\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.408 $\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\mathrm{Li}$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-3*arctanh(a*x)^2*poly
log(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arct
anh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*x/(-a^2*x^2+1)^(1/
2)+6*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*a*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2
)+arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6030, 6020, 4182, 2531, 6609, 2282, 6589, 5994, 5962, 191}

$$-3\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a*x)/Sqrt[1 - a^2*x^2] + (6*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*x*Ar
cTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2] - 2*ArcT
anh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^ArcTanh
[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyL
og[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 6*Poly
Log[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - (3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx + \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$

$$= \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 -$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right.$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right.$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right.$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right.$$

Mathematica [A] time = 0.37, size = 230, normalized size = 1.24

$$\frac{1}{8}\left(-\frac{48ax}{\sqrt{1-a^2x^2}} + \frac{8\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{24ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{48\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + 24\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (Pi^4 - (48*a*x)/Sqrt[1 - a^2*x^2] + (48*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] -
(24*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x]^3)/Sqrt[1 - a^2
*x^2] - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*
ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -E^(-
ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[
a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[
a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[a*x]])/
8
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^3}{a^4x^5-2\,a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\,(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

maple [A] time = 0.47, size = 305, normalized size = 1.65

$$-\frac{\left(\operatorname{arctanh}(ax)^3-3\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^3+3\operatorname{arctanh}(ax)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1))^
(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*(-(a*x
-1)*(a*x+1))^(1/2)/(a*x+1)-arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-
3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*poly
log(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))
+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^
(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.409 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{6a}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x}+\frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-6a\tanh^{-1}(ax)\mathrm{Li}_2$$

```
[Out] -6*a*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-6*a*arctanh(a*x)*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(
a*x+1)/(-a^2*x^2+1)^(1/2))-6*a/(-a^2*x^2+1)^(1/2)+6*a^2*x*arctanh(a*x)/(-a^
2*x^2+1)^(1/2)-3*a*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)+a^2*x*arctanh(a*x)^3/(
-a^2*x^2+1)^(1/2)-arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.42, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6030, 6008, 6020, 4182, 2531, 2282, 6589, 5962, 5958}

$$-6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6a\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - 6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x
]^2 + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x
]*PolyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLo
g[3, E^ArcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\mathrm{ta}}{x}$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax$$

Mathematica [A] time = 2.00, size = 270, normalized size = 1.44

$$-\frac{6a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3\sinh^2\left(\frac{1}{2}\tanh^{-1}(a\right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2
] - (a^2*x*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2)/(4*Sqrt[1 - a^2*x^2]) + 3
*a*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 3*a*ArcTanh[a*x]^2*Log[1 + E
^(-ArcTanh[a*x])] + 6*a*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*a*A
rcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 6*a*PolyLog[3, -E^(-ArcTanh[a*x
])] - 6*a*PolyLog[3, E^(-ArcTanh[a*x])] + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3
*Sinh[ArcTanh[a*x]/2]^2)/x
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\left(ax\right)^3}{a^4x^6-2\,a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(ax\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

maple [A] time = 0.48, size = 282, normalized size = 1.51

$$-\frac{a\left(\operatorname{arctanh}(ax)^3-3\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}-\frac{\left(\operatorname{arctanh}(ax)^3+3\operatorname{arctanh}(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1)
)^(1/2)/(a*x-1)-1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*a*(-
(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^3/x-3*
a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*arctanh(a*x)*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3
*a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog
(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.410 $\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=360

$$3a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{9}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+\frac{9}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+$$

```
[Out] -3*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-6*a^2*arctanh(a*x
)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-9/2*a^2*arctanh(a*x)^2*polylog(2,-(
a*x+1)/(-a^2*x^2+1)^(1/2))+9/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))+3*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3*a^2*polylog(2
,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+9*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*
x^2+1)^(1/2))-9*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^
2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*polylog(4,(a*x+1)/(-a^2*x^2+
1)^(1/2))-6*a^3*x/(-a^2*x^2+1)^(1/2)+6*a^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-
3*a^3*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)+a^2*arctanh(a*x)^3/(-a^2*x^2+1)^(
1/2)-3/2*a*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^3*(-a^2*x^2
+1)^(1/2)/x^2
```

Rubi [A] time = 0.98, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6030, 6026, 6008, 6018, 6020, 4182, 2531, 6609, 2282, 6589, 5994, 5962, 191}

$$3a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{9}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+\frac{9}{2}a^2\tanh$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a^3*x)/Sqrt[1 - a^2*x^2] + (6*a^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*
a^3*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x]^3)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (9*a^2*ArcTanh[a*x]^
2*PolyLog[2, -E^ArcTanh[a*x]])/2 + (9*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTa
nh[a*x]])/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*Poly
Log[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 9*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcT
anh[a*x]] - 9*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 9*a^2*PolyLog[4
, -E^ArcTanh[a*x]] + 9*a^2*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
```

```
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx + a^2$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^2\,\text{Subst}$$

$$= \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} -$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2}{}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2}{}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2}{}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2}{}$$

Mathematica [A] time = 7.87, size = 555, normalized size = 1.54

$$a^2\left(72\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right) + 144\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\text{Li}_3\left(-e^{-\tanh^{-1}(ax)}\right) - 144\sqrt{1-a^2x^2}\,\text{ta}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-96*a*x + 3*Pi^4*Sqrt[1 - a^2*x^2] + 96*ArcTanh[a*x] - 48*a*x*ArcTanh
[a*x]^2 + 16*ArcTanh[a*x]^3 - 6*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^4 - 12*Sqrt[
1 - a^2*x^2]*ArcTanh[a*x]^2*Coth[ArcTanh[a*x]/2] - 2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]^3*Csch[ArcTanh[a*x]/2]^2 + 48*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1
 - E^(-ArcTanh[a*x])] - 48*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTa
nh[a*x])] - 24*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])]
+ 24*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*Sqrt[1 -
 a^2*x^2]*(2 + 3*ArcTanh[a*x]^2)*PolyLog[2, -E^(-ArcTanh[a*x])] - 48*Sqrt[1
 - a^2*x^2]*PolyLog[2, E^(-ArcTanh[a*x])] + 72*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^2*PolyLog[2, E^ArcTanh[a*x]] + 144*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*PolyLo
g[3, -E^(-ArcTanh[a*x])] - 144*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*PolyLog[3, E^
ArcTanh[a*x]] + 144*Sqrt[1 - a^2*x^2]*PolyLog[4, -E^(-ArcTanh[a*x])] + 144*
Sqrt[1 - a^2*x^2]*PolyLog[4, E^ArcTanh[a*x]] - 2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3*Sech[ArcTanh[a*x]/2]^2 + 12*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2*Tanh[Ar
cTanh[a*x]/2]))/(16*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^3}{a^4x^7-2\,a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

maple [A] time = 0.54, size = 482, normalized size = 1.34

$$-\frac{a^2\left(\operatorname{arctanh}(ax)^3-3\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^3+3\operatorname{arctanh}(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a^2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+
1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*a^
2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x
)^2*(3*a*x+arctanh(a*x))/x^2-3/2*a^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))-9/2*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a
^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^2*polylog(4,-(a*
x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))+9/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^2*ar
ctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*polylog(4,(a*x+1)/(-
```

```
a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a
^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(3/2)),x)
[Out] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.411 $\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx=\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

fricas [A] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

maple [A] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \operatorname{arctanh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\mathrm{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^m/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \operatorname{atanh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
[Out] Integral(x**m/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

3.412 $\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Defer[Int][x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx=\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 3.20, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{arctanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^2/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}} \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

3.413 $\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

```
[Out] Shi(arctanh(a*x))/a^2
```

Rubi [A] time = 0.10, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6034, 3298}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[ArcTanh[a*x]]/a^2
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}\\ &= \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 9, normalized size = 1.00

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[ArcTanh[a*x]]/a^2
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.32, size = 26, normalized size = 2.89

$$-\frac{\mathrm{Ei}\left(1,-\operatorname{arctanh}\left(ax\right)\right)}{2a^{2}}+\frac{\mathrm{Ei}\left(1,\operatorname{arctanh}\left(ax\right)\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] -1/2*Ei(1,-arctanh(a*x))/a^2+1/2*Ei(1,arctanh(a*x))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{x}{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

3.414 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] Chi(arctanh(a*x))/a
```

Rubi [A] time = 0.06, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5968, 3301}

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

maple [A] time = 0.28, size = 10, normalized size = 1.11

$$\frac{\mathrm{X}(\operatorname{arctanh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] Chi(arctanh(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}} \operatorname{atanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

3.415 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)},x\right)$$

[Out] Unintegrable(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]

[Out] Defer[Int][1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx = \int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]

[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^5-2\,a^2x^3+x\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")

[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
[Out] int(1/(x*atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

3.416 $\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx = \int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{(a^4x^4-2\,a^2x^2+1)\,\text{artanh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\,\text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^m/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
[Out] Integral(x**m/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

3.417 $\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2},x\right)}{a^2}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^3}-\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

```
[Out] Shi(arctanh(a*x))/a^3-1/a^3/arctanh(a*x)/(-a^2*x^2+1)^(1/2)-Unintegrable(1/
(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)/a^2
```

Rubi [A] time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a^3
- Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/a^2
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx &= \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}+\frac{\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx}{a}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}
\end{aligned}$$

Mathematica [A] time = 3.26, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{arctanh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{atanh}(a\,x)^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x^2/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

3.418 $\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=36

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

```
[Out] Chi(arctanh(a*x))/a^2-x/a/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6006, 5968, 3301}

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]]/a^2
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx &= -\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx}{a}\\
&= -\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^2}\\
&= -\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 34, normalized size = 0.94

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]])/a^
2
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.35, size = 90, normalized size = 2.50

$$\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2a^2\left(ax-1\right)\text{arctanh}\left(ax\right)}-\frac{\text{Ei}\left(1,-\text{arctanh}\left(ax\right)\right)}{2a^2}+\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2a^2\left(ax+1\right)\text{arctanh}\left(ax\right)}-\frac{\text{Ei}\left(1,\text{arctanh}\left(ax\right)\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/2*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)-1/2*Ei(1,-arctanh(a*x
))/a^2+1/2*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)/arctanh(a*x)-1/2*Ei(1,arcta
nh(a*x))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x}{\text{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

3.419 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=35

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}$$

```
[Out] Shi(arctanh(a*x))/a-1/a/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5966, 6034, 3298}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^2} dx &= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} dx\\
&= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 32, normalized size = 0.91

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)-\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

maple [A] time = 0.33, size = 62, normalized size = 1.77

$$\frac{\text{arctanh}\left(ax\right)\text{Shi}\left(\text{arctanh}\left(ax\right)\right)x^2a^2-\text{Shi}\left(\text{arctanh}\left(ax\right)\right)\text{arctanh}\left(ax\right)+\sqrt{-a^2x^2+1}}{a\,\text{arctanh}\left(ax\right)\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2-Shi(arctanh(a*x))*arctanh(a*x)+
(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\text{atanh}\left(a\,x\right)^2\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

3.420 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{\text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)},x\right)}{a}-\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}+\text{Chi}\left(\tanh^{-1}(ax)\right)$$

```
[Out] Chi(arctanh(a*x))-a*x/arctanh(a*x)/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a/
x/arctanh(a*x)-Unintegrable(1/x^2/arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)/a
```

Rubi [A] time = 0.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) - Sqrt[1 - a^2*x^2]/(a*x*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]] - Defer[Int][1/(x^2*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx = a^2\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx}{a}+a\int \frac{}{(}$$

$$= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx}{a}+\text{Subst}$$

$$= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}+\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\,t}}{a}$$

Mathematica [A] time = 6.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{(a^4x^5-2a^2x^3+x)\,\text{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\operatorname{arctanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

3.421 $\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx=\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \operatorname{arctanh}\left(a x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(a x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\mathrm{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x^m/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Timed out
```

3.422 $\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3},x\right)}{a^2}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a^3}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}$$

```
[Out] 1/2*Chi(arctanh(a*x))/a^3-1/2/a^3/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2*x/a
^2/arctanh(a*x)/(-a^2*x^2+1)^(1/2)-Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctan
h(a*x)^3,x)/a^2
```

Rubi [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*a^2*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a^3) - Defer[Int][1/(Sqrt[1 - a
^2*x^2]*ArcTanh[a*x]^3), x]/a^2
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx &= \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx}{a^2}\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx}{a^2}+\frac{\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}}{2a}\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(}}{2a^2}\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,d\right.}{2a}\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 6.39, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")

[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")

[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{arctanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)

[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")

[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\text{atanh}\,(a\,x)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)

[Out] int(x^2/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\text{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

3.423 $\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=68

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}$$

```
[Out] 1/2*Shi(arctanh(a*x))/a^2-1/2*x/a/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2/a^2
/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6006, 5966, 6034, 3298}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - 1/(2*a^2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]) + SinhIntegral[ArcTanh[a*x]]/(2*a^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}dx = -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}dx}{2a}$$
$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{1}{2}\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\text{t}}$$
$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}dx,\right.}{2a^2}$$
$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A] time = 0.13, size = 43, normalized size = 0.63

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)-\frac{ax+\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((a*x + ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + SinhIntegral[
ArcTanh[a*x]])/(2*a^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.36, size = 154, normalized size = 2.26

$$\frac{\sqrt{-(ax-1)(ax+1)}}{4a^2(ax-1)\,\text{arctanh}\,(ax)^2}+\frac{\sqrt{-(ax-1)(ax+1)}}{4a^2(ax-1)\,\text{arctanh}\,(ax)}-\frac{\text{Ei}\,(1,-\text{arctanh}\,(ax))}{4a^2}+\frac{\sqrt{-(ax-1)(ax+1)}}{4a^2(ax+1)\,\text{arctanh}\,(ax)^2}-\frac{\sqrt{-}}{4a^2\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/4*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)^2+1/4*(-(a*x-1)*(a*x+
1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)-1/4*Ei(1,-arctanh(a*x))/a^2+1/4*(-(a*x-1
)*(a*x+1))^(1/2)/a^2/(a*x+1)/arctanh(a*x)^2-1/4*(-(a*x-1)*(a*x+1))^(1/2)/a^
2/(a*x+1)/arctanh(a*x)+1/4*Ei(1,arctanh(a*x))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atanh}(a\,x)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

3.424 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}+\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] 1/2*Chi(arctanh(a*x))/a-1/2/a/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2*x/arcta
nh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5966, 6006, 5968, 3301}

$$-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}+\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}+\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{1}{2}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx\right.}{2a}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Mathematica [A] time = 0.10, size = 44, normalized size = 0.68

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{ax\tanh^{-1}(ax)+1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegra
l[ArcTanh[a*x]])/(2*a)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

maple [A] time = 0.34, size = 86, normalized size = 1.32

$$\frac{\text{arctanh}\left(ax\right)^2 X\left(\text{arctanh}\left(ax\right)\right)x^2a^2+\sqrt{-a^2x^2+1}\;ax\,\text{arctanh}\left(ax\right)-X\left(\text{arctanh}\left(ax\right)\right)\text{arctanh}\left(ax\right)^2+\sqrt{-a^2x}}{2a\,\text{arctanh}\left(ax\right)^2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+(-a^2*x^2+1)^(1/2)*a*x*arct
anh(a*x)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x)^
2/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}(a\,x)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

3.425 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=124

$$-\frac{\text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2},x\right)}{2a}-\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}+\frac{1}{2}\text{Shi}\left(\tanh\right.$$

```
[Out] 1/2*Shi(arctanh(a*x))-1/2*a*x/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2/(-a^2*x
^2+1)^(1/2)/arctanh(a*x)-1/2*(-a^2*x^2+1)^(1/2)/a/x/arctanh(a*x)^2-1/2*Unin
tegrable(1/x^2/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)/a
```

Rubi [A] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -(a*x)/(2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - Sqrt[1 - a^2*x^2]/(2*a*x*ArcT
anh[a*x]^2) - 1/(2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]) + SinhIntegral[ArcTanh[a
*x]]/2 - Defer[Int][1/(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx &= a^2\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx\\
&= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{2a}\\
&= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\\
&= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\\
&= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-
\end{aligned}$$

Mathematica [A] time = 19.62, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^5-2\,a^2x^3+x\right)\operatorname{artanh}\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{arctanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\operatorname{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

3.426 $\int x^4 \sqrt{1-a^2x^2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=243

$$-\frac{i\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{i\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{8a^5}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}}{24a^2}$$

```
[Out] -7/72*(-a^2*x^2+1)^(3/2)/a^5+1/30*(-a^2*x^2+1)^(5/2)/a^5-1/8*arctan((-a*x+1
)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^5-1/16*I*polylog(2,-I*(-a*x+1)^(1/2)/
(a*x+1)^(1/2))/a^5+1/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+1/1
6*(-a^2*x^2+1)^(1/2)/a^5-1/16*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/24*x^
3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+1/6*x^5*arctanh(a*x)*(-a^2*x^2+1)^(1/
2)
```

Rubi [A] time = 0.31, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6010, 6016, 266, 43, 261, 5950}

$$-\frac{i\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{i\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a^5}-\frac{7\left(1-a^2x^2\right)^{3/2}}{72a^5}+\frac{\sqrt{1-a^2x^2}}{16a^5}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\ \mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(16*a^5) - (7*(1 - a^2*x^2)^(3/2))/(72*a^5) + (1 - a^2*x^
2)^(5/2)/(30*a^5) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^4) - (x^3*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/6 - (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^5) - ((I/16)*P
olyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + ((I/16)*PolyLog[2, (I*
Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx = \frac{1}{6}x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{6}\int\frac{x^4\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-\frac{1}{6}a\int\frac{x^5}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{24a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{\int\frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^2}+\frac{\int}{}$$

$$= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{24a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(a$$

$$= \frac{5\sqrt{1-a^2x^2}}{48a^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{9a^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a^5}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-}}{}$$

$$= \frac{\sqrt{1-a^2x^2}}{16a^5}-\frac{7\left(1-a^2x^2\right)^{3/2}}{72a^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a^5}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1}}{}$$

Mathematica [A] time = 0.72, size = 178, normalized size = 0.73

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{45i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+24\left(a^2x^2-1\right)^2+70\right.}{720a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(45 + 70*(-1 + a^2*x^2) + 24*(-1 + a^2*x^2)^2 + 45*a*x*A
rcTanh[a*x] + 210*a*x*(-1 + a^2*x^2)*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2)^
2*ArcTanh[a*x] - ((45*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
 I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcT
anh[a*x]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,x^4\,\text{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,x^4\,\mathrm{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

maple [A] time = 0.46, size = 195, normalized size = 0.80

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(120\,\mathrm{arctanh}\,(ax)\,x^5a^5+24x^4a^4-30a^3x^3\,\mathrm{arctanh}\,(ax)+22a^2x^2-45ax\,\mathrm{arctanh}\,(ax)-\right.}{720a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/720/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*x^5*a^5+24*x^4*a^4-30*
a^3*x^3*arctanh(a*x)+22*a^2*x^2-45*a*x*arctanh(a*x)-1)-1/16*I*ln(1+I*(a*x+1
)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5+1/16*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(
1/2))*arctanh(a*x)/a^5-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/1
6*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,x^4\,\mathrm{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\mathrm{atanh}\,(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{-(ax-1)(ax+1)}\,\mathrm{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

3.427 $\int x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=136

$$\frac{11\sin^{-1}(ax)}{120a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{x^3\sqrt{1-a^2x^2}}{20a}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4}+\frac{x\sqrt{1}}{2}$$

```
[Out] 11/120*arcsin(a*x)/a^4+1/24*x*(-a^2*x^2+1)^(1/2)/a^3+1/20*x^3*(-a^2*x^2+1)^
(1/2)/a-2/15*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/15*x^2*arctanh(a*x)*(-a^
2*x^2+1)^(1/2)/a^2+1/5*x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6010, 6016, 321, 216, 5994}

$$\frac{x^3\sqrt{1-a^2x^2}}{20a}+\frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^2}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4}+\frac{11\,\mathrm{si}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(24*a^3) + (x^3*Sqrt[1 - a^2*x^2])/(20*a) + (11*ArcSi
n[a*x])/(120*a^4) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^2*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/5
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
```

```
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int x^3\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)\,dx = \frac{1}{5}x^4\sqrt{1-a^2x^2}\ \tanh^{-1}(ax) + \frac{1}{5}\int \frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx - \frac{1}{5}a\int \frac{x^4}{\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{20a} - \frac{x^2\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{15a^2} + \frac{1}{5}x^4\sqrt{1-a^2x^2}\ \tanh^{-1}(ax) + \frac{2\int}{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{24a^3} + \frac{x^3\sqrt{1-a^2x^2}}{20a} - \frac{2\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{15a^4} - \frac{x^2\sqrt{1-a^2x^2}\ \tanh}{15a^2}$$

$$= \frac{x\sqrt{1-a^2x^2}}{24a^3} + \frac{x^3\sqrt{1-a^2x^2}}{20a} + \frac{11\sin^{-1}(ax)}{120a^4} - \frac{2\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{15a^4} - \frac{x^2}{}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.58

$$\frac{ax\sqrt{1-a^2x^2}\left(6a^2x^2+5\right)+8\sqrt{1-a^2x^2}\left(3a^4x^4-a^2x^2-2\right)\tanh^{-1}(ax)+11\sin^{-1}(ax)}{120a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2]*(5 + 6*a^2*x^2) + 11*ArcSin[a*x] + 8*Sqrt[1 - a^2*x^
2]*(-2 - a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/(120*a^4)
```

fricas [A] time = 0.51, size = 91, normalized size = 0.67

$$\frac{\left(6\,a^3x^3+5\,ax+4\left(3\,a^4x^4-a^2x^2-2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}-22\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*((6*a^3*x^3 + 5*a*x + 4*(3*a^4*x^4 - a^2*x^2 - 2)*log(-(a*x + 1)/(a*x
 - 1)))*sqrt(-a^2*x^2 + 1) - 22*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.39, size = 120, normalized size = 0.88

$$\frac{\sqrt{-(ax-1)(ax+1)}\left(24a^4x^4\operatorname{arctanh}(ax)+6x^3a^3-8a^2x^2\operatorname{arctanh}(ax)+5ax-16\operatorname{arctanh}(ax)\right)}{120a^4}+\frac{11i\ln\left(\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/120/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(24*a^4*x^4*arctanh(a*x)+6*x^3*a^3-8*a^2
*x^2*arctanh(a*x)+5*a*x-16*arctanh(a*x))+11/120*I*ln((a*x+1)/(-a^2*x^2+1)^(
1/2)+I)/a^4-11/120*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^4
```

maxima [A] time = 0.42, size = 128, normalized size = 0.94

$$-\frac{1}{120}a\left(\frac{3\left(\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{-a^2x^2+1}\,x}{a^2}-\frac{\arcsin(ax)}{a^3}\right)}{a^2}-\frac{8\left(\sqrt{-a^2x^2+1}\,x+\frac{\arcsin(ax)}{a}\right)}{a^4}\right)-\frac{1}{15}\left(\frac{3\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}{a^2}+\frac{2\left(-a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/120*a*(3*(2*(-a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(-a^2*x^2 + 1)*x/a^2 - arcs
in(a*x)/a^3)/a^2 - 8*(sqrt(-a^2*x^2 + 1)*x + arcsin(a*x)/a)/a^4) - 1/15*(3*
(-a^2*x^2 + 1)^(3/2)*x^2/a^2 + 2*(-a^2*x^2 + 1)^(3/2)/a^4)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{atanh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\mathrm{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

3.428 $\int x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}($$

```
[Out] -1/12*(-a^2*x^2+1)^(3/2)/a^3-1/4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arcta
nh(a*x)/a^3-1/8*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/8*I*poly
log(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/8*(-a^2*x^2+1)^(1/2)/a^3-1/8*x*
arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+1/4*x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6010, 6016, 261, 5950, 266, 43}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{12a^3}+\frac{\sqrt{1-a^2x^2}}{8a^3}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{x\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^3) - (1 - a^2*x^2)^(3/2)/(12*a^3) - (x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/4 - (ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^3) - ((I/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/8)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx &= \frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}\int\frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-\frac{1}{4}a\int\frac{x^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^2}+\frac{\int\frac{}{\sqrt{1\cdots}}}{}\\
&= -\frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\Big(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}}{}\\
&= \frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{12a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(a\cdots\end{aligned}$$

Mathematica [A] time = 0.48, size = 160, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{3i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}+2a^2x^2+6ax\left(a^2x^2-1\right)\tanh^{-1}(ax)-\frac{3i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)\cdots\right.}{\sqrt{1-a^2x^2}}\cdots\right.}{24a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2 + 3*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x^
2)*ArcTanh[a*x] - ((3*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((3*I)*(PolyLog[2, (-I)/E^ArcTanh[a
*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^3)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,x^2\,\text{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,x^2\operatorname{artanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

maple [A] time = 0.40, size = 175, normalized size = 0.90

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(6a^3x^3\operatorname{arctanh}(ax)+2a^2x^2-3ax\operatorname{arctanh}(ax)+1\right)}{24a^3}-\frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{arctanh}(ax)}{8a^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/24/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-3*a*x*a
rctanh(a*x)+1)-1/8*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/
8*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3-1/8*I*dilog(1+I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/8*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^
3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,x^2\operatorname{artanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

3.429 $\int x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1-a^2x^2}}{6a}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\sin^{-1}(ax)}{6a^2}$$

```
[Out] 1/6*arcsin(a*x)/a^2-1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)/a^2+1/6*x*(-a^2*x^2
+1)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5994, 195, 216}

$$\frac{x\sqrt{1-a^2x^2}}{6a}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\sin^{-1}(ax)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(6*a) + ArcSin[a*x]/(6*a^2) - ((1 - a^2*x^2)^(3/2)*Ar
cTanh[a*x])/(3*a^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx&=-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\int\sqrt{1-a^2x^2}\,dx}{3a}\\&=\frac{x\sqrt{1-a^2x^2}}{6a}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{6a}\\&=\frac{x\sqrt{1-a^2x^2}}{6a}+\frac{\sin^{-1}(ax)}{6a^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.83

$$\frac{ax\sqrt{1-a^2x^2}-2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\sin^{-1}(ax)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x] - 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/
(6*a^2)
```

fricas [A] time = 0.56, size = 72, normalized size = 1.22

$$\frac{\sqrt{-a^2x^2+1}\left(ax+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)-2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))) - 2
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.32, size = 99, normalized size = 1.68

$$\frac{\sqrt{-(ax-1)(ax+1)}\left(2a^2x^2\operatorname{arctanh}(ax)+ax-2\operatorname{arctanh}(ax)\right)}{6a^2}+\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{6a^2}-\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-i\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x-2*arctanh(a*x)
)+1/6*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^2-1/6*I*ln((a*x+1)/(-a^2*x^2+1)^
(1/2)-I)/a^2
```

maxima [A] time = 0.41, size = 50, normalized size = 0.85

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)}{3a^2}+\frac{\sqrt{-a^2x^2+1}\,x+\frac{\arcsin(ax)}{a}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/a^2 + 1/6*(sqrt(-a^2*x^2 + 1)*x + ar
csin(a*x)/a)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atanh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\mathrm{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

3.430 $\int \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

```
[Out] -arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-1/2*I*polylog(2,-I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)/a+1/2*(-a^2*x^2+1)^(1/2)/a+1/2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx&=\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{2}\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx\\&=\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i}{}\right.}{2a}\end{aligned}$$

Mathematica [A] time = 0.26, size = 117, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+ax\tanh^{-1}(ax)+1\right.}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^Ar
cTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]]
- PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-a^2x^2+1}\ \mathrm{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.76, size = 152, normalized size = 1.06

$$\frac{\left(ax\,\mathrm{arctanh}\left(ax\right)+1\right)\sqrt{-a^2x^2+1}}{2a}-\frac{i\,\mathrm{arctanh}\left(ax\right)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}+\frac{i\,\mathrm{arctanh}\left(ax\right)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{i\,\mathrm{dilog}\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x)
```

```
[Out] 1/2*(a*x*arctanh(a*x)+1)*(-a^2*x^2+1)^(1/2)/a-1/2*I/a*arctanh(a*x)*ln(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)
^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \mathrm{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

3.431 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=100

$$\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\sin^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

```
[Out] -arcsin(a*x)-2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2
,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+(-a
^2*x^2+1)^(1/2)*arctanh(a*x)
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6010, 6018, 216}

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\sin^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-}}{\sqrt{ax}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

```
[Out] -ArcSin[a*x] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 2*ArcTanh[a*x]*ArcTanh[Sqrt
[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - Pol
yLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}\,dx &= \sqrt{1-a^2x^2}\tanh^{-1}(ax)-a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\sin^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{Li}_2\left(-\right.\end{aligned}$$

Mathematica [A] time = 0.13, size = 91, normalized size = 0.91

$$\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)+\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\log$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

```
[Out] -2*ArcTan[Tanh[ArcTanh[a*x]/2]] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + ArcTanh[
a*x]*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] +
 PolyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\ \text{artanh}\left(ax\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.42, size = 113, normalized size = 1.13

$$\sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \text{arctanh}\left(ax\right)-2\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\text{dilog}\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\text{dilog}\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)-2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))-
dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arcta
nh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\ \text{artanh}\left(ax\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{atanh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x, x)
```

3.432 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ia\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-ia\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)t$$

```
[Out] 2*a*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)-a*arctanh((-a^2*x^2+1
)^(1/2))+I*a*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-I*a*polylog(2,I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6008, 266, 63, 208, 5950}

$$ia\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-ia\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + 2*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a
*x]]*ArcTanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + I*a*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]] - I*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]
]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
```

```
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+ia\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+ia\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+ia\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.
\end{aligned}$$

Mathematica [A] time = 0.38, size = 121, normalized size = 0.93

$$a\left(-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{ax}+i\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-i\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-i\tan\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 - I/E^A
rcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + Log[Tanh[ArcTanh[
a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh[a*x
]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.45, size = 188, normalized size = 1.45

$$-\frac{\sqrt{-(ax-1)(ax+1)}\ \operatorname{arctanh}(ax)}{x}-ia\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{arctanh}(ax)-ia\operatorname{dilog}\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)+ia\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)/x-I*a*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1
/2))*arctanh(a*x)-I*a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+I*a*ln(1+I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)+I*a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/
2))+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^2,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\ \operatorname{atanh}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**2, x)
```

3.433 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=136

$$-\frac{1}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.$$

```
[Out] a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/2*a^2*polylog(2,-(
-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)-1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.20, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6010, 6026, 264, 6018}

$$-\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\tanh^{-1}(ax)\,t$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 + (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
```

```
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2} + a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx - \int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1-a^2x^2}}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} - \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx - \frac{1}{2}a^2\int \frac{\text{tanh}}{x\sqrt{1}}$$
$$= -\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} + a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \frac{1}{2}$$

Mathematica [A] time = 0.72, size = 126, normalized size = 0.93

$$\frac{1}{8}a^2\left(-4\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) + 4\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right) + 2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right) - 4\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

```
[Out] (a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 4*Arc
Tanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a
*x])] - 4*PolyLog[2, -E^(-ArcTanh[a*x])] + 4*PolyLog[2, E^(-ArcTanh[a*x])]
- ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.43, size = 141, normalized size = 1.04

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,(ax+\text{arctanh}(ax))}{2x^2} + \frac{a^2\,\text{arctanh}(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2} + \frac{a^2\,\text{polylog}\left(2, -\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2} - \frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x+arctanh(a*x))/x^2+1/2*a^2*arctanh(a*x)*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/
2))-1/2*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^3,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**3, x)
```

3.434 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3+1/6*a^3*arctanh((-a^2*x^2+1)^(1/2)
)-1/6*a*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6008, 266, 47, 63, 208}

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(6*x^2) - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3)
 + (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
```

```
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^4}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-\frac{1}{12}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 1.13

$$-\frac{a^3x^3\log(x)+ax\sqrt{1-a^2x^2}+2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)-a^3x^3\log\left(\sqrt{1-a^2x^2}+1\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*(a*x*Sqrt[1 - a^2*x^2] + 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x] + a^3*x^3*
Log[x] - a^3*x^3*Log[1 + Sqrt[1 - a^2*x^2]])/x^3
```

fricas [A] time = 0.44, size = 75, normalized size = 1.07

$$-\frac{a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\left(ax-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(a*x - (
a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.42, size = 96, normalized size = 1.37

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(2a^2x^2\operatorname{arctanh}(ax)-ax-2\operatorname{arctanh}(ax)\right)}{6x^3}-\frac{a^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)}{6}+\frac{a^3\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] 1/6*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))/x^
3-1/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+1/6*a^3*ln(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))
```

maxima [A] time = 0.41, size = 90, normalized size = 1.29

$$\frac{1}{6}\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}\,a^2-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{x^2}\right)a-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*a
^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a - 1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^
3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^4,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**4, x)
```

3.435 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=191

$$-\frac{1}{8}a^4\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{8}a^4\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{4}a^4\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-\frac{\sqrt{1-}}{}$$

```
[Out] 1/4*a^4*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/8*a^4*polylog(
2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/8*a^4*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(
1/2))-1/12*a*(-a^2*x^2+1)^(1/2)/x^3-1/24*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*arcta
nh(a*x)*(-a^2*x^2+1)^(1/2)/x^4+1/8*a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.30, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6010, 6026, 271, 264, 6018}

$$-\frac{1}{8}a^4\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{8}a^4\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^3\sqrt{1-a^2x^2}}{24x}-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(12*x^3) - (a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*
x^2) + (a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 - (a^4*Pol
yLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 + (a^4*PolyLog[2, Sqrt[1 - a*x]/
Sqrt[1 + a*x]])/8
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^5}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^4} - \frac{1}{3}\int \frac{\tanh^{-1}(ax)}{x^5\sqrt{1-a^2x^2}}\,dx + \frac{1}{3}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a\sqrt{1-a^2x^2}}{9x^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4} - \frac{1}{12}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx - \frac{1}{4}a^2\int \frac{\text{ta}}{x^3\sqrt{}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{2a^3\sqrt{1-a^2x^2}}{9x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-}}{8x^2}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{a^3\sqrt{1-a^2x^2}}{24x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(}{8x^2}$$

Mathematica [A] time = 1.74, size = 222, normalized size = 1.16

$$\frac{1}{192}a^4\left(-\frac{ax\operatorname{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} - \frac{16\left(1-a^2x^2\right)^{3/2}\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{a^3x^3} - 24\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) + 24\text{Li}_2\left(e^{-\text{t}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

```
[Out] (a^4*(-8*Coth[ArcTanh[a*x]/2] - 6*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - (a*
x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*ArcTanh[a*x]*Csch[ArcTanh[a
*x]/2]^4 - 24*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]*Log
[1 + E^(-ArcTanh[a*x])] - 24*PolyLog[2, -E^(-ArcTanh[a*x])] + 24*PolyLog[2,
 E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 3*ArcTanh[a*x
]*Sech[ArcTanh[a*x]/2]^4 - (16*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/2]^4)/
(a^3*x^3) + 8*Tanh[ArcTanh[a*x]/2]))/192
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.45, size = 164, normalized size = 0.86

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(-x^3a^3+3a^2x^2\operatorname{arctanh}(ax)-2ax-6\operatorname{arctanh}(ax)\right)}{24x^4}+\frac{a^4\operatorname{arctanh}(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x)

[Out] 1/24*(-(a*x-1)*(a*x+1))^(1/2)*(-x^3*a^3+3*a^2*x^2*arctanh(a*x)-2*a*x-6*arctanh(a*x))/x^4+1/8*a^4*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/8*a^4*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a^4*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a^4*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^5,x)

[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**5,x)

[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**5, x)

3.436 $\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{5x^5}-\frac{a\sqrt{1-a^2x^2}}{20x^4}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15x^3}+\frac{11}{120}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{2a^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15x}$$

```
[Out] 11/120*a^5*arctanh((-a^2*x^2+1)^(1/2))-1/20*a*(-a^2*x^2+1)^(1/2)/x^4-1/24*a
^3*(-a^2*x^2+1)^(1/2)/x^2-1/5*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5+1/15*a^2*
arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3+2/15*a^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2
)/x
```

Rubi [A] time = 0.37, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6010, 6026, 266, 51, 63, 208, 6008}

$$-\frac{a^3\sqrt{1-a^2x^2}}{24x^2}-\frac{a\sqrt{1-a^2x^2}}{20x^4}+\frac{11}{120}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{2a^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15x}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(20*x^4) - (a^3*Sqrt[1 - a^2*x^2])/(24*x^2) - (Sqrt[
1 - a^2*x^2]*ArcTanh[a*x])/(5*x^5) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(
15*x^3) + (2*a^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*x) + (11*a^5*ArcTanh[S
qrt[1 - a^2*x^2]])/120
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
```

```
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^6}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^5} - \frac{1}{4}\int \frac{\tanh^{-1}(ax)}{x^6\sqrt{1-a^2x^2}}\,dx + \frac{1}{4}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} - \frac{1}{20}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}\,dx + \frac{1}{8}a\,\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt{1-a^2x}}\,d\right.$$

$$= -\frac{a\sqrt{1-a^2x^2}}{16x^4} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3} - \frac{1}{40}a\,\mathrm{Subst}\left(\right.$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3a^3\sqrt{1-a^2x^2}}{32x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(a}{15x^3}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax}{15x^3}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax}{15x^3}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax}{15x^3}$$

Mathematica [A] time = 0.14, size = 104, normalized size = 0.69

$$\frac{1}{120}\left(-11a^5\log(x) - \frac{a\sqrt{1-a^2x^2}\left(5a^2x^2+6\right)}{x^4} + 11a^5\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{8\sqrt{1-a^2x^2}\left(2a^4x^4+a^2x^2-3\right)\tanh}{x^5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```

```
[Out] (-((a*Sqrt[1 - a^2*x^2]*(6 + 5*a^2*x^2))/x^4) + (8*Sqrt[1 - a^2*x^2]*(-3 +
a^2*x^2 + 2*a^4*x^4)*ArcTanh[a*x])/x^5 - 11*a^5*Log[x] + 11*a^5*Log[1 + Sqr
t[1 - a^2*x^2]])/120
```

fricas [A] time = 0.67, size = 93, normalized size = 0.62

$$-\frac{11\,a^{5}x^{5}\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)+\left(5\,a^{3}x^{3}+6\,ax-4\left(2\,a^{4}x^{4}+a^{2}x^{2}-3\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^{2}x^{2}+1}}{120\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(11*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 + 6*a*x - 4
*(2*a^4*x^4 + a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))/x
^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.44, size = 116, normalized size = 0.77

$$\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\;\left(16a^{4}x^{4}\operatorname{arctanh}\left(ax\right)-5x^{3}a^{3}+8a^{2}x^{2}\operatorname{arctanh}\left(ax\right)-6ax-24\operatorname{arctanh}\left(ax\right)\right)}{120x^{5}}+\frac{11a^{5}\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x)
```

```
[Out] 1/120*(-(a*x-1)*(a*x+1))^(1/2)*(16*a^4*x^4*arctanh(a*x)-5*x^3*a^3+8*a^2*x^2
*arctanh(a*x)-6*a*x-24*arctanh(a*x))/x^5+11/120*a^5*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))-11/120*a^5*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

maxima [A] time = 0.41, size = 204, normalized size = 1.36

$$\frac{1}{120}\left(3\,a^{4}\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-3\sqrt{-a^{2}x^{2}+1}\,a^{4}+8\left(a^{2}\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^{2}x^{2}+1}\,a^{2}-\frac{(\cdot}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/120*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*sqrt(-a^2*x^2
+ 1)*a^4 + 8*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x
^2 + 1)*a^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a^2 - 3*(-a^2*x^2 + 1)^(3/2)*a^2/x^
2 - 6*(-a^2*x^2 + 1)^(3/2)/x^4)*a - 1/15*(2*(-a^2*x^2 + 1)^(3/2)*a^2/x^3 +
3*(-a^2*x^2 + 1)^(3/2)/x^5)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(a\,x\right)\,\sqrt{1-a^{2}\,x^{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^6,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^6, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}\left(ax\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**6, x)
```

3.437 $\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^7}\,dx$

Optimal. Leaf size=243

$$-\frac{1}{16}a^6\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{16}a^6\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{8}a^6\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}-\frac{a\sqrt{}}{}$$

```
[Out] 1/8*a^6*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/16*a^6*polylog
(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/16*a^6*polylog(2,(-a*x+1)^(1/2)/(a*x+1)
^(1/2))-1/30*a*(-a^2*x^2+1)^(1/2)/x^5-11/360*a^3*(-a^2*x^2+1)^(1/2)/x^3+1/7
20*a^5*(-a^2*x^2+1)^(1/2)/x-1/6*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6+1/24*a^
2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4+1/16*a^4*arctanh(a*x)*(-a^2*x^2+1)^(1
/2)/x^2
```

Rubi [A] time = 0.41, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6010, 6026, 271, 264, 6018}

$$-\frac{1}{16}a^6\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{16}a^6\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{a^5\sqrt{1-a^2x^2}}{720x}-\frac{11a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{a^{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(30*x^5) - (11*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (a
^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6) +
(a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) + (a^4*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x])/(16*x^2) + (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]
)/8 - (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 + (a^6*PolyLog[2,
 Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
```

```
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^7}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^6} - \frac{1}{5}\int \frac{\tanh^{-1}(ax)}{x^7\sqrt{1-a^2x^2}}\,dx + \frac{1}{5}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a\sqrt{1-a^2x^2}}{25x^5} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} - \frac{1}{30}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}\,dx - \frac{1}{6}a^2\int \frac{\tanh}{x^5\sqrt{1}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{4a^3\sqrt{1-a^2x^2}}{75x^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(a}{24x^4}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{11a^3\sqrt{1-a^2x^2}}{360x^3} - \frac{8a^5\sqrt{1-a^2x^2}}{75x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} + \frac{a^2\sqrt{}}{}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{11a^3\sqrt{1-a^2x^2}}{360x^3} + \frac{a^5\sqrt{1-a^2x^2}}{720x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} + \frac{a^2\sqrt{1}}{}$$

Mathematica [A] time = 3.70, size = 307, normalized size = 1.26

$$a^6\left(-\frac{3ax\operatorname{csch}^6\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} - \frac{26ax\operatorname{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} - \frac{416\left(1-a^2x^2\right)^{3/2}\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{a^3x^3} - 360\mathrm{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) + 360\mathrm{Li}_2\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]
```

```
[Out] (a^6*(-76*Coth[ArcTanh[a*x]/2] - 90*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - (
26*a*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 90*ArcTanh[a*x]*Csch[Arc
Tanh[a*x]/2]^4 - (3*a*x*Csch[ArcTanh[a*x]/2]^6)/Sqrt[1 - a^2*x^2] - 15*ArcT
anh[a*x]*Csch[ArcTanh[a*x]/2]^6 - 360*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x]
)] + 360*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] - 360*PolyLog[2, -E^(-ArcT
anh[a*x])] + 360*PolyLog[2, E^(-ArcTanh[a*x])] - 90*ArcTanh[a*x]*Sech[ArcTa
nh[a*x]/2]^2 + 90*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - 15*ArcTanh[a*x]*Sec
h[ArcTanh[a*x]/2]^6 - (416*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/2]^4)/(a^3
*x^3) + 76*Tanh[ArcTanh[a*x]/2] + 6*Sech[ArcTanh[a*x]/2]^4*Tanh[ArcTanh[a*x
]/2]))/5760
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.47, size = 183, normalized size = 0.75

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(x^5a^5+45a^4x^4\operatorname{arctanh}(ax)-22x^3a^3+30a^2x^2\operatorname{arctanh}(ax)-24ax-120\operatorname{arctanh}(ax)\right)}{720x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x)
```

```
[Out] 1/720*(-(a*x-1)*(a*x+1))^(1/2)*(x^5*a^5+45*a^4*x^4*arctanh(a*x)-22*x^3*a^3+
30*a^2*x^2*arctanh(a*x)-24*a*x-120*arctanh(a*x))/x^6+1/16*a^6*arctanh(a*x)*
ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(
1/2))-1/16*a^6*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^7,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**7, x)
```

3.438 $\int x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=336

$$-\frac{i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\mathrm{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{8a^5}-\frac{i\mathrm{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{8a^5}-\frac{19\sin^{-1}(a}{360a^5}$$

```
[Out] -19/360*arcsin(a*x)/a^5+1/8*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)
^2/a^5-1/8*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/8*
I*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/8*I*polylog(3,
-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5-1/8*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(
1/2))/a^5+1/18*x*(-a^2*x^2+1)^(1/2)/a^4+1/60*x^3*(-a^2*x^2+1)^(1/2)/a^2-1/3
60*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^5+11/180*x^2*arctanh(a*x)*(-a^2*x^2+1)
^(1/2)/a^3+1/15*x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a-1/16*x*arctanh(a*x)^2
*(-a^2*x^2+1)^(1/2)/a^4-1/24*x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2+1/6*
x^5*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.40, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6014, 6016, 321, 216, 5994, 5952, 4180, 2531, 2282, 6589}

$$-\frac{i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\mathrm{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{8a^5}-\frac{i\mathrm{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(18*a^4) + (x^3*Sqrt[1 - a^2*x^2])/(60*a^2) - (19*Arc
Sin[a*x])/(360*a^5) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(360*a^5) + (11*x^2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(180*a^3) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x])/(15*a) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(16*a^4) - (x^3*Sqrt[1
- a^2*x^2]*ArcTanh[a*x]^2)/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/6 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(8*a^5) - ((I/8)*ArcTanh[a*x]
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^5 + ((I/8)*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a^5 + ((I/8)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^5 - ((I/8)*P
olyLog[3, I*E^ArcTanh[a*x]])/a^5
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx = -\left(a^2\int\frac{x^6\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int\frac{x^4\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{4a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-\frac{5}{6}\int\frac{x^4\tanh^{-1}(ax}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6a^3}+\frac{x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a}-\frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(}{8a^4}$$

$$= -\frac{x\sqrt{1-a^2x^2}}{12a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{13\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{12a^5}+\frac{11x^2\sqrt{1-a^2x^2}\,\tan}{180a^3}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}+\frac{7\sin^{-1}(ax)}{6a^5}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{1}}{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{}}{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{}}{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{}}{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{}}{}$$

Mathematica [A] time = 1.69, size = 268, normalized size = 0.80

$$\sqrt{1-a^2x^2}\left(-\frac{i\left(90\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-90\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+90\mathrm{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)-90\mathrm{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right)+45\tanh^{-1}(ax)^2\right.}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(90*ArcTanh[a*x] + 140*(-1 + a^2*x^2)*ArcTanh[a*x] + 48*
(-1 + a^2*x^2)^2*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 + 6
*a*x*(-1 + a^2*x^2)*(2 + 35*ArcTanh[a*x]^2) + a*x*(52 + 45*ArcTanh[a*x]^2)
- (I*((-76*I)*ArcTan[Tanh[ArcTanh[a*x]/2]] + 45*ArcTanh[a*x]^2*Log[1 - I/E^
ArcTanh[a*x]] - 45*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 90*ArcTanh[a*
x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - 90*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh
[a*x]] + 90*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 90*PolyLog[3, I/E^ArcTanh[a*x
]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-a^2x^2+1}\,x^4\,\mathrm{artanh}\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,x^4 \operatorname{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arctanh}(ax)^2 \sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
[Out] int(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,x^4 \operatorname{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{atanh}(a\,x)^2 \sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(x^4*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

3.439 $\int x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=281

$$-\frac{11i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}+\frac{11i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}-\frac{11\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{30a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{15a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\,\text{ta}$$

```
[Out] -1/30*(-a^2*x^2+1)^(3/2)/a^4-11/30*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arc
tanh(a*x)/a^4-11/60*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4+11/60*
I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4+11/60*(-a^2*x^2+1)^(1/2)/a^
4+1/12*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^3+1/10*x^3*arctanh(a*x)*(-a^2*x^
2+1)^(1/2)/a-2/15*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4-1/15*x^2*arctanh(a*
x)^2*(-a^2*x^2+1)^(1/2)/a^2+1/5*x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6014, 6016, 261, 5950, 5994, 266, 43}

$$-\frac{11i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}+\frac{11i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}-\frac{\left(1-a^2x^2\right)^{3/2}}{30a^4}+\frac{11\sqrt{1-a^2x^2}}{60a^4}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2])/(60*a^4) - (1 - a^2*x^2)^(3/2)/(30*a^4) + (x*Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(12*a^3) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(1
0*a) - (11*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(30*a^4) - (2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(15*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^2)/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/5 - (((11*I)/60)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (((11*I)/60)*PolyLog
[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
```

```
0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\left(a^2\int\frac{x^5\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int\frac{x^3\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-\frac{4}{5}\int\frac{x^3\tanh^{-1}(}{\sqrt{1-a^2}}$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(}{3a^4}$$

$$= -\frac{\sqrt{1-a^2x^2}}{3a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{10\tan}{}$$

$$= \frac{\sqrt{1-a^2x^2}}{12a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{11\tan^{-}}{}$$

$$= \frac{11\sqrt{1-a^2x^2}}{60a^4}-\frac{\left(1-a^2x^2\right)^{3/2}}{30a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh}{10a}$$

Mathematica [A] time = 0.70, size = 175, normalized size = 0.62

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{11i\left(\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+12\left(a^2x^2-1\right)^2\mathrm{ta}\right.}{60a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(11 + 11*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x^2)*ArcTanh
[a*x] + 12*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*(1 + 10*ArcTa
nh[a*x]^2) - ((11*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E
^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[
a*x]]))/Sqrt[1 - a^2*x^2]))/(60*a^4)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,x^3\,\text{artanh}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.42, size = 211, normalized size = 0.75

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(12a^4x^4\,\text{arctanh}\,(ax)^2+6a^3x^3\,\text{arctanh}\,(ax)-4a^2x^2\,\text{arctanh}\,(ax)^2+2a^2x^2+5ax\,\text{arctanh}\,(\right.}{60a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/60/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(12*a^4*x^4*arctanh(a*x)^2+6*a^3*x^3*arct
anh(a*x)-4*a^2*x^2*arctanh(a*x)^2+2*a^2*x^2+5*a*x*arctanh(a*x)-8*arctanh(a*
x)^2+9)-11/60*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4+11/60*I
*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4-11/60*I*dilog(1+I*(a*x
+1)/(-a^2*x^2+1)^(1/2))/a^4+11/60*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,x^3\,\text{artanh}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\text{atanh}\,(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x^3*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

3.440 $\int x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=254

$$-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{4a^3}-\frac{i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{4a^3}-\frac{\sin^{-1}(ax)}{6a^3}$$

```
[Out] -1/6*arcsin(a*x)/a^3+1/4*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/
a^3-1/4*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/4*I*a
rctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/4*I*polylog(3,-I*
(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-1/4*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a^3+1/12*x*(-a^2*x^2+1)^(1/2)/a^2+1/12*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a
^3+1/6*x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a-1/8*x*arctanh(a*x)^2*(-a^2*x^2
+1)^(1/2)/a^2+1/4*x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6014, 6016, 5994, 216, 5952, 4180, 2531, 2282, 6589, 321}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{4a^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(12*a^2) - ArcSin[a*x]/(6*a^3) + (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x])/(12*a^3) + (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*a) - (x*Sqr
t[1 - a^2*x^2]*ArcTanh[a*x]^2)/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
]^2)/4 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(4*a^3) - ((I/4)*ArcTanh[a
*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + ((I/4)*ArcTanh[a*x]*PolyLog[2, I
*E^ArcTanh[a*x]])/a^3 + ((I/4)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((I/4
)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\left(a^2\int\frac{x^4\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int\frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-\frac{3}{4}\int\frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3}+\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{8a^2}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}+\frac{\sin^{-1}(ax)}{a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a}$$

Mathematica [A] time = 1.23, size = 228, normalized size = 0.90

$$\sqrt{1-a^2x^2}\left(-\frac{i\left(6\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+6\mathrm{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)-6\mathrm{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\log\left(1\right.\right.}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(6*ArcTanh[a*x] - 4*(1 - a^2*x^2)*ArcTanh[a*x] - 6*a*x*(
1 - a^2*x^2)*ArcTanh[a*x]^2 + a*x*(2 + 3*ArcTanh[a*x]^2) - (I*((-8*I)*ArcTa
n[Tanh[ArcTanh[a*x]/2]] + 3*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 3*Ar
cTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyLog[2, (-I)/E^A
rcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 6*PolyLog[3, (
-I)/E^ArcTanh[a*x]] - 6*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/
(24*a^3)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-a^2x^2+1}\,x^2\,\mathrm{artanh}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2x^2+1}\,x^2\,\mathrm{artanh}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arctanh}(ax)^2 \sqrt{-a^2x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
[Out] int(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\, x^2 \operatorname{artanh}(ax)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}(a\,x)^2 \sqrt{1-a^2\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(x^2*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-(ax-1)(ax+1)} \operatorname{atanh}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

3.441 $\int x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=175

$$-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{\sqrt{1-a^2x^2}}{3a^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2}+\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a}-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\ldots}{3a\ldots}$$

```
[Out] -2/3*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^2-1/3*(-a^2*x^2+1)
^(3/2)*arctanh(a*x)^2/a^2-1/3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/
a^2+1/3*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2+1/3*(-a^2*x^2+1)^(1
/2)/a^2+1/3*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5994, 5942, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{\sqrt{1-a^2x^2}}{3a^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2}+\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a^2) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a) - (2*A
rcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^2) - ((1 - a^2*x^2)^(
3/2)*ArcTanh[a*x]^2)/(3*a^2) - ((I/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a^2 + ((I/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2} + \frac{2\int\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx}{3a}$$

$$= \frac{\sqrt{1-a^2x^2}}{3a^2} + \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a} - \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2} + \frac{\int\frac{\tanh^{-1}}{\sqrt{1-a}}}{3a}$$

$$= \frac{\sqrt{1-a^2x^2}}{3a^2} + \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a} - \frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{3a^2} - (1-$$

Mathematica [A] time = 0.43, size = 135, normalized size = 0.77

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}} - \left(\left(1-a^2x^2\right)\tanh^{-}\right.}{3a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (1 - a^2*x^2)*ArcTanh[a*x]^2 - (
I*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + P
olyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^
2*x^2]))/(3*a^2)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,x\,\text{artanh}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.35, size = 175, normalized size = 1.00

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(a^2x^2\,\text{arctanh}\,(ax)^2+ax\,\text{arctanh}\,(ax)-\text{arctanh}\,(ax)^2+1\right)}{3a^2} - \frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\text{arctanh}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/3/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)-a
rctanh(a*x)^2+1)-1/3*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^2+
```

```
1/3*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^2-1/3*I*dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+1/3*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2 + 1}\, x\, \text{artanh}\,(ax)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\, \text{atanh}\,(a\,x)^2\, \sqrt{1 - a^2\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(x*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-(ax-1)(ax+1)}\, \text{atanh}^2\,(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

3.442 $\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=158

$$\frac{1}{2}x\sqrt{1-a^2x^2} \tanh^{-1}(ax)^2+\frac{\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{a}-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a}$$

```
[Out] -arcsin(a*x)/a+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-I*arctan
h(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*arctanh(a*x)*polylog(2,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+arctanh(a*x)*(-a^2*x^2+1)^(1/
2)/a+1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5944, 5952, 4180, 2531, 2282, 6589, 216}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx - \int$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\text{Subst}\left(\int x^2\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

Mathematica [A] time = 0.77, size = 187, normalized size = 1.18

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(2\tanh^{-1}(ax)\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+2\text{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)-2\text{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)^2\log\left(1-i\right.\right.}{\sqrt{1-a^2x^2}}\right.}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\ \text{artanh}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \ \text{arctanh}\,(ax)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \text{artanh}\,(ax)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{atanh}\,(a\,x)^2\,\sqrt{1-a^2\,x^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(ax-1)(ax+1)}\ \text{atanh}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

3.443 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=174

$$\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+2i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{Li}_2\left(\right.$$

```
[Out] 4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)-2*arctanh((a*x+1)/(-a^2
*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I*polylog(2
,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2
+1)^(1/2))+(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2
```

Rubi [A] time = 0.39, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6014, 6020, 4182, 2531, 2282, 6589, 5994, 5950}

$$2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyI}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
```

```
[Out] 4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x] + Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]^2 - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*Poly
Log[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + (2*I)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] - (2*I)*PolyLog[2, (I*Sqrt[
1 - a*x])/Sqrt[1 + a*x]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^A
rcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
```

```
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}\,dx = -\left(a^2\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$

$$= \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-(2a)\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx+\text{Subst}\left(\int x^2\text{csch}(x)\,dx,x,$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(a}\right.$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(a}\right.$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(a}\right.$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(a}\right.$$

Mathematica [A] time = 0.30, size = 203, normalized size = 1.17

$$\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+2i\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])
] + (2*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (2*I)*ArcTanh[a*x]*Log[1
 + I/E^ArcTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*ArcTan
h[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x
]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, E^(-Arc
Tanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*PolyLog[3, E^(-ArcTanh[a
*x])]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\ \text{artanh}(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\text{arctanh}(ax)^2 \sqrt{-a^2x^2+1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\ \text{artanh}(ax)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x,x)
```

```
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x, x)
```

3.444 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=197

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}+2a\mathrm{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\mathrm{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+2ia\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)-2ia\tanh^{-1}(a$$

```
[Out] -2*a*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-4*a*arctanh(a*x)*arc
tanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*I*a*arctanh(a*x)*polylog(2,-I*(a*x+1)/
(-a^2*x^2+1)^(1/2))-2*I*a*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/
2))+2*a*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/
2)/(a*x+1)^(1/2))-2*I*a*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I*a*poly
log(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.38, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6014, 6008, 6018, 5952, 4180, 2531, 2282, 6589}

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+2ia\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)-2ia\tanh^{-1}(ax)\mathrm{I}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 2*a*ArcTan[E^ArcTanh[a*x]]*ArcTan
h[a*x]^2 - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (2*I)*a*
ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (2*I)*a*ArcTanh[a*x]*PolyLog
[2, I*E^ArcTanh[a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*
a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (2*I)*a*PolyLog[3, (-I)*E^ArcTa
nh[a*x]] + (2*I)*a*PolyLog[3, I*E^ArcTanh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2}{x^2}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2}{x} - a\,\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right) + (2a)\int$$

$$= -\frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)$$

$$= -\frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)$$

$$= -\frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)$$

$$= -\frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)$$

Mathematica [A] time = 0.78, size = 223, normalized size = 1.13

$$a\left(-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{ax}+2i\tanh^{-1}(ax)\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-2i\tanh^{-1}(ax)\mathrm{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+2\mathrm{Li}_2\left(-e^{-\tanh}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(a*x)) + 2*ArcTanh[a*x]*Log[1 - E^(
-ArcTanh[a*x])] + I*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[a*
x]^2*Log[1 + I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
+ 2*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*ArcTanh[a*x]*PolyLog[2, (-I)/E^A
rcTanh[a*x]] - (2*I)*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] - 2*PolyLog[
2, E^(-ArcTanh[a*x])] + (2*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (2*I)*PolyL
og[3, I/E^ArcTanh[a*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)^2\sqrt{-a^2x^2+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^2,x)
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**2,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**2, x)
```

3.445 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=151

$$a^2\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{\tanh^{-1}(ax)}\right)-a^2\mathrm{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)+a^2\mathrm{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}($$

```
[Out] a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-a^2*x^
2+1)^(1/2))+a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arc
tanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)/(-a^
2*x^2+1)^(1/2))+a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-a*arctanh(a*x)*(-
a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.54, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6014, 6026, 6008, 266, 63, 208, 6020, 4182, 2531, 2282, 6589}

$$a^2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)-a^2\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)+a^2\mathrm{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) + a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] + a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] - a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] - a^2*PolyLog[3, -E^ArcTanh[a*x]] + a^2*Poly
Log[3, E^ArcTanh[a*x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^3}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx\right) + \int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx - a^2 \mathrm{S}$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + 2a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

Mathematica [A] time = 1.26, size = 188, normalized size = 1.25

$$\frac{1}{8}a^2\left(-8\tanh^{-1}(ax)\mathrm{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) + 8\tanh^{-1}(ax)\mathrm{Li}_2\left(e^{-\tanh^{-1}(ax)}\right) - 8\mathrm{Li}_3\left(-e^{-\tanh^{-1}(ax)}\right) + 8\mathrm{Li}_3\left(e^{-\tanh^{-1}(ax)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] (a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*
x]/2]^2 - 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]^2*Lo
g[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] - 8*ArcTanh[a*x]*Pol
yLog[2, -E^(-ArcTanh[a*x])] + 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])]
- 8*PolyLog[3, -E^(-ArcTanh[a*x])] + 8*PolyLog[3, E^(-ArcTanh[a*x])] - ArcT
anh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.46, size = 231, normalized size = 1.53

$$-\frac{\sqrt{-(ax-1)(ax+1)}\ \operatorname{arctanh}(ax)(2ax+\operatorname{arctanh}(ax))}{2x^2}+\frac{a^2\operatorname{arctanh}(ax)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+a^2\operatorname{arctanh}(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2+1/2*a^2
*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/
2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*poly
log(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
-2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^3,x)
```

```
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\ \operatorname{atanh}^2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**3, x)
```

3.446 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{1}{3}a^3\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{3}a^3\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{2}{3}a^3\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}}{3x^2}$$

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)^2/x^3+2/3*a^3*arctanh(a*x)*arctanh((-a
*x+1)^(1/2)/(a*x+1)^(1/2))-1/3*a^3*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))
+1/3*a^3*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/3*a^2*(-a^2*x^2+1)^(1/2)
/x-1/3*a*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.30, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6008, 6010, 6026, 264, 6018}

$$-\frac{1}{3}a^3\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{3}a^3\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{\left(1-a^2x^2\right)^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -(a^2*Sqrt[1 - a^2*x^2])/(3*x) - (a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*x^2)
 - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2)/(3*x^3) + (2*a^3*ArcTanh[a*x]*ArcTa
nh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3 - (a^3*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
 + a*x])])/3 + (a^3*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
```

```
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x^4}\,dx = -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a)\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -\frac{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^2} - \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3} - \frac{1}{3}(2a)\int \frac{\tanh^{-1}(a}{x^3\sqrt{1-a^2}}$$

$$= -\frac{2a^2\sqrt{1-a^2x^2}}{3x} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^2} - \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3} - \frac{1}{3}a^2$$

$$= -\frac{a^2\sqrt{1-a^2x^2}}{3x} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^2} - \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\,t$$

Mathematica [A] time = 2.06, size = 177, normalized size = 1.05

$$-\frac{1}{3}a^3\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right) - \frac{\left(1-a^2x^2\right)^{3/2}\left(\tanh^{-1}(ax)\left(\frac{\left(\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\log\left(e^{-\tanh^{-1}(ax)}+1\right)\right)\left(\sqrt{1-a^2x^2}\,\sinh\left(3\tanh^{-1}(ax)\right)-3\right.}{\sqrt{1-a^2x^2}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^3*PolyLog[2, -E^(-ArcTanh[a*x])]) - ((1 - a^2*x^2)^(3/2)*(4*ArcTanh
[a*x]^2 + 2*(-1 + Cosh[2*ArcTanh[a*x]]) - (4*a^3*x^3*PolyLog[2, E^(-ArcTanh
[a*x])])/(1 - a^2*x^2)^(3/2) + ArcTanh[a*x]*(2*Sinh[2*ArcTanh[a*x]] + ((Log
[1 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])])*(-3*a*x + Sqrt[1 - a^
2*x^2]*Sinh[3*ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2])))/(12*x^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\;\text{artanh}\,(ax)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.44, size = 171, normalized size = 1.01

$$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(a^2x^2\operatorname{arctanh}(ax)^2-a^2x^2-ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2\right)}{3x^3}+\frac{a^3\operatorname{arctanh}(ax)\ln\left(1+\frac{a}{\sqrt{-a}}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] 1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2-a^2*x^2-a*x*arctanh(a*
x)-arctanh(a*x)^2)/x^3+1/3*a^3*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2)
)+1/3*a^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*a^3*arctanh(a*x)*ln(1-
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*a^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^4,x)
```

```
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**4, x)
```

3.447 $\int x^4 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=292

$$-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}-\frac{3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{64a^5}-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{3}{16}x^5\sqrt{1-a^2x^2}$$

```
[Out] 1/192*(-a^2*x^2+1)^(3/2)/a^5-3/80*(-a^2*x^2+1)^(5/2)/a^5+1/56*(-a^2*x^2+1)^
(7/2)/a^5-3/64*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^5-3/128*
I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+3/128*I*polylog(2,I*(-a*x+
1)^(1/2)/(a*x+1)^(1/2))/a^5+3/128*(-a^2*x^2+1)^(1/2)/a^5-3/128*x*arctanh(a*
x)*(-a^2*x^2+1)^(1/2)/a^4-1/64*x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+3/16
*x^5*arctanh(a*x)*(-a^2*x^2+1)^(1/2)-1/8*a^2*x^7*arctanh(a*x)*(-a^2*x^2+1)^
(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6014, 6010, 6016, 266, 43, 261, 5950}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{192a^5}+\frac{3\sqrt{1-a^2x^2}}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(128*a^5) + (1 - a^2*x^2)^(3/2)/(192*a^5) - (3*(1 - a
^2*x^2)^(5/2))/(80*a^5) + (1 - a^2*x^2)^(7/2)/(56*a^5) - (3*x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(128*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(64*a^2
) + (3*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 - (a^2*x^7*Sqrt[1 - a^2*x^2]*
ArcTanh[a*x])/8 - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(64*
a^5) - (((3*I)/128)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (
((3*I)/128)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
```

```
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx &= -\left(a^2\int x^6\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\right)+\int x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\\
&= \frac{1}{6}x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{6}\int\frac{x^4\tanh^{-1}(a}{\sqrt{1-a^2x^2}}\\
&= -\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{24a^2}+\frac{3}{16}x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\,\mathrm{t}\\
&= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{64a^2}+\frac{3}{16}x^5\sqrt{1-a^2x^2}\,\tanh\\
&= -\frac{\sqrt{1-a^2x^2}}{48a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^5}-\frac{\left(1-a^2x^2\right)^{5/2}}{24a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}\,\mathrm{t}}{128a}\\
&= -\frac{\sqrt{1-a^2x^2}}{384a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}}{128}\\
&= \frac{3\sqrt{1-a^2x^2}}{128a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{192a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}}{128}
\end{aligned}$$

Mathematica [A] time = 1.49, size = 272, normalized size = 0.93

$$\frac{218a^2x^2\sqrt{1-a^2x^2}+121\sqrt{1-a^2x^2}-315ax\sqrt{1-a^2x^2}\tanh^{-1}(ax)-1680a^7x^7\sqrt{1-a^2x^2}\tanh^{-1}(ax)-240a^6x}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (121*Sqrt[1 - a^2*x^2] + 218*a^2*x^2*Sqrt[1 - a^2*x^2] + 216*a^4*x^4*Sqrt[1
 - a^2*x^2] - 240*a^6*x^6*Sqrt[1 - a^2*x^2] - 315*a*x*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x] - 210*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2520*a^5*x^5*Sqrt[
1 - a^2*x^2]*ArcTanh[a*x] - 1680*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (
315*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (315*I)*ArcTanh[a*x]*Log[1
+ I/E^ArcTanh[a*x]] - (315*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (315*I)*Pol
yLog[2, I/E^ArcTanh[a*x]])/(13440*a^5)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^2x^6-x^4\right)\sqrt{-a^2x^2+1}\,\mathrm{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^6 - x^4)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4\,\mathrm{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

maple [A] time = 0.46, size = 215, normalized size = 0.74

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(1680\,\mathrm{arctanh}\left(ax\right)x^7a^7+240x^6a^6-2520\,\mathrm{arctanh}\left(ax\right)x^5a^5-216x^4a^4+210a^3x^3\,\mathrm{arct}\right.}{13440a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/13440/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(1680*arctanh(a*x)*x^7*a^7+240*x^6*a^
6-2520*arctanh(a*x)*x^5*a^5-216*x^4*a^4+210*a^3*x^3*arctanh(a*x)-218*a^2*x^
2+315*a*x*arctanh(a*x)-121)-3/128*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arct
anh(a*x)/a^5+3/128*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5-3/
128*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+3/128*I*dilog(1-I*(a*x+1)/(
-a^2*x^2+1)^(1/2))/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4\,\mathrm{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{atanh}(a x) \left(1 - a^2 x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
[Out] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(-(ax-1)(ax+1)\right)^{\frac{3}{2}} \operatorname{atanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
[Out] Integral(x**4*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

3.448 $\int x^3 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=186

$$\frac{17\sin^{-1}(ax)}{560a^4}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{35a^2}-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{8}{35}x^4\sqrt{1-a^2x^2}\tanh$$

```
[Out] 17/560*arcsin(a*x)/a^4+3/112*x*(-a^2*x^2+1)^(1/2)/a^3+23/840*x^3*(-a^2*x^2+
1)^(1/2)/a-1/42*a*x^5*(-a^2*x^2+1)^(1/2)-2/35*arctanh(a*x)*(-a^2*x^2+1)^(1/
2)/a^4-1/35*x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+8/35*x^4*arctanh(a*x)*(
-a^2*x^2+1)^(1/2)-1/7*a^2*x^6*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6016, 321, 216, 5994}

$$-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}+\frac{3x\sqrt{1-a^2x^2}}{112a^3}-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{8}{35}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(112*a^3) + (23*x^3*Sqrt[1 - a^2*x^2])/(840*a) - (a
*x^5*Sqrt[1 - a^2*x^2])/42 + (17*ArcSin[a*x])/(560*a^4) - (2*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x])/(35*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(35*a^2)
+ (8*x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/35 - (a^2*x^6*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x])/7
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\int x^3\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx = -\left(a^2\int x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\right)+\int x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$$

$$= \frac{1}{5}x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{5}\int\frac{x^3\tanh^{-1}(a}{\sqrt{1-a^2x^2}}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{20a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^2}+\frac{8}{35}x^4\sqrt{1-a^2x}$$

$$= \frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4}$$

$$= \frac{3x\sqrt{1-a^2x^2}}{112a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{11\sin^{-1}(ax)}{120a^4}-\frac{2\sqrt{1-}}{}$$

$$= \frac{3x\sqrt{1-a^2x^2}}{112a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{17\sin^{-1}(ax)}{560a^4}-\frac{2\sqrt{1-}}{}$$

Mathematica [A] time = 0.09, size = 79, normalized size = 0.42

$$\frac{-48\left(5a^2x^2+2\right)\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)+ax\left(-40a^4x^4+46a^2x^2+45\right)\sqrt{1-a^2x^2}+51\sin^{-1}(ax)}{1680a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2]*(45 + 46*a^2*x^2 - 40*a^4*x^4) + 51*ArcSin[a*x] - 48
*(1 - a^2*x^2)^(5/2)*(2 + 5*a^2*x^2)*ArcTanh[a*x])/(1680*a^4)
```

fricas [A] time = 0.48, size = 106, normalized size = 0.57

$$-\frac{\left(40\,a^5x^5-46\,a^3x^3-45\,ax+24\left(5\,a^6x^6-8\,a^4x^4+a^2x^2+2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+102\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-}{ax}\right.}{1680\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/1680*((40*a^5*x^5 - 46*a^3*x^3 - 45*a*x + 24*(5*a^6*x^6 - 8*a^4*x^4 + a^
2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 102*arctan((sqrt
(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.38, size = 140, normalized size = 0.75

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(240\operatorname{arctanh}\left(ax\right)x^{6}a^{6}+40x^{5}a^{5}-384a^{4}x^{4}\operatorname{arctanh}\left(ax\right)-46x^{3}a^{3}+48a^{2}x^{2}\operatorname{arctanh}\left(a\right.\right.}{1680a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/1680/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(240*arctanh(a*x)*x^6*a^6+40*x^5*a^5-3
84*a^4*x^4*arctanh(a*x)-46*x^3*a^3+48*a^2*x^2*arctanh(a*x)-45*a*x+96*arctan
h(a*x))+17/560*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^4-17/560*I*ln((a*x+1)/(
-a^2*x^2+1)^(1/2)-I)/a^4
```

maxima [A] time = 0.41, size = 163, normalized size = 0.88

$$-\frac{1}{1680}a\left(\frac{5\left(\frac{8\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}x}{a^{2}}-\frac{2\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x}{a^{2}}-\frac{3\sqrt{-a^{2}x^{2}+1}\,x}{a^{2}}-\frac{3\arcsin(ax)}{a^{3}}\right)}{a^{2}}-\frac{12\left(2\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x+3\sqrt{-a^{2}x^{2}+1}\,x+\right.}{a^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/1680*a*(5*(8*(-a^2*x^2 + 1)^(5/2)*x/a^2 - 2*(-a^2*x^2 + 1)^(3/2)*x/a^2 -
 3*sqrt(-a^2*x^2 + 1)*x/a^2 - 3*arcsin(a*x)/a^3)/a^2 - 12*(2*(-a^2*x^2 + 1)
^(3/2)*x + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a*x)/a)/a^4) - 1/35*(5*(-a^2*x
^2 + 1)^(5/2)*x^2/a^2 + 2*(-a^2*x^2 + 1)^(5/2)/a^4)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

3.449 $\int x^2 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=243

$$-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{8a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^2}-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\tanh^{-1}$$

```
[Out] 1/72*(-a^2*x^2+1)^(3/2)/a^3-1/30*(-a^2*x^2+1)^(5/2)/a^3-1/8*arctan((-a*x+1)
^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-1/16*I*polylog(2,-I*(-a*x+1)^(1/2)/(
a*x+1)^(1/2))/a^3+1/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/16
*(-a^2*x^2+1)^(1/2)/a^3-1/16*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+7/24*x^3
*arctanh(a*x)*(-a^2*x^2+1)^(1/2)-1/6*a^2*x^5*arctanh(a*x)*(-a^2*x^2+1)^(1/2
)
```

Rubi [A] time = 0.57, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6014, 6010, 6016, 261, 5950, 266, 43}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^3}+\frac{\sqrt{1-a^2x^2}}{16a^3}-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\ \text{t}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(16*a^3) + (1 - a^2*x^2)^(3/2)/(72*a^3) - (1 - a^2*x^2)^(
5/2)/(30*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^2) + (7*x^3*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/24 - (a^2*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/6 -
 (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^3) - ((I/16)*PolyL
og[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
```

```
0]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx &= -\left(a^2\int x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\right)+\int x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\\
&= \frac{1}{4}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{4}\int\frac{x^2\tanh^{-1}(a}{\sqrt{1-a^2x^2}}\\
&= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^2}+\frac{7}{24}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\,\text{ta}\\
&= -\frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^2}+\frac{7}{24}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{6}a^2x\\
&= \frac{\sqrt{1-a^2x^2}}{48a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{36a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^2}+\frac{7}{24}x^3\\
&= \frac{\sqrt{1-a^2x^2}}{16a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^2}+\frac{7}{24}x^3
\end{aligned}$$

Mathematica [A] time = 0.97, size = 224, normalized size = 0.92

$$38a^2x^2\sqrt{1-a^2x^2}+31\sqrt{1-a^2x^2}-45ax\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-120a^5x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-24a^4x^4\sqrt{1-a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (31*Sqrt[1 - a^2*x^2] + 38*a^2*x^2*Sqrt[1 - a^2*x^2] - 24*a^4*x^4*Sqrt[1 -
a^2*x^2] - 45*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 210*a^3*x^3*Sqrt[1 - a^2
*x^2]*ArcTanh[a*x] - 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (45*I)*Ar
cTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (45*I)*ArcTanh[a*x]*Log[1 + I/E^ArcT
anh[a*x]] - (45*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (45*I)*PolyLog[2, I/E^
ArcTanh[a*x]])/(720*a^3)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2x^4 - x^2\right)\sqrt{-a^2x^2+1}\ \text{artanh}\left(ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^4 - x^2)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}} x^2\, \text{artanh}\left(ax\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

maple [A] time = 0.36, size = 195, normalized size = 0.80

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \left(120\,\text{arctanh}\left(ax\right)x^5a^5 + 24x^4a^4 - 210a^3x^3\,\text{arctanh}\left(ax\right) - 38a^2x^2 + 45ax\,\text{arctanh}\left(ax\right)\right.}{720a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/720/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*x^5*a^5+24*x^4*a^4-21
0*a^3*x^3*arctanh(a*x)-38*a^2*x^2+45*a*x*arctanh(a*x)-31)-1/16*I*ln(1+I*(a*
x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/16*I*ln(1-I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*arctanh(a*x)/a^3-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+
1/16*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}} x^2\, \text{artanh}\left(ax\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\text{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}\left(ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

3.450 $\int x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=81

$$\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{3x\sqrt{1-a^2x^2}}{40a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\sin^{-1}(ax)}{40a^2}$$

```
[Out] 1/20*x*(-a^2*x^2+1)^(3/2)/a+3/40*arcsin(a*x)/a^2-1/5*(-a^2*x^2+1)^(5/2)*arc
tanh(a*x)/a^2+3/40*x*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5994, 195, 216}

$$\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{3x\sqrt{1-a^2x^2}}{40a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\sin^{-1}(ax)}{40a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(40*a) + (x*(1 - a^2*x^2)^(3/2))/(20*a) + (3*ArcSin
[a*x])/(40*a^2) - ((1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*a^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx &= -\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{\int\left(1-a^2x^2\right)^{3/2}\,dx}{5a}\\ &= \frac{x\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\int\sqrt{1-a^2x^2}\,dx}{20a}\\ &= \frac{3x\sqrt{1-a^2x^2}}{40a}+\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{40a}\\ &= \frac{3x\sqrt{1-a^2x^2}}{40a}+\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{3\sin^{-1}(ax)}{40a^2}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.75

$$\frac{ax\left(5-2a^2x^2\right)\sqrt{1-a^2x^2}-8\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)+3\sin^{-1}(ax)}{40a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (a*x*(5 - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] + 3*ArcSin[a*x] - 8*(1 - a^2*x^2)^(5
/2)*ArcTanh[a*x])/(40*a^2)
```

fricas [A] time = 0.59, size = 90, normalized size = 1.11

$$-\frac{\left(2\,a^3x^3-5\,ax+4\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{40\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/40*((2*a^3*x^3 - 5*a*x + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1)))*sqrt(-a^2*x^2 + 1) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.31, size = 120, normalized size = 1.48

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(8a^4x^4\operatorname{arctanh}(ax)+2x^3a^3-16a^2x^2\operatorname{arctanh}(ax)-5ax+8\operatorname{arctanh}(ax)\right)}{40a^2}+\frac{3i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2}\cdots}\right.}{40a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/40/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)+2*x^3*a^3-16*a^2
*x^2*arctanh(a*x)-5*a*x+8*arctanh(a*x))+3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2
)+I)/a^2-3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^2
```

maxima [A] time = 0.41, size = 67, normalized size = 0.83

$$-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}\operatorname{artanh}(ax)}{5\,a^2}+\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}x+3\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin(ax)}{a}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/a^2 + 1/40*(2*(-a^2*x^2 + 1)^(3/2)*x
 + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a*x)/a)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
[Out] int(x*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}{\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

3.451 $\int \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=189

$$\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{3i\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\mathrm{Li}_2\left(\frac{i\sqrt{1}}{\sqrt{a}}\right.}{8a}$$

```
[Out] 1/12*(-a^2*x^2+1)^(3/2)/a+1/4*x*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-3/4*arctan(
(-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-3/8*I*polylog(2,-I*(-a*x+1)^(1
/2)/(a*x+1)^(1/2))/a+3/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+3/8*
(-a^2*x^2+1)^(1/2)/a+3/8*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{3i\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx &= \frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{4}\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx\\ &= \frac{3\sqrt{1-a^2x^2}}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh\\ &= \frac{3\sqrt{1-a^2x^2}}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh\end{aligned}$$

Mathematica [A] time = 0.61, size = 176, normalized size = 0.93

$$\frac{-2a^2x^2\sqrt{1-a^2x^2}+11\sqrt{1-a^2x^2}+15ax\sqrt{1-a^2x^2}\tanh^{-1}(ax)-6a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-9i\text{Li}_2\left(-ie^{-\tan\ldots}\right.}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTanh
[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*
x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcTanh[
a*x]])/(24*a)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.96, size = 173, normalized size = 0.92

$$-\frac{\left(6a^3x^3\,\text{arctanh}\,(ax)+2a^2x^2-15ax\,\text{arctanh}\,(ax)-11\right)\sqrt{-a^2x^2+1}}{24a}-\frac{3i\,\text{arctanh}\,(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{8a}+\frac{3i\,a\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+1
)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a*a
rctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

3.452 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=144

$$-\frac{1}{6}ax\sqrt{1-a^2x^2}+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{7}{6}\sin$$

```
[Out] -7/6*arcsin(a*x)+1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-2*arctanh(a*x)*arctanh
((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-pol
ylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/6*a*x*(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1
)^(1/2)*arctanh(a*x)
```

Rubi [A] time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6018, 216, 5994, 195}

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{6}ax\sqrt{1-a^2x^2}+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

```
[Out] -(a*x*Sqrt[1 - a^2*x^2])/6 - (7*ArcSin[a*x])/6 + Sqrt[1 - a^2*x^2]*ArcTanh[
a*x] + ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/3 - 2*ArcTanh[a*x]*ArcTanh[Sqrt[1
 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyL
og[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x}\,dx = -\left(a^2\int x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}\,dx$$
$$= \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)-\frac{1}{3}a\int \sqrt{1-a^2x^2}\,dx-a\int$$
$$= -\frac{1}{6}ax\sqrt{1-a^2x^2}-\sin^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}($$
$$= -\frac{1}{6}ax\sqrt{1-a^2x^2}-\frac{7}{6}\sin^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-}$$

Mathematica [A] time = 0.23, size = 143, normalized size = 0.99

$$\frac{1}{6}\left(-ax\sqrt{1-a^2x^2}-2a^2x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+8\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+6\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-6\text{Li}_2\left(e^{-\tanh^{-1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

```
[Out] (-(a*x*Sqrt[1 - a^2*x^2]) - 14*ArcTan[Tanh[ArcTanh[a*x]/2]] + 8*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x] - 2*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*ArcTanh[
a*x]*Log[1 - E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
 + 6*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*PolyLog[2, E^(-ArcTanh[a*x])])/6
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.41, size = 132, normalized size = 0.92

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(2a^2x^2\operatorname{arctanh}(ax)+ax-8\operatorname{arctanh}(ax)\right)}{6}-\frac{7\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3}-\operatorname{dilog}\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x)
```

```
[Out] -1/6*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x-8*arctanh(a*x))-7
/3*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-dil
og(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\operatorname{atanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x, x)
```

3.453 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=179

$$-\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+\frac{3}{2}ia\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right.$$

```
[Out] 3*a*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)-a*arctanh((-a^2*x^2+1
)^(1/2))+3/2*I*a*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*I*a*polylog
(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/2*a*(-a^2*x^2+1)^(1/2)-arctanh(a*x)*(-
a^2*x^2+1)^(1/2)/x-1/2*a^2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6014, 6008, 266, 63, 208, 5950, 5942}

$$\frac{3}{2}ia\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}ia\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-a\tanh^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (a^2*x*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/2 + 3*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*Arc
Tanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + ((3*I)/2)*a*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]] - ((3*I)/2)*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[
1 + a*x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^2}\,dx = -\left(a^2\int \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}\,dx$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-a^2\int$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+3a\,\mathrm{t}$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+3a\,\mathrm{t}$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+3a\,\mathrm{t}$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+3a\,\mathrm{t}$$

Mathematica [A] time = 0.63, size = 168, normalized size = 0.94

$$\frac{1}{2}\left(-a\sqrt{1-a^2x^2}+a^2(-x)\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+3ia\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-3ia\mathrm{Li}_2\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] (-(a*Sqrt[1 - a^2*x^2]) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - a^2*x*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x] + (3*I)*a*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]]
```

```
 - (3*I)*a*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 2*a*Log[Tanh[ArcTanh[a*
x]/2]] + (3*I)*a*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (3*I)*a*PolyLog[2, I/E^A
rcTanh[a*x]])/2
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(a^2x^2-1)\sqrt{-a^2x^2+1}\,\text{artanh}(ax)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.44, size = 205, normalized size = 1.15

$$-\frac{(a^2x^2\,\text{arctanh}(ax)+ax+2\,\text{arctanh}(ax))\sqrt{-(ax-1)(ax+1)}}{2x}+a\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)-a\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x)
```

```
[Out] -1/2*(a^2*x^2*arctanh(a*x)+a*x+2*arctanh(a*x))*(-(a*x-1)*(a*x+1))^(1/2)/x+a
*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*I*
a*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)-3/2*I*a*dilog(1-I*(a*x+1)
/(-a^2*x^2+1)^(1/2))+3/2*I*a*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x
)+3/2*I*a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\,\text{artanh}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{atanh}(a\,x)\,(1-a^2\,x^2)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^2,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**2,x)
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**2, x)
```

3.454 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=168

$$-\frac{3}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{3}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\sin$$

```
[Out] a^2*arcsin(a*x)+3*a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/
2*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/2*a^2*polylog(2,(-a*x+1)^(
1/2)/(a*x+1)^(1/2))-1/2*a*(-a^2*x^2+1)^(1/2)/x-a^2*arctanh(a*x)*(-a^2*x^2+1
)^(1/2)-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.39, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6026, 264, 6018, 216}

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\text{ ta}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) + a^2*ArcSin[a*x] - a^2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + 3*a^2*ArcTanh[a*x]*Ar
cTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt
[1 + a*x])])/2 + (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^3}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx -$$

$$= -\frac{a\sqrt{1-a^2x^2}}{x}+a^2\sin^{-1}(ax)-a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}}{2x^2}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{2x}+a^2\sin^{-1}(ax)-a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}}{2x^2}$$

Mathematica [A] time = 1.00, size = 158, normalized size = 0.94

$$\frac{1}{8}a^2\left(-8\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-12\mathrm{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)+12\mathrm{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)-12\tan$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] (a^2*(16*ArcTan[Tanh[ArcTanh[a*x]/2]] - 8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] -
2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 12*ArcTanh[a
*x]*Log[1 - E^(-ArcTanh[a*x])] + 12*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
 - 12*PolyLog[2, -E^(-ArcTanh[a*x])] + 12*PolyLog[2, E^(-ArcTanh[a*x])] - A
rcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.49, size = 145, normalized size = 0.86

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(2a^{2}x^{2}\operatorname{arctanh}\left(ax\right)+ax+\operatorname{arctanh}\left(ax\right)\right)}{2x^{2}}+2a^{2}\arctan\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)+\frac{3a^{2}\operatorname{dilog}\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+}}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x+arctanh(a*x))/x^2
+2*a^2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*dilog((a*x+1)/(-a^2*x^2+1
)^(1/2))+3/2*a^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^{2}\,x^{2}\right)}^{3/2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^3,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**3,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**3, x)
```

3.455 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=189

$$-ia^3\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)+ia^3\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a^3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}}{x}$$

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3-2*a^3*arctan((-a*x+1)^(1/2)/(a*x+1
)^(1/2))*arctanh(a*x)+7/6*a^3*arctanh((-a^2*x^2+1)^(1/2))-I*a^3*polylog(2,-
I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))+I*a^3*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))-1/6*a*(-a^2*x^2+1)^(1/2)/x^2+a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6014, 6008, 266, 47, 63, 208, 5950}

$$-ia^3\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)+ia^3\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{7}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{a^2\sqrt{1-a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(6*x^2) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (
(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3) - 2*a^3*ArcTan[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]*ArcTanh[a*x] + (7*a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6 - I*a^3*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + I*a^3*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^4}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^4}\,dx$$

$$= -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx-a^2\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx+a^4$$

$$= \frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tan$$

$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan$$

$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan$$

$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan$$

Mathematica [A] time = 1.27, size = 199, normalized size = 1.05

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)\left(\sinh\left(3\tanh^{-1}(ax)\right)-\frac{3ax}{\sqrt{1-a^2x^2}}\right)+8\tanh^{-1}(ax)+2\sinh\left(2\tanh^{-1}(ax\right.\right.}{24x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -(a^3*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 - I
/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + Log[Tanh[ArcT
anh[a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh
[a*x]])) - ((1 - a^2*x^2)^(3/2)*(8*ArcTanh[a*x] + 2*Sinh[2*ArcTanh[a*x]] +
Log[Tanh[ArcTanh[a*x]/2]]*((-3*a*x)/Sqrt[1 - a^2*x^2] + Sinh[3*ArcTanh[a*x]
])))/(24*x^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\ \operatorname{artanh}\left(ax\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.47, size = 220, normalized size = 1.16

$$\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \left(8a^2x^2\operatorname{arctanh}\left(ax\right)-ax-2\operatorname{arctanh}\left(ax\right)\right)}{6x^3}-\frac{7a^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)}{6}+\frac{7a^3\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x)
```

```
[Out] 1/6*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))/x^
3-7/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+7/6*a^3*ln(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))+I*a^3*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)+I*a^3*dilog(
1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-I*a^3*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*ar
ctanh(a*x)-I*a^3*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^4,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**4, x)
```

3.456 $\int \frac{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)}{x^5} \, dx$

Optimal. Leaf size=191

$$\frac{3}{8}a^4\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{8}a^4\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{4}a^4\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-\frac{\sqrt{1}}{}$$

```
[Out] -3/4*a^4*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/8*a^4*polylog
(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/8*a^4*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^
(1/2))-1/12*a*(-a^2*x^2+1)^(1/2)/x^3+11/24*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*arc
tanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4+5/8*a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^
2
```

Rubi [A] time = 0.57, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6026, 271, 264, 6018}

$$\frac{3}{8}a^4\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{8}a^4\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{11a^3\sqrt{1-a^2x^2}}{24x}-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(12*x^3) + (11*a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (5*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]
)/(8*x^2) - (3*a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 + (
3*a^4*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 - (3*a^4*PolyLog[2, Sqr
t[1 - a*x]/Sqrt[1 + a*x]])/8
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
```

```
, 1] && IntegerQ[q]))
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^5}\,dx &= -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^3}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^5}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^4}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}-\frac{1}{3}\int \frac{\tanh^{-1}(ax)}{x^5\sqrt{1-a^2x^2}}\,dx+\ldots\\ &= -\frac{a\sqrt{1-a^2x^2}}{9x^3}+\frac{a^3\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(\ldots}{2x^2}\\ &= -\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{5a^3\sqrt{1-a^2x^2}}{18x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{5a^2\sqrt{1-a^2x^2}\,\tanh^{\ldots}}{8x^2}\\ &= -\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{11a^3\sqrt{1-a^2x^2}}{24x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{5a^2\sqrt{1-a^2x^2}\,\tanh\ldots}{8x^2}\end{aligned}$$

Mathematica [A] time = 3.83, size = 282, normalized size = 1.48

$$\frac{1}{192}a\left(72a^3\text{Li}_2\left(-e^{-\tanh^{-1}(ax)}\right)-72a^3\text{Li}_2\left(e^{-\tanh^{-1}(ax)}\right)-40a^3\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)+72a^3\tanh^{-1}(ax)\log\left(1-e^{-\ldots}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] (a*(40*a^3*Coth[ArcTanh[a*x]/2] + 18*a^3*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^
2 - (a^4*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*a^3*ArcTanh[a*x]*C
sch[ArcTanh[a*x]/2]^4 + 72*a^3*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 72
*a^3*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + 72*a^3*PolyLog[2, -E^(-ArcTa
nh[a*x])] - 72*a^3*PolyLog[2, E^(-ArcTanh[a*x])] + 18*a^3*ArcTanh[a*x]*Sech
[ArcTanh[a*x]/2]^2 + 3*a^3*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - (16*Sqrt[1
 - a^2*x^2]*Sinh[ArcTanh[a*x]/2]^4)/x^3 + (16*a^2*Sqrt[1 - a^2*x^2]*Sinh[Ar
cTanh[a*x]/2]^4)/x - 40*a^3*Tanh[ArcTanh[a*x]/2]))/192
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\ \text{artanh}\left(ax\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.44, size = 164, normalized size = 0.86

$$\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \left(11x^3a^3+15a^2x^2\ \text{arctanh}\left(ax\right)-2ax-6\ \text{arctanh}\left(ax\right)\right)}{24x^4}-\frac{3a^4\ \text{arctanh}\left(ax\right)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2}}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x)
[Out] 1/24*(-(a*x-1)*(a*x+1))^(1/2)*(11*x^3*a^3+15*a^2*x^2*arctanh(a*x)-2*a*x-6*a
rctanh(a*x))/x^4-3/8*a^4*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*
a^4*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*a^4*arctanh(a*x)*ln(1-(a*x+1
)/(-a^2*x^2+1)^(1/2))+3/8*a^4*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\ \text{artanh}\left(ax\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="maxima")
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{atanh}\left(a\,x\right)\,\left(1-a^2\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^5,x)
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\ \text{atanh}\left(ax\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**5, x)
```

3.457 $\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=94

$$-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}$$

```
[Out] -1/20*a*(-a^2*x^2+1)^(3/2)/x^4-1/5*(-a^2*x^2+1)^(5/2)*arctanh(a*x)/x^5-3/40
*a^5*arctanh((-a^2*x^2+1)^(1/2))+3/40*a^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6008, 266, 47, 63, 208}

$$\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] (3*a^3*Sqrt[1 - a^2*x^2])/(40*x^2) - (a*(1 - a^2*x^2)^(3/2))/(20*x^4) - ((1
 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 - a^2*x^2]]
)/40
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^6}\,dx = -\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{5}a\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^5}\,dx$$

$$= -\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{10}a\,\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,\right.$$

$$= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{80}\left(3a^5\right)\text{Subst}\left(\right.$$

$$= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\right.$$

$$= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1}\right.$$

Mathematica [A] time = 0.09, size = 104, normalized size = 1.11

$$\frac{3}{40}a^5\log(x)-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)^2\tanh^{-1}(ax)}{5x^5}-\frac{3}{40}a^5\log\left(\sqrt{1-a^2x^2}+1\right)+\left(\frac{a^3}{8x^2}-\frac{a}{20x^4}\right)\sqrt{1-a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] (-1/20*a/x^4 + a^3/(8*x^2))*Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*(-1 + a^
2*x^2)^2*ArcTanh[a*x])/(5*x^5) + (3*a^5*Log[x])/40 - (3*a^5*Log[1 + Sqrt[1
- a^2*x^2]])/40
```

fricas [A] time = 0.49, size = 93, normalized size = 0.99

$$\frac{3\,a^5x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(5\,a^3x^3-2\,ax-4\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(3*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 - 2*a*x - 4*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))/x^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.41, size = 116, normalized size = 1.23

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(8a^4x^4\operatorname{arctanh}(ax)-5x^3a^3-16a^2x^2\operatorname{arctanh}(ax)+2ax+8\operatorname{arctanh}(ax)\right)}{40x^5}-\frac{3a^5\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x)
```

```
[Out] -1/40*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)-5*x^3*a^3-16*a^2*x^2
*arctanh(a*x)+2*a*x+8*arctanh(a*x))/x^5-3/40*a^5*ln(1+(a*x+1)/(-a^2*x^2+1)^
(1/2))+3/40*a^5*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

maxima [A] time = 0.42, size = 126, normalized size = 1.34

$$\frac{1}{40}\left(\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4-3\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+3\sqrt{-a^2x^2+1}\,a^4+\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2}{x^2}-\frac{2\left(-a^2x^2+1\right)^{\frac{5}{2}}}{x^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/40*((-a^2*x^2 + 1)^(3/2)*a^4 - 3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/
abs(x)) + 3*sqrt(-a^2*x^2 + 1)*a^4 + (-a^2*x^2 + 1)^(5/2)*a^2/x^2 - 2*(-a^2
*x^2 + 1)^(5/2)/x^4)*a - 1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^6,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**6,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**6, x)
```

3.458 $\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^7}\,dx$

Optimal. Leaf size=243

$$\frac{1}{16}a^6\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{16}a^6\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{8}a^6\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6}-\frac{a\sqrt{1-a}}{30x^5}$$

```
[Out] -1/8*a^6*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/16*a^6*polylo
g(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/16*a^6*polylog(2,(-a*x+1)^(1/2)/(a*x+1
)^(1/2))-1/30*a*(-a^2*x^2+1)^(1/2)/x^5+19/360*a^3*(-a^2*x^2+1)^(1/2)/x^3+31
/720*a^5*(-a^2*x^2+1)^(1/2)/x-1/6*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6+7/24*
a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4-1/16*a^4*arctanh(a*x)*(-a^2*x^2+1)^
(1/2)/x^2
```

Rubi [A] time = 0.77, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6026, 271, 264, 6018}

$$\frac{1}{16}a^6\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{16}a^6\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{31a^5\sqrt{1-a^2x^2}}{720x}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{a\sqrt{1-a^2x^2}}{30x^5}-\frac{a^4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(30*x^5) + (19*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (3
1*a^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6)
 + (7*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) - (a^4*Sqrt[1 - a^2*x^2]
*ArcTanh[a*x])/(16*x^2) - (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 +
a*x]])/8 + (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 - (a^6*PolyL
og[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
```

```
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^7}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^5}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^7}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{5x^6}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^4}-\frac{1}{5}\int \frac{\tanh^{-1}(ax)}{x^7\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1-a^2x^2}}{25x^5}+\frac{a^3\sqrt{1-a^2x^2}}{9x^3}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh}{4x^4}$$
$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{3a^3\sqrt{1-a^2x^2}}{100x^3}+\frac{2a^5\sqrt{1-a^2x^2}}{9x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}+$$
$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{13a^5\sqrt{1-a^2x^2}}{200x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}$$
$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}+\frac{31a^5\sqrt{1-a^2x^2}}{720x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}$$

Mathematica [A] time = 7.44, size = 474, normalized size = 1.95

$$\frac{64a^7x^4\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)-3a^7x^4\text{csch}^6\left(\frac{1}{2}\tanh^{-1}(ax)\right)+4a^7x^4\text{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)+82a^7x^4\text{csch}^2\left(\frac{1}{2}\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

```
[Out] (82*a^7*x^4*Csch[ArcTanh[a*x]/2]^2 + 90*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a
*x]*Csch[ArcTanh[a*x]/2]^2 + 4*a^7*x^4*Csch[ArcTanh[a*x]/2]^4 - 3*a^7*x^4*C
sch[ArcTanh[a*x]/2]^6 - 15*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Csch[ArcT
anh[a*x]/2]^6 + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-ArcT
anh[a*x])] - 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTanh
```

```
[a*x])] + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 36
0*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, E^(-ArcTanh[a*x])] + 328*a^5*x^2*(-1
 + a^2*x^2)*Sinh[ArcTanh[a*x]/2]^2 + 360*a^4*x*(1 - a^2*x^2)^(3/2)*ArcTanh[
a*x]*Sinh[ArcTanh[a*x]/2]^2 + 64*a^3*Sinh[ArcTanh[a*x]/2]^4 - 128*a^5*x^2*S
inh[ArcTanh[a*x]/2]^4 + 64*a^7*x^4*Sinh[ArcTanh[a*x]/2]^4 - (192*a*(-1 + a^
2*x^2)^3*Sinh[ArcTanh[a*x]/2]^6)/x^2 - (960*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x
]*Sinh[ArcTanh[a*x]/2]^6)/x^3)/(5760*x^3*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.44, size = 184, normalized size = 0.76

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(-31x^5a^5+45a^4x^4\,\text{artanh}\,(ax)-38x^3a^3-210a^2x^2\,\text{artanh}\,(ax)+24ax+120\,\text{artanh}\,(\right.}{720x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x)
```

```
[Out] -1/720*(-(a*x-1)*(a*x+1))^(1/2)*(-31*x^5*a^5+45*a^4*x^4*arctanh(a*x)-38*x^3
*a^3-210*a^2*x^2*arctanh(a*x)+24*a*x+120*arctanh(a*x))/x^6-1/16*a^6*arctanh
(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))+1/16*a^6*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6
*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\,\text{artanh}\,(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{atanh}\,(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^7,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**7,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**7, x)
```

3.459 $\int \left(1-a^2x^2\right)^{5/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=233

$$\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)+\frac{5}{24}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{5}{16}x\sqrt{1-}$$

```
[Out] 5/72*(-a^2*x^2+1)^(3/2)/a+1/30*(-a^2*x^2+1)^(5/2)/a+5/24*x*(-a^2*x^2+1)^(3/
2)*arctanh(a*x)+1/6*x*(-a^2*x^2+1)^(5/2)*arctanh(a*x)-5/8*arctan((-a*x+1)^(
1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-5/16*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+
1)^(1/2))/a+5/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+5/16*(-a^2*x
^2+1)^(1/2)/a+5/16*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5942, 5950}

$$-\frac{5i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}+\frac{5i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

```
[Out] (5*Sqrt[1 - a^2*x^2])/(16*a) + (5*(1 - a^2*x^2)^(3/2))/(72*a) + (1 - a^2*x^
2)^(5/2)/(30*a) + (5*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 + (5*x*(1 - a^2*x
^2)^(3/2)*ArcTanh[a*x])/24 + (x*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/6 - (5*Ar
cTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a) - (((5*I)/16)*PolyLog
[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((5*I)/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^{5/2}}{30a} + \frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax) + \frac{5}{6}\int \left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,d$$

$$= \frac{5\left(1-a^2x^2\right)^{3/2}}{72a} + \frac{\left(1-a^2x^2\right)^{5/2}}{30a} + \frac{5}{24}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax) + \frac{1}{6}x\left(1-a^2x\right.$$

$$= \frac{5\sqrt{1-a^2x^2}}{16a} + \frac{5\left(1-a^2x^2\right)^{3/2}}{72a} + \frac{\left(1-a^2x^2\right)^{5/2}}{30a} + \frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}(ax) +$$

$$= \frac{5\sqrt{1-a^2x^2}}{16a} + \frac{5\left(1-a^2x^2\right)^{3/2}}{72a} + \frac{\left(1-a^2x^2\right)^{5/2}}{30a} + \frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}(ax) +$$

Mathematica [A] time = 1.18, size = 224, normalized size = 0.96

$$\frac{-98a^2x^2\sqrt{1-a^2x^2} + 299\sqrt{1-a^2x^2} + 495ax\sqrt{1-a^2x^2}\tanh^{-1}(ax) + 120a^5x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax) + 24a^4x^4}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

```
[Out] (299*Sqrt[1 - a^2*x^2] - 98*a^2*x^2*Sqrt[1 - a^2*x^2] + 24*a^4*x^4*Sqrt[1 -
 a^2*x^2] + 495*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 390*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcTanh[a*x] + 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (225*I)
*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (225*I)*ArcTanh[a*x]*Log[1 + I/E^
ArcTanh[a*x]] - (225*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (225*I)*PolyLog[2
, I/E^ArcTanh[a*x]])/(720*a)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.14, size = 193, normalized size = 0.83

$$\frac{\left(120\,\text{arctanh}\left(ax\right)x^5a^5 + 24x^4a^4 - 390a^3x^3\,\text{arctanh}\left(ax\right) - 98a^2x^2 + 495ax\,\text{arctanh}\left(ax\right) + 299\right)\sqrt{-a^2x^2+1}}{720a} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(5/2)*arctanh(a*x),x)
```

```
[Out] 1/720*(120*arctanh(a*x)*x^5*a^5+24*x^4*a^4-390*a^3*x^3*arctanh(a*x)-98*a^2*
x^2+495*a*x*arctanh(a*x)+299)*(-a^2*x^2+1)^(1/2)/a-5/16*I/a*arctanh(a*x)*ln
(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2
*x^2+1)^(1/2))-5/16*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*dilo
g(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2x^2+1\right)^{\frac{5}{2}} \operatorname{artanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(5/2)*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-(ax-1)(ax+1))^{\frac{5}{2}} \operatorname{atanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(5/2)*atanh(a*x),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x), x)
```

3.460 $\int \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=189

$$\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{Li}_2\left(\right.}{8}$$

```
[Out] 1/12*(-a^2*x^2+1)^(3/2)/a+1/4*x*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-3/4*arctan(
(-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-3/8*I*polylog(2,-I*(-a*x+1)^(1
/2)/(a*x+1)^(1/2))/a+3/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+3/8*
(-a^2*x^2+1)^(1/2)/a+3/8*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx=\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{4}\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$$

$$=\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\text{ta}$$

$$=\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\text{ta}$$

Mathematica [A] time = 0.06, size = 176, normalized size = 0.93

$$\frac{-2a^2x^2\sqrt{1-a^2x^2}+11\sqrt{1-a^2x^2}+15ax\sqrt{1-a^2x^2}\tanh^{-1}(ax)-6a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-9i\text{Li}_2\left(-ie^{-\tanh^{-1}}\right.}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTanh
[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*
x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcTanh[
a*x]])/(24*a)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\ \text{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.91, size = 173, normalized size = 0.92

$$-\frac{\left(6a^3x^3\,\text{arctanh}\left(ax\right)+2a^2x^2-15ax\,\text{arctanh}\left(ax\right)-11\right)\sqrt{-a^2x^2+1}}{24a}-\frac{3i\,\text{arctanh}\left(ax\right)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{8a}+\frac{3i\,\text{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+1
)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a*a
rctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

3.461 $\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

```
[Out] -arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-1/2*I*polylog(2,-I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)/a+1/2*(-a^2*x^2+1)^(1/2)/a+1/2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx &= \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{2}\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-}}{\sqrt{1+}}\right.}{2a}\end{aligned}$$

Mathematica [A] time = 0.07, size = 117, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+ax\tanh^{-1}(ax)+1\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^Ar
cTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]]
- PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,\operatorname{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.75, size = 152, normalized size = 1.06

$$\frac{\left(ax\operatorname{arctanh}\left(ax\right)+1\right)\sqrt{-a^2x^2+1}}{2a}-\frac{i\operatorname{arctanh}\left(ax\right)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}+\frac{i\operatorname{arctanh}\left(ax\right)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{i\operatorname{dilog}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x)
```

```
[Out] 1/2*(a*x*arctanh(a*x)+1)*(-a^2*x^2+1)^(1/2)/a-1/2*I/a*arctanh(a*x)*ln(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)
^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\,\operatorname{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

3.462 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2}{3a\sqrt{1-a^2x^2}}-\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -1/9/a/(-a^2*x^2+1)^(3/2)+1/3*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-2/3/a/(-a^2
*x^2+1)^(1/2)+2/3*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5960, 5958}

$$-\frac{2}{3a\sqrt{1-a^2x^2}}-\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -1/(9*a*(1 - a^2*x^2)^(3/2)) - 2/(3*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x])
/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*Sqrt[1 - a^2*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx &= -\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2}{3}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx\\ &= -\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}-\frac{2}{3a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.55

$$-\frac{\left(6a^3x^3-9ax\right)\tanh^{-1}(ax)-6a^2x^2+7}{9a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -1/9*(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*x])/(a*(1 - a^2*x^2)^(
3/2))
```

fricas [A] time = 0.49, size = 73, normalized size = 0.82

$$\frac{\left(12\,a^{2}x^{2}-3\left(2\,a^{3}x^{3}-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-14\right)\sqrt{-a^{2}x^{2}+1}}{18\left(a^{5}x^{4}-2\,a^{3}x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 14)*sq
rt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 + a)
```

giac [A] time = 0.71, size = 90, normalized size = 1.01

$$-\frac{\left(2\,a^{2}x^{2}-3\right)\sqrt{-a^{2}x^{2}+1}\,x\log\left(-\frac{ax+1}{ax-1}\right)}{6\left(a^{2}x^{2}-1\right)^{2}}-\frac{6\,a^{2}x^{2}-7}{9\left(a^{2}x^{2}-1\right)\sqrt{-a^{2}x^{2}+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*x^2 - 3)*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^
2 - 1)^2 - 1/9*(6*a^2*x^2 - 7)/((a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a)
```

maple [A] time = 0.40, size = 59, normalized size = 0.66

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,\left(6a^{3}x^{3}\operatorname{arctanh}\left(ax\right)-6a^{2}x^{2}-9ax\operatorname{arctanh}\left(ax\right)+7\right)}{9a\left(a^{2}x^{2}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] -1/9/a*(-a^2*x^2+1)^(1/2)*(6*a^3*x^3*arctanh(a*x)-6*a^2*x^2-9*a*x*arctanh(a
*x)+7)/(a^2*x^2-1)^2
```

maxima [A] time = 0.31, size = 74, normalized size = 0.83

$$-\frac{1}{9}a\left(\frac{6}{\sqrt{-a^{2}x^{2}+1}\,a^{2}}+\frac{1}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}a^{2}}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^{2}x^{2}+1}}+\frac{x}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(6/(sqrt(-a^2*x^2 + 1)*a^2) + 1/((-a^2*x^2 + 1)^(3/2)*a^2)) + 1/3*(2
*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(a\,x\right)}{\left(1-a^{2}\,x^{2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.463 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{8}{15a\sqrt{1-a^2x^2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -1/25/a/(-a^2*x^2+1)^(5/2)-4/45/a/(-a^2*x^2+1)^(3/2)+1/5*x*arctanh(a*x)/(-a
^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-8/15/a/(-a^2*x^2+1)^
(1/2)+8/15*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5960, 5958}

$$-\frac{8}{15a\sqrt{1-a^2x^2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] -1/(25*a*(1 - a^2*x^2)^(5/2)) - 4/(45*a*(1 - a^2*x^2)^(3/2)) - 8/(15*a*Sqrt
[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[a*
x])/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*Sqrt[1 - a^2*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx &= -\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}+\frac{4}{5}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx\\
&= -\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/2}}+\frac{8}{15}\int \frac{\tanh^{-1}}{(1-a^2}\\
&= -\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{8}{15a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.49

$$\frac{-120a^4x^4+260a^2x^2+15ax\left(8a^4x^4-20a^2x^2+15\right)\tanh^{-1}(ax)-149}{225a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (-149 + 260*a^2*x^2 - 120*a^4*x^4 + 15*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*Ar
cTanh[a*x])/(225*a*(1 - a^2*x^2)^(5/2))
```

fricas [A] time = 0.44, size = 99, normalized size = 0.74

$$\frac{\left(240\,a^4x^4 - 520\,a^2x^2 - 15\left(8\,a^5x^5 - 20\,a^3x^3 + 15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) + 298\right)\sqrt{-a^2x^2+1}}{450\left(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*log
(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x^4 + 3*a
^3*x^2 - a)
```

giac [A] time = 0.25, size = 114, normalized size = 0.86

$$-\frac{\sqrt{-a^2x^2+1}\left(4\left(2\,a^4x^2 - 5\,a^2\right)x^2 + 15\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\left(a^2x^2-1\right)^3} + \frac{20\,a^2x^2 - 120\left(a^2x^2-1\right)^2 - 29}{225\left(a^2x^2-1\right)^2\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a^2*x^2 + 1)*(4*(2*a^4*x^2 - 5*a^2)*x^2 + 15)*x*log(-(a*x + 1)/
(a*x - 1))/(a^2*x^2 - 1)^3 + 1/225*(20*a^2*x^2 - 120*(a^2*x^2 - 1)^2 - 29)/
((a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*a)
```

maple [A] time = 0.42, size = 79, normalized size = 0.59

$$-\frac{\sqrt{-a^2x^2+1}\,\left(120\operatorname{arctanh}(ax)\,x^5a^5 - 120x^4a^4 - 300a^3x^3\operatorname{arctanh}(ax) + 260a^2x^2 + 225ax\operatorname{arctanh}(ax) - 14\right.}{225a\left(a^2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x)
```

```
[Out] -1/225/a*(-a^2*x^2+1)^(1/2)*(120*arctanh(a*x)*x^5*a^5-120*x^4*a^4-300*a^3*x
^3*arctanh(a*x)+260*a^2*x^2+225*a*x*arctanh(a*x)-149)/(a^2*x^2-1)^3
```

maxima [A] time = 0.31, size = 108, normalized size = 0.81

$$-\frac{1}{225}a\left(\frac{120}{\sqrt{-a^2x^2+1}\,a^2} + \frac{20}{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2} + \frac{9}{\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2}\right) + \frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2x^2+1}} + \frac{4\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}} + \frac{3\,x}{\left(-a^2x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/225*a*(120/(sqrt(-a^2*x^2 + 1)*a^2) + 20/((-a^2*x^2 + 1)^(3/2)*a^2) + 9/
((-a^2*x^2 + 1)^(5/2)*a^2)) + 1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2
+ 1)^(3/2) + 3*x/(-a^2*x^2 + 1)^(5/2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(1-a^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(7/2),x)
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(7/2),x)
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

3.464 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{16}{35a\sqrt{1-a^2x^2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -1/49/a/(-a^2*x^2+1)^(7/2)-6/175/a/(-a^2*x^2+1)^(5/2)-8/105/a/(-a^2*x^2+1)^
(3/2)+1/7*x*arctanh(a*x)/(-a^2*x^2+1)^(7/2)+6/35*x*arctanh(a*x)/(-a^2*x^2+1
)^(5/2)+8/35*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-16/35/a/(-a^2*x^2+1)^(1/2)+1
6/35*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5960, 5958}

$$-\frac{16}{35a\sqrt{1-a^2x^2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] -1/(49*a*(1 - a^2*x^2)^(7/2)) - 6/(175*a*(1 - a^2*x^2)^(5/2)) - 8/(105*a*(1
 - a^2*x^2)^(3/2)) - 16/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(7*(1 -
 a^2*x^2)^(7/2)) + (6*x*ArcTanh[a*x])/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTa
nh[a*x])/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*Sqrt[1 - a^2*x^
2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{9/2}}\,dx &= -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6}{7}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx\\
&= -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{5/2}}+\frac{24}{35}\int \frac{\text{ta}}{(1}\\
&= -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6x\tan}{35(1-}\\
&= -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{16}{35a\sqrt{1-a^2x^2}}+\frac{x\tanh}{7(1-}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.46

$$\frac{1680a^6x^6-5320a^4x^4+5726a^2x^2-105ax\left(16a^6x^6-56a^4x^4+70a^2x^2-35\right)\tanh^{-1}(ax)-2161}{3675a\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6 - 105*a*x*(-35 + 70*a^2
*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x])/(3675*a*(1 - a^2*x^2)^(7/2))
```

fricas [A] time = 0.46, size = 121, normalized size = 0.68

$$\frac{\left(3360\,a^6x^6-10640\,a^4x^4+11452\,a^2x^2-105\left(16\,a^7x^7-56\,a^5x^5+70\,a^3x^3-35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-4322\right)\sqrt{-a^2x^2+}}{7350\left(a^9x^8-4\,a^7x^6+6\,a^5x^4-4\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7350*(3360*a^6*x^6 - 10640*a^4*x^4 + 11452*a^2*x^2 - 105*(16*a^7*x^7 - 56
*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 4322)*sqrt(-a^2
*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

giac [A] time = 0.23, size = 138, normalized size = 0.78

$$-\frac{\sqrt{-a^2x^2+1}\left(2\left(4\left(2\,a^6x^2-7\,a^4\right)x^2+35\,a^2\right)x^2-35\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{70\left(a^2x^2-1\right)^4}-\frac{126\,a^2x^2+1680\left(a^2x^2-1\right)^3-280\left(a^2x^2-1\right.}{3675\left(a^2x^2-1\right)^3\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] -1/70*sqrt(-a^2*x^2 + 1)*(2*(4*(2*a^6*x^2 - 7*a^4)*x^2 + 35*a^2)*x^2 - 35)*
x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1)^4 - 1/3675*(126*a^2*x^2 + 1680*(a
^2*x^2 - 1)^3 - 280*(a^2*x^2 - 1)^2 - 201)/((a^2*x^2 - 1)^3*sqrt(-a^2*x^2 +
 1)*a)
```

maple [A] time = 0.44, size = 99, normalized size = 0.56

$$-\frac{\sqrt{-a^2x^2+1}\;\left(1680\operatorname{arctanh}(ax)\,x^7a^7-1680x^6a^6-5880\operatorname{arctanh}(ax)\,x^5a^5+5320x^4a^4+7350a^3x^3\operatorname{arctanh}(ax\right.}{3675a\left(a^2x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] -1/3675/a*(-a^2*x^2+1)^(1/2)*(1680*arctanh(a*x)*x^7*a^7-1680*x^6*a^6-5880*a
rctanh(a*x)*x^5*a^5+5320*x^4*a^4+7350*a^3*x^3*arctanh(a*x)-5726*a^2*x^2-367
5*a*x*arctanh(a*x)+2161)/(a^2*x^2-1)^4
```

maxima [A] time = 0.33, size = 140, normalized size = 0.79

$$-\frac{1}{3675}a\left(\frac{1680}{\sqrt{-a^2x^2+1}\,a^2}+\frac{280}{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}+\frac{126}{\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2}+\frac{75}{\left(-a^2x^2+1\right)^{\frac{7}{2}}a^2}\right)+\frac{1}{35}\left(\frac{16\,x}{\sqrt{-a^2x^2+1}}+\frac{8\,x}{\left(-a^2x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3675*a*(1680/(sqrt(-a^2*x^2 + 1)*a^2) + 280/((-a^2*x^2 + 1)^(3/2)*a^2) +
 126/((-a^2*x^2 + 1)^(5/2)*a^2) + 75/((-a^2*x^2 + 1)^(7/2)*a^2)) + 1/35*(16
*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 + 1)^(5/2)
 + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(1-a^2\,x^2\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(9/2),x)
```

```
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

3.465 $\int \left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=291

$$-\frac{3ic^2\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}+\frac{3ic^2\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}-\frac{3c^2\sqrt{1-a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a\sqrt{c-a^2cx^2}}+\frac{3c\sqrt{c-a\cdots}}{8a}$$

```
[Out] 1/12*(-a^2*c*x^2+c)^(3/2)/a+1/4*x*(-a^2*c*x^2+c)^(3/2)*arctanh(a*x)-3/4*c^2
*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/(-a
^2*c*x^2+c)^(1/2)-3/8*I*c^2*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^
2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+3/8*I*c^2*polylog(2,I*(-a*x+1)^(1/2)/
(a*x+1)^(1/2))*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+3/8*c*(-a^2*c*x^2+
c)^(1/2)/a+3/8*c*x*arctanh(a*x)*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5942, 5954, 5950}

$$-\frac{3ic^2\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}+\frac{3ic^2\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}-\frac{3c^2\sqrt{1-a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}\cdots}{4a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*c*Sqrt[c - a^2*c*x^2])/(8*a) + (c - a^2*c*x^2)^(3/2)/(12*a) + (3*c*x*Sqr
t[c - a^2*c*x^2]*ArcTanh[a*x])/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x])/4
 - (3*c^2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x
])/(4*a*Sqrt[c - a^2*c*x^2]) - (((3*I)/8)*c^2*Sqrt[1 - a^2*x^2]*PolyLog[2,
((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (((3*I)/8)*c
^2*Sqrt[1 - a^2*x^2]*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c
 - a^2*c*x^2])
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax)\,dx = \frac{\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax) + \frac{1}{4}(3c)\int \sqrt{c-a^2cx^2}\tanh^{-1}$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a} + \frac{\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax) + \frac{1}{4}x\left(c-a^2cx\right.$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a} + \frac{\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax) + \frac{1}{4}x\left(c-a^2cx\right.$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a} + \frac{\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax) + \frac{1}{4}x\left(c-a^2cx\right.$$

Mathematica [A] time = 0.68, size = 206, normalized size = 0.71

$$-\frac{c\sqrt{c-a^2cx^2}\left(2a^2x^2\sqrt{1-a^2x^2} - 11\sqrt{1-a^2x^2} - 15ax\sqrt{1-a^2x^2}\tanh^{-1}(ax) + 6a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \right.}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] -1/24*(c*Sqrt[c - a^2*c*x^2]*(-11*Sqrt[1 - a^2*x^2] + 2*a^2*x^2*Sqrt[1 - a^
2*x^2] - 15*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*a^3*x^3*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x] + (9*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (9*I)*ArcTa
nh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] -
 (9*I)*PolyLog[2, I/E^ArcTanh[a*x]]))/(a*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\,\mathrm{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.56, size = 345, normalized size = 1.19

$$-\frac{c\sqrt{-(ax-1)(ax+1)c}\;\left(6a^3x^3\,\mathrm{arctanh}\,(ax) + 2a^2x^2 - 15ax\,\mathrm{arctanh}\,(ax) - 11\right)}{24a} + \frac{3ic\sqrt{-a^2x^2+1}\;\sqrt{-(ax-1)}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/24*c/a*(-(a*x-1)*(a*x+1)*c)^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a
*x*arctanh(a*x)-11)+3/8*I*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*
(a*x+1)*c)^(1/2)*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*c/a/
(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*arctanh(a*x)*
ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*
x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I
*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1-
I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{artanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)\,\left(c-a^2\,c\,x^2\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)*(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-c(ax-1)(ax+1))^{\frac{3}{2}} \operatorname{atanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

3.466 $\int \sqrt{c-a^2cx^2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=235

$$-\frac{ic\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{ic\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)-\frac{c\sqrt{1-a^2x^2}}{}$$

```
[Out] -c*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/(
-a^2*c*x^2+c)^(1/2)-1/2*I*c*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^
2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+1/2*I*c*polylog(2,I*(-a*x+1)^(1/2)/(a
*x+1)^(1/2))*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+1/2*(-a^2*c*x^2+c)^(
1/2)/a+1/2*x*arctanh(a*x)*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5942, 5954, 5950}

$$-\frac{ic\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{ic\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a) + (x*Sqrt[c - a^2*c*x^2]*ArcTanh[a*x])/2 - (c*Sqr
t[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*Sqrt[c
- a^2*c*x^2]) - ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLo
g[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\tanh^{-1}(ax)\,dx = \frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{2}c\int \frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{\left(c\sqrt{1-a^2x^2}\right)\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)-\frac{c\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.29, size = 119, normalized size = 0.51

$$\frac{\sqrt{c\left(1-a^2x^2\right)}\left(-\frac{i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+ax\tanh^{-1}(ax)+\right.}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c*(1 - a^2*x^2)]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/
E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*
x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\ \text{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.55, size = 319, normalized size = 1.36

$$\frac{(ax\,\text{arctanh}\,(ax)+1)\sqrt{-(ax-1)(ax+1)c}}{2a}+\frac{i\sqrt{-(ax-1)(ax+1)c}\ \sqrt{-a^2x^2+1}\ \text{arctanh}\,(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a(ax+1)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x)
```

```
[Out] 1/2*(a*x*arctanh(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a+1/2*I/a*(-(a*x-1)*(a*
x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*arctanh(a*x)*ln(1+I*(a*x+1
)/(-a^2*x^2+1)^(1/2))-1/2*I/a*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2+
```

```
1)^(1/2)/(a*x-1)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*(-
(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*dilog(1+I*(a*x+
1)/(-a^2*x^2+1)^(1/2))-1/2*I/a*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2
+1)^(1/2)/(a*x-1)*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\ \operatorname{artanh}(ax)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)\,\sqrt{c-a^2\,c\,x^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c - a^2*c*x^2)^(1/2),x)
[Out] int(atanh(a*x)*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c(ax-1)(ax+1)}\ \operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*atanh(a*x),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

3.467 $\int \frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=182

$$-\frac{i\sqrt{1-a^2x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] -2*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/(
-a^2*c*x^2+c)^(1/2)-I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*x^2+
1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))
*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5954, 5950}

$$-\frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*
Sqrt[c - a^2*c*x^2]) - (I*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])
/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (I*Sqrt[1 - a^2*x^2]*PolyLog[2,
(I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{1-a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{i\sqrt{1-a^2x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 109, normalized size = 0.60

$$-\frac{i\sqrt{c\left(1-a^2x^2\right)}\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{ac\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((-I)*Sqrt[c*(1 - a^2*x^2)]*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[
1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^A
rcTanh[a*x]]))/(a*c*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\operatorname{artanh}(ax)}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arctanh(a*x)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*c*x^2 + c), x)
```

maple [A] time = 0.52, size = 290, normalized size = 1.59

$$\frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{artanh}(ax)\sqrt{-a^2x^2+1}\,\sqrt{-(ax-1)(ax+1)c}}{\left(a^2x^2-1\right)ac}-\frac{i\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{artanh}(ax)\sqrt{-a^2x^2}}{\left(a^2x^2-1\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)*(-(a*x
-1)*(a*x+1)*c)^(1/2)/(a^2*x^2-1)/a/c-I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*a
rctanh(a*x)*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a^2*x^2-1)/a/c+I
*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*x+1)
*c)^(1/2)/(a^2*x^2-1)/a/c-I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*(-a^2*x^2
+1)^(1/2)*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a^2*x^2-1)/a/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(a\,x\right)}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(1/2),x)
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(ax\right)}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atanh(a*x)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.468 $\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}$$

```
[Out] -1/a/c/(-a^2*c*x^2+c)^(1/2)+x*arctanh(a*x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5958}

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
[Out] -(1/(a*c*Sqrt[c - a^2*c*x^2])) + (x*ArcTanh[a*x])/(c*Sqrt[c - a^2*c*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{1}{ac\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.90

$$\frac{\sqrt{c-a^2cx^2}\left(1-ax\tanh^{-1}(ax)\right)}{ac^2\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[c - a^2*c*x^2]*(1 - a*x*ArcTanh[a*x]))/(a*c^2*(-1 + a^2*x^2))
```

fricas [A] time = 0.52, size = 54, normalized size = 1.12

$$-\frac{\sqrt{-a^2cx^2+c}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)-2\right)}{2\left(a^3c^2x^2-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*c^2*x^2
- a*c^2)
```

giac [A] time = 0.69, size = 70, normalized size = 1.46

$$-\frac{\sqrt{-a^2cx^2+c}\,x\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^2cx^2-c\right)c}-\frac{1}{\sqrt{-a^2cx^2+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1))/((a^2*c*x^2 - c)*c) -
 1/(sqrt(-a^2*c*x^2 + c)*a*c)
```

maple [A] time = 0.47, size = 74, normalized size = 1.54

$$-\frac{(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)c}}{2a(ax-1)c^2}-\frac{(\operatorname{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)c}}{2a(ax+1)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)/c^2-1/2*(arctanh
(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^2
```

maxima [B] time = 0.43, size = 90, normalized size = 1.88

$$-\frac{a^2\left(\frac{\sqrt{-a^2cx^2+c}}{a^4cx+a^3c}-\frac{\sqrt{-a^2cx^2+c}}{a^4cx-a^3c}\right)}{2c}+\frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2cx^2+c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(sqrt(-a^2*c*x^2 + c)/(a^4*c*x + a^3*c) - sqrt(-a^2*c*x^2 + c)/(a^
4*c*x - a^3*c))/c + x*arctanh(a*x)/(sqrt(-a^2*c*x^2 + c)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.469 $\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}-\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/9/a/c/(-a^2*c*x^2+c)^(3/2)+1/3*x*arctanh(a*x)/c/(-a^2*c*x^2+c)^(3/2)-2/3
/a/c^2/(-a^2*c*x^2+c)^(1/2)+2/3*x*arctanh(a*x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5960, 5958}

$$-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}-\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/(9*a*c*(c - a^2*c*x^2)^(3/2)) - 2/(3*a*c^2*Sqrt[c - a^2*c*x^2]) + (x*Arc
Tanh[a*x])/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c -
 a^2*c*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}+\frac{2\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{3c}\\ &= -\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.61

$$-\frac{\sqrt{c-a^2cx^2}\left(\left(6a^3x^3-9ax\right)\tanh^{-1}(ax)-6a^2x^2+7\right)}{9ac^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/9*(Sqrt[c - a^2*c*x^2]*(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*x
]))/(a*c^3*(-1 + a^2*x^2)^2)
```

fricas [A] time = 0.45, size = 84, normalized size = 0.80

$$\frac{\sqrt{-a^2cx^2+c}\left(12\,a^2x^2-3\left(2\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-14\right)}{18\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a^2*c*x^2 + c)*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1
)/(a*x - 1)) - 14)/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

giac [A] time = 0.31, size = 111, normalized size = 1.06

$$-\frac{\sqrt{-a^2cx^2+c}\left(\frac{2\,a^2x^2}{c}-\frac{3}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{6\left(a^2cx^2-c\right)^2}-\frac{6\,a^2cx^2-7\,c}{9\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a^2*c*x^2 + c)*(2*a^2*x^2/c - 3/c)*x*log(-(a*x + 1)/(a*x - 1))/(
a^2*c*x^2 - c)^2 - 1/9*(6*a^2*c*x^2 - 7*c)/((a^2*c*x^2 - c)*sqrt(-a^2*c*x^2
 + c)*a*c^2)
```

maple [A] time = 0.48, size = 160, normalized size = 1.52

$$\frac{(ax+1)(-1+3\,\mathrm{arctanh}\,(ax))\sqrt{-(ax-1)(ax+1)c}}{72a(ax-1)^2c^3}-\frac{3\,(\mathrm{arctanh}\,(ax)-1)\sqrt{-(ax-1)(ax+1)c}}{8a(ax-1)c^3}-\frac{3\,(\mathrm{arctanh}\,(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/72*(a*x+1)*(-1+3*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)^2/c^3
-3/8*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)/c^3-3/8*(arctanh
(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^3+1/72*(a*x-1)*(1+3*arctanh
(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)^2/c^3
```

maxima [A] time = 0.34, size = 90, normalized size = 0.86

$$-\frac{1}{9}\,a\left(\frac{6}{\sqrt{-a^2cx^2+c}\,a^2c^2}+\frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^2c}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^2cx^2+c}\,c^2}+\frac{x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}c}\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(6/(sqrt(-a^2*c*x^2 + c)*a^2*c^2) + 1/((-a^2*c*x^2 + c)^(3/2)*a^2*c)
) + 1/3*(2*x/(sqrt(-a^2*c*x^2 + c)*c^2) + x/((-a^2*c*x^2 + c)^(3/2)*c))*arc
tanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.470 $\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}+\frac{x\tanh^{-1}(}{5c\left(c-a^2c\right.}$$

```
[Out] -1/25/a/c/(-a^2*c*x^2+c)^(5/2)-4/45/a/c^2/(-a^2*c*x^2+c)^(3/2)+1/5*x*arctan
h(a*x)/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arctanh(a*x)/c^2/(-a^2*c*x^2+c)^(3/2)-
8/15/a/c^3/(-a^2*c*x^2+c)^(1/2)+8/15*x*arctanh(a*x)/c^3/(-a^2*c*x^2+c)^(1/2
)
```

Rubi [A] time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5960, 5958}

$$-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}+\frac{x\tanh^{-1}(}{5c\left(c-a^2c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/(25*a*c*(c - a^2*c*x^2)^(5/2)) - 4/(45*a*c^2*(c - a^2*c*x^2)^(3/2)) - 8/
(15*a*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcTanh[a*x])/(5*c*(c - a^2*c*x^2)^(5/2
)) + (4*x*ArcTanh[a*x])/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcTanh[a*x])
/(15*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{7/2}}\,dx &= -\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{4\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{5c}\\ &= -\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8}{}\\ &= -\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{4}{15c^2}\end{aligned}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.51

$$\frac{\sqrt{c-a^2cx^2}\left(120a^4x^4-260a^2x^2-15ax\left(8a^4x^4-20a^2x^2+15\right)\tanh^{-1}(ax)+149\right)}{225ac^4\left(a^2x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(149 - 260*a^2*x^2 + 120*a^4*x^4 - 15*a*x*(15 - 20*a^2
*x^2 + 8*a^4*x^4)*ArcTanh[a*x]))/(225*a*c^4*(-1 + a^2*x^2)^3)
```

fricas [A] time = 0.50, size = 112, normalized size = 0.71

$$\frac{\left(240\,a^4x^4-520\,a^2x^2-15\left(8\,a^5x^5-20\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+298\right)\sqrt{-a^2cx^2+c}}{450\left(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*log
(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*c*x^2 + c)/(a^7*c^4*x^6 - 3*a^5*c^4
*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

giac [A] time = 0.31, size = 149, normalized size = 0.95

$$-\frac{\sqrt{-a^2cx^2+c}\left(4\left(\frac{2\,a^4x^2}{c}-\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\left(a^2cx^2-c\right)^3}-\frac{120\left(a^2cx^2-c\right)^2-20\left(a^2cx^2-c\right)c+9\,c^2}{225\left(a^2cx^2-c\right)^2\sqrt{-a^2cx^2+c}\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c - 5*a^2/c)*x^2 + 15/c)*x*log(-(a
*x + 1)/(a*x - 1))/(a^2*c*x^2 - c)^3 - 1/225*(120*(a^2*c*x^2 - c)^2 - 20*(a
^2*c*x^2 - c)*c + 9*c^2)/((a^2*c*x^2 - c)^2*sqrt(-a^2*c*x^2 + c)*a*c^3)
```

maple [A] time = 0.49, size = 250, normalized size = 1.59

$$-\frac{(ax+1)^2(-1+5\operatorname{arctanh}(ax))\sqrt{-(ax-1)(ax+1)c}}{800a(ax-1)^3c^4}+\frac{5(ax+1)(-1+3\operatorname{arctanh}(ax))\sqrt{-(ax-1)(ax+1)c}}{288a(ax-1)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/800*(a*x+1)^2*(-1+5*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)^3
/c^4+5/288*(a*x+1)*(-1+3*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)
^2/c^4-5/16*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)/c^4-5/16*
(arctanh(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^4+5/288*(a*x-1)*(1+
3*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)^2/c^4-1/800*(a*x-1)^2*
(1+5*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)^3/a/c^4
```

maxima [A] time = 0.35, size = 132, normalized size = 0.84

$$-\frac{1}{225}a\left(\frac{120}{\sqrt{-a^2cx^2+c}\,a^2c^3}+\frac{20}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^2c^2}+\frac{9}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^2c}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2cx^2+c}\,c^3}+\frac{4\,x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/225*a*(120/(sqrt(-a^2*c*x^2 + c)*a^2*c^3) + 20/((-a^2*c*x^2 + c)^(3/2)*a
^2*c^2) + 9/((-a^2*c*x^2 + c)^(5/2)*a^2*c)) + 1/15*(8*x/(sqrt(-a^2*c*x^2 +
c)*c^3) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((-a^2*c*x^2 + c)^(5/2)*c)
)*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

3.471 $\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=158

$$\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a}-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(a}\right.}{a}$$

```
[Out] -arcsin(a*x)/a+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-I*arctan
h(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*arctanh(a*x)*polylog(2,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+arctanh(a*x)*(-a^2*x^2+1)^(1/
2)/a+1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5944, 5952, 4180, 2531, 2282, 6589, 216}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx - \int$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\text{Subst}\left(\int x^2\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh}\right.}{}$$

Mathematica [A] time = 0.10, size = 187, normalized size = 1.18

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(2\tanh^{-1}(ax)\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+2\text{Li}_3\left(-ie^{-\tanh^{-1}(ax)}\right)-2\text{Li}_3\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)^2\log\left(1-i\right.\right.}{\sqrt{1-a^2x^2}}}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\ \text{artanh}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \ \text{arctanh}\,(ax)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \text{artanh}\,(ax)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{atanh}\,(a\,x)^2\,\sqrt{1-a^2\,x^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(ax-1)(ax+1)}\ \text{atanh}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

3.472 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=139

$$\frac{40x}{27\sqrt{1-a^2x^2}}+\frac{2x}{27\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] 2/27*x/(-a^2*x^2+1)^(3/2)-2/9*arctanh(a*x)/a/(-a^2*x^2+1)^(3/2)+1/3*x*arcta
nh(a*x)^2/(-a^2*x^2+1)^(3/2)+40/27*x/(-a^2*x^2+1)^(1/2)-4/3*arctanh(a*x)/a/
(-a^2*x^2+1)^(1/2)+2/3*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5964, 5962, 191, 192}

$$\frac{40x}{27\sqrt{1-a^2x^2}}+\frac{2x}{27\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2*x)/(27*(1 - a^2*x^2)^(3/2)) + (40*x)/(27*Sqrt[1 - a^2*x^2]) - (2*ArcTanh
[a*x])/(9*a*(1 - a^2*x^2)^(3/2)) - (4*ArcTanh[a*x])/(3*a*Sqrt[1 - a^2*x^2])
 + (x*ArcTanh[a*x]^2)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^2)/(3*Sqr
t[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{5/2}}\,dx = -\frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}} + \frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{9}\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx + \frac{2}{3}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$$

$$= \frac{2x}{27\left(1-a^2x^2\right)^{3/2}} - \frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}} - \frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}}$$

$$= \frac{2x}{27\left(1-a^2x^2\right)^{3/2}} + \frac{40x}{27\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}} - \frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}} +$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.50

$$\frac{-40a^3x^3 - 9ax\left(2a^2x^2-3\right)\tanh^{-1}(ax)^2 + 6\left(6a^2x^2-7\right)\tanh^{-1}(ax) + 42ax}{27a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (42*a*x - 40*a^3*x^3 + 6*(-7 + 6*a^2*x^2)*ArcTanh[a*x] - 9*a*x*(-3 + 2*a^2*
x^2)*ArcTanh[a*x]^2)/(27*a*(1 - a^2*x^2)^(3/2))
```

fricas [A] time = 0.51, size = 105, normalized size = 0.76

$$-\frac{\left(160\,a^3x^3 + 9\left(2\,a^3x^3 - 3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 168\,ax - 12\left(6\,a^2x^2-7\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{108\left(a^5x^4 - 2\,a^3x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/108*(160*a^3*x^3 + 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 1
68*a*x - 12*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(
a^5*x^4 - 2*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(5/2), x)
```

maple [A] time = 0.43, size = 84, normalized size = 0.60

$$-\frac{\sqrt{-a^2x^2+1}\,\left(18\operatorname{arctanh}(ax)^2x^3a^3 + 40x^3a^3 - 36a^2x^2\operatorname{arctanh}(ax) - 27\operatorname{arctanh}(ax)^2ax - 42ax + 42\operatorname{arcta}\right.}{27a\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] -1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^2*x^3*a^3+40*x^3*a^3-36*a^2*x^2
*arctanh(a*x)-27*arctanh(a*x)^2*a*x-42*a*x+42*arctanh(a*x))/(a^2*x^2-1)^2
```

maxima [B] time = 0.50, size = 304, normalized size = 2.19

$$\frac{1}{3}\left(\frac{2x}{\sqrt{-a^2x^2+1}}+\frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\right)\operatorname{artanh}(ax)^2+\frac{1}{27}a\left(\frac{\frac{2x}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,a^2x+\sqrt{-a^2x^2+1}\,a}}{a}+\frac{\frac{2x}{\sqrt{-a^2x^2+1}}-\frac{}{\sqrt{-a^2x^2+1}\,a^2}}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)^2 + 1/27
*a*((2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 +
 1)*a))/a + (2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a
^2*x^2 + 1)*a))/a - 18*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 18*sqrt(-a^2*x^
2 + 1)/((a^2*x - a)*a) - 18*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 18*log(
-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 3*log(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a
^2) + 3*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{\left(1-a^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.473 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=208

$$\frac{4144x}{3375\sqrt{1-a^2x^2}}+\frac{272x}{3375\left(1-a^2x^2\right)^{3/2}}+\frac{2x}{125\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^2}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^2}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}}-\frac{16}{15}$$

```
[Out] 2/125*x/(-a^2*x^2+1)^(5/2)+272/3375*x/(-a^2*x^2+1)^(3/2)-2/25*arctanh(a*x)/
a/(-a^2*x^2+1)^(5/2)-8/45*arctanh(a*x)/a/(-a^2*x^2+1)^(3/2)+1/5*x*arctanh(a
*x)^2/(-a^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2)+4144/3375
*x/(-a^2*x^2+1)^(1/2)-16/15*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+8/15*x*arctan
h(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5964, 5962, 191, 192}

$$\frac{4144x}{3375\sqrt{1-a^2x^2}}+\frac{272x}{3375\left(1-a^2x^2\right)^{3/2}}+\frac{2x}{125\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^2}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^2}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}}-\frac{16}{15}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (2*x)/(125*(1 - a^2*x^2)^(5/2)) + (272*x)/(3375*(1 - a^2*x^2)^(3/2)) + (414
4*x)/(3375*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(25*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x])/(45*a*(1 - a^2*x^2)^(3/2)) - (16*ArcTanh[a*x])/(15*a*Sq
rt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTan
h[a*x]^2)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^2)/(15*Sqrt[1 - a^2*
x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
```

```
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx = -\frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} + \frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}} + \frac{2}{25}\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx + \frac{4}{5}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{5/2}}\,dx$$

$$= \frac{2x}{125\left(1-a^2x^2\right)^{5/2}} - \frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} - \frac{8\tanh^{-1}(ax)}{45a\left(1-a^2x^2\right)^{3/2}} + \frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax}{15\left(1-a^2x^2\right)^{3}}$$

$$= \frac{2x}{125\left(1-a^2x^2\right)^{5/2}} + \frac{272x}{3375\left(1-a^2x^2\right)^{3/2}} - \frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} - \frac{8\tanh^{-1}(ax)}{45a\left(1-a^2x^2\right)^{3/2}} - \frac{16\tanh^{-1}}{15a\sqrt{1-a}}$$

$$= \frac{2x}{125\left(1-a^2x^2\right)^{5/2}} + \frac{272x}{3375\left(1-a^2x^2\right)^{3/2}} + \frac{4144x}{3375\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} - \frac{8\tanh^{-1}}{45a\left(1-a^2\right.}$$

Mathematica [A] time = 0.10, size = 94, normalized size = 0.45

$$\frac{4144a^5x^5-8560a^3x^3+225ax\left(8a^4x^4-20a^2x^2+15\right)\tanh^{-1}(ax)^2-30\left(120a^4x^4-260a^2x^2+149\right)\tanh^{-1}(ax)+}{3375a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (4470*a*x - 8560*a^3*x^3 + 4144*a^5*x^5 - 30*(149 - 260*a^2*x^2 + 120*a^4*x
^4)*ArcTanh[a*x] + 225*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^2)/(3
375*a*(1 - a^2*x^2)^(5/2))
```

fricas [A] time = 0.50, size = 139, normalized size = 0.67

$$-\frac{\left(16576\,a^5x^5-34240\,a^3x^3+225\left(8\,a^5x^5-20\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+17880\,ax-60\left(120\,a^4x^4-260\,a^2x^2\right.\right.}{13500\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/13500*(16576*a^5*x^5 - 34240*a^3*x^3 + 225*(8*a^5*x^5 - 20*a^3*x^3 + 15*
a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 17880*a*x - 60*(120*a^4*x^4 - 260*a^2*x^
2 + 149)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x^4
 + 3*a^3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(7/2), x)
```

maple [A] time = 0.56, size = 118, normalized size = 0.57

$$-\frac{\sqrt{-a^2x^2+1}\,\left(1800\,\mathrm{arctanh}\,(ax)^2\,x^5a^5+4144x^5a^5-3600a^4x^4\,\mathrm{arctanh}\,(ax)-4500\,\mathrm{arctanh}\,(ax)^2\,x^3a^3-856\right.}{3375a\left(a^2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x)
```

```
[Out] -1/3375/a*(-a^2*x^2+1)^(1/2)*(1800*arctanh(a*x)^2*x^5*a^5+4144*x^5*a^5-3600
*a^4*x^4*arctanh(a*x)-4500*arctanh(a*x)^2*x^3*a^3-8560*x^3*a^3+7800*a^2*x^2
*arctanh(a*x)+3375*arctanh(a*x)^2*a*x+4470*a*x-4470*arctanh(a*x))/(a^2*x^2-
1)^3
```

maxima [B] time = 0.52, size = 514, normalized size = 2.47

$$\frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}+\frac{3\,x}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\right)\mathrm{artanh}\,(ax)^2+\frac{1}{3375}\,a\left(\frac{9\left(\frac{8\,x}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}-\frac{}{\left(-a^2x^2+1\right)}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) + 3*x/(-a^2*x^2 + 1
)^(5/2))*arctanh(a*x)^2 + 1/3375*a*(9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x
^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x + (-a^2*x^2 + 1)^(3/2)*a))/a
+ 9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^
(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a + 100*(2*x/sqrt(-a^2*x^2 + 1) - 1/
(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 + 1)*a))/a + 100*(2*x/sqrt(-a^2*x
^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a^2*x^2 + 1)*a))/a - 1800*sqr
t(-a^2*x^2 + 1)/((a^2*x + a)*a) - 1800*sqrt(-a^2*x^2 + 1)/((a^2*x - a)*a) -
 1800*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 1800*log(-a*x + 1)/(sqrt(-a^2
*x^2 + 1)*a^2) - 300*log(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) + 300*log(-a*x
 + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) - 135*log(a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a
^2) + 135*log(-a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atanh}\,(a\,x)^2}{\left(1-a^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(7/2),x)
```

```
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atanh}^2\,(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(7/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

3.474 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=277

$$\frac{413312x}{385875\sqrt{1-a^2x^2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^2}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(}{35\left(1-a^2x\right.}$$

```
[Out] 2/343*x/(-a^2*x^2+1)^(7/2)+888/42875*x/(-a^2*x^2+1)^(5/2)+30256/385875*x/(-
a^2*x^2+1)^(3/2)-2/49*arctanh(a*x)/a/(-a^2*x^2+1)^(7/2)-12/175*arctanh(a*x)
/a/(-a^2*x^2+1)^(5/2)-16/105*arctanh(a*x)/a/(-a^2*x^2+1)^(3/2)+1/7*x*arctan
h(a*x)^2/(-a^2*x^2+1)^(7/2)+6/35*x*arctanh(a*x)^2/(-a^2*x^2+1)^(5/2)+8/35*x
*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2)+413312/385875*x/(-a^2*x^2+1)^(1/2)-32/35
*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+16/35*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2
)
```

Rubi [A] time = 0.23, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5964, 5962, 191, 192}

$$\frac{413312x}{385875\sqrt{1-a^2x^2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^2}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(}{35\left(1-a^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (2*x)/(343*(1 - a^2*x^2)^(7/2)) + (888*x)/(42875*(1 - a^2*x^2)^(5/2)) + (30
256*x)/(385875*(1 - a^2*x^2)^(3/2)) + (413312*x)/(385875*Sqrt[1 - a^2*x^2])
 - (2*ArcTanh[a*x])/(49*a*(1 - a^2*x^2)^(7/2)) - (12*ArcTanh[a*x])/(175*a*(
1 - a^2*x^2)^(5/2)) - (16*ArcTanh[a*x])/(105*a*(1 - a^2*x^2)^(3/2)) - (32*A
rcTanh[a*x])/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(7*(1 - a^2*x^2)
^(7/2)) + (6*x*ArcTanh[a*x]^2)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]
^2)/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x]^2)/(35*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
```

```
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{9/2}}\,dx = -\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{x\tanh^{-1}(ax)^2}{7\left(1-a^2x^2\right)^{7/2}}+\frac{2}{49}\int \frac{1}{\left(1-a^2x^2\right)^{9/2}}\,dx+\frac{6}{7}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx$$

$$= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{12\tanh^{-1}(ax)}{175a\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)^2}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6x\tanh^{-1}}{35\left(1-a^2\right.}$$

$$= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{12\tanh^{-1}(ax)}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{16}{105a}$$

$$= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}-$$

$$= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{413312x}{385875\sqrt{1-a^2x^2}}-$$

Mathematica [A] time = 0.12, size = 120, normalized size = 0.43

$$\frac{2ax\left(-206656a^6x^6+635096a^4x^4-654220a^2x^2+226905\right)-11025ax\left(16a^6x^6-56a^4x^4+70a^2x^2-35\right)\tanh^{-1}}{385875a\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (2*a*x*(226905 - 654220*a^2*x^2 + 635096*a^4*x^4 - 206656*a^6*x^6) + 210*(-
2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh[a*x] - 11025*a*x
*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x]^2)/(385875*a*(1
- a^2*x^2)^(7/2))
```

fricas [A] time = 0.59, size = 169, normalized size = 0.61

$$-\frac{\left(1653248\,a^7x^7-5080768\,a^5x^5+5233760\,a^3x^3+11025\left(16\,a^7x^7-56\,a^5x^5+70\,a^3x^3-35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2\right.}{1543500\left(a^9x^8-4\,a^7x^6+6\,a^5x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/1543500*(1653248*a^7*x^7 - 5080768*a^5*x^5 + 5233760*a^3*x^3 + 11025*(16
*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1))^2 -
1815240*a*x - 420*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*log(-
(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 -
 4*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(9/2), x)
```

maple [A] time = 0.69, size = 152, normalized size = 0.55

$$-\frac{\sqrt{-a^2x^2+1}\,\left(176400 \operatorname{artanh}(ax)^2 x^7a^7 + 413312x^7a^7 - 352800 \operatorname{artanh}(ax)\,x^6a^6 - 617400 \operatorname{artanh}(ax)^2 x^5a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] -1/385875/a*(-a^2*x^2+1)^(1/2)*(176400*arctanh(a*x)^2*x^7*a^7+413312*x^7*a^
7-352800*arctanh(a*x)*x^6*a^6-617400*arctanh(a*x)^2*x^5*a^5-1270192*x^5*a^5
+1117200*a^4*x^4*arctanh(a*x)+771750*arctanh(a*x)^2*x^3*a^3+1308440*x^3*a^3
-1202460*a^2*x^2*arctanh(a*x)-385875*arctanh(a*x)^2*a*x-453810*a*x+453810*a
rctanh(a*x))/(a^2*x^2-1)^4
```

maxima [B] time = 0.52, size = 751, normalized size = 2.71

$$\frac{1}{35}\left(\frac{16\,x}{\sqrt{-a^2x^2+1}} + \frac{8\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}} + \frac{6\,x}{\left(-a^2x^2+1\right)^{\frac{5}{2}}} + \frac{5\,x}{\left(-a^2x^2+1\right)^{\frac{7}{2}}}\right)\operatorname{artanh}(ax)^2 + \frac{1}{385875}\,a\left(\frac{225\left(\frac{16\,x}{\sqrt{-a^2x^2+1}} + \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/35*(16*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 +
1)^(5/2) + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)^2 + 1/385875*a*(225*(16*x
/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)*a^
2*x + (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 225*(16*x/sqr
t(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)*a^2*x
- (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 882*(8*x/sqrt(-a^
2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x + (-a
^2*x^2 + 1)^(3/2)*a))/a + 882*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^
(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a + 9800*(
2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 + 1)*a
))/a + 9800*(2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a
^2*x^2 + 1)*a))/a - 176400*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 176400*sqrt
(-a^2*x^2 + 1)/((a^2*x - a)*a) - 176400*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^
2) + 176400*log(-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 29400*log(a*x + 1)/((-
a^2*x^2 + 1)^(3/2)*a^2) + 29400*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) -
13230*log(a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2) + 13230*log(-a*x + 1)/((-a^2*
x^2 + 1)^(5/2)*a^2) - 7875*log(a*x + 1)/((-a^2*x^2 + 1)^(7/2)*a^2) + 7875*l
og(-a*x + 1)/((-a^2*x^2 + 1)^(7/2)*a^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^2}{\left(1-a^2\,x^2\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(9/2),x)
```

```
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

3.475 $\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=302

$$\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a}+\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{2a}$$

```
[Out] 6*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a+arctan((a*x+1)/(-a^2*
x^2+1)^(1/2))*arctanh(a*x)^3/a-3/2*I*arctanh(a*x)^2*polylog(2,-I*(a*x+1)/(-
a^2*x^2+1)^(1/2))/a+3/2*I*arctanh(a*x)^2*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(
1/2))/a+3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-3*I*polylog(2,I*(-
a*x+1)^(1/2)/(a*x+1)^(1/2))/a+3*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a^2*x
^2+1)^(1/2))/a-3*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-3
*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+3*I*polylog(4,I*(a*x+1)/(-a^2
*x^2+1)^(1/2))/a+3/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a+1/2*x*arctanh(a*x)
^3*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5944, 5952, 4180, 2531, 6609, 2282, 6589, 5950}

$$\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{2a}+\frac{3i\tanh^{-1}(ax)^2\text{PolyL}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

```
[Out] (6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a + (3*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/2 + (ArcTan
[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, (
-I)*E^ArcTanh[a*x]])/a + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[a
*x]])/a + ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a - ((3*I)
*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + ((3*I)*ArcTanh[a*x]*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a - ((3*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[
a*x]])/a - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*PolyLog[4, I*
E^ArcTanh[a*x]])/a
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^3\,dx = \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx - 3$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}($$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}($$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}($$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}($$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}($$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}($$

Mathematica [A] time = 4.30, size = 569, normalized size = 1.88

$$-\frac{i\left(64iax\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3 + 192i\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + 192\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right) + 192i\pi\tanh}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

```
[Out] ((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 + (1
92*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 + (64*I)*
a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 - 384*ArcTanh[a*x]
*Log[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] + 384*Arc
Tanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcT
anh[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh
[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTan
h[a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*L
og[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8
*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(8 + Pi^2 - (4*I)*Pi*Ar
cTanh[a*x] - 4*ArcTanh[a*x]^2)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + 384*PolyLo
g[2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]]
 - 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2
, I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcT
anh[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)
*E^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4,
 (-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \operatorname{arctanh}(ax)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)^3\,\sqrt{1-a^2\,x^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)^3*(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(ax-1)(ax+1)}\ \operatorname{atanh}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3, x)
```

3.476 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{40}{9a\sqrt{1-a^2x^2}}-\frac{2}{27a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^3}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{3\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}}+\frac{40x\tanh^{-1}}{9\sqrt{1-a^2}}$$

```
[Out] -2/27/a/(-a^2*x^2+1)^(3/2)+2/9*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-1/3*arctan
h(a*x)^2/a/(-a^2*x^2+1)^(3/2)+1/3*x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2)-40/9/
a/(-a^2*x^2+1)^(1/2)+40/9*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-2*arctanh(a*x)^
2/a/(-a^2*x^2+1)^(1/2)+2/3*x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5964, 5962, 5958, 5960}

$$-\frac{40}{9a\sqrt{1-a^2x^2}}-\frac{2}{27a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^3}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{3\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}}+\frac{40x\tanh^{-1}}{9\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -2/(27*a*(1 - a^2*x^2)^(3/2)) - 40/(9*a*Sqrt[1 - a^2*x^2]) + (2*x*ArcTanh[a
*x])/(9*(1 - a^2*x^2)^(3/2)) + (40*x*ArcTanh[a*x])/(9*Sqrt[1 - a^2*x^2]) -
ArcTanh[a*x]^2/(3*a*(1 - a^2*x^2)^(3/2)) - (2*ArcTanh[a*x]^2)/(a*Sqrt[1 - a
^2*x^2]) + (x*ArcTanh[a*x]^3)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^3
)/(3*Sqrt[1 - a^2*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
```

```
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx = -\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}} + \frac{x\tanh^{-1}(ax)^3}{3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{3}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx + \frac{2}{3}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{2}{27a\left(1-a^2x^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{9\left(1-a^2x^2\right)^{3/2}} - \frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}} - \frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3,}}$$

$$= -\frac{2}{27a\left(1-a^2x^2\right)^{3/2}} - \frac{40}{9a\sqrt{1-a^2x^2}} + \frac{2x\tanh^{-1}(ax)}{9\left(1-a^2x^2\right)^{3/2}} + \frac{40x\tanh^{-1}(ax)}{9\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3}}$$

Mathematica [A] time = 0.10, size = 87, normalized size = 0.46

$$\frac{120a^2x^2 - 9ax\left(2a^2x^2-3\right)\tanh^{-1}(ax)^3 + 9\left(6a^2x^2-7\right)\tanh^{-1}(ax)^2 - 6ax\left(20a^2x^2-21\right)\tanh^{-1}(ax) - 122}{27a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-122 + 120*a^2*x^2 - 6*a*x*(-21 + 20*a^2*x^2)*ArcTanh[a*x] + 9*(-7 + 6*a^2
*x^2)*ArcTanh[a*x]^2 - 9*a*x*(-3 + 2*a^2*x^2)*ArcTanh[a*x]^3)/(27*a*(1 - a^
2*x^2)^(3/2))
```

fricas [A] time = 0.46, size = 134, normalized size = 0.70

$$\frac{\left(960\,a^2x^2 - 9\left(2\,a^3x^3 - 3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 18\left(6\,a^2x^2-7\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 24\left(20\,a^3x^3 - 21\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) -}{216\left(a^5x^4 - 2\,a^3x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/216*(960*a^2*x^2 - 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 18
*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1))^2 - 24*(20*a^3*x^3 - 21*a*x)*log
(-(a*x + 1)/(a*x - 1)) - 976)*sqrt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

maple [A] time = 0.44, size = 105, normalized size = 0.55

$$-\frac{\sqrt{-a^2x^2+1}\,\left(18\operatorname{arctanh}(ax)^3x^3a^3 + 120a^3x^3\operatorname{arctanh}(ax) - 54a^2x^2\operatorname{arctanh}(ax)^2 - 27\operatorname{arctanh}(ax)^3ax -\right.}{27a\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] -1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^3*x^3*a^3+120*a^3*x^3*arctanh(a
*x)-54*a^2*x^2*arctanh(a*x)^2-27*arctanh(a*x)^3*a*x-120*a^2*x^2-126*a*x*arc
tanh(a*x)+63*arctanh(a*x)^2+122)/(a^2*x^2-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{\left(1-a^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.477 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=289

$$-\frac{4144}{1125a\sqrt{1-a^2x^2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^3}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^3}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^3}{5\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -6/625/a/(-a^2*x^2+1)^(5/2)-272/3375/a/(-a^2*x^2+1)^(3/2)+6/125*x*arctanh(a
*x)/(-a^2*x^2+1)^(5/2)+272/1125*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-3/25*arct
anh(a*x)^2/a/(-a^2*x^2+1)^(5/2)-4/15*arctanh(a*x)^2/a/(-a^2*x^2+1)^(3/2)+1/
5*x*arctanh(a*x)^3/(-a^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)^3/(-a^2*x^2+1)^(3
/2)-4144/1125/a/(-a^2*x^2+1)^(1/2)+4144/1125*x*arctanh(a*x)/(-a^2*x^2+1)^(1
/2)-8/5*arctanh(a*x)^2/a/(-a^2*x^2+1)^(1/2)+8/15*x*arctanh(a*x)^3/(-a^2*x^2
+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5964, 5962, 5958, 5960}

$$-\frac{4144}{1125a\sqrt{1-a^2x^2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^3}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^3}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^3}{5\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] -6/(625*a*(1 - a^2*x^2)^(5/2)) - 272/(3375*a*(1 - a^2*x^2)^(3/2)) - 4144/(1
125*a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(125*(1 - a^2*x^2)^(5/2)) + (
272*x*ArcTanh[a*x])/(1125*(1 - a^2*x^2)^(3/2)) + (4144*x*ArcTanh[a*x])/(112
5*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(25*a*(1 - a^2*x^2)^(5/2)) - (4*A
rcTanh[a*x]^2)/(15*a*(1 - a^2*x^2)^(3/2)) - (8*ArcTanh[a*x]^2)/(5*a*Sqrt[1
- a^2*x^2]) + (x*ArcTanh[a*x]^3)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[a*x
]^3)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^3)/(15*Sqrt[1 - a^2*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx = -\frac{3\tanh^{-1}(ax)^2}{25a\left(1-a^2x^2\right)^{5/2}} + \frac{x\tanh^{-1}(ax)^3}{5\left(1-a^2x^2\right)^{5/2}} + \frac{6}{25}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx + \frac{4}{5}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx$$

$$= -\frac{6}{625a\left(1-a^2x^2\right)^{5/2}} + \frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}} - \frac{3\tanh^{-1}(ax)^2}{25a\left(1-a^2x^2\right)^{5/2}} - \frac{4\tanh^{-1}(ax)^2}{15a\left(1-a^2x^2\right)^{3/2}} + \frac{x\tanh^{-1}}{5\left(1-a^2\right.}$$

$$= -\frac{6}{625a\left(1-a^2x^2\right)^{5/2}} - \frac{272}{3375a\left(1-a^2x^2\right)^{3/2}} + \frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}} + \frac{272x\tanh^{-1}(ax)}{1125\left(1-a^2x^2\right)^{3/2}} - \frac{3\,\mathrm{ta}}{25a\,(}$$

$$= -\frac{6}{625a\left(1-a^2x^2\right)^{5/2}} - \frac{272}{3375a\left(1-a^2x^2\right)^{3/2}} - \frac{4144}{1125a\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}} + \frac{272x}{1125\,(}$$

Mathematica [A] time = 0.11, size = 119, normalized size = 0.41

$$\frac{-62160a^4x^4+125680a^2x^2+1125ax\left(8a^4x^4-20a^2x^2+15\right)\tanh^{-1}(ax)^3+30ax\left(2072a^4x^4-4280a^2x^2+2235\right)\mathrm{t}}{16875a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (-63682 + 125680*a^2*x^2 - 62160*a^4*x^4 + 30*a*x*(2235 - 4280*a^2*x^2 + 20
72*a^4*x^4)*ArcTanh[a*x] - 225*(149 - 260*a^2*x^2 + 120*a^4*x^4)*ArcTanh[a*
x]^2 + 1125*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^3)/(16875*a*(1 -
 a^2*x^2)^(5/2))
```

fricas [A] time = 0.47, size = 176, normalized size = 0.61

$$\frac{\left(497280\,a^4x^4-1005440\,a^2x^2-1125\left(8\,a^5x^5-20\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+450\left(120\,a^4x^4-260\,a^2x^2+149\right)\mathrm{l}\right.}{135000\left(a^7x^6-3\,a^5x^4+3\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/135000*(497280*a^4*x^4 - 1005440*a^2*x^2 - 1125*(8*a^5*x^5 - 20*a^3*x^3 +
 15*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 450*(120*a^4*x^4 - 260*a^2*x^2 + 149
)*log(-(a*x + 1)/(a*x - 1))^2 - 120*(2072*a^5*x^5 - 4280*a^3*x^3 + 2235*a*x
)*log(-(a*x + 1)/(a*x - 1)) + 509456)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x
^4 + 3*a^3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="giac")

[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)

maple [A] time = 0.46, size = 153, normalized size = 0.53

$$-\frac{\sqrt{-a^2x^2+1}\,\left(9000\,\mathrm{arctanh}\,(ax)^3\,x^5a^5+62160\,\mathrm{arctanh}\,(ax)\,x^5a^5-27000a^4x^4\,\mathrm{arctanh}\,(ax)^2-22500\,\mathrm{arctar}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x)

[Out] -1/16875/a*(-a^2*x^2+1)^(1/2)*(9000*arctanh(a*x)^3*x^5*a^5+62160*arctanh(a*x)*x^5*a^5-27000*a^4*x^4*arctanh(a*x)^2-22500*arctanh(a*x)^3*x^3*a^3-62160*x^4*a^4-128400*a^3*x^3*arctanh(a*x)+58500*a^2*x^2*arctanh(a*x)^2+16875*arctanh(a*x)^3*a*x+125680*a^2*x^2+67050*a*x*arctanh(a*x)-33525*arctanh(a*x)^2-63682)/(a^2*x^2-1)^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\,(ax)^3}{\left(-a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")

[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atanh}\,(a\,x)^3}{\left(1-a^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(7/2),x)

[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atanh}^3\,(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(7/2),x)

[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(7/2), x)

3.478 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{9/2}} dx$

Optimal. Leaf size=385

$$-\frac{413312}{128625a\sqrt{1-a^2x^2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^3}{35\sqrt{1-a^2x^2}}+\frac{8x}{35}$$

```
[Out] -6/2401/a/(-a^2*x^2+1)^(7/2)-2664/214375/a/(-a^2*x^2+1)^(5/2)-30256/385875/
a/(-a^2*x^2+1)^(3/2)+6/343*x*arctanh(a*x)/(-a^2*x^2+1)^(7/2)+2664/42875*x*a
rctanh(a*x)/(-a^2*x^2+1)^(5/2)+30256/128625*x*arctanh(a*x)/(-a^2*x^2+1)^(3/
2)-3/49*arctanh(a*x)^2/a/(-a^2*x^2+1)^(7/2)-18/175*arctanh(a*x)^2/a/(-a^2*x
^2+1)^(5/2)-8/35*arctanh(a*x)^2/a/(-a^2*x^2+1)^(3/2)+1/7*x*arctanh(a*x)^3/(
-a^2*x^2+1)^(7/2)+6/35*x*arctanh(a*x)^3/(-a^2*x^2+1)^(5/2)+8/35*x*arctanh(a
*x)^3/(-a^2*x^2+1)^(3/2)-413312/128625/a/(-a^2*x^2+1)^(1/2)+413312/128625*x
*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-48/35*arctanh(a*x)^2/a/(-a^2*x^2+1)^(1/2)+
16/35*x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5964, 5962, 5958, 5960}

$$-\frac{413312}{128625a\sqrt{1-a^2x^2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^3}{35\sqrt{1-a^2x^2}}+\frac{8x}{35}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] -6/(2401*a*(1 - a^2*x^2)^(7/2)) - 2664/(214375*a*(1 - a^2*x^2)^(5/2)) - 302
56/(385875*a*(1 - a^2*x^2)^(3/2)) - 413312/(128625*a*Sqrt[1 - a^2*x^2]) + (
6*x*ArcTanh[a*x])/(343*(1 - a^2*x^2)^(7/2)) + (2664*x*ArcTanh[a*x])/(42875*
(1 - a^2*x^2)^(5/2)) + (30256*x*ArcTanh[a*x])/(128625*(1 - a^2*x^2)^(3/2))
+ (413312*x*ArcTanh[a*x])/(128625*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(
49*a*(1 - a^2*x^2)^(7/2)) - (18*ArcTanh[a*x]^2)/(175*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x]^2)/(35*a*(1 - a^2*x^2)^(3/2)) - (48*ArcTanh[a*x]^2)/(35*
a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(7*(1 - a^2*x^2)^(7/2)) + (6*x*Ar
cTanh[a*x]^3)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]^3)/(35*(1 - a^2*
x^2)^(3/2)) + (16*x*ArcTanh[a*x]^3)/(35*Sqrt[1 - a^2*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
```

```
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{9/2}}\,dx = -\frac{3\tanh^{-1}(ax)^2}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{x\tanh^{-1}(ax)^3}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6}{49}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{9/2}}\,dx+\frac{6}{7}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}+\frac{6x\tanh^{-1}(ax)}{343\left(1-a^2x^2\right)^{7/2}}-\frac{3\tanh^{-1}(ax)^2}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{18\tanh^{-1}(ax)^2}{175a\left(1-a^2x^2\right)^{5/2}}+\frac{x\,\mathrm{t}}{7(}$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}+\frac{6x\tanh^{-1}(ax)}{343\left(1-a^2x^2\right)^{7/2}}+\frac{2664x\tanh^{-1}(ax)}{42875\left(1-a^2x^2\right)^{5/2}}$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}+\frac{6x\tanh^{-1}(a}{343\left(1-a^2x^2\right)}$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{413312}{128625a\sqrt{1-}}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 0.39

$$\frac{43397760a^6x^6-131252240a^4x^4+132479032a^2x^2-385875ax\left(16a^6x^6-56a^4x^4+70a^2x^2-35\right)\tanh^{-1}(ax)^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-44658302 + 132479032*a^2*x^2 - 131252240*a^4*x^4 + 43397760*a^6*x^6 - 210
*a*x*(-226905 + 654220*a^2*x^2 - 635096*a^4*x^4 + 206656*a^6*x^6)*ArcTanh[a
*x] + 11025*(-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh[a*
x]^2 - 385875*a*x*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x]
^3)/(13505625*a*(1 - a^2*x^2)^(7/2))
```

fricas [A] time = 0.52, size = 214, normalized size = 0.56

$$\frac{\left(347182080\,a^6x^6-1050017920\,a^4x^4+1059832256\,a^2x^2-385875\left(16\,a^7x^7-56\,a^5x^5+70\,a^3x^3-35\,ax\right)\log\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/108045000*(347182080*a^6*x^6 - 1050017920*a^4*x^4 + 1059832256*a^2*x^2 -
385875*(16*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x
- 1))^3 + 22050*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*log(-(a
*x + 1)/(a*x - 1))^2 - 840*(206656*a^7*x^7 - 635096*a^5*x^5 + 654220*a^3*x^
3 - 226905*a*x)*log(-(a*x + 1)/(a*x - 1)) - 357266416)*sqrt(-a^2*x^2 + 1)/(
a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)
```

maple [A] time = 0.57, size = 201, normalized size = 0.52

$$-\frac{\sqrt{-a^2x^2+1}\,\left(6174000\operatorname{arctanh}(ax)^3x^7a^7+43397760\operatorname{arctanh}(ax)x^7a^7-18522000\operatorname{arctanh}(ax)^2x^6a^6-2160\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] -1/13505625/a*(-a^2*x^2+1)^(1/2)*(6174000*arctanh(a*x)^3*x^7*a^7+43397760*a
rctanh(a*x)*x^7*a^7-18522000*arctanh(a*x)^2*x^6*a^6-21609000*arctanh(a*x)^3
*x^5*a^5-43397760*x^6*a^6-133370160*arctanh(a*x)*x^5*a^5+58653000*a^4*x^4*a
rctanh(a*x)^2+27011250*arctanh(a*x)^3*x^3*a^3+131252240*x^4*a^4+137386200*a
^3*x^3*arctanh(a*x)-63129150*a^2*x^2*arctanh(a*x)^2-13505625*arctanh(a*x)^3
*a*x-132479032*a^2*x^2-47650050*a*x*arctanh(a*x)+23825025*arctanh(a*x)^2+44
658302)/(a^2*x^2-1)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{{\left(1-a^2\,x^2\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(9/2),x)
```

```
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(9/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{(-(ax-1)(ax+1))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

3.479 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx=\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

maple [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-a^2\,x^2}}{\operatorname{atanh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/atanh(a*x),x)
[Out] int((1 - a^2*x^2)^(1/2)/atanh(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x), x)
```

3.480 $\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx=\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^2-1\right)\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\ \operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}(a\,x)\ \sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\ \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x),x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)), x)
```

3.481 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] Chi(arctanh(a*x))/a
```

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5968, 3301}

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]

[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 9, normalized size = 1.00

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]

[Out] CoshIntegral[ArcTanh[a*x]]/a
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

maple [A] time = 0.28, size = 10, normalized size = 1.11

$$\frac{\mathrm{X}\left(\operatorname{arctanh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] Chi(arctanh(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

3.482 $\int \frac{1}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

```
[Out] 3/4*Chi(arctanh(a*x))/a+1/4*Chi(3*arctanh(a*x))/a
```

Rubi [A] time = 0.09, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5968, 3312, 3301}

$$\frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
```

```
[Out] (3*CoshIntegral[ArcTanh[a*x]])/(4*a) + CoshIntegral[3*ArcTanh[a*x]]/(4*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\
&= \frac{\mathrm{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx,x,\tanh^{-1}(ax)\right)}{a}\\
&= \frac{\mathrm{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{4a}+\frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{4a}\\
&= \frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.81

$$\frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)+\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
```

```
[Out] (3*CoshIntegral[ArcTanh[a*x]] + CoshIntegral[3*ArcTanh[a*x]])/(4*a)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}}\mathrm{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

maple [A] time = 0.33, size = 21, normalized size = 0.78

$$\frac{3\mathrm{X}\left(\mathrm{arctanh}\left(ax\right)\right)+\mathrm{X}\left(3\,\mathrm{arctanh}\left(ax\right)\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x)
```

```
[Out] 1/4*(3*Chi(arctanh(a*x))+Chi(3*arctanh(a*x)))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}}\mathrm{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{1}{\mathrm{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)), x)
```

3.483 $\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{5\text{Chi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] 5/8*Chi(arctanh(a*x))/a+5/16*Chi(3*arctanh(a*x))/a+1/16*Chi(5*arctanh(a*x))
/a
```

Rubi [A] time = 0.11, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5968, 3312, 3301}

$$\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{5\text{Chi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
```

```
[Out] (5*CoshIntegral[ArcTanh[a*x]])/(8*a) + (5*CoshIntegral[3*ArcTanh[a*x]])/(16
*a) + CoshIntegral[5*ArcTanh[a*x]]/(16*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8x}+\frac{5\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{16a}+\frac{5\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{16a}\\ &= \frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{5\text{Chi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}\end{aligned}$$

Mathematica [A] time = 0.07, size = 31, normalized size = 0.76

$$\frac{10\text{Chi}\left(\tanh^{-1}(ax)\right)+5\text{Chi}\left(3\tanh^{-1}(ax)\right)+\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
```

```
[Out] (10*CoshIntegral[ArcTanh[a*x]] + 5*CoshIntegral[3*ArcTanh[a*x]] + CoshInteg
ral[5*ArcTanh[a*x]])/(16*a)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^8x^8-4\,a^6x^6+6\,a^4x^4-4\,a^2x^2+1\right)\text{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}}\text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

maple [A] time = 0.38, size = 30, normalized size = 0.73

$$\frac{10\text{X}\left(\text{arctanh}\left(ax\right)\right)+5\text{X}\left(3\,\text{arctanh}\left(ax\right)\right)+\text{X}\left(5\,\text{arctanh}\left(ax\right)\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x)
```

```
[Out] 1/16*(10*Chi(arctanh(a*x))+5*Chi(3*arctanh(a*x))+Chi(5*arctanh(a*x)))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}}\text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\text{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(7/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)), x)
```

3.484 $\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

```
[Out] 35/64*Chi(arctanh(a*x))/a+21/64*Chi(3*arctanh(a*x))/a+7/64*Chi(5*arctanh(a*
x))/a+1/64*Chi(7*arctanh(a*x))/a
```

Rubi [A] time = 0.12, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5968, 3312, 3301}

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
```

```
[Out] (35*CoshIntegral[ArcTanh[a*x]])/(64*a) + (21*CoshIntegral[3*ArcTanh[a*x]])/
(64*a) + (7*CoshIntegral[5*ArcTanh[a*x]])/(64*a) + CoshIntegral[7*ArcTanh[a
*x]]/(64*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^7(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{35\cosh(x)}{64x}+\frac{21\cosh(3x)}{64x}+\frac{7\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)dx,x,\tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{64a}+\frac{7\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{64a}+\\
&= \frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{\text{Chi}\left(7\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 0.73

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)+21\text{Chi}\left(3\tanh^{-1}(ax)\right)+7\text{Chi}\left(5\tanh^{-1}(ax)\right)+\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
```

```
[Out] (35*CoshIntegral[ArcTanh[a*x]] + 21*CoshIntegral[3*ArcTanh[a*x]] + 7*CoshIn
tegral[5*ArcTanh[a*x]] + CoshIntegral[7*ArcTanh[a*x]])/(64*a)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^{10}x^{10}-5\,a^8x^8+10\,a^6x^6-10\,a^4x^4+5\,a^2x^2-1\right)\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

maple [A] time = 0.43, size = 39, normalized size = 0.71

$$\frac{35\text{X}\,(\text{arctanh}\,(ax))+21\text{X}\,(3\,\text{arctanh}\,(ax))+7\text{X}\,(5\,\text{arctanh}\,(ax))+\text{X}\,(7\,\text{arctanh}\,(ax))}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x)
```

```
[Out] 1/64*(35*Chi(arctanh(a*x))+21*Chi(3*arctanh(a*x))+7*Chi(5*arctanh(a*x))+Chi
(7*arctanh(a*x)))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\text{atanh}\,(a\,x)\,\left(1-a^2\,x^2\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(9/2)),x)
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(9/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x),x)
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)), x)
```

3.485 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx = \int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

maple [A] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{\operatorname{arctanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{\operatorname{artanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-a^{2}\,x^{2}}}{\operatorname{atanh}\left(a\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^2,x)
[Out] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**2, x)
```

3.486 $\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx=\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^2-1\right)\text{artanh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

maple [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\ \operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\ \operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2), x)
```

3.487 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=35

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}$$

```
[Out] Shi(arctanh(a*x))/a-1/a/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5966, 6034, 3298}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx &= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx\\
&= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\
&= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.91

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)-\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
[Out] (-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

maple [A] time = 0.33, size = 62, normalized size = 1.77

$$\frac{\text{arctanh}\left(ax\right)\text{Shi}\left(\text{arctanh}\left(ax\right)\right)x^2a^2-\text{Shi}\left(\text{arctanh}\left(ax\right)\right)\text{arctanh}\left(ax\right)+\sqrt{-a^2x^2+1}}{a\,\text{arctanh}\left(ax\right)\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
[Out] 1/a*(arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2-Shi(arctanh(a*x))*arctanh(a*x)+
(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\text{atanh}\left(a\,x\right)^2\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

3.488 $\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\text{Shi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

```
[Out] -1/a/(-a^2*x^2+1)^(3/2)/arctanh(a*x)+3/4*Shi(arctanh(a*x))/a+3/4*Shi(3*arct
anh(a*x))/a
```

Rubi [A] time = 0.16, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\text{Shi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])) + (3*SinhIntegral[ArcTanh[a*x]])/
(4*a) + (3*SinhIntegral[3*ArcTanh[a*x]])/(4*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + (3a)\int \frac{x}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x}+\frac{\sinh(3x)}{4x}\right)dx, x, \tanh^{-1}(a\right.}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a} + \frac{3\,\text{Sub}}{}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\text{Shi}\left(\tanh^{-1}(ax)\right)}{4a} + \frac{3\text{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Mathematica [A] time = 0.14, size = 45, normalized size = 0.87

$$\frac{3\left(\text{Shi}\left(\tanh^{-1}(ax)\right)+\text{Shi}\left(3\tanh^{-1}(ax)\right)\right)-\frac{4}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-4/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]) + 3*(SinhIntegral[ArcTanh[a*x]] + Si
nhIntegral[3*ArcTanh[a*x]]))/(4*a)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\text{artanh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}}\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

maple [B] time = 0.43, size = 120, normalized size = 2.31

$$\frac{3\,\text{arctanh}\left(ax\right)\text{Shi}\left(\text{arctanh}\left(ax\right)\right)x^2a^2+3\,\text{arctanh}\left(ax\right)\text{Shi}\left(3\,\text{arctanh}\left(ax\right)\right)x^2a^2-\cosh\left(3\,\text{arctanh}\left(ax\right)\right)x^2a^2}{4a\,\text{arctanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/4/a*(3*arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2+3*arctanh(a*x)*Shi(3*arctan
h(a*x))*x^2*a^2-cosh(3*arctanh(a*x))*x^2*a^2-3*Shi(arctanh(a*x))*arctanh(a*
x)-3*Shi(3*arctanh(a*x))*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctanh(a
*x)))/arctanh(a*x)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}} \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}(a\,x)^2\,\left(1-a^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}} \operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)**2), x)
```

3.489 $\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{a(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{5\text{Shi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] -1/a/(-a^2*x^2+1)^(5/2)/arctanh(a*x)+5/8*Shi(arctanh(a*x))/a+15/16*Shi(3*ar
ctanh(a*x))/a+5/16*Shi(5*arctanh(a*x))/a
```

Rubi [A] time = 0.18, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{5\text{Shi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])) + (5*SinhIntegral[ArcTanh[a*x]])/
(8*a) + (15*SinhIntegral[3*ArcTanh[a*x]])/(16*a) + (5*SinhIntegral[5*ArcTan
h[a*x]])/(16*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + (5a)\int \frac{x}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x} + \frac{3\sinh(3x)}{16x} + \frac{\sinh(5x)}{16x}\right)dx, x, \text{t}\right.}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\,\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{5\,\text{Subst}}{}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a} + \frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a} + \frac{5}{}$$

Mathematica [A] time = 0.18, size = 56, normalized size = 0.85

$$\frac{5\left(2\text{Shi}\left(\tanh^{-1}(ax)\right) + 3\text{Shi}\left(3\tanh^{-1}(ax)\right) + \text{Shi}\left(5\tanh^{-1}(ax)\right)\right) - \frac{16}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]) + 5*(2*SinhIntegral[ArcTanh[a*x]] +
 3*SinhIntegral[3*ArcTanh[a*x]] + SinhIntegral[5*ArcTanh[a*x]]))/(16*a)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^8x^8 - 4\,a^6x^6 + 6\,a^4x^4 - 4\,a^2x^2 + 1\right)\operatorname{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}}\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)
```

maple [B] time = 0.53, size = 176, normalized size = 2.67

$$10\operatorname{arctanh}(ax)\,\text{Shi}(\operatorname{arctanh}(ax))\,x^2a^2 + 15\operatorname{arctanh}(ax)\,\text{Shi}(3\operatorname{arctanh}(ax))\,x^2a^2 + 5\operatorname{arctanh}(ax)\,\text{Shi}(5\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/16/a*(10*arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2+15*arctanh(a*x)*Shi(3*arc
tanh(a*x))*x^2*a^2+5*arctanh(a*x)*Shi(5*arctanh(a*x))*x^2*a^2-5*cosh(3*arct
anh(a*x))*x^2*a^2-cosh(5*arctanh(a*x))*x^2*a^2-10*Shi(arctanh(a*x))*arctanh
(a*x)-15*Shi(3*arctanh(a*x))*arctanh(a*x)-5*Shi(5*arctanh(a*x))*arctanh(a*x
)+10*(-a^2*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x))+cosh(5*arctanh(a*x)))/arctan
h(a*x)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}}\operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(7/2)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)**2), x)
```

3.490 $\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\text{Shi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{63\text{Shi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{35\text{Shi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Shi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

```
[Out] -1/a/(-a^2*x^2+1)^(7/2)/arctanh(a*x)+35/64*Shi(arctanh(a*x))/a+63/64*Shi(3*
arctanh(a*x))/a+35/64*Shi(5*arctanh(a*x))/a+7/64*Shi(7*arctanh(a*x))/a
```

Rubi [A] time = 0.19, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\text{Shi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{63\text{Shi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{35\text{Shi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Shi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x])) + (35*SinhIntegral[ArcTanh[a*x]])
/(64*a) + (63*SinhIntegral[3*ArcTanh[a*x]])/(64*a) + (35*SinhIntegral[5*Arc
Tanh[a*x]])/(64*a) + (7*SinhIntegral[7*ArcTanh[a*x]])/(64*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + (7a)\int \frac{x}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{7\,\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{7\,\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64x} + \frac{9\sinh(3x)}{64x} + \frac{5\sinh(5x)}{64x} + \right.\right.}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{7\,\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{64a} + \frac{35\,\text{S}}{}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{35\text{Shi}\left(\tanh^{-1}(ax)\right)}{64a} + \frac{63\text{Shi}\left(3\tanh^{-1}(ax)\right)}{64a}$$

Mathematica [A] time = 0.22, size = 65, normalized size = 0.81

$$\frac{7\left(5\text{Shi}\left(\tanh^{-1}(ax)\right) + 9\text{Shi}\left(3\tanh^{-1}(ax)\right) + 5\text{Shi}\left(5\tanh^{-1}(ax)\right) + \text{Shi}\left(7\tanh^{-1}(ax)\right)\right) - \frac{64}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-64/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]) + 7*(5*SinhIntegral[ArcTanh[a*x]] +
 9*SinhIntegral[3*ArcTanh[a*x]] + 5*SinhIntegral[5*ArcTanh[a*x]] + SinhInte
gral[7*ArcTanh[a*x]]))/(64*a)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^{10}x^{10} - 5\,a^8x^8 + 10\,a^6x^6 - 10\,a^4x^4 + 5\,a^2x^2 - 1\right)\text{artanh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)
```

maple [B] time = 0.62, size = 232, normalized size = 2.90

$$\frac{35\,\text{arctanh}\,(ax)\,\text{Shi}\,(\text{arctanh}\,(ax))\,x^2a^2 + 63\,\text{arctanh}\,(ax)\,\text{Shi}\,(3\,\text{arctanh}\,(ax))\,x^2a^2 + 35\,\text{arctanh}\,(ax)\,\text{Shi}\,(5\,\text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/64/a*(35*arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2+63*arctanh(a*x)*Shi(3*arc
tanh(a*x))*x^2*a^2+35*arctanh(a*x)*Shi(5*arctanh(a*x))*x^2*a^2+7*arctanh(a*
x)*Shi(7*arctanh(a*x))*x^2*a^2-21*cosh(3*arctanh(a*x))*x^2*a^2-7*cosh(5*arc
tanh(a*x))*x^2*a^2-cosh(7*arctanh(a*x))*x^2*a^2-35*Shi(arctanh(a*x))*arctan
h(a*x)-63*Shi(3*arctanh(a*x))*arctanh(a*x)-35*Shi(5*arctanh(a*x))*arctanh(a
*x)-7*Shi(7*arctanh(a*x))*arctanh(a*x)+35*(-a^2*x^2+1)^(1/2)+21*cosh(3*arct
anh(a*x))+7*cosh(5*arctanh(a*x))+cosh(7*arctanh(a*x)))/arctanh(a*x)/(a^2*x^
2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}} \operatorname{artanh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}(a\,x)^2\,\left(1-a^2\,x^2\right)^{9/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(9/2)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(9/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-(ax-1)(ax+1)\right)^{\frac{9}{2}} \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)**2), x)
```

3.491 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx = \int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Mathematica [A] time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\text{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\text{artanh}\,(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

maple [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-a^2\,x^2}}{\operatorname{atanh}(a\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^3,x)
[Out] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**3, x)
```

3.492 $\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\,dx=\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\,dx$$

Mathematica [A] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

maple [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\ \operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}(a\,x)^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\ \operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3), x)
```

3.493 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] 1/2*Chi(arctanh(a*x))/a-1/2/a/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2*x/arcta
nh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5966, 6006, 5968, 3301}

$$-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} + \frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{1}{2}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tan}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x\right.}{2a}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Mathematica [A] time = 0.12, size = 44, normalized size = 0.68

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right) - \frac{ax\tanh^{-1}(ax)+1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegra
l[ArcTanh[a*x]])/(2*a)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

maple [A] time = 0.34, size = 86, normalized size = 1.32

$$\frac{\text{arctanh}\,(ax)^2\,\mathrm{X}\,(\text{arctanh}\,(ax))\,x^2a^2 + \sqrt{-a^2x^2+1}\;ax\,\text{arctanh}\,(ax) - \mathrm{X}\,(\text{arctanh}\,(ax))\,\text{arctanh}\,(ax)^2 + \sqrt{-a^2x^2+}}{2a\,\text{arctanh}\,(ax)^2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+(-a^2*x^2+1)^(1/2)*a*x*arct
anh(a*x)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x)^
2/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

3.494 $\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=79

$$-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+\frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{9\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2-3/2*x/(-a^2*x^2+1)^(3/2)/arctanh(a
*x)+3/8*Chi(arctanh(a*x))/a+9/8*Chi(3*arctanh(a*x))/a
```

Rubi [A] time = 0.36, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+\frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{9\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) - (3*x)/(2*(1 - a^2*x^2)^(3/2)*
ArcTanh[a*x]) + (3*CoshIntegral[ArcTanh[a*x]])/(8*a) + (9*CoshIntegral[3*Ar
cTanh[a*x]])/(8*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
```

```
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+\frac{1}{2}(3a)\int \frac{x}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3}{2}\int \frac{\quad}{\left(1-a^2x\right.}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\,\mathrm{Subst}\left(\int \frac{\mathrm{co}}{}\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\,\mathrm{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\,\mathrm{Subst}\left(\int \frac{\mathrm{co}}{}\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\mathrm{Chi}\left(\tanh^{-1}\right.}{8a}$$

Mathematica [A] time = 0.22, size = 56, normalized size = 0.71

$$\frac{-\frac{4\left(3ax\tanh^{-1}(ax)+1\right)}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)+9\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```

```
[Out] ((-4*(1 + 3*a*x*ArcTanh[a*x]))/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) + 3*Cos
hIntegral[ArcTanh[a*x]] + 9*CoshIntegral[3*ArcTanh[a*x]])/(8*a)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\text{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\,\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

maple [B] time = 0.46, size = 180, normalized size = 2.28

$$\frac{3\,\text{arctanh}\,(ax)^2\,\text{X}\,(\text{arctanh}\,(ax))\,x^2a^2+9\,\text{arctanh}\,(ax)^2\,\text{X}\,(3\,\text{arctanh}\,(ax))\,x^2a^2-3\,\text{arctanh}\,(ax)\,\text{sinh}\,(3\,\text{arctanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/8/a*(3*arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+9*arctanh(a*x)^2*Chi(3*ar
ctanh(a*x))*x^2*a^2-3*arctanh(a*x)*sinh(3*arctanh(a*x))*x^2*a^2-cosh(3*arct
anh(a*x))*x^2*a^2+3*(-a^2*x^2+1)^(1/2)*a*x*arctanh(a*x)-3*Chi(arctanh(a*x))
*arctanh(a*x)^2-9*Chi(3*arctanh(a*x))*arctanh(a*x)^2+3*sinh(3*arctanh(a*x))
*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctanh(a*x)))/arctanh(a*x)^2/(a^
2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\,\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\text{atanh}\,(a\,x)^3\,(1-a^2\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)**3), x)
```

3.495 $\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=93

$$-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{16a}+\frac{45\text{Chi}\left(3\tanh^{-1}(ax)\right)}{32a}+\frac{25\text{Chi}\left(5\tanh^{-1}(ax)\right)}{32a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2-5/2*x/(-a^2*x^2+1)^(5/2)/arctanh(a
*x)+5/16*Chi(arctanh(a*x))/a+45/32*Chi(3*arctanh(a*x))/a+25/32*Chi(5*arctan
h(a*x))/a
```

Rubi [A] time = 0.40, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{16a}+\frac{45\text{Chi}\left(3\tanh^{-1}(ax)\right)}{32a}+\frac{25\text{Chi}\left(5\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (5*x)/(2*(1 - a^2*x^2)^(5/2)*
ArcTanh[a*x]) + (5*CoshIntegral[ArcTanh[a*x]])/(16*a) + (45*CoshIntegral[3*
ArcTanh[a*x]])/(32*a) + (25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
```

```
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+\frac{1}{2}(5a)\int \frac{x}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5}{2}\int \frac{}{\left(1-a^2x\right.}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\,\mathrm{Subst}\left(\int \frac{\mathrm{co}}{}\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\,\mathrm{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\,\mathrm{Subst}\left(\int \frac{\mathrm{co}}{}\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\mathrm{Chi}\left(\tanh^{-1}\right.}{16a}$$

Mathematica [A] time = 0.25, size = 79, normalized size = 0.85

$$\frac{-\frac{80ax}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}-\frac{16}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+10\mathrm{Chi}\left(\tanh^{-1}(ax)\right)+45\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)+25\mathrm{Chi}\left(5\tanh^{-1}(a\right.}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (80*a*x)/((1 - a^2*x^2)^(5/2)*A
rcTanh[a*x]) + 10*CoshIntegral[ArcTanh[a*x]] + 45*CoshIntegral[3*ArcTanh[a*
x]] + 25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^8x^8-4\,a^6x^6+6\,a^4x^4-4\,a^2x^2+1\right)\text{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}}\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

maple [B] time = 0.56, size = 272, normalized size = 2.92

$$\frac{10\,\text{arctanh}\,(ax)^2\,\text{X}\,(\text{arctanh}\,(ax))\,x^2a^2+45\,\text{arctanh}\,(ax)^2\,\text{X}\,(3\,\text{arctanh}\,(ax))\,x^2a^2+25\,\text{arctanh}\,(ax)^2\,\text{X}\,(5\,\text{arctan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/32/a*(10*arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+45*arctanh(a*x)^2*Chi(3
*arctanh(a*x))*x^2*a^2+25*arctanh(a*x)^2*Chi(5*arctanh(a*x))*x^2*a^2-15*arc
tanh(a*x)*sinh(3*arctanh(a*x))*x^2*a^2-5*arctanh(a*x)*sinh(5*arctanh(a*x))*
x^2*a^2-5*cosh(3*arctanh(a*x))*x^2*a^2-cosh(5*arctanh(a*x))*x^2*a^2+10*(-a^
2*x^2+1)^(1/2)*a*x*arctanh(a*x)-10*Chi(arctanh(a*x))*arctanh(a*x)^2-45*Chi(
3*arctanh(a*x))*arctanh(a*x)^2-25*Chi(5*arctanh(a*x))*arctanh(a*x)^2+15*sin
h(3*arctanh(a*x))*arctanh(a*x)+5*sinh(5*arctanh(a*x))*arctanh(a*x)+10*(-a^2
*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x))+cosh(5*arctanh(a*x)))/arctanh(a*x)^2/(
a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}}\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\text{atanh}\,(a\,x)^3\,\left(1-a^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(7/2)),x)
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**3,x)
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)**3), x)
```

3.496 $\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{128a}+\frac{189\text{Chi}\left(3\tanh^{-1}(ax)\right)}{128a}+\frac{175\text{Cl}}{}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2-7/2*x/(-a^2*x^2+1)^(7/2)/arctanh(a
*x)+35/128*Chi(arctanh(a*x))/a+189/128*Chi(3*arctanh(a*x))/a+175/128*Chi(5*
arctanh(a*x))/a+49/128*Chi(7*arctanh(a*x))/a
```

Rubi [A] time = 0.44, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{128a}+\frac{189\text{Chi}\left(3\tanh^{-1}(ax)\right)}{128a}+\frac{175\text{Cl}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (7*x)/(2*(1 - a^2*x^2)^(7/2)*
ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/(128*a) + (189*CoshIntegral
[3*ArcTanh[a*x]])/(128*a) + (175*CoshIntegral[5*ArcTanh[a*x]])/(128*a) + (4
9*CoshIntegral[7*ArcTanh[a*x]])/(128*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
```

```
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{1}{2}(7a)\int \frac{x}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7}{2}\int \frac{}{\left(1-a^2x\right.}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7\,\mathrm{Subst}\left(\int \frac{\mathrm{co}}{}\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7\,\mathrm{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7\,\mathrm{Subst}\left(\int \frac{\mathrm{co}}{}\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\mathrm{Chi}\left(\tanh\right.}{128a}$$

Mathematica [A] time = 0.23, size = 99, normalized size = 0.93

$$\frac{1}{128}\left(-\frac{448x}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}-\frac{64}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{35\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}+\frac{189\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-64/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (448*x)/((1 - a^2*x^2)^(7/2)*
ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/a + (189*CoshIntegral[3*Arc
Tanh[a*x]])/a + (175*CoshIntegral[5*ArcTanh[a*x]])/a + (49*CoshIntegral[7*A
rcTanh[a*x]])/a)/128
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^{10}x^{10}-5\,a^8x^8+10\,a^6x^6-10\,a^4x^4+5\,a^2x^2-1\right)\text{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

maple [B] time = 0.68, size = 364, normalized size = 3.40

$$\frac{35\,\text{arctanh}\left(ax\right)^2 \text{X}\left(\text{arctanh}\left(ax\right)\right)x^2a^2+189\,\text{arctanh}\left(ax\right)^2 \text{X}\left(3\,\text{arctanh}\left(ax\right)\right)x^2a^2+175\,\text{arctanh}\left(ax\right)^2 \text{X}\left(5\,\text{arct}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/128/a*(35*arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+189*arctanh(a*x)^2*Chi
(3*arctanh(a*x))*x^2*a^2+175*arctanh(a*x)^2*Chi(5*arctanh(a*x))*x^2*a^2+49*
arctanh(a*x)^2*Chi(7*arctanh(a*x))*x^2*a^2-63*arctanh(a*x)*sinh(3*arctanh(a
*x))*x^2*a^2-35*arctanh(a*x)*sinh(5*arctanh(a*x))*x^2*a^2-7*arctanh(a*x)*si
nh(7*arctanh(a*x))*x^2*a^2-21*cosh(3*arctanh(a*x))*x^2*a^2-7*cosh(5*arctanh
(a*x))*x^2*a^2-cosh(7*arctanh(a*x))*x^2*a^2+35*(-a^2*x^2+1)^(1/2)*a*x*arcta
nh(a*x)-35*Chi(arctanh(a*x))*arctanh(a*x)^2-189*Chi(3*arctanh(a*x))*arctanh
(a*x)^2-175*Chi(5*arctanh(a*x))*arctanh(a*x)^2-49*Chi(7*arctanh(a*x))*arcta
nh(a*x)^2+63*sinh(3*arctanh(a*x))*arctanh(a*x)+35*sinh(5*arctanh(a*x))*arct
anh(a*x)+7*sinh(7*arctanh(a*x))*arctanh(a*x)+35*(-a^2*x^2+1)^(1/2)+21*cosh(
3*arctanh(a*x))+7*cosh(5*arctanh(a*x))+cosh(7*arctanh(a*x)))/arctanh(a*x)^2
/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\text{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(9/2)),x)
```

```
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(9/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)**3), x)
```

3.497 $\int \frac{(d+ex)\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=122

$$\frac{be\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^2}+\frac{d\left(a+b\tanh^{-1}(cx\right.}{3bc}$$

```
[Out] 1/3*d*(a+b*arctanh(c*x))^3/b/c-1/3*e*(a+b*arctanh(c*x))^3/b/c^2+e*(a+b*arct
anh(c*x))^2*ln(2/(-c*x+1))/c^2+b*e*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1
))/c^2-1/2*b^2*e*polylog(3,1-2/(-c*x+1))/c^2
```

Rubi [A] time = 0.32, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6048, 5948, 5984, 5918, 6058, 6610}

$$\frac{be\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}-\frac{b^2e\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^2}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\log\left(\frac{2}{1-cx}\right)\left(a+b\right.}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```

```
[Out] (d*(a + b*ArcTanh[c*x])^3)/(3*b*c) - (e*(a + b*ArcTanh[c*x])^3)/(3*b*c^2) +
 (e*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c^2 + (b*e*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 - c*x)])/c^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2
*c^2)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
```

```
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}+\frac{ex\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx + e\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-cx}\,dx}{c}$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)^2 \log}{c^2}$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)^2 \log}{c^2}$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)^2 \log}{c^2}$$

Mathematica [A] time = 0.41, size = 193, normalized size = 1.58

$$-3a^2cd\log(1-cx)+3a^2cd\log(cx+1)-3a^2e\log(1-cx)-3a^2e\log(cx+1)+6abcd\tanh^{-1}(cx)^2-6be\mathrm{Li}_2\left(-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```

```
[Out] (6*a*b*c*d*ArcTanh[c*x]^2 + 6*a*b*e*ArcTanh[c*x]^2 + 2*b^2*c*d*ArcTanh[c*x]
^3 + 2*b^2*e*ArcTanh[c*x]^3 + 12*a*b*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*b^2*e*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 3*a^2*c*d*Log
[1 - c*x] - 3*a^2*e*Log[1 - c*x] + 3*a^2*c*d*Log[1 + c*x] - 3*a^2*e*Log[1 +
 c*x] - 6*b*e*(a + b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 3*b^2
*e*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(6*c^2)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{a^2ex+a^2d+\left(b^2ex+b^2d\right)\mathrm{artanh}\,(cx)^2+2\,(abex+abd)\,\mathrm{artanh}\,(cx)}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*e*x
+ a*b*d)*arctanh(c*x))/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ex+d)(b\,\text{artanh}\,(cx)+a)^2}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(e*x + d)*(b*arctanh(c*x) + a)^2/(c^2*x^2 - 1), x)
```

maple [C] time = 0.70, size = 1871, normalized size = 15.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x)
```

```
[Out] -1/2*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^2*Pi*d+1/4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*d-1/4*
I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+
1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*d-1/4*I/c*b^2*arctanh(c*x)^2*c
sgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*d+1/4*
I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1
)^2/(c^2*x^2-1))*Pi*e+1/2*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi*e-1/4*I/c^2*b^2*arctanh(c*x)^
2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(
-c^2*x^2+1)))^2*Pi*e+1/4*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*e-1
/c^2*a*b*arctanh(c*x)*ln(c*x+1)*e-1/2/c^2*a*b*ln(c*x+1)*ln(-1/2*c*x+1/2)*e+
1/2/c^2*a*b*ln(c*x-1)*ln(1/2+1/2*c*x)*e+1/c*a*b*arctanh(c*x)*ln(c*x+1)*d+1/
2/c*a*b*ln(c*x-1)*ln(1/2+1/2*c*x)*d+1/2/c*a*b*ln(c*x+1)*ln(-1/2*c*x+1/2)*d-
1/2/c*a*b*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*d-1/c*a*b*arctanh(c*x)*ln(c*x-1)
*d+1/4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^
2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)*Pi*d-1/4*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x
^2+1)))*Pi*e-1/2/c^2*b^2*e*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/2/c^2*a^2*l
n(c*x-1)*e+1/3/c*b^2*d*arctanh(c*x)^3+1/2/c*a^2*ln(c*x+1)*d-1/2/c*a^2*ln(c*
x-1)*d+1/2*I/c^2*b^2*arctanh(c*x)^2*Pi*e+1/2*I/c*b^2*arctanh(c*x)^2*Pi*d-1/
c^2*a*b*arctanh(c*x)*ln(c*x-1)*e+1/2/c^2*a*b*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/
2)*e-1/3/c^2*b^2*e*arctanh(c*x)^3-1/2/c^2*a^2*ln(c*x+1)*e-1/4/c*a*b*ln(c*x+
1)^2*d+1/4*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1)))^3*Pi*e+1/2*I/c^2*b^2*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(
-c^2*x^2+1)))^3*Pi*e+1/2*I/c*b^2*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x
^2+1)))^3*Pi*d-1/2*I/c*b^2*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)^2*Pi*d-1/4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*d-1/
4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^3*Pi*d-1/2*I/c^2*b^2*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)
))^2*Pi*e+1/4*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e
-1/2/c*b^2*arctanh(c*x)^2*ln(c*x-1)*d+1/2/c*b^2*arctanh(c*x)^2*ln(c*x+1)*d-
1/2/c^2*b^2*arctanh(c*x)^2*ln(c*x+1)*e-1/4/c*a*b*ln(c*x-1)^2*d-1/4/c^2*a*b*
ln(c*x-1)^2*e+1/c^2*b^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))*e+1/c
^2*b^2*e*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/2/c^2*b^2*arctan
h(c*x)^2*ln(c*x-1)*e+1/c^2*a*b*dilog(1/2+1/2*c*x)*e+1/c^2*b^2*arctanh(c*x)^
2*ln(2)*e+1/4/c^2*a*b*ln(c*x+1)^2*e-1/c*b^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2
*x^2+1)^(1/2))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$abd\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\operatorname{artanh}(cx)+\frac{1}{2}a^2d\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)-\frac{\left(\log(cx+1)^2-2\log(cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] a*b*d*(log(c*x + 1)/c - log(c*x - 1)/c)*arctanh(c*x) + 1/2*a^2*d*(log(c*x +
 1)/c - log(c*x - 1)/c) - 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1)
 + log(c*x - 1)^2)*a*b*d/c - 1/2*a^2*e*log(c^2*x^2 - 1)/c^2 + 1/24*(3*(c*d
- e)*b^2*log(c*x + 1)*log(-c*x + 1)^2 - (c*d + e)*b^2*log(-c*x + 1)^3)/c^2
- integrate(1/4*(4*a*b*c*e*x*log(c*x + 1) + (b^2*c*e*x + b^2*c*d)*log(c*x +
 1)^2 - (4*a*b*c*e*x - ((c^2*d - 3*c*e)*b^2*x - (c*d + e)*b^2)*log(c*x + 1)
)*log(-c*x + 1))/(c^3*x^2 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+e\,x)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a + b*atanh(c*x))^2*(d + e*x))/(c^2*x^2 - 1),x)
```

```
[Out] int(-((a + b*atanh(c*x))^2*(d + e*x))/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2d}{c^2x^2-1}\,dx-\int\frac{a^2ex}{c^2x^2-1}\,dx-\int\frac{b^2d\operatorname{atanh}^2(cx)}{c^2x^2-1}\,dx-\int\frac{2abd\operatorname{atanh}(cx)}{c^2x^2-1}\,dx-\int\frac{b^2ex\operatorname{atanh}^2(cx)}{c^2x^2-1}\,dx-\int\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2*d/(c**2*x**2 - 1), x) - Integral(a**2*e*x/(c**2*x**2 - 1), x
) - Integral(b**2*d*atanh(c*x)**2/(c**2*x**2 - 1), x) - Integral(2*a*b*d*at
anh(c*x)/(c**2*x**2 - 1), x) - Integral(b**2*e*x*atanh(c*x)**2/(c**2*x**2 -
 1), x) - Integral(2*a*b*e*x*atanh(c*x)/(c**2*x**2 - 1), x)
```

3.498 $\int \left(c+dx^2\right)^4 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=245

$$\frac{d^3x^6\left(36a^2c+7d\right)}{378a^3}+\frac{d^2x^4\left(378a^4c^2+180a^2cd+35d^2\right)}{1260a^5}+\frac{dx^2\left(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3\right)}{630a^7}+\frac{\left(315a^8c^4\right.}{}$$

```
[Out] 1/630*d*(420*a^6*c^3+378*a^4*c^2*d+180*a^2*c*d^2+35*d^3)*x^2/a^7+1/1260*d^2
*(378*a^4*c^2+180*a^2*c*d+35*d^2)*x^4/a^5+1/378*d^3*(36*a^2*c+7*d)*x^6/a^3+
1/72*d^4*x^8/a+c^4*x*arctanh(a*x)+4/3*c^3*d*x^3*arctanh(a*x)+6/5*c^2*d^2*x^
5*arctanh(a*x)+4/7*c*d^3*x^7*arctanh(a*x)+1/9*d^4*x^9*arctanh(a*x)+1/630*(3
15*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4)*ln(-a^2*x^2+
1)/a^9
```

Rubi [A] time = 0.18, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {194, 5976, 1810, 260}

$$\frac{d^2x^4\left(378a^4c^2+180a^2cd+35d^2\right)}{1260a^5}+\frac{dx^2\left(378a^4c^2d+420a^6c^3+180a^2cd^2+35d^3\right)}{630a^7}+\frac{\left(378a^4c^2d^2+420a^6c^3d+315\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcTanh[a*x],x]
```

```
[Out] (d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + (d^3*(36*a^2*c
+ 7*d)*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcTanh[a*x] + (4*c^3*d*x^
3*ArcTanh[a*x])/3 + (6*c^2*d^2*x^5*ArcTanh[a*x])/5 + (4*c*d^3*x^7*ArcTanh[a
*x])/7 + (d^4*x^9*ArcTanh[a*x])/9 + ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(630*a^9)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \tanh^{-1}(ax)\,dx = c^4x\tanh^{-1}(ax) + \frac{4}{3}c^3dx^3\tanh^{-1}(ax) + \frac{6}{5}c^2d^2x^5\tanh^{-1}(ax) + \frac{4}{7}cd^3x^7\tanh^{-1}(a$$

$$= c^4x\tanh^{-1}(ax) + \frac{4}{3}c^3dx^3\tanh^{-1}(ax) + \frac{6}{5}c^2d^2x^5\tanh^{-1}(ax) + \frac{4}{7}cd^3x^7\tanh^{-1}(a$$

$$= \frac{d\left(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3\right)x^2}{630a^7} + \frac{d^2\left(378a^4c^2+180a^2cd+35d^2\right.}{1260a^5}$$

$$= \frac{d\left(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3\right)x^2}{630a^7} + \frac{d^2\left(378a^4c^2+180a^2cd+35d^2\right.}{1260a^5}$$

Mathematica [A] time = 0.12, size = 213, normalized size = 0.87

$$\frac{24a^9x\tanh^{-1}(ax)\left(315c^4+420c^3dx^2+378c^2d^2x^4+180cd^3x^6+35d^4x^8\right)+a^2dx^2\left(3a^6\left(1680c^3+756c^2dx^2+2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 270
*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 +
35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*
d^3*x^6 + 35*d^4*x^8)*ArcTanh[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*
a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

fricas [A] time = 0.49, size = 248, normalized size = 1.01

$$\frac{105\,a^8d^4x^8+20\left(36\,a^8cd^3+7\,a^6d^4\right)x^6+6\left(378\,a^8c^2d^2+180\,a^6cd^3+35\,a^4d^4\right)x^4+12\left(420\,a^8c^3d+378\,a^6c^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 + 7*a^6*d^4)*x^6 + 6*(378*a^8*c^
2*d^2 + 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d + 378*a^6*c^2*d
^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^2 + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 37
8*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 - 1) + 12*(35*a^9*d^4*x
^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*
c^4*x)*log(-(a*x + 1)/(a*x - 1)))/a^9
```

giac [B] time = 1.24, size = 1471, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/945*a*(3*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 +
 35*d^4)*log(abs(-a*x - 1)/abs(a*x - 1))/a^10 - 3*(315*a^8*c^4 + 420*a^6*c^
3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(abs(-(a*x + 1)/(a*x - 1
) + 1))/a^10 + 8*(3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 + 35*d
^4)*(a*x + 1)^7/(a*x - 1)^7 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*
d^3 + 7*d^4)*(a*x + 1)^6/(a*x - 1)^6 + (4725*a^6*c^3*d + 6237*a^4*c^2*d^2 +
 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^5/(a*x - 1)^5 - 2*(3150*a^6*c^3*d + 39
69*a^4*c^2*d^2 + 2340*a^2*c*d^3 + 455*d^4)*(a*x + 1)^4/(a*x - 1)^4 + (4725*
```

```
a^6*c^3*d + 6237*a^4*c^2*d^2 + 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^3/(a*x -
 1)^3 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*d^3 + 7*d^4)*(a*x + 1)
^2/(a*x - 1)^2 + 3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 + 35*d^
4)*(a*x + 1)/(a*x - 1))/(a^10*((a*x + 1)/(a*x - 1) - 1)^8) + 3*(315*(a*x +
1)^8*a^8*c^4/(a*x - 1)^8 - 2520*(a*x + 1)^7*a^8*c^4/(a*x - 1)^7 + 8820*(a*x
 + 1)^6*a^8*c^4/(a*x - 1)^6 - 17640*(a*x + 1)^5*a^8*c^4/(a*x - 1)^5 + 22050
*(a*x + 1)^4*a^8*c^4/(a*x - 1)^4 - 17640*(a*x + 1)^3*a^8*c^4/(a*x - 1)^3 +
8820*(a*x + 1)^2*a^8*c^4/(a*x - 1)^2 - 2520*(a*x + 1)*a^8*c^4/(a*x - 1) + 3
15*a^8*c^4 + 1260*(a*x + 1)^8*a^6*c^3*d/(a*x - 1)^8 - 7560*(a*x + 1)^7*a^6*
c^3*d/(a*x - 1)^7 + 19320*(a*x + 1)^6*a^6*c^3*d/(a*x - 1)^6 - 27720*(a*x +
1)^5*a^6*c^3*d/(a*x - 1)^5 + 25200*(a*x + 1)^4*a^6*c^3*d/(a*x - 1)^4 - 1596
0*(a*x + 1)^3*a^6*c^3*d/(a*x - 1)^3 + 7560*(a*x + 1)^2*a^6*c^3*d/(a*x - 1)^
2 - 2520*(a*x + 1)*a^6*c^3*d/(a*x - 1) + 420*a^6*c^3*d + 1890*(a*x + 1)^8*a
^4*c^2*d^2/(a*x - 1)^8 - 7560*(a*x + 1)^7*a^4*c^2*d^2/(a*x - 1)^7 + 15120*(
a*x + 1)^6*a^4*c^2*d^2/(a*x - 1)^6 - 22680*(a*x + 1)^5*a^4*c^2*d^2/(a*x - 1
)^5 + 24948*(a*x + 1)^4*a^4*c^2*d^2/(a*x - 1)^4 - 16632*(a*x + 1)^3*a^4*c^2
*d^2/(a*x - 1)^3 + 6048*(a*x + 1)^2*a^4*c^2*d^2/(a*x - 1)^2 - 1512*(a*x + 1
)*a^4*c^2*d^2/(a*x - 1) + 378*a^4*c^2*d^2 + 1260*(a*x + 1)^8*a^2*c*d^3/(a*x
 - 1)^8 - 2520*(a*x + 1)^7*a^2*c*d^3/(a*x - 1)^7 + 7560*(a*x + 1)^6*a^2*c*d
^3/(a*x - 1)^6 - 12600*(a*x + 1)^5*a^2*c*d^3/(a*x - 1)^5 + 10080*(a*x + 1)^
4*a^2*c*d^3/(a*x - 1)^4 - 7560*(a*x + 1)^3*a^2*c*d^3/(a*x - 1)^3 + 3960*(a*
x + 1)^2*a^2*c*d^3/(a*x - 1)^2 - 360*(a*x + 1)*a^2*c*d^3/(a*x - 1) + 180*a^
2*c*d^3 + 315*(a*x + 1)^8*d^4/(a*x - 1)^8 + 2940*(a*x + 1)^6*d^4/(a*x - 1)^
6 + 4410*(a*x + 1)^4*d^4/(a*x - 1)^4 + 1260*(a*x + 1)^2*d^4/(a*x - 1)^2 + 3
5*d^4)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(
a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^10*((a*x +
 1)/(a*x - 1) - 1)^9))
```

maple [A] time = 0.03, size = 334, normalized size = 1.36

$$\frac{6c^2d^2x^5\operatorname{arctanh}(ax)}{5}+\frac{4c\,d^3x^7\operatorname{arctanh}(ax)}{7}+\frac{2c\,d^3x^6}{21a}+\frac{2x^2c\,d^3}{7a^5}+\frac{3c^2d^2x^4}{10a}+\frac{2c^3d\,x^2}{3a}+\frac{4c^3d\,x^3\operatorname{arctanh}(ax)}{3}+\frac{x^4c\,d^3}{7a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arctanh(a*x),x)
```

```
[Out] 2/21/a*c*d^3*x^6+2/7/a^5*x^2*c*d^3+3/10/a*c^2*d^2*x^4+2/3/a*c^3*d*x^2+1/7/a
^3*x^4*c*d^3+2/3/a^3*ln(a*x-1)*c^3*d+3/5/a^3*c^2*d^2*x^2+2/7/a^7*ln(a*x-1)*
c*d^3+2/3/a^3*ln(a*x+1)*c^3*d+3/5/a^5*ln(a*x+1)*c^2*d^2+2/7/a^7*ln(a*x+1)*c
*d^3+3/5/a^5*ln(a*x-1)*c^2*d^2+1/36/a^5*x^4*d^4+1/54/a^3*x^6*d^4+1/2/a*ln(a
*x+1)*c^4+1/18/a^9*ln(a*x+1)*d^4+1/2/a*ln(a*x-1)*c^4+1/18/a^9*ln(a*x-1)*d^4
+1/18/a^7*x^2*d^4+c^4*x*arctanh(a*x)+1/72*d^4*x^8/a+1/9*d^4*x^9*arctanh(a*x
)+4/3*c^3*d*x^3*arctanh(a*x)+6/5*c^2*d^2*x^5*arctanh(a*x)+4/7*c*d^3*x^7*arc
tanh(a*x)
```

maxima [A] time = 0.31, size = 276, normalized size = 1.13

$$\frac{1}{7560}a\left(\frac{105\,a^6d^4x^8+20\left(36\,a^6cd^3+7\,a^4d^4\right)x^6+6\left(378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4\right)x^4+12\left(420\,a^6c^3d+378\,a^4\right.}{a^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^6
*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*c^
2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315*a^8
*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x -
```

```
1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*
x^3 + 315*c^4*x)*arctanh(a*x)
```

mupad [B] time = 1.28, size = 288, normalized size = 1.18

$$x^2\left(\frac{\frac{\frac{d^4}{9a^3}+\frac{4cd^3}{7a}}{a^2}+\frac{6c^2d^2}{5a}}{2a^2}+\frac{2c^3d}{3a}\right)+x^6\left(\frac{d^4}{54a^3}+\frac{2cd^3}{21a}\right)+\ln(ax+1)\left(\frac{c^4x}{2}+\frac{2c^3dx^3}{3}+\frac{3c^2d^2x^5}{5}+\frac{2cd^3x^7}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^4,x)
```

```
[Out] x^2*(((d^4/(9*a^3) + (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a))/(2*a^2) + (2
*c^3*d)/(3*a)) + x^6*(d^4/(54*a^3) + (2*c*d^3)/(21*a)) + log(a*x + 1)*((c^4
*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^7)/7 + (3*c^2*d^2*x^5)/
5) - log(1 - a*x)*((c^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^
7)/7 + (3*c^2*d^2*x^5)/5) + x^4*((d^4/(9*a^3) + (4*c*d^3)/(7*a))/(4*a^2) +
(3*c^2*d^2)/(10*a)) + (log(a^2*x^2 - 1)*(35*d^4 + 315*a^8*c^4 + 180*a^2*c*d
^3 + 420*a^6*c^3*d + 378*a^4*c^2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

sympy [A] time = 5.55, size = 372, normalized size = 1.52

$$\begin{cases} c^4x\,\mathrm{atanh}(ax)+\frac{4c^3dx^3\,\mathrm{atanh}(ax)}{3}+\frac{6c^2d^2x^5\,\mathrm{atanh}(ax)}{5}+\frac{4cd^3x^7\,\mathrm{atanh}(ax)}{7}+\frac{d^4x^9\,\mathrm{atanh}(ax)}{9}+\frac{c^4\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^4\,\mathrm{atanh}(ax)}{a}+ \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*atanh(a*x),x)
```

```
[Out] Piecewise((c**4*x*atanh(a*x) + 4*c**3*d*x**3*atanh(a*x)/3 + 6*c**2*d**2*x**
5*atanh(a*x)/5 + 4*c*d**3*x**7*atanh(a*x)/7 + d**4*x**9*atanh(a*x)/9 + c**4
*log(x - 1/a)/a + c**4*atanh(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**
4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1/a)/
(3*a**3) + 4*c**3*d*atanh(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**
3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a**5) +
 6*c**2*d**2*atanh(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**4/(36*a
**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*atanh(a*x)/(7*a**7) + d**4
*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*atanh(a*x)/(9*a**9), Ne
(a, 0)), (0, True))
```

3.499 $\int \left(c+dx^2\right)^3 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=169

$$\frac{d^2x^4\left(21a^2c+5d\right)}{140a^3}+\frac{dx^2\left(35a^4c^2+21a^2cd+5d^2\right)}{70a^5}+\frac{\left(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3\right)\log\left(1-a^2x^2\right)}{70a^7}+c^3x\tanh$$

```
[Out] 1/70*d*(35*a^4*c^2+21*a^2*c*d+5*d^2)*x^2/a^5+1/140*d^2*(21*a^2*c+5*d)*x^4/a
^3+1/42*d^3*x^6/a+c^3*x*arctanh(a*x)+c^2*d*x^3*arctanh(a*x)+3/5*c*d^2*x^5*a
rctanh(a*x)+1/7*d^3*x^7*arctanh(a*x)+1/70*(35*a^6*c^3+35*a^4*c^2*d+21*a^2*c
*d^2+5*d^3)*ln(-a^2*x^2+1)/a^7
```

Rubi [A] time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {194, 5976, 1810, 260}

$$\frac{dx^2\left(35a^4c^2+21a^2cd+5d^2\right)}{70a^5}+\frac{\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)\log\left(1-a^2x^2\right)}{70a^7}+\frac{d^2x^4\left(21a^2c+5d\right)}{140a^3}+c^2dx^3\tan$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcTanh[a*x] + c^2*d*x^3*ArcTanh[
a*x] + (3*c*d^2*x^5*ArcTanh[a*x])/5 + (d^3*x^7*ArcTanh[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \tanh^{-1}(ax)\,dx = c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}(ax)+\frac{1}{7}d^3x^7\tanh^{-1}(ax)-$$

$$= c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}(ax)+\frac{1}{7}d^3x^7\tanh^{-1}(ax)-$$

$$= \frac{d\left(35a^4c^2+21a^2cd+5d^2\right)x^2}{70a^5}+\frac{d^2\left(21a^2c+5d\right)x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\tanh^{-1}(ax)+$$

$$= \frac{d\left(35a^4c^2+21a^2cd+5d^2\right)x^2}{70a^5}+\frac{d^2\left(21a^2c+5d\right)x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\tanh^{-1}(ax)+$$

Mathematica [A] time = 0.08, size = 150, normalized size = 0.89

$$\frac{12a^7x\tanh^{-1}(ax)\left(35c^3+35c^2dx^2+21cd^2x^4+5d^3x^6\right)+a^2dx^2\left(a^4\left(210c^2+63cdx^2+10d^2x^4\right)+3a^2d\left(42c+5\right.\right.}{420a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcTanh[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1
- a^2*x^2])/(420*a^7)
```

fricas [A] time = 0.52, size = 178, normalized size = 1.05

$$\frac{10\,a^6d^3x^6+3\left(21\,a^6cd^2+5\,a^4d^3\right)x^4+6\left(35\,a^6c^2d+21\,a^4cd^2+5\,a^2d^3\right)x^2+6\left(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+\right.}{420\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
+ 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d
^2 + 5*d^3)*log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7
*c^2*d*x^3 + 35*a^7*c^3*x)*log(-(a*x + 1)/(a*x - 1)))/a^7
```

giac [B] time = 0.25, size = 930, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*log(abs(-a*x
- 1)/abs(a*x - 1))/a^8 - 3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^
3)*log(abs(-(a*x + 1)/(a*x - 1) + 1))/a^8 + 2*(3*(35*a^4*c^2*d + 42*a^2*c*d
^2 + 15*d^3)*(a*x + 1)^5/(a*x - 1)^5 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*
d^3)*(a*x + 1)^4/(a*x - 1)^4 + 2*(315*a^4*c^2*d + 252*a^2*c*d^2 + 85*d^3)*(
a*x + 1)^3/(a*x - 1)^3 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*d^3)*(a*x + 1)
^2/(a*x - 1)^2 + 3*(35*a^4*c^2*d + 42*a^2*c*d^2 + 15*d^3)*(a*x + 1)/(a*x -
1))/(a^8*((a*x + 1)/(a*x - 1) - 1)^6) + 3*(35*(a*x + 1)^6*a^6*c^3/(a*x - 1)
^6 - 210*(a*x + 1)^5*a^6*c^3/(a*x - 1)^5 + 525*(a*x + 1)^4*a^6*c^3/(a*x - 1
)^4 - 700*(a*x + 1)^3*a^6*c^3/(a*x - 1)^3 + 525*(a*x + 1)^2*a^6*c^3/(a*x -
1)^2 - 210*(a*x + 1)*a^6*c^3/(a*x - 1) + 35*a^6*c^3 + 105*(a*x + 1)^6*a^4*c
```

```
^2*d/(a*x - 1)^6 - 420*(a*x + 1)^5*a^4*c^2*d/(a*x - 1)^5 + 665*(a*x + 1)^4*
a^4*c^2*d/(a*x - 1)^4 - 560*(a*x + 1)^3*a^4*c^2*d/(a*x - 1)^3 + 315*(a*x +
1)^2*a^4*c^2*d/(a*x - 1)^2 - 140*(a*x + 1)*a^4*c^2*d/(a*x - 1) + 35*a^4*c^2
*d + 105*(a*x + 1)^6*a^2*c*d^2/(a*x - 1)^6 - 210*(a*x + 1)^5*a^2*c*d^2/(a*x
 - 1)^5 + 315*(a*x + 1)^4*a^2*c*d^2/(a*x - 1)^4 - 420*(a*x + 1)^3*a^2*c*d^2
/(a*x - 1)^3 + 231*(a*x + 1)^2*a^2*c*d^2/(a*x - 1)^2 - 42*(a*x + 1)*a^2*c*d
^2/(a*x - 1) + 21*a^2*c*d^2 + 35*(a*x + 1)^6*d^3/(a*x - 1)^6 + 175*(a*x + 1
)^4*d^3/(a*x - 1)^4 + 105*(a*x + 1)^2*d^3/(a*x - 1)^2 + 5*d^3)*log(-(a*((a*
x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x -
 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^8*((a*x + 1)/(a*x - 1) - 1)^7
))
```

maple [A] time = 0.03, size = 233, normalized size = 1.38

$$\frac{d^3x^7\,\mathrm{arctanh}\,(ax)}{7}+\frac{3c\,d^2x^5\,\mathrm{arctanh}\,(ax)}{5}+c^2d\,x^3\,\mathrm{arctanh}\,(ax)+c^3x\,\mathrm{arctanh}\,(ax)+\frac{d^3x^6}{42a}+\frac{3c\,d^2x^2}{10a^3}+\frac{x^2c^2d}{2a}+\frac{3c\,d^2x^4}{20a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arctanh(a*x),x)
```

```
[Out] 1/7*d^3*x^7*arctanh(a*x)+3/5*c*d^2*x^5*arctanh(a*x)+c^2*d*x^3*arctanh(a*x)+
c^3*x*arctanh(a*x)+1/42*d^3*x^6/a+3/10/a^3*c*d^2*x^2+1/2/a*x^2*c^2*d+3/20/a
*c*d^2*x^4+1/2/a*ln(a*x+1)*c^3+1/2/a^3*ln(a*x+1)*c^2*d+3/10/a^5*ln(a*x+1)*c
*d^2+1/14/a^7*ln(a*x+1)*d^3+1/14/a^5*d^3*x^2+1/28/a^3*x^4*d^3+1/2/a*ln(a*x-
1)*c^3+1/2/a^3*ln(a*x-1)*c^2*d+3/10/a^5*ln(a*x-1)*c*d^2+1/14/a^7*ln(a*x-1)*
d^3
```

maxima [A] time = 0.31, size = 198, normalized size = 1.17

$$\frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\left(21\,a^4cd^2+5\,a^2d^3\right)x^4+6\left(35\,a^4c^2d+21\,a^2cd^2+5\,d^3\right)x^2}{a^6}+\frac{6\left(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2\right.}{a^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 + 5*a^2*d^3)*x^4 + 6*(35*a^4*c^2
*d + 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2
*c*d^2 + 5*d^3)*log(a*x + 1)/a^8 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*
d^2 + 5*d^3)*log(a*x - 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*
x^3 + 35*c^3*x)*arctanh(a*x)
```

mupad [B] time = 1.38, size = 190, normalized size = 1.12

$$c^3\,x\,\mathrm{atanh}\left(a\,x\right)+\frac{d^3\,x^7\,\mathrm{atanh}\left(a\,x\right)}{7}+\frac{c^3\,\ln\left(a^2\,x^2-1\right)}{2\,a}+\frac{d^3\,\ln\left(a^2\,x^2-1\right)}{14\,a^7}+\frac{d^3\,x^6}{42\,a}+\frac{d^3\,x^4}{28\,a^3}+\frac{d^3\,x^2}{14\,a^5}+\frac{c^2\,d\,\ln\left(a^2\,x^2-1\right)}{2\,a^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^3,x)
```

```
[Out] c^3*x*atanh(a*x) + (d^3*x^7*atanh(a*x))/7 + (c^3*log(a^2*x^2 - 1))/(2*a) +
(d^3*log(a^2*x^2 - 1))/(14*a^7) + (d^3*x^6)/(42*a) + (d^3*x^4)/(28*a^3) + (
d^3*x^2)/(14*a^5) + (c^2*d*log(a^2*x^2 - 1))/(2*a^3) + (3*c*d^2*log(a^2*x^2
 - 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) + (3*c*d^2*x^2)/
(10*a^3) + c^2*d*x^3*atanh(a*x) + (3*c*d^2*x^5*atanh(a*x))/5
```

sympy [A] time = 3.19, size = 245, normalized size = 1.45

$$\begin{cases} c^3x\,\mathrm{atanh}\,(ax)+c^2dx^3\,\mathrm{atanh}\,(ax)+\frac{3cd^2x^5\,\mathrm{atanh}\,(ax)}{5}+\frac{d^3x^7\,\mathrm{atanh}\,(ax)}{7}+\frac{c^3\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^3\,\mathrm{atanh}\,(ax)}{a}+\frac{c^2dx^2}{2a}+\frac{3cd^2x^4}{20a}+\ldots \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*atanh(a*x),x)
```

```
[Out] Piecewise((c**3*x*atanh(a*x) + c**2*d*x**3*atanh(a*x) + 3*c*d**2*x**5*atanh
(a*x)/5 + d**3*x**7*atanh(a*x)/7 + c**3*log(x - 1/a)/a + c**3*atanh(a*x)/a
+ c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*log(
x - 1/a)/a**3 + c**2*d*atanh(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3*x**
4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*atanh(a*x)/(5*a**5)
 + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*atanh(a*x)/(7*a*
*7), Ne(a, 0)), (0, True))
```

3.500 $\int \left(c+dx^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=110

$$\frac{dx^2\left(10a^2c+3d\right)}{30a^3}+\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\log\left(1-a^2x^2\right)}{30a^5}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax$$

```
[Out] 1/30*d*(10*a^2*c+3*d)*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arctanh(a*x)+2/3*c*d*x^3
*arctanh(a*x)+1/5*d^2*x^5*arctanh(a*x)+1/30*(15*a^4*c^2+10*a^2*c*d+3*d^2)*l
n(-a^2*x^2+1)/a^5
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {194, 5976, 1594, 1247, 698}

$$\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\log\left(1-a^2x^2\right)}{30a^5}+\frac{dx^2\left(10a^2c+3d\right)}{30a^3}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{d^2x^4}{20a}+\frac{1}{5}d^2x^5\tan$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (d*(10*a^2*c + 3*d)*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcTanh[a*x] +
 (2*c*d*x^3*ArcTanh[a*x])/3 + (d^2*x^5*ArcTanh[a*x])/5 + ((15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/(30*a^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^2 \tanh^{-1}(ax)\,dx = c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-a\int \frac{c^2x+\frac{2}{3}cdx^3+}{1-a^2x^2}$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-a\int \frac{x\left(c^2+\frac{2}{3}cdx^2+\right.}{1-a^2x^2}$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{c^2+}{1}\right.$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-\frac{1}{2}a\,\mathrm{Subst}\left(\int\left(-\frac{d(1}{}\right.\right.$$

$$= \frac{d\left(10a^2c+3d\right)x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh$$

Mathematica [A] time = 0.05, size = 98, normalized size = 0.89

$$\frac{4a^5x\tanh^{-1}(ax)\left(15c^2+10cdx^2+3d^2x^4\right)+a^2dx^2\left(a^2\left(20c+3dx^2\right)+6d\right)+\left(30a^4c^2+20a^2cd+6d^2\right)\log\left(1-\right.}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3*
d^2*x^4)*ArcTanh[a*x] + (30*a^4*c^2 + 20*a^2*c*d + 6*d^2)*Log[1 - a^2*x^2])
/(60*a^5)
```

fricas [A] time = 0.49, size = 119, normalized size = 1.08

$$\frac{3\,a^4d^2x^4+2\left(10\,a^4cd+3\,a^2d^2\right)x^2+2\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\log\left(a^2x^2-1\right)+2\left(3\,a^5d^2x^5+10\,a^5cdx^3+15\right.}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d + 3*a^2*d^2)*x^2 + 2*(15*a^4*c^2 + 10*a
^2*c*d + 3*d^2)*log(a^2*x^2 - 1) + 2*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a
^5*c^2*x)*log(-(a*x + 1)/(a*x - 1)))/a^5
```

giac [B] time = 0.20, size = 527, normalized size = 4.79

$$\frac{1}{15}a\left(\frac{\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6}-\frac{\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^6}+\frac{4\left(\frac{\left(5\,a^2cd+3\,d^2\right)(ax-}{(ax-1)^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(-a*x - 1)/abs(a*x - 1))/a
^6 - (15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(-(a*x + 1)/(a*x - 1) + 1))/a
^6 + 4*((5*a^2*c*d + 3*d^2)*(a*x + 1)^3/(a*x - 1)^3 - (10*a^2*c*d + 3*d^2)*
(a*x + 1)^2/(a*x - 1)^2 + (5*a^2*c*d + 3*d^2)*(a*x + 1)/(a*x - 1))/(a^6*((a
*x + 1)/(a*x - 1) - 1)^4) + (15*(a*x + 1)^4*a^4*c^2/(a*x - 1)^4 - 60*(a*x +
 1)^3*a^4*c^2/(a*x - 1)^3 + 90*(a*x + 1)^2*a^4*c^2/(a*x - 1)^2 - 60*(a*x +
1)*a^4*c^2/(a*x - 1) + 15*a^4*c^2 + 30*(a*x + 1)^4*a^2*c*d/(a*x - 1)^4 - 60
*(a*x + 1)^3*a^2*c*d/(a*x - 1)^3 + 40*(a*x + 1)^2*a^2*c*d/(a*x - 1)^2 - 20*
(a*x + 1)*a^2*c*d/(a*x - 1) + 10*a^2*c*d + 15*(a*x + 1)^4*d^2/(a*x - 1)^4 +
 30*(a*x + 1)^2*d^2/(a*x - 1)^2 + 3*d^2)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/
((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/
(a*x - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^5))
```

maple [A] time = 0.03, size = 148, normalized size = 1.35

$$\frac{d^2x^5\operatorname{arctanh}(ax)}{5}+\frac{2cd\,x^3\operatorname{arctanh}(ax)}{3}+c^2x\operatorname{arctanh}(ax)+\frac{d^2x^4}{20a}+\frac{x^2cd}{3a}+\frac{x^2d^2}{10a^3}+\frac{\ln(ax-1)\,c^2}{2a}+\frac{\ln(ax-1)\,cd}{3a^3}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arctanh(a*x),x)
```

```
[Out] 1/5*d^2*x^5*arctanh(a*x)+2/3*c*d*x^3*arctanh(a*x)+c^2*x*arctanh(a*x)+1/20*d
^2*x^4/a+1/3/a*x^2*c*d+1/10/a^3*x^2*d^2+1/2/a*ln(a*x-1)*c^2+1/3/a^3*ln(a*x-
1)*c*d+1/10/a^5*ln(a*x-1)*d^2+1/2/a*ln(a*x+1)*c^2+1/3/a^3*ln(a*x+1)*c*d+1/1
0/a^5*ln(a*x+1)*d^2
```

maxima [A] time = 0.30, size = 131, normalized size = 1.19

$$\frac{1}{60}a\left(\frac{3a^2d^2x^4+2\left(10a^2cd+3d^2\right)x^2}{a^4}+\frac{2\left(15a^4c^2+10a^2cd+3d^2\right)\log(ax+1)}{a^6}+\frac{2\left(15a^4c^2+10a^2cd+3d^2\right)\mathrm{lo}}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d + 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 +
10*a^2*c*d + 3*d^2)*log(a*x + 1)/a^6 + 2*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*
log(a*x - 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*arctanh(a*x)
```

mupad [B] time = 1.09, size = 118, normalized size = 1.07

$$c^2\,x\,\mathrm{atanh}\left(a\,x\right)+\frac{d^2\,x^5\,\mathrm{atanh}\left(a\,x\right)}{5}+\frac{c^2\,\ln\left(a^2\,x^2-1\right)}{2\,a}+\frac{d^2\,\ln\left(a^2\,x^2-1\right)}{10\,a^5}+\frac{d^2\,x^4}{20\,a}+\frac{d^2\,x^2}{10\,a^3}+\frac{2\,c\,d\,x^3\,\mathrm{atanh}\left(a\,x\right)}{3}+\frac{c\,d\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^2,x)
```

```
[Out] c^2*x*atanh(a*x) + (d^2*x^5*atanh(a*x))/5 + (c^2*log(a^2*x^2 - 1))/(2*a) +
(d^2*log(a^2*x^2 - 1))/(10*a^5) + (d^2*x^4)/(20*a) + (d^2*x^2)/(10*a^3) + (
2*c*d*x^3*atanh(a*x))/3 + (c*d*log(a^2*x^2 - 1))/(3*a^3) + (c*d*x^2)/(3*a)
```

sympy [A] time = 1.74, size = 155, normalized size = 1.41

$$\begin{cases} c^2x\operatorname{atanh}(ax)+\frac{2cdx^3\operatorname{atanh}(ax)}{3}+\frac{d^2x^5\operatorname{atanh}(ax)}{5}+\frac{c^2\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^2\operatorname{atanh}(ax)}{a}+\frac{cdx^2}{3a}+\frac{d^2x^4}{20a}+\frac{2cd\log\left(x-\frac{1}{a}\right)}{3a^3}+\frac{2cd\operatorname{atanh}(ax)}{3a^3} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*atanh(a*x),x)
```

```
[Out] Piecewise((c**2*x*atanh(a*x) + 2*c*d*x**3*atanh(a*x)/3 + d**2*x**5*atanh(a*
x)/5 + c**2*log(x - 1/a)/a + c**2*atanh(a*x)/a + c*d*x**2/(3*a) + d**2*x**4
/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*atanh(a*x)/(3*a**3) + d**2*x*
*2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*atanh(a*x)/(5*a**5), Ne(a,
 0)), (0, True))
```

3.501 $\int (c + dx^2) \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=57

$$\frac{(3a^2c + d)\log(1 - a^2x^2)}{6a^3} + cx\tanh^{-1}(ax) + \frac{1}{3}dx^3\tanh^{-1}(ax) + \frac{dx^2}{6a}$$

```
[Out] 1/6*d*x^2/a+c*x*arctanh(a*x)+1/3*d*x^3*arctanh(a*x)+1/6*(3*a^2*c+d)*ln(-a^2
*x^2+1)/a^3
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5976, 1593, 444, 43}

$$\frac{(3a^2c + d)\log(1 - a^2x^2)}{6a^3} + cx\tanh^{-1}(ax) + \frac{dx^2}{6a} + \frac{1}{3}dx^3\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcTanh[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + ((3*a^2*c + d)*
Log[1 - a^2*x^2])/(6*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)\tanh^{-1}(ax)\,dx = cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-a\int\frac{cx+\frac{dx^3}{3}}{1-a^2x^2}\,dx$$
$$= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-a\int\frac{x\left(c+\frac{dx^2}{3}\right)}{1-a^2x^2}\,dx$$
$$= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\frac{c+\frac{dx}{3}}{1-a^2x}\,dx,x,x^2\right)$$
$$= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\left(-\frac{d}{3a^2}+\frac{-3a^2c-d}{3a^2\left(-1+a^2x\right)}\right)dx,x\right.$$
$$= \frac{dx^2}{6a}+cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)+\frac{\left(3a^2c+d\right)\log\left(1-a^2x^2\right)}{6a^3}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.21

$$\frac{c\log\left(1-a^2x^2\right)}{2a}+\frac{d\log\left(1-a^2x^2\right)}{6a^3}+cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)+\frac{dx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcTanh[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + (c*Log[1 - a^2*
x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

fricas [A] time = 0.50, size = 65, normalized size = 1.14

$$\frac{a^2dx^2+\left(3\,a^2c+d\right)\log\left(a^2x^2-1\right)+\left(a^3dx^3+3\,a^3cx\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d*x^2 + (3*a^2*c + d)*log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*l
og(-(a*x + 1)/(a*x - 1)))/a^3
```

giac [B] time = 0.21, size = 266, normalized size = 4.67

$$\frac{1}{3}a\left(\frac{\left(3\,a^2c+d\right)\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4}-\frac{\left(3\,a^2c+d\right)\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^4}+\frac{2\,(ax+1)d}{(ax-1)a^4\left(\frac{ax+1}{ax-1}-1\right)^2}+\frac{\left(\frac{3\,(ax+1)^2a^2c}{(ax-1)^2}-\frac{6\,(ax+1)a^2c}{ax-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c + d)*log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - (3*a^2*c + d)*lo
g(abs(-(a*x + 1)/(a*x - 1) + 1))/a^4 + 2*(a*x + 1)*d/((a*x - 1)*a^4*((a*x +
 1)/(a*x - 1) - 1)^2) + (3*(a*x + 1)^2*a^2*c/(a*x - 1)^2 - 6*(a*x + 1)*a^2*
```

```
c/(a*x - 1) + 3*a^2*c + 3*(a*x + 1)^2*d/(a*x - 1)^2 + d)*log(-(a*((a*x + 1)
/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) +
1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^3))
```

maple [A] time = 0.03, size = 76, normalized size = 1.33

$$\frac{d\,x^3\,\mathrm{arctanh}\,(ax)}{3}+cx\,\mathrm{arctanh}\,(ax)+\frac{d\,x^2}{6a}+\frac{\ln\,(ax-1)\,c}{2a}+\frac{\ln\,(ax-1)\,d}{6a^3}+\frac{\ln\,(ax+1)\,c}{2a}+\frac{\ln\,(ax+1)\,d}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arctanh(a*x),x)
```

```
[Out] 1/3*d*x^3*arctanh(a*x)+c*x*arctanh(a*x)+1/6*d*x^2/a+1/2/a*ln(a*x-1)*c+1/6/a
^3*ln(a*x-1)*d+1/2/a*ln(a*x+1)*c+1/6/a^3*ln(a*x+1)*d
```

maxima [A] time = 0.32, size = 65, normalized size = 1.14

$$\frac{1}{6}\,a\left(\frac{dx^2}{a^2}+\frac{\left(3\,a^2c+d\right)\log\,(ax+1)}{a^4}+\frac{\left(3\,a^2c+d\right)\log\,(ax-1)}{a^4}\right)+\frac{1}{3}\left(dx^3+3\,cx\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*a*(d*x^2/a^2 + (3*a^2*c + d)*log(a*x + 1)/a^4 + (3*a^2*c + d)*log(a*x -
 1)/a^4) + 1/3*(d*x^3 + 3*c*x)*arctanh(a*x)
```

mupad [B] time = 0.94, size = 60, normalized size = 1.05

$$\frac{\frac{d\,\ln\left(a^2\,x^2-1\right)}{6}+a^2\,\left(\frac{c\,\ln\left(a^2\,x^2-1\right)}{2}+\frac{d\,x^2}{6}\right)}{a^3}+\frac{d\,x^3\,\mathrm{atanh}\,(a\,x)}{3}+c\,x\,\mathrm{atanh}\,(a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2),x)
```

```
[Out] ((d*log(a^2*x^2 - 1))/6 + a^2*((c*log(a^2*x^2 - 1))/2 + (d*x^2)/6))/a^3 + (
d*x^3*atanh(a*x))/3 + c*x*atanh(a*x)
```

sympy [A] time = 0.79, size = 73, normalized size = 1.28

$$\begin{cases} cx\,\mathrm{atanh}\,(ax)+\frac{dx^3\,\mathrm{atanh}\,(ax)}{3}+\frac{c\log\left(x-\frac{1}{a}\right)}{a}+\frac{c\,\mathrm{atanh}\,(ax)}{a}+\frac{dx^2}{6a}+\frac{d\log\left(x-\frac{1}{a}\right)}{3a^3}+\frac{d\,\mathrm{atanh}\,(ax)}{3a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*atanh(a*x),x)
```

```
[Out] Piecewise((c*x*atanh(a*x) + d*x**3*atanh(a*x)/3 + c*log(x - 1/a)/a + c*atan
h(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*atanh(a*x)/(3*a**3),
Ne(a, 0)), (0, True))
```

3.502 $\int \frac{\tanh^{-1}(ax)}{c+dx^2}\,dx$

Optimal. Leaf size=429

$$-\frac{\text{Li}_2\left(-\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}(1-ax)}{\sqrt{-c}a+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\text{Li}_2\left(-\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}(ax+1)}{\sqrt{-c}a+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\log(ax}{}$$

```
[Out] -1/4*ln(-a*x+1)*ln(a*((-c)^(1/2)-x*d^(1/2))/(a*(-c)^(1/2)-d^(1/2)))/(-c)^(1
/2)/d^(1/2)+1/4*ln(a*x+1)*ln(a*((-c)^(1/2)-x*d^(1/2))/(a*(-c)^(1/2)+d^(1/2)
))/(-c)^(1/2)/d^(1/2)-1/4*ln(a*x+1)*ln(a*((-c)^(1/2)+x*d^(1/2))/(a*(-c)^(1/
2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*ln(-a*x+1)*ln(a*((-c)^(1/2)+x*d^(1/2))/
(a*(-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-(-a*x+1)*d^(1/2)/
(a*(-c)^(1/2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-(a*x+1)*d^(1/2)/(
a*(-c)^(1/2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,(-a*x+1)*d^(1/2)/(a
*(-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,(a*x+1)*d^(1/2)/(a*(
-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5972, 2409, 2394, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\log(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2),x]
```

```
[Out] -(Log[1 - a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(4*S
qrt[-c]*Sqrt[d]) + (Log[1 + a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c]
 + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - (Log[1 + a*x]*Log[(a*(Sqrt[-c] + Sqrt[
d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) + (Log[1 - a*x]*Log[(a
*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqrt[-c] + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - Po
lyLog[2, -((Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[d]
) + PolyLog[2, (Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sqrt[-c]*Sqrt
[d]) - PolyLog[2, -((Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[-c
]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sqrt
[-c]*Sqrt[d])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-ax)}{c+dx^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+ax)}{c+dx^2}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\sqrt{-c}\,\log(1-ax)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\log(1-ax)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\sqrt{-c}\,\log(1+ax)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\log(1$$

$$= \frac{\int \frac{\log(1-ax)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}}+\frac{\int \frac{\log(1-ax)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}}-\frac{\int \frac{\log(1+ax)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}}-\frac{\int \frac{\log(1+ax)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}}$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}\right.}{a\sqrt{-c}-\sqrt{}}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}\right.}{a\sqrt{-c}-\sqrt{}}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}\right.}{a\sqrt{-c}-\sqrt{}}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

Mathematica [C] time = 1.46, size = 662, normalized size = 1.54

$$-\frac{a\left(i\left(\text{Li}_2\left(\frac{\left(-ca^2+d+2i\sqrt{a^2cd}\right)\left(iac+\sqrt{a^2cd}\,x\right)}{\left(ca^2+d\right)\left(\sqrt{a^2cd}\,x-iac\right)}\right)-\text{Li}_2\left(\frac{\left(-ca^2+d-2i\sqrt{a^2cd}\right)\left(iac+\sqrt{a^2cd}\,x\right)}{\left(ca^2+d\right)\left(\sqrt{a^2cd}\,x-iac\right)}\right)\right)-2i\cos^{-1}\left(\frac{d-a^2c}{a^2c+d}\right)\tan^{-1}\left(\frac{adx}{\sqrt{a^2cd}}\right)+4\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2),x]
```

```
[Out] -1/4*(a*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*
c*d]] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c)
+ d)/(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d +
Sqrt[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (ArcC
os[(-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*
c*(d + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))]
 + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
 + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c
+ d]*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] +
(ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] +
```

```
ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/
(Sqrt[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(
-PolyLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x)
)/((a^2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] + PolyLog[2, ((-(a^2*c) + d +
 (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + S
qrt[a^2*c*d]*x))])))/Sqrt[a^2*c*d]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(d*x^2 + c), x)
```

maple [B] time = 0.62, size = 833, normalized size = 1.94

$$\frac{a^3\ln\left(1-\frac{(a^2c+d)(ax+1)^2}{(-a^2x^2+1)\left(-a^2c-2\sqrt{-a^2cd}+d\right)}\right)\operatorname{artanh}(ax)\sqrt{-a^2cd}\;c}{2d\left(a^4c^2+2a^2cd+d^2\right)}+\frac{a\ln\left(1-\frac{(a^2c+d)(ax+1)^2}{(-a^2x^2+1)\left(-a^2c-2\sqrt{-a^2cd}+d\right)}\right)\operatorname{artanh}(ax)\sqrt{}}{a^4c^2+2a^2cd+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c),x)
```

```
[Out] 1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a
^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)*c+a/(a^4*c^2+2*a^
2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)
+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)-1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*arctan
h(a*x)^2*(-a^2*c*d)^(1/2)*c-a/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^2*
c*d)^(1/2)+1/4*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/
(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*c+1/2*a/(a^4*c
^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^
2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)+1/2/a/c/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2
*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-
a^2*c*d)^(1/2)*d-1/2/a/c/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^2*c*d)^
(1/2)*d+1/4/a/c/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2
*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*d-1/2/a*(-a^2*c*d)^
(1/2)/c/d*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^
2*c*d)^(1/2)+d))+1/2/a*(-a^2*c*d)^(1/2)/c/d*arctanh(a*x)^2-1/4/a*(-a^2*c*d)
^(1/2)/c/d*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^
(1/2)+d))
```

maxima [C] time = 0.57, size = 406, normalized size = 0.95

$$\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)\operatorname{artanh}(ax)}{\sqrt{cd}}+\frac{\left(\arctan\left(\frac{(a^2x+a)\sqrt{c}\sqrt{d}}{a^2c+d},\frac{adx+d}{a^2c+d}\right)-\arctan\left(\frac{(a^2x-a)\sqrt{c}\sqrt{d}}{a^2c+d},-\frac{adx-d}{a^2c+d}\right)\right)\log\left(dx^2+c\right)-\operatorname{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] arctan(d*x/sqrt(c*d))*arctanh(a*x)/sqrt(c*d) + 1/4*((arctan2((a^2*x + a)*sq
rt(c)*sqrt(d)/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - arctan2((a^2*x - a)*s
qrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c) - arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) + arctan(
sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - I*dilog((a^
2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(
d) - d)) - I*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c
 + 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c + a*d*x + (I*a^2*x - I*a)*s
qrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c - a*d
*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)))
/sqrt(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2), x)
```

3.503 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=590

$$\frac{a\log\left(1-a^2x^2\right)}{4c\left(a^2c+d\right)}-\frac{a\log\left(c+dx^2\right)}{4c\left(a^2c+d\right)}+\frac{i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{c}a+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{a\left(i\sqrt{d}x+\sqrt{c}\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{Li}_2\left(\frac{a\left(i\sqrt{d}x+\sqrt{c}\right)}{\sqrt{c}a+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}$$

```
[Out] 1/2*x*arctanh(a*x)/c/(d*x^2+c)+1/4*a*ln(-a^2*x^2+1)/c/(a^2*c+d)-1/4*a*ln(d*
x^2+c)/c/(a^2*c+d)+1/2*arctan(x*d^(1/2)/c^(1/2))*arctanh(a*x)/c^(3/2)/d^(1/
2)-1/8*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2
))/c^(3/2)/d^(1/2)+1/8*I*ln((-a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1-I*
x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)-1/8*I*ln(-(-a*x+1)*d^(1/2)/(I*a*c^(1/2)-
d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*ln((a*x+1)*d^(1/2
)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*po
lylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1/2)-1/8*
I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)/d^(1/2)+
1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1
/2)-1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)/
d^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {199, 205, 5976, 6725, 517, 444, 36, 31, 4908, 2409, 2394, 2393, 2391}

$$\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (x*ArcTanh[a*x])/(2*c*(c + d*x^2)) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTanh[a
*x])/(2*c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] + Sq
rt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((S
qrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])
/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] - Sqrt[d
]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c
] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2
)*Sqrt[d])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - a\int \frac{\frac{x}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1-a^2x^2}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - a\int\left(-\frac{x}{2c(-1+ax)(1+ax)\left(c+dx^2\right)} - \frac{\tan^{-1}}{2c^{3/2}\sqrt{d}\,(}\right.$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{(-1+ax)(1+ax)\left(c+dx^2\right)}\,dx}{2c} + \frac{a\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{-1+a^2x^2}\,dx}{2c^{3/2}\sqrt{d}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{\left(-1+a^2x^2\right)\left(c+dx^2\right)}\,dx}{2c} + \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{-1+a^2x^2}\,dx}{4c^{3/2}\sqrt{d}} - \frac{(i}{}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\left(-1+a^2x\right)(c+dx)}\,dx, x, x^2\right)}{4c} + \frac{(ia)\int\left(-\frac{\log(}{2}\right.}{}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{1-ax}\,dx}{8c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{1+ax}\,dx}{8c^{3/2}\sqrt{d}} + \frac{(ia)}{}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}\,(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}\,(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right.}{8c^{3/}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}\,(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}\,(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right.}{8c^{3/}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}\,(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}\,(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right.}{8c^{3/}}$$

Mathematica [A] time = 7.79, size = 746, normalized size = 1.26

$$a\left(\frac{i\left(\text{Li}_2\left(\frac{\left(-ca^2+d-2i\sqrt{a^2cd}\right)\left(iac+\sqrt{a^2cd}\,x\right)}{\left(ca^2+d\right)\left(\sqrt{a^2cd}\,x-iac\right)}\right)-\text{Li}_2\left(\frac{\left(-ca^2+d+2i\sqrt{a^2cd}\right)\left(iac+\sqrt{a^2cd}\,x\right)}{\left(ca^2+d\right)\left(\sqrt{a^2cd}\,x-iac\right)}\right)\right)+2i\cos^{-1}\left(\frac{d-a^2c}{a^2c+d}\right)\tan^{-1}\left(\frac{adx}{\sqrt{a^2cd}}\right)-4\tanh^{-1}(ax)\tan^{-1}\left(\frac{ac}{x\sqrt{a^2cd}}\right)+\log\left(\frac{2ia}{(a^2}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (a*((-2*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)])/(a^2*c + d
) + ((2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]
 - 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] + (ArcCos[(-(a^2*c) + d)/
(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + Sqrt[
a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] + (ArcCos[(-
(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*c*(d
+ I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (A
rcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + Ar
cTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*
E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] - (ArcC
os[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTa
n[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt
[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(PolyL
og[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^
2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] - PolyLog[2, ((-(a^2*c) + d + (2*I)
*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^
2*c*d]*x))]))/Sqrt[a^2*c*d] + (4*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/(a^2*c
- d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]])))/(8*c)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{(dx^2 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(d*x^2 + c)^2, x)
```

maple [B] time = 1.35, size = 2346, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^2,x)
```

```
[Out] -1/4*a^4*(c*d)^(1/2)/d/(a^2*c+d)^2*arctan(a/d*(c*d)^(1/2))-1/4*a^4*(c*d)^(1
/2)/d/(a^2*c+d)^2*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^
(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))+a/(a^2*c+d)^
2/c*d*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*(c*d)^(1/2)/c^2*d/(a^2*c+d)^2*arct
an(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d
)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-1/4*a^3/(a^2*c+d)^2*ln(a^2*c*(a*x+
1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)
^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)+1/4*a^2*(c*d)^(1/2)/c/d/(a^2*c+d)*ar
ctan(a/d*(c*d)^(1/2))-1/4/a*(-a^2*c*d)^(1/2)/(a^2*c+d)/c^2*arctanh(a*x)*ln(
1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+3/4*a^3/(
a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^
2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)+1/4*a*(-a^2*c*d)^(
```

```
1/2)/c/d/(a^2*c+d)*arctanh(a*x)^2-1/8*a*(-a^2*c*d)^(1/2)/c/d/(a^2*c+d)*poly
log(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+1/4*a
^2*(c*d)^(1/2)/c/d/(a^2*c+d)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+
d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-1
/4*(c*d)^(1/2)/c^2/(a^2*c+d)*arctan(a/d*(c*d)^(1/2))-1/2*a^3*arctanh(a*x)/(
a^2*c+d)/(a^2*d*x^2+a^2*c)-1/4*(c*d)^(1/2)/c^2/(a^2*c+d)*arctan(1/(a^2*c+d)
*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*
x-a/(a^2*c+d)*(c*d)^(1/2))+3/4*a/(a^2*c+d)/c/(a^4*c^2+2*a^2*c*d+d^2)*d*ln(1
-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*
x)*(-a^2*c*d)^(1/2)+1/4*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c
+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^
2*c*d)^(1/2)*c+1/4/a/(a^2*c+d)/c^2*d^2/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+
d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2
*c*d)^(1/2)+a^3/(a^2*c+d)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*a/(a^2*c+d)/
c*d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^2*c*d)^(1/2)+1/8*a^5/d/(a^2*
c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a
^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*c-1/4*a*(-a^2*c*d)^(1/2)/c/d/(
a^2*c+d)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2
*c*d)^(1/2)+d))-1/2*a^3*arctanh(a*x)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)*d*x^2+1/
2*a^2*arctanh(a*x)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)*x*d+1/8/a/(a^2*c+d)/c^2*d^
2/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*
c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)-1/4/a/(a^2*c+d)/c^2*d^2/(a^4*c^2+
2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^2*c*d)^(1/2)+3/8*a/(a^2*c+d)/c*d/(a^4*c^2
+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*
c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)-1/4*a/(a^2*c+d)^2/c*d*ln(a^2*c*(a*x+1)^4/(-
a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2+a^2*
c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)-1/4*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)
*arctanh(a*x)^2*(-a^2*c*d)^(1/2)*c+1/4*(c*d)^(1/2)/c^2*d/(a^2*c+d)^2*arctan
(a/d*(c*d)^(1/2))+1/4/a*(-a^2*c*d)^(1/2)/(a^2*c+d)/c^2*arctanh(a*x)^2-1/8/a
*(-a^2*c*d)^(1/2)/(a^2*c+d)/c^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/
(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+1/2*a^4*arctanh(a*x)/(a^2*c+d)/(a^2*d*x^2+a^
2*c)*x-3/4*a^3/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*arctanh(a*x)^2*(-a^2*c*d)^
(1/2)+3/8*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)
^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)
```

maxima [A] time = 0.53, size = 550, normalized size = 0.93

$$\frac{1}{2}\left(\frac{x}{cdx^2+c^2}+\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}\right)\text{artanh}\,(ax)-\frac{\left(2\,acd\log\left(dx^2+c\right)-2\,acd\log\left(ax+1\right)-2\,acd\log\left(ax-1\right)+\left(\left(a^2c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arctanh(a*x)
- 1/8*(2*a*c*d*log(d*x^2 + c) - 2*a*c*d*log(a*x + 1) - 2*a*c*d*log(a*x - 1)
 + ((a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^
2*c + d)) - (a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x
+ d)/(a^2*c + d)) + (I*a^2*c + I*d)*dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*
sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (I*a^2*c + I*d)*dil
og((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)
*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilog((a^2*c + a*d*x + (I*a^2*x - I*a)*sq
rt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilo
g((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*
sqrt(d) - d)) - ((a^2*c + d)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*c + d
), (a*d*x + d)/(a^2*c + d)) - (a^2*c + d)*arctan2((a^2*x - a)*sqrt(c)*sqrt(
d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d))
*a/(a^3*c^3*d + a*c^2*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^2,x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.504 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^3} dx$

Optimal. Leaf size=657

$$\frac{a\left(5a^2c+3d\right)\log\left(1-a^2x^2\right)}{16c^2\left(a^2c+d\right)^2}-\frac{a\left(5a^2c+3d\right)\log\left(c+dx^2\right)}{16c^2\left(a^2c+d\right)^2}+\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{3i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{Li}_2\left(\frac{a(\sqrt{}\ldots}{\sqrt{}\ldots}\right)}{32c^{5/}\ldots}$$

```
[Out] 1/8*a/c/(a^2*c+d)/(d*x^2+c)+1/4*x*arctanh(a*x)/c/(d*x^2+c)^2+3/8*x*arctanh(
a*x)/c^2/(d*x^2+c)+1/16*a*(5*a^2*c+3*d)*ln(-a^2*x^2+1)/c^2/(a^2*c+d)^2-1/16
*a*(5*a^2*c+3*d)*ln(d*x^2+c)/c^2/(a^2*c+d)^2+3/8*arctan(x*d^(1/2)/c^(1/2))*
arctanh(a*x)/c^(5/2)/d^(1/2)-3/32*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2
)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32*I*ln((-a*x+1)*d^(1/2)/(I
*a*c^(1/2)+d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32*I*ln(-(
-a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^
(1/2)+3/32*I*ln((a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(
1/2))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)-I
*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1
/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a
*c^(1/2)-I*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2
))/(a*c^(1/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {199, 205, 5976, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391}

$$\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^3,x]
```

```
[Out] a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*ArcTanh[a*x])/(4*c*(c + d*x^2)^2) + (3
*x*ArcTanh[a*x])/(8*c^2*(c + d*x^2)) + (3*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTa
nh[a*x])/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) +
 (((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2])
/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
 + d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)])/(c^(5/2)*Sqrt[d])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
```

```
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^3}\,dx = \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}-a\int\frac{\frac{x}{4c\left(c+dx^2\right)^2}+\frac{3x}{8c^2\left(c+dx^2\right)}+\ldots}{1-a^2x^2}$$

$$=\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}-a\int\left(-\frac{x\left(5c+3dx^2\right)}{8c^2\left(-1+a^2x^2\right)\left(c+dx\ldots\right.}\right.$$

$$=\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{a\int\frac{x\left(5c+3dx^2\right)}{\left(-1+a^2x^2\right)\left(c+dx^2\right)^2}\,dx}{8c^2}+\frac{(3a)\ldots}{}$$

$$=\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{a\,\mathrm{Subst}\left(\int\frac{5c+3dx}{\left(-1+a^2x\right)(c+dx)^2}\,dx,x\ldots\right.}{16c^2}$$

$$=\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{a\,\mathrm{Subst}\left(\int\left(\frac{a^2\left(5a^2c+3d\right)}{\left(a^2c+d\right)^2\left(-1+a^2x\right)}-\ldots\right.\right.}{}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{a\,(5a\ldots}{}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\ldots}{}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\ldots}{}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\ldots}{}$$

Mathematica [B] time = 12.94, size = 1828, normalized size = 2.78

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^3,x]
```

```
[Out] a^5*((-5*Log[1 - ((-(a^2*c) - d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)])/(16*a^
2*c*(a^2*c + d)^2) - (3*d*Log[1 - ((-(a^2*c) - d)*Cosh[2*ArcTanh[a*x]])/(a^
2*c - d)])/(16*a^4*c^2*(a^2*c + d)^2) - (3*((-2*I)*ArcCos[(-(a^2*c) + d)/(a
^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)
]*ArcTanh[a*x] - (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqr
t[a^2*c*d]])*Log[1 - ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*
Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))] + (-Ar
cCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[1 -
 ((a^2*c - d + (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((
a^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))] + (ArcCos[(-(a^2*c) + d)/(
a^2*c + d)] + (2*I)*((-I)*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x
)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcTanh[a
*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + (ArcCos[(-(a^2*c
) + d)/(a^2*c + d)] - (2*I)*((-I)*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTa
n[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt
[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(PolyL
og[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x
))/((a^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))] - PolyLog[2, ((a^2*c
- d + (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d
)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))])))/(32*a^2*c*Sqrt[a^2*c*d]*(a^2*c +
 d)) - (3*d*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[
a^2*c*d]] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2
*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[1 - ((a^2*c -
d - (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*
(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))] + (-ArcCos[(-(a^2*c) + d)/(a^2*c + d)
] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[1 - ((a^2*c - d + (2*I)*Sqrt[a^2*c
*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I)*a*S
qrt[a^2*c*d]*x))] + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + (2*I)*((-I)*ArcTa
n[(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]
*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d
)*Cosh[2*ArcTanh[a*x]]])] + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - (2*I)*((-
I)*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[
(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt[a^2*c + d]*Sqrt[a^2*c - d + (a
^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(PolyLog[2, ((a^2*c - d - (2*I)*Sqrt[
a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I
)*a*Sqrt[a^2*c*d]*x))] - PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*c*d])*(2*a
^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c
*d]*x))])))/(32*a^4*c^2*Sqrt[a^2*c*d]*(a^2*c + d)) + (d*ArcTanh[a*x]*Sinh[2
*ArcTanh[a*x]])/(2*a^2*c*(a^2*c + d)*(a^2*c - d + a^2*c*Cosh[2*ArcTanh[a*x]
] + d*Cosh[2*ArcTanh[a*x]])^2) + (2*a^2*c*d + 5*a^4*c^2*ArcTanh[a*x]*Sinh[2
*ArcTanh[a*x]] + 8*a^2*c*d*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]] + 3*d^2*ArcTan
h[a*x]*Sinh[2*ArcTanh[a*x]])/(8*a^4*c^2*(a^2*c + d)^2*(a^2*c - d + a^2*c*Co
sh[2*ArcTanh[a*x]] + d*Cosh[2*ArcTanh[a*x]])))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\,(ax)}{d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(d*x^2 + c)^3, x)
```

maple [B] time = 1.41, size = 4311, normalized size = 6.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^3,x)
```

```
[Out] 1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d+5/4*a^5/(a^4*c^2+2*a^
2*c*d+d^2)/(a^2*c+d)*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*a^5/(a^4*c^2+2*a^2*
c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^
(1/2)+d))*(-a^2*c*d)^(1/2)-3/4*a^5/(a^4*c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2
*(-a^2*c*d)^(1/2)-5/16*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*ln(a^2*c*(a*x+
1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)
^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)+5/16*a^4*(c*d)^(1/2)/c/d/(a^4*c^2+2*
a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-3/16*a^4*(c*d)^(1/2)/c/(a^2*c+d)/(a^4*
c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-5/16*a^6*(c*d)^(1/2)/d/(a^2*c+d)
/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))+3/16*(c*d)^(1/2)/c^3*d^2/(
a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-1/8*a^7/(a^4*c^2+2
*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*d^2*x^4+1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)
/(a^2*d*x^2+a^2*c)^2/c*d^2*x^2+9/16*a^3/c/(a^4*c^2+2*a^2*c*d+d^2)^2*d*polyl
og(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*
c*d)^(1/2)+5/16*a^4*(c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c
+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/
2)*x-a/(a^2*c+d)*(c*d)^(1/2))-3/4*a/(a^4*c^2+2*a^2*c*d+d^2)^2/c^2*d^2*arcta
nh(a*x)^2*(-a^2*c*d)^(1/2)-3/16*a/c^2/(a^4*c^2+2*a^2*c*d+d^2)*d^2/(a^2*c+d)
*ln(a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)
^4/(-a^2*x^2+1)^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)-1/2*a^3/c/(a^4*c^2+2*
a^2*c*d+d^2)*d/(a^2*c+d)*ln(a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^
2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+
d)+3/4*a/c^2/(a^4*c^2+2*a^2*c*d+d^2)*d^2/(a^2*c+d)*ln((a*x+1)/(-a^2*x^2+1)^
(1/2))-3/16*a^4*(c*d)^(1/2)/c/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a
^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)
^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))+5/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^
2+a^2*c)^2*c*arctanh(a*x)*x-3/8*a*(-a^2*c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d
^2)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)
^(1/2)+d))+5/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arctanh(a*x)
*x*d+3/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arctanh(a*x)*x^3*d
-5/4*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arctanh(a*x)*x^2*d-3/1
6/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2*(-a^2*c*d)^(1/2)-3/32/
a*(-a^2*c*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1
)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+3/16/a*(-a^2*c*d)^(1/2)/c^3
*d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2+3/32/a/c^3*d^3/(a^4*c^2+2*a^2*c*d
+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/
2)+d))*(-a^2*c*d)^(1/2)-3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2
*(-a^2*c*d)^(1/2)*c-9/8*a^3/c/(a^4*c^2+2*a^2*c*d+d^2)^2*d*arctanh(a*x)^2*(-
a^2*c*d)^(1/2)-3/32*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*polylo
g(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+3/16*a^
```

```
3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2+3/32*a^7/d/(a
^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-
2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*c+2*a^3/c/(a^4*c^2+2*a^2*c*d+d^2)*d
/(a^2*c+d)*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-5/16*a^6*(c*d)^(1/2)/d/(a^2*c+d)/
(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/
(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))+3/8*a/
(a^4*c^2+2*a^2*c*d+d^2)^2/c^2*d^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1
)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)+3/16*(c*d)^(1/2)/c^3*d^2/
(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a
^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/
2))-5/8*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*c*arctanh(a*x)-1/8*
a^2*(c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1
/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)
*(c*d)^(1/2))-1/8*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d*x^2+5/1
6*a^2*(c*d)^(1/2)/c^2*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d
)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)
*x-a/(a^2*c+d)*(c*d)^(1/2))-3/4*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*
c)^2/c*arctanh(a*x)*x^2*d^2+5/8*a^4/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*
c)^2/c*arctanh(a*x)*x*d^2+3/4*a/c^2*d^2/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2
*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-
a^2*c*d)^(1/2)+3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^
2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)
*c-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c^2*arctanh(a*x)*x^4
*d^3+3/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arctanh(a*x)*x^3
*d^2-3/16*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)*ln(
1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+3/8*a^4/(
a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c^2*arctanh(a*x)*x^3*d^3+3/16/a/
c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a
^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)-3/16/a*(-a^2*c*d)
^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/
(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+9/8*a^3/c/(a^4*c^2+2*a^2*c*d+d^
2)^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*a
rctanh(a*x)*(-a^2*c*d)^(1/2)*d-5/8*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a
^2*c)^2/c*arctanh(a*x)*x^4*d^2+5/16*a^2*(c*d)^(1/2)/c^2*d/(a^2*c+d)/(a^4*c^
2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-3/16*(c*d)^(1/2)/c^3*d/(a^4*c^2+2*
a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-1/8*a^2*(c*d)^(1/2)/c^2/(a^4*c^2+2*a^2
*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-3/16*(c*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d
+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^
2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))+3/8*a*(-a^2*c*d)^(1/2)/c
^2/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2-3/16*a*(-a^2*c*d)^(1/2)/c^2/(a^4*
c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a
^2*c*d)^(1/2)+d))+3/4*a^5/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^
2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)
-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d*arctanh(a*x)
```

maxima [B] time = 0.60, size = 1084, normalized size = 1.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*d*x^3 + 5*c*x)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4) + 3*arctan(d*x/sqr
t(c*d))/(sqrt(c*d)*c^2))*arctanh(a*x) + 1/32*(4*a^3*c^3*d + 4*a*c^2*d^2 - (
3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) - 3*(a^4*
c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan(sqr
t(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) + (3*I*a^4*c^3 +
 6*I*a^2*c^2*d + 3*I*c*d^2 + (3*I*a^4*c^2*d + 6*I*a^2*c*d^2 + 3*I*d^3)*x^2)
*dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqr
```

```
t(c)*sqrt(d) - d)) + (3*I*a^4*c^3 + 6*I*a^2*c^2*d + 3*I*c*d^2 + (3*I*a^4*c^
2*d + 6*I*a^2*c*d^2 + 3*I*d^3)*x^2)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*
sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (-3*I*a^4*c^3 - 6*I
*a^2*c^2*d - 3*I*c*d^2 + (-3*I*a^4*c^2*d - 6*I*a^2*c*d^2 - 3*I*d^3)*x^2)*di
log((a^2*c + a*d*x + (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c
)*sqrt(d) - d)) + (-3*I*a^4*c^3 - 6*I*a^2*c^2*d - 3*I*c*d^2 + (-3*I*a^4*c^2
*d - 6*I*a^2*c*d^2 - 3*I*d^3)*x^2)*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*s
qrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) - 3*((a^4*c^3 + 2*a^2*
c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x + a)*sq
rt(c)*sqrt(d)/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - (a^4*c^3 + 2*a^2*c^2*
d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x - a)*sqrt(c
)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*s
qrt(d) - 2*(5*a^3*c^3*d + 3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*lo
g(d*x^2 + c) + 2*(5*a^3*c^3*d + 3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x
^2)*log(a*x + 1) + 2*(5*a^3*c^3*d + 3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^
3)*x^2)*log(a*x - 1))*a/(a^5*c^6*d + 2*a^3*c^5*d^2 + a*c^4*d^3 + (a^5*c^5*d
^2 + 2*a^3*c^4*d^3 + a*c^3*d^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(d\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^3,x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.505 $\int \frac{1}{(a-ax^2)(b-2b\tanh^{-1}(x))}\,dx$

Optimal. Leaf size=17

$$-\frac{\log\left(1-2\tanh^{-1}(x)\right)}{2ab}$$

```
[Out] -1/2*ln(1-2*arctanh(x))/a/b
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5946}

$$-\frac{\log\left(1-2\tanh^{-1}(x)\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

```
[Out] -Log[1 - 2*ArcTanh[x]]/(2*a*b)
```

Rule 5946

```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-ax^2\right)\left(b-2b\tanh^{-1}(x)\right)}\,dx = -\frac{\log\left(1-2\tanh^{-1}(x)\right)}{2ab}$$

Mathematica [A] time = 0.06, size = 17, normalized size = 1.00

$$-\frac{\log\left(2\tanh^{-1}(x)-1\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

```
[Out] -1/2*Log[-1 + 2*ArcTanh[x]]/(a*b)
```

fricas [A] time = 0.49, size = 22, normalized size = 1.29

$$-\frac{\log\left(\log\left(-\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(log(-(x + 1)/(x - 1)) - 1)/(a*b)
```

giac [B] time = 0.23, size = 48, normalized size = 2.82

$$-\frac{\log\left(\frac{1}{4}\pi^2\left(\mathrm{sgn}\left(x-1\right)\mathrm{sgn}\left(-x-1\right)-1\right)^2+\left(\log\left(\frac{|-x-1|}{|x-1|}\right)-1\right)^2\right)}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(1/4*pi^2*(sgn(x - 1)*sgn(-x - 1) - 1)^2 + (log(abs(-x - 1)/abs(x -
 1)) - 1)^2)/(a*b)
```

maple [A] time = 0.06, size = 19, normalized size = 1.12

$$-\frac{\ln(2b\,\mathrm{arctanh}(x)-b)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x)
```

```
[Out] -1/2/a*ln(2*b*arctanh(x)-b)/b
```

maxima [A] time = 0.35, size = 23, normalized size = 1.35

$$-\frac{\log\left(-\log(x+1)+\log(-x+1)+1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(-log(x + 1) + log(-x + 1) + 1)/(a*b)
```

mupad [B] time = 0.98, size = 15, normalized size = 0.88

$$-\frac{\ln(2\,\mathrm{atanh}(x)-1)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x^2)*(b - 2*b*atanh(x))),x)
```

```
[Out] -log(2*atanh(x) - 1)/(2*a*b)
```

sympy [A] time = 0.56, size = 14, normalized size = 0.82

$$-\frac{\log\left(\mathrm{atanh}(x)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x**2+a)/(b-2*b*atanh(x)),x)
```

```
[Out] -log(atanh(x) - 1/2)/(2*a*b)
```

3.506 $\int \frac{\tanh^{-1}(bx)}{1-x^2}\,dx$

Optimal. Leaf size=171

$$\frac{1}{4}\text{Li}_2\left(\frac{1-bx}{1-b}\right)-\frac{1}{4}\text{Li}_2\left(\frac{1-bx}{b+1}\right)+\frac{1}{4}\text{Li}_2\left(\frac{bx+1}{1-b}\right)-\frac{1}{4}\text{Li}_2\left(\frac{bx+1}{b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(x+1)}{b+1}\right.$$

```
[Out] 1/4*ln(-b*(1-x)/(1-b))*ln(-b*x+1)-1/4*ln(b*(1+x)/(1+b))*ln(-b*x+1)-1/4*ln(b
*(1-x)/(1+b))*ln(b*x+1)+1/4*ln(-b*(1+x)/(1-b))*ln(b*x+1)+1/4*polylog(2,(-b*
x+1)/(1-b))-1/4*polylog(2,(-b*x+1)/(1+b))+1/4*polylog(2,(b*x+1)/(1-b))-1/4*
polylog(2,(b*x+1)/(1+b))
```

Rubi [A] time = 0.24, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5972, 2409, 2394, 2393, 2391}

$$\frac{1}{4}\text{PolyLog}\left(2,\frac{1-bx}{1-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1-bx}{b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{bx+1}{1-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{bx+1}{b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-}{1-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - b))]*Log[1 - b*x])/4 - (Log[(b*(1 + x))/(1 + b)]*Lo
g[1 - b*x])/4 - (Log[(b*(1 - x))/(1 + b)]*Log[1 + b*x])/4 + (Log[-((b*(1 +
x))/(1 - b))]*Log[1 + b*x])/4 + PolyLog[2, (1 - b*x)/(1 - b)]/4 - PolyLog[2
, (1 - b*x)/(1 + b)]/4 + PolyLog[2, (1 + b*x)/(1 - b)]/4 - PolyLog[2, (1 +
b*x)/(1 + b)]/4
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
```

```
x], x] /; FreeQ[{c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(bx)}{1-x^2}\,dx &= -\left(\frac{1}{2}\int \frac{\log(1-bx)}{1-x^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+bx)}{1-x^2}\,dx\\ &= -\left(\frac{1}{2}\int \left(\frac{\log(1-bx)}{2(1-x)}+\frac{\log(1-bx)}{2(1+x)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+bx)}{2(1-x)}+\frac{\log(1+bx)}{2(1+x)}\right)dx\\ &= -\left(\frac{1}{4}\int \frac{\log(1-bx)}{1-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-bx)}{1+x}\,dx+\frac{1}{4}\int \frac{\log(1+bx)}{1-x}\,dx+\frac{1}{4}\int \frac{\log(1+bx)}{1+x}\,dx\\ &= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx)\\ &= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx)\\ &= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx)\end{aligned}$$

Mathematica [C] time = 0.94, size = 576, normalized size = 3.37

$$-\frac{b\left(i\left(\mathrm{Li}_2\left(\frac{\left(b^2-2i\sqrt{-b^2}+1\right)\left(b-i\sqrt{-b^2}\,x\right)}{\left(b^2-1\right)\left(b+i\sqrt{-b^2}\,x\right)}\right)-\mathrm{Li}_2\left(\frac{\left(b^2+2i\sqrt{-b^2}+1\right)\left(b-i\sqrt{-b^2}\,x\right)}{\left(b^2-1\right)\left(b+i\sqrt{-b^2}\,x\right)}\right)\right)+2i\cos^{-1}\left(\frac{b^2+1}{1-b^2}\right)\tan^{-1}\left(\frac{bx}{\sqrt{-b^2}}\right)-4\tan^{-1}\left(\frac{\sqrt{-b^2}}{bx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[b*x]/(1 - x^2),x]
```

```
[Out] -1/4*(b*((2*I)*ArcCos[(1 + b^2)/(1 - b^2)]*ArcTan[(b*x)/Sqrt[-b^2]] - 4*Arc
Tan[Sqrt[-b^2]/(b*x)]*ArcTanh[b*x] - (ArcCos[(1 + b^2)/(1 - b^2)] - 2*ArcTa
n[(b*x)/Sqrt[-b^2]])*Log[(2*b*(-I + Sqrt[-b^2])*(-1 + b*x))/((-1 + b^2)*((-
I)*b + Sqrt[-b^2]*x))] - (ArcCos[(1 + b^2)/(1 - b^2)] + 2*ArcTan[(b*x)/Sqrt
[-b^2]])*Log[(2*b*(I + Sqrt[-b^2])*(1 + b*x))/((-1 + b^2)*((-I)*b + Sqrt[-b
^2]*x))] + (ArcCos[(1 + b^2)/(1 - b^2)] - 2*(ArcTan[Sqrt[-b^2]/(b*x)] + Arc
Tan[(b*x)/Sqrt[-b^2]]))*Log[(Sqrt[2]*Sqrt[-b^2])/(Sqrt[-1 + b^2]*E^ArcTanh[
b*x]*Sqrt[1 + b^2 + (-1 + b^2)*Cosh[2*ArcTanh[b*x]]])] + (ArcCos[(1 + b^2)/
(1 - b^2)] + 2*(ArcTan[Sqrt[-b^2]/(b*x)] + ArcTan[(b*x)/Sqrt[-b^2]]))*Log[(
Sqrt[2]*Sqrt[-b^2]*E^ArcTanh[b*x])/(Sqrt[-1 + b^2]*Sqrt[1 + b^2 + (-1 + b^2
)*Cosh[2*ArcTanh[b*x]]])] + I*(PolyLog[2, ((1 + b^2 - (2*I)*Sqrt[-b^2])*(b
- I*Sqrt[-b^2]*x))/((-1 + b^2)*(b + I*Sqrt[-b^2]*x))] - PolyLog[2, ((1 + b^
2 + (2*I)*Sqrt[-b^2])*(b - I*Sqrt[-b^2]*x))/((-1 + b^2)*(b + I*Sqrt[-b^2]*x
))])))/Sqrt[-b^2]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\operatorname{artanh}(bx)}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(b*x)/(x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(bx)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(b*x)/(x^2 - 1), x)
```

maple [A] time = 0.06, size = 176, normalized size = 1.03

$$\frac{\operatorname{arctanh}(bx)\ln(bx+b)}{2}-\frac{\operatorname{arctanh}(bx)\ln(bx-b)}{2}+\frac{\operatorname{dilog}\left(\frac{bx-1}{-b-1}\right)}{4}+\frac{\ln(bx+b)\ln\left(\frac{bx-1}{-b-1}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{bx+1}{1-b}\right)}{4}-\frac{\ln(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x)/(-x^2+1),x)
```

```
[Out] 1/2*arctanh(b*x)*ln(b*x+b)-1/2*arctanh(b*x)*ln(b*x-b)+1/4*dilog((b*x-1)/(-b
-1))+1/4*ln(b*x+b)*ln((b*x-1)/(-b-1))-1/4*dilog((b*x+1)/(1-b))-1/4*ln(b*x+b
)*ln((b*x+1)/(1-b))-1/4*dilog((b*x-1)/(-1+b))-1/4*ln(b*x-b)*ln((b*x-1)/(-1+
b))+1/4*dilog((b*x+1)/(1+b))+1/4*ln(b*x-b)*ln((b*x+1)/(1+b))
```

maxima [A] time = 0.31, size = 180, normalized size = 1.05

$$\frac{1}{4}b\left(\frac{\log(x+1)\log\left(-\frac{bx+b}{b+1}+1\right)+\operatorname{Li}_2\left(\frac{bx+b}{b+1}\right)}{b}+\frac{\log(x-1)\log\left(\frac{bx-b}{b+1}+1\right)+\operatorname{Li}_2\left(-\frac{bx-b}{b+1}\right)}{b}-\frac{\log(x+1)\log\left(-\frac{bx}{b-}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*b*((log(x + 1)*log(-(b*x + b)/(b + 1) + 1) + dilog((b*x + b)/(b + 1)))/
b + (log(x - 1)*log((b*x - b)/(b + 1) + 1) + dilog(-(b*x - b)/(b + 1)))/b -
 (log(x + 1)*log(-(b*x + b)/(b - 1) + 1) + dilog((b*x + b)/(b - 1)))/b - (l
og(x - 1)*log((b*x - b)/(b - 1) + 1) + dilog(-(b*x - b)/(b - 1)))/b) + 1/2*
(log(x + 1) - log(x - 1))*arctanh(b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(b\,x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(b*x)/(x^2 - 1),x)
```

```
[Out] -int(atanh(b*x)/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(bx)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x)/(-x**2+1),x)
```

```
[Out] -Integral(atanh(b*x)/(x**2 - 1), x)
```

3.507 $\int \frac{\tanh^{-1}(a+bx)}{1-x^2}\,dx$

Optimal. Leaf size=203

$$\frac{1}{4}\text{Li}_2\left(\frac{-a-bx+1}{-a-b+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{-a-bx+1}{-a+b+1}\right)+\frac{1}{4}\text{Li}_2\left(\frac{a+bx+1}{a-b+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{a+bx+1}{a+b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-x)}{-a-b+1}\right)\log(-a-$$

```
[Out] 1/4*ln(-b*(1-x)/(1-a-b))*ln(-b*x-a+1)-1/4*ln(b*(1+x)/(1-a+b))*ln(-b*x-a+1)-
1/4*ln(b*(1-x)/(1+a+b))*ln(b*x+a+1)+1/4*ln(-b*(1+x)/(1+a-b))*ln(b*x+a+1)+1/
4*polylog(2,(-b*x-a+1)/(1-a-b))-1/4*polylog(2,(-b*x-a+1)/(1-a+b))+1/4*polyl
og(2,(b*x+a+1)/(1+a-b))-1/4*polylog(2,(b*x+a+1)/(1+a+b))
```

Rubi [A] time = 0.26, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6115, 2409, 2394, 2393, 2391}

$$\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a+b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a+b+1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - a - b))]*Log[1 - a - b*x])/4 - (Log[(b*(1 + x))/(1
- a + b)]*Log[1 - a - b*x])/4 - (Log[(b*(1 - x))/(1 + a + b)]*Log[1 + a + b
*x])/4 + (Log[-((b*(1 + x))/(1 + a - b))]*Log[1 + a + b*x])/4 + PolyLog[2,
(1 - a - b*x)/(1 - a - b)]/4 - PolyLog[2, (1 - a - b*x)/(1 - a + b)]/4 + Po
lyLog[2, (1 + a + b*x)/(1 + a - b)]/4 - PolyLog[2, (1 + a + b*x)/(1 + a + b
)]/4
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6115

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
```

```
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{1-x^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{1-x^2}\,dx\right) + \frac{1}{2}\int \frac{\log(1+a+bx)}{1-x^2}\,dx$$
$$= -\left(\frac{1}{2}\int \left(\frac{\log(1-a-bx)}{2(1-x)} + \frac{\log(1-a-bx)}{2(1+x)}\right)dx\right) + \frac{1}{2}\int \left(\frac{\log(1+a+bx)}{2(1-x)} + \frac{\log(1+a}{2(1+}\right.$$
$$= -\left(\frac{1}{4}\int \frac{\log(1-a-bx)}{1-x}\,dx\right) - \frac{1}{4}\int \frac{\log(1-a-bx)}{1+x}\,dx + \frac{1}{4}\int \frac{\log(1+a+bx)}{1-x}\,dx + \frac{1}{4}$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx) - \frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx) - \frac{1}{4}\log\left(\frac{b(1-}{1+a}\right.$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx) - \frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx) - \frac{1}{4}\log\left(\frac{b(1-}{1+a}\right.$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx) - \frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx) - \frac{1}{4}\log\left(\frac{b(1-}{1+a}\right.$$

Mathematica [A] time = 0.04, size = 203, normalized size = 1.00

$$\frac{1}{4}\text{Li}_2\left(\frac{-a-bx+1}{-a-b+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{-a-bx+1}{-a+b+1}\right)+\frac{1}{4}\text{Li}_2\left(\frac{a+bx+1}{a-b+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{a+bx+1}{a+b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-x)}{-a-b+1}\right)\log(-$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - a - b))]*Log[1 - a - b*x])/4 - (Log[(b*(1 + x))/(1
- a + b)]*Log[1 - a - b*x])/4 - (Log[(b*(1 - x))/(1 + a + b)]*Log[1 + a + b
*x])/4 + (Log[-((b*(1 + x))/(1 + a - b))]*Log[1 + a + b*x])/4 + PolyLog[2,
(1 - a - b*x)/(1 - a - b)]/4 - PolyLog[2, (1 - a - b*x)/(1 - a + b)]/4 + Po
lyLog[2, (1 + a + b*x)/(1 + a - b)]/4 - PolyLog[2, (1 + a + b*x)/(1 + a + b
)]/4
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{artanh}(bx+a)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(b*x + a)/(x^2 - 1), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.08, size = 196, normalized size = 0.97

$$\frac{\operatorname{arctanh}(bx+a)\ln(bx+b)}{2}-\frac{\operatorname{arctanh}(bx+a)\ln(bx-b)}{2}+\frac{\operatorname{dilog}\left(\frac{bx+a-1}{-b-1+a}\right)}{4}+\frac{\ln(bx+b)\ln\left(\frac{bx+a-1}{-b-1+a}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{bx+a\cdots}{1+a\cdots}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(-x^2+1),x)
```

```
[Out] 1/2*arctanh(b*x+a)*ln(b*x+b)-1/2*arctanh(b*x+a)*ln(b*x-b)+1/4*dilog((b*x+a-
1)/(-b-1+a))+1/4*ln(b*x+b)*ln((b*x+a-1)/(-b-1+a))-1/4*dilog((b*x+a+1)/(1+a-
b))-1/4*ln(b*x+b)*ln((b*x+a+1)/(1+a-b))+1/4*dilog((b*x+a+1)/(1+a+b))+1/4*ln
(b*x-b)*ln((b*x+a+1)/(1+a+b))-1/4*dilog((b*x+a-1)/(b-1+a))-1/4*ln(b*x-b)*ln
((b*x+a-1)/(b-1+a))
```

maxima [A] time = 0.32, size = 198, normalized size = 0.98

$$\frac{1}{4}b\left(\frac{\log(x-1)\log\left(\frac{bx-b}{a+b+1}+1\right)+\operatorname{Li}_2\left(-\frac{bx-b}{a+b+1}\right)}{b}-\frac{\log(x-1)\log\left(\frac{bx-b}{a+b-1}+1\right)+\operatorname{Li}_2\left(-\frac{bx-b}{a+b-1}\right)}{b}-\frac{\log(x+1)\log\left(\frac{b\cdots}{a-\cdots}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*b*((log(x - 1)*log((b*x - b)/(a + b + 1) + 1) + dilog(-(b*x - b)/(a + b
 + 1)))/b - (log(x - 1)*log((b*x - b)/(a + b - 1) + 1) + dilog(-(b*x - b)/(
a + b - 1)))/b - (log(x + 1)*log((b*x + b)/(a - b + 1) + 1) + dilog(-(b*x +
 b)/(a - b + 1)))/b + (log(x + 1)*log((b*x + b)/(a - b - 1) + 1) + dilog(-(
b*x + b)/(a - b - 1)))/b) + 1/2*(log(x + 1) - log(x - 1))*arctanh(b*x + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\operatorname{atanh}(a+b\,x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a + b*x)/(x^2 - 1),x)
```

```
[Out] -int(atanh(a + b*x)/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(a+bx)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(-x**2+1),x)
```

```
[Out] -Integral(atanh(a + b*x)/(x**2 - 1), x)
```

3.508 $\int \frac{\tanh^{-1}(x)}{a+bx} \, dx$

Optimal. Leaf size=86

$$-\frac{\text{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(x+1)}\right)}{2b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(x+1)(a+b)}\right)}{b}+\frac{\text{Li}_2\left(1-\frac{2}{x+1}\right)}{2b}-\frac{\log\left(\frac{2}{x+1}\right)\tanh^{-1}(x)}{b}$$

```
[Out] -arctanh(x)*ln(2/(1+x))/b+arctanh(x)*ln(2*(b*x+a)/(a+b)/(1+x))/b+1/2*polylo
g(2,1-2/(1+x))/b-1/2*polylog(2,1-2*(b*x+a)/(a+b)/(1+x))/b
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5920, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2(a+bx)}{(x+1)(a+b)}\right)}{2b}+\frac{\text{PolyLog}\left(2,1-\frac{2}{x+1}\right)}{2b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(x+1)(a+b)}\right)}{b}-\frac{\log\left(\frac{2}{x+1}\right)\tanh^{-1}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x),x]
```

```
[Out] -((ArcTanh[x]*Log[2/(1 + x)])/b) + (ArcTanh[x]*Log[(2*(a + b*x))/((a + b)*(
1 + x))])/b + PolyLog[2, 1 - 2/(1 + x)]/(2*b) - PolyLog[2, 1 - (2*(a + b*x)
)/((a + b)*(1 + x))]/(2*b)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{a+bx}\,dx = -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b} + \frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b} + \frac{\int \frac{\log\left(\frac{2}{1+x}\right)}{1-x^2}\,dx}{b} - \frac{\int \frac{\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{1-x^2}\,dx}{b}$$
$$= -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b} + \frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b} - \frac{\operatorname{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(1+x)}\right)}{2b} + \frac{\operatorname{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx\right.}{b}$$
$$= -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b} + \frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b} + \frac{\operatorname{Li}_2\left(1-\frac{2}{1+x}\right)}{2b} - \frac{\operatorname{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(1+x)}\right)}{2b}$$

Mathematica [C] time = 0.09, size = 260, normalized size = 3.02

$$\frac{-4\operatorname{Li}_2\left(e^{-2\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)}\right) + 8\tanh^{-1}(x)\tanh^{-1}\left(\frac{a}{b}\right) + 8\tanh^{-1}\left(\frac{a}{b}\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)}\right) + 8\tanh^{-1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[x]/(a + b*x),x]
```

```
[Out] (-Pi^2 + 4*ArcTanh[a/b]^2 + (4*I)*Pi*ArcTanh[x] + 8*ArcTanh[a/b]*ArcTanh[x]
 + 8*ArcTanh[x]^2 - (4*I)*Pi*Log[1 + E^(2*ArcTanh[x])] - 8*ArcTanh[x]*Log[1
 + E^(2*ArcTanh[x])] + 8*ArcTanh[a/b]*Log[1 - E^(-2*(ArcTanh[a/b] + ArcTanh
[x]))] + 8*ArcTanh[x]*Log[1 - E^(-2*(ArcTanh[a/b] + ArcTanh[x]))] + (4*I)*P
i*Log[2/Sqrt[1 - x^2]] + 8*ArcTanh[x]*Log[2/Sqrt[1 - x^2]] + 4*ArcTanh[x]*L
og[1 - x^2] + 8*ArcTanh[x]*Log[I*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 8*ArcTa
nh[a/b]*Log[(2*I)*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 8*ArcTanh[x]*Log[(2*I)
*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 4*PolyLog[2, -E^(2*ArcTanh[x])] - 4*Pol
yLog[2, E^(-2*(ArcTanh[a/b] + ArcTanh[x]))])/(8*b)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{artanh}(x)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(x)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(b*x + a), x)
```

maple [A] time = 0.07, size = 110, normalized size = 1.28

$$\frac{\ln(bx+a)\operatorname{arctanh}(x)}{b} - \frac{\ln(bx+a)\ln\left(\frac{bx+b}{-a+b}\right)}{2b} - \frac{\operatorname{dilog}\left(\frac{bx+b}{-a+b}\right)}{2b} + \frac{\ln(bx+a)\ln\left(\frac{bx-b}{-a-b}\right)}{2b} + \frac{\operatorname{dilog}\left(\frac{bx-b}{-a-b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b*arctanh(x)-1/2/b*ln(b*x+a)*ln((b*x+b)/(-a+b))-1/2/b*dilog((b*x+
b)/(-a+b))+1/2/b*ln(b*x+a)*ln((b*x-b)/(-a-b))+1/2/b*dilog((b*x-b)/(-a-b))
```

maxima [A] time = 0.30, size = 119, normalized size = 1.38

$$-\frac{\left(\log\left(x+1\right)-\log\left(x-1\right)\right)\log\left(bx+a\right)}{2\,b}+\frac{\operatorname{artanh}(x)\log\left(bx+a\right)}{b}-\frac{\log\left(x-1\right)\log\left(\frac{bx-b}{a+b}+1\right)+\operatorname{Li}_2\left(-\frac{bx-b}{a+b}\right)}{2\,b}+\frac{\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(x + 1) - log(x - 1))*log(b*x + a)/b + arctanh(x)*log(b*x + a)/b -
 1/2*(log(x - 1)*log((b*x - b)/(a + b) + 1) + dilog(-(b*x - b)/(a + b)))/b
+ 1/2*(log(x + 1)*log((b*x + b)/(a - b) + 1) + dilog(-(b*x + b)/(a - b)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(x)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a + b*x),x)
```

```
[Out] int(atanh(x)/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(x\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(b*x+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x), x)
```

3.509 $\int \frac{\tanh^{-1}(x)}{a+bx^2}\,dx$

Optimal. Leaf size=397

$$-\frac{\text{Li}_2\left(-\frac{\sqrt{b}(1-x)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{Li}_2\left(\frac{\sqrt{b}(1-x)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\text{Li}_2\left(-\frac{\sqrt{b}(x+1)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{Li}_2\left(\frac{\sqrt{b}(x+1)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(x+1)\log\left(\cdots\right.}{4\sqrt{-a}\sqrt{\cdots}}$$

```
[Out] -1/4*ln(1-x)*ln(((-a)^(1/2)-x*b^(1/2))/((-a)^(1/2)-b^(1/2)))/(-a)^(1/2)/b^(
1/2)+1/4*ln(1+x)*ln(((-a)^(1/2)-x*b^(1/2))/((-a)^(1/2)+b^(1/2)))/(-a)^(1/2)
/b^(1/2)-1/4*ln(1+x)*ln(((-a)^(1/2)+x*b^(1/2))/((-a)^(1/2)-b^(1/2)))/(-a)^(
1/2)/b^(1/2)+1/4*ln(1-x)*ln(((-a)^(1/2)+x*b^(1/2))/((-a)^(1/2)+b^(1/2)))/(-
a)^(1/2)/b^(1/2)-1/4*polylog(2,-(1-x)*b^(1/2)/((-a)^(1/2)-b^(1/2)))/(-a)^(1
/2)/b^(1/2)-1/4*polylog(2,-(1+x)*b^(1/2)/((-a)^(1/2)-b^(1/2)))/(-a)^(1/2)/b
^(1/2)+1/4*polylog(2,(1-x)*b^(1/2)/((-a)^(1/2)+b^(1/2)))/(-a)^(1/2)/b^(1/2)
+1/4*polylog(2,(1+x)*b^(1/2)/((-a)^(1/2)+b^(1/2)))/(-a)^(1/2)/b^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5972, 2409, 2394, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{b}(1-x)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(1-x)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{b}(x+1)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(x+1)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(1-x)\,\text{lo}\cdots}{4\sqrt{-}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x^2),x]
```

```
[Out] -(Log[1 - x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] - Sqrt[b])])/(4*Sqrt[-a]*
Sqrt[b]) + (Log[1 + x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] + Sqrt[b])])/(4
*Sqrt[-a]*Sqrt[b]) - (Log[1 + x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sqrt[-a] - Sqr
t[b])])/(4*Sqrt[-a]*Sqrt[b]) + (Log[1 - x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sqrt
[-a] + Sqrt[b])])/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 - x))/(Sq
rt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 - x))/(Sq
rt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 + x))/(S
qrt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 + x))/(S
qrt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{a+bx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-x)}{a+bx^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+x)}{a+bx^2}\,dx$$

$$= -\left(\frac{1}{2}\int\left(\frac{\sqrt{-a}\,\log(1-x)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{-a}\,\log(1-x)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx\right)+\frac{1}{2}\int\left(\frac{\sqrt{-a}\,\log(1+x)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{-a}\,\mathrm{lo}}{2a\left(\sqrt{-a}\right.}\right.$$

$$= \frac{\int \frac{\log(1-x)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}}+\frac{\int \frac{\log(1-x)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}}-\frac{\int \frac{\log(1+x)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}}-\frac{\int \frac{\log(1+x)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{\log(1+x)\log\left(\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\mathrm{lo}}{}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{\log(1+x)\log\left(\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\mathrm{lo}}{}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{\log(1+x)\log\left(\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\mathrm{lo}}{}$$

Mathematica [C] time = 1.13, size = 485, normalized size = 1.22

$$-\frac{i\left(\mathrm{Li}_2\left(\frac{\left(-a+b+2i\sqrt{ab}\right)\left(ia+\sqrt{ab}\,x\right)}{(a+b)\left(\sqrt{ab}\,x-ia\right)}\right)-\mathrm{Li}_2\left(\frac{\left(-a+b-2i\sqrt{ab}\right)\left(ia+\sqrt{ab}\,x\right)}{(a+b)\left(\sqrt{ab}\,x-ia\right)}\right)\right)-2i\cos^{-1}\left(\frac{b-a}{a+b}\right)\tan^{-1}\left(\frac{bx}{\sqrt{ab}}\right)+4\tanh^{-1}(x)\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a + b*x^2),x]
```

```
[Out] -1/4*((-2*I)*ArcCos[(-a + b)/(a + b)]*ArcTan[(b*x)/Sqrt[a*b]] + 4*ArcTan[a/
(Sqrt[a*b]*x)]*ArcTanh[x] - (ArcCos[(-a + b)/(a + b)] + 2*ArcTan[(b*x)/Sqrt
[a*b]])*Log[((2*I)*a*(I*b + Sqrt[a*b])*(-1 + x))/((a + b)*(a + I*Sqrt[a*b]*
x))] - (ArcCos[(-a + b)/(a + b)] - 2*ArcTan[(b*x)/Sqrt[a*b]])*Log[(2*a*(b +
 I*Sqrt[a*b])*(1 + x))/((a + b)*(a + I*Sqrt[a*b]*x))] + (ArcCos[(-a + b)/(a
 + b)] + 2*(ArcTan[a/(Sqrt[a*b]*x)] + ArcTan[(b*x)/Sqrt[a*b]]))*Log[(Sqrt[2
]*Sqrt[a*b])/(Sqrt[a + b]*E^ArcTanh[x]*Sqrt[a - b + (a + b)*Cosh[2*ArcTanh[
x]]])] + (ArcCos[(-a + b)/(a + b)] - 2*(ArcTan[a/(Sqrt[a*b]*x)] + ArcTan[(b
*x)/Sqrt[a*b]]))*Log[(Sqrt[2]*Sqrt[a*b]*E^ArcTanh[x])/(Sqrt[a + b]*Sqrt[a -
 b + (a + b)*Cosh[2*ArcTanh[x]]])] + I*(-PolyLog[2, ((-a + b - (2*I)*Sqrt[a
*b])*(I*a + Sqrt[a*b]*x))/((a + b)*((-I)*a + Sqrt[a*b]*x))] + PolyLog[2, ((
```

```
-a + b + (2*I)*Sqrt[a*b])*(I*a + Sqrt[a*b]*x))/((a + b)*((-I)*a + Sqrt[a*b]
*x))]))/Sqrt[a*b]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(x)}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(x)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(b*x^2 + a), x)
```

maple [B] time = 0.62, size = 606, normalized size = 1.53

$$-\frac{\sqrt{-ab}\ \operatorname{arctanh}(x)\ln\left(1-\frac{(a+b)(1+x)^2}{(-x^2+1)\left(2\sqrt{-ab}-a+b\right)}\right)}{2ab}+\frac{\sqrt{-ab}\ \operatorname{arctanh}(x)^2}{2ab}-\frac{\sqrt{-ab}\ \operatorname{polylog}\left(2,\frac{(a+b)(1+x)^2}{(-x^2+1)\left(2\sqrt{-ab}-a+b\right)}\right)}{4ab}-\frac{(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(b*x^2+a),x)
```

```
[Out] -1/2*(-a*b)^(1/2)/a/b*arctanh(x)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(2*(-a*b)^(1/2
)-a+b))+1/2*(-a*b)^(1/2)/a/b*arctanh(x)^2-1/4*(-a*b)^(1/2)/a/b*polylog(2,(a
+b)*(1+x)^2/(-x^2+1)/(2*(-a*b)^(1/2)-a+b))-(-2*(-a*b)^(1/2)+a-b)/(a^2+2*a*b
+b^2)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*arctanh(x)+1/2*(2*
a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/b/(a^2+2*a*b+b^2)*ln(1-(a+b)*(1+x)^2/(-x
^2+1)/(-2*(-a*b)^(1/2)-a+b))*arctanh(x)-1/2*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1
/2)*b)/a/(a^2+2*a*b+b^2)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))
*arctanh(x)-1/(a^2+2*a*b+b^2)*arctanh(x)^2*(-a*b)^(1/2)-1/2/b/(a^2+2*a*b+b^
2)*arctanh(x)^2*a*(-a*b)^(1/2)-1/2/a/(a^2+2*a*b+b^2)*arctanh(x)^2*b*(-a*b)^
(1/2)+1/2/(a^2+2*a*b+b^2)*polylog(2,(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)
-a+b))*(-a*b)^(1/2)+1/4/b/(a^2+2*a*b+b^2)*polylog(2,(a+b)*(1+x)^2/(-x^2+1)/
(-2*(-a*b)^(1/2)-a+b))*a*(-a*b)^(1/2)+1/4/a/(a^2+2*a*b+b^2)*polylog(2,(a+b)
*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*b*(-a*b)^(1/2)
```

maxima [C] time = 0.52, size = 304, normalized size = 0.77

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{artanh}(x)}{\sqrt{ab}}+\frac{\left(\arctan\left(\frac{\sqrt{a}\sqrt{b}(x+1)}{a+b},\frac{bx+b}{a+b}\right)-\arctan\left(\frac{\sqrt{a}\sqrt{b}(x-1)}{a+b},-\frac{bx-b}{a+b}\right)\right)\log\left(bx^2+a\right)-\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))*arctanh(x)/sqrt(a*b) + 1/4*((arctan2(sqrt(a)*sqrt(b)*
(x + 1)/(a + b), (b*x + b)/(a + b)) - arctan2(sqrt(a)*sqrt(b)*(x - 1)/(a +
b), -(b*x - b)/(a + b)))*log(b*x^2 + a) - arctan(sqrt(b)*x/sqrt(a))*log((b*
x^2 + 2*b*x + b)/(a + b)) + arctan(sqrt(b)*x/sqrt(a))*log((b*x^2 - 2*b*x +
```

```
b)/(a + b)) - I*dilog(-(b*x - sqrt(a)*sqrt(b)*(I*x + I) - a)/(a + 2*I*sqrt(
a)*sqrt(b) - b)) - I*dilog((b*x - sqrt(a)*sqrt(b)*(I*x - I) + a)/(a + 2*I*s
qrt(a)*sqrt(b) - b)) + I*dilog(-(b*x + sqrt(a)*sqrt(b)*(I*x + I) - a)/(a -
2*I*sqrt(a)*sqrt(b) - b)) + I*dilog((b*x + sqrt(a)*sqrt(b)*(I*x - I) + a)/(
a - 2*I*sqrt(a)*sqrt(b) - b)))/sqrt(a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(x)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a + b*x^2),x)
```

```
[Out] int(atanh(x)/(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(b*x**2+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x**2), x)
```

3.510 $\int \frac{\tanh^{-1}(x)}{a+bx+cx^2} \, dx$

Optimal. Leaf size=258

$$-\frac{\text{Li}_2\left(1-\frac{2\left(b+2cx-\sqrt{b^2-4ac}\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(x+1)}\right)}{2\sqrt{b^2-4ac}}+\frac{\text{Li}_2\left(1-\frac{2\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(x+1)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tanh^{-1}(x)\log\left(\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\tanh^{-1}(x)\log}{\sqrt{}}$$

```
[Out] arctanh(x)*ln(2*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1+x)/(b+2*c-(-4*a*c+b^2)^(1/2
)))/(-4*a*c+b^2)^(1/2)-arctanh(x)*ln(2*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(1+x)/(
b+2*c+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)-1/2*polylog(2,1-2*(b+2*c*x-(-
4*a*c+b^2)^(1/2))/(1+x)/(b+2*c-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+1/2*
polylog(2,1-2*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(1+x)/(b+2*c+(-4*a*c+b^2)^(1/2))
)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {618, 206, 6728, 5920, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(\sqrt{b^2-4ac}+b+2c\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tanh^{-1}(x)\log\left(\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcTanh[x]*Log[(2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2 -
4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c] - (ArcTanh[x]*Log[(2*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c]
 - PolyLog[2, 1 - (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2
- 4*a*c])*(1 + x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 - (2*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))]/(2*Sqrt[b^2 -
4*a*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(x)}{a+bx+cx^2}\,dx &= \int \left(\frac{2c\tanh^{-1}(x)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)}-\frac{2c\tanh^{-1}(x)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx\\
&= \frac{(2c)\int \frac{\tanh^{-1}(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{\tanh^{-1}(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}}-\frac{\tanh^{-1}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}}-\frac{\int \frac{\log\left(\frac{2\left(b-\sqrt{b^2\cdots}\right.}{\left(b+2c-\sqrt{\cdots}\right.}\right.}{1-\cdots}}{\sqrt{b^2\cdots}}\\
&= \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}}-\frac{\tanh^{-1}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}}-\frac{\text{Li}_2\left(1-\frac{2(\cdots}{(b+2\cdots}\right.}{2\sqrt{b\cdots}}
\end{aligned}
$$

Mathematica [C] time = 19.32, size = 874, normalized size = 3.39

$$
\frac{2\sqrt{4ac-b^2}\left(b\left(\sqrt{\frac{c(a+b+c)}{4ac-b^2}}\,e^{i\tan^{-1}\left(\frac{-b-2c}{\sqrt{4ac-b^2}}\right)}-\sqrt{\frac{c(a-b+c)}{4ac-b^2}}\,e^{i\tan^{-1}\left(\frac{2c-b}{\sqrt{4ac-b^2}}\right)}\right)-2c\left(e^{i\tan^{-1}\left(\frac{2c-b}{\sqrt{4ac-b^2}}\right)}\sqrt{\frac{c(a-b+c)}{4ac-b^2}}+\sqrt{\frac{c(a+b+c)}{4ac-b^2}}\,e^{i\tan^{-1}\left(\frac{-b-2c}{\sqrt{4ac-b^2}}\right)}-1\right)\right)\tan\cdots}{b^2-4c^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[x]/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[-b^2 + 4*a*c]*(b*(Sqrt[(c*(a + b + c))/(-b^2 + 4*a*c)]*E^(I*ArcTan
[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]) - Sqrt[(c*(a - b + c))/(-b^2 + 4*a*c)]*E^(
I*ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]])) - 2*c*(-1 + Sqrt[(c*(a + b + c))/
(-b^2 + 4*a*c)]*E^(I*ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]) + Sqrt[(c*(a -
b + c))/(-b^2 + 4*a*c)]*E^(I*ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]])))*ArcTa
```

```
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]^2)/(b^2 - 4*c^2) + 2*ArcTan[(b + 2*c*x)/S
qrt[-b^2 + 4*a*c]]*((-I)*ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + I*ArcTan[(
-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + 2*ArcTanh[x] + Log[1 - E^((2*I)*(ArcTan[(-b
 - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]))] - L
og[1 - E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]]))]) + 2*(ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]*(Log[1
- E^((2*I)*(ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]]))] - Log[Sin[ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]]]) + ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]]
*(-Log[1 - E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*
c*x)/Sqrt[-b^2 + 4*a*c]]))] + Log[Sin[ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]]
 + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]]])) - I*PolyLog[2, E^((2*I)*(ArcT
an[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
)] + I*PolyLog[2, E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]))])/(2*Sqrt[-b^2 + 4*a*c])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(x)}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(x)}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(c*x^2 + b*x + a), x)
```

maple [B] time = 0.61, size = 1599, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(c*x^2+b*x+a),x)
```

```
[Out] -1/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*a^2*(-4*a*c+b^2)^(1/2)-1/(a
^2+2*a*c-b^2+c^2)*arctanh(x)^2*(-4*a*c+b^2)^(1/2)+1/(a^2+2*a*c-b^2+c^2)*arc
tanh(x)^2*a-1/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*c+1/2/(a^2+2*a*c-b^2+c^2)*po
lylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*(-4*a*c+b^2)^(1
/2)-1/2/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^
2)^(1/2)-a+c))*a-1/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*c*b^2+1/(4*
a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*a*b^2-1/(4*a*c-b^2)/(a^2+2*a*c-b^
2+c^2)*arctanh(x)^2*c^2*(-4*a*c+b^2)^(1/2)+1/2/(a^2+2*a*c-b^2+c^2)*polylog(
2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*c-1/2*(-4*a*c+b^2)^(1
/2)/(4*a*c-b^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/((-4*a*c+b^2)^(1/2)-a+c)
)+(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*arctanh(x)^2+2/(4*a*c-b^2)/(a^2+2*a*c-b^2+
c^2)*arctanh(x)^2*a*c*(-4*a*c+b^2)^(1/2)-(4*a*c-b^2+(-4*a*c+b^2)^(1/2)*a-(-
4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*ln(1-(a+b+c)*(1+x)^2/(-
x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*arctanh(x)*c-1/(4*a*c-b^2)/(a^2+2*a*c-b^2
+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*a*c*(-4
*a*c+b^2)^(1/2)+(4*a*c-b^2+(-4*a*c+b^2)^(1/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*
```

```
c-b^2)/(a^2+2*a*c-b^2+c^2)*ln(1-(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/
2)-a+c))*arctanh(x)*a+1/2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)
*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*a^2*(-4*a*c+b^2)^(1/2)+2/(4*a*
c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2
)^(1/2)-a+c))*a^2*c-1/2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(
1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*a*b^2+1/2/(4*a*c-b^2)/(a^2+2*a*c
-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*c*b
^2+1/2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(
-(-4*a*c+b^2)^(1/2)-a+c))*c^2*(-4*a*c+b^2)^(1/2)-2/(4*a*c-b^2)/(a^2+2*a*c-b
^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*c^2*a
-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*arctanh(x)*ln(1-(a+b+c)*(1+x)^2/(-x^2+1)/((
-4*a*c+b^2)^(1/2)-a+c))-(-(-4*a*c+b^2)^(1/2)+a-c)/(a^2+2*a*c-b^2+c^2)*ln(1-
(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*arctanh(x)+4/(4*a*c-b^2
)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*c^2*a-4/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*
arctanh(x)^2*a^2*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(x)}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a + b*x + c*x^2),x)
```

```
[Out] int(atanh(x)/(a + b*x + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}{\left(x \right)}}{a + bx + cx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x + c*x**2), x)
```

3.511 $\int \sqrt{c+dx^2} \tanh^{-1}(ax) \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\tanh^{-1}(ax)\sqrt{c+dx^2},x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{c+dx^2} \tanh^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2} \tanh^{-1}(ax) \, dx = \int \sqrt{c+dx^2} \tanh^{-1}(ax) \, dx$$

Mathematica [A] time = 4.25, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2} \tanh^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
[Out] Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{dx^2+c} \, \text{artanh}(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2+c} \, \text{artanh}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

maple [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \sqrt{d\,x^2+c} \, \operatorname{arctanh}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

```
[Out] int((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c}\, \operatorname{artanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{atanh}(a\,x)\,\sqrt{d\,x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)*(c + d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2}\, \operatorname{atanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*atanh(a*x),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*atanh(a*x), x)
```

3.512 $\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}}\, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
[Out] Defer[Int][ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}}\, dx = \int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}}\, dx$$

Mathematica [A] time = 4.10, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
[Out] Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\,(ax)}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\,(ax)}{\sqrt{dx^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

maple [A] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/sqrt(c + d*x**2), x)
```

3.513 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=62

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

```
[Out] -arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c/(a^2*c+d)^(1/2)+x*arctanh(a*x
)/c/(d*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {191, 5976, 12, 444, 63, 208}

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcTanh[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c + d]]/(c*Sqrt[a^2*c + d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
```

```
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\left(1-a^2x^2\right)\sqrt{c+dx^2}}\,dx\\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{\left(1-a^2x^2\right)\sqrt{c+dx^2}}\,dx}{c}\\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\left(1-a^2x\right)\sqrt{c+dx}}\,dx, x, x^2\right)}{2c}\\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{cd}\\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 119, normalized size = 1.92

$$\frac{\frac{-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx\right)-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx\right)+\log(1-ax)+\log(ax+1)}{\sqrt{a^2c+d}}+\frac{2x\tanh^{-1}(ax)}{\sqrt{c+dx^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*x*ArcTanh[a*x])/Sqrt[c + d*x^2] + (Log[1 - a*x] + Log[1 + a*x] - Log[a*
c - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]] - Log[a*c + d*x + Sqrt[a^2*c + d
]*Sqrt[c + d*x^2]])/Sqrt[a^2*c + d])/(2*c)
```

fricas [B] time = 0.82, size = 356, normalized size = 5.74

$$\left[\frac{2\left(a^2c+d\right)\sqrt{dx^2+c}\,x\log\left(-\frac{ax+1}{ax-1}\right)+\sqrt{a^2c+d}\left(dx^2+c\right)\log\left(\frac{a^4d^2x^4+8a^4c^2+8a^2cd+2\left(4a^4cd+3a^2d^2\right)x^2-4\left(a^3dx^2+2a^3c+a\right.}{a^4x^4-2a^2x^2+1}\right.}{4\left(a^2c^3+c^2d+\left(a^2c^2d+cd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)) + sqrt(a^2*
c + d)*(d*x^2 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d
+ 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2
 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)))/(a^2*c^3 + c^2*d + (a^2*c^2*d + c*
d^2)*x^2), 1/2*((a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)) + s
qrt(-a^2*c - d)*(d*x^2 + c)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*
c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)))/(a^2*c^3
 + c^2*d + (a^2*c^2*d + c*d^2)*x^2)]
```

giac [A] time = 0.18, size = 71, normalized size = 1.15

$$\frac{x \log\left(-\frac{ax+1}{ax-1}\right)}{2 \sqrt{dx^2+c}\,c}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{\sqrt{-a^2c-d}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*log(-(a*x + 1)/(a*x - 1))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d*x^2 + c
)*a/sqrt(-a^2*c - d))/(sqrt(-a^2*c - d)*c)
```

maple [F] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)}{\left(d\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

maxima [B] time = 0.35, size = 153, normalized size = 2.47

$$\frac{a^2\left(\frac{\operatorname{arsinh}\left(-\frac{2\,a^2c}{\sqrt{cd}\,|2\,a^2x+2\,a|}+\frac{2\,adx}{\sqrt{cd}\,|2\,a^2x+2\,a|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}-\frac{\operatorname{arsinh}\left(\frac{2\,a^2c}{\sqrt{cd}\,|2\,a^2x-2\,a|}+\frac{2\,adx}{\sqrt{cd}\,|2\,a^2x-2\,a|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}\right)}{2\,c}+\frac{x\operatorname{artanh}(ax)}{\sqrt{dx^2+c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(arcsinh(-2*a^2*c/(sqrt(c*d)*abs(2*a^2*x + 2*a)) + 2*a*d*x/(sqrt(c*
d)*abs(2*a^2*x + 2*a)))/(a^3*sqrt(c + d/a^2)) - arcsinh(2*a^2*c/(sqrt(c*d)*
abs(2*a^2*x - 2*a)) + 2*a*d*x/(sqrt(c*d)*abs(2*a^2*x - 2*a)))/(a^3*sqrt(c +
 d/a^2)))/c + x*arctanh(a*x)/(sqrt(d*x^2 + c)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(3/2), x)
```

3.514 $\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{\left(3a^2c+2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\left(a^2c+d\right)^{3/2}}+\frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

```
[Out] 1/3*x*arctanh(a*x)/c/(d*x^2+c)^(3/2)-1/3*(3*a^2*c+2*d)*arctanh(a*(d*x^2+c)^
(1/2)/(a^2*c+d)^(1/2))/c^2/(a^2*c+d)^(3/2)+1/3*a/c/(a^2*c+d)/(d*x^2+c)^(1/2
)+2/3*x*arctanh(a*x)/c^2/(d*x^2+c)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5976, 6688, 12, 571, 78, 63, 208}

$$-\frac{\left(3a^2c+2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\left(a^2c+d\right)^{3/2}}+\frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] a/(3*c*(a^2*c + d)*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(3*c*(c + d*x^2)^(3/
2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c + 2*d)*ArcTanh
[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(3*c^2*(a^2*c + d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{\frac{x}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1-a^2x^2}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x\left(3c+2dx^2\right)}{3c^2\left(1-a^2x^2\right)\left(c+dx^2\right)^{3/2}}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(3c+2dx^2\right)}{\left(1-a^2x^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \frac{3c+2dx}{\left(1-a^2x\right)(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\left(3a^2c+2d\right)\right)\text{Subst}\left(\int \frac{}{(1-\cdots}\right.}{6c^2\left(a^2c+\cdots\right.}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\left(3a^2c+2d\right)\right)\text{Subst}\left(\int \frac{}{1+\cdots}\right.}{3c^2d\left(a^2c\cdots\right.}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(3a^2c+2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right.}{3c^2\left(a^2c+d\right)^{3/2}}$$

Mathematica [A] time = 0.31, size = 226, normalized size = 1.77

$$\frac{\frac{2ac}{\left(a^2c+d\right)\sqrt{c+dx^2}} - \frac{\left(3a^2c+2d\right)\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx\right)}{\left(a^2c+d\right)^{3/2}} - \frac{\left(3a^2c+2d\right)\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx\right)}{\left(a^2c+d\right)^{3/2}} + \frac{\left(3a^2c+2d\right)\log(1-ax)}{\left(a^2c+d\right)^{3/2}} + \frac{\left(3a^2c+2a\cdots\right.}{\left(a^2c\cdots\right.}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*x*(3*c + 2*d*x^2)*ArcTanh[a*x])
/(c + d*x^2)^(3/2) + ((3*a^2*c + 2*d)*Log[1 - a*x])/(a^2*c + d)^(3/2) + ((3
*a^2*c + 2*d)*Log[1 + a*x])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d)*Log[a*c -
d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d
)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2))/(6*c
^2)
```

fricas [B] time = 0.60, size = 730, normalized size = 5.70

$$\left[\frac{\left(3\,a^2c^3 + \left(3\,a^2cd^2 + 2\,d^3\right)x^4 + 2\,c^2d + 2\left(3\,a^2c^2d + 2\,cd^2\right)x^2\right)\sqrt{a^2c+d}\,\log\left(\frac{a^4d^2x^4 + 8\,a^4c^2 + 8\,a^2cd + 2\left(4\,a^4cd + 3\,a^2d^2\right)x^2 - \cdots}{a^4x^4 - 2\,a\cdots}\right.}{12\left(a^4c^6 + 2\,a^2c^5d + \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d +
2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*
(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)
*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a*c^2
*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 +
 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2
 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^
2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*a^2*c^3
+ (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x^2)*sqrt
(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x
^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3 + 2*a*c^2*d
 + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 + 3
*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 +
 c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2*
d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

giac [A] time = 0.21, size = 135, normalized size = 1.05

$$\frac{1}{3}\,a\left(\frac{\left(3\,a^2c+2\,d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{\left(a^2c^3+c^2d\right)\sqrt{-a^2c-d}\,a}+\frac{1}{\left(a^2c^2+cd\right)\sqrt{dx^2+c}}\right)+\frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c + 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^2*c^3
 + c^2*d)*sqrt(-a^2*c - d)*a) + 1/((a^2*c^2 + c*d)*sqrt(d*x^2 + c))) + 1/6*
x*(2*d*x^2/c^2 + 3/c)*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(3/2)
```

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(ax\right)}{\left(d\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

maxima [B] time = 0.42, size = 223, normalized size = 1.74

$$\frac{1}{6}\,a\left(\frac{\frac{ad\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\left(a^2c^2+cd\right)\sqrt{a^2c+d}}+\frac{2\,d}{\left(a^2c^2+cd\right)\sqrt{dx^2+c}}}{d}+\frac{2\,\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\sqrt{a^2c+d}\,ac^2}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{dx^2+c}\,c^2}+\frac{x}{\left(dx^2+c\right)^{\frac{3}{2}}c}\right)\operatorname{artanl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*
a^2 + sqrt(a^2*c + d)*a))/((a^2*c^2 + c*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c^2
 + c*d)*sqrt(d*x^2 + c)))/d + 2*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*
a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^2)) + 1/
3*(2*x/(sqrt(d*x^2 + c)*c^2) + x/((d*x^2 + c)^(3/2)*c))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(c+dx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(5/2), x)
```

3.515 $\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}-\frac{\left(15a^4c^2+20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3\left(a^2c+d\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

```
[Out] 1/15*a/c/(a^2*c+d)/(d*x^2+c)^(3/2)+1/5*x*arctanh(a*x)/c/(d*x^2+c)^(5/2)+4/1
5*x*arctanh(a*x)/c^2/(d*x^2+c)^(3/2)-1/15*(15*a^4*c^2+20*a^2*c*d+8*d^2)*arc
tanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c^3/(a^2*c+d)^(5/2)+1/15*a*(7*a^2*c
+4*d)/c^2/(a^2*c+d)^2/(d*x^2+c)^(1/2)+8/15*x*arctanh(a*x)/c^3/(d*x^2+c)^(1/
2)
```

Rubi [A] time = 1.08, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {192, 191, 5976, 6688, 12, 6715, 897, 1261, 208}

$$-\frac{\left(15a^4c^2+20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3\left(a^2c+d\right)^{5/2}}+\frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] a/(15*c*(a^2*c + d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c + 4*d))/(15*c^2*(a^2*c
 + d)^2*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcTanh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*c^3*Sqrt[
c + d*x^2]) - ((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(15*c^3*(a^2*c + d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{\frac{x}{5c(c+dx^2)^{5/2}} + \frac{4x}{15c^2(c+dx^2)^{3/2}} + \frac{8}{15c^3\sqrt{}}}{1-a^2x^2}$$

$$= \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{15c^3(1-a^2x^2)(c+dx^2)^{5/2}}dx$$

$$= \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1-a^2x^2)(c+dx^2)^{5/2}}dx}{15c^3}$$

$$= \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1-a^2x)(c+dx)^{5/2}}dx, x, x^2\right)}{30c^3}$$

$$= \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{a^2c+d}{d}-\frac{a^2x^2}{d}\right)}dx, x, \sqrt{c+}\right.}{15c^3d}$$

$$= \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \left(\frac{3c^2d}{(a^2c+d)x^4} + \frac{cd(7a^2c+4d)}{(a^2c+d)^2x^2} + \right.\right.}{15c}$$

$$= \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}$$

$$= \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}$$

Mathematica [A] time = 0.59, size = 329, normalized size = 1.64

$$\frac{2x(a^2c+d)^{5/2}\tanh^{-1}(ax)(15c^2+20cdx^2+8d^2x^4) + 2ac\sqrt{a^2c+d}\,(c+dx^2)(a^2c(8c+7dx^2)+d(5c+4dx^2)) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x^
2)) + 2*(a^2*c + d)^(5/2)*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTanh[a*x]
+ (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 - a*x] + (15*a^
4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 + a*x] - (15*a^4*c^2 +
20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sqrt[
c + d*x^2]] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c +
 d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(30*c^3*(a^2*c + d)^(5/2)*(c + d*x
^2)^(5/2))
```

fricas [B] time = 0.71, size = 1280, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x
^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 +
 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2
*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(
a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^
4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2
 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*
a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3
 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*
d^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 2
0*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 +
 8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^
2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 +
 c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c^5 + 26*a^3*c^4*d
 + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5
*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2
*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d
^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*l
og(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c
^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*
x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^
6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

giac [A] time = 0.22, size = 218, normalized size = 1.09

$$\frac{1}{15}\,a\left(\frac{\left(15\,a^4c^2+20\,a^2cd+8\,d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{\left(a^4c^5+2\,a^2c^4d+c^3d^2\right)\sqrt{-a^2c-d}\,a}+\frac{7\left(dx^2+c\right)a^2c+a^2c^2+4\left(dx^2+c\right)d+cd}{\left(a^4c^4+2\,a^2c^3d+c^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}\right)+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\cdots\right.\right.}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^
2*c - d))/((a^4*c^5 + 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c - d)*a) + (7*(d*x^
2 + c)*a^2*c + a^2*c^2 + 4*(d*x^2 + c)*d + c*d)/((a^4*c^4 + 2*a^2*c^3*d + c
^2*d^2)*(d*x^2 + c)^(3/2))) + 1/30*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)
*x*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(5/2)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(ax\right)}{\left(d\,x^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

maxima [B] time = 0.43, size = 401, normalized size = 2.00

$$\frac{1}{30}\,a\left(\frac{\frac{3\,a^3d\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\left(a^4c^3+2\,a^2c^2d+cd^2\right)\sqrt{a^2c+d}}+\frac{2\left(3\left(dx^2+c\right)a^2d+a^2cd+d^2\right)}{\left(a^4c^3+2\,a^2c^2d+cd^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}}{d}+\frac{4\left(\frac{ad\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\left(a^2c^3+c^2d\right)\sqrt{a^2c+d}}+\frac{2\,d}{\left(a^2c^3+c^2d\right)\sqrt{dx^2+c}}\right)}{d}+\frac{8\,\log\left(\cdots\right.}{\sqrt{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*a*((3*a^3*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2
+ c)*a^2 + sqrt(a^2*c + d)*a))/((a^4*c^3 + 2*a^2*c^2*d + c*d^2)*sqrt(a^2*c
+ d)) + 2*(3*(d*x^2 + c)*a^2*d + a^2*c*d + d^2)/((a^4*c^3 + 2*a^2*c^2*d + c
*d^2)*(d*x^2 + c)^(3/2)))/d + 4*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c
+ d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^3 + c^2*d)*sqrt(
a^2*c + d)) + 2*d/((a^2*c^3 + c^2*d)*sqrt(d*x^2 + c)))/d + 8*log((sqrt(d*x^
2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/
(sqrt(a^2*c + d)*a*c^3)) + 1/15*(8*x/(sqrt(d*x^2 + c)*c^3) + 4*x/((d*x^2 +
c)^(3/2)*c^2) + 3*x/((d*x^2 + c)^(5/2)*c))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(a\,x\right)}{{\left(d\,x^2+c\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(7/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}{\left(ax \right)}}{\left(c + dx^{2}\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(7/2), x)
```

3.516 $\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{9/2}} dx$

Optimal. Leaf size=283

$$\frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}+\frac{a}{35c\left(a^2c+d\right)\left(c+dx^2\right)^{5/2}}+\frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}}-\frac{\left(35a^6c^3+70a^4c^2d+56\right.}{35c^4}$$

```
[Out] 1/35*a/c/(a^2*c+d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c+6*d)/c^2/(a^2*c+d)^2/(
d*x^2+c)^(3/2)+1/7*x*arctanh(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arctanh(a*x)/c^2
/(d*x^2+c)^(5/2)+8/35*x*arctanh(a*x)/c^3/(d*x^2+c)^(3/2)-1/35*(35*a^6*c^3+7
0*a^4*c^2*d+56*a^2*c*d^2+16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))
/c^4/(a^2*c+d)^(7/2)+1/35*a*(19*a^4*c^2+22*a^2*c*d+8*d^2)/c^3/(a^2*c+d)^3/(
d*x^2+c)^(1/2)+16/35*x*arctanh(a*x)/c^4/(d*x^2+c)^(1/2)
```

Rubi [A] time = 1.51, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5976, 6688, 12, 6715, 1619, 63, 208}

$$\frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}}-\frac{\left(70a^4c^2d+35a^6c^3+56a^2cd^2+16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4\left(a^2c+d\right)^{7/2}}+\frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] a/(35*c*(a^2*c + d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c + 6*d))/(105*c^2*(a^2
*c + d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c + d)^3*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcTanh[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcTanh[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35
*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(35*c^4*(a^2*c + d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
```

```
&& NeQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{9/2}}\,dx = \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x}{7c\left(c+dx^2\right)}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x\left(35c^3\right.}{35}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(35c^3+70c\right.}{(1-}}{}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{3}{}\right.}{}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int\left(\right.\right.}{}$$

$$= \frac{a}{35c\left(a^2c+d\right)\left(c+dx^2\right)^{5/2}} + \frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}} +$$

$$= \frac{a}{35c\left(a^2c+d\right)\left(c+dx^2\right)^{5/2}} + \frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}} +$$

$$= \frac{a}{35c\left(a^2c+d\right)\left(c+dx^2\right)^{5/2}} + \frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}} +$$

Mathematica [A] time = 0.95, size = 431, normalized size = 1.52

$$\frac{6x\left(a^2c+d\right)^{7/2}\tanh^{-1}(ax)\left(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6\right)+2ac\sqrt{a^2c+d}\left(c+dx^2\right)\left(3c^2\left(a^2c+d\right)^2+c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(3*c^2*(a^2*c + d)^2 + c*(a^2*c + d)*(11
*a^2*c + 6*d)*(c + d*x^2) + 3*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2)*(c + d*x^2)
^2) + 6*(a^2*c + d)^(7/2)*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*
x^6)*ArcTanh[a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(
c + d*x^2)^(7/2)*Log[1 - a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2
 + 16*d^3)*(c + d*x^2)^(7/2)*Log[1 + a*x] - 3*(35*a^6*c^3 + 70*a^4*c^2*d +
56*a^2*c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sq
rt[c + d*x^2]] - 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(c +
 d*x^2)^(7/2)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(210*c^4*(a
^2*c + d)^(7/2)*(c + d*x^2)^(7/2))
```

fricas [B] time = 0.92, size = 2006, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 +
2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2
+ 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 + 6
*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(18
2*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2*(
196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3*(
16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7 +
56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*x^
5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d^5
)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d^4
)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d +
 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 +
6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*d
^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^6
*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d + 4
*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35
*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5
 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3
*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3
+ 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*
a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a
^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*
d^2)*x^2)) + (142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3
+ 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*
(182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 +
2*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 +
3*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7
 + 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)
*x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*
d^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*
d^4)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*
d + 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5
 + 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^
8*d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*
a^6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d
+ 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

giac [A] time = 0.23, size = 349, normalized size = 1.23

$$\frac{1}{105}a\left(\frac{3\left(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{\left(a^6c^7+3a^4c^6d+3a^2c^5d^2+c^4d^3\right)\sqrt{-a^2c-d}\,a}+\frac{57\left(dx^2+c\right)^2a^4c^2+11\left(dx^2+c\right)a^4c^3+3a^4c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*arctan(sqrt(
d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 + c^
4*d^3)*sqrt(-a^2*c - d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*a^4
*c^3 + 3*a^4*c^4 + 66*(d*x^2 + c)^2*a^2*c*d + 17*(d*x^2 + c)*a^2*c^2*d + 6*
```

```
a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((a^6*c
^6 + 3*a^4*c^5*d + 3*a^2*c^4*d^2 + c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/70*(2*(
4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*log(-(a*x + 1)/
(a*x - 1))/(d*x^2 + c)^(7/2)
```

maple [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)}{\left(d\,x^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

maxima [B] time = 0.46, size = 639, normalized size = 2.26

$$\frac{1}{210}a\left(\frac{\frac{15\,a^5 d\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\left(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3\right)\sqrt{a^2c+d}}+\frac{2\left(15\left(dx^2+c\right)^2a^4d+3\,a^4c^2d+6\,a^2cd^2+3\,d^3+5\left(a^4cd+a^2d^2\right)\left(dx^2+c\right)\right)}{\left(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3\right)\left(dx^2+c\right)^{\frac{5}{2}}}}{d}+\frac{6\left(\frac{3\,a^3d\log\left(\frac{\sqrt{dx^2+c}\,a^2 \cdots}{\sqrt{dx^2+c}\,a^2 \cdots}\right)}{\left(a^4c^4+2\,a^2c^3d+c^2 \cdots\right.}\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/210*a*((15*a^5*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^
2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^6*c^4 + 3*a^4*c^3*d + 3*a^2*c^2*d^2 +
c*d^3)*sqrt(a^2*c + d)) + 2*(15*(d*x^2 + c)^2*a^4*d + 3*a^4*c^2*d + 6*a^2*c
*d^2 + 3*d^3 + 5*(a^4*c*d + a^2*d^2)*(d*x^2 + c))/((a^6*c^4 + 3*a^4*c^3*d +
 3*a^2*c^2*d^2 + c*d^3)*(d*x^2 + c)^(5/2)))/d + 6*(3*a^3*d*log((sqrt(d*x^2
+ c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((
a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*sqrt(a^2*c + d)) + 2*(3*(d*x^2 + c)*a^2*d
+ a^2*c*d + d^2)/((a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*(d*x^2 + c)^(3/2)))/d +
 24*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2
 + sqrt(a^2*c + d)*a))/((a^2*c^4 + c^3*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c^4
+ c^3*d)*sqrt(d*x^2 + c)))/d + 48*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d
)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^4)) +
1/35*(16*x/(sqrt(d*x^2 + c)*c^4) + 8*x/((d*x^2 + c)^(3/2)*c^3) + 6*x/((d*x^
2 + c)^(5/2)*c^2) + 5*x/((d*x^2 + c)^(7/2)*c))*arctanh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(d\,x^2+c\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(9/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(c+dx^2\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(9/2), x)
```

3.517 $\int \sqrt{a-ax^2}\, \tanh^{-1}(x)\, dx$

Optimal. Leaf size=186

$$-\frac{ia\sqrt{1-x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)-\frac{a\sqrt{1-x^2}\,\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}$$

```
[Out] -a*arctan((1-x)^(1/2)/(1+x)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/
2)-1/2*I*a*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^
(1/2)+1/2*I*a*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a
)^(1/2)+1/2*(-a*x^2+a)^(1/2)+1/2*x*arctanh(x)*(-a*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5942, 5954, 5950}

$$-\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)-\frac{a\sqrt{1-}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

```
[Out] Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcTanh[x])/2 - (a*Sqrt[1 - x^2]*Arc
Tan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^2] - ((I/2)*a*Sqrt[1
- x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2] + ((I/2)
*a*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] && !GtQ[d, 0]
```

Rubi steps

$$\int \sqrt{a-ax^2}\,\tanh^{-1}(x)\,dx = \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)+\frac{1}{2}a\int\frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx$$
$$=\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)+\frac{\left(a\sqrt{1-x^2}\right)\int\frac{\tanh^{-1}(x)}{\sqrt{1-x^2}}\,dx}{2\sqrt{a-ax^2}}$$
$$=\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)-\frac{a\sqrt{1-x^2}\,\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-}}{2}$$

Mathematica [A] time = 0.32, size = 97, normalized size = 0.52

$$\frac{1}{2}\sqrt{a\left(1-x^2\right)}\left(-\frac{i\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(x)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(x)}\right)+\tanh^{-1}(x)\left(\log\left(1-ie^{-\tanh^{-1}(x)}\right)-\log\left(1+ie^{-\tanh^{-1}(x)}\right.\right.\right.}{\sqrt{1-x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

```
[Out] (Sqrt[a*(1 - x^2)]*(1 + x*ArcTanh[x] - (I*(ArcTanh[x]*(Log[1 - I/E^ArcTanh[
x]] - Log[1 + I/E^ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[2,
 I/E^ArcTanh[x]]))/Sqrt[1 - x^2]))/2
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-ax^2+a}\,\text{artanh}(x),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*x^2 + a)*arctanh(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-ax^2+a}\,\text{artanh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + a)*arctanh(x), x)
```

maple [A] time = 0.61, size = 229, normalized size = 1.23

$$\frac{(\text{arctanh}(x)x+1)\sqrt{-(-1+x)(1+x)a}}{2}+\frac{i\sqrt{-(-1+x)(1+x)a}\,\sqrt{-x^2+1}\,\text{arctanh}(x)\ln\left(1+\frac{i(1+x)}{\sqrt{-x^2+1}}\right)}{2(1+x)(-1+x)}-\frac{i\sqrt{-(-1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+a)^(1/2)*arctanh(x),x)
```

```
[Out] 1/2*(arctanh(x)*x+1)*(-(-1+x)*(1+x)*a)^(1/2)+1/2*I*(-(-1+x)*(1+x)*a)^(1/2)/
(1+x)*(-x^2+1)^(1/2)/(-1+x)*arctanh(x)*ln(1+I*(1+x)/(-x^2+1)^(1/2))-1/2*I*(
-(-1+x)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)^(1/2)/(-1+x)*arctanh(x)*ln(1-I*(1+x)/
(-x^2+1)^(1/2))+1/2*I*(-(-1+x)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)^(1/2)/(-1+x)*d
```

```
ilog(1+I*(1+x)/(-x^2+1)^(1/2))-1/2*I*(-(-1+x)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)
^(1/2)/(-1+x)*dilog(1-I*(1+x)/(-x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-ax^2 + a} \operatorname{artanh}(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="maxima")
[Out] integrate(sqrt(-a*x^2 + a)*arctanh(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(x) \sqrt{a - a x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)*(a - a*x^2)^(1/2),x)
[Out] int(atanh(x)*(a - a*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a (x - 1) (x + 1)} \operatorname{atanh}(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+a)**(1/2)*atanh(x),x)
[Out] Integral(sqrt(-a*(x - 1)*(x + 1))*atanh(x), x)
```

3.518 $\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}-\frac{2\sqrt{1-x^2}\,\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}$$

```
[Out] -2*arctan((1-x)^(1/2)/(1+x)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/
2)-I*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2)+
I*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5954, 5950}

$$-\frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}-\frac{2\sqrt{1-x^2}\,\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

```
[Out] (-2*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^
2] - (I*Sqrt[1 - x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
a*x^2] + (I*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
 a*x^2]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{\tanh^{-1}(x)}{\sqrt{1-x^2}}\,dx}{\sqrt{a-ax^2}}\\ &= -\frac{2\sqrt{1-x^2}\,\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 90, normalized size = 0.62

$$-\frac{i\sqrt{a\left(1-x^2\right)}\left(\mathrm{Li}_2\left(-ie^{-\tanh^{-1}(x)}\right)-\mathrm{Li}_2\left(ie^{-\tanh^{-1}(x)}\right)+\tanh^{-1}(x)\left(\log\left(1-ie^{-\tanh^{-1}(x)}\right)-\log\left(1+ie^{-\tanh^{-1}(x)}\right)\right)\right)}{a\sqrt{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

```
[Out] ((-I)*Sqrt[a*(1 - x^2)]*(ArcTanh[x]*(Log[1 - I/E^ArcTanh[x]] - Log[1 + I/E^
ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[2, I/E^ArcTanh[x]]))
/(a*Sqrt[1 - x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-ax^2+a}\,\operatorname{artanh}(x)}{ax^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x^2 + a)*arctanh(x)/(a*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(x)}{\sqrt{-ax^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/sqrt(-a*x^2 + a), x)
```

maple [A] time = 0.50, size = 210, normalized size = 1.46

$$\frac{i\ln\left(1+\frac{i(1+x)}{\sqrt{-x^2+1}}\right)\operatorname{arctanh}(x)\sqrt{-x^2+1}\,\sqrt{-(-1+x)(1+x)a}}{a\left(x^2-1\right)}-\frac{i\ln\left(1-\frac{i(1+x)}{\sqrt{-x^2+1}}\right)\operatorname{arctanh}(x)\sqrt{-x^2+1}\,\sqrt{-(-1}{a\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(1/2),x)
```

```
[Out] I*ln(1+I*(1+x)/(-x^2+1)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)*(-(-1+x)*(1+x)*a)^
(1/2)/a/(x^2-1)-I*ln(1-I*(1+x)/(-x^2+1)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)*(-
(-1+x)*(1+x)*a)^(1/2)/a/(x^2-1)+I*dilog(1+I*(1+x)/(-x^2+1)^(1/2))*(-x^2+1)^
(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/a/(x^2-1)-I*dilog(1-I*(1+x)/(-x^2+1)^(1/2))*(
-x^2+1)^(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/a/(x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(x)}{\sqrt{-ax^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(x)/sqrt(-a*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(x)}{\sqrt{a-a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(1/2),x)
```

```
[Out] int(atanh(x)/(a - a*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{\sqrt{-a(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(1/2),x)
```

```
[Out] Integral(atanh(x)/sqrt(-a*(x - 1)*(x + 1)), x)
```

3.519 $\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

```
[Out] -1/a/(-a*x^2+a)^(1/2)+x*arctanh(x)/a/(-a*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5958}

$$\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[a - a*x^2])) + (x*ArcTanh[x])/(a*Sqrt[a - a*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx=-\frac{1}{a\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.81

$$\frac{\sqrt{a-ax^2}\left(1-x\tanh^{-1}(x)\right)}{a^2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a - a*x^2]*(1 - x*ArcTanh[x]))/(a^2*(-1 + x^2))
```

fricas [A] time = 0.47, size = 42, normalized size = 1.14

$$-\frac{\sqrt{-ax^2+a}\left(x\log\left(-\frac{x+1}{x-1}\right)-2\right)}{2\left(a^2x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*(x*log(-(x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)
```

giac [A] time = 0.20, size = 54, normalized size = 1.46

$$-\frac{\sqrt{-ax^2+a}\,x\log\left(-\frac{x+1}{x-1}\right)}{2\left(ax^2-a\right)a}-\frac{1}{\sqrt{-ax^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*x*log(-(x + 1)/(x - 1))/((a*x^2 - a)*a) - 1/(sqrt(-a*
x^2 + a)*a)
```

maple [A] time = 0.45, size = 52, normalized size = 1.41

$$-\frac{(\operatorname{arctanh}(x)-1)\sqrt{-(-1+x)(1+x)a}}{2(-1+x)a^2}-\frac{(1+\operatorname{arctanh}(x))\sqrt{-(-1+x)(1+x)a}}{2(1+x)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(3/2),x)
```

```
[Out] -1/2*(arctanh(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^2-1/2*(1+arctanh(x))*(
-(-1+x)*(1+x)*a)^(1/2)/(1+x)/a^2
```

maxima [A] time = 0.42, size = 63, normalized size = 1.70

$$\frac{x\,\operatorname{artanh}(x)}{\sqrt{-ax^2+a}\,a}-\frac{\frac{\sqrt{-ax^2+a}}{ax+a}-\frac{\sqrt{-ax^2+a}}{ax-a}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(x)/(sqrt(-a*x^2 + a)*a) - 1/2*(sqrt(-a*x^2 + a)/(a*x + a) - sqrt(
-a*x^2 + a)/(a*x - a))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atanh}(x)}{\left(a-a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(3/2),x)
```

```
[Out] int(atanh(x)/(a - a*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{(-a(x-1)(x+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(3/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(3/2), x)
```

3.520 $\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

```
[Out] -1/9/a/(-a*x^2+a)^(3/2)+1/3*x*arctanh(x)/a/(-a*x^2+a)^(3/2)-2/3/a^2/(-a*x^2
+a)^(1/2)+2/3*x*arctanh(x)/a^2/(-a*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5960, 5958}

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/(9*a*(a - a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a - a*x^2]) + (x*ArcTanh[x])/(3*
a*(a - a*x^2)^(3/2)) + (2*x*ArcTanh[x])/(3*a^2*Sqrt[a - a*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{5/2}}\,dx &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.54

$$-\frac{\sqrt{a-ax^2}\left(\left(6x^3-9x\right)\tanh^{-1}(x)-6x^2+7\right)}{9a^3\left(x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/9*(Sqrt[a - a*x^2]*(7 - 6*x^2 + (-9*x + 6*x^3)*ArcTanh[x]))/(a^3*(-1 + x
^2)^2)
```

fricas [A] time = 0.48, size = 62, normalized size = 0.75

$$\frac{\sqrt{-ax^2+a}\left(12\,x^2-3\left(2\,x^3-3\,x\right)\log\left(-\frac{x+1}{x-1}\right)-14\right)}{18\left(a^3x^4-2\,a^3x^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a*x^2 + a)*(12*x^2 - 3*(2*x^3 - 3*x)*log(-(x + 1)/(x - 1)) - 14)
/(a^3*x^4 - 2*a^3*x^2 + a^3)
```

giac [A] time = 0.19, size = 86, normalized size = 1.04

$$-\frac{\sqrt{-ax^2+a}\,x\left(\frac{2\,x^2}{a}-\frac{3}{a}\right)\log\left(-\frac{x+1}{x-1}\right)}{6\left(ax^2-a\right)^2}-\frac{6\,ax^2-7\,a}{9\left(ax^2-a\right)\sqrt{-ax^2+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a*x^2 + a)*x*(2*x^2/a - 3/a)*log(-(x + 1)/(x - 1))/(a*x^2 - a)^2
 - 1/9*(6*a*x^2 - 7*a)/((a*x^2 - a)*sqrt(-a*x^2 + a)*a^2)
```

maple [A] time = 0.47, size = 112, normalized size = 1.35

$$\frac{(1+x)(-1+3\,\mathrm{arctanh}(x))\sqrt{-(-1+x)(1+x)\,a}}{72\,(-1+x)^2\,a^3}-\frac{3\,(\mathrm{arctanh}(x)-1)\sqrt{-(-1+x)(1+x)\,a}}{8\,(-1+x)\,a^3}-\frac{3\,(1+\mathrm{arctanh}(x))}{8\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(5/2),x)
```

```
[Out] 1/72*(1+x)*(-1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^2/a^3-3/8*(arct
anh(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^3-3/8*(1+arctanh(x))*(-(-1+x)*(1
+x)*a)^(1/2)/(1+x)/a^3+1/72*(-1+x)*(1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)
/(1+x)^2/a^3
```

maxima [A] time = 0.33, size = 67, normalized size = 0.81

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{-ax^2+a}\,a^2}+\frac{x}{\left(-ax^2+a\right)^{\frac{3}{2}}a}\right)\mathrm{artanh}(x)-\frac{2}{3\sqrt{-ax^2+a}\,a^2}-\frac{1}{9\left(-ax^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(-a*x^2 + a)*a^2) + x/((-a*x^2 + a)^(3/2)*a))*arctanh(x) - 2/
3/(sqrt(-a*x^2 + a)*a^2) - 1/9/((-a*x^2 + a)^(3/2)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(x)}{\left(a-a\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(5/2),x)
```

```
[Out] int(atanh(x)/(a - a*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{(-a(x-1)(x+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(5/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

3.521 $\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{7/2}} \, dx$

Optimal. Leaf size=124

$$-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{8x\tanh^{-1}(x)}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}-\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}$$

```
[Out] -1/25/a/(-a*x^2+a)^(5/2)-4/45/a^2/(-a*x^2+a)^(3/2)+1/5*x*arctanh(x)/a/(-a*x
^2+a)^(5/2)+4/15*x*arctanh(x)/a^2/(-a*x^2+a)^(3/2)-8/15/a^3/(-a*x^2+a)^(1/2
)+8/15*x*arctanh(x)/a^3/(-a*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5960, 5958}

$$-\frac{8}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\tanh^{-1}(x)}{15a^3\sqrt{a-ax^2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}-\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(7/2),x]
```

```
[Out] -1/(25*a*(a - a*x^2)^(5/2)) - 4/(45*a^2*(a - a*x^2)^(3/2)) - 8/(15*a^3*Sqrt
[a - a*x^2]) + (x*ArcTanh[x])/(5*a*(a - a*x^2)^(5/2)) + (4*x*ArcTanh[x])/(1
5*a^2*(a - a*x^2)^(3/2)) + (8*x*ArcTanh[x])/(15*a^3*Sqrt[a - a*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{7/2}} \, dx = -\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{5/2}} \, dx}{5a}$$

$$= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}+\frac{8\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{3/2}} \, dx}{15a^2}$$

$$= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.44

$$\frac{\sqrt{a-ax^2}\left(120x^4-260x^2-15\left(8x^4-20x^2+15\right)x\tanh^{-1}(x)+149\right)}{225a^4\left(x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(7/2),x]
```

```
[Out] (Sqrt[a - a*x^2]*(149 - 260*x^2 + 120*x^4 - 15*x*(15 - 20*x^2 + 8*x^4)*ArcT
anh[x]))/(225*a^4*(-1 + x^2)^3)
```

fricas [A] time = 0.56, size = 82, normalized size = 0.66

$$\frac{\left(240\,x^4-520\,x^2-15\left(8\,x^5-20\,x^3+15\,x\right)\log\left(-\frac{x+1}{x-1}\right)+298\right)\sqrt{-ax^2+a}}{450\left(a^4x^6-3\,a^4x^4+3\,a^4x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*x^4 - 520*x^2 - 15*(8*x^5 - 20*x^3 + 15*x)*log(-(x + 1)/(x - 1))
 + 298)*sqrt(-a*x^2 + a)/(a^4*x^6 - 3*a^4*x^4 + 3*a^4*x^2 - a^4)
```

giac [A] time = 0.19, size = 118, normalized size = 0.95

$$-\frac{\sqrt{-ax^2+a}\left(4\,x^2\left(\frac{2\,x^2}{a}-\frac{5}{a}\right)+\frac{15}{a}\right)x\log\left(-\frac{x+1}{x-1}\right)}{30\left(ax^2-a\right)^3}-\frac{120\left(ax^2-a\right)^2-20\left(ax^2-a\right)a+9\,a^2}{225\left(ax^2-a\right)^2\sqrt{-ax^2+a}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a*x^2 + a)*(4*x^2*(2*x^2/a - 5/a) + 15/a)*x*log(-(x + 1)/(x - 1
))/(a*x^2 - a)^3 - 1/225*(120*(a*x^2 - a)^2 - 20*(a*x^2 - a)*a + 9*a^2)/((a
*x^2 - a)^2*sqrt(-a*x^2 + a)*a^3)
```

maple [A] time = 0.48, size = 176, normalized size = 1.42

$$-\frac{(1+x)^2\left(-1+5\,\mathrm{arctanh}(x)\right)\sqrt{-(-1+x)(1+x)\,a}}{800\,(-1+x)^3\,a^4}+\frac{5\,(1+x)\left(-1+3\,\mathrm{arctanh}(x)\right)\sqrt{-(-1+x)(1+x)\,a}}{288\,(-1+x)^2\,a^4}-\frac{5\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(7/2),x)
```

```
[Out] -1/800*(1+x)^2*(-1+5*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^3/a^4+5/288
*(1+x)*(-1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^2/a^4-5/16*(arctanh
(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^4-5/16*(1+arctanh(x))*(-(-1+x)*(1+x
)*a)^(1/2)/(1+x)/a^4+5/288*(-1+x)*(1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/
(1+x)^2/a^4-1/800*(-1+x)^2*(1+5*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(1+x)^3
/a^4
```

maxima [A] time = 0.34, size = 99, normalized size = 0.80

$$\frac{1}{15}\left(\frac{8\,x}{\sqrt{-ax^2+a}\,a^3}+\frac{4\,x}{\left(-ax^2+a\right)^{\frac{3}{2}}a^2}+\frac{3\,x}{\left(-ax^2+a\right)^{\frac{5}{2}}a}\right)\mathrm{artanh}(x)-\frac{8}{15\sqrt{-ax^2+a}\,a^3}-\frac{4}{45\left(-ax^2+a\right)^{\frac{3}{2}}a^2}-\frac{}{25\left(-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/(sqrt(-a*x^2 + a)*a^3) + 4*x/((-a*x^2 + a)^(3/2)*a^2) + 3*x/((-a*
x^2 + a)^(5/2)*a))*arctanh(x) - 8/15/(sqrt(-a*x^2 + a)*a^3) - 4/45/((-a*x^2
 + a)^(3/2)*a^2) - 1/25/((-a*x^2 + a)^(5/2)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(x)}{\left(a-a\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(7/2),x)
```

```
[Out] int(atanh(x)/(a - a*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{\left(-a\,(x-1)\,(x+1)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(7/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(7/2), x)
```

3.522 $\int x^4 \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=315

$$-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{2aex}{5c^4}-\frac{2aex^3}{15c^2}-$$

```
[Out] -2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9/200*b*e*x^4/c-2/25*a*e
*x^5-2/5*b*e*x*arctanh(c*x)/c^4-2/15*b*e*x^3*arctanh(c*x)/c^2-2/25*b*e*x^5*
arctanh(c*x)+1/5*b*e*arctanh(c*x)^2/c^5-1/20*(4*a+3*b)*e*ln(-c*x+1)/c^5+1/2
0*(4*a-3*b)*e*ln(c*x+1)/c^5-23/75*b*e*ln(-c^2*x^2+1)/c^5-1/20*b*e*ln(-c^2*x
^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(-c^2*x^2+1))/c^3+1/20*b*x^4*(d+e*ln(-c^2*x^2
+1))/c+1/5*x^5*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))+1/10*b*ln(-c^2*x^2+1
)*(d+e*ln(-c^2*x^2+1))/c^5
```

Rubi [A] time = 0.78, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5916, 266, 43, 6085, 6725, 1802, 633, 31, 5980, 5910, 260, 5948, 2475, 2390, 2301}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcTanh[c*x])/(5*c^4) - (2*b*e*x^
3*ArcTanh[c*x])/(15*c^2) - (2*b*e*x^5*ArcTanh[c*x])/25 + (b*e*ArcTanh[c*x]^
2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 +
 c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2
]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*L
og[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

```
]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^4 \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx = \frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+$$

$$=-\frac{be\log^2\left(1-c^2x^2\right)}{20c^5}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d\right.}{}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2}{25}bex^5\tanh$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex^3\tanh^{-1}}{15c^2}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex\tanh^{-1}(}{5c^4}$$

$$=-\frac{2aex}{5c^4}-\frac{19bex^2}{100c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\tanh}{5c^4}$$

$$=-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\tanh}{5c^4}$$

Mathematica [A] time = 0.17, size = 236, normalized size = 0.75

$$\frac{30c^2ex^2\log\left(1-c^2x^2\right)\left(4ac^3x^3+4bc^3x^3\tanh^{-1}(cx)+b\left(c^2x^2+2\right)\right)+2\log(1-cx)(-60ae+30bd-137be)+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
- 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15 +
5*c^2*x^2 + 3*c^4*x^4))*ArcTanh[c*x] + 120*b*e*ArcTanh[c*x]^2 + 2*(30*b*d -
 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 + c*x
] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcTanh[c*x])
*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

fricas [A] time = 0.53, size = 251, normalized size = 0.80

$$-\frac{80\,ac^3ex^3-24\left(5\,ac^5d-2\,ac^5e\right)x^5-3\left(10\,bc^4d-9\,bc^4e\right)x^4+240\,acex-30\,be\log\left(-c^2x^2+1\right)^2-30\,be\log\left(-\frac{cx+}{cx-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9
*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*log(-c^2*x^2 + 1)^2 - 30*b*e*log(-(c*x
 + 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5 +
15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*log(-c^2*x^2 + 1) - 4*(
15*b*c^5*e*x^5*log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b*c^5*
e)*x^5 - 30*b*c*e*x + 30*a*e)*log(-(c*x + 1)/(c*x - 1)))/c^5
```

giac [A] time = 0.48, size = 455, normalized size = 1.44

$$\frac{60\,bc^5x^5e\log\left(cx+1\right)^2-60\,bc^5x^5e\log\left(-cx+1\right)^2+120\,ac^5x^5e\log\left(cx+1\right)-24\,bc^5x^5e\log\left(cx+1\right)+120\,ac^5x^5e\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/600*(60*b*c^5*x^5*e*log(c*x + 1)^2 - 60*b*c^5*x^5*e*log(-c*x + 1)^2 + 120
*a*c^5*x^5*e*log(c*x + 1) - 24*b*c^5*x^5*e*log(c*x + 1) + 120*a*c^5*x^5*e*l
og(-c*x + 1) + 24*b*c^5*x^5*e*log(-c*x + 1) + 60*b*c^5*d*x^5*log(-(c*x + 1)
/(c*x - 1)) + 120*a*c^5*d*x^5 - 48*a*c^5*x^5*e + 30*b*c^4*x^4*e*log(c*x + 1
) + 30*b*c^4*x^4*e*log(-c*x + 1) + 30*b*c^4*d*x^4 - 27*b*c^4*x^4*e - 40*b*c
^3*x^3*e*log(c*x + 1) + 40*b*c^3*x^3*e*log(-c*x + 1) - 80*a*c^3*x^3*e + 60*
b*c^2*x^2*e*log(c*x + 1) + 60*b*c^2*x^2*e*log(-c*x + 1) + 60*b*c^2*d*x^2 -
154*b*c^2*x^2*e - 120*b*c*x*e*log(c*x + 1) + 120*b*c*x*e*log(-c*x + 1) - 24
0*a*c*x*e + 60*b*e*log(c*x + 1)^2 - 60*b*e*log(c*x - 1)^2 + 120*b*e*log(c*x
 - 1)*log(-c*x + 1) + 60*b*d*log(c^2*x^2 - 1) + 120*a*e*log(c*x + 1) - 274*
b*e*log(c*x + 1) - 120*a*e*log(c*x - 1) - 274*b*e*log(c*x - 1))/c^5
```

maple [C] time = 6.09, size = 4757, normalized size = 15.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -2/25*a*e*x^5+181/600*e/c^5*b-2/5*b*e*x*arctanh(c*x)/c^4-2/15*b*e*x^3*arcta
nh(c*x)/c^2+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+
1/10*I/c^5*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*e*arctanh(c*x)+1/40*
I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^4*e
+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)
^3*x^2*e+1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^4*e-3/40*I/c^5*b*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+
1)^2/(-c^2*x^2+1))^2)*e*Pi-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^
```

```
2-9/200*b*e*x^4/c-2/25*b*e*x^5*arctanh(c*x)+1/10/c^5*b*e*(4*arctanh(c*x)*x^
5*c^5+c^4*x^4+2*c^2*x^2+4*arctanh(c*x)-4*ln(1+(c*x+1)^2/(-c^2*x^2+1))-3)*ln
((c*x+1)/(-c^2*x^2+1)^(1/2))-1/10*I*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/
(-c^2*x^2+1))^2)*arctanh(c*x)*x^5*e-1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(
c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)-1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^4*e-1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^2*e+1/10*I/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/10*I/c^5*b*cs
gn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*e*l
n(1+(c*x+1)^2/(-c^2*x^2+1))*Pi+1/5*I/c^5*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*
e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2
+3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)-1/10*
I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/10*I/c^5*b*arctanh(c*x)*Pi*e*cs
gn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^
2/(-c^2*x^2+1))^2)^2-1/5*I/c^5*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csg
n(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*e*arctanh(c*x)+1/5*I/c^5*b*arctanh(c*x)*
Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/1
0*I/c^3*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x
^2+1))^2)^2*x^2*e+1/40*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I
*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(1+(c*x+1)^2/(-
c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^2*e+1/10*I/c^5*b*arct
anh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^
2-1))-1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^4*e-1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/
(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x
^2*e+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*cs
gn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)+1/40*I/c*b*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^4
*e+1/20*I/c^3*b*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^2*e+1/20*I/c*b*Pi*csgn(I*(c*x+1
)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*x^4*e+1/10*I/c^3*b*Pi
*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*x^2*e+1
/40*I/c*b*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2
-1))*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*x^2*e-1/20*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*c
sgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^4*e-1/10*I*b*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*arc
tanh(c*x)*x^5*e+1/10*I*b*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*arctanh(c*x)*x^5*e+1/10*
I*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1
))^2)*arctanh(c*x)*x^5*e+1/10*I*b*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)*x^5*e-1/5*I*b*Pi*csgn(I*(1+(c*x+1
)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*arctanh(c*x)*x^5*
e+1/5*I*b*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1
))^2*arctanh(c*x)*x^5*e+1/10*I/c^5*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1))^2)^2-1/10*I/c^5*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I/(1+(c*x+1
)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
)^2)^2-1/10*I/c^5*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/5*I/c^5*b*Pi*ln(1+(c*x+1)^2
/(-c^2*x^2+1))*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x
^2-1))^2+1/5/c^5*b*d*arctanh(c*x)-46/75/c^5*b*arctanh(c*x)*e-1/5/c^5*b*d*ln
```

```
(1+(c*x+1)^2/(-c^2*x^2+1))+1/5/c^5*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))^2*e+137/1
50/c^5*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))-1/5*a*e/c^5*ln(c*x-1)+1/5*a*e*x^5*l
n(-c^2*x^2+1)+1/5*a*e/c^5*ln(c*x+1)+1/20/c*b*x^4*d+1/5*b*arctanh(c*x)*x^5*d
+1/5*x^5*a*d+1/5/c^3*b*ln(2)*x^2*e-2/5/c^5*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))
*ln(2)-1/10/c*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^4*e+2/5*b*ln(2)*arctanh(c*x)
*x^5*e-2/5*b*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^5*e-1/5/c^3*b*ln(1
+(c*x+1)^2/(-c^2*x^2+1))*x^2*e+2/5/c^5*b*arctanh(c*x)*ln(2)*e+1/10/c*b*ln(2
)*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^2*e+1/40*I/c*b*Pi
*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(1+(c*
x+1)^2/(-c^2*x^2+1))^2)^3*x^2*e+1/10*I*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
+(c*x+1)^2/(-c^2*x^2+1))^2)^3*arctanh(c*x)*x^5*e+1/10*I*b*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^3*arctanh(c*x)*x^5*e+1/10*I*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*
x^2+1))^2)^3*arctanh(c*x)*x^5*e+1/10/c^3*b*x^2*d-3/40*I/c^5*b*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*e*Pi-3/40*I/c^5*b*Pi*e*csgn
(I*(c*x+1)^2/(c^2*x^2-1))^3-3/40*I/c^5*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^
2)^3*e*Pi-3/10/c^5*b*e*ln(2)-3/20*I/c^5*b*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^
(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)/(-c
^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/10*I/c^5*b*Pi*ln(1+(c*x+
1)^2/(-c^2*x^2+1))*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/10*I/c^5*b*Pi
*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/10*I/c^5*
b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1))^2)^3+3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(
I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+3/20*I/c^5*b*e*Pi*c
sgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/
10*I/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*
e*arctanh(c*x)-3/40*I/c^5*b*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(
I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)-3/20/c^5*b*d
```

maxima [C] time = 0.34, size = 314, normalized size = 1.00

$$\frac{1}{5}\,adx^5+\frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)be\,\mathrm{artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*e*arctanh(c*x
) + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1
)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e + 1/600*((3
0*I*pi*c^4 - 27*c^4)*x^4 + (60*I*pi*c^2 - 154*c^2)*x^2 + (60*I*pi + 30*c^4*
x^4 + 60*c^2*x^2 + 120*log(c*x - 1) - 274)*log(c*x + 1) - 2*(-30*I*pi - 15*
c^4*x^4 - 30*c^2*x^2 + 137)*log(c*x - 1))*b*e/c^5
```

mupad [B] time = 5.59, size = 599, normalized size = 1.90

$$\frac{a\,d\,x^5}{5}-\frac{2\,a\,e\,x^5}{25}+\frac{b\,d\,x^5\,\ln\left(c\,x+1\right)}{10}-\frac{b\,d\,x^5\,\ln\left(1-c\,x\right)}{10}-\frac{b\,e\,x^5\,\ln\left(c\,x+1\right)}{25}+\frac{b\,e\,x^5\,\ln\left(1-c\,x\right)}{25}+\frac{b\,e\,{\ln\left(c\,x+1\right)}^2}{10\,c^5}+\frac{b\,e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] (a*d*x^5)/5 - (2*a*e*x^5)/25 + (b*d*x^5*log(c*x + 1))/10 - (b*d*x^5*log(1 -
 c*x))/10 - (b*e*x^5*log(c*x + 1))/25 + (b*e*x^5*log(1 - c*x))/25 + (b*e*lo
g(c*x + 1)^2)/(10*c^5) + (b*e*log(1 - c*x)^2)/(10*c^5) - (2*a*e*x)/(5*c^4)
- (2*a*e*x^3)/(15*c^2) + (b*d*x^4)/(20*c) + (b*d*x^2)/(10*c^3) - (9*b*e*x^4
```

```
)/(200*c) - (77*b*e*x^2)/(300*c^3) + (a*e*x^5*log(1 - c^2*x^2))/5 - (a*e*lo
g(c*x - 1))/(5*c^5) + (a*e*log(c*x + 1))/(5*c^5) + (b*d*log(c*x - 1))/(10*c
^5) + (b*d*log(c*x + 1))/(10*c^5) - (137*b*e*log(c*x - 1))/(300*c^5) - (137
*b*e*log(c*x + 1))/(300*c^5) - (b*e*log(c*x + 1)*log(-(2*a*e - 2*a*c*e*x)/(
5*c^4)))/(10*c^5) - (b*e*log(c*x + 1)*log(-(2*a*e + 2*a*c*e*x)/(5*c^4)))/(1
0*c^5) - (b*e*log(1 - c*x)*log(-(2*a*e - 2*a*c*e*x)/(5*c^4)))/(10*c^5) - (b
*e*log(1 - c*x)*log(-(2*a*e + 2*a*c*e*x)/(5*c^4)))/(10*c^5) - (b*e*x*log(c*
x + 1))/(5*c^4) + (b*e*x*log(1 - c*x))/(5*c^4) + (b*e*x^4*log(1 - c^2*x^2))
/(20*c) + (b*e*x^2*log(1 - c^2*x^2))/(10*c^3) + (b*e*log(-(2*a*e - 2*a*c*e*
x)/(5*c^4))*log(1 - c^2*x^2))/(10*c^5) + (b*e*log(-(2*a*e + 2*a*c*e*x)/(5*c
^4))*log(1 - c^2*x^2))/(10*c^5) - (b*e*x^3*log(c*x + 1))/(15*c^2) + (b*e*x^
3*log(1 - c*x))/(15*c^2) + (b*e*x^5*log(c*x + 1)*log(1 - c^2*x^2))/10 - (b*
e*x^5*log(1 - c*x)*log(1 - c^2*x^2))/10
```

sympy [A] time = 15.18, size = 338, normalized size = 1.07

$$\begin{cases} \frac{adx^5}{5} + \frac{aex^5\log\left(-c^2x^2+1\right)}{5} - \frac{2aex^5}{25} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} + \frac{2ae\,\mathrm{atanh}\,(cx)}{5c^5} + \frac{bdx^5\,\mathrm{atanh}\,(cx)}{5} + \frac{bex^5\log\left(-c^2x^2+1\right)\mathrm{atanh}\,(cx)}{5} - \frac{2bex^5\,\mathrm{atanl}}{25} \\ \frac{adx^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atanh(c*x)/(5*c**5) + b*d*x**
5*atanh(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*atanh(c*x)/5 - 2*b*e*x**5*ata
nh(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*b*e*
x**4/(200*c) - 2*b*e*x**3*atanh(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x
**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*atanh(
c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x**2 + 1)
**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*atanh(c*x)**2/
(5*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

3.523 $\int x^3 \left(a + b \tanh^{-1}(cx)\right) \left(d + e \log\left(1 - c^2x^2\right)\right) dx$

Optimal. Leaf size=225

$$\frac{1}{4}x^4 \left(a + b \tanh^{-1}(cx)\right) \left(e \log\left(1 - c^2x^2\right) + d\right) - \frac{ex^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{e \log\left(1 - c^2x^2\right) \left(a + b \tanh^{-1}(cx)\right)}{4c^4} - \frac{1}{8}ex^4 \left(\right.$$

```
[Out] 1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3+1/24*b*(2*d-e)*x^3/c-1/18*b*e*x^3/c-1/8
*b*(2*d-3*e)*arctanh(c*x)/c^4+2/3*b*e*arctanh(c*x)/c^4-1/4*e*x^2*(a+b*arcta
nh(c*x))/c^2-1/8*e*x^4*(a+b*arctanh(c*x))+1/4*b*e*x*ln(-c^2*x^2+1)/c^3+1/12
*b*e*x^3*ln(-c^2*x^2+1)/c-1/4*e*(a+b*arctanh(c*x))*ln(-c^2*x^2+1)/c^4+1/4*x
^4*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))
```

Rubi [A] time = 0.27, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2454, 2395, 43, 6083, 459, 321, 206, 2471, 2448, 2455, 302}

$$\frac{1}{4}x^4 \left(a + b \tanh^{-1}(cx)\right) \left(e \log\left(1 - c^2x^2\right) + d\right) - \frac{ex^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{e \log\left(1 - c^2x^2\right) \left(a + b \tanh^{-1}(cx)\right)}{4c^4} - \frac{1}{8}ex^4 \left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) + (b*(2*d - e)*x^3)/(24*c) -
(b*e*x^3)/(18*c) - (b*(2*d - 3*e)*ArcTanh[c*x])/(8*c^4) + (2*b*e*ArcTanh[c*
x])/(3*c^4) - (e*x^2*(a + b*ArcTanh[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTanh[
c*x]))/8 + (b*e*x*Log[1 - c^2*x^2])/(4*c^3) + (b*e*x^3*Log[1 - c^2*x^2])/(1
2*c) - (e*(a + b*ArcTanh[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcT
anh[c*x])*(d + e*Log[1 - c^2*x^2]))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx = -\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\tanh^{-1}(cx)\right) - \frac{e\left(a+b\right.}{}$$

$$= -\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\tanh^{-1}(cx)\right) - \frac{e\left(a+b\right.}{}$$

$$= \frac{b(2d-e)x^3}{24c} - \frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\tanh^{-1}(c\right.$$

$$= \frac{b(2d-3e)x}{8c^3} + \frac{b(2d-e)x^3}{24c} - \frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(\right.$$

$$= \frac{b(2d-3e)x}{8c^3} + \frac{b(2d-e)x^3}{24c} - \frac{b(2d-3e)\tanh^{-1}(cx)}{8c^4} - \frac{ex^2\left(a\right.}{}$$

$$= \frac{b(2d-3e)x}{8c^3} - \frac{bex}{2c^3} + \frac{b(2d-e)x^3}{24c} - \frac{b(2d-3e)\tanh^{-1}(cx)}{8c^4} - $$

$$= \frac{b(2d-3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d-e)x^3}{24c} - \frac{bex^3}{18c} - \frac{b(2d-3e)\tanh^{-1}}{8c^4}$$

$$= \frac{b(2d-3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d-e)x^3}{24c} - \frac{bex^3}{18c} - \frac{b(2d-3e)\tanh^{-1}}{8c^4}$$

Mathematica [A] time = 0.16, size = 192, normalized size = 0.85

$$\frac{12e\log\left(1-c^2x^2\right)\left(3ac^4x^4+3b\left(c^4x^4-1\right)\tanh^{-1}(cx)+bcx\left(c^2x^2+3\right)\right)+3\log(1-cx)(-12ae+6bd-25be)-3\,l}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^4
*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcTanh[c*x]
 + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e)*L
og[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x^4)*
ArcTanh[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

fricas [A] time = 0.65, size = 196, normalized size = 0.87

$$-\frac{36\,ac^2ex^2-18\left(2\,ac^4d-ac^4e\right)x^4-2\left(6\,bc^3d-7\,bc^3e\right)x^3-6\,(6\,bcd-25\,bce)x-12\left(3\,ac^4ex^4+bc^3ex^3+3\,bcex-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/144*(36*a*c^2*e*x^2 - 18*(2*a*c^4*d - a*c^4*e)*x^4 - 2*(6*b*c^3*d - 7*b*
c^3*e)*x^3 - 6*(6*b*c*d - 25*b*c*e)*x - 12*(3*a*c^4*e*x^4 + b*c^3*e*x^3 + 3
*b*c*e*x - 3*a*e)*log(-c^2*x^2 + 1) + 3*(6*b*c^2*e*x^2 - 3*(2*b*c^4*d - b*c
^4*e)*x^4 + 6*b*d - 25*b*e - 6*(b*c^4*e*x^4 - b*e)*log(-c^2*x^2 + 1))*log(-
(c*x + 1)/(c*x - 1)))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 6.00, size = 3739, normalized size = 16.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/4/c^4*b*d*arctanh(c*x)+1/4*b*arctanh(c*x)*x^4*d-1/8*b*arctanh(c*x)*x^4*e
-1/4*a*e/c^2*x^2-1/4*a*e/c^4*ln(c^2*x^2-1)-1/8*I/c^4*b*csgn(I*(c*x+1)^2/(c^
2*x^2-1))^3*Pi*e*arctanh(c*x)-1/8*I/c^4*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))
^2)^3*Pi*e*arctanh(c*x)-1/6*I/c^4*b*e*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^
2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/6*I/c^4*b
*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1))^2)^2-1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2
*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/8*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*
Pi*x*e+1/24*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2
)^3*Pi*x^3*e+1/24*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*x^3*e+1/24*I/c*b
*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*x^3*e+1/8*I/c^3*b*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*x*e-1/3*I/c^4*b*e*Pi*csgn
(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/6*I/c^4*b*
e*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))
^2)+1/3*I/c^4*b*e*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2
/(-c^2*x^2+1))^2)^2+1/8*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*x^4*e+1/8*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(
c^2*x^2-1))^3*Pi*x^4*e+1/8*I*b*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1
))^2)^3*Pi*x^4*e-25/24*b*e*x/c^3-7/72*b*e*x^3/c+41/24*b*e*arctanh(c*x)/c^4+
1/4*b*d*x/c^3+1/12*b*d*x^3/c+1/8*I/c^3*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^
2)^3*Pi*x*e-1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(
c*x+1)^2/(-c^2*x^2+1))^2)^3+1/2*b*ln(2)*arctanh(c*x)*x^4*e-1/2*b*arctanh(c*
x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^4*e+41/36*e/c^4*b-2/3/c^4*b*e*ln(2)-1/6*I
/c^4*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/
6*I/c^4*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/6*I/c^4*b*e*Pi*csgn(I*(1+(
c*x+1)^2/(-c^2*x^2+1))^2)^3+1/4*x^4*a*e*ln(-c^2*x^2+1)+1/4*x^4*a*d-1/8*a*e*
x^4+1/8*I/c^4*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arc
tanh(c*x)-1/24*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
)^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*
x^3*e-1/8*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*
x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1
))*Pi*x^4*e-1/8*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2
+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
Pi*x*e+1/8*I*b*arctanh(c*x)*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*Pi*x^4*e+1/4*I*b*arctanh(c*x)*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi*x^4*e+1/8*I*b*arctanh(c*x)*csg
n(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*x
^4*e-1/4*I*b*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x
+1)^2/(-c^2*x^2+1))^2)^2*Pi*x^4*e+1/4*I/c^4*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2
+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*e*arctanh(c*x)+1/6*I/c^4*b*
e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)+1/24*I/c*b*csgn(I*(
c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-
c^2*x^2+1))^2)*Pi*x^3*e-1/24*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^
2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*x^3*e+1/24*I/c*b*csgn
(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*x^3*e+1/1
```

```
2*I/c*b*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*
Pi*x^3*e+1/24*I/c*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^
2/(-c^2*x^2+1))^2)*Pi*x^3*e-1/12*I/c*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*c
sgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*Pi*x^3*e+1/8*I/c^3*b*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+
1))^2)*Pi*x*e-1/8*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*x*e+1/8*I/c^3*b*csgn(I*(c*x+1)
/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*x*e+1/4*I/c^3*b*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi*x*e+1/8*
I/c^3*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+
1))^2)*Pi*x*e-1/4*I/c^3*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x
+1)^2/(-c^2*x^2+1))^2)^2*Pi*x*e-1/8*I/c^4*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*e*arc
tanh(c*x)+1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(
I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2-1/8*I/c^4*b*csgn(I*
(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*e*arctanh(c*
x)-1/4*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*
(c*x+1)^2/(c^2*x^2-1))^2-1/8*I/c^4*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*c
sgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*e*arctanh(c*x)+1/8*I*b*arctanh(c*x)*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)*Pi*x^4*e-1/8*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*x^4
*e-1/3/c^4*b*d+1/6/c^4*b*e*(3*arctanh(c*x)*x^3*c^3+3*arctanh(c*x)*x^2*c^2+c
^2*x^2+3*arctanh(c*x)*x*c+c*x+3*arctanh(c*x)+4)*(c*x-1)*ln((c*x+1)/(-c^2*x^
2+1)^(1/2))+1/6/c*b*ln(2)*x^3*e-1/6/c*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^3*e-
1/4/c^2*b*arctanh(c*x)*x^2*e+1/2/c^3*b*ln(2)*x*e-1/2/c^3*b*ln(1+(c*x+1)^2/(
-c^2*x^2+1))*x*e-1/2/c^4*b*ln(2)*arctanh(c*x)*e+1/2/c^4*b*arctanh(c*x)*e*ln
(1+(c*x+1)^2/(-c^2*x^2+1))
```

maxima [C] time = 0.33, size = 269, normalized size = 1.20

$$\frac{1}{4}adx^4+\frac{1}{8}\left(2x^4\log\left(-c^2x^2+1\right)-c^2\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)be\,\mathrm{artanh}(cx)+\frac{1}{24}\left(6x^4\,\mathrm{artanh}(cx)+c\left(\frac{2}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2
*log(c^2*x^2 - 1)/c^6))*b*e*arctanh(c*x) + 1/24*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/8*(
2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c
^6))*a*e + 1/144*((12*I*pi*c^3 - 14*c^3)*x^3 + (36*I*pi*c - 150*c)*x - 3*(6
*I*pi - 4*c^3*x^3 - 12*c*x - 25)*log(c*x + 1) - 3*(-6*I*pi - 4*c^3*x^3 - 12
*c*x + 25)*log(c*x - 1))*b*e/c^4
```

mupad [B] time = 1.70, size = 851, normalized size = 3.78

$$\ln(1-cx)^2\left(\frac{be}{8c^4}-\frac{bex^4}{8}\right)-\ln(cx+1)^2\left(\frac{be}{8c^4}-\frac{bex^4}{8}\right)+\ln(1-cx)\left(\frac{x^4\left(ae-\frac{bd}{2}+\frac{be}{4}+\frac{be\left(\ln(cx+1)+\ln(1-cx)-\ln(1\ldots\right.}{2}\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1 - c*x)^2*((b*e)/(8*c^4) - (b*e*x^4)/8) - log(c*x + 1)^2*((b*e)/(8*c^4
) - (b*e*x^4)/8) + log(1 - c*x)*((x^4*(a*e - (b*d)/2 + (b*e)/4 + (b*e*(log(
c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/2))/4 - (x^2*((16*a*e - 8*b*d
```

```
+ 8*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/c - (16*a*e - 8*b
*d + 4*b*e + 8*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/c))/(3
2*c) + (b*e*x)/(4*c^3) + (b*e*x^3)/(12*c)) - x^2*((a*(e - 2*d + 2*e*(log(c*
x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(4*c^2) + (a*(d - e*(log(c*x +
1) + log(1 - c*x) - log(1 - c^2*x^2))))/(2*c^2)) - x*((b*(7*e - 6*d + 6*e*(
log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(24*c^3) + (3*b*e)/(4*c^3
)) - (a*x^4*(e - 2*d + 2*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))
))/8 - (log((x*(12*a*e - 6*b*d + 25*b*e + 6*b*e*(log(c*x + 1) + log(1 - c*x
) - log(1 - c^2*x^2))))/(24*c^2) - (25*b*e - 6*b*d + 6*b*e*(log(c*x + 1) +
log(1 - c*x) - log(1 - c^2*x^2)))/(24*c^3) - (a*e*x)/(2*c^2))*(12*a*e - 6*b
*d + 25*b*e + 6*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(48*
c^4) - (log((x*(12*a*e + 6*b*d - 25*b*e - 6*b*e*(log(c*x + 1) + log(1 - c*x
) - log(1 - c^2*x^2))))/(24*c^2) - (25*b*e - 6*b*d + 6*b*e*(log(c*x + 1) +
log(1 - c*x) - log(1 - c^2*x^2)))/(24*c^3) - (a*e*x)/(2*c^2))*(12*a*e + 6*b
*d - 25*b*e - 6*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(48*
c^4) + c*log(c*x + 1)*((x^4*(4*a*e + 2*b*d - b*e - 2*b*e*(log(c*x + 1) + lo
g(1 - c*x) - log(1 - c^2*x^2))))/(16*c) + (b*e*x)/(4*c^4) + (b*e*x^3)/(12*c
^2) - (b*e*x^2)/(8*c^3)) - (b*x^3*(7*e - 6*d + 6*e*(log(c*x + 1) + log(1 -
c*x) - log(1 - c^2*x^2))))/(72*c)
```

sympy [A] time = 9.74, size = 279, normalized size = 1.24

$$\begin{cases} \frac{adx^4}{4} + \frac{aex^4 \log\left(-c^2x^2+1\right)}{4} - \frac{aex^4}{8} - \frac{aex^2}{4c^2} - \frac{ae\log\left(-c^2x^2+1\right)}{4c^4} + \frac{bdx^4\operatorname{atanh}(cx)}{4} + \frac{bex^4\log\left(-c^2x^2+1\right)\operatorname{atanh}(cx)}{4} - \frac{bex^4\operatorname{atanh}(cx)}{8} + \\ \frac{adx^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*x
**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atanh(c*x)/4 + b
*e*x**4*log(-c**2*x**2 + 1)*atanh(c*x)/4 - b*e*x**4*atanh(c*x)/8 + b*d*x**3
/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e*x**
2*atanh(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*c**3)
 - 25*b*e*x/(24*c**3) - b*d*atanh(c*x)/(4*c**4) - b*e*log(-c**2*x**2 + 1)*a
tanh(c*x)/(4*c**4) + 25*b*e*atanh(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, T
rue))
```

3.524 $\int x^2 \left(a + b \tanh^{-1}(cx)\right) \left(d + e \log \left(1 - c^2x^2\right)\right) dx$

Optimal. Leaf size=247

$$-\frac{e(2a+b)\log(1-cx)}{6c^3}+\frac{e(2a-b)\log(cx+1)}{6c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{2aex}{3c^2}-\frac{2}{9}aex^3+\frac{be\tan$$

```
[Out] -2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*b*e*x*arctanh(c*x)/c^2-2/9*b*
e*x^3*arctanh(c*x)+1/3*b*e*arctanh(c*x)^2/c^3-1/6*(2*a+b)*e*ln(-c*x+1)/c^3+
1/6*(2*a-b)*e*ln(c*x+1)/c^3-4/9*b*e*ln(-c^2*x^2+1)/c^3-1/12*b*e*ln(-c^2*x^2
+1)^2/c^3+1/6*b*x^2*(d+e*ln(-c^2*x^2+1))/c+1/3*x^3*(a+b*arctanh(c*x))*(d+e*
ln(-c^2*x^2+1))+1/6*b*ln(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/c^3
```

Rubi [A] time = 0.63, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5916, 266, 43, 6085, 6725, 801, 633, 31, 5980, 5910, 260, 5948, 2475, 2390, 2301}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{e(2a+b)\log(1-cx)}{6c^3}+\frac{e(2a-b)\log(cx+1)}{6c^3}-\frac{2aex}{3c^2}-\frac{2}{9}aex^3+\frac{bx^2\left(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcTanh[
c*x])/(3*c^2) - (2*b*e*x^3*ArcTanh[c*x])/9 + (b*e*ArcTanh[c*x]^2)/(3*c^3) -
 ((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1
 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^2 \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx = \frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\right.$$

$$= \frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\right.$$

$$= \frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\right.$$

$$= \frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\right.$$

$$= -\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\right.$$

$$= -\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2}{9}bex^3\tanh^{-1}(cx)-\frac{be\log^2\left(1-c^2\right.}{12c^3}$$

$$= -\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2bex\tanh^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tanh^{-1}(c$$

$$= -\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2bex\tanh^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tanh^{-1}(c$$

$$= -\frac{2aex}{3c^2}-\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2bex\tanh^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tanh^{-1}($$

Mathematica [A] time = 0.14, size = 183, normalized size = 0.74

$$6c^2ex^2\log\left(1-c^2x^2\right)\left(2acx+2bcx\tanh^{-1}(cx)+b\right)+2\log(1-cx)(-6ae+3bd-11be)+2\log(cx+1)(6ae+3bd$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x*
(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcTanh[c*x] + 12*b*e*ArcTanh[c*x]^2 + 2*
(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1 +
c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcTanh[c*x])*Log[1 - c^2*x^2] +
3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

fricas [A] time = 0.68, size = 200, normalized size = 0.81

$$-\frac{24\,acex-4\left(3\,ac^3d-2\,ac^3e\right)x^3-3\,be\log\left(-c^2x^2+1\right)^2-3\,be\log\left(-\frac{cx+1}{cx-1}\right)^2-2\left(3\,bc^2d-5\,bc^2e\right)x^2-2\left(6\,ac^3e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/36*(24*a*c*e*x - 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 3*b*e*log(-c^2*x^2 + 1)
^2 - 3*b*e*log(-(c*x + 1)/(c*x - 1))^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 - 2*
(6*a*c^3*e*x^3 + 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(-c^2*x^2 + 1) - 2*(3*b
*c^3*e*x^3*log(-c^2*x^2 + 1) - 6*b*c*e*x + (3*b*c^3*d - 2*b*c^3*e)*x^3 + 6*
a*e)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

giac [A] time = 0.40, size = 353, normalized size = 1.43

$$\frac{3\,bc^3x^3e\log\left(cx+1\right)^2-3\,bc^3x^3e\log\left(-cx+1\right)^2+6\,ac^3x^3e\log\left(cx+1\right)-2\,bc^3x^3e\log\left(cx+1\right)+6\,ac^3x^3e\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/18*(3*b*c^3*x^3*e*log(c*x + 1)^2 - 3*b*c^3*x^3*e*log(-c*x + 1)^2 + 6*a*c^
3*x^3*e*log(c*x + 1) - 2*b*c^3*x^3*e*log(c*x + 1) + 6*a*c^3*x^3*e*log(-c*x
+ 1) + 2*b*c^3*x^3*e*log(-c*x + 1) + 3*b*c^3*d*x^3*log(-(c*x + 1)/(c*x - 1)
) + 6*a*c^3*d*x^3 - 4*a*c^3*x^3*e + 3*b*c^2*x^2*e*log(c*x + 1) + 3*b*c^2*x^
2*e*log(-c*x + 1) + 3*b*c^2*d*x^2 - 5*b*c^2*x^2*e - 6*b*c*x*e*log(c*x + 1)
+ 6*b*c*x*e*log(-c*x + 1) - 12*a*c*x*e + 3*b*e*log(c*x + 1)^2 - 3*b*e*log(c
*x - 1)^2 + 6*b*e*log(c*x - 1)*log(-c*x + 1) + 3*b*d*log(c^2*x^2 - 1) + 6*a
*e*log(c*x + 1) - 11*b*e*log(c*x + 1) - 6*a*e*log(c*x - 1) - 11*b*e*log(c*x
 - 1))/c^3
```

maple [C] time = 4.13, size = 3994, normalized size = 16.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/6/c^3*b*d+5/18*e/c^3*b-1/3/c^3*b*e*ln(2)-2/9*a*e*x^3-2/3*b*e*x*arctanh(c
*x)/c^2-2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/9*b*e*x^3*arctanh(c*x)+1/6*b*d*x^2/c
-1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/12*I/c^3*b*e*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/12*I/c^3*b*e*Pi*cs
gn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3+1/3/c^3*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))
^2*e+1/3/c^3*b*arctanh(c*x)*d-8/9/c^3*b*arctanh(c*x)*e-1/3/c^3*b*d*ln(1+(c*
x+1)^2/(-c^2*x^2+1))+11/9/c^3*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))+1/3*b*arctan
h(c*x)*x^3*d+1/3*x^3*a*d+1/3*a*e*x^3*ln(-c^2*x^2+1)+1/3*a*e/c^3*ln(c*x+1)-1
/3*a*e/c^3*ln(c*x-1)+1/3/c^3*b*e*(2*arctanh(c*x)*x^3*c^3+c^2*x^2+2*arctanh(
c*x)-2*ln(1+(c*x+1)^2/(-c^2*x^2+1))-1)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/6*I
*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*x^3*e
-1/6*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctan
h(c*x)+1/6*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^
2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*e*ln(
1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/12*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(
```

```
I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(
-c^2*x^2+1))^2)*Pi*x^2*e+1/6*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(
c*x+1)/(-c^2*x^2+1)^(1/2))*Pi*x^2*e-1/12*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*Pi*x^2*e+1/1
2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*
Pi*x^2*e+1/12*I/c*b*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*Pi*x^2*e-1/6*I/c*b*csgn(I*(1+(c*
x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*x^2*e+1/12*
I/c*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)
))^2*Pi*x^2*e+1/3*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-
c^2*x^2+1)^(1/2))*Pi*e*arctanh(c*x)-1/6*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctan
h(c*x)-1/6*I/c^3*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2
/(-c^2*x^2+1)))^2*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e+1/3*I*b*arctanh(c*x)*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi*x^3*e-1
/6*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*Pi*x^3*e+1/6*I*b*arctanh(c*x)*csgn(I*(c
*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*x^3*e+1/6*I*b*
arctanh(c*x)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*Pi*x^3*e-1/3*I*b*arctanh(c*x)*csgn(I*(1
+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*x^3*e+1
/6*I*b*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*Pi*x^3*e+1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))
*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+
1)^2/(-c^2*x^2+1))^2)+1/6*I/c^3*b*Pi*arctanh(c*x)*e*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/6*I/c^3*b*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
^2)*Pi*e*arctanh(c*x)-1/3*I/c^3*b*Pi*arctanh(c*x)*e*csgn(I*(1+(c*x+1)^2/(-c
^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/6*I/c^3*b*csgn(I*(1+(c
*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*e*arctanh(
c*x)-1/3*I/c^3*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/6*I/c^3*b*Pi*ln(1+(c*x+1)^2/(-
c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
+(c*x+1)^2/(-c^2*x^2+1))^2)^2-1/6*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e-1/6*I/c
^3*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/3*I/c^3*b*P
i*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*
(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/6*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*
x^2-1))^3*Pi*x^3*e+1/6*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*
x+1)^2/(-c^2*x^2+1))^2)^3*Pi*x^3*e+1/6*I*b*arctanh(c*x)*csgn(I*(1+(c*x+1)^2
/(-c^2*x^2+1))^2)^3*Pi*x^3*e+1/12*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*
x^2*e+1/12*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)
^3*Pi*x^2*e-1/3/c*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^2*e+1/3/c*b*ln(2)*x^2*e+
2/3/c^3*b*arctanh(c*x)*ln(2)*e-2/3/c^3*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*ln(
2)-2/3*b*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^3*e+2/3*b*arctanh(c*x)
*ln(2)*x^3*e+1/12*I/c*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*x^2*e+1/6
*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e*arctanh(c*x)+1/6*I/c^3*b*Pi*a
rctanh(c*x)*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3+
1/6*I/c^3*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*e*arctanh(c*x)-1/6*I/
c^3*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/6
*I/c^3*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+
(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/6*I/c^3*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*
csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/6*I/c^3*b*Pi*e*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/12*I/c^3*b*e*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
)^2)^2-1/12*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))^2*Pi*e-1/12*I/c^3*b*e*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/6*I/c^3*b*e*Pi*
```

```
csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2-1
/12*I/c^3*b*e*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(
-c^2*x^2+1))^2)
```

maxima [C] time = 0.34, size = 252, normalized size = 1.02

$$\frac{1}{3}adx^3+\frac{1}{9}\left(3x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(c^2x^3+3x\right)}{c^4}-\frac{3\log\left(cx+1\right)}{c^5}+\frac{3\log\left(cx-1\right)}{c^5}\right)\right)be\,\mathrm{artanh}\left(cx\right)+\frac{1}{6}\left(2x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*e*arctanh(c*x) + 1/6*(2*x^3*arct
anh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2*x^
2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c
^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*log(c*x
 - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*e/c^3
```

mupad [B] time = 2.81, size = 515, normalized size = 2.09

$$\frac{a\,d\,x^3}{3}-\frac{2\,a\,e\,x^3}{9}+\frac{b\,d\,x^3\,\ln\left(c\,x+1\right)}{6}-\frac{b\,d\,x^3\,\ln\left(1-c\,x\right)}{6}-\frac{b\,e\,x^3\,\ln\left(c\,x+1\right)}{9}+\frac{b\,e\,x^3\,\ln\left(1-c\,x\right)}{9}+\frac{b\,e\ln\left(c\,x+1\right)^2}{6\,c^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] (a*d*x^3)/3 - (2*a*e*x^3)/9 + (b*d*x^3*log(c*x + 1))/6 - (b*d*x^3*log(1 - c
*x))/6 - (b*e*x^3*log(c*x + 1))/9 + (b*e*x^3*log(1 - c*x))/9 + (b*e*log(c*x
 + 1)^2)/(6*c^3) + (b*e*log(1 - c*x)^2)/(6*c^3) - (2*a*e*x)/(3*c^2) + (b*d*
x^2)/(6*c) - (5*b*e*x^2)/(18*c) + (a*e*x^3*log(1 - c^2*x^2))/3 - (a*e*log(c
*x - 1))/(3*c^3) + (a*e*log(c*x + 1))/(3*c^3) + (b*d*log(c*x - 1))/(6*c^3)
+ (b*d*log(c*x + 1))/(6*c^3) - (11*b*e*log(c*x - 1))/(18*c^3) - (11*b*e*log
(c*x + 1))/(18*c^3) - (b*e*log(c*x + 1)*log(-(2*a*e - 2*a*c*e*x)/(3*c^2)))/
(6*c^3) - (b*e*log(c*x + 1)*log(-(2*a*e + 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b
*e*log(1 - c*x)*log(-(2*a*e - 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b*e*log(1 - c
*x)*log(-(2*a*e + 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b*e*x*log(c*x + 1))/(3*c^
2) + (b*e*x*log(1 - c*x))/(3*c^2) + (b*e*x^2*log(1 - c^2*x^2))/(6*c) + (b*e
*log(-(2*a*e - 2*a*c*e*x)/(3*c^2))*log(1 - c^2*x^2))/(6*c^3) + (b*e*log(-(2
*a*e + 2*a*c*e*x)/(3*c^2))*log(1 - c^2*x^2))/(6*c^3) + (b*e*x^3*log(c*x + 1
)*log(1 - c^2*x^2))/6 - (b*e*x^3*log(1 - c*x)*log(1 - c^2*x^2))/6
```

sympy [A] time = 6.54, size = 258, normalized size = 1.04

$$\begin{cases}\frac{adx^3}{3}+\frac{aex^3\log\left(-c^2x^2+1\right)}{3}-\frac{2aex^3}{9}-\frac{2aex}{3c^2}+\frac{2ae\,\mathrm{atanh}\left(cx\right)}{3c^3}+\frac{bdx^3\,\mathrm{atanh}\left(cx\right)}{3}+\frac{bex^3\log\left(-c^2x^2+1\right)\mathrm{atanh}\left(cx\right)}{3}-\frac{2bex^3\,\mathrm{atanh}\left(cx\right)}{9}+ \\ \frac{adx^3}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*a
*e*x/(3*c**2) + 2*a*e*atanh(c*x)/(3*c**3) + b*d*x**3*atanh(c*x)/3 + b*e*x**
3*log(-c**2*x**2 + 1)*atanh(c*x)/3 - 2*b*e*x**3*atanh(c*x)/9 + b*d*x**2/(6*
c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*atanh
(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2 + 1)
**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*atanh(c*x)**2/(3
*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

3.525 $\int x \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=140

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b(d-e)\tanh}{2c^2}$$

```
[Out] 1/2*b*(d-e)*x/c-b*e*x/c-1/2*b*(d-e)*arctanh(c*x)/c^2+b*e*arctanh(c*x)/c^2+1
/2*d*x^2*(a+b*arctanh(c*x))-1/2*e*x^2*(a+b*arctanh(c*x))+1/2*b*e*x*ln(-c^2*
x^2+1)/c-1/2*e*(-c^2*x^2+1)*(a+b*arctanh(c*x))*ln(-c^2*x^2+1)/c^2
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2454, 2389, 2295, 6083, 321, 207, 2448, 206}

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b(d-e)\tanh}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (b*e*A
rcTanh[c*x])/c^2 + (d*x^2*(a + b*ArcTanh[c*x]))/2 - (e*x^2*(a + b*ArcTanh[c
*x]))/2 + (b*e*x*Log[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*ArcTanh[
c*x])*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{e(1-$$

$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{e(1-$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{be\tanh^{-1}(cx)}{c^2}+\frac{1}{2}$$

Mathematica [A] time = 0.11, size = 129, normalized size = 0.92

$$\frac{2e\log\left(1-c^2x^2\right)\left(cx(acx+b)+b\left(c^2x^2-1\right)\tanh^{-1}(cx)\right)+\log(1-cx)(b(d-3e)-2ae)-\log(cx+1)(2ae+b(}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcTanh[c*x]
 + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x]
+ 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcTanh[c*x])*Log[1 - c^2*x^2])/(
4*c^2)
```

fricas [A] time = 0.63, size = 139, normalized size = 0.99

$$\frac{2\left(ac^2d-ac^2e\right)x^2+2\left(bcd-3\,bce\right)x+2\left(ac^2ex^2+bcex-ae\right)\log\left(-c^2x^2+1\right)+\left(\left(bc^2d-bc^2e\right)x^2-bd+3\,be+\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*c^2*d - a*c^2*e)*x^2 + 2*(b*c*d - 3*b*c*e)*x + 2*(a*c^2*e*x^2 + b
*c*e*x - a*e)*log(-c^2*x^2 + 1) + ((b*c^2*d - b*c^2*e)*x^2 - b*d + 3*b*e +
(b*c^2*e*x^2 - b*e)*log(-c^2*x^2 + 1))*log(-(c*x + 1)/(c*x - 1)))/c^2
```

giac [B] time = 0.35, size = 305, normalized size = 2.18

$$bc^2x^2e\log(cx+1)^2 - bc^2x^2e\log(-cx+1)^2 + 2\,ac^2x^2e\log(cx+1) - bc^2x^2e\log(cx+1) + 2\,ac^2x^2e\log(-cx+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/4*(b*c^2*x^2*e*log(c*x + 1)^2 - b*c^2*x^2*e*log(-c*x + 1)^2 + 2*a*c^2*x^2
*e*log(c*x + 1) - b*c^2*x^2*e*log(c*x + 1) + 2*a*c^2*x^2*e*log(-c*x + 1) +
b*c^2*x^2*e*log(-c*x + 1) + b*c^2*d*x^2*log(-(c*x + 1)/(c*x - 1)) + 2*a*c^2
*d*x^2 - 2*a*c^2*x^2*e + 2*b*c*x*e*log(c*x + 1) + 2*b*c*x*e*log(-c*x + 1) +
 2*b*c*d*x - 6*b*c*x*e - b*e*log(c*x + 1)^2 - b*e*log(c*x - 1)^2 + 2*b*e*lo
g(c*x - 1)*log(-c*x + 1) - b*d*log(c*x + 1) - 2*a*e*log(c*x + 1) + 3*b*e*lo
g(c*x + 1) + b*d*log(c*x - 1) - 2*a*e*log(c*x - 1) - 3*b*e*log(c*x - 1))/c^
2
```

maple [C] time = 3.85, size = 2951, normalized size = 21.08

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)
*x^2*e-1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x*e+
1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn
(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*e*arctanh(c*
x)*Pi-1/c^2*b*e*ln(2)+3/2*e/c^2*b+1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x*e+1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
^2)*x*e+1/2*a*e/c^2-1/2/c^2*b*d*arctanh(c*x)+1/2*b*arctanh(c*x)*x^2*d-1/2*b
*arctanh(c*x)*x^2*e+1/2*a*e*ln(-c^2*x^2+1)*x^2-1/2*a*e/c^2*ln(-c^2*x^2+1)-1
/2/c^2*b*d+5/2*b*e*arctanh(c*x)/c^2-1/2*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*
x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*x*e+1/4*I/c*b*Pi*csgn(I*(1+
(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x*e-1/2*I/c
^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*e*a
rctanh(c*x)*Pi+1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*
x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh(c*x)-1/4*I/c^2*b*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*e*arctanh(
c*x)*Pi+1/2*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x
+1)/(-c^2*x^2+1)^(1/2))*x^2*e-1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^2*e
+1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2
*x^2+1)^(1/2))^2*x^2*e+1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^2*e
-1/2*I*b*arctanh(c*x)*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(
c*x+1)^2/(-c^2*x^2+1)))*x^2*e+1/4*I*b*arctanh(c*x)*Pi*csgn(I*(1+(c*x+1)^2/(
-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^2*e-1/4*I/c^2*b*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1))^2)*Pi*e*arctanh(c*x)+1/2*I/c^2*b*csgn(I*(1+(c*x+1)^2/(-c^2*
```

```
x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*e*arctanh(c*x)*Pi-1/4*I/c^2
*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^
2*e*arctanh(c*x)*Pi+1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(
-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*e*Pi+1/2*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-
c^2*x^2+1)^(1/2))*x*e-1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x*e-1/2*a*e*x^2+1/2*a*x^
2*d+1/2*b*d*x/c-3/2*b*e*x/c+1/c*b*ln(2)*x*e-1/c*b*ln(1+(c*x+1)^2/(-c^2*x^2+
1))*x*e-1/c^2*b*ln(2)*e*arctanh(c*x)+1/c^2*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e
*arctanh(c*x)-b*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^2*e+b*arctanh(c
*x)*ln(2)*x^2*e-1/4*I/c^2*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/
(-c^2*x^2+1))^2)^3-1/4*I/c^2*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*e*Pi+
1/c^2*b*e*(arctanh(c*x)*x*c+arctanh(c*x)+1)*(c*x-1)*ln((c*x+1)/(-c^2*x^2+1)
^(1/2))-1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/2*I/c^2*b*Pi*e*c
sgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/4*I/c^2
*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*Pi*e-1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))
^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*e+1/
2*I/c^2*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*
x^2+1)))*Pi*e-1/4*I/c^2*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c
*x+1)^2/(-c^2*x^2+1)))^2*Pi*e+1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1))^3*x^2*e+1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
*x+1)^2/(-c^2*x^2+1))^2)^3*x^2*e+1/4*I*b*arctanh(c*x)*Pi*csgn(I*(1+(c*x+1)^
2/(-c^2*x^2+1))^2)^3*x^2*e+1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x*e
+1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*
x*e+1/4*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x*e-1/4*I/c^2*b*csg
n(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e*arctanh(c*x)-1/4*I/c^2*b*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*e*arctanh(c*x)*Pi-1/4*I/c^2*b*
csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*e*arctanh(c*x)
```

maxima [A] time = 0.32, size = 171, normalized size = 1.22

$$\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\left(cx\right)+c\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+1\right)}{c^3}+\frac{\log\left(cx-1\right)}{c^3}\right)\right)bd-\frac{\left(c^2x^2-\left(c^2x^2-1\right)\log\left(-c^2x^2+1\right)-1\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*b
*e*arctanh(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*a
*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*b*e
/c^2
```

mupad [B] time = 1.42, size = 557, normalized size = 3.98

$$\ln\left(1-c\,x\right)^2\,\left(\frac{b\,e}{4\,c^2}-\frac{b\,e\,x^2}{4}\right)-\ln\left(c\,x+1\right)^2\,\left(\frac{b\,e}{4\,c^2}-\frac{b\,e\,x^2}{4}\right)+\ln\left(1-c\,x\right)\left(\frac{x^2\,\left(a\,e-\frac{b\,d}{2}+\frac{b\,e}{2}+\frac{b\,e\left(\ln(c\,x+1)+\ln(1-c\,x)-\right.}{2}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1 - c*x)^2*((b*e)/(4*c^2) - (b*e*x^2)/4) - log(c*x + 1)^2*((b*e)/(4*c^2
) - (b*e*x^2)/4) + log(1 - c*x)*((x^2*(a*e - (b*d)/2 + (b*e)/2 + (b*e*(log(
c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/2))/2 + (b*e*x)/(2*c)) + c*log
(c*x + 1)*((x^2*(2*a*e + b*d - b*e - b*e*(log(c*x + 1) + log(1 - c*x) - log
```

```
(1 - c^2*x^2))))/(4*c) + (b*e*x)/(2*c^2)) - (a*x^2*(e - d + e*(log(c*x + 1)
 + log(1 - c*x) - log(1 - c^2*x^2))))/2 - (log((x*(2*a*e + b*d - 3*b*e - b*
e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/2 - (3*b*e - b*d + b*e
*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/(2*c) - a*e*x)*(2*a*e +
b*d - 3*b*e - b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(4*c^2
) - (log((x*(2*a*e - b*d + 3*b*e + b*e*(log(c*x + 1) + log(1 - c*x) - log(1
 - c^2*x^2))))/2 - (3*b*e - b*d + b*e*(log(c*x + 1) + log(1 - c*x) - log(1
- c^2*x^2)))/(2*c) - a*e*x)*(2*a*e - b*d + 3*b*e + b*e*(log(c*x + 1) + log(
1 - c*x) - log(1 - c^2*x^2))))/(4*c^2) - (b*x*(3*e - d + e*(log(c*x + 1) +
log(1 - c*x) - log(1 - c^2*x^2))))/(2*c)
```

sympy [A] time = 3.78, size = 202, normalized size = 1.44

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^2\log\left(-c^2x^2+1\right)}{2} - \frac{aex^2}{2} - \frac{ae\log\left(-c^2x^2+1\right)}{2c^2} + \frac{bdx^2\,\mathrm{atanh}\,(cx)}{2} + \frac{bex^2\log\left(-c^2x^2+1\right)\mathrm{atanh}\,(cx)}{2} - \frac{bex^2\,\mathrm{atanh}\,(cx)}{2} + \frac{bdx}{2c} + \frac{bex\,\mathrm{l}}{} \\ \frac{adx^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*l
og(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atanh(c*x)/2 + b*e*x**2*log(-c**2*x*
*2 + 1)*atanh(c*x)/2 - b*e*x**2*atanh(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-c**
2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*atanh(c*x)/(2*c**2) - b*e*log(-c**2
*x**2 + 1)*atanh(c*x)/(2*c**2) + 3*b*e*atanh(c*x)/(2*c**2), Ne(c, 0)), (a*d
*x**2/2, True))
```

3.526 $\int \left(a+b \tanh^{-1}(cx)\right) \left(d+e \log \left(1-c^2x^2\right)\right) \, dx$

Optimal. Leaf size=104

$$x \left(a+b \tanh^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)+\frac{e \left(a+b \tanh^{-1}(cx)\right)^2}{bc}-2aex+\frac{b \left(e \log \left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be \log \left(1-\right.}{c}$$

```
[Out] -2*a*e*x-2*b*e*x*arctanh(c*x)+e*(a+b*arctanh(c*x))^2/b/c-b*e*ln(-c^2*x^2+1)
/c+x*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))+1/4*b*(d+e*ln(-c^2*x^2+1))^2/c
/e
```

Rubi [A] time = 0.20, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6073, 2475, 2390, 2301, 5980, 5910, 260, 5948}

$$x \left(a+b \tanh^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)+\frac{e \left(a+b \tanh^{-1}(cx)\right)^2}{bc}-2aex+\frac{b \left(e \log \left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be \log \left(1-\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcTanh[c*x] + (e*(a + b*ArcTanh[c*x])^2)/(b*c) - (b*e*L
og[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(
d + e*Log[1 - c^2*x^2])^2)/(4*c*e)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6073

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[
2*e*g, Int[(x^2*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx &= x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-(bc)\int \frac{x\left(d+e\log\right.}{1-}\\ &= x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{d}{}\right.\\ &= -2aex+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}+x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\right.\\ &= -2aex-2bex\tanh^{-1}(cx)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}+x\left(a+b\tan\right.\\ &= -2aex-2bex\tanh^{-1}(cx)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}-\frac{be\log\left(1-\right.}{c}\end{aligned}$$

Mathematica [A] time = 0.02, size = 144, normalized size = 1.38

$$aex\log\left(1-c^2x^2\right)+\frac{2ae\tanh^{-1}(cx)}{c}+adx-2aex+\frac{bd\log\left(1-c^2x^2\right)}{2c}+\frac{be\log^2\left(1-c^2x^2\right)}{4c}-\frac{be\log\left(1-c^2x^2\right)}{c}+bex\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*ArcTanh[c*x])/c + b*d*x*ArcTanh[c*x] - 2*b*e*x*Arc
Tanh[c*x] + (b*e*ArcTanh[c*x]^2)/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*Lo
g[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcTanh[c*x]*Log[1 - c^2
*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

fricas [A] time = 0.53, size = 132, normalized size = 1.27

$$\frac{be\log\left(-c^2x^2+1\right)^2+be\log\left(-\frac{cx+1}{cx-1}\right)^2+4\,(acd-2\,ace)x+2\,(2\,acex+bd-2\,be)\log\left(-c^2x^2+1\right)+2\left(bcex\log\left(-\right.\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*e*log(-c^2*x^2 + 1)^2 + b*e*log(-(c*x + 1)/(c*x - 1))^2 + 4*(a*c*d -
 2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*log(-c^2*x^2 + 1) + 2*(b*c*e*x*lo
g(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

giac [B] time = 0.18, size = 223, normalized size = 2.14

$$bcxe\log(cx+1)^2 - bcxe\log(-cx+1)^2 + 2\,acxe\log(cx+1) - 2\,bcxe\log(cx+1) + 2\,acxe\log(-cx+1) + 2\,bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x*e*log(c*x + 1)^2 - b*c*x*e*log(-c*x + 1)^2 + 2*a*c*x*e*log(c*x +
 1) - 2*b*c*x*e*log(c*x + 1) + 2*a*c*x*e*log(-c*x + 1) + 2*b*c*x*e*log(-c*x
 + 1) + b*c*d*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*d*x - 4*a*c*x*e + b*e*log
(c*x + 1)^2 - b*e*log(c*x - 1)^2 + 2*b*e*log(c*x - 1)*log(-c*x + 1) + b*d*l
og(c^2*x^2 - 1) + 2*a*e*log(c*x + 1) - 2*b*e*log(c*x + 1) - 2*a*e*log(c*x -
 1) - 2*b*e*log(c*x - 1))/c
```

maple [C] time = 1.48, size = 2529, normalized size = 24.32

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] 1/2*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))^2*x*e+2*b*arctanh(c*x)*ln(2)*x*e+2/c*b*ln(2)*e*arctanh(c*x)-2
/c*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*ln(2)-2*b*arctanh(c*x)*ln(1+(c*x+1)^2/(
-c^2*x^2+1))*x*e+1/2*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
*x+1)^2/(-c^2*x^2+1))^2)^3*x*e+1/2*I*b*arctanh(c*x)*Pi*csgn(I*(1+(c*x+1)^2/
(-c^2*x^2+1))^2)^3*x*e+1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e*arcta
nh(c*x)+1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2
)^3*e*arctanh(c*x)*Pi+1/2*I/c*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*Pi*e
*arctanh(c*x)-1/2*I/c*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/
(c^2*x^2-1))^3-1/2*I/c*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/2*I/c*b*Pi*ln(1+(c*x+1)^2/(-
c^2*x^2+1))*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-2*a*x*e+a*x*e*ln(-c^2*
x^2+1)-1/2*I/c*b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(
-c^2*x^2+1)))^2*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/2*I/c*b*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*e*ln(1+(c*x+1)^2/(-c^2*
x^2+1))*Pi+1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))^2*e*arctanh(c*x)*Pi+I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I
*(c*x+1)/(-c^2*x^2+1)^(1/2))*e*arctanh(c*x)*Pi+I/c*b*csgn(I*(1+(c*x+1)^2/(-
c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*e*ln(1+(c*x+1)^2/(-c^2*
x^2+1))*Pi+I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*x*e-1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1
)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh(c*x)-I*b*
arctanh(c*x)*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/
(-c^2*x^2+1)))*x*e+1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^
2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))
*Pi-1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-I/c*
b*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))
*e*arctanh(c*x)*Pi+1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^
2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*e*arctanh(c*x)+1/2*I
*b*arctanh(c*x)*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c
```

```
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x*e+1/2*I/c*b*csgn(I*(1+(c*x+1)^2
/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*e*arctanh(c*x)*Pi+1/
2*I*b*arctanh(c*x)*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1
)^2/(-c^2*x^2+1)))^2*x*e-1/2*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x*e-I/c*b*
csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*e*ln(1+(
c*x+1)^2/(-c^2*x^2+1))*Pi+a*x*d-a*e/c*ln(c*x-1)+1/c*b*ln(1+(c*x+1)^2/(-c^2*
x^2+1))^2*e+1/c*b*d*arctanh(c*x)-2/c*b*e*arctanh(c*x)-1/c*b*d*ln(1+(c*x+1)^
2/(-c^2*x^2+1))+2/c*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))+b*arctanh(c*x)*x*d-2*b
*e*x*arctanh(c*x)+a*e/c*ln(c*x+1)+2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))*(arctanh
(c*x)*x*c+arctanh(c*x)-ln(1+(c*x+1)^2/(-c^2*x^2+1)))*b*e/c-1/2*I*b*arctanh(
c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x*e-1/2*I/c*b*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*e*arctanh(c*x)*Pi+1/2*I/c*
b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*e*ln(1+(c*x+1)^2/(-c^2
*x^2+1))*Pi+1/2*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x*e
```

maxima [C] time = 0.32, size = 178, normalized size = 1.71

$$-\left(c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) - x\log\left(-c^2x^2+1\right)\right)be\,\text{artanh}(cx) - \left(c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] -(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1)
)*b*e*arctanh(c*x) - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) -
 x*log(-c^2*x^2 + 1))*a*e + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2)*lo
g(c*x - 1))*b*e/c
```

mupad [B] time = 1.79, size = 385, normalized size = 3.70

$$a\,d\,x - 2\,a\,e\,x + \frac{b\,e\ln(c\,x+1)^2}{2\,c} + \frac{b\,e\ln(1-c\,x)^2}{2\,c} + a\,e\,x\,\ln\left(1-c^2\,x^2\right) + \frac{b\,d\,x\,\ln(c\,x+1)}{2} - \frac{b\,d\,x\,\ln(1-c\,x)}{2} - b\,e\,x\,\ln(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] a*d*x - 2*a*e*x + (b*e*log(c*x + 1)^2)/(2*c) + (b*e*log(1 - c*x)^2)/(2*c) +
 a*e*x*log(1 - c^2*x^2) + (b*d*x*log(c*x + 1))/2 - (b*d*x*log(1 - c*x))/2 -
 b*e*x*log(c*x + 1) + b*e*x*log(1 - c*x) - (a*e*log(c*x - 1))/c + (a*e*log(
c*x + 1))/c + (b*d*log(c*x - 1))/(2*c) + (b*d*log(c*x + 1))/(2*c) - (b*e*lo
g(c*x - 1))/c - (b*e*log(c*x + 1))/c + (b*e*x*log(c*x + 1)*log(1 - c^2*x^2)
)/2 - (b*e*x*log(1 - c*x)*log(1 - c^2*x^2))/2 + (b*e*log(1 - c^2*x^2)*log(-
 2*a*e - 2*a*c*e*x))/(2*c) + (b*e*log(1 - c^2*x^2)*log(2*a*c*e*x - 2*a*e))/
(2*c) - (b*e*log(c*x + 1)*log(- 2*a*e - 2*a*c*e*x))/(2*c) - (b*e*log(c*x +
1)*log(2*a*c*e*x - 2*a*e))/(2*c) - (b*e*log(1 - c*x)*log(- 2*a*e - 2*a*c*e*
x))/(2*c) - (b*e*log(1 - c*x)*log(2*a*c*e*x - 2*a*e))/(2*c)
```

sympy [A] time = 2.26, size = 148, normalized size = 1.42

$$\begin{cases} adx + aex\log\left(-c^2x^2+1\right) - 2aex + \frac{2ae\,\text{atanh}(cx)}{c} + bdx\,\text{atanh}(cx) + bex\log\left(-c^2x^2+1\right)\text{atanh}(cx) - 2bex\,\text{atanh} \\ adx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atanh(c*x)/c
 + b*d*x*atanh(c*x) + b*e*x*log(-c**2*x**2 + 1)*atanh(c*x) - 2*b*e*x*atanh(
c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c) - b
*e*log(-c**2*x**2 + 1)/c + b*e*atanh(c*x)**2/c, Ne(c, 0)), (a*d*x, True))
```

3.527 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$

Optimal. Leaf size=216

$$-\frac{1}{2}ae\text{Li}_2\left(c^2x^2\right)+ad\log(x)+\frac{1}{2}be\text{Li}_2(-cx)\left(-\log\left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)-\frac{1}{2}be\text{Li}_2(cx)\left(-\log\left(1-c^2\right.\right.$$

```
[Out] a*d*ln(x)-1/2*b*e*ln(c*x)*ln(-c*x+1)^2+1/2*b*e*ln(-c*x)*ln(c*x+1)^2-1/2*b*d
*polylog(2,-c*x)+1/2*b*e*(ln(-c*x+1)+ln(c*x+1)-ln(-c^2*x^2+1))*polylog(2,-c
*x)+1/2*b*d*polylog(2,c*x)-1/2*b*e*(ln(-c*x+1)+ln(c*x+1)-ln(-c^2*x^2+1))*po
lylog(2,c*x)-1/2*a*e*polylog(2,c^2*x^2)-b*e*ln(-c*x+1)*polylog(2,-c*x+1)+b*
e*ln(c*x+1)*polylog(2,c*x+1)+b*e*polylog(3,-c*x+1)-b*e*polylog(3,c*x+1)
```

Rubi [A] time = 0.28, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6079, 5912, 6077, 2391, 6075, 2396, 2433, 2374, 6589}

$$-\frac{1}{2}ae\text{PolyLog}\left(2,c^2x^2\right)+\frac{1}{2}be\left(-\log\left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\text{PolyLog}(2,-cx)-\frac{1}{2}be\left(-\log\left(1-c^2x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] a*d*Log[x] - (b*e*Log[c*x]*Log[1 - c*x]^2)/2 + (b*e*Log[-(c*x)]*Log[1 + c*x
]^2)/2 - (b*d*PolyLog[2, -(c*x)])/2 + (b*e*(Log[1 - c*x] + Log[1 + c*x] - L
og[1 - c^2*x^2])*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2 - (b*e*(Lo
g[1 - c*x] + Log[1 + c*x] - Log[1 - c^2*x^2])*PolyLog[2, c*x])/2 - (a*e*Pol
yLog[2, c^2*x^2])/2 - b*e*Log[1 - c*x]*PolyLog[2, 1 - c*x] + b*e*Log[1 + c*
x]*PolyLog[2, 1 + c*x] + b*e*PolyLog[3, 1 - c*x] - b*e*PolyLog[3, 1 + c*x]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6075

```
Int[(ArcTanh[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist
[Log[f + g*x^2] - Log[1 - c*x] - Log[1 + c*x], Int[ArcTanh[c*x]/x, x], x] +
 (-Dist[1/2, Int[Log[1 - c*x]^2/x, x], x] + Dist[1/2, Int[Log[1 + c*x]^2/x,
 x], x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

Rule 6077

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTanh[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[(Log[f + g*x^
2]*ArcTanh[c*x])/x, x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 6079

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Dist[d, Int[(a + b*ArcTanh[c*x])/x, x], x] + Dis
t[e, Int[(Log[f + g*x^2]*(a + b*ArcTanh[c*x]))/x, x], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(1-c\right.}{x}$$

$$= ad\log(x) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd\text{Li}_2(cx) + (ae)\int \frac{\log\left(1-c^2\right.}{x}$$

$$= ad\log(x) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd\text{Li}_2(cx) - \frac{1}{2}ae\text{Li}_2\left(c^2x^2\right) - \frac{1}{2}($$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1-$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1-$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1-$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1-$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd\,\text{artanh}\,(cx)+ad+(be\,\text{artanh}\,(cx)+ae)\log\left(-c^2x^2+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

maple [C] time = 10.98, size = 1638, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x,x)
```

```
[Out] b*e*polylog(3,-c*x+1)-b*e*polylog(3,c*x+1)+ln(c*x)*ln(c*x-1)*a*e-1/2*ln(c*x
)*ln(c*x-1)*b*d-1/2*ln(c*x)*ln(c*x-1)^2*b*e-polylog(2,-c*x+1)*ln(c*x-1)*b*e
+1/2*b*e*ln(-c*x)*ln(c*x+1)^2+b*e*ln(c*x+1)*polylog(2,c*x+1)-1/4*I*ln(c*x)*
ln(c*x-1)*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*dilog(c*x)
*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/2*I*ln(c*
x)*Pi*a*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/4*I*ln(
c*x)*ln(c*x-1)*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+dilog(c*x)*
a*e-1/2*dilog(c*x)*b*d+ln(c*x)*a*d+1/2*ln(c*x)*Pi^2*b*e-1/4*I*dilog(c*x)*Pi
*b*e*csgn(I*(c*x-1)*(c*x+1))^3+1/4*I*ln(c*x)*ln(c*x-1)*Pi*b*e*csgn(I*(c*x-1
))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x
-1))^3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+I*ln(c*x)*Pi*a*e-1/4*I*ln(
c*x)*ln(c*x-1)*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^3+1/4*ln(c*x)*Pi^2*b*e*csgn(I
*(c*x-1))^3*csgn(I*(c*x-1)*(c*x+1))^3+1/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1))^
3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-(1/2*I*Pi*b*e*csgn(I*(c*x-1))^2-1
/2*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/2*I*Pi*b*e*csgn(I*(c*x-1))^3-1/4*I*
Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/4*I*Pi*b*e
*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x+1))*csg
n(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^3+a*e+1/2*b*d)*
dilog(c*x+1)-1/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c
*x-1)*(c*x+1))+1/2*I*ln(c*x)*ln(c*x-1)*Pi*b*e*csgn(I*(c*x-1))^2+1/2*I*ln(c*
x)*ln(c*x-1)*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/2*I*ln(c*x)*ln(c*x-1)*Pi*b*
e*csgn(I*(c*x-1))^3-1/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1))^2*csgn(I*(c*x-1)*(
c*x+1))^3-3/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x-1)*(c*x+1))^2-
1/2*I*ln(c*x)*Pi*b*d-I*dilog(c*x)*Pi*b*e-1/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1
))^2*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*ln(c*x)*Pi^2*b*e*csgn(I*
```

```
(c*x-1))^4*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/4*I*dilog(c*x)*Pi*b*e*
csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*dilog(c*x)*Pi*b*e*csgn(I*(c
*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*ln(c*x)*Pi*a*e*csgn(I*(c*x-1))*csgn(
I*(c*x-1)*(c*x+1))^2+1/2*I*ln(c*x)*Pi*a*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c
*x+1))^2+1/2*I*ln(c*x)*Pi*a*e*csgn(I*(c*x-1)*(c*x+1))^3-I*ln(c*x)*ln(c*x-1)
*Pi*b*e+1/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*
ln(c*x)*Pi^2*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*ln(c*x)*Pi^2
*b*e*csgn(I*(c*x-1))^4*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*dilog(c*x)*Pi*b*e*cs
gn(I*(c*x-1))^2+1/2*I*dilog(c*x)*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2-I*ln(c*x)
*Pi*a*e*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*ln(c*x)*Pi*b*d*csgn(I*(c*x-1))^2-1/
2*I*ln(c*x)*Pi*b*d*csgn(I*(c*x-1))^3-1/2*I*dilog(c*x)*Pi*b*e*csgn(I*(c*x-1)
)^3+1/2*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1))^3+1/4*ln(c*x)*Pi^2*b*e*csgn(I*(c*x
-1)*(c*x+1))^3+1/2*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1))^2*csgn(I*(c*x-1)*(c*x+1
))^2-1/2*ln(c*x)*Pi^2*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/2*ln(c*x)*Pi^2*b*e*cs
gn(I*(c*x-1))^2
```

maxima [A] time = 0.47, size = 152, normalized size = 0.70

$$-\frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,\mathrm{Li}_{2}\left(-cx+1\right)\log\left(-cx+1\right)-2\,\mathrm{Li}_{3}(-cx+1)\right)be+\frac{1}{2}\left(\log\left(cx+1\right)^{2}\log\left(-cx\right)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylo
g(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*dilog(c*x + 1)*log(
c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*(b*d - 2*a*e)*(log
(c*x)*log(-c*x + 1) + dilog(-c*x + 1)) + 1/2*(b*d + 2*a*e)*(log(c*x + 1)*lo
g(-c*x) + dilog(c*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(cx))\left(d+e\log\left(-c^2x^2+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

3.528 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=105

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}+\frac{1}{2}bc\log\left(1-\frac{1}{1-c^2x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)$$

```
[Out] -c*e*(a+b*arctanh(c*x))^2/b-(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x+1/2*b
*c*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(-c^2*x^
2+1))
```

Rubi [A] time = 0.26, antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6081, 2475, 2411, 2344, 2301, 2316, 2315, 5948}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{bc\left(e\log\left(1-c^2x^2\right)+\right.}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -((c*e*(a + b*ArcTanh[c*x])^2)/b) + b*c*d*Log[x] - ((a + b*ArcTanh[c*x])*(d
 + e*Log[1 - c^2*x^2]))/x - (b*c*(d + e*Log[1 - c^2*x^2])^2)/(4*e) - (b*c*e
*PolyLog[2, c^2*x^2])/2
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}+(bc)\int\frac{d+e\log}{x\left(1\right.}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-\right.\right.}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-\right.\right.}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-\right.\right.}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}+bcd\log(x)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(\right.}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}+bcd\log(x)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(\right.}{x}$$

Mathematica [B] time = 0.21, size = 332, normalized size = 3.16

$$-\frac{4ae\log\left(1-c^2x^2\right)+8acex\tanh^{-1}(cx)+4ad+2bcdx\log\left(1-c^2x^2\right)-4bcex\log(x)\log\left(1-c^2x^2\right)+2bcex\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -1/4*(4*a*d + 4*b*d*ArcTanh[c*x] + 8*a*c*e*x*ArcTanh[c*x] + 4*b*c*e*x*ArcTa
nh[c*x]^2 - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c^(
```

```
-1) + x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[x]*
Log[1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*Log[
x]*Log[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2] + 4*b
*e*ArcTanh[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2] + 2*b*
c*e*x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*Log[1 -
 c^2*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x] - 2*b*
c*e*x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2])/x
```

<u>**fricas**</u> **[F]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd\,\text{artanh}\,(cx)+ad+(be\,\text{artanh}\,(cx)+ae)\log\left(-c^2x^2+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^2, x)
```

<u>**giac**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

<u>**maple**</u> **[F]** time = 8.29, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\text{arctanh}\,(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

<u>**maxima**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\,(cx)}{x}\right)bd-\left(c^2\left(\frac{\log\,(cx+1)}{c}-\frac{\log\,(cx-1)}{c}\right)+\frac{\log\left(-c^2x^2+1\right)}{x}\right)ae+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - (c^2*(log(c
*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e + 1/2*b*e*(log(-c*x
+ 1)^2/x - integrate(-((c*x - 1)*log(c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(c*x
^3 - x^2), x)) - a*d/x
```

<u>**mupad**</u> **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\text{atanh}\,(c\,x))\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^2,x)
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\operatorname{atanh}(cx))\left(d+e\log\left(-c^2x^2+1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

3.529 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^3}\,dx$

Optimal. Leaf size=157

$$-\frac{(a+b\tanh^{-1}(cx))(e\log(1-c^2x^2)+d)}{2x^2}+\frac{1}{2}c^2e(a+b)\log(1-cx)+\frac{1}{2}c^2e(a-b)\log(cx+1)-ac^2e\log(x)-\frac{bc(e\log(1\ldots}{2\ldots}$$

```
[Out] -a*c^2*e*ln(x)+1/2*(a+b)*c^2*e*ln(-c*x+1)+1/2*(a-b)*c^2*e*ln(c*x+1)-1/2*b*c
*(d+e*ln(-c^2*x^2+1))/x+1/2*b*c^2*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/2*(a+
b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2+1/2*b*c^2*e*polylog(2,-c*x)-1/2*b*
c^2*e*polylog(2,c*x)
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5916, 325, 206, 6085, 801, 5912}

$$\frac{1}{2}bc^2e\text{PolyLog}(2,-cx)-\frac{1}{2}bc^2e\text{PolyLog}(2,cx)-\frac{(a+b\tanh^{-1}(cx))(e\log(1-c^2x^2)+d)}{2x^2}+\frac{1}{2}c^2e(a+b)\log(1-cx)+\frac{1}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] -(a*c^2*e*Log[x]) + ((a + b)*c^2*e*Log[1 - c*x])/2 + ((a - b)*c^2*e*Log[1 +
 c*x])/2 - (b*c*(d + e*Log[1 - c^2*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d +
e*Log[1 - c^2*x^2]))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(2
*x^2) + (b*c^2*e*PolyLog[2, -(c*x)])/2 - (b*c^2*e*PolyLog[2, c*x])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(1-\right.\right.$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(1-\right.\right.$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(1-\right.\right.$$

$$= -ac^2e\log(x)+\frac{1}{2}(a+b)c^2e\log(1-cx)+\frac{1}{2}(a-b)c^2e\log(1+$$

Mathematica [A] time = 0.17, size = 152, normalized size = 0.97

$$\frac{1}{2}\left(-\frac{e\log\left(1-c^2x^2\right)\left(a+\left(b-bc^2x^2\right)\tanh^{-1}(cx)+bcx\right)}{x^2}+c^2e(a+b)\log(1-cx)+c^2e(a-b)\log(cx+1)-2ac\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] (-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^2
*e*Log[1 + c*x] - (b*d*(2*ArcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Lo
g[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcTanh[c*x])*Log[1
 - c^2*x^2])/x^2 + b*c^2*e*(PolyLog[2, -(c*x)] - PolyLog[2, c*x]))/2
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd\operatorname{artanh}(cx)+ad+(be\operatorname{artanh}(cx)+ae)\log\left(-c^2x^2+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

maple [F] time = 26.70, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arctanh}(cx)) \left(d + e \ln\left(-c^2x^2 + 1\right)\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(\left(c \log(cx + 1) - c \log(cx - 1) - \frac{2}{x}\right)c - \frac{2 \operatorname{artanh}(cx)}{x^2}\right)bd + \frac{1}{2}\left(c^2\left(\log\left(c^2x^2 - 1\right) - \log\left(x^2\right)\right) - \frac{\log\left(-c^2x^2 + 1\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d +
1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e + 1/4*b
*e*(log(-c*x + 1)^2/x^2 - 2*integrate(-((c*x - 1)*log(c*x + 1)^2 - c*x*log(
-c*x + 1))/(c*x^4 - x^3), x)) - 1/2*a*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\,x))\,\left(d + e\,\ln\left(1 - c^2\,x^2\right)\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx)) \left(d + e \log\left(-c^2x^2 + 1\right)\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

3.530 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx$

Optimal. Leaf size=197

$$-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-bc^3e\log(x)-\frac{bc(}{}$$

```
[Out] 2/3*c^2*e*(a+b*arctanh(c*x))/x-1/3*c^3*e*(a+b*arctanh(c*x))^2/b-b*c^3*e*ln(
x)+1/3*b*c^3*e*ln(-c^2*x^2+1)-1/6*b*c*(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/x^2
-1/3*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3+1/6*b*c^3*(d+e*ln(-c^2*x^2
+1))*ln(1-1/(-c^2*x^2+1))-1/6*b*c^3*e*polylog(2,1/(-c^2*x^2+1))
```

Rubi [A] time = 0.46, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {6081, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 5982, 5916, 266, 36, 29, 5948}

$$-\frac{1}{6}bc^3e\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{2c^2e\left(a+b\,\text{tanl}\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] (2*c^2*e*(a + b*ArcTanh[c*x]))/(3*x) - (c^3*e*(a + b*ArcTanh[c*x])^2)/(3*b)
 + (b*c^3*d*Log[x])/3 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*
c*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcTanh[c*x])*(
d + e*Log[1 - c^2*x^2]))/(3*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2])^2)/(12*e
) - (b*c^3*e*PolyLog[2, c^2*x^2])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{3}(bc)\int\frac{d+e\,\mathrm{l}}{x^3}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{6}(bc)\,\mathrm{Subst}\Big(\int$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{bc\left(1-\right.}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3d\,\mathrm{l}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3d\,\mathrm{l}$$

Mathematica [B] time = 0.37, size = 460, normalized size = 2.34

$$\frac{1}{6}\left(-4ac^3e\tanh^{-1}(cx)-\frac{2ae\log\left(1-c^2x^2\right)}{x^3}+\frac{4ac^2e}{x}-\frac{2ad}{x^3}+2bc^3d\log(x)-2bc^3e\mathrm{Li}_2(-cx)-2bc^3e\mathrm{Li}_2(cx)+bc\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] ((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - 4*a*c^3*e*ArcTanh[c*x] - (2*b
*d*ArcTanh[c*x])/x^3 + (4*b*c^2*e*ArcTanh[c*x])/x - 2*b*c^3*e*ArcTanh[c*x]^
```

```
2 + 2*b*c^3*d*Log[x] - 2*b*c^3*e*Log[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 +
(b*c^3*e*Log[c^(-1) + x]^2)/2 + b*c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] -
2*b*c^3*e*Log[x]*Log[1 - c*x] + b*c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] -
 2*b*c^3*e*Log[x]*Log[1 + c*x] - 4*b*c^3*e*Log[(c*x)/Sqrt[1 - c^2*x^2]] - b
*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 - c^2*x^2
])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcTanh[c*x]*Log[1 - c^2*x^2
])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) + x]*Log[1
 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*e*PolyLog[
2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2 - (c*x)/2]
+ b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd\,\text{artanh}(cx) + ad + (be\,\text{artanh}(cx) + ae)\log\left(-c^2x^2 + 1\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}(cx) + a)\left(e\log\left(-c^2x^2 + 1\right) + d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{arctanh}(cx))\left(d + e\ln\left(-c^2x^2 + 1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2 - 1\right) - c^2\log\left(x^2\right) + \frac{1}{x^2}\right)c + \frac{2\,\text{artanh}(cx)}{x^3}\right)bd - \frac{1}{3}\left(\left(c\log(cx + 1) - c\log(cx - 1) - \frac{2}{x}\right)c^2 + \frac{\log(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 + 1)
/x^3)*a*e + 1/6*b*e*(log(-c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x - 1)*lo
g(c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(c*x^5 - x^4), x)) - 1/3*a*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}\,(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^4,x)
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(cx))\,\left(d+e\log\left(-c^2x^2+1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

3.531 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx$

Optimal. Leaf size=244

$$\frac{1}{12}c^4e(3a+4b)\log(1-cx)+\frac{1}{12}c^4e(3a-4b)\log(cx+1)-\frac{(a+b\tanh^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}-\frac{1}{2}ac^4e\log(x)+\frac{ac^2e}{4x^2}$$

```
[Out] 1/4*a*c^2*e/x^2+5/12*b*c^3*e/x-1/4*b*c^4*e*arctanh(c*x)+1/4*b*c^2*e*arctanh
(c*x)/x^2-1/2*a*c^4*e*ln(x)+1/12*(3*a+4*b)*c^4*e*ln(-c*x+1)+1/12*(3*a-4*b)*
c^4*e*ln(c*x+1)-1/12*b*c*(d+e*ln(-c^2*x^2+1))/x^3-1/4*b*c^3*(d+e*ln(-c^2*x^
2+1))/x+1/4*b*c^4*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/4*(a+b*arctanh(c*x))*
(d+e*ln(-c^2*x^2+1))/x^4+1/4*b*c^4*e*polylog(2,-c*x)-1/4*b*c^4*e*polylog(2,
c*x)
```

Rubi [A] time = 0.26, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5916, 325, 206, 6085, 1802, 6044, 5912}

$$\frac{1}{4}bc^4e\text{PolyLog}(2,-cx)-\frac{1}{4}bc^4e\text{PolyLog}(2,cx)-\frac{(a+b\tanh^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{12}c^4e(3a+4b)\log(1-cx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] (a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) - (b*c^4*e*ArcTanh[c*x])/4 + (b*c^2*
e*ArcTanh[c*x])/(4*x^2) - (a*c^4*e*Log[x])/2 + ((3*a + 4*b)*c^4*e*Log[1 - c
*x])/12 + ((3*a - 4*b)*c^4*e*Log[1 + c*x])/12 - (b*c*(d + e*Log[1 - c^2*x^2
]))/(12*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2]))/(4*x) + (b*c^4*ArcTanh[c*x]
*(d + e*Log[1 - c^2*x^2]))/4 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/(4*x^4) + (b*c^4*e*PolyLog[2, -(c*x)])/4 - (b*c^4*e*PolyLog[2, c*x])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6044

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3}-\frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x}+\frac{1}{4}bc$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3}-\frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x}+\frac{1}{4}bc$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3}-\frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x}+\frac{1}{4}bc$$

$$= \frac{ac^2e}{4x^2}+\frac{bc^3e}{6x}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}(3a+4b)c^4e\log(1-cx)+$$

$$= \frac{ac^2e}{4x^2}+\frac{bc^3e}{6x}+\frac{bc^2e\tanh^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}(3a+4b$$

$$= \frac{ac^2e}{4x^2}+\frac{5bc^3e}{12x}+\frac{bc^2e\tanh^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}(3a+4$$

$$= \frac{ac^2e}{4x^2}+\frac{5bc^3e}{12x}-\frac{1}{4}bc^4e\tanh^{-1}(cx)+\frac{bc^2e\tanh^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e$$

Mathematica [A] time = 0.16, size = 299, normalized size = 1.23

$$\frac{1}{12}\log(1-cx)\left(3ac^4e+4bc^4e\right)+\frac{1}{12}\log(cx+1)\left(3ac^4e-4bc^4e\right)+\frac{e\log\left(1-c^2x^2\right)\left(-3a+3bc^4x^4\tanh^{-1}(cx)-3bc\right.}{12x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] -1/4*(a*d)/x^4 + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2 +
 ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-1/2*ArcTanh[c*x]/(c
^2*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d*(-
1/4*ArcTanh[c*x]/(c^4*x^4) + (-1/3*1/(c^3*x^3) - 1/(c*x) - Log[1 - c*x]/2 +
 Log[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(-3*a
```

```
- b*c*x - 3*b*c^3*x^3 - 3*b*ArcTanh[c*x] + 3*b*c^4*x^4*ArcTanh[c*x])*Log[1
- c^2*x^2])/(12*x^4) - (b*c^4*e*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/4
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd\,\text{artanh}\,(cx)+ad+(be\,\text{artanh}\,(cx)+ae)\log\left(-c^2x^2+1\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arctanh}\,(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}\left(\left(3\,c^3\log(cx+1)-3\,c^3\log(cx-1)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\text{artanh}\,(cx)}{x^4}\right)bd+\frac{1}{4}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxima
")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^
2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e + 1/8*b*e*(log(-c*x + 1)^2/x^4 - 4*inte
grate(-1/2*(2*(c*x - 1)*log(c*x + 1)^2 - c*x*log(-c*x + 1))/(c*x^6 - x^5),
x)) - 1/4*a*d/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\text{atanh}\,(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^5,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx)) \left(d + e \log\left(-c^2x^2 + 1\right)\right)}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

3.532 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

Optimal. Leaf size=256

$$-\frac{c^5e\left(a+b\tanh^{-1}(cx)\right)^2}{5b}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}+\frac{2c^2e\left(a+b\tanh^{-1}\right.}{15x^3}$$

```
[Out] 7/60*b*c^3*e/x^2+2/15*c^2*e*(a+b*arctanh(c*x))/x^3+2/5*c^4*e*(a+b*arctanh(c
*x))/x-1/5*c^5*e*(a+b*arctanh(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*ln(
-c^2*x^2+1)-1/20*b*c*(d+e*ln(-c^2*x^2+1))/x^4-1/10*b*c^3*(-c^2*x^2+1)*(d+e*
ln(-c^2*x^2+1))/x^2-1/5*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5+1/10*b*
c^5*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1/(-c^
2*x^2+1))
```

Rubi [A] time = 0.66, antiderivative size = 250, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 18, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6081, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 5982, 5916, 266, 36, 29, 5948}

$$-\frac{1}{10}bc^5e\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}-\frac{c^5e\left(a+b\tanh^{-1}\right.}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] (7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*ArcTanh[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcTanh[c*x]))/(5*x) - (c^5*e*(a + b*ArcTanh[c*x])^2)/(5*b) + (b*c^5*
d*Log[x])/5 - (5*b*c^5*e*Log[x])/6 + (19*b*c^5*e*Log[1 - c^2*x^2])/60 - (b*
c*(d + e*Log[1 - c^2*x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*Log[1 -
c^2*x^2]))/(10*x^2) - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(5*x^
5) - (b*c^5*(d + e*Log[1 - c^2*x^2])^2)/(20*e) - (b*c^5*e*PolyLog[2, c^2*x^
2])/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{5}(bc)\int\frac{d+e\,\mathrm{l}}{x^5}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{10}(bc)\,\mathrm{Subst}\Big($$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-\right.\right.}{5x^5}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+\right.}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+\right.}{}$$

$$= \frac{bc^3e}{15x^2}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bd\,\mathrm{artanh}\,(cx)+ad+(be\,\mathrm{artanh}\,(cx)+ae)\log\left(-c^2x^2+1\right)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arctanh}(cx)) \left(d + e \ln \left(-c^2 x^2 + 1\right)\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{20}\left(\left(2\,c^4 \log\left(c^2x^2-1\right) - 2\,c^4 \log\left(x^2\right) + \frac{2\,c^2x^2+1}{x^4}\right)c + \frac{4\ \operatorname{artanh}(cx)}{x^5}\right)bd - \frac{1}{15}\left(\left(3\,c^3 \log(cx+1) - 3\,c^3 \log(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxima
")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arctanh(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) -
2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e + 1/10*b*e*(log(-
c*x + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x - 1)*log(c*x + 1)^2 - 2*c*x*log(-
c*x + 1))/(c*x^7 - x^6), x)) - 1/5*a*d/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c\,x)) \left(d + e\,\ln\left(1 - c^2\,x^2\right)\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^6,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx)) \left(d + e \log\left(-c^2x^2 + 1\right)\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**6, x)
```

3.533 $\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$

Optimal. Leaf size=512

$$\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\log\left(f+gx^2\right)\left(a+b\tanh^{-1}(cx)\right)}{2g}-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b(d-e)\tanh}{2c^2}$$

```
[Out] 1/2*b*(d-e)*x/c-b*e*x/c-1/2*b*(d-e)*arctanh(c*x)/c^2+1/2*d*x^2*(a+b*arctanh
(c*x))-1/2*e*x^2*(a+b*arctanh(c*x))-b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+1)
)/c^2/g+1/2*b*e*x*ln(g*x^2+f)/c-1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(g*x^2+f)/
c^2/g+1/2*e*(g*x^2+f)*(a+b*arctanh(c*x))*ln(g*x^2+f)/g+1/2*b*e*(c^2*f+g)*ar
ctanh(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/c^
2/g+1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c
*(-f)^(1/2)+g^(1/2)))/c^2/g+1/2*b*e*(c^2*f+g)*polylog(2,1-2/(c*x+1))/c^2/g-
1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1
/2)-g^(1/2)))/c^2/g-1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)+x*g^(1/2)
)/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/c^2/g+b*e*arctan(x*g^(1/2)/f^(1/2))*f^(1/
2)/c/g^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {2454, 2389, 2295, 6083, 321, 207, 517, 2528, 2448, 205, 2470, 12, 5992, 5920, 2402, 2315, 2447}

$$\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,\right.}{4c^2g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])
/(c*Sqrt[g]) - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (d*x^2*(a + b*ArcTanh[c*x
]))/2 - (e*x^2*(a + b*ArcTanh[c*x]))/2 - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[
2/(1 + c*x)])/(c^2*g) + (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] -
Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(2*c^2*g) + (b*e*(c^2*f +
g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1
 + c*x))])/(2*c^2*g) + (b*e*x*Log[f + g*x^2])/(2*c) - (b*e*(c^2*f + g)*ArcT
anh[c*x]*Log[f + g*x^2])/(2*c^2*g) + (e*(f + g*x^2)*(a + b*ArcTanh[c*x])*Lo
g[f + g*x^2])/(2*g) + (b*e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*
g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[
-f] - Sqrt[g])*(1 + c*x))])/(4*c^2*g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*
c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*c^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e\left(f+g\right.}{}$$

$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e\left(f+g\right.}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(c\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+$$

Mathematica [C] time = 5.81, size = 1376, normalized size = 2.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (-4*b*c*e*g*x + 2*a*c^2*(d - e)*g*x^2 + 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqr
t[g]*x)/Sqrt[f]] + 2*a*c^2*e*f*Log[f + g*x^2] + 2*e*g*(c*x*(b + a*c*x) + b*
(-1 + c^2*x^2)*ArcTanh[c*x])*Log[f + g*x^2] + b*d*(2*(c*g*x + c^2*f*ArcTanh
[c*x]^2 + ArcTanh[c*x]*(-g + c^2*g*x^2 + 2*c^2*f*Log[1 + E^(-2*ArcTanh[c*x]
)]) - c^2*f*PolyLog[2, -E^(-2*ArcTanh[c*x])]]) - c^2*f*(2*ArcTanh[c*x]*(-Arc
Tanh[c*x] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*
Sqrt[g] - g)] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[
-f]*Sqrt[g] - g)]) + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f -
```

```
 2*c*Sqrt[-f]*Sqrt[g] - g))] + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g)
)/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))])) + b*e*(-2*(c*g*x + c^2*f*ArcTanh[c
*x]^2 + ArcTanh[c*x]*(-g + c^2*g*x^2 + 2*c^2*f*Log[1 + E^(-2*ArcTanh[c*x])]
) - c^2*f*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + c^2*f*(2*ArcTanh[c*x]*(-ArcTa
nh[c*x] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sq
rt[g] - g)] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f
]*Sqrt[g] - g)]) + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2
*c*Sqrt[-f]*Sqrt[g] - g))] + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/
(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))])) + b*c^2*d*f*(2*ArcTanh[c*x]^2 - (4*I
)*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - 2*A
rcTanh[c*x]*(ArcTanh[c*x] + 2*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*((-I)*ArcSi
n[Sqrt[(c^2*f)/(c^2*f + g)]] + ArcTanh[c*x])*Log[(c^2*(1 + E^(2*ArcTanh[c*x
]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x
])*(c^2*f + g))] + 2*(I*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]] + ArcTanh[c*x])*L
og[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-
(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*PolyLog[2, -E^(-2*ArcTanh
[c*x])] - PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x]
)*(c^2*f + g))] - PolyLog[2, (-(c^2*f) + g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcT
anh[c*x])*(c^2*f + g))]) + b*e*g*(2*ArcTanh[c*x]^2 - (4*I)*ArcSin[Sqrt[(c^2
*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - 2*ArcTanh[c*x]*(ArcTa
nh[c*x] + 2*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*((-I)*ArcSin[Sqrt[(c^2*f)/(c^
2*f + g)]] + ArcTanh[c*x])*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2
*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] +
 2*(I*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]] + ArcTanh[c*x])*Log[(c^2*(1 + E^(2*
ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*
ArcTanh[c*x])*(c^2*f + g))] + 2*PolyLog[2, -E^(-2*ArcTanh[c*x])] - PolyLog[
2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] -
PolyLog[2, (-(c^2*f) + g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f +
 g))]))/(4*c^2*g)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(bdx\,\text{artanh}\,(cx) + adx + (bex\,\text{artanh}\,(cx) + aex)\log\left(gx^2 + f\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*x*arctanh(c*x) + a*d*x + (b*e*x*arctanh(c*x) + a*e*x)*log(g*x^
2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

maple [C] time = 2.91, size = 10161, normalized size = 19.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log{(cx+1)}}{c^3}+\frac{\log{(cx-1)}}{c^3}\right)\right)bd-\frac{1}{4}\left(2\,c^2g\int\frac{x^3\log{(cx+1)}}{c^2gx^2+c^2f}\,dx-2\,c^2g\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/4*(2*c^2*g*integrate(x^3*log(c*x + 1)/(c^2*g*x^2 +
c^2*f), x) - 2*c^2*g*integrate(x^3*log(-c*x + 1)/(c^2*g*x^2 + c^2*f), x) -
2*c*g*(-I*f*(log(I*g*x/sqrt(f*g) + 1) - log(-I*g*x/sqrt(f*g) + 1))/(sqrt(f*
g)*c^2*g) - 2*x/(c^2*g)) - 2*g*integrate(x*log(c*x + 1)/(c^2*g*x^2 + c^2*f)
, x) + 2*g*integrate(x*log(-c*x + 1)/(c^2*g*x^2 + c^2*f), x) - (2*c*x + (c^
2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log(-c*x + 1))*log(g*x^2 + f)/c^2)*
b*e - 1/2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int(x*(a + b*atanh(c*x))*(d + e*log(f + g*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

3.534 $\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=599

$$x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2aex+\frac{b\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c}$$

```
[Out] -2*a*e*x-2*b*e*x*arctanh(c*x)-b*e*ln(-c^2*x^2+1)/c+x*(a+b*arctanh(c*x))*(d+
e*ln(g*x^2+f))+1/2*b*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))/c+1/2*b
*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g))/c+1/2*b*e*ln(-c*x+1)*ln(c*((-f)^(1/2)
-x*g^(1/2))/(c*(-f)^(1/2)-g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*b*e*ln(c*x+1)*ln
(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*b*
e*ln(c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)-g^(1/2)))*(-f)^(1/2)/
g^(1/2)-1/2*b*e*ln(-c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)+g^(1/2
)))*(-f)^(1/2)/g^(1/2)+1/2*b*e*polylog(2,-(-c*x+1)*g^(1/2)/(c*(-f)^(1/2)-g^
(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*b*e*polylog(2,-(c*x+1)*g^(1/2)/(c*(-f)^(1/2)
-g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*b*e*polylog(2,(-c*x+1)*g^(1/2)/(c*(-f)^(1
/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*b*e*polylog(2,(c*x+1)*g^(1/2)/(c*(-f)^
(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*f^(1/2)/
g^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 599, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6073, 2475, 2394, 2393, 2391, 5980, 5910, 260, 5974, 205, 5972, 2409}

$$\frac{be\,\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)}{2c}+\frac{be\sqrt{-f}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}\,(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\,\text{PolyLog}\left(2,\frac{\sqrt{g}\,(1-cx)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\sqrt{-f}\,\text{PolyLog}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - 2*b*e*x*Ar
cTanh[c*x] + (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*S
qrt[-f] - Sqrt[g])])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*Log[1 + c*x]*Log[(c*(Sqrt[
-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*Log[
1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[g]
) - (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] +
 Sqrt[g])])/(2*Sqrt[g]) - (b*e*Log[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])
*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[
f + g*x^2]))/(2*c) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt
[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 - c*x
))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[
g]*(1 + c*x))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog
[2, (Sqrt[g]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*PolyLog
[2, (c^2*(f + g*x^2))/(c^2*f + g)])/(2*c)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5974

```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTanh[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

```
]
```

Rule 6073

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[
2*e*g, Int[(x^2*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-(bc)\int \frac{x\left(d+e\,l\ldots\right.}{1\ldots}$$

$$= x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int\ldots\right.$$

$$= -2aex+x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{b\log\left(\ldots\right.}{\ldots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+x\left(a+b\,\mathrm{ta}\ldots\right.$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)-\frac{be\log\left(1-\ldots\right.}{c}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)-\frac{be\log\left(1-\ldots\right.}{c}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\sqrt{-f}\,\mathrm{lo}\ldots}{\ldots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\sqrt{-f}\,\mathrm{lo}\ldots}{\ldots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\sqrt{-f}\,\mathrm{lo}\ldots}{\ldots}$$

Mathematica [C] time = 3.36, size = 1251, normalized size = 2.09

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + b*d
*x*ArcTanh[c*x] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*e
```

```
*(x*ArcTanh[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] - (b*e*g*(((-Log[
-c^(-1) + x] - Log[c^(-1) + x] + Log[1 - c^2*x^2])*Log[f + g*x^2])/(2*g) +
(Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c
)] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)])/(2*g)
+ (Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)
] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)])/(2*g) + (L
og[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)] +
 PolyLog[2, (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)])/(2*g) + (Lo
g[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)] + Pol
yLog[2, (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)])/(2*g)))/c - (b*e*(
4*c*x*ArcTanh[c*x] - 4*Log[1/Sqrt[1 - c^2*x^2]] + (Sqrt[c^2*f*g]*((-2*I)*Ar
cCos[(-(c^2*f) + g)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] + 4*ArcTan[S
qrt[c^2*f*g]/(c*g*x)]*ArcTanh[c*x] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] -
2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2*f*(g + I*Sqrt[c^2*f*g])*(1 + c*
x))/((c^2*f + g)*(c^2*f + I*c*Sqrt[c^2*f*g]*x))] - (ArcCos[(-(c^2*f) + g)/(
c^2*f + g)] + 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2*f*(I*g + Sqrt[c^2
*f*g])*(-1 + c*x))/((c^2*f + g)*((-I)*c^2*f + c*Sqrt[c^2*f*g]*x))] + (ArcCo
s[(-(c^2*f) + g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(
c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcTanh[c*x]*Sqrt[c^
2*f + g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + (ArcCos[(-(
c^2*f) + g)/(c^2*f + g)] - 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(c*g*x
)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcTanh[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f +
 g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + I*(-PolyLog[2, (
(-(c^2*f) + g - (2*I)*Sqrt[c^2*f*g])*(I*c^2*f + c*Sqrt[c^2*f*g]*x))/((c^2*f
 + g)*((-I)*c^2*f + c*Sqrt[c^2*f*g]*x))] + PolyLog[2, ((-(c^2*f) + g + (2*I
)*Sqrt[c^2*f*g])*(I*c^2*f + c*Sqrt[c^2*f*g]*x))/((c^2*f + g)*((-I)*c^2*f +
c*Sqrt[c^2*f*g]*x))])))/g))/(2*c)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(bd\,\text{artanh}\,(cx) + ad + (be\,\text{artanh}\,(cx) + ae)\log\left(gx^2 + f\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

maple [F] time = 2.63, size = 0, normalized size = 0.00

$$\int (a + b\,\text{arctanh}\,(cx))\left(d + e\ln\left(g\,x^2 + f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2g\left(\frac{f\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{\sqrt{fg}g}-\frac{x}{g}\right)+x\log\left(gx^2+f\right)\right)ae+adx+\frac{1}{2}be\left(\frac{\left((cx+1)\log(cx+1)-(cx-1)\log(-cx+1)\right)\log}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] (2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 + f))*a*e
+ a*d*x + 1/2*b*e*(((c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*log(g
*x^2 + f)/c + integrate(-2*((c*g*x^2 + g*x)*log(c*x + 1) - (c*g*x^2 - g*x)*
log(-c*x + 1))/(c*g*x^2 + c*f), x)) + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{atanh}\,(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int((a + b*atanh(c*x))*(d + e*log(f + g*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

3.535 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal. Leaf size=93

$$be\mathrm{Int}\left(\frac{\tanh^{-1}(cx)\log\left(f+gx^2\right)}{x},x\right)+ad\log(x)+\frac{1}{2}ae\mathrm{Li}_2\left(\frac{gx^2}{f}+1\right)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\mathrm{Li}_2(-cx)+$$

```
[Out] b*e*CannotIntegrate(arctanh(c*x)*ln(g*x^2+f)/x,x)+a*d*ln(x)+1/2*a*e*ln(-g*x
^2/f)*ln(g*x^2+f)-1/2*b*d*polylog(2,-c*x)+1/2*b*d*polylog(2,c*x)+1/2*a*e*po
lylog(2,1+g*x^2/f)
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 - (b*d*PolyLog[2, -(c
*x)])/2 + (b*d*PolyLog[2, c*x])/2 + (a*e*PolyLog[2, 1 + (g*x^2)/f])/2 + b*e
*Defer[Int][(ArcTanh[c*x]*Log[f + g*x^2])/x, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(f+gx^2\right.}{x}$$

$$= ad\log(x)-\frac{1}{2}bd\mathrm{Li}_2(-cx)+\frac{1}{2}bd\mathrm{Li}_2(cx)+(ae)\int \frac{\log\left(f+gx^2\right)}{x}$$

$$= ad\log(x)-\frac{1}{2}bd\mathrm{Li}_2(-cx)+\frac{1}{2}bd\mathrm{Li}_2(cx)+\frac{1}{2}(ae)\,\mathrm{Subst}\left(\int \frac{\log(f}{}\right.$$

$$= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\mathrm{Li}_2(-cx)+\frac{1}{2}b$$

$$= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\mathrm{Li}_2(-cx)+\frac{1}{2}b$$

Mathematica [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

fricas [A] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bd\,\mathrm{artanh}\,(cx)+ad+(be\,\mathrm{artanh}\,(cx)+ae)\log\left(gx^2+f\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f))
/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)\left(e \log\left(gx^2 + f\right) + d\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

maple [A] time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arctanh}(cx))\left(d + e \ln\left(g x^2 + f\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$ad \log(x) + \int \frac{be\left(\log(cx+1) - \log(-cx+1)\right)\log\left(gx^2 + f\right)}{2x} + \frac{bd\left(\log(cx+1) - \log(-cx+1)\right)}{2x} + \frac{ae \log\left(gx^2 + \right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + integrate(1/2*b*e*(log(c*x + 1) - log(-c*x + 1))*log(g*x^2 + f
)/x + 1/2*b*d*(log(c*x + 1) - log(-c*x + 1))/x + a*e*log(g*x^2 + f)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\,x))\,\left(d + e\,\ln\left(g\,x^2 + f\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Timed out
```

3.536 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

Optimal. Leaf size=613

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{1}{2}bc\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}$$

```
[Out] -(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+
f))-1/2*b*c*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))-1/2*b*c*e*polylo
g(2,c^2*(g*x^2+f)/(c^2*f+g))+1/2*b*c*e*polylog(2,1+g*x^2/f)-1/2*b*e*ln(-c*x
+1)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)-g^(1/2)))*g^(1/2)/(-f)^(1/2)+
1/2*b*e*ln(c*x+1)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+g^(1/2)))*g^(1/
2)/(-f)^(1/2)-1/2*b*e*ln(c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)-g
^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*b*e*ln(-c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/
(c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-f)^(1/2)-1/2*b*e*polylog(2,-(-c*x+1)*g^(1
/2)/(c*(-f)^(1/2)-g^(1/2)))*g^(1/2)/(-f)^(1/2)-1/2*b*e*polylog(2,-(c*x+1)*g
^(1/2)/(c*(-f)^(1/2)-g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*b*e*polylog(2,(-c*x+1
)*g^(1/2)/(c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*b*e*polylog(2,(c*x
+1)*g^(1/2)/(c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-f)^(1/2)+2*a*e*arctan(x*g^(1/
2)/f^(1/2))*g^(1/2)/f^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 613, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6081, 2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 5974, 205, 5972, 2409}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)+\frac{1}{2}bce\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)-\frac{be\sqrt{g}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\,\text{PolyLog}\left(2\right.}{2\sqrt{-f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*Log[1 -
c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) +
 (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqr
t[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]
*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*Log[1 - c*x]*Log[
(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - ((a + b*
ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-((g*x^2)/f)]*(d + e*Log
[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*
x^2]))/2 - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] - Sqrt
[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]*(1 - c*x))/(c*Sqrt[
-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 + c*x
))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g
]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*c*e*PolyLog[2, (c^2
*(f + g*x^2))/(c^2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5974

```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTanh[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+(bc)\int \frac{d+e\log\left(f\right.}{x\left(1-c\right.}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int \frac{d}{\ }\right.$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be}{\ }$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be}{\ }$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be}{\ }$$

Mathematica [C] time = 3.27, size = 1226, normalized size = 2.00

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]

[Out] -((a*d)/x) - (b*d*ArcTanh[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2])/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2]/x) + (b*e*(-(((2*ArcTanh[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[f + g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*Sqrt[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g]*x))/(c*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log[1 - c^2*x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x))/(I*c*Sqrt[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt[f] + Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g])] + PolyLog[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]) + (c*g*((2*I)*ArcCos[(-(c^2*f) + g)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - 4*ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)]*ArcTanh[c*x] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] + 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[((2*I)*c*f*(I*g + Sqrt[c^2*f*g])*(-1 + c*x))/((c^2*f + g)*(c*f + I*Sqrt[c^2*f*g]*x))] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] - 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c*f*(g + I*Sqrt[c^2*f*g])*(1 + c*x))/((c^2*f + g)*(c*f + I*Sqrt[c^2*f*g]*x))] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] + 2*(ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcTanh[c*x]*Sqrt[c^2*f + g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] - 2*(ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcTanh[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f + g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + I*(PolyLog[2, ((-(c^2*f) + g - (2*I)*Sqrt[c^2*f*g])*(I*c*f + Sqrt[c^2*f*g]*x))/((c^2*f + g)*((-I)*c*f + Sqrt[c^2*f*g]*x))] - PolyLog[2, ((-(c^2*f) + g + (2*I)*Sqrt[c^2*f*g])*(I*c*f + Sqrt[c^2*f*g]*x))/((c^2*f + g)*((-I)*c*f + Sqrt[c^2*f*g]*x))])))/Sqrt[c^2*f*g]))/2

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd\,\text{artanh}\left(cx\right)+ad+\left(be\,\text{artanh}\left(cx\right)+ae\right)\log\left(gx^2+f\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")

[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f))/x^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\text{artanh}\left(cx\right)+a\right)\left(e\log\left(gx^2+f\right)+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)

maple [F] time = 2.02, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\text{arctanh}\left(cx\right)\right)\left(d+e\ln\left(g\,x^2+f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\operatorname{artanh}(cx)}{x}\right)bd+\left(\frac{2\,g\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{\sqrt{fg}}-\frac{\log\left(gx^2+f\right)}{x}\right)ae+\frac{1}{2}\,be\int\frac{(\log(cx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d + (2*g*arctan
(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + 1/2*b*e*integrate((log(
c*x + 1) - log(-c*x + 1))*log(g*x^2 + f)/x^2, x) - a*d/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\operatorname{atanh}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

3.537 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

Optimal. Leaf size=470

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}-\frac{aeg\log\left(f+gx^2\right)}{2f}+\frac{aeg\log(x)}{f}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+gx\right.\right.$$

```
[Out] a*e*g*ln(x)/f+b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+1))/f-1/2*a*e*g*ln(g*x^2
+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x+1/2*b*c^2*arctanh(c*x)*(d+e*ln(g*x^2+f))-
1/2*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2-1/2*b*e*(c^2*f+g)*arctanh(c*x)
*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/f-1/2*b*e*(c
^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^
(1/2)))/f-1/2*b*e*g*polylog(2,-c*x)/f+1/2*b*e*g*polylog(2,c*x)/f-1/2*b*e*(c
^2*f+g)*polylog(2,1-2/(c*x+1))/f+1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1
/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/f+1/4*b*e*(c^2*f+g)*polylog(
2,1-2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/f+b*c*e*arct
an(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {5916, 325, 206, 6085, 801, 635, 205, 260, 446, 72, 6725, 5912, 5992, 5920, 2402, 2315, 2447}

$$-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2f}+\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4f}+\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*x)])/f - (b*e*(c^2*f + g)*ArcTanh[
c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/
(2*f) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*
Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(2*f) - (a*e*g*Log[f + g*x^2])/(2*f) - (b*
c*(d + e*Log[f + g*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g*x^2]
))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) - (b*e*g*PolyL
og[2, -(c*x)])/(2*f) + (b*e*g*PolyLog[2, c*x])/(2*f) - (b*e*(c^2*f + g)*Pol
yLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqr
t[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(4*f) + (b*e*(c^2*
f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*
(1 + c*x))])/(4*f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+gx^2\right.\right.$$

$$= -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+gx^2\right.\right.$$

$$= -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+gx^2\right.\right.$$

$$= \frac{aeg\log(x)}{f}-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\right.$$

$$= \frac{aeg\log(x)}{f}-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\right.$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}-\frac{aeg\log\left(f+gx^2\right)}{2f}-\frac{bc\left(d+\right.}{}$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log}{f}$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log}{f}$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log}{f}$$

Mathematica [C] time = 4.97, size = 982, normalized size = 2.09

$$-\frac{-4beg\tanh^{-1}(cx)^2x^2-4bce\sqrt{f}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)x^2-2bc^2df\tanh^{-1}(cx)x^2-4ibc^2ef\sin^{-1}\left(\sqrt{\frac{c^2f}{fc^2+g}}\right)\tanh^{-1}\left(-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] -1/4*(2*a*d*f + 2*b*c*d*f*x - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x
)/Sqrt[f]] + 2*b*d*f*ArcTanh[c*x] - 2*b*c^2*d*f*x^2*ArcTanh[c*x] - 4*b*e*g*
x^2*ArcTanh[c*x]^2 - (4*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*
ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - 4*b*e*g*x^2*ArcTanh[c*x]*Log[1 - E^(-2*
ArcTanh[c*x])] - 4*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])]
- (2*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*
ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*
ArcTanh[c*x])*(c^2*f + g))] + 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[(c^2*(1 + E^
(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^
(2*ArcTanh[c*x])*(c^2*f + g))] + (2*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c
^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))
*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*c^2*e*f*x^
2*ArcTanh[c*x]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x
]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*e*g*x^2
*ArcTanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f
```

```
]*Sqrt[g] - g)] + 2*b*e*g*x^2*ArcTanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*(c^2
*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)] - 4*a*e*g*x^2*Log[x] + 2*a*e*f
*Log[f + g*x^2] + 2*b*c*e*f*x*Log[f + g*x^2] + 2*a*e*g*x^2*Log[f + g*x^2] +
 2*b*e*f*ArcTanh[c*x]*Log[f + g*x^2] - 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[f +
 g*x^2] + 2*b*c^2*e*f*x^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*b*e*g*x^2*Po
lyLog[2, E^(-2*ArcTanh[c*x])] + b*e*g*x^2*PolyLog[2, -((E^(2*ArcTanh[c*x])*
(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g))] + b*e*g*x^2*PolyLog[2, -(
(E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))] - b*c
^2*e*f*x^2*PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x
])*(c^2*f + g))] - b*c^2*e*f*x^2*PolyLog[2, (-(c^2*f) + g + 2*Sqrt[-(c^2*f*
g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))])/(f*x^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd\,\text{artanh}\,(cx)+ad+(be\,\text{artanh}\,(cx)+ae)\log\left(gx^2+f\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f))
/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{artanh}\,(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

maple [B] time = 3.59, size = 960, normalized size = 2.04

$$-\frac{db\,c^2\ln(cx)}{4}+\frac{db\,c^2\ln(cx+1)}{4}-\frac{db\ln(cx+1)}{4x^2}+\frac{db\,c^2\ln(-cx)}{4}+\frac{db\ln(-cx+1)}{4x^2}+\left(-\frac{be\ln(cx+1)}{4x^2}+\frac{e\left(b\,c^2\ln(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

```
[Out] -1/4*d*b*c^2*ln(c*x)+1/4*d*b*c^2*ln(c*x+1)-1/4*d*b*ln(c*x+1)/x^2+1/4*d*b*c^
2*ln(-c*x)+1/4*d*b*ln(-c*x+1)/x^2+(-1/4*b*e/x^2*ln(c*x+1)+1/4*e*(b*c^2*ln(c
*x+1)*x^2-b*c^2*ln(-c*x+1)*x^2-2*x*b*c+b*ln(-c*x+1)-2*a)/x^2)*ln(g*x^2+f)+a
*e*g*ln(x)/f-1/2*a*e*g*ln(g*x^2+f)/f-1/2/x^2*d*a-1/4*b*c^2*d*ln(-c*x+1)-1/2
*b*c*d/x+1/4*b*e*ln(-c*x+1)*ln((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2
)-g))*c^2+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+g
))+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))-1/2*
g*b*e*dilog(c*x+1)/f+1/2*g*b*e*dilog(-c*x+1)/f+1/4*b*e*ln(-c*x+1)*ln((c*(-f
*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e*dilog((c*(-f*g)^(1/
2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e*dilog((c*(-f*g)^(1/2)+(c*x+
1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)-(-c*x+1)*g+g)
/(c*(-f*g)^(1/2)+g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f
*g)^(1/2)-g))*c^2+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(
c*(-f*g)^(1/2)-g))-1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f
*g)^(1/2)+g))*c^2-1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*
g)^(1/2)-g))*c^2-1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(
```

```
1/2)+g))-1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))
-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-
1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+g
*e*b*c/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-f
*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)bd-\frac{1}{2}\left(g\left(\frac{\log\left(gx^2+f\right)}{f}-\frac{\log\left(x^2\right)}{f}\right)+\frac{\log\left(gx^2+f\right)}{x^2}\right)ae$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d -
1/2*(g*(log(g*x^2 + f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e - 1/4*(2*c
^2*g*integrate(x^2*log(c*x + 1)/(g*x^3 + f*x), x) - 2*c^2*g*integrate(x^2*l
og(-c*x + 1)/(g*x^3 + f*x), x) + 2*I*c*g*(log(I*g*x/sqrt(f*g) + 1) - log(-I
*g*x/sqrt(f*g) + 1))/sqrt(f*g) - 2*g*integrate(log(c*x + 1)/(g*x^3 + f*x),
x) + 2*g*integrate(log(-c*x + 1)/(g*x^3 + f*x), x) + (2*c*x - (c^2*x^2 - 1)
*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*x + 1))*log(g*x^2 + f)/x^2)*b*e - 1/2*
a*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}\left(c\,x\right))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

3.538 $\int \frac{\tanh^{-1}(cx)\left(a+b\tanh^{-1}(cx)\right)}{(1+cx)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{a+b}{2c(cx+1)}+\frac{(a+b)\tanh^{-1}(cx)}{2c}-\frac{(a+b)\tanh^{-1}(cx)}{c(cx+1)}-\frac{b(1-cx)\tanh^{-1}(cx)^2}{2c(cx+1)}$$

```
[Out] 1/2*(-a-b)/c/(c*x+1)+1/2*(a+b)*arctanh(c*x)/c-(a+b)*arctanh(c*x)/c/(c*x+1)-
1/2*b*(-c*x+1)*arctanh(c*x)^2/c/(c*x+1)
```

Rubi [A] time = 0.29, antiderivative size = 122, normalized size of antiderivative = 1.56, number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5926, 627, 44, 207, 6742, 5928, 5948}

$$-\frac{a}{2c(cx+1)}+\frac{a\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(cx+1)}-\frac{b}{2c(cx+1)}+\frac{b\tanh^{-1}(cx)^2}{2c}-\frac{b\tanh^{-1}(cx)^2}{c(cx+1)}+\frac{b\tanh^{-1}(cx)}{2c}-\frac{b\tanh^{-1}}{c(cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

```
[Out] -a/(2*c*(1 + c*x)) - b/(2*c*(1 + c*x)) + (a*ArcTanh[c*x])/(2*c) + (b*ArcTan
h[c*x])/(2*c) - (a*ArcTanh[c*x])/(c*(1 + c*x)) - (b*ArcTanh[c*x])/(c*(1 + c
*x)) + (b*ArcTanh[c*x]^2)/(2*c) - (b*ArcTanh[c*x]^2)/(c*(1 + c*x))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\tanh^{-1}(cx)\left(a+b\tanh^{-1}(cx)\right)}{(1+cx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)\left(a+b\tanh^{-1}(x)\right)}{(1+x)^2}\,dx, x, cx\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a\tanh^{-1}(x)}{(1+x)^2}+\frac{b\tanh^{-1}(x)^2}{(1+x)^2}\right)dx, x, cx\right)}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{(1+x)^2}\,dx, x, cx\right)}{c} + \frac{b\,\text{Subst}\left(\int \frac{\tanh^{-1}(x)^2}{(1+x)^2}\,dx, x, cx\right)}{c}$$

$$= -\frac{a\tanh^{-1}(cx)}{c(1+cx)} - \frac{b\tanh^{-1}(cx)^2}{c(1+cx)} + \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x)(1-x^2)}\,dx, x, cx\right)}{c} + \frac{(2b)\,\text{S}}{}$$

$$= -\frac{a\tanh^{-1}(cx)}{c(1+cx)} - \frac{b\tanh^{-1}(cx)^2}{c(1+cx)} + \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x)(1+x)^2}\,dx, x, cx\right)}{c} + \frac{b\,\text{Sub}}{}$$

$$= -\frac{a\tanh^{-1}(cx)}{c(1+cx)} - \frac{b\tanh^{-1}(cx)}{c(1+cx)} + \frac{b\tanh^{-1}(cx)^2}{2c} - \frac{b\tanh^{-1}(cx)^2}{c(1+cx)} + \frac{a\,\text{Subs}}{}$$

$$= -\frac{a}{2c(1+cx)} - \frac{a\tanh^{-1}(cx)}{c(1+cx)} - \frac{b\tanh^{-1}(cx)}{c(1+cx)} + \frac{b\tanh^{-1}(cx)^2}{2c} - \frac{b\tanh^{-1}(c}{c(1+cx}$$

$$= -\frac{a}{2c(1+cx)} + \frac{a\tanh^{-1}(cx)}{2c} - \frac{a\tanh^{-1}(cx)}{c(1+cx)} - \frac{b\tanh^{-1}(cx)}{c(1+cx)} + \frac{b\tanh^{-1}(c}{2c}$$

$$= -\frac{a}{2c(1+cx)} - \frac{b}{2c(1+cx)} + \frac{a\tanh^{-1}(cx)}{2c} - \frac{a\tanh^{-1}(cx)}{c(1+cx)} - \frac{b\tanh^{-1}(cx)}{c(1+cx)}$$

$$= -\frac{a}{2c(1+cx)} - \frac{b}{2c(1+cx)} + \frac{a\tanh^{-1}(cx)}{2c} + \frac{b\tanh^{-1}(cx)}{2c} - \frac{a\tanh^{-1}(cx)}{c(1+cx)}$$

Mathematica [A] time = 0.09, size = 70, normalized size = 0.90

$$-\frac{(a+b)((cx+1)\log(1-cx)-(cx+1)\log(cx+1)+2)+4(a+b)\tanh^{-1}(cx)-2b(cx-1)\tanh^{-1}(cx)^2}{4c(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

```
[Out] -1/4*(4*(a + b)*ArcTanh[c*x] - 2*b*(-1 + c*x)*ArcTanh[c*x]^2 + (a + b)*(2 +
 (1 + c*x)*Log[1 - c*x] - (1 + c*x)*Log[1 + c*x]))/(c*(1 + c*x))
```

fricas [A] time = 0.45, size = 74, normalized size = 0.95

$$\frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)^2+2\left((a+b)cx-a-b\right)\log\left(-\frac{cx+1}{cx-1}\right)-4\,a-4\,b}{8\left(c^2x+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1))^2 + 2*((a + b)*c*x - a - b)*log(
-(c*x + 1)/(c*x - 1)) - 4*a - 4*b)/(c^2*x + c)
```

giac [A] time = 0.17, size = 93, normalized size = 1.19

$$\frac{1}{8}c\left(\frac{(cx-1)b\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2}+\frac{2\,(cx-1)(a+b)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2}+\frac{2\,(cx-1)(a+b)}{(cx+1)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*c*((c*x - 1)*b*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2) + 2*(c*x - 1
)*(a + b)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2) + 2*(c*x - 1)*(a + b)/(
(c*x + 1)*c^2))
```

maple [B] time = 0.07, size = 247, normalized size = 3.17

$$-\frac{a\operatorname{arctanh}(cx)}{c\,(cx+1)}-\frac{a\ln(cx-1)}{4c}-\frac{a}{2c\,(cx+1)}+\frac{a\ln(cx+1)}{4c}-\frac{b\operatorname{arctanh}(cx)^2}{c\,(cx+1)}-\frac{b\operatorname{arctanh}(cx)\ln(cx-1)}{2c}-\frac{b\operatorname{arcta}}{c\,(cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x)
```

```
[Out] -a*arctanh(c*x)/c/(c*x+1)-1/4*a/c*ln(c*x-1)-1/2*a/c/(c*x+1)+1/4*a/c*ln(c*x+
1)-b*arctanh(c*x)^2/c/(c*x+1)-1/2/c*b*arctanh(c*x)*ln(c*x-1)-b*arctanh(c*x)
/c/(c*x+1)+1/2/c*b*arctanh(c*x)*ln(c*x+1)-1/8/c*b*ln(c*x-1)^2+1/4/c*b*ln(c*
x-1)*ln(1/2+1/2*c*x)-1/4/c*b*ln(c*x-1)-1/2*b/c/(c*x+1)+1/4/c*b*ln(c*x+1)-1/
8/c*b*ln(c*x+1)^2-1/4/c*b*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/4/c*b*ln(-1/2*
c*x+1/2)*ln(c*x+1)
```

maxima [C] time = 0.40, size = 226, normalized size = 2.90

$$-\frac{1}{8}\left(bc\left(\frac{2}{c^4x+c^3}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)+2\,a\left(\frac{2}{c^3x+c^2}-\frac{\log(cx+1)}{c^2}+\frac{\log(cx-1)}{c^2}\right)+\frac{-2i\,\pi b+(i\,\pi b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b*c*(2/(c^4*x + c^3) - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) + 2*a*(2/
(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + (-2*I*pi*b + (I*pi*b
 + (I*pi*b*c - b*c)*x + b)*log(c*x + 1) + (-I*pi*b + (-I*pi*b*c + b*c)*x -
b)*log(c*x - 1) + 2*b)/(c^3*x + c^2))*c - 1/4*((c*(2/(c^3*x + c^2) - log(c*
x + 1)/c^2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*b + 4*a/(c^2*x
 + c))*arctanh(c*x)
```

mupad [B] time = 1.15, size = 67, normalized size = 0.86

$$\frac{a\,\mathrm{atanh}(c\,x)+b\,\mathrm{atanh}(c\,x)+b\,\mathrm{atanh}(c\,x)^2}{2\,c}-\frac{a+b+2\,a\,\mathrm{atanh}(c\,x)+2\,b\,\mathrm{atanh}(c\,x)+2\,b\,\mathrm{atanh}(c\,x)^2}{2\,x\,c^2+2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(c*x)*(a + b*atanh(c*x)))/(c*x + 1)^2,x)
```

```
[Out] (a*atanh(c*x) + b*atanh(c*x) + b*atanh(c*x)^2)/(2*c) - (a + b + 2*a*atanh(c
*x) + 2*b*atanh(c*x) + 2*b*atanh(c*x)^2)/(2*c + 2*c^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atanh}(cx))\operatorname{atanh}(cx)}{(cx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c*x)*(a+b*atanh(c*x))/(c*x+1)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))*atanh(c*x)/(c*x + 1)**2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```