Computer algebra independent integration tests

8-Special-functions/8.5-Hyperbolic-integral-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [136]. This is test number [207].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (136)	% 0.00 (0)
Mathematica	% 100.00 (136)	% 0.00 (0)
Maple	% 76.47 (104)	% 23.53 (32)
Maxima	% 25.00 (34)	% 75.00 (102)
Fricas	% 25.00 (34)	% 75.00 (102)
Sympy	% 36.76 (50)	% 63.24 (86)
Giac	% 25.00 (34)	% 75.00 (102)
Mupad	% 25.00 (34)	% 75.00 (102)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

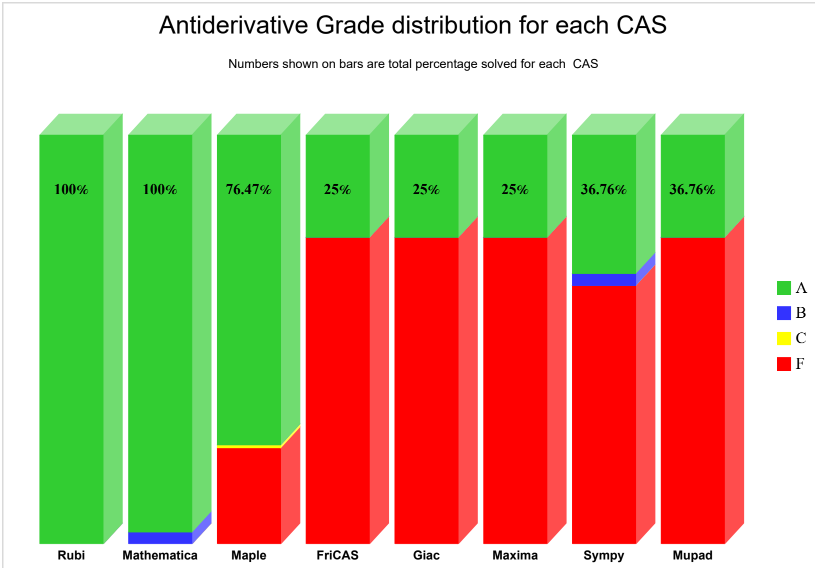
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

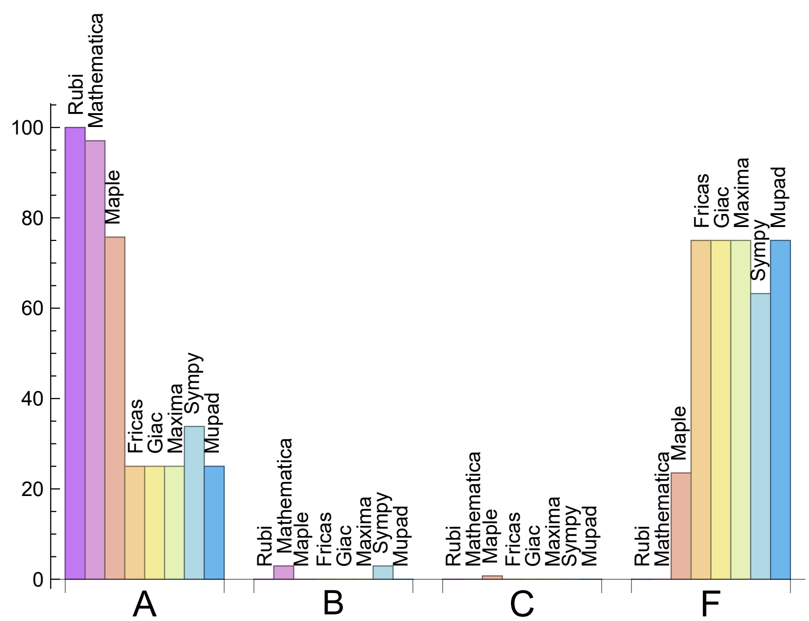
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	97.06	2.94	0.00	0.00
Maple	75.74	0.00	0.74	23.53
Maxima	25.00	0.00	0.00	75.00
Fricas	25.00	0.00	0.00	75.00
Sympy	33.82	2.94	0.00	63.24
Giac	25.00	0.00	0.00	75.00
Mupad	25.00	0.00	0.00	75.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	32	100.00 %	0.00 %	0.00 %
Maxima	102	100.00 %	0.00 %	0.00 %
Fricas	102	96.08 %	0.00 %	3.92 %
Sympy	86	97.67 %	0.00 %	2.33 %
Giac	102	100.00 %	0.00 %	0.00 %
Mupad	102	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

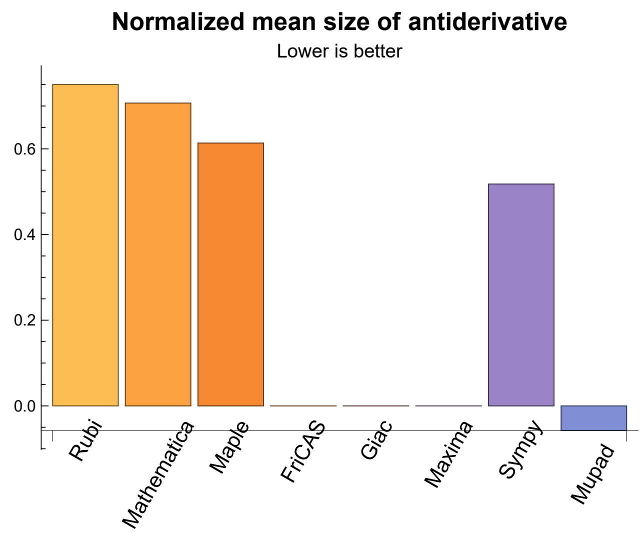
1.3 Performance

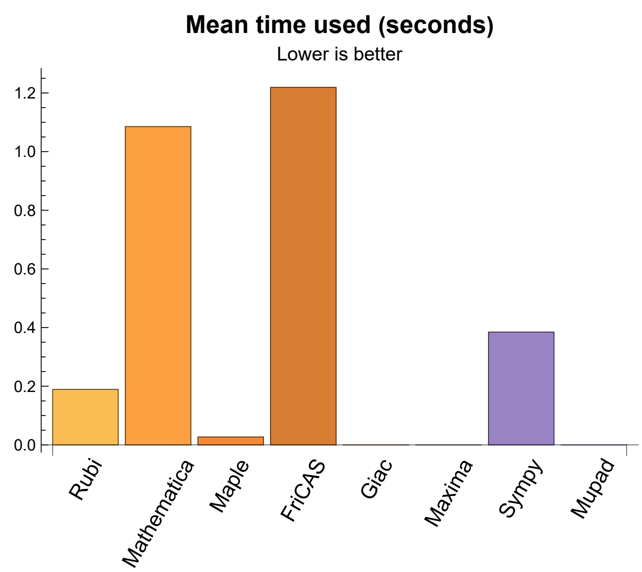
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	77.30	0.75	49.00	1.00
Mathematica	1.09	80.07	0.71	45.00	0.74
Maple	0.03	46.55	0.61	32.00	0.85
Maxima	0.00	0.00	0.00	0.00	0.00
Fricas	1.22	0.00	0.00	0.00	0.00
Sympy	0.38	25.86	0.52	0.00	0.00
Giac	0.00	0.00	0.00	0.00	0.00
Mupad	0.00	-1.00	-0.06	-1.00	-0.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{9, 14, 15, 16, 17, 22, 25, 29, 30, 31, 40, 46, 48, 58, 62, 65, 68, 77, 82, 83, 84, 85, 90, 93, 97, 98, 99, 108, 114, 116, 126, 130, 133, 136}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {36, 37, 104, 105}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

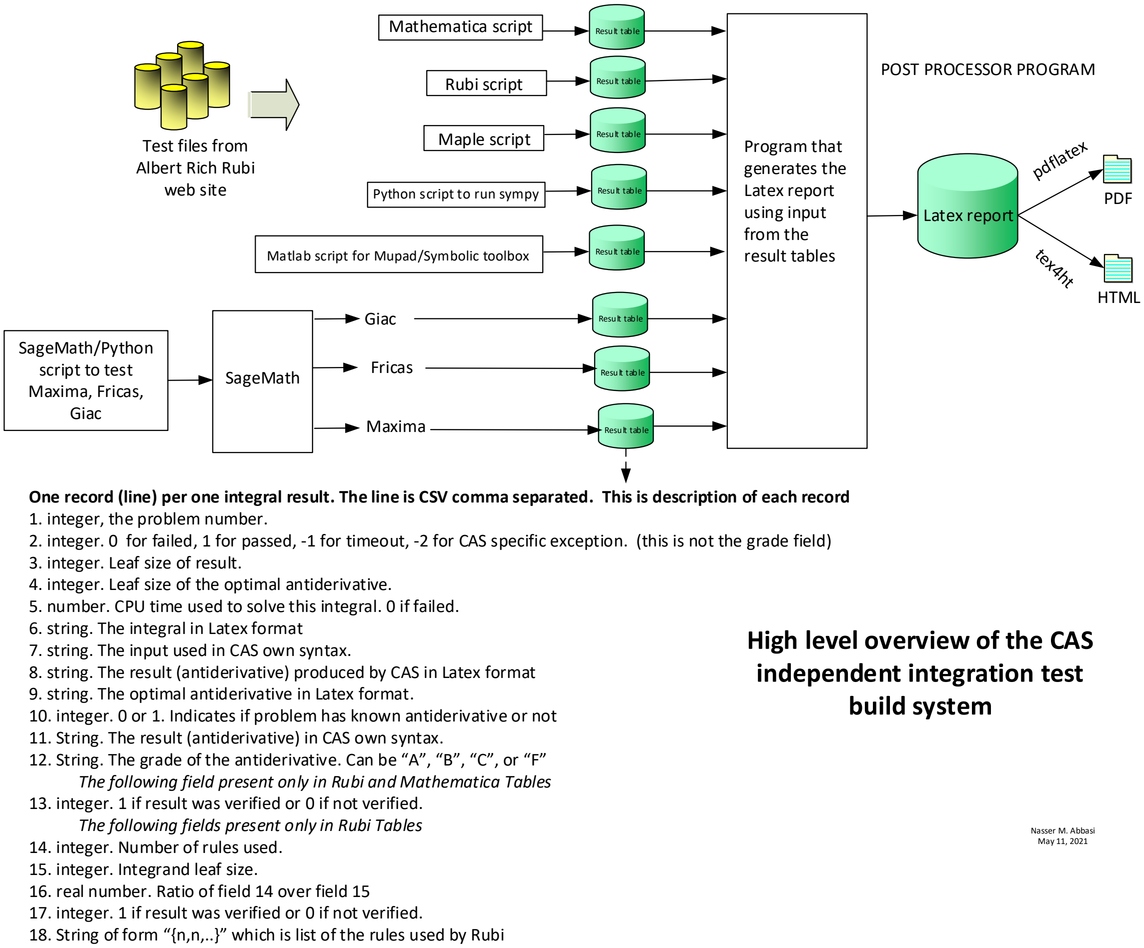
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 135, 136 }

B grade: { 63, 66, 131, 134 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 27, 28, 29, 30, 31, 35, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 68, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 95, 96, 97, 98, 99, 103, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 136 }

B grade: { }

C grade: { 1 }

F grade: { 23, 24, 26, 32, 33, 34, 36, 37, 38, 39, 47, 63, 64, 66, 67, 69, 74, 91, 92, 94, 100, 101, 102, 104, 105, 106, 107, 115, 131, 132, 134, 135 }

2.1.4 Maxima

A grade: { 9, 14, 15, 16, 17, 22, 25, 29, 30, 31, 40, 46, 48, 58, 62, 65, 68, 77, 82, 83, 84, 85, 90, 93, 97, 98, 99, 108, 114, 116, 126, 130, 133, 136 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

2.1.5 FriCAS

A grade: { 9, 14, 15, 16, 17, 22, 25, 29, 30, 31, 40, 46, 48, 58, 62, 65, 68, 77, 82, 83, 84, 85, 90, 93, 97, 98, 99, 108, 114, 116, 126, 130, 133, 136 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 14, 15, 16, 17, 22, 25, 29, 30, 31, 40, 41, 46, 48, 58, 62, 65, 68, 70, 71, 72, 77, 82, 83, 84, 85, 90, 93, 97, 98, 99, 108, 109, 114, 116, 126, 130, 133, 136 }

B grade: { 69, 73, 75, 76 }

C grade: { }

F grade: { 6, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 74, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

2.1.7 Giac

A grade: { 9, 14, 15, 16, 17, 22, 25, 29, 30, 31, 40, 46, 48, 58, 62, 65, 68, 77, 82, 83, 84, 85, 90, 93, 97, 98, 99, 108, 114, 116, 126, 130, 133, 136 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

2.1.8 Mupad

A grade: { 9, 14, 15, 16, 17, 22, 25, 29, 30, 31, 40, 46, 48, 58, 62, 65, 68, 77, 82, 83, 84, 85, 90, 93, 97, 98, 99, 108, 114, 116, 126, 130, 133, 136 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 23, 24, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 86, 87, 88, 89, 91, 92, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 134, 135 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	56	37	0	0	44	0	-1
normalized size	1	1.00	0.74	0.49	0.00	0.00	0.58	0.00	-0.01
time (sec)	N/A	0.074	0.087	0.036	0.000	0.726	0.907	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	0	0	61	0	-1
normalized size	1	1.00	0.84	0.89	0.00	0.00	0.97	0.00	-0.02
time (sec)	N/A	0.084	0.034	0.019	0.000	1.407	1.077	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	44	0	0	46	0	-1
normalized size	1	1.00	0.90	0.90	0.00	0.00	0.94	0.00	-0.02
time (sec)	N/A	0.056	0.025	0.020	0.000	0.850	1.236	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	0	0	29	0	-1
normalized size	1	1.00	1.00	0.91	0.00	0.00	0.83	0.00	-0.03
time (sec)	N/A	0.026	0.007	0.018	0.000	0.497	0.698	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	0	0	12	0	-1
normalized size	1	1.00	1.00	1.19	0.00	0.00	0.75	0.00	-0.06
time (sec)	N/A	0.004	0.002	0.014	0.000	1.489	0.738	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	20	0	0	0	0	-1
normalized size	1	1.00	1.00	0.53	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.006	0.037	0.000	0.536	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	0	0	34	0	-1
normalized size	1	1.00	1.00	1.28	0.00	0.00	1.36	0.00	-0.04
time (sec)	N/A	0.049	0.011	0.020	0.000	0.936	0.870	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	48	0	0	39	0	-1
normalized size	1	1.00	1.00	1.04	0.00	0.00	0.85	0.00	-0.02
time (sec)	N/A	0.071	0.012	0.021	0.000	0.566	0.791	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.031	0.636	0.022	0.000	1.124	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	107	138	0	0	0	0	-1
normalized size	1	1.00	0.72	0.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.118	0.046	0.000	0.801	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	78	84	0	0	0	0	-1
normalized size	1	1.00	0.70	0.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.074	0.040	0.000	0.758	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	69	0	0	0	0	-1
normalized size	1	1.00	0.78	0.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.051	0.026	0.000	2.432	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	0	0	0	0	-1
normalized size	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.008	0.020	0.000	0.763	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.023	0.146	0.022	0.000	2.916	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.025	0.165	0.023	0.000	1.092	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.025	0.197	0.018	0.000	0.825	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.280	10.308	0.035	0.000	3.021	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	94	156	0	0	0	0	-1
normalized size	1	1.00	0.51	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.380	0.199	0.023	0.000	1.359	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	64	101	0	0	0	0	-1
normalized size	1	1.00	0.54	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.180	0.023	0.000	0.536	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	47	60	0	0	0	0	-1
normalized size	1	1.00	0.66	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.101	0.021	0.000	2.234	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	26	0	0	0	0	-1
normalized size	1	1.00	1.56	0.96	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.037	0.013	0.000	2.221	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.023	0.350	0.026	0.000	2.120	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.228	0.103	0.031	0.000	0.600	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	86	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.353	0.277	0.030	0.000	2.938	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.043	5.275	0.033	0.000	2.147	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	158	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.490	1.378	0.020	0.000	0.527	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	95	135	0	0	0	0	-1
normalized size	1	1.00	0.62	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.327	0.264	0.027	0.000	0.762	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	43	0	0	0	0	-1
normalized size	1	1.00	0.85	0.90	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.009	0.013	0.000	0.604	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	0.637	0.020	0.000	1.348	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.034	1.628	0.028	0.000	0.504	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.035	1.373	0.029	0.000	2.749	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	98	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.269	1.648	0.074	0.000	0.930	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	98	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	1.619	0.064	0.000	0.541	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	95	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	1.570	0.058	0.000	2.025	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	96	73	0	0	0	0	-1
normalized size	1	1.00	1.75	1.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.080	0.036	0.000	0.576	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	146	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	1.784	0.066	0.000	2.026	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	148	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	1.841	0.080	0.000	0.619	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	120	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	2.828	0.056	0.000	0.688	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.014	0.023	0.000	0.654	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.151	0.248	0.026	0.000	0.693	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	0	0	7	0	-1
normalized size	1	1.00	1.00	0.90	0.00	0.00	0.70	0.00	-0.10
time (sec)	N/A	0.021	0.004	0.003	0.000	0.939	0.746	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	0	0	0	-1
normalized size	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	0.010	0.014	0.000	1.014	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	58	0	0	0	0	-1
normalized size	1	1.00	0.72	0.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.082	0.014	0.000	1.974	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	64	68	0	0	0	0	-1
normalized size	1	1.00	0.71	0.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.073	0.013	0.000	1.336	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	93	126	0	0	0	0	-1
normalized size	1	1.00	0.74	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.158	0.018	0.000	1.123	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.443	0.037	0.000	0.647	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.008	0.029	0.000	0.465	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	0.256	0.027	0.000	1.457	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	33	0	0	0	0	-1
normalized size	1	1.00	1.06	0.97	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	0.011	0.023	0.000	0.584	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	46	0	0	0	0	-1
normalized size	1	1.00	0.74	0.74	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.050	0.029	0.000	0.570	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	72	95	0	0	0	0	-1
normalized size	1	1.00	0.73	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.092	0.032	0.000	0.939	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	94	104	0	0	0	0	-1
normalized size	1	1.00	0.73	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.103	0.033	0.000	0.520	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	0	0	0	0	-1
normalized size	1	1.00	0.86	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	0.028	0.119	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	0	0	0	0	-1
normalized size	1	1.00	0.86	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	0.025	0.116	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	123	170	0	0	0	0	-1
normalized size	1	1.00	0.66	0.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.584	0.323	0.033	0.000	0.627	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	89	0	0	0	0	-1
normalized size	1	1.00	0.75	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.175	0.027	0.000	0.641	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	0	0	0	0	-1
normalized size	1	1.00	0.97	0.91	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.062	0.016	0.004	0.000	0.568	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.128	0.630	0.033	0.000	1.149	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	134	198	0	0	0	0	-1
normalized size	1	1.00	0.61	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.697	0.325	0.044	0.000	1.499	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	78	106	0	0	0	0	-1
normalized size	1	1.00	0.72	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.189	0.031	0.000	0.920	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	43	0	0	0	0	-1
normalized size	1	1.00	0.98	0.93	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.025	0.014	0.000	1.418	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.114	0.968	0.037	0.000	0.528	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	887	0	0	0	0	0	-1
normalized size	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.964	16.336	0.599	0.000	0.543	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	209	0	0	0	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	2.152	0.201	0.000	0.559	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.131	10.415	0.044	0.000	0.400	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	887	0	0	0	0	0	-1
normalized size	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.317	7.248	0.585	0.000	0.512	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	209	0	0	0	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.565	0.210	0.000	1.916	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.103	10.419	0.059	0.000	0.425	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	74	0	0	0	649	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	8.54	0.00	-0.01
time (sec)	N/A	0.077	0.056	0.032	0.000	0.511	1.564	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	0	0	85	0	-1
normalized size	1	1.00	0.84	0.89	0.00	0.00	1.35	0.00	-0.02
time (sec)	N/A	0.081	0.036	0.010	0.000	0.553	2.226	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	44	0	0	70	0	-1
normalized size	1	1.00	0.90	0.90	0.00	0.00	1.43	0.00	-0.02
time (sec)	N/A	0.055	0.025	0.010	0.000	1.061	1.898	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	0	0	53	0	-1
normalized size	1	1.00	1.00	0.91	0.00	0.00	1.51	0.00	-0.03
time (sec)	N/A	0.027	0.007	0.012	0.000	0.471	1.197	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	0	0	31	0	-1
normalized size	1	1.00	1.00	1.19	0.00	0.00	1.94	0.00	-0.06
time (sec)	N/A	0.004	0.001	0.010	0.000	0.494	1.276	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.006	0.056	0.000	2.023	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	0	0	39	0	-1
normalized size	1	1.00	1.00	1.28	0.00	0.00	1.56	0.00	-0.04
time (sec)	N/A	0.051	0.010	0.013	0.000	1.871	0.973	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	48	0	0	87	0	-1
normalized size	1	1.00	1.00	1.04	0.00	0.00	1.89	0.00	-0.02
time (sec)	N/A	0.073	0.012	0.011	0.000	2.027	2.299	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.032	0.568	0.021	0.000	0.642	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	107	138	0	0	0	0	-1
normalized size	1	1.00	0.65	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.095	0.031	0.000	0.436	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	78	84	0	0	0	0	-1
normalized size	1	1.00	0.70	0.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.073	0.031	0.000	1.429	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	69	0	0	0	0	-1
normalized size	1	1.00	0.77	0.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.043	0.028	0.000	0.451	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	0	0	0	0	-1
normalized size	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.007	0.010	0.000	0.481	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.022	0.135	0.014	0.000	0.988	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.026	0.159	0.020	0.000	0.968	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.026	0.189	0.017	0.000	2.113	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.258	9.652	0.029	0.000	0.469	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	94	156	0	0	0	0	-1
normalized size	1	1.00	0.51	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.388	0.195	0.016	0.000	0.520	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	64	101	0	0	0	0	-1
normalized size	1	1.00	0.54	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.176	0.017	0.000	0.727	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	47	60	0	0	0	0	-1
normalized size	1	1.00	0.66	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.102	0.018	0.000	2.101	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	26	0	0	0	0	-1
normalized size	1	1.00	1.56	0.96	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.035	0.003	0.000	3.293	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.020	0.265	0.028	0.000	0.865	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.216	0.101	0.030	0.000	1.020	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	80	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.303	0.030	0.000	1.360	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.043	3.454	0.027	0.000	1.937	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	158	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.306	1.078	0.017	0.000	1.149	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	95	135	0	0	0	0	-1
normalized size	1	1.00	0.62	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.327	0.188	0.029	0.000	1.426	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	43	0	0	0	0	-1
normalized size	1	1.00	0.85	0.90	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.010	0.004	0.000	0.984	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	0.508	0.016	0.000	1.463	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.035	1.103	0.025	0.000	0.698	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.035	1.341	0.026	0.000	0.953	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	97	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.922	0.094	0.000	0.453	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	97	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.893	0.085	0.000	1.404	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	93	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.858	0.079	0.000	2.216	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	96	73	0	0	0	0	-1
normalized size	1	1.00	1.75	1.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.079	0.025	0.000	0.517	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	144	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	1.102	0.096	0.000	0.497	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	146	0	0	0	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	1.161	0.090	0.000	2.965	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	119	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	1.915	0.043	0.000	0.563	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.014	0.026	0.000	0.443	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.148	0.204	0.024	0.000	0.496	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	0	0	7	0	-1
normalized size	1	1.00	1.00	0.90	0.00	0.00	0.70	0.00	-0.10
time (sec)	N/A	0.020	0.003	0.012	0.000	0.494	0.730	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	0	0	0	-1
normalized size	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	0.011	0.022	0.000	0.414	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	58	0	0	0	0	-1
normalized size	1	1.00	0.75	0.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.049	0.017	0.000	0.519	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	64	68	0	0	0	0	-1
normalized size	1	1.00	0.71	0.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.077	0.018	0.000	1.021	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	94	125	0	0	0	0	-1
normalized size	1	1.00	0.66	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.100	0.022	0.000	0.515	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.428	0.026	0.000	0.622	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.006	0.025	0.000	2.619	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.026	0.256	0.027	0.000	0.764	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	33	0	0	0	0	-1
normalized size	1	1.00	1.06	0.97	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	0.011	0.019	0.000	2.279	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	44	46	0	0	0	0	-1
normalized size	1	1.00	0.71	0.74	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.072	0.030	0.000	1.955	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	72	96	0	0	0	0	-1
normalized size	1	1.00	0.66	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.109	0.031	0.000	1.355	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	94	104	0	0	0	0	-1
normalized size	1	1.00	0.64	0.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.094	0.042	0.000	1.276	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	0	0	0	0	-1
normalized size	1	1.00	0.93	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	0.025	0.094	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	0	0	0	0	-1
normalized size	1	1.00	0.93	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	0.022	0.070	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	134	199	0	0	0	0	-1
normalized size	1	1.00	0.61	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.697	0.377	0.034	0.000	1.975	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	78	106	0	0	0	0	-1
normalized size	1	1.00	0.72	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.186	0.036	0.000	1.534	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	43	0	0	0	0	-1
normalized size	1	1.00	0.98	0.93	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.025	0.008	0.000	4.054	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.134	0.893	0.030	0.000	2.002	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	123	171	0	0	0	0	-1
normalized size	1	1.00	0.66	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.572	0.279	0.038	0.000	0.563	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	89	0	0	0	0	-1
normalized size	1	1.00	0.75	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.160	0.036	0.000	0.616	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	0	0	0	0	-1
normalized size	1	1.00	0.97	0.91	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.061	0.016	0.016	0.000	0.589	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.077	0.515	0.032	0.000	0.591	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	916	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.989	8.721	0.561	0.000	0.963	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	209	0	0	0	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.594	0.229	0.000	0.613	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.132	9.456	0.026	0.000	1.357	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	916	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.857	2.249	0.563	0.000	0.895	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	209	0	0	0	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.550	0.187	0.000	0.600	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.087	8.768	0.036	0.000	1.376	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [26] had the largest ratio of [1.583]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	8	0.500
2	A	6	4	1.00	8	0.500
3	A	5	4	1.00	8	0.500
4	A	4	4	1.00	6	0.667
5	A	1	1	1.00	4	0.250
6	A	1	1	1.00	8	0.125
7	A	4	4	1.00	8	0.500
8	A	5	4	1.00	8	0.500
9	A	0	0	0.00	0	0.000
10	A	19	11	1.00	10	1.100
11	A	15	10	1.00	10	1.000
12	A	10	8	1.00	8	1.000
13	A	6	5	1.00	6	0.833
14	A	0	0	0.00	0	0.000
15	A	0	0	0.00	0	0.000
16	A	0	0	0.00	0	0.000
17	A	0	0	0.00	0	0.000
18	A	14	6	1.00	10	0.600
19	A	10	6	1.00	10	0.600
20	A	7	6	1.00	8	0.750
21	A	1	1	1.00	6	0.167
22	A	0	0	0.00	0	0.000
23	A	7	5	1.00	10	0.500
24	A	11	6	1.00	10	0.600
25	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	39	19	1.00	12	1.583
27	A	17	14	1.00	10	1.400
28	A	5	5	1.00	8	0.625
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	0	0	0.00	0	0.000
32	A	7	5	1.00	17	0.294
33	A	7	5	1.00	15	0.333
34	A	7	5	1.00	13	0.385
35	A	3	1	1.00	17	0.059
36	A	7	5	1.00	17	0.294
37	A	7	5	1.00	17	0.294
38	A	7	5	1.00	19	0.263
39	A	14	10	1.00	12	0.833
40	A	0	0	0.00	0	0.000
41	A	1	1	1.00	12	0.083
42	A	5	4	1.00	9	0.444
43	A	9	7	1.00	10	0.700
44	A	14	9	1.00	12	0.750
45	A	18	10	1.00	12	0.833
46	A	0	0	0.00	0	0.000
47	A	7	6	1.00	12	0.500
48	A	0	0	0.00	0	0.000
49	A	5	4	1.00	9	0.444
50	A	9	7	1.00	10	0.700
51	A	13	9	1.00	12	0.750

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	20	11	1.00	12	0.917
53	A	6	4	1.00	9	0.444
54	A	6	4	1.00	9	0.444
55	A	21	16	1.00	16	1.000
56	A	11	9	1.00	14	0.643
57	A	4	4	1.00	13	0.308
58	A	0	0	0.00	0	0.000
59	A	21	14	1.00	16	0.875
60	A	12	10	1.00	14	0.714
61	A	4	3	1.00	13	0.231
62	A	0	0	0.00	0	0.000
63	A	24	10	1.00	14	0.714
64	A	9	5	1.00	13	0.385
65	A	0	0	0.00	0	0.000
66	A	24	9	1.00	14	0.643
67	A	9	5	1.00	13	0.385
68	A	0	0	0.00	0	0.000
69	A	5	4	1.00	8	0.500
70	A	6	4	1.00	8	0.500
71	A	5	4	1.00	8	0.500
72	A	4	4	1.00	6	0.667
73	A	1	1	1.00	4	0.250
74	A	1	1	1.00	8	0.125
75	A	4	4	1.00	8	0.500
76	A	5	4	1.00	8	0.500
77	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	19	11	1.00	10	1.100
79	A	15	10	1.00	10	1.000
80	A	10	8	1.00	8	1.000
81	A	6	5	1.00	6	0.833
82	A	0	0	0.00	0	0.000
83	A	0	0	0.00	0	0.000
84	A	0	0	0.00	0	0.000
85	A	0	0	0.00	0	0.000
86	A	14	6	1.00	10	0.600
87	A	10	6	1.00	10	0.600
88	A	7	6	1.00	8	0.750
89	A	1	1	1.00	6	0.167
90	A	0	0	0.00	0	0.000
91	A	7	5	1.00	10	0.500
92	A	11	6	1.00	10	0.600
93	A	0	0	0.00	0	0.000
94	A	39	19	1.00	12	1.583
95	A	17	14	1.00	10	1.400
96	A	5	5	1.00	8	0.625
97	A	0	0	0.00	0	0.000
98	A	0	0	0.00	0	0.000
99	A	0	0	0.00	0	0.000
100	A	7	5	1.00	17	0.294
101	A	7	5	1.00	15	0.333
102	A	7	5	1.00	13	0.385
103	A	3	1	1.00	17	0.059

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
104	A	7	5	1.00	17	0.294
105	A	7	5	1.00	17	0.294
106	A	7	5	1.00	19	0.263
107	A	14	10	1.00	12	0.833
108	A	0	0	0.00	0	0.000
109	A	1	1	1.00	12	0.083
110	A	5	4	1.00	9	0.444
111	A	9	7	1.00	10	0.700
112	A	14	9	1.00	12	0.750
113	A	18	10	1.00	12	0.833
114	A	0	0	0.00	0	0.000
115	A	7	6	1.00	12	0.500
116	A	0	0	0.00	0	0.000
117	A	5	4	1.00	9	0.444
118	A	9	7	1.00	10	0.700
119	A	13	9	1.00	12	0.750
120	A	20	11	1.00	12	0.917
121	A	6	4	1.00	9	0.444
122	A	6	4	1.00	9	0.444
123	A	21	14	1.00	16	0.875
124	A	12	10	1.00	14	0.714
125	A	4	3	1.00	13	0.231
126	A	0	0	0.00	0	0.000
127	A	21	16	1.00	16	1.000
128	A	11	9	1.00	14	0.643
129	A	4	4	1.00	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	0	0	0.00	0	0.000
131	A	24	9	1.00	14	0.643
132	A	9	5	1.00	13	0.385
133	A	0	0	0.00	0	0.000
134	A	24	10	1.00	14	0.714
135	A	9	5	1.00	13	0.385
136	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^m \mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=76

$$\frac{x^{m+1}\text{Shi}(bx)}{m+1}-\frac{x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b(m+1)}-\frac{x^m(bx)^{-m}\Gamma(m+1,bx)}{2b(m+1)}$$

```
[Out] -1/2*x^m*GAMMA(1+m,-b*x)/b/(1+m)/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/(1+m)/
((b*x)^m)+x^(1+m)*Shi(b*x)/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6532, 12, 3308, 2181}

$$-\frac{x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b(m+1)}-\frac{x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{2b(m+1)}+\frac{x^{m+1}\text{Shi}(bx)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*SinhIntegral[b*x],x]
```

```
[Out] -(x^m*Gamma[1 + m, -(b*x)])/(2*b*(1 + m)*(-(b*x))^m) - (x^m*Gamma[1 + m, b*
x])/(2*b*(1 + m)*(b*x)^m) + (x^(1 + m)*SinhIntegral[b*x])/(1 + m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^m \text{Shi}(bx)\,dx &= \frac{x^{1+m}\text{Shi}(bx)}{1+m} - \frac{b\int \frac{x^m \sinh(bx)}{b}\,dx}{1+m} \\
&= \frac{x^{1+m}\text{Shi}(bx)}{1+m} - \frac{\int x^m \sinh(bx)\,dx}{1+m} \\
&= \frac{x^{1+m}\text{Shi}(bx)}{1+m} + \frac{\int e^{-bx}x^m\,dx}{2(1+m)} - \frac{\int e^{bx}x^m\,dx}{2(1+m)} \\
&= -\frac{x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b(1+m)} - \frac{x^m(bx)^{-m}\Gamma(1+m,bx)}{2b(1+m)} + \frac{x^{1+m}\text{Shi}(bx)}{1+m}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 56, normalized size = 0.74

$$-\frac{x^m\left((-bx)^{-m}\Gamma(m+1,-bx)+(bx)^{-m}\Gamma(m+1,bx)-2bx\text{Shi}(bx)\right)}{2b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*SinhIntegral[b*x],x]
```

```
[Out] -1/2*(x^m*(Gamma[1 + m, -(b*x)]/(-(b*x))^m + Gamma[1 + m, b*x]/(b*x)^m - 2*
b*x*SinhIntegral[b*x]))/(b*(1 + m))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^{m}\,\mathrm{Shi}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x),x, algorithm="fricas")
[Out] integral(x^m*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\mathrm{Shi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x),x, algorithm="giac")
[Out] integrate(x^m*Shi(b*x), x)
```

maple [C] time = 0.04, size = 37, normalized size = 0.49

$$\frac{b\,x^{m+2}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+\frac{m}{2}\right],\left[\frac{3}{2},\frac{3}{2},2+\frac{m}{2}\right],\frac{b^{2}x^{2}}{4}\right)}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Shi(b*x),x)
[Out] b/(m+2)*x^(m+2)*hypergeom([1/2,1+1/2*m],[3/2,3/2,2+1/2*m],1/4*b^2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\mathrm{Shi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x),x, algorithm="maxima")
[Out] integrate(x^m*Shi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m}\,\mathrm{sinhint}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinhint(b*x),x)
```

```
[Out] int(x^m*sinhint(b*x), x)
```

sympy [A] time = 0.91, size = 44, normalized size = 0.58

$$\frac{bx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_3\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{3}{2},\frac{3}{2},\frac{m}{2}+2\end{matrix}\middle|\frac{b^2x^2}{4}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Shi(b*x),x)
```

```
[Out] b*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (3/2, 3/2, m/2 + 2), b**2*
x**2/4)/(2*gamma(m/2 + 2))
```

3.2 $\int x^3\textbf{Shi}(bx)\,dx$

Optimal. Leaf size=63

$$\frac{3\sinh(bx)}{2b^4}-\frac{3x\cosh(bx)}{2b^3}+\frac{3x^2\sinh(bx)}{4b^2}+\frac{1}{4}x^4\mathrm{Shi}(bx)-\frac{x^3\cosh(bx)}{4b}$$

```
[Out] -3/2*x*cosh(b*x)/b^3-1/4*x^3*cosh(b*x)/b+1/4*x^4*Shi(b*x)+3/2*sinh(b*x)/b^4
+3/4*x^2*sinh(b*x)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6532, 12, 3296, 2637}

$$\frac{3x^2\sinh(bx)}{4b^2}+\frac{3\sinh(bx)}{2b^4}-\frac{3x\cosh(bx)}{2b^3}+\frac{1}{4}x^4\mathrm{Shi}(bx)-\frac{x^3\cosh(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*SinhIntegral[b*x],x]
```

```
[Out] (-3*x*Cosh[b*x])/(2*b^3) - (x^3*Cosh[b*x])/(4*b) + (3*Sinh[b*x])/(2*b^4) +
(3*x^2*Sinh[b*x])/(4*b^2) + (x^4*SinhIntegral[b*x])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
```

```
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\text{Shi}(bx)\,dx &= \frac{1}{4}x^4\text{Shi}(bx) - \frac{1}{4}b\int \frac{x^3\sinh(bx)}{b}\,dx \\
&= \frac{1}{4}x^4\text{Shi}(bx) - \frac{1}{4}\int x^3\sinh(bx)\,dx \\
&= -\frac{x^3\cosh(bx)}{4b} + \frac{1}{4}x^4\text{Shi}(bx) + \frac{3\int x^2\cosh(bx)\,dx}{4b} \\
&= -\frac{x^3\cosh(bx)}{4b} + \frac{3x^2\sinh(bx)}{4b^2} + \frac{1}{4}x^4\text{Shi}(bx) - \frac{3\int x\sinh(bx)\,dx}{2b^2} \\
&= -\frac{3x\cosh(bx)}{2b^3} - \frac{x^3\cosh(bx)}{4b} + \frac{3x^2\sinh(bx)}{4b^2} + \frac{1}{4}x^4\text{Shi}(bx) + \frac{3\int\cosh(bx)\,dx}{2b^3} \\
&= -\frac{3x\cosh(bx)}{2b^3} - \frac{x^3\cosh(bx)}{4b} + \frac{3\sinh(bx)}{2b^4} + \frac{3x^2\sinh(bx)}{4b^2} + \frac{1}{4}x^4\text{Shi}(bx)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{3\left(b^2x^2+2\right)\sinh(bx)}{4b^4} - \frac{x\left(b^2x^2+6\right)\cosh(bx)}{4b^3} + \frac{1}{4}x^4\text{Shi}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*SinhIntegral[b*x],x]
```

```
[Out] -1/4*(x*(6 + b^2*x^2)*Cosh[b*x])/b^3 + (3*(2 + b^2*x^2)*Sinh[b*x])/(4*b^4)
+ (x^4*SinhIntegral[b*x])/4
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\,\text{Shi}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Shi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*Shi(b*x), x)
```

maple [A] time = 0.02, size = 56, normalized size = 0.89

$$\frac{\frac{b^4x^4 \operatorname{Shi}(bx)}{4} - \frac{b^3x^3 \cosh(bx)}{4} + \frac{3b^2x^2 \sinh(bx)}{4} - \frac{3bx \cosh(bx)}{2} + \frac{3 \sinh(bx)}{2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Shi(b*x),x)
```

```
[Out] 1/b^4*(1/4*b^4*x^4*Shi(b*x)-1/4*b^3*x^3*cosh(b*x)+3/4*b^2*x^2*sinh(b*x)-3/2
*b*x*cosh(b*x)+3/2*sinh(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{Shi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*Shi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \operatorname{sinhint}(b\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinhint(b*x),x)
```

```
[Out] int(x^3*sinhint(b*x), x)
```

sympy [A] time = 1.08, size = 61, normalized size = 0.97

$$\frac{x^4 \operatorname{Shi}(bx)}{4} - \frac{x^3 \cosh(bx)}{4b} + \frac{3x^2 \sinh(bx)}{4b^2} - \frac{3x \cosh(bx)}{2b^3} + \frac{3 \sinh(bx)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Shi(b*x),x)
```

```
[Out] x**4*Shi(b*x)/4 - x**3*cosh(b*x)/(4*b) + 3*x**2*sinh(b*x)/(4*b**2) - 3*x*co
sh(b*x)/(2*b**3) + 3*sinh(b*x)/(2*b**4)
```

3.3 $\int x^2 \mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=49

$$-\frac{2\cosh(bx)}{3b^3}+\frac{2x\sinh(bx)}{3b^2}+\frac{1}{3}x^3\mathrm{Shi}(bx)-\frac{x^2\cosh(bx)}{3b}$$

```
[Out] -2/3*cosh(b*x)/b^3-1/3*x^2*cosh(b*x)/b+1/3*x^3*Shi(b*x)+2/3*x*sinh(b*x)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6532, 12, 3296, 2638}

$$\frac{2x\sinh(bx)}{3b^2}-\frac{2\cosh(bx)}{3b^3}+\frac{1}{3}x^3\mathrm{Shi}(bx)-\frac{x^2\cosh(bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*SinhIntegral[b*x],x]
```

```
[Out] (-2*Cosh[b*x])/(3*b^3) - (x^2*Cosh[b*x])/(3*b) + (2*x*Sinh[b*x])/(3*b^2) +
(x^3*SinhIntegral[b*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\text{Shi}(bx)\,dx &= \frac{1}{3}x^3\text{Shi}(bx)-\frac{1}{3}b\int\frac{x^2\sinh(bx)}{b}\,dx\\
&= \frac{1}{3}x^3\text{Shi}(bx)-\frac{1}{3}\int x^2\sinh(bx)\,dx\\
&= -\frac{x^2\cosh(bx)}{3b}+\frac{1}{3}x^3\text{Shi}(bx)+\frac{2\int x\cosh(bx)\,dx}{3b}\\
&= -\frac{x^2\cosh(bx)}{3b}+\frac{2x\sinh(bx)}{3b^2}+\frac{1}{3}x^3\text{Shi}(bx)-\frac{2\int\sinh(bx)\,dx}{3b^2}\\
&= -\frac{2\cosh(bx)}{3b^3}-\frac{x^2\cosh(bx)}{3b}+\frac{2x\sinh(bx)}{3b^2}+\frac{1}{3}x^3\text{Shi}(bx)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.90

$$\frac{2x\sinh(bx)}{3b^2}-\frac{\left(b^2x^2+2\right)\cosh(bx)}{3b^3}+\frac{1}{3}x^3\text{Shi}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*SinhIntegral[b*x],x]
```

```
[Out] -1/3*((2 + b^2*x^2)*Cosh[b*x])/b^3 + (2*x*Sinh[b*x])/(3*b^2) + (x^3*SinhInt
egral[b*x])/3
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{Shi}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{Shi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x), x)
```

maple [A] time = 0.02, size = 44, normalized size = 0.90

$$\frac{\frac{b^3x^3\,\mathrm{Shi}(bx)}{3}-\frac{b^2x^2\cosh(bx)}{3}+\frac{2bx\sinh(bx)}{3}-\frac{2\cosh(bx)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Shi(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*Shi(b*x)-1/3*b^2*x^2*cosh(b*x)+2/3*b*x*sinh(b*x)-2/3*cos
h(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{x^3\,\mathrm{sinhint}\,(b\,x)}{3}-\frac{\frac{2\cosh(b\,x)}{3}+\frac{b^2\,x^2\cosh(b\,x)}{3}-\frac{2\,b\,x\sinh(b\,x)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(b*x),x)
```

```
[Out] (x^3*sinhint(b*x))/3 - ((2*cosh(b*x))/3 + (b^2*x^2*cosh(b*x))/3 - (2*b*x*si
nh(b*x))/3)/b^3
```

sympy [A] time = 1.24, size = 46, normalized size = 0.94

$$\frac{x^3\,\mathrm{Shi}\,(bx)}{3}-\frac{x^2\cosh\,(bx)}{3b}+\frac{2x\sinh\,(bx)}{3b^2}-\frac{2\cosh\,(bx)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Shi(b*x),x)
```

```
[Out] x**3*Shi(b*x)/3 - x**2*cosh(b*x)/(3*b) + 2*x*sinh(b*x)/(3*b**2) - 2*cosh(b*
x)/(3*b**3)
```

3.4 $\int x\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=35

$$\frac{\sinh(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)-\frac{x\cosh(bx)}{2b}$$

```
[Out] -1/2*x*cosh(b*x)/b+1/2*x^2*Shi(b*x)+1/2*sinh(b*x)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6532, 12, 3296, 2637}

$$\frac{\sinh(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)-\frac{x\cosh(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*SinhIntegral[b*x],x]
```

```
[Out] -(x*Cosh[b*x])/(2*b) + Sinh[b*x]/(2*b^2) + (x^2*SinhIntegral[b*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\text{Shi}(bx)\,dx &= \frac{1}{2}x^2\text{Shi}(bx) - \frac{1}{2}b\int \frac{x\sinh(bx)}{b}\,dx \\ &= \frac{1}{2}x^2\text{Shi}(bx) - \frac{1}{2}\int x\sinh(bx)\,dx \\ &= -\frac{x\cosh(bx)}{2b} + \frac{1}{2}x^2\text{Shi}(bx) + \frac{\int \cosh(bx)\,dx}{2b} \\ &= -\frac{x\cosh(bx)}{2b} + \frac{\sinh(bx)}{2b^2} + \frac{1}{2}x^2\text{Shi}(bx)\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{\sinh(bx)}{2b^2} + \frac{1}{2}x^2\text{Shi}(bx) - \frac{x\cosh(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*SinhIntegral[b*x],x]
[Out] -1/2*(x*Cosh[b*x])/b + Sinh[b*x]/(2*b^2) + (x^2*SinhIntegral[b*x])/2
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Shi}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x),x, algorithm="fricas")
[Out] integral(x*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x),x, algorithm="giac")
[Out] integrate(x*Shi(b*x), x)
```

maple [A] time = 0.02, size = 32, normalized size = 0.91

$$\frac{\frac{b^2x^2\,\text{Shi}(bx)}{2} - \frac{bx\cosh(bx)}{2} + \frac{\sinh(bx)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*Shi(b*x)-1/2*b*x*cosh(b*x)+1/2*sinh(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\frac{\frac{\sinh(b\,x)}{2}-\frac{b\,x\cosh(b\,x)}{2}}{b^2}+\frac{x^2\,\mathrm{sinhint}\left(b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(b*x),x)
```

```
[Out] (sinh(b*x)/2 - (b*x*cosh(b*x))/2)/b^2 + (x^2*sinhint(b*x))/2
```

sympy [A] time = 0.70, size = 29, normalized size = 0.83

$$\frac{x^2\,\mathrm{Shi}\left(bx\right)}{2}-\frac{x\cosh\left(bx\right)}{2b}+\frac{\sinh\left(bx\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x),x)
```

```
[Out] x**2*Shi(b*x)/2 - x*cosh(b*x)/(2*b) + sinh(b*x)/(2*b**2)
```

3.5 $\int \text{Shi}(bx)\,dx$

Optimal. Leaf size=16

$$x\text{Shi}(bx) - \frac{\cosh(bx)}{b}$$

```
[Out] -cosh(b*x)/b+x*Shi(b*x)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6528}

$$x\text{Shi}(bx) - \frac{\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[b*x],x]
[Out] -(Cosh[b*x]/b) + x*SinhIntegral[b*x]
```

Rule 6528

```
Int[SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*SinhInte
gral[a + b*x])/b, x] - Simp[Cosh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{Shi}(bx)\,dx = -\frac{\cosh(bx)}{b} + x\text{Shi}(bx)$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$x\text{Shi}(bx) - \frac{\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[b*x],x]
[Out] -(Cosh[b*x]/b) + x*SinhIntegral[b*x]
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{Shi}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x),x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Shi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x),x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x), x)
```

maple [A] time = 0.01, size = 19, normalized size = 1.19

$$\frac{bx\operatorname{Shi}(bx)-\cosh(bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x),x)
```

```
[Out] 1/b*(b*x*Shi(b*x)-cosh(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Shi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$x\,\mathrm{sinhint}(b\,x)-\frac{\cosh(b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x),x)
```

```
[Out] x*sinhint(b*x) - cosh(b*x)/b
```

sympy [A] time = 0.74, size = 12, normalized size = 0.75

$$x\operatorname{Shi}(bx) - \frac{\cosh(bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x),x)
```

```
[Out] x*Shi(b*x) - cosh(b*x)/b
```

3.6 $\int \frac{\text{Shi}(bx)}{x} \, dx$

Optimal. Leaf size=38

$$\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)$$

```
[Out] 1/2*b*x*HypergeometricPFQ([1, 1, 1],[2, 2, 2],-b*x)+1/2*b*x*HypergeometricP
FQ([1, 1, 1],[2, 2, 2],b*x)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6530}

$$\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[b*x]/x,x]
```

```
[Out] (b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)])/2 + (b*x*Hypergeometr
icPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2
```

Rule 6530

```
Int[SinhIntegral[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(1*b*x*HypergeometricP
FQ[{1, 1, 1}, {2, 2, 2}, -(b*x)])/2, x] + Simp[(1*b*x*HypergeometricPFQ[{1,
 1, 1}, {2, 2, 2}, b*x])/2, x] /; FreeQ[b, x]
```

Rubi steps

$$\int \frac{\text{Shi}(bx)}{x} \, dx = \frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[b*x]/x,x]
```

```
[Out] (b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)])/2 + (b*x*Hypergeometr
icPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\,(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)/x, x)
```

maple [A] time = 0.04, size = 20, normalized size = 0.53

$$bx\,\text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}\right], \left[\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right], \frac{b^2x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)/x,x)
```

```
[Out] b*x*hypergeom([1/2,1/2],[3/2,3/2,3/2],1/4*b^2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\text{sinhint}\,(b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)/x,x)
[Out] int(sinhint(b*x)/x, x)
```

sympy [**F**(**-2**)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x,x)
[Out] Exception raised: AttributeError
```

3.7 $\int \frac{\textbf{Shi}(bx)}{x^2} \, dx$

Optimal. Leaf size=25

$$b\text{Chi}(bx) - \frac{\text{Shi}(bx)}{x} - \frac{\sinh(bx)}{x}$$

```
[Out] b*Chi(b*x)-Shi(b*x)/x-sinh(b*x)/x
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6532, 12, 3297, 3301}

$$b\text{Chi}(bx) - \frac{\text{Shi}(bx)}{x} - \frac{\sinh(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[b*x]/x^2,x]
```

```
[Out] b*CoshIntegral[b*x] - Sinh[b*x]/x - SinhIntegral[b*x]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\mathrm{Shi}(bx)}{x^2}\,dx &= -\frac{\mathrm{Shi}(bx)}{x} + b\int \frac{\sinh(bx)}{bx^2}\,dx \\
&= -\frac{\mathrm{Shi}(bx)}{x} + \int \frac{\sinh(bx)}{x^2}\,dx \\
&= -\frac{\sinh(bx)}{x} - \frac{\mathrm{Shi}(bx)}{x} + b\int \frac{\cosh(bx)}{x}\,dx \\
&= b\mathrm{Chi}(bx) - \frac{\sinh(bx)}{x} - \frac{\mathrm{Shi}(bx)}{x}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$
b\mathrm{Chi}(bx) - \frac{\mathrm{Shi}(bx)}{x} - \frac{\sinh(bx)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[b*x]/x^2,x]
[Out] b*CoshIntegral[b*x] - Sinh[b*x]/x - SinhIntegral[b*x]/x
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\mathrm{Shi}\,(bx)}{x^2}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x^2,x, algorithm="fricas")
[Out] integral(sinh_integral(b*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\mathrm{Shi}\,(bx)}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x^2,x, algorithm="giac")
[Out] integrate(Shi(b*x)/x^2, x)
```

maple [A] time = 0.02, size = 32, normalized size = 1.28

$$b\left(-\frac{\mathrm{Shi}\,(bx)}{bx}-\frac{\sinh\,(bx)}{bx}+\mathrm{X}\,(bx)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)/x^2,x)
```

```
[Out] b*(-Shi(b*x)/b/x-sinh(b*x)/b/x+Chi(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{sinhint}\,(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)/x^2,x)
```

```
[Out] int(sinhint(b*x)/x^2, x)
```

sympy [A] time = 0.87, size = 34, normalized size = 1.36

$$\frac{b^3x^2{}_3F_4\left(\begin{matrix} 1,1,\frac{3}{2} \\ 2,2,\frac{5}{2},\frac{5}{2} \end{matrix}\middle|\frac{b^2x^2}{4}\right)}{36}+\frac{b\log\left(b^2x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x**2,x)
```

```
[Out] b**3*x**2*hyper((1, 1, 3/2), (2, 2, 5/2, 5/2), b**2*x**2/4)/36 + b*log(b**2
*x**2)/2
```

3.8 $\int \frac{\textbf{Shi}(bx)}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{4}b^2\text{Shi}(bx)-\frac{\text{Shi}(bx)}{2x^2}-\frac{\sinh(bx)}{4x^2}-\frac{b\cosh(bx)}{4x}$$

```
[Out] -1/4*b*cosh(b*x)/x+1/4*b^2*Shi(b*x)-1/2*Shi(b*x)/x^2-1/4*sinh(b*x)/x^2
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6532, 12, 3297, 3298}

$$\frac{1}{4}b^2\text{Shi}(bx)-\frac{\text{Shi}(bx)}{2x^2}-\frac{\sinh(bx)}{4x^2}-\frac{b\cosh(bx)}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[b*x]/x^3,x]
```

```
[Out] -(b*Cosh[b*x])/(4*x) - Sinh[b*x]/(4*x^2) + (b^2*SinhIntegral[b*x])/4 - Sinh
Integral[b*x]/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
```

```
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Shi}(bx)}{x^3}\,dx &= -\frac{\text{Shi}(bx)}{2x^2}+\frac{1}{2}b\int \frac{\sinh(bx)}{bx^3}\,dx\\
&= -\frac{\text{Shi}(bx)}{2x^2}+\frac{1}{2}\int \frac{\sinh(bx)}{x^3}\,dx\\
&= -\frac{\sinh(bx)}{4x^2}-\frac{\text{Shi}(bx)}{2x^2}+\frac{1}{4}b\int \frac{\cosh(bx)}{x^2}\,dx\\
&= -\frac{b\cosh(bx)}{4x}-\frac{\sinh(bx)}{4x^2}-\frac{\text{Shi}(bx)}{2x^2}+\frac{1}{4}b^2\int \frac{\sinh(bx)}{x}\,dx\\
&= -\frac{b\cosh(bx)}{4x}-\frac{\sinh(bx)}{4x^2}+\frac{1}{4}b^2\text{Shi}(bx)-\frac{\text{Shi}(bx)}{2x^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$
\frac{1}{4}b^2\text{Shi}(bx)-\frac{\text{Shi}(bx)}{2x^2}-\frac{\sinh(bx)}{4x^2}-\frac{b\cosh(bx)}{4x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[b*x]/x^3,x]
```

```
[Out] -1/4*(b*Cosh[b*x])/x - Sinh[b*x]/(4*x^2) + (b^2*SinhIntegral[b*x])/4 - Sinh
Integral[b*x]/(2*x^2)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\text{Shi}\,(bx)}{x^3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\text{Shi}\,(bx)}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)/x^3, x)
```

maple [A] time = 0.02, size = 48, normalized size = 1.04

$$b^2 \left(-\frac{\mathrm{Shi}\,(bx)}{2b^2x^2} - \frac{\sinh\,(bx)}{4b^2x^2} - \frac{\cosh\,(bx)}{4bx} + \frac{\mathrm{Shi}\,(bx)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2*Shi(b*x)/b^2/x^2-1/4*sinh(b*x)/b^2/x^2-1/4/b/x*cosh(b*x)+1/4*Shi(
b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{b^2\,\mathrm{sinhint}\,(b\,x)}{4} - \frac{\frac{\mathrm{sinhint}(b\,x)}{2} + \frac{\sinh(b\,x)}{4} + \frac{b\,x\cosh(b\,x)}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)/x^3,x)
```

```
[Out] (b^2*sinhint(b*x))/4 - (sinhint(b*x)/2 + sinh(b*x)/4 + (b*x*cosh(b*x))/4)/x
^2
```

sympy [A] time = 0.79, size = 39, normalized size = 0.85

$$\frac{b^2\,\mathrm{Shi}\,(bx)}{4} - \frac{b\cosh\,(bx)}{4x} - \frac{\sinh\,(bx)}{4x^2} - \frac{\mathrm{Shi}\,(bx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)/x**3,x)
```

```
[Out] b**2*Shi(b*x)/4 - b*cosh(b*x)/(4*x) - sinh(b*x)/(4*x**2) - Shi(b*x)/(2*x**2
)
```

3.9 $\int x^m \mathbf{Shi}(bx)^2 \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(x^m \text{Shi}(bx)^2, x\right)$$

```
[Out] CannotIntegrate(x^m*Shi(b*x)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \text{Shi}(bx)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*SinhIntegral[b*x]^2,x]
```

```
[Out] Defer[Int][x^m*SinhIntegral[b*x]^2, x]
```

Rubi steps

$$\int x^m \text{Shi}(bx)^2 \, dx = \int x^m \text{Shi}(bx)^2 \, dx$$

Mathematica [A] time = 0.64, size = 0, normalized size = 0.00

$$\int x^m \text{Shi}(bx)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*SinhIntegral[b*x]^2,x]
```

```
[Out] Integrate[x^m*SinhIntegral[b*x]^2, x]
```

fricas [A] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \,\text{Shi}\,(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sinh_integral(b*x)^2, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x)^2,x, algorithm="giac")
[Out] integrate(x^m*Shi(b*x)^2, x)
```

maple [**A**] time = 0.02, size = 0, normalized size = 0.00

$$\int x^m \,\mathrm{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Shi(b*x)^2,x)
[Out] int(x^m*Shi(b*x)^2,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^m*Shi(b*x)^2, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.08

$$\int x^m \,\mathrm{sinhint}\,(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinhint(b*x)^2,x)
[Out] int(x^m*sinhint(b*x)^2, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \,\mathrm{Shi}^2\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Shi(b*x)**2,x)
[Out] Integral(x**m*Shi(b*x)**2, x)
```

3.10 $\int x^3\mathbf{Shi}(bx)^2\,dx$

Optimal. Leaf size=149

$$-\frac{3\text{Chi}(2bx)}{2b^4}+\frac{3\text{Shi}(bx)\sinh(bx)}{b^4}+\frac{3\log(x)}{2b^4}+\frac{2\sinh^2(bx)}{b^4}-\frac{3x\text{Shi}(bx)\cosh(bx)}{b^3}-\frac{x\sinh(bx)\cosh(bx)}{b^3}+\frac{3x^2\text{Shi}(bx)}{2b^2}$$

```
[Out] 1/2*x^2/b^2-3/2*Chi(2*b*x)/b^4+3/2*ln(x)/b^4-3*x*cosh(b*x)*Shi(b*x)/b^3-1/2
*x^3*cosh(b*x)*Shi(b*x)/b+1/4*x^4*Shi(b*x)^2-x*cosh(b*x)*sinh(b*x)/b^3+3*Sh
i(b*x)*sinh(b*x)/b^4+3/2*x^2*Shi(b*x)*sinh(b*x)/b^2+2*sinh(b*x)^2/b^4+1/4*x
^2*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.23, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6536, 6542, 12, 5372, 3310, 30, 6548, 2564, 6546, 3312, 3301}

$$-\frac{3\text{Chi}(2bx)}{2b^4}+\frac{3x^2\text{Shi}(bx)\sinh(bx)}{2b^2}+\frac{3\text{Shi}(bx)\sinh(bx)}{b^4}-\frac{3x\text{Shi}(bx)\cosh(bx)}{b^3}+\frac{x^2}{2b^2}+\frac{x^2\sinh^2(bx)}{4b^2}+\frac{3\log(x)}{2b^4}+\frac{2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*SinhIntegral[b*x]^2,x]
```

```
[Out] x^2/(2*b^2) - (3*CoshIntegral[2*b*x])/(2*b^4) + (3*Log[x])/(2*b^4) - (x*Cos
h[b*x]*Sinh[b*x])/b^3 + (2*Sinh[b*x]^2)/b^4 + (x^2*Sinh[b*x]^2)/(4*b^2) - (
3*x*Cosh[b*x]*SinhIntegral[b*x])/b^3 - (x^3*Cosh[b*x]*SinhIntegral[b*x])/(2
*b) + (3*Sinh[b*x]*SinhIntegral[b*x])/b^4 + (3*x^2*Sinh[b*x]*SinhIntegral[b
*x])/(2*b^2) + (x^4*SinhIntegral[b*x]^2)/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6536

```
Int[(x_)^(m_.)*SinhIntegral[(b_.)*(x_)]^2, x_Symbol] :> Simp[(x^(m + 1)*Sin
hIntegral[b*x]^2)/(m + 1), x] - Dist[2/(m + 1), Int[x^m*Sinh[b*x]*SinhInteg
ral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\text{Shi}(bx)^2\,dx &= \frac{1}{4}x^4\text{Shi}(bx)^2 - \frac{1}{2}\int x^3\sinh(bx)\text{Shi}(bx)\,dx \\
&= -\frac{x^3\cosh(bx)\text{Shi}(bx)}{2b} + \frac{1}{4}x^4\text{Shi}(bx)^2 + \frac{1}{2}\int\frac{x^2\cosh(bx)\sinh(bx)}{b}\,dx + \frac{3\int x^2\cosh(bx)\text{Shi}(b
\\
&= -\frac{x^3\cosh(bx)\text{Shi}(bx)}{2b} + \frac{3x^2\sinh(bx)\text{Shi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Shi}(bx)^2 - \frac{3\int x\sinh(bx)\text{Shi}(bx)\,dx}{b^2} + \frac{\int
\\
&= \frac{x^2\sinh^2(bx)}{4b^2} - \frac{3x\cosh(bx)\text{Shi}(bx)}{b^3} - \frac{x^3\cosh(bx)\text{Shi}(bx)}{2b} + \frac{3x^2\sinh(bx)\text{Shi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Shi}(
\\
&= -\frac{x\cosh(bx)\sinh(bx)}{b^3} + \frac{\sinh^2(bx)}{2b^4} + \frac{x^2\sinh^2(bx)}{4b^2} - \frac{3x\cosh(bx)\text{Shi}(bx)}{b^3} - \frac{x^3\cosh(bx)\text{Shi}(b}{2b}
\\
&= \frac{x^2}{2b^2} - \frac{x\cosh(bx)\sinh(bx)}{b^3} + \frac{\sinh^2(bx)}{2b^4} + \frac{x^2\sinh^2(bx)}{4b^2} - \frac{3x\cosh(bx)\text{Shi}(bx)}{b^3} - \frac{x^3\cosh(bx)}{2b}
\\
&= \frac{x^2}{2b^2} - \frac{x\cosh(bx)\sinh(bx)}{b^3} + \frac{2\sinh^2(bx)}{b^4} + \frac{x^2\sinh^2(bx)}{4b^2} - \frac{3x\cosh(bx)\text{Shi}(bx)}{b^3} - \frac{x^3\cosh(b}{2b}
\\
&= \frac{x^2}{2b^2} + \frac{3\log(x)}{2b^4} - \frac{x\cosh(bx)\sinh(bx)}{b^3} + \frac{2\sinh^2(bx)}{b^4} + \frac{x^2\sinh^2(bx)}{4b^2} - \frac{3x\cosh(bx)\text{Shi}(bx)}{b^3} -
\\
&= \frac{x^2}{2b^2} - \frac{3\text{Chi}(2bx)}{2b^4} + \frac{3\log(x)}{2b^4} - \frac{x\cosh(bx)\sinh(bx)}{b^3} + \frac{2\sinh^2(bx)}{b^4} + \frac{x^2\sinh^2(bx)}{4b^2} - 3x\cosh
\end{aligned}$$

Mathematica [A] time = 0.12, size = 107, normalized size = 0.72

$$\frac{2b^4x^4\text{Shi}(bx)^2 - 4\text{Shi}(bx)\left(bx\left(b^2x^2+6\right)\cosh(bx) - 3\left(b^2x^2+2\right)\sinh(bx)\right) + 3b^2x^2 + b^2x^2\cosh(2bx) - 12\text{Chi}(2b}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*SinhIntegral[b*x]^2,x]
```

```
[Out] (3*b^2*x^2 + 8*Cosh[2*b*x] + b^2*x^2*Cosh[2*b*x] - 12*CoshIntegral[2*b*x] +
 12*Log[x] - 4*b*x*Sinh[2*b*x] - 4*(b*x*(6 + b^2*x^2)*Cosh[b*x] - 3*(2 + b^
2*x^2)*Sinh[b*x])*SinhIntegral[b*x] + 2*b^4*x^4*SinhIntegral[b*x]^2)/(8*b^4
)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\,\text{Shi}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*sinh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*Shi(b*x)^2, x)
```

maple [A] time = 0.05, size = 138, normalized size = 0.93

$$\frac{x^4\,\text{Shi}\,(bx)^2}{4}-\frac{x^3\cosh\,(bx)\,\text{Shi}\,(bx)}{2b}+\frac{3x^2\,\text{Shi}\,(bx)\sinh\,(bx)}{2b^2}-\frac{3x\cosh\,(bx)\,\text{Shi}\,(bx)}{b^3}+\frac{3\,\text{Shi}\,(bx)\sinh\,(bx)}{b^4}+\frac{x^2\left(\cos\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Shi(b*x)^2,x)
```

```
[Out] 1/4*x^4*Shi(b*x)^2-1/2*x^3*cosh(b*x)*Shi(b*x)/b+3/2*x^2*Shi(b*x)*sinh(b*x)/
b^2-3*x*cosh(b*x)*Shi(b*x)/b^3+3*Shi(b*x)*sinh(b*x)/b^4+1/4/b^2*x^2*cosh(b*
x)^2-x*cosh(b*x)*sinh(b*x)/b^3+1/4*x^2/b^2+2*cosh(b*x)^2/b^4+3/2/b^4*ln(b*x
)-3/2*Chi(2*b*x)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*Shi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{sinhint}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinhint(b*x)^2,x)
```

```
[Out] int(x^3*sinhint(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{Shi}^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Shi(b*x)**2,x)
```

```
[Out] Integral(x**3*Shi(b*x)**2, x)
```

3.11 $\int x^2\mathbf{Shi}(bx)^2\,dx$

Optimal. Leaf size=112

$$\frac{2\text{Shi}(2bx)}{3b^3}-\frac{4\text{Shi}(bx)\cosh(bx)}{3b^3}-\frac{5\sinh(bx)\cosh(bx)}{6b^3}+\frac{4x\text{Shi}(bx)\sinh(bx)}{3b^2}+\frac{5x}{6b^2}+\frac{x\sinh^2(bx)}{3b^2}+\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2x}{}$$

```
[Out] 5/6*x/b^2-4/3*cosh(b*x)*Shi(b*x)/b^3-2/3*x^2*cosh(b*x)*Shi(b*x)/b+1/3*x^3*S
hi(b*x)^2+2/3*Shi(2*b*x)/b^3-5/6*cosh(b*x)*sinh(b*x)/b^3+4/3*x*Shi(b*x)*sin
h(b*x)/b^2+1/3*x*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6536, 6542, 12, 5372, 2635, 8, 6548, 6540, 5448, 3298}

$$\frac{2\text{Shi}(2bx)}{3b^3}+\frac{4x\text{Shi}(bx)\sinh(bx)}{3b^2}-\frac{4\text{Shi}(bx)\cosh(bx)}{3b^3}+\frac{5x}{6b^2}+\frac{x\sinh^2(bx)}{3b^2}-\frac{5\sinh(bx)\cosh(bx)}{6b^3}+\frac{1}{3}x^3\text{Shi}(bx)^2-\frac{2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*SinhIntegral[b*x]^2,x]
```

```
[Out] (5*x)/(6*b^2) - (5*Cosh[b*x]*Sinh[b*x])/(6*b^3) + (x*Sinh[b*x]^2)/(3*b^2) -
 (4*Cosh[b*x]*SinhIntegral[b*x])/(3*b^3) - (2*x^2*Cosh[b*x]*SinhIntegral[b*
x])/(3*b) + (4*x*Sinh[b*x]*SinhIntegral[b*x])/(3*b^2) + (x^3*SinhIntegral[b
*x]^2)/3 + (2*SinhIntegral[2*b*x])/(3*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6536

```
Int[(x_)^(m_.)*SinhIntegral[(b_.)*(x_)]^2, x_Symbol] :> Simp[(x^(m + 1)*Sin
hIntegral[b*x]^2)/(m + 1), x] - Dist[2/(m + 1), Int[x^m*Sinh[b*x]*SinhInteg
ral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\text{Shi}(bx)^2\,dx = \frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{2}{3}\int x^2\sinh(bx)\text{Shi}(bx)\,dx$$

$$= -\frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b} + \frac{1}{3}x^3\text{Shi}(bx)^2 + \frac{2}{3}\int\frac{x\cosh(bx)\sinh(bx)}{b}\,dx + \frac{4\int x\cosh(bx)\text{Shi}(b}{3b}$$

$$= -\frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b} + \frac{4x\sinh(bx)\text{Shi}(bx)}{3b^2} + \frac{1}{3}x^3\text{Shi}(bx)^2 - \frac{4\int\sinh(bx)\text{Shi}(bx)\,dx}{3b^2} + \frac{2}{}$$

$$= \frac{x\sinh^2(bx)}{3b^2} - \frac{4\cosh(bx)\text{Shi}(bx)}{3b^3} - \frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b} + \frac{4x\sinh(bx)\text{Shi}(bx)}{3b^2} + \frac{1}{3}x^3\text{Shi}(b$$

$$= -\frac{5\cosh(bx)\sinh(bx)}{6b^3} + \frac{x\sinh^2(bx)}{3b^2} - \frac{4\cosh(bx)\text{Shi}(bx)}{3b^3} - \frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b} + \frac{4x\sinh}{}$$

$$= \frac{5x}{6b^2} - \frac{5\cosh(bx)\sinh(bx)}{6b^3} + \frac{x\sinh^2(bx)}{3b^2} - \frac{4\cosh(bx)\text{Shi}(bx)}{3b^3} - \frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b} + \frac{4x}{}$$

$$= \frac{5x}{6b^2} - \frac{5\cosh(bx)\sinh(bx)}{6b^3} + \frac{x\sinh^2(bx)}{3b^2} - \frac{4\cosh(bx)\text{Shi}(bx)}{3b^3} - \frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b} + \frac{4x}{}$$

$$= \frac{5x}{6b^2} - \frac{5\cosh(bx)\sinh(bx)}{6b^3} + \frac{x\sinh^2(bx)}{3b^2} - \frac{4\cosh(bx)\text{Shi}(bx)}{3b^3} - \frac{2x^2\cosh(bx)\text{Shi}(bx)}{3b} + \frac{4x}{}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.70

$$\frac{4b^3x^3\text{Shi}(bx)^2 - 8\text{Shi}(bx)\left(\left(b^2x^2+2\right)\cosh(bx) - 2bx\sinh(bx)\right) + 8\text{Shi}(2bx) + 8bx - 5\sinh(2bx) + 2bx\cosh(2b}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*SinhIntegral[b*x]^2,x]
```

```
[Out] (8*b*x + 2*b*x*Cosh[2*b*x] - 5*Sinh[2*b*x] - 8*((2 + b^2*x^2)*Cosh[b*x] - 2
*b*x*Sinh[b*x])*SinhIntegral[b*x] + 4*b^3*x^3*SinhIntegral[b*x]^2 + 8*SinhI
ntegral[2*b*x])/(12*b^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{Shi}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x)^2, x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.75

$$\frac{\frac{b^3x^3\,\mathrm{Shi}(bx)^2}{3} - 2\,\mathrm{Shi}\,(bx)\left(\frac{b^2x^2\cosh(bx)}{3} - \frac{2bx\sinh(bx)}{3} + \frac{2\cosh(bx)}{3}\right) + \frac{bx\left(\cosh^2(bx)\right)}{3} - \frac{5\sinh(bx)\cosh(bx)}{6} + \frac{bx}{2} + \frac{2\,\mathrm{Shi}(2bx)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Shi(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*Shi(b*x)^2-2*Shi(b*x)*(1/3*b^2*x^2*cosh(b*x)-2/3*b*x*sin
h(b*x)+2/3*cosh(b*x))+1/3*b*x*cosh(b*x)^2-5/6*sinh(b*x)*cosh(b*x)+1/2*b*x+2
/3*Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{sinhint}\,(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(b*x)^2,x)
```

```
[Out] int(x^2*sinhint(b*x)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Shi}^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Shi(b*x)**2,x)
```

```
[Out] Integral(x**2*Shi(b*x)**2, x)
```

3.12 $\int x\mathbf{Shi}(bx)^2\,dx$

Optimal. Leaf size=74

$$-\frac{\text{Chi}(2bx)}{2b^2}+\frac{\text{Shi}(bx)\sinh(bx)}{b^2}+\frac{\log(x)}{2b^2}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

```
[Out] -1/2*Chi(2*b*x)/b^2+1/2*ln(x)/b^2-x*cosh(b*x)*Shi(b*x)/b+1/2*x^2*Shi(b*x)^2
+Shi(b*x)*sinh(b*x)/b^2+1/2*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6536, 6542, 12, 2564, 30, 6546, 3312, 3301}

$$-\frac{\text{Chi}(2bx)}{2b^2}+\frac{\text{Shi}(bx)\sinh(bx)}{b^2}+\frac{\log(x)}{2b^2}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(bx)^2-\frac{x\text{Shi}(bx)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*SinhIntegral[b*x]^2,x]
```

```
[Out] -CoshIntegral[2*b*x]/(2*b^2) + Log[x]/(2*b^2) + Sinh[b*x]^2/(2*b^2) - (x*Co
sh[b*x]*SinhIntegral[b*x])/b + (Sinh[b*x]*SinhIntegral[b*x])/b^2 + (x^2*Sin
hIntegral[b*x]^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6536

```
Int[(x_)^(m_.)*SinhIntegral[(b_.)*(x_)]^2, x_Symbol] :> Simp[(x^(m + 1)*Sin
hIntegral[b*x]^2)/(m + 1), x] - Dist[2/(m + 1), Int[x^m*Sinh[b*x]*SinhInteg
ral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int x\text{Shi}(bx)^2\,dx = \frac{1}{2}x^2\text{Shi}(bx)^2 - \int x\sinh(bx)\text{Shi}(bx)\,dx$$
$$= -\frac{x\cosh(bx)\text{Shi}(bx)}{b} + \frac{1}{2}x^2\text{Shi}(bx)^2 + \frac{\int \cosh(bx)\text{Shi}(bx)\,dx}{b} + \int \frac{\cosh(bx)\sinh(bx)}{b}\,dx$$
$$= -\frac{x\cosh(bx)\text{Shi}(bx)}{b} + \frac{\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{1}{2}x^2\text{Shi}(bx)^2 + \frac{\int \cosh(bx)\sinh(bx)\,dx}{b} - \frac{\int \frac{\sinh^2(bx)}{bx}}{b}$$
$$= -\frac{x\cosh(bx)\text{Shi}(bx)}{b} + \frac{\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{1}{2}x^2\text{Shi}(bx)^2 - \frac{\int \frac{\sinh^2(bx)}{x}\,dx}{b^2} - \frac{\text{Subst}(\int x\,dx, x, i\sin}{b^2}$$
$$= \frac{\sinh^2(bx)}{2b^2} - \frac{x\cosh(bx)\text{Shi}(bx)}{b} + \frac{\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{1}{2}x^2\text{Shi}(bx)^2 + \frac{\int \left(\frac{1}{2x} - \frac{\cosh(2bx)}{2x}\right)dx}{b^2}$$
$$= \frac{\log(x)}{2b^2} + \frac{\sinh^2(bx)}{2b^2} - \frac{x\cosh(bx)\text{Shi}(bx)}{b} + \frac{\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{1}{2}x^2\text{Shi}(bx)^2 - \frac{\int \frac{\cosh(2bx)}{x}\,dx}{2b^2}$$
$$= -\frac{\text{Chi}(2bx)}{2b^2} + \frac{\log(x)}{2b^2} + \frac{\sinh^2(bx)}{2b^2} - \frac{x\cosh(bx)\text{Shi}(bx)}{b} + \frac{\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{1}{2}x^2\text{Shi}(bx)^2$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.78

$$\frac{2b^2x^2\text{Shi}(bx)^2 - 2\text{Chi}(2bx) + \text{Shi}(bx)(4\sinh(bx) - 4bx\cosh(bx)) + \cosh(2bx) + 2\log(x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*SinhIntegral[b*x]^2,x]
```

```
[Out] (Cosh[2*b*x] - 2*CoshIntegral[2*b*x] + 2*Log[x] + (-4*b*x*Cosh[b*x] + 4*Sin
h[b*x])*SinhIntegral[b*x] + 2*b^2*x^2*SinhIntegral[b*x]^2)/(4*b^2)
```

fricas [F] time = 2.43, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Shi}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x*sinh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*Shi(b*x)^2, x)
```

maple [A] time = 0.03, size = 69, normalized size = 0.93

$$\frac{x^2 \operatorname{Shi}(bx)^2}{2} - \frac{x \cosh(bx) \operatorname{Shi}(bx)}{b} + \frac{\operatorname{Shi}(bx) \sinh(bx)}{b^2} + \frac{\cosh^2(bx)}{2b^2} + \frac{\ln(bx)}{2b^2} - \frac{\mathrm{X}(2bx)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(b*x)^2,x)
```

```
[Out] 1/2*x^2*Shi(b*x)^2-x*cosh(b*x)*Shi(b*x)/b+Shi(b*x)*sinh(b*x)/b^2+1/2/b^2*co
sh(b*x)^2+1/2/b^2*ln(b*x)-1/2*Chi(2*b*x)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{Shi}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{sinhint}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(b*x)^2,x)
```

```
[Out] int(x*sinhint(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Shi}^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)**2,x)
```

```
[Out] Integral(x*Shi(b*x)**2, x)
```

3.13 $\int \mathrm{Shi}(bx)^2\, dx$

Optimal. Leaf size=31

$$x\mathrm{Shi}(bx)^2 + \frac{\mathrm{Shi}(2bx)}{b} - \frac{2\mathrm{Shi}(bx)\cosh(bx)}{b}$$

```
[Out] -2*cosh(b*x)*Shi(b*x)/b+x*Shi(b*x)^2+Shi(2*b*x)/b
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6534, 6540, 12, 5448, 3298}

$$x\mathrm{Shi}(bx)^2 + \frac{\mathrm{Shi}(2bx)}{b} - \frac{2\mathrm{Shi}(bx)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[b*x]^2,x]
```

```
[Out] (-2*Cosh[b*x]*SinhIntegral[b*x])/b + x*SinhIntegral[b*x]^2 + SinhIntegral[2
*b*x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6534

```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*SinhIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Sinh[a + b*x]*SinhIntegral[a + b*x],
 x], x] /; FreeQ[{a, b}, x]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Shi}(bx)^2\,dx &= x\mathrm{Shi}(bx)^2 - 2\int \sinh(bx)\mathrm{Shi}(bx)\,dx\\
&= -\frac{2\cosh(bx)\mathrm{Shi}(bx)}{b} + x\mathrm{Shi}(bx)^2 + 2\int \frac{\cosh(bx)\sinh(bx)}{bx}\,dx\\
&= -\frac{2\cosh(bx)\mathrm{Shi}(bx)}{b} + x\mathrm{Shi}(bx)^2 + \frac{2\int \frac{\cosh(bx)\sinh(bx)}{x}\,dx}{b}\\
&= -\frac{2\cosh(bx)\mathrm{Shi}(bx)}{b} + x\mathrm{Shi}(bx)^2 + \frac{2\int \frac{\sinh(2bx)}{2x}\,dx}{b}\\
&= -\frac{2\cosh(bx)\mathrm{Shi}(bx)}{b} + x\mathrm{Shi}(bx)^2 + \frac{\int \frac{\sinh(2bx)}{x}\,dx}{b}\\
&= -\frac{2\cosh(bx)\mathrm{Shi}(bx)}{b} + x\mathrm{Shi}(bx)^2 + \frac{\mathrm{Shi}(2bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$x\mathrm{Shi}(bx)^2 + \frac{\mathrm{Shi}(2bx)}{b} - \frac{2\mathrm{Shi}(bx)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[b*x]^2,x]
```

```
[Out] (-2*Cosh[b*x]*SinhIntegral[b*x])/b + x*SinhIntegral[b*x]^2 + SinhIntegral[2
*b*x]/b
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{Shi}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\,(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)^2, x)
```

maple [A] time = 0.02, size = 30, normalized size = 0.97

$$\frac{bx\,\mathrm{Shi}\,(bx)^2 - 2\cosh\,(bx)\,\mathrm{Shi}\,(bx) + \mathrm{Shi}\,(2bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)^2,x)
```

```
[Out] 1/b*(b*x*Shi(b*x)^2-2*cosh(b*x)*Shi(b*x)+Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\,(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{sinhint}\,(b\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)^2,x)
```

```
[Out] int(sinhint(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}^2\,(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)**2,x)
```

```
[Out] Integral(Shi(b*x)**2, x)
```

3.14 $\int \frac{\textbf{Shi}(bx)^2}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\text{Shi}(bx)^2}{x},x\right)$$

```
[Out] CannotIntegrate(Shi(b*x)^2/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[SinhIntegral[b*x]^2/x,x]
```

```
[Out] Defer[Int][SinhIntegral[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx=\int \frac{\text{Shi}(bx)^2}{x}\,dx$$

Mathematica [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[SinhIntegral[b*x]^2/x,x]
```

```
[Out] Integrate[SinhIntegral[b*x]^2/x, x]
```

fricas [A] time = 2.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\,(bx)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)^2/x, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)^2/x,x)
```

```
[Out] int(Shi(b*x)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)^2/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{sinhint}\,(b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)^2/x,x)
```

```
[Out] int(sinhint(b*x)^2/x, x)
```

<u>**sympy**</u> **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)**2/x,x)
```

```
[Out] Integral(Shi(b*x)**2/x, x)
```

3.15 $\int \frac{\mathbf{Shi}(bx)^2}{x^2}\,dx$

Optimal. Leaf size=13

$$\operatorname{Int}\left(\frac{\operatorname{Shi}(bx)^2}{x^2},x\right)$$

```
[Out] CannotIntegrate(Shi(b*x)^2/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{Shi}(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[SinhIntegral[b*x]^2/x^2,x]
```

```
[Out] Defer[Int][SinhIntegral[b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{Shi}(bx)^2}{x^2}\,dx=\int \frac{\operatorname{Shi}(bx)^2}{x^2}\,dx$$

Mathematica [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[SinhIntegral[b*x]^2/x^2,x]
```

```
[Out] Integrate[SinhIntegral[b*x]^2/x^2, x]
```

fricas [A] time = 1.09, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{Shi}\left(bx\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)^2/x^2, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)^2/x^2,x)
```

```
[Out] int(Shi(b*x)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)^2/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\text{sinhint}(b\,x)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)^2/x^2,x)
```

```
[Out] int(sinhint(b*x)^2/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)**2/x**2,x)
```

```
[Out] Integral(Shi(b*x)**2/x**2, x)
```

3.16 $\int \frac{\mathbf{Shi}(bx)^2}{x^3} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\text{Shi}(bx)^2}{x^3},x\right)$$

```
[Out] CannotIntegrate(Shi(b*x)^2/x^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Shi}(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[SinhIntegral[b*x]^2/x^3,x]
[Out] Defer[Int][SinhIntegral[b*x]^2/x^3, x]
```

Rubi steps

$$\int \frac{\text{Shi}(bx)^2}{x^3} dx = \int \frac{\text{Shi}(bx)^2}{x^3} dx$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[SinhIntegral[b*x]^2/x^3,x]
[Out] Integrate[SinhIntegral[b*x]^2/x^3, x]
```

fricas [A] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\,(bx)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x)^2/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)^2/x^3, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)^2/x^3,x)
```

```
[Out] int(Shi(b*x)^2/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)^2/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{sinhint}\,(b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)^2/x^3,x)
```

```
[Out] int(sinhint(b*x)^2/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)**2/x**3,x)
```

```
[Out] Integral(Shi(b*x)**2/x**3, x)
```

3.17 $\int x^m \mathbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=48

$$\frac{x^{m+1}\mathrm{Shi}(a+bx)}{m+1}-\frac{b\mathrm{Int}\left(\frac{x^{m+1}\sinh(a+bx)}{a+bx},x\right)}{m+1}$$

```
[Out] -b*CannotIntegrate(x^(1+m)*sinh(b*x+a)/(b*x+a),x)/(1+m)+x^(1+m)*Shi(b*x+a)/
(1+m)
```

Rubi [A] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \mathrm{Shi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*SinhIntegral[a + b*x],x]
```

```
[Out] (x^(1 + m)*SinhIntegral[a + b*x])/(1 + m) - (b*Defer[Int][(x^(1 + m)*Sinh[a
 + b*x])/(a + b*x), x])/(1 + m)
```

Rubi steps

$$\int x^m \mathrm{Shi}(a+bx)\,dx=\frac{x^{1+m}\mathrm{Shi}(a+bx)}{1+m}-\frac{b\int\frac{x^{1+m}\sinh(a+bx)}{a+bx}\,dx}{1+m}$$

Mathematica [A] time = 10.31, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Shi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*SinhIntegral[a + b*x],x]
```

```
[Out] Integrate[x^m*SinhIntegral[a + b*x], x]
```

fricas [A] time = 3.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\,\mathrm{Shi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*sinh_integral(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Shi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*Shi(b*x + a), x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int x^m \,\mathrm{Shi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Shi(b*x+a),x)
```

```
[Out] int(x^m*Shi(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Shi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*Shi(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \,\mathrm{sinhint}\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinhint(a + b*x),x)
```

```
[Out] int(x^m*sinhint(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \,\mathrm{Shi}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Shi(b*x+a),x)
```

```
[Out] Integral(x**m*Shi(a + b*x), x)
```

3.18 $\int x^3 \mathbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=184

$$-\frac{a^4\text{Shi}(a+bx)}{4b^4}+\frac{a^3\cosh(a+bx)}{4b^4}+\frac{a^2\sinh(a+bx)}{4b^4}-\frac{a^2x\cosh(a+bx)}{4b^3}+\frac{3\sinh(a+bx)}{2b^4}+\frac{a\cosh(a+bx)}{2b^4}-\frac{ax\sinh}{}$$

```
[Out] 1/2*a*cosh(b*x+a)/b^4+1/4*a^3*cosh(b*x+a)/b^4-3/2*x*cosh(b*x+a)/b^3-1/4*a^2
*x*cosh(b*x+a)/b^3+1/4*a*x^2*cosh(b*x+a)/b^2-1/4*x^3*cosh(b*x+a)/b-1/4*a^4*
Shi(b*x+a)/b^4+1/4*x^4*Shi(b*x+a)+3/2*sinh(b*x+a)/b^4+1/4*a^2*sinh(b*x+a)/b
^4-1/2*a*x*sinh(b*x+a)/b^3+3/4*x^2*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.38, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6532, 6742, 2638, 3296, 2637, 3298}

$$-\frac{a^4\text{Shi}(a+bx)}{4b^4}+\frac{a^2\sinh(a+bx)}{4b^4}+\frac{a^3\cosh(a+bx)}{4b^4}-\frac{a^2x\cosh(a+bx)}{4b^3}+\frac{3x^2\sinh(a+bx)}{4b^2}+\frac{ax^2\cosh(a+bx)}{4b^2}-\frac{ax}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*SinhIntegral[a + b*x],x]
```

```
[Out] (a*Cosh[a + b*x])/(2*b^4) + (a^3*Cosh[a + b*x])/(4*b^4) - (3*x*Cosh[a + b*x
])/(2*b^3) - (a^2*x*Cosh[a + b*x])/(4*b^3) + (a*x^2*Cosh[a + b*x])/(4*b^2)
- (x^3*Cosh[a + b*x])/(4*b) + (3*Sinh[a + b*x])/(2*b^4) + (a^2*Sinh[a + b*x
])/(4*b^4) - (a*x*Sinh[a + b*x])/(2*b^3) + (3*x^2*Sinh[a + b*x])/(4*b^2) -
(a^4*SinhIntegral[a + b*x])/(4*b^4) + (x^4*SinhIntegral[a + b*x])/4
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^3\text{Shi}(a+bx)\,dx &= \frac{1}{4}x^4\text{Shi}(a+bx)-\frac{1}{4}b\int\frac{x^4\sinh(a+bx)}{a+bx}\,dx\\ &= \frac{1}{4}x^4\text{Shi}(a+bx)-\frac{1}{4}b\int\left(-\frac{a^3\sinh(a+bx)}{b^4}+\frac{a^2x\sinh(a+bx)}{b^3}-\frac{ax^2\sinh(a+bx)}{b^2}+\frac{x^3\sin\right.\\ &= \frac{1}{4}x^4\text{Shi}(a+bx)-\frac{1}{4}\int x^3\sinh(a+bx)\,dx+\frac{a^3\int\sinh(a+bx)\,dx}{4b^3}-\frac{a^4\int\frac{\sinh(a+bx)}{a+bx}\,dx}{4b^3}-\frac{a^2\\ &= \frac{a^3\cosh(a+bx)}{4b^4}-\frac{a^2x\cosh(a+bx)}{4b^3}+\frac{ax^2\cosh(a+bx)}{4b^2}-\frac{x^3\cosh(a+bx)}{4b}-\frac{a^4\text{Shi}(a+bx}{4b^4}\\ &= \frac{a^3\cosh(a+bx)}{4b^4}-\frac{a^2x\cosh(a+bx)}{4b^3}+\frac{ax^2\cosh(a+bx)}{4b^2}-\frac{x^3\cosh(a+bx)}{4b}+\frac{a^2\sinh(a+}{4b^4}\\ &= \frac{a\cosh(a+bx)}{2b^4}+\frac{a^3\cosh(a+bx)}{4b^4}-\frac{3x\cosh(a+bx)}{2b^3}-\frac{a^2x\cosh(a+bx)}{4b^3}+\frac{ax^2\cosh(a+b}{4b^2}\\ &= \frac{a\cosh(a+bx)}{2b^4}+\frac{a^3\cosh(a+bx)}{4b^4}-\frac{3x\cosh(a+bx)}{2b^3}-\frac{a^2x\cosh(a+bx)}{4b^3}+\frac{ax^2\cosh(a+b}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.20, size = 94, normalized size = 0.51

$$\frac{\left(b^4x^4-a^4\right)\text{Shi}(a+bx)+\left(a^2-2abx+3b^2x^2+6\right)\sinh(a+bx)+\left(a^3-a^2bx+ab^2x^2+2a-b^3x^3-6bx\right)\cosh(a}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*SinhIntegral[a + b*x],x]
```

```
[Out] ((2*a + a^3 - 6*b*x - a^2*b*x + a*b^2*x^2 - b^3*x^3)*Cosh[a + b*x] + (6 + a
^2 - 2*a*b*x + 3*b^2*x^2)*Sinh[a + b*x] + (-a^4 + b^4*x^4)*SinhIntegral[a +
 b*x])/(4*b^4)
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\,\text{Shi}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^3*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Shi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*Shi(b*x + a), x)
```

maple [A] time = 0.02, size = 156, normalized size = 0.85

$$\frac{\frac{\text{Shi}(bx+a)b^4x^4}{4}-\frac{(bx+a)^3\cosh(bx+a)}{4}+\frac{3(bx+a)^2\sinh(bx+a)}{4}-\frac{3(bx+a)\cosh(bx+a)}{2}+\frac{3\sinh(bx+a)}{2}+a\left((bx+a)^2\cosh(bx+a)-2\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Shi(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*Shi(b*x+a)*b^4*x^4-1/4*(b*x+a)^3*cosh(b*x+a)+3/4*(b*x+a)^2*sinh(
b*x+a)-3/2*(b*x+a)*cosh(b*x+a)+3/2*sinh(b*x+a)+a*((b*x+a)^2*cosh(b*x+a)-2*(
b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))-3/2*a^2*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))
+a^3*cosh(b*x+a)-1/4*a^4*Shi(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Shi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*Shi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{sinhint}(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinhint(a + b*x),x)
```

```
[Out] int(x^3*sinhint(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{Shi}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Shi(b*x+a),x)
```

```
[Out] Integral(x**3*Shi(a + b*x), x)
```

3.19 $\int x^2\mathbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=118

$$\frac{a^3\text{Shi}(a+bx)}{3b^3}-\frac{a^2\cosh(a+bx)}{3b^3}-\frac{a\sinh(a+bx)}{3b^3}-\frac{2\cosh(a+bx)}{3b^3}+\frac{2x\sinh(a+bx)}{3b^2}+\frac{ax\cosh(a+bx)}{3b^2}+\frac{1}{3}x^3\text{Shi}(a$$

```
[Out] -2/3*cosh(b*x+a)/b^3-1/3*a^2*cosh(b*x+a)/b^3+1/3*a*x*cosh(b*x+a)/b^2-1/3*x^
2*cosh(b*x+a)/b+1/3*a^3*Shi(b*x+a)/b^3+1/3*x^3*Shi(b*x+a)-1/3*a*sinh(b*x+a)
/b^3+2/3*x*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6532, 6742, 2638, 3296, 2637, 3298}

$$\frac{a^3\text{Shi}(a+bx)}{3b^3}-\frac{a^2\cosh(a+bx)}{3b^3}-\frac{a\sinh(a+bx)}{3b^3}+\frac{2x\sinh(a+bx)}{3b^2}+\frac{ax\cosh(a+bx)}{3b^2}-\frac{2\cosh(a+bx)}{3b^3}+\frac{1}{3}x^3\text{Shi}(a$$

Antiderivative was successfully verified.

```
[In] Int[x^2*SinhIntegral[a + b*x],x]
```

```
[Out] (-2*Cosh[a + b*x])/(3*b^3) - (a^2*Cosh[a + b*x])/(3*b^3) + (a*x*Cosh[a + b*
x])/(3*b^2) - (x^2*Cosh[a + b*x])/(3*b) - (a*Sinh[a + b*x])/(3*b^3) + (2*x*
Sinh[a + b*x])/(3*b^2) + (a^3*SinhIntegral[a + b*x])/(3*b^3) + (x^3*SinhInt
egral[a + b*x])/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x^2\text{Shi}(a+bx)\,dx &= \frac{1}{3}x^3\text{Shi}(a+bx)-\frac{1}{3}b\int\frac{x^3\sinh(a+bx)}{a+bx}\,dx\\
&= \frac{1}{3}x^3\text{Shi}(a+bx)-\frac{1}{3}b\int\left(\frac{a^2\sinh(a+bx)}{b^3}-\frac{ax\sinh(a+bx)}{b^2}+\frac{x^2\sinh(a+bx)}{b}-\frac{a^3\sinh(a}{b^3(a+}\right.\\
&= \frac{1}{3}x^3\text{Shi}(a+bx)-\frac{1}{3}\int x^2\sinh(a+bx)\,dx-\frac{a^2\int\sinh(a+bx)\,dx}{3b^2}+\frac{a^3\int\frac{\sinh(a+bx)}{a+bx}\,dx}{3b^2}+\frac{a}{}\\
&= -\frac{a^2\cosh(a+bx)}{3b^3}+\frac{ax\cosh(a+bx)}{3b^2}-\frac{x^2\cosh(a+bx)}{3b}+\frac{a^3\text{Shi}(a+bx)}{3b^3}+\frac{1}{3}x^3\text{Shi}(a+bx\\
&= -\frac{a^2\cosh(a+bx)}{3b^3}+\frac{ax\cosh(a+bx)}{3b^2}-\frac{x^2\cosh(a+bx)}{3b}-\frac{a\sinh(a+bx)}{3b^3}+\frac{2x\sinh(a+b}{3b^2}\\
&= -\frac{2\cosh(a+bx)}{3b^3}-\frac{a^2\cosh(a+bx)}{3b^3}+\frac{ax\cosh(a+bx)}{3b^2}-\frac{x^2\cosh(a+bx)}{3b}-\frac{a\sinh(a+bx}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 64, normalized size = 0.54

$$-\frac{-\left(a^3+b^3x^3\right)\text{Shi}(a+bx)+\left(a^2-abx+b^2x^2+2\right)\cosh(a+bx)+(a-2bx)\sinh(a+bx)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*SinhIntegral[a + b*x],x]
```

```
[Out] -1/3*((2 + a^2 - a*b*x + b^2*x^2)*Cosh[a + b*x] + (a - 2*b*x)*Sinh[a + b*x]
 - (a^3 + b^3*x^3)*SinhIntegral[a + b*x])/b^3
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2 \,\mathrm{Shi}\,(bx + a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Shi}\,(bx + a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x + a), x)
```

maple [A] time = 0.02, size = 101, normalized size = 0.86

$$\frac{\frac{b^3x^3\,\mathrm{Shi}(bx+a)}{3} - \frac{(bx+a)^2\cosh(bx+a)}{3} + \frac{2(bx+a)\sinh(bx+a)}{3} - \frac{2\cosh(bx+a)}{3} + a\left((bx+a)\cosh(bx+a) - \sinh(bx+a)\right) - a^2 \mathrm{c}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Shi(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*Shi(b*x+a)-1/3*(b*x+a)^2*cosh(b*x+a)+2/3*(b*x+a)*sinh(b*
x+a)-2/3*cosh(b*x+a)+a*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-a^2*cosh(b*x+a)+1/
3*a^3*Shi(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Shi}\,(bx + a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \,\mathrm{sinhint}\,(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(a + b*x),x)
```

```
[Out] int(x^2*sinhint(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Shi}(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Shi(b*x+a),x)
```

```
[Out] Integral(x**2*Shi(a + b*x), x)
```

3.20 $\int x\mathbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=71

$$-\frac{a^2\text{Shi}(a+bx)}{2b^2}+\frac{\sinh(a+bx)}{2b^2}+\frac{a\cosh(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(a+bx)-\frac{x\cosh(a+bx)}{2b}$$

```
[Out] 1/2*a*cosh(b*x+a)/b^2-1/2*x*cosh(b*x+a)/b-1/2*a^2*Shi(b*x+a)/b^2+1/2*x^2*Sh
i(b*x+a)+1/2*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.21, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6532, 6742, 2638, 3296, 2637, 3298}

$$-\frac{a^2\text{Shi}(a+bx)}{2b^2}+\frac{\sinh(a+bx)}{2b^2}+\frac{a\cosh(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(a+bx)-\frac{x\cosh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*SinhIntegral[a + b*x],x]
```

```
[Out] (a*Cosh[a + b*x])/(2*b^2) - (x*Cosh[a + b*x])/(2*b) + Sinh[a + b*x]/(2*b^2)
 - (a^2*SinhIntegral[a + b*x])/(2*b^2) + (x^2*SinhIntegral[a + b*x])/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x\text{Shi}(a+bx)\,dx &= \frac{1}{2}x^2\text{Shi}(a+bx)-\frac{1}{2}b\int\frac{x^2\sinh(a+bx)}{a+bx}\,dx\\
&= \frac{1}{2}x^2\text{Shi}(a+bx)-\frac{1}{2}b\int\left(-\frac{a\sinh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}+\frac{a^2\sinh(a+bx)}{b^2(a+bx)}\right)dx\\
&= \frac{1}{2}x^2\text{Shi}(a+bx)-\frac{1}{2}\int x\sinh(a+bx)\,dx+\frac{a\int\sinh(a+bx)\,dx}{2b}-\frac{a^2\int\frac{\sinh(a+bx)}{a+bx}\,dx}{2b}\\
&= \frac{a\cosh(a+bx)}{2b^2}-\frac{x\cosh(a+bx)}{2b}-\frac{a^2\text{Shi}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(a+bx)+\frac{\int\cosh(a+bx)\,dx}{2b}\\
&= \frac{a\cosh(a+bx)}{2b^2}-\frac{x\cosh(a+bx)}{2b}+\frac{\sinh(a+bx)}{2b^2}-\frac{a^2\text{Shi}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Shi}(a+bx)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 47, normalized size = 0.66

$$\frac{(b^2x^2-a^2)\,\text{Shi}(a+bx)+\sinh(a+bx)+(a-bx)\cosh(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*SinhIntegral[a + b*x],x]
```

```
[Out] ((a - b*x)*Cosh[a + b*x] + Sinh[a + b*x] + (-a^2 + b^2*x^2)*SinhIntegral[a
+ b*x])/(2*b^2)
```

fricas [F] time = 2.23, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Shi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*Shi(b*x + a), x)
```

maple [A] time = 0.02, size = 60, normalized size = 0.85

$$\frac{\mathrm{Shi}\left(bx+a\right)\left(\frac{(bx+a)^2}{2}-a\left(bx+a\right)\right)-\frac{(bx+a)\cosh(bx+a)}{2}+\frac{\sinh(bx+a)}{2}+a\cosh\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(b*x+a),x)
```

```
[Out] 1/b^2*(Shi(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))-1/2*(b*x+a)*cosh(b*x+a)+1/2*sin
h(b*x+a)+a*cosh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\frac{\frac{\mathrm{e}^{-a-b\,x}\,\left(a+\mathrm{e}^{2\,a+2\,b\,x}+a\,\mathrm{e}^{2\,a+2\,b\,x}-2\,a^2\,\mathrm{sinhint}(a+b\,x)\,\mathrm{e}^{a+b\,x}-1\right)}{4}-\frac{b\,\mathrm{e}^{-a-b\,x}\,\left(x+x\,\mathrm{e}^{2\,a+2\,b\,x}\right)}{4}}{b^2}+\frac{x^2\,\mathrm{sinhint}\left(a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(a + b*x),x)
```

```
[Out] ((exp(- a - b*x)*(a + exp(2*a + 2*b*x) + a*exp(2*a + 2*b*x) - 2*a^2*sinhint
(a + b*x)*exp(a + b*x) - 1))/4 - (b*exp(- a - b*x)*(x + x*exp(2*a + 2*b*x))
)/4)/b^2 + (x^2*sinhint(a + b*x))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Shi}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a),x)
```

```
[Out] Integral(x*Shi(a + b*x), x)
```

3.21 $\int \text{Shi}(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{(a+bx)\text{Shi}(a+bx)}{b}-\frac{\cosh(a+bx)}{b}$$

```
[Out] -cosh(b*x+a)/b+(b*x+a)*Shi(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6528}

$$\frac{(a+bx)\text{Shi}(a+bx)}{b}-\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[a + b*x],x]
```

```
[Out] -(Cosh[a + b*x]/b) + ((a + b*x)*SinhIntegral[a + b*x])/b
```

Rule 6528

```
Int[SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*SinhInte
gral[a + b*x])/b, x] - Simp[Cosh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{Shi}(a+bx)\,dx=-\frac{\cosh(a+bx)}{b}+\frac{(a+bx)\text{Shi}(a+bx)}{b}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.56

$$x\text{Shi}(a+bx)+\frac{a\text{Shi}(a+bx)}{b}-\frac{\sinh(a)\sinh(bx)}{b}-\frac{\cosh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[a + b*x],x]
```

```
[Out] -((Cosh[a]*Cosh[b*x])/b) - (Sinh[a]*Sinh[b*x])/b + (a*SinhIntegral[a + b*x]
)/b + x*SinhIntegral[a + b*x]
```

fricas [F] time = 2.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{Shi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Shi}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x + a), x)
```

maple [A] time = 0.01, size = 26, normalized size = 0.96

$$\frac{(bx + a)\operatorname{Shi}(bx + a) - \cosh(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*Shi(b*x+a)-cosh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Shi}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$x\operatorname{sinhint}(a + b\,x) - \frac{e^{a+b\,x}}{2\,b} - \frac{e^{-a-b\,x}}{2\,b} + \frac{a\operatorname{sinhint}(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x),x)
```

```
[Out] x*sinhint(a + b*x) - exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) + (a*sinhint
(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a),x)
```

```
[Out] Integral(Shi(a + b*x), x)
```

3.22 $\int \frac{\mathbf{Shi}(a+bx)}{x} dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{\mathrm{Shi}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(Shi(b*x+a)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[SinhIntegral[a + b*x]/x,x]
```

```
[Out] Defer[Int][SinhIntegral[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx=\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[SinhIntegral[a + b*x]/x,x]
```

```
[Out] Integrate[SinhIntegral[a + b*x]/x, x]
```

fricas [A] time = 2.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Shi}\,(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x + a)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)/x,x)
```

```
[Out] int(Shi(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\text{sinhint}\,(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)/x,x)
```

```
[Out] int(sinhint(a + b*x)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x,x)
```

```
[Out] Integral(Shi(a + b*x)/x, x)
```

3.23 $\int \frac{\text{Shi}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=46

$$\frac{b\sinh(a)\text{Chi}(bx)}{a}-\frac{b\text{Shi}(a+bx)}{a}-\frac{\text{Shi}(a+bx)}{x}+\frac{b\cosh(a)\text{Shi}(bx)}{a}$$

```
[Out] b*cosh(a)*Shi(b*x)/a-b*Shi(b*x+a)/a-Shi(b*x+a)/x+b*Chi(b*x)*sinh(a)/a
```

Rubi [A] time = 0.23, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6532, 6742, 3303, 3298, 3301}

$$\frac{b\sinh(a)\text{Chi}(bx)}{a}-\frac{b\text{Shi}(a+bx)}{a}-\frac{\text{Shi}(a+bx)}{x}+\frac{b\cosh(a)\text{Shi}(bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[a + b*x]/x^2,x]
```

```
[Out] (b*CoshIntegral[b*x]*Sinh[a])/a + (b*Cosh[a]*SinhIntegral[b*x])/a - (b*Sinh
Integral[a + b*x])/a - SinhIntegral[a + b*x]/x
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
```

```
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\mathrm{Shi}(a+bx)}{x^2}\,dx &= -\frac{\mathrm{Shi}(a+bx)}{x}+b\int \frac{\sinh(a+bx)}{x(a+bx)}\,dx\\
&= -\frac{\mathrm{Shi}(a+bx)}{x}+b\int \left(\frac{\sinh(a+bx)}{ax}-\frac{b\sinh(a+bx)}{a(a+bx)}\right)dx\\
&= -\frac{\mathrm{Shi}(a+bx)}{x}+\frac{b\int \frac{\sinh(a+bx)}{x}\,dx}{a}-\frac{b^2\int \frac{\sinh(a+bx)}{a+bx}\,dx}{a}\\
&= -\frac{b\mathrm{Shi}(a+bx)}{a}-\frac{\mathrm{Shi}(a+bx)}{x}+\frac{(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx}{a}+\frac{(b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx}{a}\\
&= \frac{b\mathrm{Chi}(bx)\sinh(a)}{a}+\frac{b\cosh(a)\mathrm{Shi}(bx)}{a}-\frac{b\mathrm{Shi}(a+bx)}{a}-\frac{\mathrm{Shi}(a+bx)}{x}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 39, normalized size = 0.85

$$\frac{bx\sinh(a)\mathrm{Chi}(bx)-(a+bx)\mathrm{Shi}(a+bx)+bx\cosh(a)\mathrm{Shi}(bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[a + b*x]/x^2,x]
```

```
[Out] (b*x*CoshIntegral[b*x]*Sinh[a] + b*x*Cosh[a]*SinhIntegral[b*x] - (a + b*x)*
SinhIntegral[a + b*x])/(a*x)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Shi}\,(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\left(bx + a\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x + a)/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\left(bx + a\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)/x^2,x)
```

```
[Out] int(Shi(b*x+a)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\left(bx + a\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{sinhint}\left(a + b\,x\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)/x^2,x)
```

```
[Out] int(sinhint(a + b*x)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\left(a + bx\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x**2,x)
```

```
[Out] Integral(Shi(a + b*x)/x**2, x)
```

3.24 $\int \frac{\mathbf{Shi}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{b^2\sinh(a)\mathrm{Chi}(bx)}{2a^2}+\frac{b^2\mathrm{Shi}(a+bx)}{2a^2}-\frac{b^2\cosh(a)\mathrm{Shi}(bx)}{2a^2}+\frac{b^2\cosh(a)\mathrm{Chi}(bx)}{2a}+\frac{b^2\sinh(a)\mathrm{Shi}(bx)}{2a}-\frac{\mathrm{Shi}(a+bx)}{2x^2}-$$

```
[Out] 1/2*b^2*Chi(b*x)*cosh(a)/a-1/2*b^2*cosh(a)*Shi(b*x)/a^2+1/2*b^2*Shi(b*x+a)/
a^2-1/2*Shi(b*x+a)/x^2-1/2*b^2*Chi(b*x)*sinh(a)/a^2+1/2*b^2*Shi(b*x)*sinh(a
)/a-1/2*b*sinh(b*x+a)/a/x
```

Rubi [A] time = 0.35, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6532, 6742, 3297, 3303, 3298, 3301}

$$-\frac{b^2\sinh(a)\mathrm{Chi}(bx)}{2a^2}+\frac{b^2\mathrm{Shi}(a+bx)}{2a^2}-\frac{b^2\cosh(a)\mathrm{Shi}(bx)}{2a^2}+\frac{b^2\cosh(a)\mathrm{Chi}(bx)}{2a}+\frac{b^2\sinh(a)\mathrm{Shi}(bx)}{2a}-\frac{\mathrm{Shi}(a+bx)}{2x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[a + b*x]/x^3,x]
```

```
[Out] (b^2*Cosh[a]*CoshIntegral[b*x])/(2*a) - (b^2*CoshIntegral[b*x]*Sinh[a])/(2*
a^2) - (b*Sinh[a + b*x])/(2*a*x) - (b^2*Cosh[a]*SinhIntegral[b*x])/(2*a^2)
+ (b^2*Sinh[a]*SinhIntegral[b*x])/(2*a) + (b^2*SinhIntegral[a + b*x])/(2*a^
2) - SinhIntegral[a + b*x]/(2*x^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6532

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*SinhIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Sinh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\text{Shi}(a+bx)}{x^3}\,dx = -\frac{\text{Shi}(a+bx)}{2x^2} + \frac{1}{2}b\int \frac{\sinh(a+bx)}{x^2(a+bx)}\,dx$$
$$= -\frac{\text{Shi}(a+bx)}{2x^2} + \frac{1}{2}b\int \left(\frac{\sinh(a+bx)}{ax^2} - \frac{b\sinh(a+bx)}{a^2x} + \frac{b^2\sinh(a+bx)}{a^2(a+bx)}\right)dx$$
$$= -\frac{\text{Shi}(a+bx)}{2x^2} + \frac{b\int \frac{\sinh(a+bx)}{x^2}\,dx}{2a} - \frac{b^2\int \frac{\sinh(a+bx)}{x}\,dx}{2a^2} + \frac{b^3\int \frac{\sinh(a+bx)}{a+bx}\,dx}{2a^2}$$
$$= -\frac{b\sinh(a+bx)}{2ax} + \frac{b^2\text{Shi}(a+bx)}{2a^2} - \frac{\text{Shi}(a+bx)}{2x^2} + \frac{b^2\int \frac{\cosh(a+bx)}{x}\,dx}{2a} - \frac{\left(b^2\cosh(a)\right)\int \frac{\sinh(bx}{x}$$
$$= -\frac{b^2\text{Chi}(bx)\sinh(a)}{2a^2} - \frac{b\sinh(a+bx)}{2ax} - \frac{b^2\cosh(a)\text{Shi}(bx)}{2a^2} + \frac{b^2\text{Shi}(a+bx)}{2a^2} - \frac{\text{Shi}(a+bx)}{2x^2} +$$
$$= \frac{b^2\cosh(a)\text{Chi}(bx)}{2a} - \frac{b^2\text{Chi}(bx)\sinh(a)}{2a^2} - \frac{b\sinh(a+bx)}{2ax} - \frac{b^2\cosh(a)\text{Shi}(bx)}{2a^2} + \frac{b^2\sinh(a)}{2a}$$

Mathematica [A] time = 0.28, size = 86, normalized size = 0.77

$$\frac{a^2(-\text{Shi}(a+bx)) + b^2x^2(a\cosh(a)-\sinh(a))\text{Chi}(bx) + b^2x^2\text{Shi}(a+bx) + b^2x^2(a\sinh(a)-\cosh(a))\text{Shi}(bx) - ab}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[a + b*x]/x^3,x]
```

```
[Out] (b^2*x^2*CoshIntegral[b*x]*(a*Cosh[a] - Sinh[a]) - a*b*x*Sinh[a + b*x] + b^
2*x^2*(-Cosh[a] + a*Sinh[a])*SinhIntegral[b*x] - a^2*SinhIntegral[a + b*x]
+ b^2*x^2*SinhIntegral[a + b*x])/(2*a^2*x^2)
```

fricas [F] time = 2.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\,(bx+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x + a)/x^3, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)/x^3,x)
```

```
[Out] int(Shi(b*x+a)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{sinhint}(a + b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)/x^3,x)
```

```
[Out] int(sinhint(a + b*x)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}(a + bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)/x**3,x)
```

```
[Out] Integral(Shi(a + b*x)/x**3, x)
```

3.25 $\int x^m \mathbf{Shi}(a+bx)^2\,dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(x^m \operatorname{Shi}(a+bx)^2, x\right)$$

```
[Out] CannotIntegrate(x^m*Shi(b*x+a)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{Shi}(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*SinhIntegral[a + b*x]^2,x]
[Out] Defer[Int][x^m*SinhIntegral[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \operatorname{Shi}(a+bx)^2\,dx = \int x^m \operatorname{Shi}(a+bx)^2\,dx$$

Mathematica [A] time = 5.28, size = 0, normalized size = 0.00

$$\int x^m \operatorname{Shi}(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*SinhIntegral[a + b*x]^2,x]
[Out] Integrate[x^m*SinhIntegral[a + b*x]^2, x]
```

fricas [A] time = 2.15, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^m \operatorname{Shi}\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*sinh_integral(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Shi}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*Shi(b*x + a)^2, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int x^m\,\mathrm{Shi}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Shi(b*x+a)^2,x)
[Out] int(x^m*Shi(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Shi}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Shi(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*Shi(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int x^m\,\mathrm{sinhint}\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinhint(a + b*x)^2,x)
[Out] int(x^m*sinhint(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\,\mathrm{Shi}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Shi(b*x+a)**2,x)
[Out] Integral(x**m*Shi(a + b*x)**2, x)
```

3.26 $\int x^2\mathbf{Shi}(a+bx)^2\,dx$

Optimal. Leaf size=328

$$\frac{a^2(a+bx)\mathrm{Shi}(a+bx)^2}{3b^3}+\frac{a^2\mathrm{Shi}(2a+2bx)}{b^3}-\frac{2a^2\mathrm{Shi}(a+bx)\cosh(a+bx)}{3b^3}+\frac{a\mathrm{Chi}(2a+2bx)}{b^3}+\frac{2\mathrm{Shi}(2a+2bx)}{3b^3}-\frac{2a}{}$$

```
[Out] 2/3*x/b^2+a*Chi(2*b*x+2*a)/b^3-1/3*a*cosh(2*b*x+2*a)/b^3+1/6*x*cosh(2*b*x+2
*a)/b^2-a*ln(b*x+a)/b^3-4/3*cosh(b*x+a)*Shi(b*x+a)/b^3-2/3*a^2*cosh(b*x+a)*
Shi(b*x+a)/b^3+2/3*a*x*cosh(b*x+a)*Shi(b*x+a)/b^2-2/3*x^2*cosh(b*x+a)*Shi(b
*x+a)/b+1/3*a^2*(b*x+a)*Shi(b*x+a)^2/b^3-1/3*a*x*(b*x+a)*Shi(b*x+a)^2/b^2+1
/3*x^2*(b*x+a)*Shi(b*x+a)^2/b+2/3*Shi(2*b*x+2*a)/b^3+a^2*Shi(2*b*x+2*a)/b^3
-2/3*cosh(b*x+a)*sinh(b*x+a)/b^3-2/3*a*Shi(b*x+a)*sinh(b*x+a)/b^3+4/3*x*Shi
(b*x+a)*sinh(b*x+a)/b^2-1/12*sinh(2*b*x+2*a)/b^3
```

Rubi [A] time = 1.49, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 19, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.583, Rules used = {6538, 6542, 5617, 6741, 6742, 2638, 3296, 2637, 3298, 6548, 2635, 8, 3312, 3301, 6540, 5448, 12, 6546, 6534}

$$\frac{a^2(a+bx)\mathrm{Shi}(a+bx)^2}{3b^3}+\frac{a^2\mathrm{Shi}(2a+2bx)}{b^3}-\frac{2a^2\mathrm{Shi}(a+bx)\cosh(a+bx)}{3b^3}+\frac{a\mathrm{Chi}(2a+2bx)}{b^3}-\frac{ax(a+bx)\mathrm{Shi}(a+b}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*SinhIntegral[a + b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2) - (a*Cosh[2*a + 2*b*x])/(3*b^3) + (x*Cosh[2*a + 2*b*x])/(6*b^
2) + (a*CoshIntegral[2*a + 2*b*x])/b^3 - (a*Log[a + b*x])/b^3 - (2*Cosh[a +
 b*x]*Sinh[a + b*x])/(3*b^3) - Sinh[2*a + 2*b*x]/(12*b^3) - (4*Cosh[a + b*x
]*SinhIntegral[a + b*x])/(3*b^3) - (2*a^2*Cosh[a + b*x]*SinhIntegral[a + b*
x])/(3*b^3) + (2*a*x*Cosh[a + b*x]*SinhIntegral[a + b*x])/(3*b^2) - (2*x^2*
Cosh[a + b*x]*SinhIntegral[a + b*x])/(3*b) - (2*a*Sinh[a + b*x]*SinhIntegra
l[a + b*x])/(3*b^3) + (4*x*Sinh[a + b*x]*SinhIntegral[a + b*x])/(3*b^2) + (
a^2*(a + b*x)*SinhIntegral[a + b*x]^2)/(3*b^3) - (a*x*(a + b*x)*SinhIntegra
l[a + b*x]^2)/(3*b^2) + (x^2*(a + b*x)*SinhIntegral[a + b*x]^2)/(3*b) + (2*
SinhIntegral[2*a + 2*b*x])/(3*b^3) + (a^2*SinhIntegral[2*a + 2*b*x])/b^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6534

```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*SinhIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Sinh[a + b*x]*SinhIntegral[a + b*x],
 x], x] /; FreeQ[{a, b}, x]
```

Rule 6538

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_) + (b_.)*(x_)]^2, x_Symbol]
 :> Simp[((a + b*x)*(c + d*x)^m*SinhIntegral[a + b*x]^2)/(b*(m + 1)), x] +
(-Dist[2/(m + 1), Int[(c + d*x)^m*Sinh[a + b*x]*SinhIntegral[a + b*x], x],
x] + Dist[((b*c - a*d)*m)/(b*(m + 1)), Int[(c + d*x)^(m - 1)*SinhIntegral[a
 + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\text{Shi}(a+bx)^2\,dx = \frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b} - \frac{2}{3}\int x^2\sinh(a+bx)\text{Shi}(a+bx)\,dx - \frac{(2a)\int x\text{Shi}(a+bx)^2\,dx}{3b}$$

$$= -\frac{2x^2\cosh(a+bx)\text{Shi}(a+bx)}{3b} - \frac{ax(a+bx)\text{Shi}(a+bx)^2}{3b^2} + \frac{x^2(a+bx)\text{Shi}(a+bx)^2}{3b} + \frac{2}{3}\int$$

$$= \frac{2ax\cosh(a+bx)\text{Shi}(a+bx)}{3b^2} - \frac{2x^2\cosh(a+bx)\text{Shi}(a+bx)}{3b} + \frac{4x\sinh(a+bx)\text{Shi}(a+b}{3b^2}$$

$$= -\frac{4\cosh(a+bx)\text{Shi}(a+bx)}{3b^3} - \frac{2a^2\cosh(a+bx)\text{Shi}(a+bx)}{3b^3} + \frac{2ax\cosh(a+bx)\text{Shi}(a+b}{3b^2}$$

$$= -\frac{4\cosh(a+bx)\text{Shi}(a+bx)}{3b^3} - \frac{2a^2\cosh(a+bx)\text{Shi}(a+bx)}{3b^3} + \frac{2ax\cosh(a+bx)\text{Shi}(a+b}{3b^2}$$

$$= -\frac{a\log(a+bx)}{3b^3} - \frac{2\cosh(a+bx)\sinh(a+bx)}{3b^3} - \frac{4\cosh(a+bx)\text{Shi}(a+bx)}{3b^3} - \frac{2a^2\cosh(a}{}$$

$$= \frac{2x}{3b^2} - \frac{a\cosh(2a+2bx)}{6b^3} + \frac{x\cosh(2a+2bx)}{6b^2} + \frac{a\text{Chi}(2a+2bx)}{3b^3} - \frac{a\log(a+bx)}{b^3} - \frac{2\cosh}{}$$

$$= \frac{2x}{3b^2} - \frac{a\cosh(2a+2bx)}{3b^3} + \frac{x\cosh(2a+2bx)}{6b^2} + \frac{a\text{Chi}(2a+2bx)}{b^3} - \frac{a\log(a+bx)}{b^3} - \frac{2\cosh}{}$$

Mathematica [A] time = 1.38, size = 158, normalized size = 0.48

$$\frac{4\left(a^3+b^3x^3\right)\mathrm{Shi}(a+bx)^2-8\mathrm{Shi}(a+bx)\left(\left(a^2-abx+b^2x^2+2\right)\cosh(a+bx)+(a-2bx)\sinh(a+bx)\right)+12a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*SinhIntegral[a + b*x]^2,x]
```

```
[Out] (8*a + 8*b*x - 4*a*Cosh[2*(a + b*x)] + 2*b*x*Cosh[2*(a + b*x)] + 12*a*CoshI
ntegral[2*(a + b*x)] - 12*a*Log[a + b*x] - 5*Sinh[2*(a + b*x)] - 8*((2 + a^
2 - a*b*x + b^2*x^2)*Cosh[a + b*x] + (a - 2*b*x)*Sinh[a + b*x])*SinhIntegra
l[a + b*x] + 4*(a^3 + b^3*x^3)*SinhIntegral[a + b*x]^2 + 8*SinhIntegral[2*(
a + b*x)] + 12*a^2*SinhIntegral[2*(a + b*x)])/(12*b^3)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\,\mathrm{Shi}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh_integral(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x + a)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{Shi}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Shi(b*x+a)^2,x)
```

```
[Out] int(x^2*Shi(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^2*Shi(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{sinhint}(a + b\, x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(a + b*x)^2,x)
[Out] int(x^2*sinhint(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Shi}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Shi(b*x+a)**2,x)
[Out] Integral(x**2*Shi(a + b*x)**2, x)
```

3.27 $\int x\mathbf{Shi}(a+bx)^2\,dx$

Optimal. Leaf size=154

$$-\frac{\text{Chi}(2a+2bx)}{2b^2}-\frac{a(a+bx)\text{Shi}(a+bx)^2}{2b^2}-\frac{a\text{Shi}(2a+2bx)}{b^2}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b^2}+\frac{a\text{Shi}(a+bx)\cosh(a+bx}{b^2}$$

```
[Out] -1/2*Chi(2*b*x+2*a)/b^2+1/4*cosh(2*b*x+2*a)/b^2+1/2*ln(b*x+a)/b^2+a*cosh(b*
x+a)*Shi(b*x+a)/b^2-x*cosh(b*x+a)*Shi(b*x+a)/b-1/2*a*(b*x+a)*Shi(b*x+a)^2/b
^2+1/2*x*(b*x+a)*Shi(b*x+a)^2/b-a*Shi(2*b*x+2*a)/b^2+Shi(b*x+a)*sinh(b*x+a)
/b^2
```

Rubi [A] time = 0.33, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {6538, 6542, 5617, 6741, 6742, 2638, 3298, 6546, 3312, 3301, 6534, 6540, 5448, 12}

$$-\frac{\text{Chi}(2a+2bx)}{2b^2}-\frac{a(a+bx)\text{Shi}(a+bx)^2}{2b^2}-\frac{a\text{Shi}(2a+2bx)}{b^2}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b^2}+\frac{a\text{Shi}(a+bx)\cosh(a+bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*SinhIntegral[a + b*x]^2,x]
```

```
[Out] Cosh[2*a + 2*b*x]/(4*b^2) - CoshIntegral[2*a + 2*b*x]/(2*b^2) + Log[a + b*x
]/(2*b^2) + (a*Cosh[a + b*x]*SinhIntegral[a + b*x])/b^2 - (x*Cosh[a + b*x]*
SinhIntegral[a + b*x])/b + (Sinh[a + b*x]*SinhIntegral[a + b*x])/b^2 - (a*(
a + b*x)*SinhIntegral[a + b*x]^2)/(2*b^2) + (x*(a + b*x)*SinhIntegral[a + b
*x]^2)/(2*b) - (a*SinhIntegral[2*a + 2*b*x])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6534

```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*SinhIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Sinh[a + b*x]*SinhIntegral[a + b*x],
 x], x] /; FreeQ[{a, b}, x]
```

Rule 6538

```
Int[((c_.) + (d_.)*(x_))^(m_.)*SinhIntegral[(a_) + (b_.)*(x_)]^2, x_Symbol]
 :> Simp[((a + b*x)*(c + d*x)^m*SinhIntegral[a + b*x]^2)/(b*(m + 1)), x] +
(-Dist[2/(m + 1), Int[(c + d*x)^m*Sinh[a + b*x]*SinhIntegral[a + b*x], x],
x] + Dist[((b*c - a*d)*m)/(b*(m + 1)), Int[(c + d*x)^(m - 1)*SinhIntegral[a
 + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\text{Shi}(a+bx)^2\,dx = \frac{x(a+bx)\text{Shi}(a+bx)^2}{2b} - \frac{a\int \text{Shi}(a+bx)^2\,dx}{2b} - \int x\sinh(a+bx)\text{Shi}(a+bx)\,dx$$

$$= -\frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{a(a+bx)\text{Shi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Shi}(a+bx)^2}{2b} + \frac{\int \cosh(\cdots$$

$$= \frac{a\cosh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} + \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{a(\cdots$$

$$= \frac{a\cosh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} + \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{a(\cdots$$

$$= \frac{\log(a+bx)}{2b^2} + \frac{a\cosh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} + \frac{\sinh(a+bx)\text{Sh}\cdots}{b^2}$$

$$= -\frac{\text{Chi}(2a+2bx)}{2b^2} + \frac{\log(a+bx)}{2b^2} + \frac{a\cosh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{x\cosh(a+bx)\text{Shi}(a+b\cdots}{b}$$

$$= \frac{\cosh(2a+2bx)}{4b^2} - \frac{\text{Chi}(2a+2bx)}{2b^2} + \frac{\log(a+bx)}{2b^2} + \frac{a\cosh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{x\cosh(\cdots$$

Mathematica [A] time = 0.26, size = 95, normalized size = 0.62

$$\frac{-2\left(a^2-b^2x^2\right)\mathrm{Shi}(a+bx)^2-2\mathrm{Chi}(2(a+bx))-4a\mathrm{Shi}(2(a+bx))+4\mathrm{Shi}(a+bx)(\sinh(a+bx)+(a-bx)\cosh(a}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*SinhIntegral[a + b*x]^2,x]
```

```
[Out] (Cosh[2*(a + b*x)] - 2*CoshIntegral[2*(a + b*x)] + 2*Log[a + b*x] + 4*((a -
 b*x)*Cosh[a + b*x] + Sinh[a + b*x])*SinhIntegral[a + b*x] - 2*(a^2 - b^2*x
^2)*SinhIntegral[a + b*x]^2 - 4*a*SinhIntegral[2*(a + b*x)])/(4*b^2)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x\,\mathrm{Shi}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*sinh_integral(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*Shi(b*x + a)^2, x)
```

maple [A] time = 0.03, size = 135, normalized size = 0.88

$$\frac{x^2\,\mathrm{Shi}\left(bx+a\right)^2}{2}-\frac{\mathrm{Shi}\left(bx+a\right)^2a^2}{2b^2}-\frac{x\cosh\left(bx+a\right)\mathrm{Shi}\left(bx+a\right)}{b}+\frac{a\cosh\left(bx+a\right)\mathrm{Shi}\left(bx+a\right)}{b^2}+\frac{\mathrm{Shi}\left(bx+a\right)\sinh\left(\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(b*x+a)^2,x)
```

```
[Out] 1/2*x^2*Shi(b*x+a)^2-1/2/b^2*Shi(b*x+a)^2*a^2-x*cosh(b*x+a)*Shi(b*x+a)/b+a*
cosh(b*x+a)*Shi(b*x+a)/b^2+Shi(b*x+a)*sinh(b*x+a)/b^2+1/2/b^2*cosh(b*x+a)^2
+1/2*ln(b*x+a)/b^2-1/2*Chi(2*b*x+2*a)/b^2-a*Shi(2*b*x+2*a)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\text{sinhint}\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(a + b*x)^2,x)
```

```
[Out] int(x*sinhint(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{Shi}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)**2,x)
```

```
[Out] Integral(x*Shi(a + b*x)**2, x)
```

3.28 $\int \mathbf{Shi}(a+bx)^2\,dx$

Optimal. Leaf size=48

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}+\frac{\text{Shi}(2a+2bx)}{b}-\frac{2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

```
[Out] -2*cosh(b*x+a)*Shi(b*x+a)/b+(b*x+a)*Shi(b*x+a)^2/b+Shi(2*b*x+2*a)/b
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6534, 6540, 5448, 12, 3298}

$$\frac{(a+bx)\text{Shi}(a+bx)^2}{b}+\frac{\text{Shi}(2a+2bx)}{b}-\frac{2\text{Shi}(a+bx)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x]*SinhIntegral[a + b*x])/b + ((a + b*x)*SinhIntegral[a + b*
x]^2)/b + SinhIntegral[2*a + 2*b*x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6534

```
Int[SinhIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*SinhIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Sinh[a + b*x]*SinhIntegral[a + b*x],
```

```
x], x] /; FreeQ[{a, b}, x]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{Shi}(a+bx)^2\,dx &= \frac{(a+bx)\mathrm{Shi}(a+bx)^2}{b}-2\int \sinh(a+bx)\mathrm{Shi}(a+bx)\,dx\\ &= -\frac{2\cosh(a+bx)\mathrm{Shi}(a+bx)}{b}+\frac{(a+bx)\mathrm{Shi}(a+bx)^2}{b}+2\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}\,dx\\ &= -\frac{2\cosh(a+bx)\mathrm{Shi}(a+bx)}{b}+\frac{(a+bx)\mathrm{Shi}(a+bx)^2}{b}+2\int \frac{\sinh(2a+2bx)}{2(a+bx)}\,dx\\ &= -\frac{2\cosh(a+bx)\mathrm{Shi}(a+bx)}{b}+\frac{(a+bx)\mathrm{Shi}(a+bx)^2}{b}+\int \frac{\sinh(2a+2bx)}{a+bx}\,dx\\ &= -\frac{2\cosh(a+bx)\mathrm{Shi}(a+bx)}{b}+\frac{(a+bx)\mathrm{Shi}(a+bx)^2}{b}+\frac{\mathrm{Shi}(2a+2bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.85

$$\frac{(a+bx)\mathrm{Shi}(a+bx)^2+\mathrm{Shi}(2(a+bx))-2\mathrm{Shi}(a+bx)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x]*SinhIntegral[a + b*x] + (a + b*x)*SinhIntegral[a + b*x]^2
 + SinhIntegral[2*(a + b*x)])/b
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{Shi}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2,x, algorithm="giac")
[Out] integrate(Shi(b*x + a)^2, x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.90

$$\frac{(bx+a)\,\mathrm{Shi}\,(bx+a)^2-2\cosh(bx+a)\,\mathrm{Shi}\,(bx+a)+\mathrm{Shi}\,(2bx+2a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)^2,x)
[Out] 1/b*((b*x+a)*Shi(b*x+a)^2-2*cosh(b*x+a)*Shi(b*x+a)+Shi(2*b*x+2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(Shi(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{sinhint}\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)^2,x)
[Out] int(sinhint(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)**2,x)
[Out] Integral(Shi(a + b*x)**2, x)
```

3.29 $\int \frac{\text{Shi}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\text{Shi}(a+bx)^2}{x},x\right)$$

[Out] CannotIntegrate(Shi(b*x+a)^2/x,x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[SinhIntegral[a + b*x]^2/x,x]

[Out] Defer[Int][SinhIntegral[a + b*x]^2/x, x]

Rubi steps

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx=\int \frac{\text{Shi}(a+bx)^2}{x}\,dx$$

Mathematica [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(a+bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[SinhIntegral[a + b*x]^2/x,x]

[Out] Integrate[SinhIntegral[a + b*x]^2/x, x]

fricas [A] time = 1.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\left(bx+a\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x + a)^2/x, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)^2/x,x)
```

```
[Out] int(Shi(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)^2/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{sinhint}(a+b\,x)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)^2/x,x)
```

```
[Out] int(sinhint(a + b*x)^2/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)**2/x,x)
```

```
[Out] Integral(Shi(a + b*x)**2/x, x)
```

3.30 $\int \frac{\mathrm{Shi}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{\mathrm{Shi}(a+bx)^2}{x^2},x\right)$$

```
[Out] CannotIntegrate(Shi(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[SinhIntegral[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][SinhIntegral[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^2}\,dx=\int \frac{\mathrm{Shi}(a+bx)^2}{x^2}\,dx$$

Mathematica [A] time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}(a+bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[SinhIntegral[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[SinhIntegral[a + b*x]^2/x^2, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Shi}\left(bx+a\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(bx+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x + a)^2/x^2, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(bx+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)^2/x^2,x)
```

```
[Out] int(Shi(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(bx+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)^2/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\text{sinhint}\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)^2/x^2,x)
```

```
[Out] int(sinhint(a + b*x)^2/x^2, x)
```

<u>**sympy**</u> **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)**2/x**2,x)
```

```
[Out] Integral(Shi(a + b*x)**2/x**2, x)
```

3.31 $\int \frac{\text{Shi}(a+bx)^2}{x^3} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\text{Shi}(a+bx)^2}{x^3},x\right)$$

[Out] CannotIntegrate(Shi(b*x+a)^2/x^3,x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Shi}(a+bx)^2}{x^3} \, dx$$

Verification is Not applicable to the result.

[In] Int[SinhIntegral[a + b*x]^2/x^3,x]

[Out] Defer[Int][SinhIntegral[a + b*x]^2/x^3, x]

Rubi steps

$$\int \frac{\text{Shi}(a+bx)^2}{x^3} \, dx = \int \frac{\text{Shi}(a+bx)^2}{x^3} \, dx$$

Mathematica [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(a+bx)^2}{x^3} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[SinhIntegral[a + b*x]^2/x^3,x]

[Out] Integrate[SinhIntegral[a + b*x]^2/x^3, x]

fricas [A] time = 2.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\left(bx+a\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*x + a)^2/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x + a)^2/x^3, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)^2/x^3,x)
```

```
[Out] int(Shi(b*x+a)^2/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)^2/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\text{sinhint}\,(a+b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)^2/x^3,x)
```

```
[Out] int(sinhint(a + b*x)^2/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)**2/x**3,x)
```

```
[Out] Integral(Shi(a + b*x)**2/x**3, x)
```

3.32 $\int x^2\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=128

$$\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(3-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(bdn+3)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\frac{1}{3}x^3\text{Shi}\left(d\left(\right.\right.$$

```
[Out] 1/6*x^3*Ei((-b*d*n+3)*(a+b*ln(c*x^n))/b/n)/exp(3*a/b/n)/((c*x^n)^(3/n))-1/6
*x^3*Ei((b*d*n+3)*(a+b*ln(c*x^n))/b/n)/exp(3*a/b/n)/((c*x^n)^(3/n))+1/3*x^3
*Shi(d*(a+b*ln(c*x^n)))
```

Rubi [A] time = 0.27, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6555, 12, 5539, 2310, 2178}

$$\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(3-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(bdn+3)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\frac{1}{3}x^3\text{Shi}\left(d\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*ExpIntegralEi[((3 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(6*E^((3*a)/(b*
n))*(c*x^n)^(3/n)) - (x^3*ExpIntegralEi[((3 + b*d*n)*(a + b*Log[c*x^n]))/(b
*n)])/(6*E^((3*a)/(b*n))*(c*x^n)^(3/n)) + (x^3*SinhIntegral[d*(a + b*Log[c*
x^n])])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5539

```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Dist[(i*x)
^r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*L
og[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*
n)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6555

```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[((e*x)^(m + 1)*SinhIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Sinh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx &= \frac{1}{3}x^3\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}(bdn)\int\frac{x^2\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}\,dx\\ &= \frac{1}{3}x^3\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}(bn)\int\frac{x^2\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}\,dx\\ &= \frac{1}{3}x^3\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{1}{6}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int\frac{x^{2-bdn}}{a+b\log\left(cx^n\right)}\,dx\\ &= \frac{1}{3}x^3\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{1}{6}\left(be^{-ad}x^3\left(cx^n\right)^{-bd-\frac{3-bdn}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(3-bdn)x}{n}}}{a+bx}\right.\\ &= \frac{1}{6}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(3-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{6}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\text{Ei}\left(\right.\end{aligned}$$

Mathematica [A] time = 1.65, size = 98, normalized size = 0.77

$$\frac{1}{6}x^3\left(e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\left(\text{Ei}\left(-\frac{(bdn-3)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\text{Ei}\left(\frac{(bdn+3)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\right)+2\text{Shi}\left(d\left(a+b\log\left(c\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*((ExpIntegralEi[-(((-3 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))] - ExpInteg
ralEi[((3 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(E^((3*a)/(b*n))*(c*x^n)^(3/
n)) + 2*SinhIntegral[d*(a + b*Log[c*x^n])]))/6
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{Shi}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh_integral(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{Shi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^2\,\text{Shi}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Shi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*Shi(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{Shi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{sinhint}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^2*sinhint(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Shi}\left(ad + bd \log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Shi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*Shi(a*d + b*d*log(c*x**n)), x)
```

3.33 $\int x\textbf{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=128

$$\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(bdn+2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\frac{1}{2}x^2\text{Shi}\left(d\left(\right.\right.$$

```
[Out] 1/4*x^2*Ei((-b*d*n+2)*(a+b*ln(c*x^n))/b/n)/exp(2*a/b/n)/((c*x^n)^(2/n))-1/4
*x^2*Ei((b*d*n+2)*(a+b*ln(c*x^n))/b/n)/exp(2*a/b/n)/((c*x^n)^(2/n))+1/2*x^2
*Shi(d*(a+b*ln(c*x^n)))
```

Rubi [A] time = 0.25, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6555, 12, 5539, 2310, 2178}

$$\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(bdn+2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\frac{1}{2}x^2\text{Shi}\left(d\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*ExpIntegralEi[((2 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(4*E^((2*a)/(b*
n))*(c*x^n)^(2/n)) - (x^2*ExpIntegralEi[((2 + b*d*n)*(a + b*Log[c*x^n]))/(b
*n)])/(4*E^((2*a)/(b*n))*(c*x^n)^(2/n)) + (x^2*SinhIntegral[d*(a + b*Log[c*
x^n])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5539

```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Dist[(i*x)
^r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*L
og[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*
n)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6555

```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[((e*x)^(m + 1)*SinhIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Sinh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx &= \frac{1}{2}x^2\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int\frac{x\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}dx\\ &= \frac{1}{2}x^2\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bn)\int\frac{x\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}dx\\ &= \frac{1}{2}x^2\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{1}{4}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int\frac{x^{1-bdn}}{a+b\log\left(cx^n\right)}dx -\\ &= \frac{1}{2}x^2\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{1}{4}\left(be^{-ad}x^2\left(cx^n\right)^{-bd-\frac{2-bdn}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(2-bdn)x}{n}}}{a+bx}d\right.\\ &= \frac{1}{4}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{4}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\text{Ei}\left((2-\right.\end{aligned}$$

Mathematica [A] time = 1.62, size = 98, normalized size = 0.77

$$\frac{1}{4}x^2\left(e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\left(\text{Ei}\left(-\frac{(bdn-2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\text{Ei}\left(\frac{(bdn+2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\right)+2\text{Shi}\left(d\left(a+b\log\left(c\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*((ExpIntegralEi[-(((-2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))] - ExpInteg
ralEi[((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(E^((2*a)/(b*n))*(c*x^n)^(2/
n)) + 2*SinhIntegral[d*(a + b*Log[c*x^n])]))/4
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Shi}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x*sinh_integral(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*Shi((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\,\text{Shi}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(d*(a+b*ln(c*x^n))),x)
[Out] int(x*Shi(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x*Shi((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{sinhint}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x*sinhint(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Shi}\left(ad + bd \log\left(cx^{n}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*Shi(a*d + b*d*log(c*x**n)), x)
```

3.34 $\int \text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=119

$$\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\text{Ei}\left(\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\text{Ei}\left(\frac{(bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+x\text{Shi}\left(d\left(a+b\right.\right.$$

```
[Out] 1/2*x*Ei((-b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/n)/((c*x^n)^(1/n))-1/2*x*E
i((b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/n)/((c*x^n)^(1/n))+x*Shi(d*(a+b*ln
(c*x^n)))
```

Rubi [A] time = 0.23, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6552, 12, 5537, 2310, 2178}

$$\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\text{Ei}\left(\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\text{Ei}\left(\frac{(bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+x\text{Shi}\left(d\left(a+b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*ExpIntegralEi[((1 - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*E^(a/(b*n))*(c
*x^n)^n^(-1)) - (x*ExpIntegralEi[((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(
2*E^(a/(b*n))*(c*x^n)^n^(-1)) + x*SinhIntegral[d*(a + b*Log[c*x^n])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5537

```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*Sinh[((a_.) + Log[(
c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Dist[1/(E^(a*d)*(c*x^n)^(b*d)
*(2/x^(b*d*n))), Int[(h*(e + f*Log[g*x^m]))^q/x^(b*d*n), x], x] + Dist[(E^(
a*d)*(c*x^n)^(b*d))/(2*x^(b*d*n)), Int[x^(b*d*n)*(h*(e + f*Log[g*x^m]))^q,
x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, q}, x]
```

Rule 6552

```
Int[SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :>
 Simp[x*SinhIntegral[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Sinh[d*(a
+ b*Log[c*x^n])]/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\int \text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx &= x\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-(bdn)\int \frac{\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}\,dx\\ &= x\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-(bn)\int \frac{\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}\,dx\\ &= x\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{1}{2}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int \frac{x^{-bdn}}{a+b\log\left(cx^n\right)}\,dx-\frac{1}{2}\Big(\\ &= x\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{1}{2}\left(be^{-ad}x\left(cx^n\right)^{-bd-\frac{1-bdn}{n}}\right)\text{Subst}\left(\int \frac{e^{\frac{(1-bdn)x}{n}}}{a+bx}\,dx,x,\mathrm{l}\right.\\ &= \frac{1}{2}e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\text{Ei}\left(\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{2}e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\text{Ei}\left(\frac{(1+bd}{}\right.\end{aligned}$$

Mathematica [A] time = 1.57, size = 95, normalized size = 0.80

$$\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\left(\text{Ei}\left(-\frac{(bdn-1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\text{Ei}\left(\frac{(bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\right)+x\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*(ExpIntegralEi[-(((-1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))] - ExpIntegral
Ei[((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)]))/(2*E^(a/(b*n))*(c*x^n)^n^(-1))
 + x*SinhIntegral[d*(a + b*Log[c*x^n])]
```

fricas [F] time = 2.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{Shi}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(sinh_integral(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(Shi((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(d*(a+b*ln(c*x^n))),x)
[Out] int(Shi(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(Shi((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{sinhint}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(sinhint(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(Shi(d*(a + b*log(c*x**n))), x)
```

3.35 $\int \frac{\text{Shi}\left(d\left(a+b\log(cx^n)\right)\right)}{x}\,dx$

Optimal. Leaf size=55

$$\frac{(a+b\log(cx^n))\,\text{Shi}\left(d\left(a+b\log(cx^n)\right)\right)}{bn}-\frac{\cosh\left(d\left(a+b\log(cx^n)\right)\right)}{bdn}$$

```
[Out] -cosh(d*(a+b*ln(c*x^n)))/b/d/n+(a+b*ln(c*x^n))*Shi(d*(a+b*ln(c*x^n)))/b/n
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6528}

$$\frac{(a+b\log(cx^n))\,\text{Shi}\left(d\left(a+b\log(cx^n)\right)\right)}{bn}-\frac{\cosh\left(d\left(a+b\log(cx^n)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] -(Cosh[d*(a + b*Log[c*x^n])]/(b*d*n)) + ((a + b*Log[c*x^n])*SinhIntegral[d*
(a + b*Log[c*x^n])])/(b*n)
```

Rule 6528

```
Int[SinhIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*SinhInte
gral[a + b*x])/b, x] - Simp[Cosh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Shi}\left(d\left(a+b\log(cx^n)\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \text{Shi}(d(a+bx))\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int \text{Shi}(x)\,dx,x,ad+bd\log(cx^n)\right)}{bdn}\\ &= -\frac{\cosh\left(ad+bd\log(cx^n)\right)}{bdn}+\frac{(a+b\log(cx^n))\,\text{Shi}\left(ad+bd\log(cx^n)\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 1.75

$$\frac{\log(cx^n)\,\text{Shi}\left(d\left(a+b\log(cx^n)\right)\right)}{n}+\frac{a\text{Shi}\left(ad+b\log(cx^n)\,d\right)}{bn}-\frac{\sinh(ad)\sinh\left(bd\log(cx^n)\right)}{bdn}-\frac{\cosh(ad)\cosh\left(bd\log\right.}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[SinhIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] -((Cosh[a*d]*Cosh[b*d*Log[c*x^n]])/(b*d*n)) - (Sinh[a*d]*Sinh[b*d*Log[c*x^n
]])/(b*d*n) + (Log[c*x^n]*SinhIntegral[d*(a + b*Log[c*x^n])])/n + (a*SinhIn
tegral[a*d + b*d*Log[c*x^n]])/(b*n)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\left(bd\log\left(cx^{n}\right)+ad\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*d*log(c*x^n) + a*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] integrate(Shi((b*log(c*x^n) + a)*d)/x, x)
```

maple [A] time = 0.04, size = 73, normalized size = 1.33

$$\frac{\ln\left(c\,x^{n}\right)\text{Shi}\left(ad+bd\ln\left(c\,x^{n}\right)\right)}{n}+\frac{\text{Shi}\left(ad+bd\ln\left(c\,x^{n}\right)\right)a}{nb}-\frac{\cosh\left(ad+bd\ln\left(c\,x^{n}\right)\right)}{nbd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*Shi(a*d+b*d*ln(c*x^n))+1/n/b*Shi(a*d+b*d*ln(c*x^n))*a-1/n/b/d
*cosh(a*d+b*d*ln(c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] integrate(Shi((b*log(c*x^n) + a)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{\mathrm{sinhint}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)\ \ln\left(c\,x^n\right)}{n}+\frac{a\,\mathrm{sinhint}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)}{b\,n}-\frac{\mathrm{e}^{a\,d}\,{\left(c\,x^n\right)}^{b\,d}}{2\,b\,d\,n}-\frac{\mathrm{e}^{-a\,d}}{2\,b\,d\,n\,{\left(c\,x^n\right)}^{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] (sinhint(d*(a + b*log(c*x^n)))*log(c*x^n))/n + (a*sinhint(d*(a + b*log(c*x^
n))))/(b*n) - (exp(a*d)*(c*x^n)^(b*d))/(2*b*d*n) - exp(-a*d)/(2*b*d*n*(c*x^
n)^(b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}\left(ad+bd\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(Shi(a*d + b*d*log(c*x**n))/x, x)
```

3.36 $\int \frac{\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$

Optimal. Leaf size=122

$$\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2x}-\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{(bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2x}-\frac{\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

```
[Out] 1/2*exp(a/b/n)*(c*x^n)^(1/n)*Ei(-(-b*d*n+1)*(a+b*ln(c*x^n))/b/n)/x-1/2*exp(
a/b/n)*(c*x^n)^(1/n)*Ei(-(b*d*n+1)*(a+b*ln(c*x^n))/b/n)/x-Shi(d*(a+b*ln(c*x
^n)))/x
```

Rubi [A] time = 0.25, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6555, 12, 5539, 2310, 2178}

$$\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2x}-\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{(bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2x}-\frac{\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-(((1 - b*d*n)*(a + b*Log[c*x^n])
)/(b*n))])/(2*x) - (E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-(((1 + b*d*n)
*(a + b*Log[c*x^n]))/(b*n))])/(2*x) - SinhIntegral[d*(a + b*Log[c*x^n])]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5539

```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Dist[(i*x)
^r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*L
og[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*
n)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6555

```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[((e*x)^(m + 1)*SinhIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Sinh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\int \frac{\mathrm{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx = -\frac{\mathrm{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+(bdn)\int \frac{\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{dx^2\left(a+b\log\left(cx^n\right)\right)}\,dx$$

$$= -\frac{\mathrm{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+(bn)\int \frac{\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2\left(a+b\log\left(cx^n\right)\right)}\,dx$$

$$= -\frac{\mathrm{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}-\frac{1}{2}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int \frac{x^{-2-bdn}}{a+b\log\left(cx^n\right)}\,dx+\frac{1}{2}\Big(b$$

$$= -\frac{\mathrm{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}-\frac{\left(be^{-ad}\left(cx^n\right)^{-bd-\frac{-1-bdn}{n}}\right)\mathrm{Subst}\left(\int \frac{e^{\frac{(-1-bdn)x}{n}}}{a+bx}\,dx,x,\log\right.}{2x}$$

$$= \frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2x}-\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{(1+bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2x}-\frac{\mathrm{Shi}\Big(}{}$$

Mathematica [A] time = 1.78, size = 146, normalized size = 1.20

$$\frac{1}{2}\exp\left(-\frac{(bdn-1)\left(a+b\left(\log\left(cx^n\right)-n\log(x)\right)\right)}{bn}\right)\left(\mathrm{Ei}\left(\frac{(bdn-1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\mathrm{Ei}\left(-\frac{(bdn+1)\left(a+b\log\left(c\right.\right.}{bn}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[SinhIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((ExpIntegralEi[((-1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)] - ExpIntegralEi[-(
((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + Log
[c*x^n]))] + Sinh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(2*E^(((-1 + b*d*
n)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))) - SinhIntegral[d*(a + b*Log[
c*x^n])]/x
```

fricas [F] time = 2.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\left(bd\log\left(cx^{n}\right)+ad\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*d*log(c*x^n) + a*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(Shi((b*log(c*x^n) + a)*d)/x^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\text{Shi}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(Shi(d*(a+b*ln(c*x^n)))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{Shi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
[Out] integrate(Shi((b*log(c*x^n) + a)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{sinhint}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(d*(a + b*log(c*x^n)))/x^2,x)
[Out] int(sinhint(d*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\left(ad+bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*ln(c*x**n)))/x**2,x)
[Out] Integral(Shi(a*d + b*d*log(c*x**n))/x**2, x)
```

3.37 $\int \frac{\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$

Optimal. Leaf size=130

$$\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{Ei}\left(-\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{4x^2}-\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{Ei}\left(-\frac{(bdn+2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{4x^2}-\frac{\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

```
[Out] 1/4*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(-b*d*n+2)*(a+b*ln(c*x^n))/b/n)/x^2-1/4*
exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(b*d*n+2)*(a+b*ln(c*x^n))/b/n)/x^2-1/2*Shi(d
*(a+b*ln(c*x^n)))/x^2
```

Rubi [A] time = 0.25, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6555, 12, 5539, 2310, 2178}

$$\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{Ei}\left(-\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{4x^2}-\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{Ei}\left(-\frac{(bdn+2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{4x^2}-\frac{\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[SinhIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[-(((2 - b*d*n)*(a + b*Log[c*x^
n]))/(b*n))])/(4*x^2) - (E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[-(((2
+ b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(4*x^2) - SinhIntegral[d*(a + b*Log[c
*x^n])]/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5539

```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Dist[(i*x)
^r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*L
og[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*
n)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6555

```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[((e*x)^(m + 1)*SinhIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Sinh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx &= -\frac{\operatorname{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{2}(bdn)\int\frac{\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{dx^3\left(a+b\log\left(cx^n\right)\right)}\,dx\\
&= -\frac{\operatorname{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{2}(bn)\int\frac{\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3\left(a+b\log\left(cx^n\right)\right)}\,dx\\
&= -\frac{\operatorname{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}-\frac{1}{4}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int\frac{x^{-3-bdn}}{a+b\log\left(cx^n\right)}\,dx+\frac{1}{4}\Big(b\\
&= -\frac{\operatorname{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}-\frac{\left(be^{-ad}\left(cx^n\right)^{-bd-\frac{-2-bdn}{n}}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{(-2-bdn)x}{n}}}{a+bx}\,dx,x,\log\right.}{4x^2}\\
&= \frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\operatorname{Ei}\left(-\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{4x^2}-\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\operatorname{Ei}\left(-\frac{(2+bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{4x^2}-\frac{\operatorname{Sh}}{}
\end{aligned}$$

Mathematica [A] time = 1.84, size = 148, normalized size = 1.14

$$\frac{1}{4}\exp\left(-\frac{(bdn-2)\left(a+b\left(\log\left(cx^n\right)-n\log(x)\right)\right)}{bn}\right)\left(\operatorname{Ei}\left(\frac{(bdn-2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\operatorname{Ei}\left(-\frac{(bdn+2)\left(a+b\log\left(c\right.\right.}{bn}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[SinhIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((ExpIntegralEi[((-2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)] - ExpIntegralEi[-(
((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + Log
[c*x^n]))] + Sinh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(4*E^(((-2 + b*d*
n)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))) - SinhIntegral[d*(a + b*Log[
c*x^n])]/(2*x^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Shi}\left(bd\log\left(cx^n\right)+ad\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(sinh_integral(b*d*log(c*x^n) + a*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{Shi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(Shi((b*log(c*x^n) + a)*d)/x^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{\text{Shi}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(Shi(d*(a+b*ln(c*x^n)))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{Shi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
[Out] integrate(Shi((b*log(c*x^n) + a)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{sinhint}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(d*(a + b*log(c*x^n)))/x^3,x)
[Out] int(sinhint(d*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Shi}\left(ad+bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*(a+b*ln(c*x**n)))/x**3,x)
[Out] Integral(Shi(a*d + b*d*log(c*x**n))/x**3, x)
```

3.38 $\int (ex)^m \textbf{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=167

$$\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m-bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2(m+1)}-\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2(m+1)}+\frac{(ex)^{m+1}\mathrm{S}}{}$$

```
[Out] 1/2*x*(e*x)^m*Ei((-b*d*n+m+1)*(a+b*ln(c*x^n))/b/n)/exp(a*(1+m)/b/n)/(1+m)/(
(c*x^n)^((1+m)/n))-1/2*x*(e*x)^m*Ei((b*d*n+m+1)*(a+b*ln(c*x^n))/b/n)/exp(a*
(1+m)/b/n)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*Shi(d*(a+b*ln(c*x^n)))/e/(
1+m)
```

Rubi [A] time = 0.30, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6555, 12, 5539, 2310, 2178}

$$\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m-bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2(m+1)}-\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2(m+1)}+\frac{(ex)^{m+1}\mathrm{S}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*(e*x)^m*ExpIntegralEi[((1 + m - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*E^
((a*(1 + m))/(b*n))*(1 + m)*(c*x^n)^((1 + m)/n)) - (x*(e*x)^m*ExpIntegralEi
[((1 + m + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*E^((a*(1 + m))/(b*n))*(1 +
 m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*SinhIntegral[d*(a + b*Log[c*x^n])
])/(e*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5539

```
Int[(((e_.) + Log[(g_.)*(x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.)*
Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Dist[(i*x)
^r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*L
og[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*
n)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6555

```
Int[((e_.)*(x_))^(m_.)*SinhIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(
d_.)], x_Symbol] :> Simp[((e*x)^(m + 1)*SinhIntegral[d*(a + b*Log[c*x^n)])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Sinh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m \text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{(ex)^{1+m}\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bdn)\int \frac{(ex)^m\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bn)\int \frac{(ex)^m\sinh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} + \frac{\left(be^{-ad}nx^{-m+bdn}(ex)^m\left(cx^n\right)^{-bd}\right)\int \frac{x^{m-bd}}{a+b\log(}$$

$$= \frac{(ex)^{1+m}\text{Shi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} + \frac{\left(be^{-ad}x(ex)^m\left(cx^n\right)^{-bd-\frac{1+m-bdn}{n}}\right)\text{Subst}\Big(\int}{2(1+m)}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2(1+m)} - \frac{e^{-\frac{a(1+m)}{bn}}x(ex)^m\left(cx^n\right)^{-\frac{1}{}}}{2}$$

Mathematica [A] time = 2.83, size = 120, normalized size = 0.72

$$\frac{(ex)^m \left(x^{-m} \exp\left(-\frac{(m+1)\left(a+b\log(cx^n)-bn\log(x)\right)}{bn}\right)\left(\text{Ei}\left(\frac{(m-bdn+1)\left(a+b\log(cx^n)\right)}{bn}\right)-\text{Ei}\left(\frac{(m+bdn+1)\left(a+b\log(cx^n)\right)}{bn}\right)\right)+2x\text{Shi}\left(d\left(\right.\right.\right.}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*SinhIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*((ExpIntegralEi[((1 + m - b*d*n)*(a + b*Log[c*x^n]))/(b*n)] - ExpI
ntegralEi[((1 + m + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(E^(((1 + m)*(a - b*
n*Log[x] + b*Log[c*x^n]))/(b*n))*x^m) + 2*x*SinhIntegral[d*(a + b*Log[c*x^n
])]))/(2*(1 + m))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \text{Shi}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*Shi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh_integral(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \text{Shi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*Shi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*Shi((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex)^m \text{Shi}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*Shi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*Shi(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \text{Shi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*Shi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate((e*x)^m*Shi((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{sinhint}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)\,\left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(d*(a + b*log(c*x^n)))*(e*x)^m,x)
[Out] int(sinhint(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \operatorname{Shi}\left(ad + bd\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*Shi(d*(a+b*ln(c*x**n))),x)
[Out] Integral((e*x)**m*Shi(a*d + b*d*log(c*x**n)), x)
```

3.39 $\int \frac{\sinh(bx)\mathbf{Shi}(bx)}{x^3}\,dx$

Optimal. Leaf size=96

$$b^2\text{Chi}(2bx)+\frac{1}{4}b^2\text{Shi}(bx)^2-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}-\frac{b\text{Shi}(bx)\cosh(bx)}{2x}-\frac{\sinh^2(bx)}{4x^2}-\frac{b\sinh(2bx)}{4x}-\frac{b\sinh(bx)\cosh(bx)}{2x}$$

```
[Out] b^2*Chi(2*b*x)-1/2*b*cosh(b*x)*Shi(b*x)/x+1/4*b^2*Shi(b*x)^2-1/2*b*cosh(b*x
)*sinh(b*x)/x-1/2*Shi(b*x)*sinh(b*x)/x^2-1/4*sinh(b*x)^2/x^2-1/4*b*sinh(2*b
*x)/x
```

Rubi [A] time = 0.22, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6544, 6550, 6686, 12, 5448, 3297, 3301, 3314, 29, 3312}

$$b^2\text{Chi}(2bx)+\frac{1}{4}b^2\text{Shi}(bx)^2-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}-\frac{b\text{Shi}(bx)\cosh(bx)}{2x}-\frac{\sinh^2(bx)}{4x^2}-\frac{b\sinh(2bx)}{4x}-\frac{b\sinh(bx)\cosh(bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[b*x]*SinhIntegral[b*x])/x^3,x]
```

```
[Out] b^2*CoshIntegral[2*b*x] - (b*Cosh[b*x]*Sinh[b*x])/(2*x) - Sinh[b*x]^2/(4*x^
2) - (b*Sinh[2*b*x])/(4*x) - (b*Cosh[b*x]*SinhIntegral[b*x])/(2*x) - (Sinh[
b*x]*SinhIntegral[b*x])/(2*x^2) + (b^2*SinhIntegral[b*x]^2)/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6544

```
Int[((e_.) + (f_.)*(x_))^(m_)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) +
 (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^(m + 1)*Sinh[a + b*x]*SinhIntegr
al[c + d*x])/(f*(m + 1)), x] + (-Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)*
Cosh[a + b*x]*SinhIntegral[c + d*x], x], x] - Dist[d/(f*(m + 1)), Int[((e +
 f*x)^(m + 1)*Sinh[a + b*x]*Sinh[c + d*x])/(c + d*x), x], x]) /; FreeQ[{a,
b, c, d, e, f}, x] && ILtQ[m, -1]
```

Rule 6550

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^(m + 1)*Cosh[a + b*x]*SinhInteg
ral[c + d*x])/(f*(m + 1)), x] + (-Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)
*Sinh[a + b*x]*SinhIntegral[c + d*x], x], x] - Dist[d/(f*(m + 1)), Int[((e
+ f*x)^(m + 1)*Cosh[a + b*x]*Sinh[c + d*x])/(c + d*x), x], x]) /; FreeQ[{a,
 b, c, d, e, f}, x] && ILtQ[m, -1]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^3}\,dx = -\frac{\sinh(bx)\text{Shi}(bx)}{2x^2} + \frac{1}{2}b\int \frac{\sinh^2(bx)}{bx^3}\,dx + \frac{1}{2}b\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx$$

$$= -\frac{b\cosh(bx)\text{Shi}(bx)}{2x} - \frac{\sinh(bx)\text{Shi}(bx)}{2x^2} + \frac{1}{2}\int \frac{\sinh^2(bx)}{x^3}\,dx + \frac{1}{2}b^2\int \frac{\cosh(bx)\sinh(}{bx^2}$$

$$= -\frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{\sinh^2(bx)}{4x^2} - \frac{b\cosh(bx)\text{Shi}(bx)}{2x} - \frac{\sinh(bx)\text{Shi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Shi}$$

$$= \frac{1}{2}b^2\log(x) - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{\sinh^2(bx)}{4x^2} - \frac{b\cosh(bx)\text{Shi}(bx)}{2x} - \frac{\sinh(bx)\text{Shi}(bx}{2x^2}$$

$$= -\frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{\sinh^2(bx)}{4x^2} - \frac{b\cosh(bx)\text{Shi}(bx)}{2x} - \frac{\sinh(bx)\text{Shi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Shi}$$

$$= \frac{1}{2}b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{\sinh^2(bx)}{4x^2} - \frac{b\sinh(2bx)}{4x} - \frac{b\cosh(bx)\text{Shi}(bx)}{2x}$$

$$= b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{\sinh^2(bx)}{4x^2} - \frac{b\sinh(2bx)}{4x} - \frac{b\cosh(bx)\text{Shi}(bx)}{2x} -$$

Mathematica [A] time = 0.01, size = 96, normalized size = 1.00

$$b^2\text{Chi}(2bx)+\frac{1}{4}b^2\text{Shi}(bx)^2-\frac{\text{Shi}(bx)\sinh(bx)}{2x^2}-\frac{b\text{Shi}(bx)\cosh(bx)}{2x}-\frac{\sinh^2(bx)}{4x^2}-\frac{b\sinh(2bx)}{4x}-\frac{b\sinh(bx)\cosh(bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x^3,x]
```

```
[Out] b^2*CoshIntegral[2*b*x] - (b*Cosh[b*x]*Sinh[b*x])/(2*x) - Sinh[b*x]^2/(4*x^
2) - (b*Sinh[2*b*x])/(4*x) - (b*Cosh[b*x]*SinhIntegral[b*x])/(2*x) - (Sinh[
b*x]*SinhIntegral[b*x])/(2*x^2) + (b^2*SinhIntegral[b*x]^2)/4
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(bx)\,\text{Shi}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x)*sinh_integral(b*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)\sinh\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)*sinh(b*x)/x^3, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)\sinh\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)*sinh(b*x)/x^3,x)
```

```
[Out] int(Shi(b*x)*sinh(b*x)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)\sinh\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)*sinh(b*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{sinhint}\,(b\,x)\,\sinh\,(b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(b*x)*sinh(b*x))/x^3,x)
```

```
[Out] int((sinhint(b*x)*sinh(b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\,(bx)\,\mathrm{Shi}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x**3,x)
```

```
[Out] Integral(sinh(b*x)*Shi(b*x)/x**3, x)
```

3.40 $\int \frac{\sinh(bx)\mathbf{Shi}(bx)}{x^2} dx$

Optimal. Leaf size=49

$$b\text{Int}\left(\frac{\text{Shi}(bx)\cosh(bx)}{x},x\right)+b\text{Shi}(2bx)-\frac{\text{Shi}(bx)\sinh(bx)}{x}-\frac{\sinh^2(bx)}{x}$$

```
[Out] b*CannotIntegrate(cosh(b*x)*Shi(b*x)/x,x)+b*Shi(2*b*x)-Shi(b*x)*sinh(b*x)/x
-sinh(b*x)^2/x
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[b*x]*SinhIntegral[b*x])/x^2,x]
```

```
[Out] -(Sinh[b*x]^2/x) - (Sinh[b*x]*SinhIntegral[b*x])/x + b*SinhIntegral[2*b*x]
+ b*Defer[Int][(Cosh[b*x]*SinhIntegral[b*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}\,dx &= -\frac{\sinh(bx)\text{Shi}(bx)}{x}+b\int \frac{\sinh^2(bx)}{bx^2}\,dx+b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx\\
&= -\frac{\sinh(bx)\text{Shi}(bx)}{x}+b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx+\int \frac{\sinh^2(bx)}{x^2}\,dx\\
&= -\frac{\sinh^2(bx)}{x}-\frac{\sinh(bx)\text{Shi}(bx)}{x}-(2ib)\int \frac{i\sinh(2bx)}{2x}\,dx+b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx\\
&= -\frac{\sinh^2(bx)}{x}-\frac{\sinh(bx)\text{Shi}(bx)}{x}+b\int \frac{\sinh(2bx)}{x}\,dx+b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx\\
&= -\frac{\sinh^2(bx)}{x}-\frac{\sinh(bx)\text{Shi}(bx)}{x}+b\text{Shi}(2bx)+b\int \frac{\cosh(bx)\text{Shi}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x^2,x]
[Out] Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x^2, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(bx)\,\text{Shi}(bx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x^2,x, algorithm="fricas")
[Out] integral(sinh(b*x)*sinh_integral(b*x)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)\sinh(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x^2,x, algorithm="giac")
[Out] integrate(Shi(b*x)*sinh(b*x)/x^2, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)\sinh(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)*sinh(b*x)/x^2,x)
[Out] int(Shi(b*x)*sinh(b*x)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}(bx)\sinh(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x^2,x, algorithm="maxima")
[Out] integrate(Shi(b*x)*sinh(b*x)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{sinhint}\left(b\,x\right)\,\sinh\left(b\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(b*x)*sinh(b*x))/x^2,x)
```

```
[Out] int((sinhint(b*x)*sinh(b*x))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx\right)\mathrm{Shi}\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x**2,x)
```

```
[Out] Integral(sinh(b*x)*Shi(b*x)/x**2, x)
```

3.41 $\int \frac{\sinh(bx)\mathbf{Shi}(bx)}{x} \, dx$

Optimal. Leaf size=10

$$\frac{\text{Shi}(bx)^2}{2}$$

```
[Out] 1/2*Shi(b*x)^2
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.083, Rules used = {6686}

$$\frac{\text{Shi}(bx)^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[b*x]*SinhIntegral[b*x])/x,x]
```

```
[Out] SinhIntegral[b*x]^2/2
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh(bx)\text{Shi}(bx)}{x} \, dx = \frac{\text{Shi}(bx)^2}{2}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\text{Shi}(bx)^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[b*x]*SinhIntegral[b*x])/x,x]
```

```
[Out] SinhIntegral[b*x]^2/2
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(bx)\,\text{Shi}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x)*sinh_integral(b*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)\sinh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)*sinh(b*x)/x, x)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\mathrm{Shi}\,(bx)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)*sinh(b*x)/x,x)
```

```
[Out] 1/2*Shi(b*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\,(bx)\sinh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)*sinh(b*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\frac{\mathrm{sinhint}\,(b\,x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(b*x)*sinh(b*x))/x,x)
```

```
[Out] sinhint(b*x)^2/2
```

sympy [A] time = 0.75, size = 7, normalized size = 0.70

$$\frac{\mathrm{Shi}^2(bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x)/x,x)
```

```
[Out] Shi(b*x)**2/2
```

3.42 $\int \sinh(bx)\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=25

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}$$

[Out] cosh(b*x)*Shi(b*x)/b-1/2*Shi(2*b*x)/b

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6540, 12, 5448, 3298}

$$\frac{\text{Shi}(bx)\cosh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[b*x]*SinhIntegral[b*x],x]

[Out] (Cosh[b*x]*SinhIntegral[b*x])/b - SinhIntegral[2*b*x]/(2*b)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(bx)\mathrm{Shi}(bx)\,dx &= \frac{\cosh(bx)\mathrm{Shi}(bx)}{b} - \int \frac{\cosh(bx)\sinh(bx)}{bx}\,dx \\ &= \frac{\cosh(bx)\mathrm{Shi}(bx)}{b} - \frac{\int \frac{\cosh(bx)\sinh(bx)}{x}\,dx}{b} \\ &= \frac{\cosh(bx)\mathrm{Shi}(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{2x}\,dx}{b} \\ &= \frac{\cosh(bx)\mathrm{Shi}(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{x}\,dx}{2b} \\ &= \frac{\cosh(bx)\mathrm{Shi}(bx)}{b} - \frac{\mathrm{Shi}(2bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\mathrm{Shi}(bx)\cosh(bx)}{b} - \frac{\mathrm{Shi}(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[b*x]*SinhIntegral[b*x],x]
[Out] (Cosh[b*x]*SinhIntegral[b*x])/b - SinhIntegral[2*b*x]/(2*b)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sinh\left(bx\right)\mathrm{Shi}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x),x, algorithm="fricas")
[Out] integral(sinh(b*x)*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x),x, algorithm="giac")
[Out] integrate(Shi(b*x)*sinh(b*x), x)
```

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{\cosh\left(bx\right)\mathrm{Shi}\left(bx\right)-\frac{\mathrm{Shi}(2bx)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x)*sinh(b*x),x)
[Out] 1/b*(cosh(b*x)*Shi(b*x)-1/2*Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x),x, algorithm="maxima")
[Out] integrate(Shi(b*x)*sinh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{sinhint}\left(b\,x\right)\,\sinh\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)*sinh(b*x),x)
[Out] int(sinhint(b*x)*sinh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx\right)\mathrm{Shi}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x)*sinh(b*x),x)
[Out] Integral(sinh(b*x)*Shi(b*x), x)
```

3.43 $\int x \sinh(bx)\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=61

$$\frac{\mathrm{Chi}(2bx)}{2b^2}-\frac{\mathrm{Shi}(bx)\sinh(bx)}{b^2}-\frac{\log(x)}{2b^2}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\mathrm{Shi}(bx)\cosh(bx)}{b}$$

```
[Out] 1/2*Chi(2*b*x)/b^2-1/2*ln(x)/b^2+x*cosh(b*x)*Shi(b*x)/b-Shi(b*x)*sinh(b*x)/
b^2-1/2*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6542, 12, 2564, 30, 6546, 3312, 3301}

$$\frac{\mathrm{Chi}(2bx)}{2b^2}-\frac{\mathrm{Shi}(bx)\sinh(bx)}{b^2}-\frac{\log(x)}{2b^2}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\mathrm{Shi}(bx)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] CoshIntegral[2*b*x]/(2*b^2) - Log[x]/(2*b^2) - Sinh[b*x]^2/(2*b^2) + (x*Cos
h[b*x]*SinhIntegral[b*x])/b - (Sinh[b*x]*SinhIntegral[b*x])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x\sinh(bx)\text{Shi}(bx)\,dx &= \frac{x\cosh(bx)\text{Shi}(bx)}{b} - \frac{\int \cosh(bx)\text{Shi}(bx)\,dx}{b} - \int \frac{\cosh(bx)\sinh(bx)}{b}\,dx \\
&= \frac{x\cosh(bx)\text{Shi}(bx)}{b} - \frac{\sinh(bx)\text{Shi}(bx)}{b^2} - \frac{\int \cosh(bx)\sinh(bx)\,dx}{b} + \frac{\int \frac{\sinh^2(bx)}{bx}\,dx}{b} \\
&= \frac{x\cosh(bx)\text{Shi}(bx)}{b} - \frac{\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{\int \frac{\sinh^2(bx)}{x}\,dx}{b^2} + \frac{\text{Subst}(\int x\,dx, x, i\sinh(bx))}{b^2} \\
&= -\frac{\sinh^2(bx)}{2b^2} + \frac{x\cosh(bx)\text{Shi}(bx)}{b} - \frac{\sinh(bx)\text{Shi}(bx)}{b^2} - \frac{\int \left(\frac{1}{2x} - \frac{\cosh(2bx)}{2x}\right)dx}{b^2} \\
&= -\frac{\log(x)}{2b^2} - \frac{\sinh^2(bx)}{2b^2} + \frac{x\cosh(bx)\text{Shi}(bx)}{b} - \frac{\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{\int \frac{\cosh(2bx)}{x}\,dx}{2b^2} \\
&= \frac{\text{Chi}(2bx)}{2b^2} - \frac{\log(x)}{2b^2} - \frac{\sinh^2(bx)}{2b^2} + \frac{x\cosh(bx)\text{Shi}(bx)}{b} - \frac{\sinh(bx)\text{Shi}(bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 44, normalized size = 0.72

$$-\frac{-2\mathrm{Chi}(2bx)+\mathrm{Shi}(bx)(4\sinh(bx)-4bx\cosh(bx))+\cosh(2bx)+2\log(x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] -1/4*(Cosh[2*b*x] - 2*CoshIntegral[2*b*x] + 2*Log[x] + (-4*b*x*Cosh[b*x] +
4*Sinh[b*x])*SinhIntegral[b*x])/b^2
```

fricas [F] time = 1.97, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x\sinh\left(bx\right)\mathrm{Shi}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)*sinh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x)*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)*sinh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*Shi(b*x)*sinh(b*x), x)
```

maple [A] time = 0.01, size = 58, normalized size = 0.95

$$\frac{x\cosh\left(bx\right)\mathrm{Shi}\left(bx\right)}{b}-\frac{\mathrm{Shi}\left(bx\right)\sinh\left(bx\right)}{b^2}-\frac{\cosh^2\left(bx\right)}{2b^2}-\frac{\ln\left(bx\right)}{2b^2}+\frac{\mathrm{X}\left(2bx\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(b*x)*sinh(b*x),x)
```

```
[Out] x*cosh(b*x)*Shi(b*x)/b-Shi(b*x)*sinh(b*x)/b^2-1/2/b^2*cosh(b*x)^2-1/2/b^2*l
n(b*x)+1/2*Chi(2*b*x)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)*sinh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x)*sinh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \,\mathrm{sinhint}\,(b\,x)\;\sinh\,(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(b*x)*sinh(b*x),x)
```

```
[Out] int(x*sinhint(b*x)*sinh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh{\left(bx \right)} \operatorname{Shi}{\left(bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x)*sinh(b*x),x)
```

```
[Out] Integral(x*sinh(b*x)*Shi(b*x), x)
```

3.44 $\int x^2 \sinh(bx)\textbf{Shi}(bx)\,dx$

Optimal. Leaf size=90

$$-\frac{\text{Shi}(2bx)}{b^3}+\frac{2\text{Shi}(bx)\cosh(bx)}{b^3}+\frac{5\sinh(bx)\cosh(bx)}{4b^3}-\frac{2x\text{Shi}(bx)\sinh(bx)}{b^2}-\frac{5x}{4b^2}-\frac{x\sinh^2(bx)}{2b^2}+\frac{x^2\text{Shi}(bx)\cosh(}{b}$$

```
[Out] -5/4*x/b^2+2*cosh(b*x)*Shi(b*x)/b^3+x^2*cosh(b*x)*Shi(b*x)/b-Shi(2*b*x)/b^3
+5/4*cosh(b*x)*sinh(b*x)/b^3-2*x*Shi(b*x)*sinh(b*x)/b^2-1/2*x*sinh(b*x)^2/b
^2
```

Rubi [A] time = 0.13, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6542, 12, 5372, 2635, 8, 6548, 6540, 5448, 3298}

$$-\frac{\text{Shi}(2bx)}{b^3}-\frac{2x\text{Shi}(bx)\sinh(bx)}{b^2}+\frac{2\text{Shi}(bx)\cosh(bx)}{b^3}-\frac{5x}{4b^2}-\frac{x\sinh^2(bx)}{2b^2}+\frac{5\sinh(bx)\cosh(bx)}{4b^3}+\frac{x^2\text{Shi}(bx)\cosh(}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] (-5*x)/(4*b^2) + (5*Cosh[b*x]*Sinh[b*x])/(4*b^3) - (x*Sinh[b*x]^2)/(2*b^2)
+ (2*Cosh[b*x]*SinhIntegral[b*x])/b^3 + (x^2*Cosh[b*x]*SinhIntegral[b*x])/b
 - (2*x*Sinh[b*x]*SinhIntegral[b*x])/b^2 - SinhIntegral[2*b*x]/b^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2 \sinh(bx)\text{Shi}(bx)\,dx = \frac{x^2\cosh(bx)\text{Shi}(bx)}{b} - \frac{2\int x\cosh(bx)\text{Shi}(bx)\,dx}{b} - \int \frac{x\cosh(bx)\sinh(bx)}{b}\,dx$$

$$= \frac{x^2\cosh(bx)\text{Shi}(bx)}{b} - \frac{2x\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{2\int \sinh(bx)\text{Shi}(bx)\,dx}{b^2} - \frac{\int x\cosh(bx)}{}$$

$$= -\frac{x\sinh^2(bx)}{2b^2} + \frac{2\cosh(bx)\text{Shi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Shi}(bx)}{b} - \frac{2x\sinh(bx)\text{Shi}(bx)}{b^2} +$$

$$= \frac{5\cosh(bx)\sinh(bx)}{4b^3} - \frac{x\sinh^2(bx)}{2b^2} + \frac{2\cosh(bx)\text{Shi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Shi}(bx)}{b} -$$

$$= -\frac{5x}{4b^2} + \frac{5\cosh(bx)\sinh(bx)}{4b^3} - \frac{x\sinh^2(bx)}{2b^2} + \frac{2\cosh(bx)\text{Shi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Shi}(}{b}$$

$$= -\frac{5x}{4b^2} + \frac{5\cosh(bx)\sinh(bx)}{4b^3} - \frac{x\sinh^2(bx)}{2b^2} + \frac{2\cosh(bx)\text{Shi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Shi}(}{b}$$

$$= -\frac{5x}{4b^2} + \frac{5\cosh(bx)\sinh(bx)}{4b^3} - \frac{x\sinh^2(bx)}{2b^2} + \frac{2\cosh(bx)\text{Shi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Shi}(}{b}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.71

$$\frac{8\text{Shi}(bx)\left(\left(b^2x^2+2\right)\cosh(bx)-2bx\sinh(bx)\right)-8\text{Shi}(2bx)-8bx+5\sinh(2bx)-2bx\cosh(2bx)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] (-8*b*x - 2*b*x*Cosh[2*b*x] + 5*Sinh[2*b*x] + 8*((2 + b^2*x^2)*Cosh[b*x] -
2*b*x*Sinh[b*x])*SinhIntegral[b*x] - 8*SinhIntegral[2*b*x])/(8*b^3)
```

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\sinh(bx)\,\text{Shi}(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x)*sinh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x)*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{Shi}(bx)\sinh(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x)*sinh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x)*sinh(b*x), x)
```

maple [A] time = 0.01, size = 68, normalized size = 0.76

$$\frac{\mathrm{Shi}\left(bx\right)\left(b^{2}x^{2}\cosh\left(bx\right)-2bx\sinh\left(bx\right)+2\cosh\left(bx\right)\right)-\frac{bx\left(\cosh^{2}\left(bx\right)\right)}{2}+\frac{5\sinh\left(bx\right)\cosh\left(bx\right)}{4}-\frac{3bx}{4}-\mathrm{Shi}\left(2bx\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Shi(b*x)*sinh(b*x),x)
```

```
[Out] 1/b^3*(Shi(b*x)*(b^2*x^2*cosh(b*x)-2*b*x*sinh(b*x)+2*cosh(b*x))-1/2*b*x*cos
h(b*x)^2+5/4*sinh(b*x)*cosh(b*x)-3/4*b*x-Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\mathrm{Shi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x)*sinh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi(b*x)*sinh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\,\mathrm{sinhint}\left(b\,x\right)\,\sinh\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(b*x)*sinh(b*x),x)
```

```
[Out] int(x^2*sinhint(b*x)*sinh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sinh\left(bx\right)\mathrm{Shi}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Shi(b*x)*sinh(b*x),x)
```

```
[Out] Integral(x**2*sinh(b*x)*Shi(b*x), x)
```

3.45 $\int x^3 \sinh(bx)\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=125

$$\frac{3\text{Chi}(2bx)}{b^4}-\frac{6\text{Shi}(bx)\sinh(bx)}{b^4}-\frac{3\log(x)}{b^4}-\frac{4\sinh^2(bx)}{b^4}+\frac{6x\text{Shi}(bx)\cosh(bx)}{b^3}+\frac{2x\sinh(bx)\cosh(bx)}{b^3}-\frac{3x^2\text{Shi}(bx}{b}$$

```
[Out] -x^2/b^2+3*Chi(2*b*x)/b^4-3*ln(x)/b^4+6*x*cosh(b*x)*Shi(b*x)/b^3+x^3*cosh(b
*x)*Shi(b*x)/b+2*x*cosh(b*x)*sinh(b*x)/b^3-6*Shi(b*x)*sinh(b*x)/b^4-3*x^2*S
hi(b*x)*sinh(b*x)/b^2-4*sinh(b*x)^2/b^4-1/2*x^2*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.20, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6542, 12, 5372, 3310, 30, 6548, 2564, 6546, 3312, 3301}

$$\frac{3\text{Chi}(2bx)}{b^4}-\frac{3x^2\text{Shi}(bx)\sinh(bx)}{b^2}-\frac{6\text{Shi}(bx)\sinh(bx)}{b^4}+\frac{6x\text{Shi}(bx)\cosh(bx)}{b^3}-\frac{x^2}{b^2}-\frac{x^2\sinh^2(bx)}{2b^2}-\frac{3\log(x)}{b^4}-\frac{4\sinh}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] -(x^2/b^2) + (3*CoshIntegral[2*b*x])/b^4 - (3*Log[x])/b^4 + (2*x*Cosh[b*x]*
Sinh[b*x])/b^3 - (4*Sinh[b*x]^2)/b^4 - (x^2*Sinh[b*x]^2)/(2*b^2) + (6*x*Cos
h[b*x]*SinhIntegral[b*x])/b^3 + (x^3*Cosh[b*x]*SinhIntegral[b*x])/b - (6*Si
nh[b*x]*SinhIntegral[b*x])/b^4 - (3*x^2*Sinh[b*x]*SinhIntegral[b*x])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3 \sinh(bx)\text{Shi}(bx)\,dx = \frac{x^3\cosh(bx)\text{Shi}(bx)}{b} - \frac{3\int x^2\cosh(bx)\text{Shi}(bx)\,dx}{b} - \int \frac{x^2\cosh(bx)\sinh(bx)}{b}\,dx$$

$$= \frac{x^3\cosh(bx)\text{Shi}(bx)}{b} - \frac{3x^2\sinh(bx)\text{Shi}(bx)}{b^2} + \frac{6\int x\sinh(bx)\text{Shi}(bx)\,dx}{b^2} - \frac{\int x^2\cosh}{}$$

$$= -\frac{x^2\sinh^2(bx)}{2b^2} + \frac{6x\cosh(bx)\text{Shi}(bx)}{b^3} + \frac{x^3\cosh(bx)\text{Shi}(bx)}{b} - \frac{3x^2\sinh(bx)\text{Shi}(bx)}{b^2}$$

$$= \frac{2x\cosh(bx)\sinh(bx)}{b^3} - \frac{\sinh^2(bx)}{b^4} - \frac{x^2\sinh^2(bx)}{2b^2} + \frac{6x\cosh(bx)\text{Shi}(bx)}{b^3} + \frac{x^3\cosh}{}$$

$$= -\frac{x^2}{b^2} + \frac{2x\cosh(bx)\sinh(bx)}{b^3} - \frac{\sinh^2(bx)}{b^4} - \frac{x^2\sinh^2(bx)}{2b^2} + \frac{6x\cosh(bx)\text{Shi}(bx)}{b^3} +$$

$$= -\frac{x^2}{b^2} + \frac{2x\cosh(bx)\sinh(bx)}{b^3} - \frac{4\sinh^2(bx)}{b^4} - \frac{x^2\sinh^2(bx)}{2b^2} + \frac{6x\cosh(bx)\text{Shi}(bx)}{b^3} +$$

$$= -\frac{x^2}{b^2} - \frac{3\log(x)}{b^4} + \frac{2x\cosh(bx)\sinh(bx)}{b^3} - \frac{4\sinh^2(bx)}{b^4} - \frac{x^2\sinh^2(bx)}{2b^2} + \frac{6x\cosh(bx}{b^3}$$

$$= -\frac{x^2}{b^2} + \frac{3\text{Chi}(2bx)}{b^4} - \frac{3\log(x)}{b^4} + \frac{2x\cosh(bx)\sinh(bx)}{b^3} - \frac{4\sinh^2(bx)}{b^4} - \frac{x^2\sinh^2(bx)}{2b^2}$$

Mathematica [A] time = 0.16, size = 93, normalized size = 0.74

$$-\frac{-4\text{Shi}(bx)\left(bx\left(b^2x^2+6\right)\cosh(bx)-3\left(b^2x^2+2\right)\sinh(bx)\right)+3b^2x^2+b^2x^2\cosh(2bx)-12\text{Chi}(2bx)-4bx\sin}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] -1/4*(3*b^2*x^2 + 8*Cosh[2*b*x] + b^2*x^2*Cosh[2*b*x] - 12*CoshIntegral[2*b
*x] + 12*Log[x] - 4*b*x*Sinh[2*b*x] - 4*(b*x*(6 + b^2*x^2)*Cosh[b*x] - 3*(2
 + b^2*x^2)*Sinh[b*x])*SinhIntegral[b*x])/b^4
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\sinh(bx)\,\text{Shi}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x)*sinh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*sinh(b*x)*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{Shi}(bx) \sinh(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x)*sinh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*Shi(b*x)*sinh(b*x), x)
```

maple [A] time = 0.02, size = 126, normalized size = 1.01

$$\frac{x^3 \cosh(bx) \mathrm{Shi}(bx)}{b} - \frac{3x^2 \mathrm{Shi}(bx) \sinh(bx)}{b^2} + \frac{6x \cosh(bx) \mathrm{Shi}(bx)}{b^3} - \frac{6 \mathrm{Shi}(bx) \sinh(bx)}{b^4} - \frac{x^2 \left(\cosh^2(bx)\right)}{2b^2} + \frac{2x \mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Shi(b*x)*sinh(b*x),x)
```

```
[Out] x^3*cosh(b*x)*Shi(b*x)/b-3*x^2*Shi(b*x)*sinh(b*x)/b^2+6*x*cosh(b*x)*Shi(b*x
)/b^3-6*Shi(b*x)*sinh(b*x)/b^4-1/2/b^2*x^2*cosh(b*x)^2+2*x*cosh(b*x)*sinh(b
*x)/b^3-1/2*x^2/b^2-4*cosh(b*x)^2/b^4-3/b^4*ln(b*x)+3*Chi(2*b*x)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{Shi}(bx) \sinh(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Shi(b*x)*sinh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*Shi(b*x)*sinh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \,\mathrm{sinhint}(b\,x)\, \sinh(b\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinhint(b*x)*sinh(b*x),x)
```

```
[Out] int(x^3*sinhint(b*x)*sinh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sinh(bx) \operatorname{Shi}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Shi(b*x)*sinh(b*x),x)
```

```
[Out] Integral(x**3*sinh(b*x)*Shi(b*x), x)
```

3.46 $\int \frac{\cosh(bx)\textbf{Shi}(bx)}{x^3}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}b^2\mathrm{Int}\left(\frac{\mathrm{Shi}(bx)\cosh(bx)}{x},x\right)+b^2\mathrm{Shi}(2bx)-\frac{\mathrm{Shi}(bx)\cosh(bx)}{2x^2}-\frac{b\mathrm{Shi}(bx)\sinh(bx)}{2x}-\frac{\sinh(2bx)}{8x^2}-\frac{b\sinh^2(bx)}{2x}-\frac{b\cos}{}$$

```
[Out] 1/2*b^2*CannotIntegrate(cosh(b*x)*Shi(b*x)/x,x)-1/4*b*cosh(2*b*x)/x-1/2*cos
h(b*x)*Shi(b*x)/x^2+b^2*Shi(2*b*x)-1/2*b*Shi(b*x)*sinh(b*x)/x-1/2*b*sinh(b*
x)^2/x-1/8*sinh(2*b*x)/x^2
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[b*x]*SinhIntegral[b*x])/x^3,x]
```

```
[Out] -(b*Cosh[2*b*x])/(4*x) - (b*Sinh[b*x]^2)/(2*x) - Sinh[2*b*x]/(8*x^2) - (Cos
h[b*x]*SinhIntegral[b*x])/(2*x^2) - (b*Sinh[b*x]*SinhIntegral[b*x])/(2*x) +
 b^2*SinhIntegral[2*b*x] + (b^2*Defer[Int][(Cosh[b*x]*SinhIntegral[b*x])/x,
 x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx &= -\frac{\cosh(bx)\mathrm{Shi}(bx)}{2x^2}+\frac{1}{2}b\int \frac{\cosh(bx)\sinh(bx)}{bx^3}\,dx+\frac{1}{2}b\int \frac{\sinh(bx)\mathrm{Shi}(bx)}{x^2}\,dx\\
&= -\frac{\cosh(bx)\mathrm{Shi}(bx)}{2x^2}-\frac{b\sinh(bx)\mathrm{Shi}(bx)}{2x}+\frac{1}{2}\int \frac{\cosh(bx)\sinh(bx)}{x^3}\,dx+\frac{1}{2}b^2\int \frac{\sinh^2(b}{bx^2}\\
&= -\frac{\cosh(bx)\mathrm{Shi}(bx)}{2x^2}-\frac{b\sinh(bx)\mathrm{Shi}(bx)}{2x}+\frac{1}{2}\int \frac{\sinh(2bx)}{2x^3}\,dx+\frac{1}{2}b\int \frac{\sinh^2(bx)}{x^2}\,dx+\frac{1}{2}\\
&= -\frac{b\sinh^2(bx)}{2x}-\frac{\cosh(bx)\mathrm{Shi}(bx)}{2x^2}-\frac{b\sinh(bx)\mathrm{Shi}(bx)}{2x}+\frac{1}{4}\int \frac{\sinh(2bx)}{x^3}\,dx-\left(ib^2\right)\int \frac{i}{}\\
&= -\frac{b\sinh^2(bx)}{2x}-\frac{\sinh(2bx)}{8x^2}-\frac{\cosh(bx)\mathrm{Shi}(bx)}{2x^2}-\frac{b\sinh(bx)\mathrm{Shi}(bx)}{2x}+\frac{1}{4}b\int \frac{\cosh(2bx)}{x^2}\\
&= -\frac{b\cosh(2bx)}{4x}-\frac{b\sinh^2(bx)}{2x}-\frac{\sinh(2bx)}{8x^2}-\frac{\cosh(bx)\mathrm{Shi}(bx)}{2x^2}-\frac{b\sinh(bx)\mathrm{Shi}(bx)}{2x}+\frac{1}{2}\\
&= -\frac{b\cosh(2bx)}{4x}-\frac{b\sinh^2(bx)}{2x}-\frac{\sinh(2bx)}{8x^2}-\frac{\cosh(bx)\mathrm{Shi}(bx)}{2x^2}-\frac{b\sinh(bx)\mathrm{Shi}(bx)}{2x}+b^2
\end{aligned}$$

Mathematica [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx)\mathrm{Shi}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x^3,x]
```

```
[Out] Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x^3, x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cosh(bx)\,\mathrm{Shi}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x)*sinh_integral(b*x)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}(bx)\cosh(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)*cosh(b*x)/x^3, x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx)\,\mathrm{Shi}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x)*Shi(b*x)/x^3,x)
```

```
[Out] int(cosh(b*x)*Shi(b*x)/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}(bx)\cosh(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)*cosh(b*x)/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{sinhint}(b\,x)\,\cosh(b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(b*x)*cosh(b*x))/x^3,x)
```

```
[Out] int((sinhint(b*x)*cosh(b*x))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx)\operatorname{Shi}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x**3,x)
```

```
[Out] Integral(cosh(b*x)*Shi(b*x)/x**3, x)
```

3.47 $\int \frac{\cosh(bx)\mathbf{Shi}(bx)}{x^2}\,dx$

Optimal. Leaf size=44

$$b\text{Chi}(2bx)+\frac{1}{2}b\text{Shi}(bx)^2-\frac{\text{Shi}(bx)\cosh(bx)}{x}-\frac{\sinh(2bx)}{2x}$$

```
[Out] b*Chi(2*b*x)-cosh(b*x)*Shi(b*x)/x+1/2*b*Shi(b*x)^2-1/2*sinh(2*b*x)/x
```

Rubi [A] time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6550, 6686, 12, 5448, 3297, 3301}

$$b\text{Chi}(2bx)+\frac{1}{2}b\text{Shi}(bx)^2-\frac{\text{Shi}(bx)\cosh(bx)}{x}-\frac{\sinh(2bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[b*x]*SinhIntegral[b*x])/x^2,x]
```

```
[Out] b*CoshIntegral[2*b*x] - Sinh[2*b*x]/(2*x) - (Cosh[b*x]*SinhIntegral[b*x])/x
 + (b*SinhIntegral[b*x]^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 6550

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^(m + 1)*Cosh[a + b*x]*SinhInteg
ral[c + d*x])/(f*(m + 1)), x] + (-Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)
*Sinh[a + b*x]*SinhIntegral[c + d*x], x], x] - Dist[d/(f*(m + 1)), Int[((e
+ f*x)^(m + 1)*Cosh[a + b*x]*Sinh[c + d*x])/(c + d*x), x], x]) /; FreeQ[{a,
 b, c, d, e, f}, x] && ILtQ[m, -1]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(bx)\text{Shi}(bx)}{x^2}\,dx &= -\frac{\cosh(bx)\text{Shi}(bx)}{x} + b\int \frac{\cosh(bx)\sinh(bx)}{bx^2}\,dx + b\int \frac{\sinh(bx)\text{Shi}(bx)}{x}\,dx \\
&= -\frac{\cosh(bx)\text{Shi}(bx)}{x} + \frac{1}{2}b\text{Shi}(bx)^2 + \int \frac{\cosh(bx)\sinh(bx)}{x^2}\,dx \\
&= -\frac{\cosh(bx)\text{Shi}(bx)}{x} + \frac{1}{2}b\text{Shi}(bx)^2 + \int \frac{\sinh(2bx)}{2x^2}\,dx \\
&= -\frac{\cosh(bx)\text{Shi}(bx)}{x} + \frac{1}{2}b\text{Shi}(bx)^2 + \frac{1}{2}\int \frac{\sinh(2bx)}{x^2}\,dx \\
&= -\frac{\sinh(2bx)}{2x} - \frac{\cosh(bx)\text{Shi}(bx)}{x} + \frac{1}{2}b\text{Shi}(bx)^2 + b\int \frac{\cosh(2bx)}{x}\,dx \\
&= b\text{Chi}(2bx) - \frac{\sinh(2bx)}{2x} - \frac{\cosh(bx)\text{Shi}(bx)}{x} + \frac{1}{2}b\text{Shi}(bx)^2
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$b\text{Chi}(2bx) + \frac{1}{2}b\text{Shi}(bx)^2 - \frac{\text{Shi}(bx)\cosh(bx)}{x} - \frac{\sinh(2bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x^2,x]
```

```
[Out] b*CoshIntegral[2*b*x] - Sinh[2*b*x]/(2*x) - (Cosh[b*x]*SinhIntegral[b*x])/x
 + (b*SinhIntegral[b*x]^2)/2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(bx\right)\text{Shi}\left(bx\right)}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x)*sinh_integral(b*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(bx\right)\cosh\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)*cosh(b*x)/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx\right)\text{Shi}\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x)*Shi(b*x)/x^2,x)
```

```
[Out] int(cosh(b*x)*Shi(b*x)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(bx\right)\cosh\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x)*cosh(b*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{sinhint}\left(b\,x\right)\,\cosh\left(b\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(b*x)*cosh(b*x))/x^2,x)
```

```
[Out] int((sinhint(b*x)*cosh(b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx)\operatorname{Shi}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x**2,x)
```

```
[Out] Integral(cosh(b*x)*Shi(b*x)/x**2, x)
```

3.48 $\int \frac{\cosh(bx)\mathbf{Shi}(bx)}{x}\,dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\operatorname{Shi}(bx)\cosh(bx)}{x},x\right)$$

```
[Out] CannotIntegrate(cosh(b*x)*Shi(b*x)/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(bx)\operatorname{Shi}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[b*x]*SinhIntegral[b*x])/x,x]
[Out] Defer[Int][(Cosh[b*x]*SinhIntegral[b*x])/x, x]
```

Rubi steps

$$\int \frac{\cosh(bx)\operatorname{Shi}(bx)}{x}\,dx = \int \frac{\cosh(bx)\operatorname{Shi}(bx)}{x}\,dx$$

Mathematica [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx)\operatorname{Shi}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x,x]
[Out] Integrate[(Cosh[b*x]*SinhIntegral[b*x])/x, x]
```

fricas [A] time = 1.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cosh(bx)\operatorname{Shi}(bx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x)*sinh_integral(b*x)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx)\cosh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x,x, algorithm="giac")
[Out] integrate(Shi(b*x)*cosh(b*x)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cosh\,(bx)\,\text{Shi}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x)*Shi(b*x)/x,x)
[Out] int(cosh(b*x)*Shi(b*x)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx)\cosh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x,x, algorithm="maxima")
[Out] integrate(Shi(b*x)*cosh(b*x)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\text{sinhint}\,(b\,x)\,\cosh\,(b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(b*x)*cosh(b*x))/x,x)
[Out] int((sinhint(b*x)*cosh(b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\,(bx)\,\text{Shi}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x)/x,x)
```

```
[Out] Integral(cosh(b*x)*Shi(b*x)/x, x)
```

3.49 $\int \cosh(bx)\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\text{Chi}(2bx)}{2b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\log(x)}{2b}$$

```
[Out] -1/2*Chi(2*b*x)/b+1/2*ln(x)/b+Shi(b*x)*sinh(b*x)/b
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6546, 12, 3312, 3301}

$$-\frac{\text{Chi}(2bx)}{2b}+\frac{\text{Shi}(bx)\sinh(bx)}{b}+\frac{\log(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] -CoshIntegral[2*b*x]/(2*b) + Log[x]/(2*b) + (Sinh[b*x]*SinhIntegral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh(bx)\text{Shi}(bx)\,dx &= \frac{\sinh(bx)\text{Shi}(bx)}{b} - \int \frac{\sinh^2(bx)}{bx}\,dx \\
&= \frac{\sinh(bx)\text{Shi}(bx)}{b} - \frac{\int \frac{\sinh^2(bx)}{x}\,dx}{b} \\
&= \frac{\sinh(bx)\text{Shi}(bx)}{b} + \frac{\int \left(\frac{1}{2x} - \frac{\cosh(2bx)}{2x}\right)dx}{b} \\
&= \frac{\log(x)}{2b} + \frac{\sinh(bx)\text{Shi}(bx)}{b} - \frac{\int \frac{\cosh(2bx)}{x}\,dx}{2b} \\
&= -\frac{\text{Chi}(2bx)}{2b} + \frac{\log(x)}{2b} + \frac{\sinh(bx)\text{Shi}(bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.06

$$
-\frac{\text{Chi}(2bx)}{2b} + \frac{\text{Shi}(bx)\sinh(bx)}{b} + \frac{\log(bx)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] -1/2*CoshIntegral[2*b*x]/b + Log[b*x]/(2*b) + (Sinh[b*x]*SinhIntegral[b*x])
/b
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$
\text{integral}\left(\cosh\left(bx\right)\text{Shi}\left(bx\right), x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x)*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \text{Shi}\left(bx\right)\cosh\left(bx\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x),x, algorithm="giac")
```

```
[Out] integrate(Shi(b*x)*cosh(b*x), x)
```

maple [A] time = 0.02, size = 33, normalized size = 0.97

$$\frac{\mathrm{Shi}\left(bx\right)\sinh\left(bx\right)}{b}+\frac{\ln\left(bx\right)}{2b}-\frac{\mathrm{X}\left(2bx\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x)*Shi(b*x),x)
[Out] Shi(b*x)*sinh(b*x)/b+1/2/b*ln(b*x)-1/2*Chi(2*b*x)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x),x, algorithm="maxima")
[Out] integrate(Shi(b*x)*cosh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{sinhint}\left(b\,x\right)\,\cosh\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(b*x)*cosh(b*x),x)
[Out] int(sinhint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx\right)\mathrm{Shi}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x)*Shi(b*x),x)
[Out] Integral(cosh(b*x)*Shi(b*x), x)
```

3.50 $\int x \cosh(bx)\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=62

$$\frac{\text{Shi}(2bx)}{2b^2} - \frac{\text{Shi}(bx)\cosh(bx)}{b^2} - \frac{\sinh(bx)\cosh(bx)}{2b^2} + \frac{x\text{Shi}(bx)\sinh(bx)}{b} + \frac{x}{2b}$$

```
[Out] 1/2*x/b-cosh(b*x)*Shi(b*x)/b^2+1/2*Shi(2*b*x)/b^2-1/2*cosh(b*x)*sinh(b*x)/b
^2+x*Shi(b*x)*sinh(b*x)/b
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.700, Rules used = {6548, 12, 2635, 8, 6540, 5448, 3298}

$$\frac{\text{Shi}(2bx)}{2b^2} - \frac{\text{Shi}(bx)\cosh(bx)}{b^2} - \frac{\sinh(bx)\cosh(bx)}{2b^2} + \frac{x\text{Shi}(bx)\sinh(bx)}{b} + \frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] x/(2*b) - (Cosh[b*x]*Sinh[b*x])/(2*b^2) - (Cosh[b*x]*SinhIntegral[b*x])/b^2
 + (x*Sinh[b*x]*SinhIntegral[b*x])/b + SinhIntegral[2*b*x]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh(bx)\text{Shi}(bx)\,dx &= \frac{x\sinh(bx)\text{Shi}(bx)}{b} - \frac{\int \sinh(bx)\text{Shi}(bx)\,dx}{b} - \int \frac{\sinh^2(bx)}{b}\,dx\\
&= -\frac{\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{x\sinh(bx)\text{Shi}(bx)}{b} + \frac{\int \frac{\cosh(bx)\sinh(bx)}{bx}\,dx}{b} - \frac{\int \sinh^2(bx)\,dx}{b}\\
&= -\frac{\cosh(bx)\sinh(bx)}{2b^2} - \frac{\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{x\sinh(bx)\text{Shi}(bx)}{b} + \frac{\int \frac{\cosh(bx)\sinh(bx)}{x}\,dx}{b^2} +\\
&= \frac{x}{2b} - \frac{\cosh(bx)\sinh(bx)}{2b^2} - \frac{\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{x\sinh(bx)\text{Shi}(bx)}{b} + \frac{\int \frac{\sinh(2bx)}{2x}\,dx}{b^2}\\
&= \frac{x}{2b} - \frac{\cosh(bx)\sinh(bx)}{2b^2} - \frac{\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{x\sinh(bx)\text{Shi}(bx)}{b} + \frac{\int \frac{\sinh(2bx)}{x}\,dx}{2b^2}\\
&= \frac{x}{2b} - \frac{\cosh(bx)\sinh(bx)}{2b^2} - \frac{\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{x\sinh(bx)\text{Shi}(bx)}{b} + \frac{\text{Shi}(2bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 0.74

$$\frac{2\text{Shi}(2bx) + 4\text{Shi}(bx)(bx\sinh(bx) - \cosh(bx)) + 2bx - \sinh(2bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] (2*b*x - Sinh[2*b*x] + 4*(-Cosh[b*x] + b*x*Sinh[b*x])*SinhIntegral[b*x] + 2
*SinhIntegral[2*b*x])/(4*b^2)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cosh\left(bx\right)\text{Shi}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x)*Shi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*cosh(b*x)*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x)*Shi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*Shi(b*x)*cosh(b*x), x)
```

maple [A] time = 0.03, size = 46, normalized size = 0.74

$$\frac{\text{Shi}\left(bx\right)\left(bx\sinh\left(bx\right)-\cosh\left(bx\right)\right)-\frac{\sinh(bx)\cosh(bx)}{2}+\frac{bx}{2}+\frac{\text{Shi}(2bx)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x)*Shi(b*x),x)
```

```
[Out] 1/b^2*(Shi(b*x)*(b*x*sinh(b*x)-cosh(b*x))-1/2*sinh(b*x)*cosh(b*x)+1/2*b*x+1
/2*Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x)*Shi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x)*cosh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{sinhint}(b x) \cosh(b x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(b*x)*cosh(b*x),x)
[Out] int(x*sinhint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx) \operatorname{Shi}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x)*Shi(b*x),x)
[Out] Integral(x*cosh(b*x)*Shi(b*x), x)
```

3.51 $\int x^2 \cosh(bx)\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=98

$$-\frac{\text{Chi}(2bx)}{b^3}+\frac{2\text{Shi}(bx)\sinh(bx)}{b^3}+\frac{\log(x)}{b^3}+\frac{5\sinh^2(bx)}{4b^3}-\frac{2x\text{Shi}(bx)\cosh(bx)}{b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b^2}+\frac{x^2\text{Shi}(bx)\sin}{b}$$

```
[Out] 1/4*x^2/b-Chi(2*b*x)/b^3+ln(x)/b^3-2*x*cosh(b*x)*Shi(b*x)/b^2-1/2*x*cosh(b*
x)*sinh(b*x)/b^2+2*Shi(b*x)*sinh(b*x)/b^3+x^2*Shi(b*x)*sinh(b*x)/b+5/4*sinh
(b*x)^2/b^3
```

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6548, 12, 3310, 30, 6542, 2564, 6546, 3312, 3301}

$$-\frac{\text{Chi}(2bx)}{b^3}+\frac{2\text{Shi}(bx)\sinh(bx)}{b^3}-\frac{2x\text{Shi}(bx)\cosh(bx)}{b^2}+\frac{\log(x)}{b^3}+\frac{5\sinh^2(bx)}{4b^3}-\frac{x\sinh(bx)\cosh(bx)}{2b^2}+\frac{x^2\text{Shi}(bx)\sin}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] x^2/(4*b) - CoshIntegral[2*b*x]/b^3 + Log[x]/b^3 - (x*Cosh[b*x]*Sinh[b*x])/
(2*b^2) + (5*Sinh[b*x]^2)/(4*b^3) - (2*x*Cosh[b*x]*SinhIntegral[b*x])/b^2 +
 (2*Sinh[b*x]*SinhIntegral[b*x])/b^3 + (x^2*Sinh[b*x]*SinhIntegral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\cosh(bx)\text{Shi}(bx)\,dx = \frac{x^2\sinh(bx)\text{Shi}(bx)}{b} - \frac{2\int x\sinh(bx)\text{Shi}(bx)\,dx}{b} - \int \frac{x\sinh^2(bx)}{b}\,dx$$

$$= -\frac{2x\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{x^2\sinh(bx)\text{Shi}(bx)}{b} + \frac{2\int\cosh(bx)\text{Shi}(bx)\,dx}{b^2} - \frac{\int x\sinh^2(}{b}$$

$$= -\frac{x\cosh(bx)\sinh(bx)}{2b^2} + \frac{\sinh^2(bx)}{4b^3} - \frac{2x\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{2\sinh(bx)\text{Shi}(bx)}{b^3} + \frac{x}{}$$

$$= \frac{x^2}{4b} - \frac{x\cosh(bx)\sinh(bx)}{2b^2} + \frac{\sinh^2(bx)}{4b^3} - \frac{2x\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{2\sinh(bx)\text{Shi}(bx)}{b^3}$$

$$= \frac{x^2}{4b} - \frac{x\cosh(bx)\sinh(bx)}{2b^2} + \frac{5\sinh^2(bx)}{4b^3} - \frac{2x\cosh(bx)\text{Shi}(bx)}{b^2} + \frac{2\sinh(bx)\text{Shi}(bx}{b^3}$$

$$= \frac{x^2}{4b} + \frac{\log(x)}{b^3} - \frac{x\cosh(bx)\sinh(bx)}{2b^2} + \frac{5\sinh^2(bx)}{4b^3} - \frac{2x\cosh(bx)\text{Shi}(bx)}{b^2} + 2\sinh($$

$$= \frac{x^2}{4b} - \frac{\text{Chi}(2bx)}{b^3} + \frac{\log(x)}{b^3} - \frac{x\cosh(bx)\sinh(bx)}{2b^2} + \frac{5\sinh^2(bx)}{4b^3} - \frac{2x\cosh(bx)\text{Shi}(b}{b^2}$$

Mathematica [A] time = 0.09, size = 72, normalized size = 0.73

$$\frac{8\text{Shi}(bx)\left(\left(b^2x^2+2\right)\sinh(bx)-2bx\cosh(bx)\right)+2b^2x^2-8\text{Chi}(2bx)-2bx\sinh(2bx)+5\cosh(2bx)+8\log(x)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] (2*b^2*x^2 + 5*Cosh[2*b*x] - 8*CoshIntegral[2*b*x] + 8*Log[x] - 2*b*x*Sinh[
2*b*x] + 8*(-2*b*x*Cosh[b*x] + (2 + b^2*x^2)*Sinh[b*x])*SinhIntegral[b*x])/
(8*b^3)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\cosh(bx)\,\text{Shi}(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x)*Shi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh(b*x)*sinh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}(bx)\cosh(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x)*Shi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x)*cosh(b*x), x)
```

maple [A] time = 0.03, size = 95, normalized size = 0.97

$$\frac{x^2\,\mathrm{Shi}(bx)\sinh(bx)}{b}-\frac{2x\cosh(bx)\,\mathrm{Shi}(bx)}{b^2}+\frac{2\,\mathrm{Shi}(bx)\sinh(bx)}{b^3}-\frac{x\cosh(bx)\sinh(bx)}{2b^2}+\frac{x^2}{4b}+\frac{5\left(\cosh^2(bx)\right)}{4b^3}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x)*Shi(b*x),x)
```

```
[Out] x^2*Shi(b*x)*sinh(b*x)/b-2*x*cosh(b*x)*Shi(b*x)/b^2+2*Shi(b*x)*sinh(b*x)/b^
3-1/2*x*cosh(b*x)*sinh(b*x)/b^2+1/4*x^2/b+5/4*cosh(b*x)^2/b^3+1/b^3*ln(b*x)
-Chi(2*b*x)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}(bx)\cosh(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x)*Shi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi(b*x)*cosh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{sinhint}(b\,x)\,\cosh(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(b*x)*cosh(b*x),x)
```

```
[Out] int(x^2*sinhint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh(bx)\,\mathrm{Shi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x)*Shi(b*x),x)
```

```
[Out] Integral(x**2*cosh(b*x)*Shi(b*x), x)
```

3.52 $\int x^3 \cosh(bx)\mathbf{Shi}(bx)\,dx$

Optimal. Leaf size=128

$$\frac{3\mathrm{Shi}(2bx)}{b^4}-\frac{6\mathrm{Shi}(bx)\cosh(bx)}{b^4}-\frac{4\sinh(bx)\cosh(bx)}{b^4}+\frac{6x\mathrm{Shi}(bx)\sinh(bx)}{b^3}+\frac{4x}{b^3}+\frac{2x\sinh^2(bx)}{b^3}-\frac{3x^2\mathrm{Shi}(bx)\cosh(}{b^2}$$

```
[Out] 4*x/b^3+1/6*x^3/b-6*cosh(b*x)*Shi(b*x)/b^4-3*x^2*cosh(b*x)*Shi(b*x)/b^2+3*S
hi(2*b*x)/b^4-4*cosh(b*x)*sinh(b*x)/b^4-1/2*x^2*cosh(b*x)*sinh(b*x)/b^2+6*x
*Shi(b*x)*sinh(b*x)/b^3+x^3*Shi(b*x)*sinh(b*x)/b+2*x*sinh(b*x)^2/b^3
```

Rubi [A] time = 0.19, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6548, 12, 3311, 30, 2635, 8, 6542, 5372, 6540, 5448, 3298}

$$-\frac{3x^2\mathrm{Shi}(bx)\cosh(bx)}{b^2}+\frac{3\mathrm{Shi}(2bx)}{b^4}+\frac{6x\mathrm{Shi}(bx)\sinh(bx)}{b^3}-\frac{6\mathrm{Shi}(bx)\cosh(bx)}{b^4}-\frac{x^2\sinh(bx)\cosh(bx)}{2b^2}+\frac{4x}{b^3}+\frac{2x\sinh}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[b*x]*SinhIntegral[b*x],x]
```

```
[Out] (4*x)/b^3 + x^3/(6*b) - (4*Cosh[b*x]*Sinh[b*x])/b^4 - (x^2*Cosh[b*x]*Sinh[b
*x])/(2*b^2) + (2*x*Sinh[b*x]^2)/b^3 - (6*Cosh[b*x]*SinhIntegral[b*x])/b^4
- (3*x^2*Cosh[b*x]*SinhIntegral[b*x])/b^2 + (6*x*Sinh[b*x]*SinhIntegral[b*x
])/b^3 + (x^3*Sinh[b*x]*SinhIntegral[b*x])/b + (3*SinhIntegral[2*b*x])/b^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\cosh(bx)\mathrm{Shi}(bx)\,dx &= \frac{x^3\sinh(bx)\mathrm{Shi}(bx)}{b} - \frac{3\int x^2\sinh(bx)\mathrm{Shi}(bx)\,dx}{b} - \int \frac{x^2\sinh^2(bx)}{b}\,dx \\
&= -\frac{3x^2\cosh(bx)\mathrm{Shi}(bx)}{b^2} + \frac{x^3\sinh(bx)\mathrm{Shi}(bx)}{b} + \frac{6\int x\cosh(bx)\mathrm{Shi}(bx)\,dx}{b^2} - \frac{\int x^2 \sinh}{b} \\
&= -\frac{x^2\cosh(bx)\sinh(bx)}{2b^2} + \frac{x\sinh^2(bx)}{2b^3} - \frac{3x^2\cosh(bx)\mathrm{Shi}(bx)}{b^2} + \frac{6x\sinh(bx)\mathrm{Shi}(bx)}{b^3} \\
&= \frac{x^3}{6b} - \frac{\cosh(bx)\sinh(bx)}{4b^4} - \frac{x^2\cosh(bx)\sinh(bx)}{2b^2} + \frac{2x\sinh^2(bx)}{b^3} - \frac{6\cosh(bx)\mathrm{Shi}(bx)}{b^4} \\
&= \frac{x}{4b^3} + \frac{x^3}{6b} - \frac{4\cosh(bx)\sinh(bx)}{b^4} - \frac{x^2\cosh(bx)\sinh(bx)}{2b^2} + \frac{2x\sinh^2(bx)}{b^3} - \frac{6\cosh(bx}{b} \\
&= \frac{4x}{b^3} + \frac{x^3}{6b} - \frac{4\cosh(bx)\sinh(bx)}{b^4} - \frac{x^2\cosh(bx)\sinh(bx)}{2b^2} + \frac{2x\sinh^2(bx)}{b^3} - \frac{6\cosh(bx)}{b^4} \\
&= \frac{4x}{b^3} + \frac{x^3}{6b} - \frac{4\cosh(bx)\sinh(bx)}{b^4} - \frac{x^2\cosh(bx)\sinh(bx)}{2b^2} + \frac{2x\sinh^2(bx)}{b^3} - \frac{6\cosh(bx)}{b^4} \\
&= \frac{4x}{b^3} + \frac{x^3}{6b} - \frac{4\cosh(bx)\sinh(bx)}{b^4} - \frac{x^2\cosh(bx)\sinh(bx)}{2b^2} + \frac{2x\sinh^2(bx)}{b^3} - \frac{6\cosh(bx)}{b^4}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 94, normalized size = 0.73

$$\frac{2b^3x^3 + 12\mathrm{Shi}(bx)\left(bx\left(b^2x^2+6\right)\sinh(bx) - 3\left(b^2x^2+2\right)\cosh(bx)\right) - 3b^2x^2\sinh(2bx) + 36\mathrm{Shi}(2bx) + 36bx - 24}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[b*x]*SinhIntegral[b*x],x]
```

[Out] (36*b*x + 2*b^3*x^3 + 12*b*x*Cosh[2*b*x] - 24*Sinh[2*b*x] - 3*b^2*x^2*Sinh[2*b*x] + 12*(-3*(2 + b^2*x^2)*Cosh[b*x] + b*x*(6 + b^2*x^2)*Sinh[b*x])*SinhIntegral[b*x] + 36*SinhIntegral[2*b*x])/(12*b^4)

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{3} \cosh\left(bx\right) \text{Shi}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(b*x)*Shi(b*x),x, algorithm="fricas")

[Out] integral(x^3*cosh(b*x)*sinh_integral(b*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\text{Shi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(b*x)*Shi(b*x),x, algorithm="giac")

[Out] integrate(x^3*Shi(b*x)*cosh(b*x), x)

maple [A] time = 0.03, size = 104, normalized size = 0.81

$$\frac{\text{Shi}\left(bx\right)\left(\sinh\left(bx\right) b^{3} x^{3} - 3 b^{2} x^{2} \cosh\left(bx\right) + 6 b x \sinh\left(bx\right) - 6 \cosh\left(bx\right)\right) - \frac{b^{2} x^{2} \cosh(bx) \sinh(bx)}{2} + \frac{b^{3} x^{3}}{6} + 2 b x \left(\cosh\right.}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(b*x)*Shi(b*x),x)

[Out] 1/b^4*(Shi(b*x)*(sinh(b*x)*b^3*x^3-3*b^2*x^2*cosh(b*x)+6*b*x*sinh(b*x)-6*cosh(b*x))-1/2*b^2*x^2*cosh(b*x)*sinh(b*x)+1/6*b^3*x^3+2*b*x*cosh(b*x)^2-4*sinh(b*x)*cosh(b*x)+2*b*x+3*Shi(2*b*x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\text{Shi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(b*x)*Shi(b*x),x, algorithm="maxima")

[Out] integrate(x^3*Shi(b*x)*cosh(b*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{sinhint}(b\, x)\, \cosh(b\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinhint(b*x)*cosh(b*x),x)
```

```
[Out] int(x^3*sinhint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx) \operatorname{Shi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x)*Shi(b*x),x)
```

```
[Out] Integral(x**3*cosh(b*x)*Shi(b*x), x)
```

3.53 $\int \sinh(5x)\textbf{Shi}(2x)\,dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}(3x)}{10}-\frac{\text{Shi}(7x)}{10}+\frac{1}{5}\text{Shi}(2x)\cosh(5x)$$

```
[Out] 1/5*cosh(5*x)*Shi(2*x)+1/10*Shi(3*x)-1/10*Shi(7*x)
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6540, 12, 5472, 3298}

$$\frac{\text{Shi}(3x)}{10}-\frac{\text{Shi}(7x)}{10}+\frac{1}{5}\text{Shi}(2x)\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[5*x]*SinhIntegral[2*x],x]
```

```
[Out] (Cosh[5*x]*SinhIntegral[2*x])/5 + SinhIntegral[3*x]/10 - SinhIntegral[7*x]/
10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(5x)\text{Shi}(2x)\,dx &= \frac{1}{5}\cosh(5x)\text{Shi}(2x) - \frac{2}{5}\int \frac{\cosh(5x)\sinh(2x)}{2x}\,dx\\ &= \frac{1}{5}\cosh(5x)\text{Shi}(2x) - \frac{1}{5}\int \frac{\cosh(5x)\sinh(2x)}{x}\,dx\\ &= \frac{1}{5}\cosh(5x)\text{Shi}(2x) - \frac{1}{5}\int \left(-\frac{\sinh(3x)}{2x} + \frac{\sinh(7x)}{2x}\right)dx\\ &= \frac{1}{5}\cosh(5x)\text{Shi}(2x) + \frac{1}{10}\int \frac{\sinh(3x)}{x}\,dx - \frac{1}{10}\int \frac{\sinh(7x)}{x}\,dx\\ &= \frac{1}{5}\cosh(5x)\text{Shi}(2x) + \frac{\text{Shi}(3x)}{10} - \frac{\text{Shi}(7x)}{10}\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.86

$$\frac{1}{10}(\text{Shi}(3x) - \text{Shi}(7x) + 2\text{Shi}(2x)\cosh(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[5*x]*SinhIntegral[2*x],x]
```

```
[Out] (2*Cosh[5*x]*SinhIntegral[2*x] + SinhIntegral[3*x] - SinhIntegral[7*x])/10
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(2*x)*sinh(5*x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> An error occurred when FriCAS evaluated '(op
erator(Shi)((x)*(2)))*(sinh((x)*(5)))':   There are 1 exposed and 1 unexpos
ed library operations named elt having      1 argument(s) but none was dete
rmined to be applicable. Use HyperDoc      Browse, or issue
                )display op elt      to learn more about the available oper
ations. Perhaps       package-calling the operation or using coercions on t
he arguments       will allow you to apply the operation.    Cannot find ap
plication of object of type BasicOperator to argument(s)       of type(s)
                             Polynomial(Integer)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\,(2\,x)\sinh\,(5\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(2*x)*sinh(5*x),x, algorithm="giac")
```

```
[Out] integrate(Shi(2*x)*sinh(5*x), x)
```

maple [A] time = 0.12, size = 24, normalized size = 0.83

$$\frac{\cosh(5x)\,\mathrm{Shi}(2x)}{5} + \frac{\mathrm{Shi}(3x)}{10} - \frac{\mathrm{Shi}(7x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(2*x)*sinh(5*x),x)
```

```
[Out] 1/5*cosh(5*x)*Shi(2*x)+1/10*Shi(3*x)-1/10*Shi(7*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}(2\,x)\sinh(5\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(2*x)*sinh(5*x),x, algorithm="maxima")
```

```
[Out] integrate(Shi(2*x)*sinh(5*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{sinhint}(2\,x)\;\sinh(5\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(2*x)*sinh(5*x),x)
```

```
[Out] int(sinhint(2*x)*sinh(5*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(5x)\,\mathrm{Shi}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(2*x)*sinh(5*x),x)
```

```
[Out] Integral(sinh(5*x)*Shi(2*x), x)
```

3.54 $\int \cosh(5x)\mathbf{Shi}(2x)\,dx$

Optimal. Leaf size=29

$$\frac{\text{Chi}(3x)}{10}-\frac{\text{Chi}(7x)}{10}+\frac{1}{5}\text{Shi}(2x)\sinh(5x)$$

```
[Out] 1/10*Chi(3*x)-1/10*Chi(7*x)+1/5*Shi(2*x)*sinh(5*x)
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6546, 12, 5470, 3301}

$$\frac{\text{Chi}(3x)}{10}-\frac{\text{Chi}(7x)}{10}+\frac{1}{5}\text{Shi}(2x)\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[5*x]*SinhIntegral[2*x],x]
```

```
[Out] CoshIntegral[3*x]/10 - CoshIntegral[7*x]/10 + (Sinh[5*x]*SinhIntegral[2*x])
/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5470

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh(5x)\mathrm{Shi}(2x)\,dx &= \frac{1}{5}\sinh(5x)\mathrm{Shi}(2x)-\frac{2}{5}\int \frac{\sinh(2x)\sinh(5x)}{2x}\,dx\\
&= \frac{1}{5}\sinh(5x)\mathrm{Shi}(2x)-\frac{1}{5}\int \frac{\sinh(2x)\sinh(5x)}{x}\,dx\\
&= \frac{1}{5}\sinh(5x)\mathrm{Shi}(2x)-\frac{1}{5}\int \left(-\frac{\cosh(3x)}{2x}+\frac{\cosh(7x)}{2x}\right)dx\\
&= \frac{1}{5}\sinh(5x)\mathrm{Shi}(2x)+\frac{1}{10}\int \frac{\cosh(3x)}{x}\,dx-\frac{1}{10}\int \frac{\cosh(7x)}{x}\,dx\\
&= \frac{\mathrm{Chi}(3x)}{10}-\frac{\mathrm{Chi}(7x)}{10}+\frac{1}{5}\sinh(5x)\mathrm{Shi}(2x)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.86

$$\frac{1}{10}(\mathrm{Chi}(3x)-\mathrm{Chi}(7x)+2\mathrm{Shi}(2x)\sinh(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[5*x]*SinhIntegral[2*x],x]
```

```
[Out] (CoshIntegral[3*x] - CoshIntegral[7*x] + 2*Sinh[5*x]*SinhIntegral[2*x])/10
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(5*x)*Shi(2*x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> An error occurred when FriCAS evaluated '(op
erator(Shi)((x)*(2)))*(cosh((x)*(5)))':   There are 1 exposed and 1 unexpos
ed library operations named elt having      1 argument(s) but none was dete
rmined to be applicable. Use HyperDoc      Browse, or issue
              )display op elt      to learn more about the available oper
ations. Perhaps       package-calling the operation or using coercions on t
he arguments       will allow you to apply the operation.    Cannot find ap
plication of object of type BasicOperator to argument(s)       of type(s)
                              Polynomial(Integer)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\,(2\,x)\cosh\,(5\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(5*x)*Shi(2*x),x, algorithm="giac")
```

```
[Out] integrate(Shi(2*x)*cosh(5*x), x)
```

maple [A] time = 0.12, size = 24, normalized size = 0.83

$$\frac{\mathrm{X}(3x)}{10} - \frac{\mathrm{X}(7x)}{10} + \frac{\mathrm{Shi}(2x)\sinh(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(5*x)*Shi(2*x),x)
```

```
[Out] 1/10*Chi(3*x)-1/10*Chi(7*x)+1/5*Shi(2*x)*sinh(5*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}(2\,x)\cosh(5\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(5*x)*Shi(2*x),x, algorithm="maxima")
```

```
[Out] integrate(Shi(2*x)*cosh(5*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{sinhint}(2\,x)\,\cosh(5\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(2*x)*cosh(5*x),x)
```

```
[Out] int(sinhint(2*x)*cosh(5*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(5x)\,\mathrm{Shi}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(5*x)*Shi(2*x),x)
```

```
[Out] Integral(cosh(5*x)*Shi(2*x), x)
```

3.55 $\int x^2 \sinh(a + bx)\mathbf{Shi}(a + bx)\, dx$

Optimal. Leaf size=186

$$-\frac{a^2\text{Shi}(2a+2bx)}{2b^3}-\frac{a\text{Chi}(2a+2bx)}{b^3}-\frac{\text{Shi}(2a+2bx)}{b^3}+\frac{2\text{Shi}(a+bx)\cosh(a+bx)}{b^3}+\frac{a\log(a+bx)}{b^3}+\frac{\sinh(2a+2bx}{8b^3}$$

```
[Out] -x/b^2-a*Chi(2*b*x+2*a)/b^3+1/4*a*cosh(2*b*x+2*a)/b^3-1/4*x*cosh(2*b*x+2*a)
/b^2+a*ln(b*x+a)/b^3+2*cosh(b*x+a)*Shi(b*x+a)/b^3+x^2*cosh(b*x+a)*Shi(b*x+a
)/b-Shi(2*b*x+2*a)/b^3-1/2*a^2*Shi(2*b*x+2*a)/b^3+cosh(b*x+a)*sinh(b*x+a)/b
^3-2*x*Shi(b*x+a)*sinh(b*x+a)/b^2+1/8*sinh(2*b*x+2*a)/b^3
```

Rubi [A] time = 0.58, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6542, 5617, 6741, 6742, 2638, 3296, 2637, 3298, 6548, 2635, 8, 3312, 3301, 6540, 5448, 12}

$$-\frac{a^2\text{Shi}(2a+2bx)}{2b^3}-\frac{a\text{Chi}(2a+2bx)}{b^3}-\frac{\text{Shi}(2a+2bx)}{b^3}-\frac{2x\text{Shi}(a+bx)\sinh(a+bx)}{b^2}+\frac{2\text{Shi}(a+bx)\cosh(a+bx)}{b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] -(x/b^2) + (a*Cosh[2*a + 2*b*x])/(4*b^3) - (x*Cosh[2*a + 2*b*x])/(4*b^2) -
(a*CoshIntegral[2*a + 2*b*x])/b^3 + (a*Log[a + b*x])/b^3 + (Cosh[a + b*x]*S
inh[a + b*x])/b^3 + Sinh[2*a + 2*b*x]/(8*b^3) + (2*Cosh[a + b*x]*SinhIntegr
al[a + b*x])/b^3 + (x^2*Cosh[a + b*x]*SinhIntegral[a + b*x])/b - (2*x*Sinh[
a + b*x]*SinhIntegral[a + b*x])/b^2 - SinhIntegral[2*a + 2*b*x]/b^3 - (a^2*
SinhIntegral[2*a + 2*b*x])/(2*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \sinh(a+bx)\text{Shi}(a+bx)\,dx = \frac{x^2\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{2\int x\cosh(a+bx)\text{Shi}(a+bx)\,dx}{b} - \int \frac{x^2\cosh}{}$$

$$= \frac{x^2\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{2x\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{1}{2}\int \frac{x^2\sinh(2(a}{a+b}$$

$$= \frac{2\cosh(a+bx)\text{Shi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{2x\sinh(a+bx)\text{S}}{b^2}$$

$$= \frac{2\cosh(a+bx)\text{Shi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{2x\sinh(a+bx)\text{S}}{b^2}$$

$$= \frac{\cosh(a+bx)\sinh(a+bx)}{b^3} + \frac{2\cosh(a+bx)\text{Shi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{Sh}}{b}$$

$$= -\frac{x}{b^2} + \frac{a\cosh(2a+2bx)}{4b^3} - \frac{x\cosh(2a+2bx)}{4b^2} + \frac{a\log(a+bx)}{b^3} + \frac{\cosh(a+bx)}{b}$$

$$= -\frac{x}{b^2} + \frac{a\cosh(2a+2bx)}{4b^3} - \frac{x\cosh(2a+2bx)}{4b^2} - \frac{a\text{Chi}(2a+2bx)}{b^3} + \frac{a\log(a+b}{b^3}$$

Mathematica [A] time = 0.32, size = 123, normalized size = 0.66

$$\frac{-4a^2\text{Shi}(2(a+bx)) + 8\text{Shi}(a+bx)\left(\left(b^2x^2+2\right)\cosh(a+bx) - 2bx\sinh(a+bx)\right) - 8a\text{Chi}(2(a+bx)) - 8\text{Shi}(2(a}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] (-8*b*x + 2*a*Cosh[2*(a + b*x)] - 2*b*x*Cosh[2*(a + b*x)] - 8*a*CoshIntegra
l[2*(a + b*x)] + 8*a*Log[a + b*x] + 5*Sinh[2*(a + b*x)] + 8*((2 + b^2*x^2)*
Cosh[a + b*x] - 2*b*x*Sinh[a + b*x])*SinhIntegral[a + b*x] - 8*SinhIntegral
[2*(a + b*x)] - 4*a^2*SinhIntegral[2*(a + b*x)])/(8*b^3)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\sinh\left(bx+a\right)\text{Shi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x + a)*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}\,(bx+a)\sinh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.03, size = 170, normalized size = 0.91

$$\frac{x^2\cosh\,(bx+a)\,\mathrm{Shi}\,(bx+a)}{b}-\frac{2x\,\mathrm{Shi}\,(bx+a)\sinh\,(bx+a)}{b^2}+\frac{2\cosh\,(bx+a)\,\mathrm{Shi}\,(bx+a)}{b^3}-\frac{\left(\cosh^2(bx+a)\right)x}{2b^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Shi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] x^2*cosh(b*x+a)*Shi(b*x+a)/b-2*x*Shi(b*x+a)*sinh(b*x+a)/b^2+2*cosh(b*x+a)*S
hi(b*x+a)/b^3-1/2/b^2*cosh(b*x+a)^2*x+1/2/b^3*a*cosh(b*x+a)^2+5/4*cosh(b*x+
a)*sinh(b*x+a)/b^3-3/4*x/b^2-3/4/b^3*a-Shi(2*b*x+2*a)/b^3+a*ln(b*x+a)/b^3-a
*Chi(2*b*x+2*a)/b^3-1/2*a^2*Shi(2*b*x+2*a)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Shi}\,(bx+a)\sinh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Shi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi(b*x + a)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{sinhint}\,(a+b\,x)\;\sinh\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(a + b*x)*sinh(a + b*x),x)
```

```
[Out] int(x^2*sinhint(a + b*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh\,(a+bx)\,\mathrm{Shi}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Shi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*Shi(a + b*x), x)
```

3.56 $\int x\sinh(a+bx)\mathbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=97

$$\frac{\text{Chi}(2a+2bx)}{2b^2}+\frac{a\text{Shi}(2a+2bx)}{2b^2}-\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b^2}-\frac{\log(a+bx)}{2b^2}-\frac{\cosh(2a+2bx)}{4b^2}+\frac{x\text{Shi}(a+bx)\cosh(a}{b}$$

```
[Out] 1/2*Chi(2*b*x+2*a)/b^2-1/4*cosh(2*b*x+2*a)/b^2-1/2*ln(b*x+a)/b^2+x*cosh(b*x
+a)*Shi(b*x+a)/b+1/2*a*Shi(2*b*x+2*a)/b^2-Shi(b*x+a)*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.25, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6542, 5617, 6741, 6742, 2638, 3298, 6546, 3312, 3301}

$$\frac{\text{Chi}(2a+2bx)}{2b^2}+\frac{a\text{Shi}(2a+2bx)}{2b^2}-\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b^2}-\frac{\log(a+bx)}{2b^2}-\frac{\cosh(2a+2bx)}{4b^2}+\frac{x\text{Shi}(a+bx)\cosh(a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] -Cosh[2*a + 2*b*x]/(4*b^2) + CoshIntegral[2*a + 2*b*x]/(2*b^2) - Log[a + b*
x]/(2*b^2) + (x*Cosh[a + b*x]*SinhIntegral[a + b*x])/b - (Sinh[a + b*x]*Sin
hIntegral[a + b*x])/b^2 + (a*SinhIntegral[2*a + 2*b*x])/(2*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\sinh(a+bx)\text{Shi}(a+bx)\,dx = \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\int\cosh(a+bx)\text{Shi}(a+bx)\,dx}{b} - \int\frac{x\cosh(a+
$$

$$= \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{1}{2}\int\frac{x\sinh(2(a+b}{a+bx}$$

$$= \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{1}{2}\int\frac{x\sinh(2a+2b}{a+bx}$$

$$= -\frac{\log(a+bx)}{2b^2} + \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b^2} - \frac{1}{2}\int$$

$$= \frac{\text{Chi}(2a+2bx)}{2b^2} - \frac{\log(a+bx)}{2b^2} + \frac{x\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\sinh(a+bx)\text{Sh}}{b^2}$$

$$= -\frac{\cosh(2a+2bx)}{4b^2} + \frac{\text{Chi}(2a+2bx)}{2b^2} - \frac{\log(a+bx)}{2b^2} + \frac{x\cosh(a+bx)\text{Shi}(a+b}{b}$$

Mathematica [A] time = 0.17, size = 73, normalized size = 0.75

$$\frac{2\text{Chi}(2(a+bx)) + 2a\text{Shi}(2(a+bx)) + 4\text{Shi}(a+bx)(bx\cosh(a+bx) - \sinh(a+bx)) - 2\log(a+bx) - \cosh(2(a}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] (-Cosh[2*(a + b*x)] + 2*CoshIntegral[2*(a + b*x)] - 2*Log[a + b*x] + 4*(b*x
*Cosh[a + b*x] - Sinh[a + b*x])*SinhIntegral[a + b*x] + 2*a*SinhIntegral[2*
(a + b*x)])/(4*b^2)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(x\sinh\left(bx+a\right)\text{Shi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x + a)*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*Shi(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.03, size = 89, normalized size = 0.92

$$\frac{x\cosh(bx+a)\operatorname{Shi}(bx+a)}{b}-\frac{\operatorname{Shi}(bx+a)\sinh(bx+a)}{b^2}-\frac{\cosh^2(bx+a)}{2b^2}-\frac{\ln(bx+a)}{2b^2}+\frac{\mathrm{X}(2bx+2a)}{2b^2}+\frac{a\operatorname{Shi}(2bx+}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] x*cosh(b*x+a)*Shi(b*x+a)/b-Shi(b*x+a)*sinh(b*x+a)/b^2-1/2/b^2*cosh(b*x+a)^2
-1/2*ln(b*x+a)/b^2+1/2*Chi(2*b*x+2*a)/b^2+1/2*a*Shi(2*b*x+2*a)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{Shi}(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x + a)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{sinhint}(a+b\,x)\,\sinh(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(a + b*x)*sinh(a + b*x),x)
```

```
[Out] int(x*sinhint(a + b*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh(a+bx)\operatorname{Shi}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*Shi(a + b*x), x)
```

3.57 $\int \sinh(a+bx)\mathbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}$$

```
[Out] cosh(b*x+a)*Shi(b*x+a)/b-1/2*Shi(2*b*x+2*a)/b
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6540, 5448, 12, 3298}

$$\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] (Cosh[a + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh(a+bx)\text{Shi}(a+bx)\,dx &= \frac{\cosh(a+bx)\text{Shi}(a+bx)}{b} - \int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}\,dx \\
&= \frac{\cosh(a+bx)\text{Shi}(a+bx)}{b} - \int \frac{\sinh(2a+2bx)}{2(a+bx)}\,dx \\
&= \frac{\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}\,dx \\
&= \frac{\cosh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\text{Shi}(2a+2bx)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.97

$$
\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b} - \frac{\text{Shi}(2(a+bx))}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*SinhIntegral[a + b*x],x]
[Out] (Cosh[a + b*x]*SinhIntegral[a + b*x])/b - SinhIntegral[2*(a + b*x)]/(2*b)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$
\text{integral}\left(\sinh\left(bx+a\right)\text{Shi}\left(bx+a\right),x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] integral(sinh(b*x + a)*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \text{Shi}\left(bx+a\right)\sinh\left(bx+a\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(Shi(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.00, size = 30, normalized size = 0.91

$$
\frac{\cosh\left(bx+a\right)\text{Shi}\left(bx+a\right) - \frac{\text{Shi}(2bx+2a)}{2}}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b*(cosh(b*x+a)*Shi(b*x+a)-1/2*Shi(2*b*x+2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \text{sinhint}\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)*sinh(a + b*x),x)
```

```
[Out] int(sinhint(a + b*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\text{Shi}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*Shi(a + b*x), x)
```

3.58 $\int \frac{\sinh(a+bx)\mathbf{Shi}(a+bx)}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\mathrm{Shi}(a+bx)\sinh(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(Shi(b*x+a)*sinh(b*x+a)/x,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*SinhIntegral[a + b*x])/x,x]
[Out] Defer[Int][(Sinh[a + b*x]*SinhIntegral[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*SinhIntegral[a + b*x])/x,x]
[Out] Integrate[(Sinh[a + b*x]*SinhIntegral[a + b*x])/x, x]
```

fricas [A] time = 1.15, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sinh\left(bx+a\right)\mathrm{Shi}\left(bx+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)*sinh_integral(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)\sinh\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(Shi(b*x + a)*sinh(b*x + a)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)\sinh\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(b*x+a)*sinh(b*x+a)/x,x)
[Out] int(Shi(b*x+a)*sinh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(bx+a)\sinh\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(Shi(b*x + a)*sinh(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\text{sinhint}\,(a+b\,x)\,\sinh\,(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(a + b*x)*sinh(a + b*x))/x,x)
[Out] int((sinhint(a + b*x)*sinh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\,(a+bx)\,\text{Shi}\,(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(b*x+a)*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*Shi(a + b*x)/x, x)
```

3.59 $\int x^2 \cosh(a+bx)\textbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=219

$$-\frac{a^2\text{Chi}(2a+2bx)}{2b^3}+\frac{a^2\log(a+bx)}{2b^3}-\frac{\text{Chi}(2a+2bx)}{b^3}-\frac{a\text{Shi}(2a+2bx)}{b^3}+\frac{2\text{Shi}(a+bx)\sinh(a+bx)}{b^3}+\frac{\log(a+bx)}{b^3}+$$

```
[Out] -1/2*a*x/b^2+1/4*x^2/b-Chi(2*b*x+2*a)/b^3-1/2*a^2*Chi(2*b*x+2*a)/b^3+1/2*co
sh(2*b*x+2*a)/b^3+ln(b*x+a)/b^3+1/2*a^2*ln(b*x+a)/b^3-2*x*cosh(b*x+a)*Shi(b
*x+a)/b^2-a*Shi(2*b*x+2*a)/b^3+1/2*a*cosh(b*x+a)*sinh(b*x+a)/b^3-1/2*x*cosh
(b*x+a)*sinh(b*x+a)/b^2+2*Shi(b*x+a)*sinh(b*x+a)/b^3+x^2*Shi(b*x+a)*sinh(b*
x+a)/b+1/4*sinh(b*x+a)^2/b^3
```

Rubi [A] time = 0.70, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6548, 6742, 2635, 8, 3310, 30, 3312, 3301, 6542, 5617, 6741, 2638, 3298, 6546}

$$-\frac{a^2\text{Chi}(2a+2bx)}{2b^3}+\frac{a^2\log(a+bx)}{2b^3}-\frac{\text{Chi}(2a+2bx)}{b^3}-\frac{a\text{Shi}(2a+2bx)}{b^3}+\frac{2\text{Shi}(a+bx)\sinh(a+bx)}{b^3}-\frac{2x\text{Shi}(a+bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] -(a*x)/(2*b^2) + x^2/(4*b) + Cosh[2*a + 2*b*x]/(2*b^3) - CoshIntegral[2*a +
 2*b*x]/b^3 - (a^2*CoshIntegral[2*a + 2*b*x])/(2*b^3) + Log[a + b*x]/b^3 +
(a^2*Log[a + b*x])/(2*b^3) + (a*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^3) - (x*C
osh[a + b*x]*Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) - (2*x*Cosh[a
 + b*x]*SinhIntegral[a + b*x])/b^2 + (2*Sinh[a + b*x]*SinhIntegral[a + b*x]
)/b^3 + (x^2*Sinh[a + b*x]*SinhIntegral[a + b*x])/b - (a*SinhIntegral[2*a +
 2*b*x])/b^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\cosh(a+bx)\text{Shi}(a+bx)\,dx = \frac{x^2\sinh(a+bx)\text{Shi}(a+bx)}{b} - \frac{2\int x\sinh(a+bx)\text{Shi}(a+bx)\,dx}{b} - \int \frac{x^2\sinh^2}{a+}$$

$$= -\frac{2x\cosh(a+bx)\text{Shi}(a+bx)}{b^2} + \frac{x^2\sinh(a+bx)\text{Shi}(a+bx)}{b} + \frac{2\int\cosh(a+b}{}$$

$$= -\frac{2x\cosh(a+bx)\text{Shi}(a+bx)}{b^2} + \frac{2\sinh(a+bx)\text{Shi}(a+bx)}{b^3} + \frac{x^2\sinh(a+bx)}{b}$$

$$= \frac{a\cosh(a+bx)\sinh(a+bx)}{2b^3} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3}$$

$$= -\frac{ax}{2b^2} + \frac{x^2}{4b} + \frac{\log(a+bx)}{b^3} + \frac{a^2\log(a+bx)}{2b^3} + \frac{a\cosh(a+bx)\sinh(a+bx)}{2b^3} -$$

$$= -\frac{ax}{2b^2} + \frac{x^2}{4b} - \frac{\text{Chi}(2a+2bx)}{b^3} - \frac{a^2\text{Chi}(2a+2bx)}{2b^3} + \frac{\log(a+bx)}{b^3} + \frac{a^2\log(a+}{2b^3}$$

$$= -\frac{ax}{2b^2} + \frac{x^2}{4b} + \frac{\cosh(2a+2bx)}{2b^3} - \frac{\text{Chi}(2a+2bx)}{b^3} - \frac{a^2\text{Chi}(2a+2bx)}{2b^3} + \frac{\log(a}{b^3}$$

Mathematica [A] time = 0.33, size = 134, normalized size = 0.61

$$\frac{-4\left(a^2+2\right)\text{Chi}(2(a+bx)) + 4a^2\log(a+bx) + 8\text{Shi}(a+bx)\left(\left(b^2x^2+2\right)\sinh(a+bx) - 2bx\cosh(a+bx)\right) - 8a\text{S}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] (-4*a*b*x + 2*b^2*x^2 + 5*Cosh[2*(a + b*x)] - 4*(2 + a^2)*CoshIntegral[2*(a
 + b*x)] + 8*Log[a + b*x] + 4*a^2*Log[a + b*x] + 2*a*Sinh[2*(a + b*x)] - 2*
b*x*Sinh[2*(a + b*x)] + 8*(-2*b*x*Cosh[a + b*x] + (2 + b^2*x^2)*Sinh[a + b*
x])*SinhIntegral[a + b*x] - 8*a*SinhIntegral[2*(a + b*x)])/(8*b^3)
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\cosh\left(bx+a\right)\text{Shi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh(b*x + a)*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \text{Shi}\,(bx + a) \cosh\,(bx + a)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*Shi(b*x + a)*cosh(b*x + a), x)
```

maple [A] time = 0.04, size = 198, normalized size = 0.90

$$\frac{x^2 \,\text{Shi}\,(bx + a) \sinh\,(bx + a)}{b} - \frac{2x \cosh\,(bx + a)\,\text{Shi}\,(bx + a)}{b^2} + \frac{2 \,\text{Shi}\,(bx + a) \sinh\,(bx + a)}{b^3} - \frac{x \cosh\,(bx + a) \sinh\,(}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*Shi(b*x+a),x)
```

```
[Out] x^2*Shi(b*x+a)*sinh(b*x+a)/b-2*x*cosh(b*x+a)*Shi(b*x+a)/b^2+2*Shi(b*x+a)*si
nh(b*x+a)/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*a*cosh(b*x+a)*sinh(b*x+
a)/b^3+1/4*x^2/b-1/2*a*x/b^2-3/4/b^3*a^2+5/4*cosh(b*x+a)^2/b^3+ln(b*x+a)/b^
3-Chi(2*b*x+2*a)/b^3-a*Shi(2*b*x+2*a)/b^3+1/2*a^2*ln(b*x+a)/b^3-1/2*a^2*Chi
(2*b*x+2*a)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \text{Shi}\,(bx + a) \cosh\,(bx + a)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Shi(b*x + a)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \,\text{sinhint}\,(a + b\,x)\; \cosh\,(a + b\,x)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinhint(a + b*x)*cosh(a + b*x),x)
```

```
[Out] int(x^2*sinhint(a + b*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(a+bx) \operatorname{Shi}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*Shi(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x)*Shi(a + b*x), x)
```

3.60 $\int x \cosh(a+bx)\mathbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{a\text{Chi}(2a+2bx)}{2b^2}+\frac{\text{Shi}(2a+2bx)}{2b^2}-\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b^2}-\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x\text{Shi}(a$$

```
[Out] 1/2*x/b+1/2*a*Chi(2*b*x+2*a)/b^2-1/2*a*ln(b*x+a)/b^2-cosh(b*x+a)*Shi(b*x+a)
/b^2+1/2*Shi(2*b*x+2*a)/b^2-1/2*cosh(b*x+a)*sinh(b*x+a)/b^2+x*Shi(b*x+a)*si
nh(b*x+a)/b
```

Rubi [A] time = 0.21, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6548, 6742, 2635, 8, 3312, 3301, 6540, 5448, 12, 3298}

$$\frac{a\text{Chi}(2a+2bx)}{2b^2}+\frac{\text{Shi}(2a+2bx)}{2b^2}-\frac{\text{Shi}(a+bx)\cosh(a+bx)}{b^2}-\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x\text{Shi}(a$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] x/(2*b) + (a*CoshIntegral[2*a + 2*b*x])/(2*b^2) - (a*Log[a + b*x])/(2*b^2)
- (Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) - (Cosh[a + b*x]*SinhIntegral[a + b
*x])/b^2 + (x*Sinh[a + b*x]*SinhIntegral[a + b*x])/b + SinhIntegral[2*a + 2
*b*x]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\cosh(a+bx)\text{Shi}(a+bx)\,dx = \frac{x\sinh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\int \sinh(a+bx)\text{Shi}(a+bx)\,dx}{b} - \int \frac{x\sinh^2(a+}{a+bx}$$
$$= -\frac{\cosh(a+bx)\text{Shi}(a+bx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}(a+bx)}{b} + \frac{\int \frac{\cosh(a+bx)\sinh(a+}{a+bx}}{b}$$
$$= -\frac{\cosh(a+bx)\text{Shi}(a+bx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}(a+bx)}{b} - \frac{\int \sinh^2(a+bx)\,d}{b}$$
$$= -\frac{\cosh(a+bx)\sinh(a+bx)}{2b^2} - \frac{\cosh(a+bx)\text{Shi}(a+bx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}}{b}$$
$$= \frac{x}{2b} - \frac{a\log(a+bx)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+bx)}{2b^2} - \frac{\cosh(a+bx)\text{Shi}(a+bx)}{b^2}$$
$$= \frac{x}{2b} + \frac{a\text{Chi}(2a+2bx)}{2b^2} - \frac{a\log(a+bx)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+bx)}{2b^2} - \cosh($$

Mathematica [A] time = 0.19, size = 78, normalized size = 0.72

$$\frac{2a\text{Chi}(2(a+bx)) + 2\text{Shi}(2(a+bx)) + 4\text{Shi}(a+bx)(bx\sinh(a+bx) - \cosh(a+bx)) - 2a\log(a+bx) - \sinh(2(}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] (2*b*x + 2*a*CoshIntegral[2*(a + b*x)] - 2*a*Log[a + b*x] - Sinh[2*(a + b*x
)] + 4*(-Cosh[a + b*x] + b*x*Sinh[a + b*x])*SinhIntegral[a + b*x] + 2*SinhI
ntegral[2*(a + b*x)])/(4*b^2)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cosh\left(bx+a\right)\text{Shi}\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*cosh(b*x + a)*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(bx+a\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*Shi(b*x + a)*cosh(b*x + a), x)
```

maple [A] time = 0.03, size = 106, normalized size = 0.97

$$\frac{x\,\mathrm{Shi}\left(bx+a\right)\sinh\left(bx+a\right)}{b}-\frac{\cosh\left(bx+a\right)\mathrm{Shi}\left(bx+a\right)}{b^{2}}-\frac{\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2b^{2}}+\frac{x}{2b}+\frac{a}{2b^{2}}+\frac{\mathrm{Shi}\left(2bx+2a\right)}{2b^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*Shi(b*x+a),x)
```

```
[Out] x*Shi(b*x+a)*sinh(b*x+a)/b-cosh(b*x+a)*Shi(b*x+a)/b^2-1/2*cosh(b*x+a)*sinh(
b*x+a)/b^2+1/2*x/b+1/2/b^2*a+1/2*Shi(2*b*x+2*a)/b^2-1/2*a*ln(b*x+a)/b^2+1/2
*a*Chi(2*b*x+2*a)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(bx+a\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(b*x + a)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{sinhint}\left(a+b\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(a + b*x)*cosh(a + b*x),x)
```

```
[Out] int(x*sinhint(a + b*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+bx\right)\mathrm{Shi}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(b*x+a),x)
```

```
[Out] Integral(x*cosh(a + b*x)*Shi(a + b*x), x)
```

3.61 $\int \cosh(a+bx)\textbf{Shi}(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}$$

```
[Out] -1/2*Chi(2*b*x+2*a)/b+1/2*ln(b*x+a)/b+Shi(b*x+a)*sinh(b*x+a)/b
```

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6546, 3312, 3301}

$$-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Shi}(a+bx)\sinh(a+bx)}{b}+\frac{\log(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*SinhIntegral[a + b*x],x]
```

```
[Out] -CoshIntegral[2*a + 2*b*x]/(2*b) + Log[a + b*x]/(2*b) + (Sinh[a + b*x]*Sinh
Integral[a + b*x])/b
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\text{Shi}(a+bx)\,dx = \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b} - \int \frac{\sinh^2(a+bx)}{a+bx}\,dx$$
$$= \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b} + \int \left(\frac{1}{2(a+bx)} - \frac{\cosh(2a+2bx)}{2(a+bx)}\right)dx$$
$$= \frac{\log(a+bx)}{2b} + \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b} - \frac{1}{2}\int \frac{\cosh(2a+2bx)}{a+bx}\,dx$$
$$= -\frac{\text{Chi}(2a+2bx)}{2b} + \frac{\log(a+bx)}{2b} + \frac{\sinh(a+bx)\text{Shi}(a+bx)}{b}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.98

$$-\frac{\text{Chi}(2(a+bx))}{2b} + \frac{\text{Shi}(a+bx)\sinh(a+bx)}{b} + \frac{\log(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*SinhIntegral[a + b*x],x]
[Out] -1/2*CoshIntegral[2*(a + b*x)]/b + Log[a + b*x]/(2*b) + (Sinh[a + b*x]*Sinh
Integral[a + b*x])/b
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\text{Shi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(b*x+a),x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*sinh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(bx+a\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(b*x+a),x, algorithm="giac")
[Out] integrate(Shi(b*x + a)*cosh(b*x + a), x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.93

$$-\frac{\text{X}\left(2bx+2a\right)}{2b} + \frac{\ln\left(bx+a\right)}{2b} + \frac{\text{Shi}\left(bx+a\right)\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*Shi(b*x+a),x)
```

```
[Out] -1/2*Chi(2*b*x+2*a)/b+1/2*ln(b*x+a)/b+Shi(b*x+a)*sinh(b*x+a)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\left(bx+a\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Shi(b*x + a)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{sinhint}\left(a+b\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(a + b*x)*cosh(a + b*x),x)
```

```
[Out] int(sinhint(a + b*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\mathrm{Shi}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)*Shi(a + b*x), x)
```

3.62 $\int \frac{\cosh(a+bx)\mathbf{Shi}(a+bx)}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\mathrm{Shi}(a+bx)\cosh(a+bx)}{x},x\right)$$

[Out] CannotIntegrate(cosh(b*x+a)*Shi(b*x+a)/x,x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[a + b*x]*SinhIntegral[a + b*x])/x,x]

[Out] Defer[Int][(Cosh[a + b*x]*SinhIntegral[a + b*x])/x, x]

Rubi steps

$$\int \frac{\cosh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\mathrm{Shi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cosh[a + b*x]*SinhIntegral[a + b*x])/x,x]

[Out] Integrate[(Cosh[a + b*x]*SinhIntegral[a + b*x])/x, x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cosh(bx+a)\,\mathrm{Shi}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)*Shi(b*x+a)/x,x, algorithm="fricas")

```
[Out] integral(cosh(b*x + a)*sinh_integral(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\left(bx+a\right)\cosh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(b*x+a)/x,x, algorithm="giac")
[Out] integrate(Shi(b*x + a)*cosh(b*x + a)/x, x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)\mathrm{Shi}\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*Shi(b*x+a)/x,x)
[Out] int(cosh(b*x+a)*Shi(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Shi}\left(bx+a\right)\cosh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(Shi(b*x + a)*cosh(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{sinhint}\left(a+b\,x\right)\,\cosh\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(a + b*x)*cosh(a + b*x))/x,x)
[Out] int((sinhint(a + b*x)*cosh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+bx\right)\mathrm{Shi}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(b*x+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x)*Shi(a + b*x)/x, x)
```

3.63 $\int x \sinh(a+bx)\mathbf{Shi}(c+dx)\,dx$

Optimal. Leaf size=371

$$-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-}{d}\right.}{2b^2}$$

```
[Out] -1/2*Chi(c*(b-d)/d+(b-d)*x)*cosh(a-b*c/d)/b^2+1/2*Chi(c*(b+d)/d+(b+d)*x)*co
sh(a-b*c/d)/b^2+1/2*cosh(a-c+(b-d)*x)/b/(b-d)-1/2*cosh(a+c+(b+d)*x)/b/(b+d)
-1/2*c*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b/d+x*cosh(b*x+a)*Shi(d*x+c)/b+
1/2*c*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b/d-1/2*c*Chi(c*(b-d)/d+(b-d)*x)
*sinh(a-b*c/d)/b/d+1/2*c*Chi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b/d-1/2*Shi(c
*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b^2+1/2*Shi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d
)/b^2-Shi(d*x+c)*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.96, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6542, 6742, 5618, 2638, 5472, 3303, 3298, 3301, 6546, 5470}

$$-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-}{d}\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] Cosh[a - c + (b - d)*x]/(2*b*(b - d)) - Cosh[a + c + (b + d)*x]/(2*b*(b + d
)) - (Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))/d + (b - d)*x])/(2*b^2) +
(Cosh[a - (b*c)/d]*CoshIntegral[(c*(b + d))/d + (b + d)*x])/(2*b^2) - (c*Co
shIntegral[(c*(b - d))/d + (b - d)*x]*Sinh[a - (b*c)/d])/(2*b*d) + (c*CoshI
ntegral[(c*(b + d))/d + (b + d)*x]*Sinh[a - (b*c)/d])/(2*b*d) - (c*Cosh[a -
 (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b*d) - (Sinh[a - (b*c
)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b^2) + (x*Cosh[a + b*x]*Si
nhIntegral[c + d*x])/b - (Sinh[a + b*x]*SinhIntegral[c + d*x])/b^2 + (c*Cos
h[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b*d) + (Sinh[a -
 (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5470

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 6542

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*SinhIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\sinh(a+bx)\text{Shi}(c+dx)\,dx=\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{\int\cosh(a+bx)\text{Shi}(c+dx)\,dx}{b}-\frac{d\int\frac{x\cosh(a+bx)\ldots}{c+d\ldots}}{b}\ldots$$

$$=\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b^2}+\frac{d\int\frac{\sinh(a+bx)\sinh(c+d\ldots}{c+dx}}{b^2}\ldots$$

$$=\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b^2}-\frac{\int\cosh(a+bx)\sinh\ldots}{b}\ldots$$

$$=\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b^2}-\frac{\int\left(-\frac{1}{2}\sinh(a-c+\ldots\right.}{\ldots}\ldots$$

$$=\frac{x\cosh(a+bx)\text{Shi}(c+dx)}{b}-\frac{\sinh(a+bx)\text{Shi}(c+dx)}{b^2}+\frac{\int\sinh(a-c+(b-\ldots}{2b}\ldots$$

$$=\frac{\cosh(a-c+(b-d)x)}{2b(b-d)}-\frac{\cosh(a+c+(b+d)x)}{2b(b+d)}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-d)}{d}\ldots\right.}{2b^2}\ldots$$

$$=\frac{\cosh(a-c+(b-d)x)}{2b(b-d)}-\frac{\cosh(a+c+(b+d)x)}{2b(b+d)}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-d)}{d}\ldots\right.}{2b^2}\ldots$$

Mathematica [B] time = 16.34, size = 887, normalized size = 2.39

$$\frac{4dx\cosh(a+bx)\text{Shi}(c+dx)b^3-c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3-c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3+2c\cos\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] (2*b^2*d*Cosh[a - c + b*x - d*x] + 2*b*d^2*Cosh[a - c + b*x - d*x] - 2*b^2*
d*Cosh[a + c + (b + d)*x] + 2*b*d^2*Cosh[a + c + (b + d)*x] - 2*(b^2 - d^2)
*CoshIntegral[-(((b - d)*(c + d*x))/d)]*(d*Cosh[a - (b*c)/d] + b*c*Sinh[a -
 (b*c)/d]) + 2*(b^2 - d^2)*CoshIntegral[((b + d)*(c + d*x))/d]*(d*Cosh[a -
(b*c)/d] + b*c*Sinh[a - (b*c)/d]) + 4*b^3*d*x*Cosh[a + b*x]*SinhIntegral[c
+ d*x] - 4*b*d^3*x*Cosh[a + b*x]*SinhIntegral[c + d*x] - 4*b^2*d*Sinh[a + b
*x]*SinhIntegral[c + d*x] + 4*d^3*Sinh[a + b*x]*SinhIntegral[c + d*x] - b^3
*c*Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] - b^2*d*Cosh[a - (
b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b*c*d^2*Cosh[a - (b*c)/d]*Sin
hIntegral[((b - d)*(c + d*x))/d] + d^3*Cosh[a - (b*c)/d]*SinhIntegral[((b -
 d)*(c + d*x))/d] - b^3*c*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x)
)/d] - b^2*d*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b*c*d^
2*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + d^3*Sinh[a - (b*c
)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + 2*b^3*c*Cosh[a - (b*c)/d]*SinhIn
tegral[((b + d)*(c + d*x))/d] - 2*b*c*d^2*Cosh[a - (b*c)/d]*SinhIntegral[((
b + d)*(c + d*x))/d] + 2*b^2*d*Sinh[a - (b*c)/d]*SinhIntegral[((b + d)*(c +
 d*x))/d] - 2*d^3*Sinh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] + b
^3*c*Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] - b^2*d*Cosh[a
 - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] - b*c*d^2*Cosh[a - (b*c)/
d]*SinhIntegral[c - (b*c)/d - b*x + d*x] + d^3*Cosh[a - (b*c)/d]*SinhIntegr
al[c - (b*c)/d - b*x + d*x] - b^3*c*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c
)/d - b*x + d*x] + b^2*d*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x +
 d*x] + b*c*d^2*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] - d
^3*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x])/(4*b^2*(b - d)*
d*(b + d))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(x\sinh\left(bx+a\right)\text{Shi}\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x + a)*sinh_integral(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*x+c)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*Shi(d*x + c)*sinh(b*x + a), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int x \operatorname{Shi}(dx+c)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Shi(d*x+c)*sinh(b*x+a),x)
[Out] int(x*Shi(d*x+c)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}(dx+c)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*Shi(d*x + c)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{sinhint}(c+d\,x)\,\sinh(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(c + d*x)*sinh(a + b*x),x)
[Out] int(x*sinhint(c + d*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh(a+bx)\operatorname{Shi}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Shi(d*x+c)*sinh(b*x+a),x)
[Out] Integral(x*sinh(a + b*x)*Shi(c + d*x), x)
```

3.64 $\int \sinh(a+bx)\mathbf{Shi}(c+dx)\,dx$

Optimal. Leaf size=153

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}$$

```
[Out] 1/2*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b+cosh(b*x+a)*Shi(d*x+c)/b-1/2*cos
h(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b+1/2*Chi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d
)/b-1/2*Chi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b
```

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6540, 5472, 3303, 3298, 3301}

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] (CoshIntegral[(c*(b - d))/d + (b - d)*x]*Sinh[a - (b*c)/d])/(2*b) - (CoshIn
tegral[(c*(b + d))/d + (b + d)*x]*Sinh[a - (b*c)/d])/(2*b) + (Cosh[a - (b*c
)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b) + (Cosh[a + b*x]*SinhIn
tegral[c + d*x])/b - (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d
)*x])/(2*b)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\text{Shi}(c+dx)\,dx &= \frac{\cosh(a+bx)\text{Shi}(c+dx)}{b} - \frac{d\int \frac{\cosh(a+bx)\sinh(c+dx)}{c+dx}\,dx}{b}\\
&= \frac{\cosh(a+bx)\text{Shi}(c+dx)}{b} - \frac{d\int \left(-\frac{\sinh(a-c+(b-d)x)}{2(c+dx)} + \frac{\sinh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}\\
&= \frac{\cosh(a+bx)\text{Shi}(c+dx)}{b} + \frac{d\int \frac{\sinh(a-c+(b-d)x)}{c+dx}\,dx}{2b} - \frac{d\int \frac{\sinh(a+c+(b+d)x)}{c+dx}\,dx}{2b}\\
&= \frac{\cosh(a+bx)\text{Shi}(c+dx)}{b} + \frac{\left(d\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{c(b-d)}{d}+(b-d)x\right)}{c+dx}\,dx}{2b} - \frac{\left(d\cosh\right.}{}\\
&= \frac{\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b} - \frac{\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b} +\end{aligned}$$

Mathematica [A] time = 2.15, size = 209, normalized size = 1.37

$$\frac{2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(-\frac{(b-d)(c+dx)}{d}\right) - 2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{(b+d)(c+dx)}{d}\right) + \sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{(b-d)(c+dx)}{d}\right) + \sinh\left(a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] (2*CoshIntegral[-(((b - d)*(c + d*x))/d)]*Sinh[a - (b*c)/d] - 2*CoshIntegra
l[((b + d)*(c + d*x))/d]*Sinh[a - (b*c)/d] + 4*Cosh[a + b*x]*SinhIntegral[c
 + d*x] + Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + Sinh[a -
(b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] - 2*Cosh[a - (b*c)/d]*SinhInte
gral[((b + d)*(c + d*x))/d] - Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d -
b*x + d*x] + Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x])/(4*b)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sinh\left(bx+a\right)\mathrm{Shi}\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)*sinh_integral(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(Shi(d*x + c)*sinh(b*x + a), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(d*x+c)*sinh(b*x+a),x)
```

```
[Out] int(Shi(d*x+c)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Shi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Shi(d*x + c)*sinh(b*x + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{sinhint}\left(c+d\,x\right)\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(c + d*x)*sinh(a + b*x),x)
```

```
[Out] int(sinhint(c + d*x)*sinh(a + b*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\mathrm{Shi}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*Shi(c + d*x), x)
```

3.65 $\int \frac{\sinh(a+bx)\mathbf{Shi}(c+dx)}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\sinh(a+bx)\mathrm{Shi}(c+dx)}{x},x\right)$$

[Out] CannotIntegrate(Shi(d*x+c)*sinh(b*x+a)/x,x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(c+dx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Sinh[a + b*x]*SinhIntegral[c + d*x])/x,x]

[Out] Defer[Int][(Sinh[a + b*x]*SinhIntegral[c + d*x])/x, x]

Rubi steps

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(c+dx)}{x}\,dx = \int \frac{\sinh(a+bx)\mathrm{Shi}(c+dx)}{x}\,dx$$

Mathematica [A] time = 10.41, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\mathrm{Shi}(c+dx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sinh[a + b*x]*SinhIntegral[c + d*x])/x,x]

[Out] Integrate[(Sinh[a + b*x]*SinhIntegral[c + d*x])/x, x]

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sinh(bx+a)\,\mathrm{Shi}(dx+c)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(Shi(d*x+c)*sinh(b*x+a)/x,x, algorithm="fricas")

```
[Out] integral(sinh(b*x + a)*sinh_integral(d*x + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(dx+c)\sinh\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*x+c)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(Shi(d*x + c)*sinh(b*x + a)/x, x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(dx+c)\sinh\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Shi(d*x+c)*sinh(b*x+a)/x,x)
[Out] int(Shi(d*x+c)*sinh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\,(dx+c)\sinh\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*x+c)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(Shi(d*x + c)*sinh(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\text{sinhint}\,(c+d\,x)\ \sinh\,(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(c + d*x)*sinh(a + b*x))/x,x)
[Out] int((sinhint(c + d*x)*sinh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\,(a+bx)\,\text{Shi}\,(c+dx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Shi(d*x+c)*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*Shi(c + d*x)/x, x)
```

3.66 $\int x \cosh(a+bx)\textbf{Shi}(c+dx)\,dx$

Optimal. Leaf size=371

$$-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-}{d}\right.}{2b^2}$$

```
[Out] -1/2*c*Chi(c*(b-d)/d+(b-d)*x)*cosh(a-b*c/d)/b/d+1/2*c*Chi(c*(b+d)/d+(b+d)*x
)*cosh(a-b*c/d)/b/d-1/2*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b^2-cosh(b*x+a
)*Shi(d*x+c)/b^2+1/2*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b^2-1/2*Chi(c*(b-
d)/d+(b-d)*x)*sinh(a-b*c/d)/b^2+1/2*Chi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b^
2-1/2*c*Shi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b/d+1/2*c*Shi(c*(b+d)/d+(b+d)*
x)*sinh(a-b*c/d)/b/d+x*Shi(d*x+c)*sinh(b*x+a)/b+1/2*sinh(a-c+(b-d)*x)/b/(b-
d)-1/2*sinh(a+c+(b+d)*x)/b/(b+d)
```

Rubi [A] time = 1.32, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6548, 5642, 6742, 2637, 3303, 3298, 3301, 6540, 5472}

$$-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-}{d}\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] -(c*Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))/d + (b - d)*x])/(2*b*d) + (c
*Cosh[a - (b*c)/d]*CoshIntegral[(c*(b + d))/d + (b + d)*x])/(2*b*d) - (Cosh
Integral[(c*(b - d))/d + (b - d)*x]*Sinh[a - (b*c)/d])/(2*b^2) + (CoshInteg
ral[(c*(b + d))/d + (b + d)*x]*Sinh[a - (b*c)/d])/(2*b^2) + Sinh[a - c + (b
 - d)*x]/(2*b*(b - d)) - Sinh[a + c + (b + d)*x]/(2*b*(b + d)) - (Cosh[a -
(b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b^2) - (c*Sinh[a - (b*
c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b*d) - (Cosh[a + b*x]*Sin
hIntegral[c + d*x])/b^2 + (x*Sinh[a + b*x]*SinhIntegral[c + d*x])/b + (Cosh
[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b^2) + (c*Sinh[a
- (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b*d)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 5642

```
Int[(u_.)*Sinh[(a_.) + (b_.)*(x_)]^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.), x_
Symbol] :> Int[ExpandTrigReduce[u, Sinh[a + b*x]^m*Sinh[c + d*x]^n, x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6540

```
Int[Sinh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6548

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*SinhIntegral[(c_.)
+ (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*SinhIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Sinh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*SinhIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\cosh(a+bx)\text{Shi}(c+dx)\,dx = \frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b} - \frac{\int \sinh(a+bx)\text{Shi}(c+dx)\,dx}{b} - \frac{d\int \frac{x\sinh(a+bx)\,\text{s}}{c+dx}}{b}$$

$$= -\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b} + \frac{d\int \frac{\cosh(a+bx)\sinh(c}{c+dx}}{b^2}$$

$$= -\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b} + \frac{d\int\left(-\frac{\sinh(a-c+(b-d}{2(c+dx)}\right.}{}$$

$$= -\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b} - \frac{d\int \frac{\sinh(a-c+(b-d)x)}{c+dx}}{2b^2}$$

$$= -\frac{\cosh(a+bx)\text{Shi}(c+dx)}{b^2} + \frac{x\sinh(a+bx)\text{Shi}(c+dx)}{b} + \frac{\int \cosh(a-c+(b}{2b}$$

$$= -\frac{\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b^2} + \frac{\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right.}{2b^2}$$

$$= -\frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} + \frac{c\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b+d)}{d}+(b+\right.}{2bd}$$

Mathematica [B] time = 7.25, size = 887, normalized size = 2.39

$$\frac{4dx\sinh(a+bx)\text{Shi}(c+dx)b^3 - c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3 - c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3 + 2c\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] (-2*(b^2 - d^2)*CoshIntegral[-(((b - d)*(c + d*x))/d)]*(b*c*Cosh[a - (b*c)/
d] + d*Sinh[a - (b*c)/d]) + 2*(b^2 - d^2)*CoshIntegral[((b + d)*(c + d*x))/
d]*(b*c*Cosh[a - (b*c)/d] + d*Sinh[a - (b*c)/d]) + 2*b^2*d*Sinh[a - c + b*x
 - d*x] + 2*b*d^2*Sinh[a - c + b*x - d*x] - 2*b^2*d*Sinh[a + c + (b + d)*x]
```

```
 + 2*b*d^2*Sinh[a + c + (b + d)*x] - 4*b^2*d*Cosh[a + b*x]*SinhIntegral[c +
 d*x] + 4*d^3*Cosh[a + b*x]*SinhIntegral[c + d*x] + 4*b^3*d*x*Sinh[a + b*x]
*SinhIntegral[c + d*x] - 4*b*d^3*x*Sinh[a + b*x]*SinhIntegral[c + d*x] - b^
3*c*Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] - b^2*d*Cosh[a -
(b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b*c*d^2*Cosh[a - (b*c)/d]*Si
nhIntegral[((b - d)*(c + d*x))/d] + d^3*Cosh[a - (b*c)/d]*SinhIntegral[((b
- d)*(c + d*x))/d] - b^3*c*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x
))/d] - b^2*d*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b*c*d
^2*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + d^3*Sinh[a - (b*
c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + 2*b^2*d*Cosh[a - (b*c)/d]*SinhI
ntegral[((b + d)*(c + d*x))/d] - 2*d^3*Cosh[a - (b*c)/d]*SinhIntegral[((b +
 d)*(c + d*x))/d] + 2*b^3*c*Sinh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*
x))/d] - 2*b*c*d^2*Sinh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] -
b^3*c*Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] + b^2*d*Cosh[
a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] + b*c*d^2*Cosh[a - (b*c)
/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] - d^3*Cosh[a - (b*c)/d]*SinhInteg
ral[c - (b*c)/d - b*x + d*x] + b^3*c*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*
c)/d - b*x + d*x] - b^2*d*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x
+ d*x] - b*c*d^2*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] +
d^3*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x])/(4*b^2*(b - d)
*d*(b + d))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cosh\left(bx+a\right)\text{Shi}\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x*cosh(b*x + a)*sinh_integral(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Shi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(x*Shi(d*x + c)*cosh(b*x + a), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\text{Shi}\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*Shi(d*x+c),x)
```

```
[Out] int(x*cosh(b*x+a)*Shi(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Shi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(x*Shi(d*x + c)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{sinhint}\left(c+d\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinhint(c + d*x)*cosh(a + b*x),x)
```

```
[Out] int(x*sinhint(c + d*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+bx\right)\mathrm{Shi}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*Shi(d*x+c),x)
```

```
[Out] Integral(x*cosh(a + b*x)*Shi(c + d*x), x)
```

3.67 $\int \cosh(a+bx)\mathbf{Shi}(c+dx)\,dx$

Optimal. Leaf size=153

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}$$

```
[Out] 1/2*Chi(c*(b-d)/d+(b-d)*x)*cosh(a-b*c/d)/b-1/2*Chi(c*(b+d)/d+(b+d)*x)*cosh(
a-b*c/d)/b+1/2*Shi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b-1/2*Shi(c*(b+d)/d+(b+
d)*x)*sinh(a-b*c/d)/b+Shi(d*x+c)*sinh(b*x+a)/b
```

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6546, 5470, 3303, 3298, 3301}

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}-\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] (Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))/d + (b - d)*x])/(2*b) - (Cosh[a
 - (b*c)/d]*CoshIntegral[(c*(b + d))/d + (b + d)*x])/(2*b) + (Sinh[a - (b*c
)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b) + (Sinh[a + b*x]*SinhIn
tegral[c + d*x])/b - (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d
)*x])/(2*b)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5470

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rule 6546

```
Int[Cosh[(a_.) + (b_.)*(x_)]*SinhIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*SinhIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Sinh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\text{Shi}(c+dx)\,dx = \frac{\sinh(a+bx)\text{Shi}(c+dx)}{b} - \frac{d\int \frac{\sinh(a+bx)\sinh(c+dx)}{c+dx}\,dx}{b}$$
$$= \frac{\sinh(a+bx)\text{Shi}(c+dx)}{b} - \frac{d\int \left(-\frac{\cosh(a-c+(b-d)x)}{2(c+dx)} + \frac{\cosh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}$$
$$= \frac{\sinh(a+bx)\text{Shi}(c+dx)}{b} + \frac{d\int \frac{\cosh(a-c+(b-d)x)}{c+dx}\,dx}{2b} - \frac{d\int \frac{\cosh(a+c+(b+d)x)}{c+dx}\,dx}{2b}$$
$$= \frac{\sinh(a+bx)\text{Shi}(c+dx)}{b} + \frac{\left(d\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{c(b-d)}{d}+(b-d)x\right)}{c+dx}\,dx}{2b} - \frac{\left(d\cosh\right.}{}$$
$$= \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} - \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)}{2b} +$$

Mathematica [A] time = 0.57, size = 209, normalized size = 1.37

$$\frac{2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(-\frac{(b-d)(c+dx)}{d}\right) - 2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{(b+d)(c+dx)}{d}\right) + 4\sinh(a+bx)\text{Shi}(c+dx) + \sinh\left(a-\frac{bc}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*SinhIntegral[c + d*x],x]
```

```
[Out] (2*Cosh[a - (b*c)/d]*CoshIntegral[-(((b - d)*(c + d*x))/d)] - 2*Cosh[a - (b
*c)/d]*CoshIntegral[((b + d)*(c + d*x))/d] + 4*Sinh[a + b*x]*SinhIntegral[c
 + d*x] + Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + Sinh[a -
(b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] - 2*Sinh[a - (b*c)/d]*SinhInte
gral[((b + d)*(c + d*x))/d] + Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d -
b*x + d*x] - Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x])/(4*b)
```

fricas [F] time = 1.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\text{Shi}\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*sinh_integral(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(Shi(d*x + c)*cosh(b*x + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)\text{Shi}\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*Shi(d*x+c),x)
```

```
[Out] int(cosh(b*x+a)*Shi(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Shi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(Shi(d*x + c)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{sinhint}\left(c+d\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinhint(c + d*x)*cosh(a + b*x),x)
```

```
[Out] int(sinhint(c + d*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\mathrm{Shi}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*Shi(c + d*x), x)
```

3.68 $\int \frac{\cosh(a+bx)\textbf{Shi}(c+dx)}{x}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cosh(a+bx)\text{Shi}(c+dx)}{x},x\right)$$

```
[Out] CannotIntegrate(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*SinhIntegral[c + d*x])/x,x]
```

```
[Out] Defer[Int][(Cosh[a + b*x]*SinhIntegral[c + d*x])/x, x]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx = \int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx$$

Mathematica [A] time = 10.42, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\text{Shi}(c+dx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*SinhIntegral[c + d*x])/x,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*SinhIntegral[c + d*x])/x, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)\,\text{Shi}(dx+c)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*sinh_integral(d*x + c)/x, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(dx + c\right) \cosh\left(bx + a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c)/x,x, algorithm="giac")
[Out] integrate(Shi(d*x + c)*cosh(b*x + a)/x, x)
```

maple [**A**] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx + a\right) \text{Shi}\left(dx + c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*Shi(d*x+c)/x,x)
[Out] int(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Shi}\left(dx + c\right) \cosh\left(bx + a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c)/x,x, algorithm="maxima")
[Out] integrate(Shi(d*x + c)*cosh(b*x + a)/x, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{sinhint}\left(c + d\,x\right)\,\cosh\left(a + b\,x\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinhint(c + d*x)*cosh(a + b*x))/x,x)
[Out] int((sinhint(c + d*x)*cosh(a + b*x))/x, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a + bx\right) \text{Shi}\left(c + dx\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*Shi(d*x+c)/x,x)
```

```
[Out] Integral(cosh(a + b*x)*Shi(c + d*x)/x, x)
```

3.69 $\int x^m \mathbf{Chi}(bx)\, dx$

Optimal. Leaf size=76

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b(m+1)}+\frac{x^m(bx)^{-m}\Gamma(m+1,bx)}{2b(m+1)}$$

```
[Out] x^(1+m)*Chi(b*x)/(1+m)-1/2*x^m*GAMMA(1+m,-b*x)/b/(1+m)/((-b*x)^m)+1/2*x^m*G
AMMA(1+m,b*x)/b/(1+m)/((b*x)^m)
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6533, 12, 3307, 2181}

$$-\frac{x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b(m+1)}+\frac{x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{2b(m+1)}+\frac{x^{m+1}\text{Chi}(bx)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*CoshIntegral[b*x],x]
```

```
[Out] (x^(1 + m)*CoshIntegral[b*x])/(1 + m) - (x^m*Gamma[1 + m, -(b*x)])/(2*b*(1
+ m)*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(2*b*(1 + m)*(b*x)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m\text{Chi}(bx)\,dx &= \frac{x^{1+m}\text{Chi}(bx)}{1+m}-\frac{b\int\frac{x^m\cosh(bx)}{b}\,dx}{1+m}\\ &= \frac{x^{1+m}\text{Chi}(bx)}{1+m}-\frac{\int x^m\cosh(bx)\,dx}{1+m}\\ &= \frac{x^{1+m}\text{Chi}(bx)}{1+m}-\frac{\int e^{-bx}x^m\,dx}{2(1+m)}-\frac{\int e^{bx}x^m\,dx}{2(1+m)}\\ &= \frac{x^{1+m}\text{Chi}(bx)}{1+m}-\frac{x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b(1+m)}+\frac{x^m(bx)^{-m}\Gamma(1+m,bx)}{2b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.97

$$\frac{x^{m+1}\text{Chi}(bx)}{m+1}-\frac{-x^{m+1}(-bx)^{-m-1}\Gamma(m+1,-bx)-x^{m+1}(bx)^{-m-1}\Gamma(m+1,bx)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*CoshIntegral[b*x],x]
```

```
[Out] (x^(1 + m)*CoshIntegral[b*x])/(1 + m) - (-(x^(1 + m)*(-(b*x))^(-1 - m)*Gamm
a[1 + m, -(b*x)]) - x^(1 + m)*(b*x)^(-1 - m)*Gamma[1 + m, b*x])/(2*(1 + m))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{Chi}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\text{Chi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x),x, algorithm="giac")
[Out] integrate(x^m*Chi(b*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^m \mathrm{X}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Chi(b*x),x)
[Out] int(x^m*Chi(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Chi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x),x, algorithm="maxima")
[Out] integrate(x^m*Chi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \,\mathrm{coshint}(b\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*coshint(b*x),x)
[Out] int(x^m*coshint(b*x), x)
```

sympy [B] time = 1.56, size = 649, normalized size = 8.54

$$\frac{4\cdot 2^m b^{-m} m x \sqrt{e^{-2m\log(2)} e^{m\log\left(b^2x^2\right)}}\,\log\left(b^2x^2\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{8m^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+16m\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{8\cdot 2^m \gamma b^{-m} m x \sqrt{e^{-2m\log(2)} e^{m\log\left(b^2x^2\right)}}\,\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{8m^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+16m\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{4\cdot 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Chi(b*x),x)
```

```
[Out] 4*2**m*b**(-m)*m*x*sqrt(exp(-2*m*log(2))*exp(m*log(b**2*x**2)))*log(b**2*x*
*2)*gamma(m/2 + 5/2)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/2) + 8*g
amma(m/2 + 5/2)) + 8*2**m*EulerGamma*b**(-m)*m*x*sqrt(exp(-2*m*log(2))*exp(
m*log(b**2*x**2)))*gamma(m/2 + 5/2)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m
/2 + 5/2) + 8*gamma(m/2 + 5/2)) + 4*2**m*b**(-m)*x*sqrt(exp(-2*m*log(2))*ex
p(m*log(b**2*x**2)))*log(b**2*x**2)*gamma(m/2 + 5/2)/(8*m**2*gamma(m/2 + 5/
2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2)) - 8*2**m*b**(-m)*x*sqrt(ex
p(-2*m*log(2))*exp(m*log(b**2*x**2)))*gamma(m/2 + 5/2)/(8*m**2*gamma(m/2 +
5/2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2)) + 8*2**m*EulerGamma*b**(
-m)*x*sqrt(exp(-2*m*log(2))*exp(m*log(b**2*x**2)))*gamma(m/2 + 5/2)/(8*m**2
*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2)) + b**2*m**2
*x**3*x**m*gamma(m/2 + 3/2)*hyper((1, 1, m/2 + 3/2), (3/2, 2, 2, m/2 + 5/2)
, b**2*x**2/4)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m
/2 + 5/2)) + 2*b**2*m*x**3*x**m*gamma(m/2 + 3/2)*hyper((1, 1, m/2 + 3/2), (
3/2, 2, 2, m/2 + 5/2), b**2*x**2/4)/(8*m**2*gamma(m/2 + 5/2) + 16*m*gamma(m
/2 + 5/2) + 8*gamma(m/2 + 5/2)) + b**2*x**3*x**m*gamma(m/2 + 3/2)*hyper((1,
 1, m/2 + 3/2), (3/2, 2, 2, m/2 + 5/2), b**2*x**2/4)/(8*m**2*gamma(m/2 + 5/
2) + 16*m*gamma(m/2 + 5/2) + 8*gamma(m/2 + 5/2))
```

3.70 $\int x^3 \mathbf{Chi}(bx)\, dx$

Optimal. Leaf size=63

$$\frac{3\cosh(bx)}{2b^4} - \frac{3x\sinh(bx)}{2b^3} + \frac{3x^2\cosh(bx)}{4b^2} + \frac{1}{4}x^4\text{Chi}(bx) - \frac{x^3\sinh(bx)}{4b}$$

```
[Out] 1/4*x^4*Chi(b*x)+3/2*cosh(b*x)/b^4+3/4*x^2*cosh(b*x)/b^2-3/2*x*sinh(b*x)/b^
3-1/4*x^3*sinh(b*x)/b
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6533, 12, 3296, 2638}

$$\frac{3x^2\cosh(bx)}{4b^2} - \frac{3x\sinh(bx)}{2b^3} + \frac{3\cosh(bx)}{2b^4} + \frac{1}{4}x^4\text{Chi}(bx) - \frac{x^3\sinh(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*CoshIntegral[b*x],x]
```

```
[Out] (3*Cosh[b*x])/(2*b^4) + (3*x^2*Cosh[b*x])/(4*b^2) + (x^4*CoshIntegral[b*x])
/4 - (3*x*Sinh[b*x])/(2*b^3) - (x^3*Sinh[b*x])/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
```

```
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3\text{Chi}(bx)\,dx &= \frac{1}{4}x^4\text{Chi}(bx) - \frac{1}{4}b\int \frac{x^3\cosh(bx)}{b}\,dx \\
&= \frac{1}{4}x^4\text{Chi}(bx) - \frac{1}{4}\int x^3\cosh(bx)\,dx \\
&= \frac{1}{4}x^4\text{Chi}(bx) - \frac{x^3\sinh(bx)}{4b} + \frac{3\int x^2\sinh(bx)\,dx}{4b} \\
&= \frac{3x^2\cosh(bx)}{4b^2} + \frac{1}{4}x^4\text{Chi}(bx) - \frac{x^3\sinh(bx)}{4b} - \frac{3\int x\cosh(bx)\,dx}{2b^2} \\
&= \frac{3x^2\cosh(bx)}{4b^2} + \frac{1}{4}x^4\text{Chi}(bx) - \frac{3x\sinh(bx)}{2b^3} - \frac{x^3\sinh(bx)}{4b} + \frac{3\int \sinh(bx)\,dx}{2b^3} \\
&= \frac{3\cosh(bx)}{2b^4} + \frac{3x^2\cosh(bx)}{4b^2} + \frac{1}{4}x^4\text{Chi}(bx) - \frac{3x\sinh(bx)}{2b^3} - \frac{x^3\sinh(bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.84

$$
\frac{3\left(b^2x^2+2\right)\cosh(bx)}{4b^4} - \frac{x\left(b^2x^2+6\right)\sinh(bx)}{4b^3} + \frac{1}{4}x^4\text{Chi}(bx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*CoshIntegral[b*x],x]
```

```
[Out] (3*(2 + b^2*x^2)*Cosh[b*x])/(4*b^4) + (x^4*CoshIntegral[b*x])/4 - (x*(6 + b
^2*x^2)*Sinh[b*x])/(4*b^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$
\text{integral}\left(x^3\,\text{Chi}\,(bx)\,,x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int x^3\text{Chi}\,(bx)\;dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x),x, algorithm="giac")
[Out] integrate(x^3*Chi(b*x), x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$\frac{\frac{b^4x^4\mathrm{X}(bx)}{4} - \frac{\sinh(bx)b^3x^3}{4} + \frac{3b^2x^2\cosh(bx)}{4} - \frac{3bx\sinh(bx)}{2} + \frac{3\cosh(bx)}{2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Chi(b*x),x)
[Out] 1/b^4*(1/4*b^4*x^4*Chi(b*x)-1/4*sinh(b*x)*b^3*x^3+3/4*b^2*x^2*cosh(b*x)-3/2
*b*x*sinh(b*x)+3/2*cosh(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\mathrm{Chi}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x),x, algorithm="maxima")
[Out] integrate(x^3*Chi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\mathrm{coshint}\,(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coshint(b*x),x)
[Out] int(x^3*coshint(b*x), x)
```

sympy [A] time = 2.23, size = 85, normalized size = 1.35

$$-\frac{x^4\log(bx)}{4} + \frac{x^4\log\left(b^2x^2\right)}{8} + \frac{x^4\,\mathrm{Chi}\,(bx)}{4} - \frac{x^3\sinh(bx)}{4b} + \frac{3x^2\cosh(bx)}{4b^2} - \frac{3x\sinh(bx)}{2b^3} + \frac{3\cosh(bx)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Chi(b*x),x)
[Out] -x**4*log(b*x)/4 + x**4*log(b**2*x**2)/8 + x**4*Chi(b*x)/4 - x**3*sinh(b*x)
/(4*b) + 3*x**2*cosh(b*x)/(4*b**2) - 3*x*sinh(b*x)/(2*b**3) + 3*cosh(b*x)/(
2*b**4)
```

3.71 $\int x^2 \mathbf{Chi}(bx)\,dx$

Optimal. Leaf size=49

$$-\frac{2\sinh(bx)}{3b^3}+\frac{2x\cosh(bx)}{3b^2}+\frac{1}{3}x^3\mathrm{Chi}(bx)-\frac{x^2\sinh(bx)}{3b}$$

```
[Out] 1/3*x^3*Chi(b*x)+2/3*x*cosh(b*x)/b^2-2/3*sinh(b*x)/b^3-1/3*x^2*sinh(b*x)/b
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6533, 12, 3296, 2637}

$$-\frac{2\sinh(bx)}{3b^3}+\frac{2x\cosh(bx)}{3b^2}+\frac{1}{3}x^3\mathrm{Chi}(bx)-\frac{x^2\sinh(bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*CoshIntegral[b*x],x]
```

```
[Out] (2*x*Cosh[b*x])/(3*b^2) + (x^3*CoshIntegral[b*x])/3 - (2*Sinh[b*x])/(3*b^3)
 - (x^2*Sinh[b*x])/(3*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\text{Chi}(bx)\,dx &= \frac{1}{3}x^3\text{Chi}(bx)-\frac{1}{3}b\int \frac{x^2\cosh(bx)}{b}\,dx\\ &= \frac{1}{3}x^3\text{Chi}(bx)-\frac{1}{3}\int x^2\cosh(bx)\,dx\\ &= \frac{1}{3}x^3\text{Chi}(bx)-\frac{x^2\sinh(bx)}{3b}+\frac{2\int x\sinh(bx)\,dx}{3b}\\ &= \frac{2x\cosh(bx)}{3b^2}+\frac{1}{3}x^3\text{Chi}(bx)-\frac{x^2\sinh(bx)}{3b}-\frac{2\int\cosh(bx)\,dx}{3b^2}\\ &= \frac{2x\cosh(bx)}{3b^2}+\frac{1}{3}x^3\text{Chi}(bx)-\frac{2\sinh(bx)}{3b^3}-\frac{x^2\sinh(bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.90

$$\frac{2x\cosh(bx)}{3b^2}-\frac{(b^2x^2+2)\sinh(bx)}{3b^3}+\frac{1}{3}x^3\text{Chi}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*CoshIntegral[b*x],x]
```

```
[Out] (2*x*Cosh[b*x])/(3*b^2) + (x^3*CoshIntegral[b*x])/3 - ((2 + b^2*x^2)*Sinh[b
*x])/(3*b^3)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{Chi}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{Chi}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*Chi(b*x), x)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{\frac{b^3x^3\mathrm{X}(bx)}{3}-\frac{b^2x^2\sinh(bx)}{3}+\frac{2bx\cosh(bx)}{3}-\frac{2\sinh(bx)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*Chi(b*x)-1/3*b^2*x^2*sinh(b*x)+2/3*b*x*cosh(b*x)-2/3*sin
h(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Chi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Chi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{x^3\,\mathrm{coshint}\,(b\,x)}{3}-\frac{\frac{2\sinh(b\,x)}{3}+\frac{b^2\,x^2\sinh(b\,x)}{3}-\frac{2\,b\,x\cosh(b\,x)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(b*x),x)
```

```
[Out] (x^3*coshint(b*x))/3 - ((2*sinh(b*x))/3 + (b^2*x^2*sinh(b*x))/3 - (2*b*x*co
sh(b*x))/3)/b^3
```

sympy [A] time = 1.90, size = 70, normalized size = 1.43

$$-\frac{x^3\log{(bx)}}{3}+\frac{x^3\log{\left(b^2x^2\right)}}{6}+\frac{x^3\,\mathrm{Chi}\,(bx)}{3}-\frac{x^2\sinh{(bx)}}{3b}+\frac{2x\cosh{(bx)}}{3b^2}-\frac{2\sinh{(bx)}}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x),x)
```

```
[Out] -x**3*log(b*x)/3 + x**3*log(b**2*x**2)/6 + x**3*Chi(b*x)/3 - x**2*sinh(b*x)
/(3*b) + 2*x*cosh(b*x)/(3*b**2) - 2*sinh(b*x)/(3*b**3)
```

3.72 $\int x\,\mathrm{Chi}(bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cosh(bx)}{2b^2}+\frac{1}{2}x^2\mathrm{Chi}(bx)-\frac{x\sinh(bx)}{2b}$$

```
[Out] 1/2*x^2*Chi(b*x)+1/2*cosh(b*x)/b^2-1/2*x*sinh(b*x)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6533, 12, 3296, 2638}

$$\frac{\cosh(bx)}{2b^2}+\frac{1}{2}x^2\mathrm{Chi}(bx)-\frac{x\sinh(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[b*x],x]
```

```
[Out] Cosh[b*x]/(2*b^2) + (x^2*CoshIntegral[b*x])/2 - (x*Sinh[b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\text{Chi}(bx)\,dx &= \frac{1}{2}x^2\text{Chi}(bx) - \frac{1}{2}b\int \frac{x\cosh(bx)}{b}\,dx\\ &= \frac{1}{2}x^2\text{Chi}(bx) - \frac{1}{2}\int x\cosh(bx)\,dx\\ &= \frac{1}{2}x^2\text{Chi}(bx) - \frac{x\sinh(bx)}{2b} + \frac{\int \sinh(bx)\,dx}{2b}\\ &= \frac{\cosh(bx)}{2b^2} + \frac{1}{2}x^2\text{Chi}(bx) - \frac{x\sinh(bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{\cosh(bx)}{2b^2} + \frac{1}{2}x^2\text{Chi}(bx) - \frac{x\sinh(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[b*x],x]
[Out] Cosh[b*x]/(2*b^2) + (x^2*CoshIntegral[b*x])/2 - (x*Sinh[b*x])/(2*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Chi}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x),x, algorithm="fricas")
[Out] integral(x*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x),x, algorithm="giac")
[Out] integrate(x*Chi(b*x), x)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{\frac{b^2x^2\text{X}(bx)}{2} - \frac{bx\sinh(bx)}{2} + \frac{\cosh(bx)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*Chi(b*x)-1/2*b*x*sinh(b*x)+1/2*cosh(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\frac{\frac{\cosh(b\,x)}{2}-\frac{b\,x\sinh(b\,x)}{2}}{b^2}+\frac{x^2\,\mathrm{coshint}\,(b\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(b*x),x)
```

```
[Out] (cosh(b*x)/2 - (b*x*sinh(b*x))/2)/b^2 + (x^2*coshint(b*x))/2
```

sympy [A] time = 1.20, size = 53, normalized size = 1.51

$$-\frac{x^2\log{(bx)}}{2}+\frac{x^2\log{\left(b^2x^2\right)}}{4}+\frac{x^2\,\mathrm{Chi}\,(bx)}{2}-\frac{x\sinh{(bx)}}{2b}+\frac{\cosh{(bx)}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x),x)
```

```
[Out] -x**2*log(b*x)/2 + x**2*log(b**2*x**2)/4 + x**2*Chi(b*x)/2 - x*sinh(b*x)/(2
*b) + cosh(b*x)/(2*b**2)
```

3.73 $\int \mathrm{Chi}(bx)\,dx$

Optimal. Leaf size=16

$$x\mathrm{Chi}(bx)-\frac{\sinh(bx)}{b}$$

```
[Out] x*Chi(b*x)-sinh(b*x)/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6529}

$$x\mathrm{Chi}(bx)-\frac{\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[b*x],x]
```

```
[Out] x*CoshIntegral[b*x] - Sinh[b*x]/b
```

Rule 6529

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*CoshInte
gral[a + b*x])/b, x] - Simp[Sinh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \mathrm{Chi}(bx)\,dx=x\mathrm{Chi}(bx)-\frac{\sinh(bx)}{b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$x\mathrm{Chi}(bx)-\frac{\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[b*x],x]
```

```
[Out] x*CoshIntegral[b*x] - Sinh[b*x]/b
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{Chi}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x),x, algorithm="fricas")
[Out] integral(cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x),x, algorithm="giac")
[Out] integrate(Chi(b*x), x)
```

maple [A] time = 0.01, size = 19, normalized size = 1.19

$$\frac{bx\mathrm{X}\,(bx) - \sinh\,(bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x),x)
[Out] 1/b*(b*x*Chi(b*x)-sinh(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x),x, algorithm="maxima")
[Out] integrate(Chi(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$x\,\mathrm{coshint}\,(b\,x) - \frac{\sinh\,(b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x),x)
[Out] x*coshint(b*x) - sinh(b*x)/b
```

sympy [B] time = 1.28, size = 31, normalized size = 1.94

$$-x\log(bx) + \frac{x\log\left(b^2x^2\right)}{2} + x\,\mathrm{Chi}(bx) - \frac{\sinh(bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x),x)
```

```
[Out] -x*log(b*x) + x*log(b**2*x**2)/2 + x*Chi(b*x) - sinh(b*x)/b
```

3.74 $\int \frac{\mathbf{Chi}(bx)}{x}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)+\frac{1}{2}\log^2(bx)+\gamma\log(x)$$

```
[Out] -1/2*b*x*HypergeometricPFQ([1, 1, 1],[2, 2, 2],-b*x)+1/2*b*x*Hypergeometric
PFQ([1, 1, 1],[2, 2, 2],b*x)+EulerGamma*ln(x)+1/2*ln(b*x)^2
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6531}

$$-\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)+\frac{1}{2}\log^2(bx)+\gamma\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[b*x]/x,x]
```

```
[Out] -(b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)])/2 + (b*x*Hypergeomet
ricPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2 + EulerGamma*Log[x] + Log[b*x]^2/2
```

Rule 6531

```
Int[CoshIntegral[(b_.)*(x_)]/(x_), x_Symbol] :> -Simp[(b*x*HypergeometricPF
Q[{1, 1, 1}, {2, 2, 2}, -(b*x)])/2, x] + (Simp[(1*b*x*HypergeometricPFQ[{1,
 1, 1}, {2, 2, 2}, b*x])/2, x] + Simp[EulerGamma*Log[x], x] + Simp[(1*Log[b
*x]^2)/2, x]) /; FreeQ[b, x]
```

Rubi steps

$$\int \frac{\text{Chi}(bx)}{x}\,dx=-\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)+\gamma\log(x)+\frac{1}{2}\log^2(bx)$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.00

$$-\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;-bx)+\frac{1}{2}bx\,{}_3F_3(1,1,1;2,2,2;bx)+\frac{1}{2}\log^2(bx)+\gamma\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[b*x]/x,x]
```

```
[Out] -1/2*(b*x*HypergeometricPFQ[{1, 1, 1}, {2, 2, 2}, -(b*x)]) + (b*x*Hypergeom
etricPFQ[{1, 1, 1}, {2, 2, 2}, b*x])/2 + EulerGamma*Log[x] + Log[b*x]^2/2
```

fricas [F] time = 2.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\,(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x)/x, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)/x,x)
```

```
[Out] int(Chi(b*x)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{coshint}\,(b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)/x,x)
```

```
[Out] int(coshint(b*x)/x, x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

3.75 $\int \frac{\mathbf{Chi}(bx)}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\text{Chi}(bx)}{x}+b\text{Shi}(bx)-\frac{\cosh(bx)}{x}$$

```
[Out] -Chi(b*x)/x-cosh(b*x)/x+b*Shi(b*x)
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6533, 12, 3297, 3298}

$$-\frac{\text{Chi}(bx)}{x}+b\text{Shi}(bx)-\frac{\cosh(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[b*x]/x^2,x]
```

```
[Out] -(Cosh[b*x]/x) - CoshIntegral[b*x]/x + b*SinhIntegral[b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Chi}(bx)}{x^2}\,dx &= -\frac{\text{Chi}(bx)}{x}+b\int \frac{\cosh(bx)}{bx^2}\,dx\\ &= -\frac{\text{Chi}(bx)}{x}+\int \frac{\cosh(bx)}{x^2}\,dx\\ &= -\frac{\cosh(bx)}{x}-\frac{\text{Chi}(bx)}{x}+b\int \frac{\sinh(bx)}{x}\,dx\\ &= -\frac{\cosh(bx)}{x}-\frac{\text{Chi}(bx)}{x}+b\text{Shi}(bx)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\text{Chi}(bx)}{x}+b\text{Shi}(bx)-\frac{\cosh(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[b*x]/x^2,x]
[Out] -(Cosh[b*x]/x) - CoshIntegral[b*x]/x + b*SinhIntegral[b*x]
```

fricas [F] time = 1.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\,(bx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x^2,x, algorithm="fricas")
[Out] integral(cosh_integral(b*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x^2,x, algorithm="giac")
[Out] integrate(Chi(b*x)/x^2, x)
```

maple [A] time = 0.01, size = 32, normalized size = 1.28

$$b\left(-\frac{\mathrm{X}(bx)}{bx}-\frac{\cosh(bx)}{bx}+\mathrm{Shi}(bx)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)/x^2,x)
```

```
[Out] b*(-Chi(b*x)/b/x-1/b/x*cosh(b*x)+Shi(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Chi}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\mathrm{coshint}(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)/x^2,x)
```

```
[Out] int(coshint(b*x)/x^2, x)
```

sympy [B] time = 0.97, size = 39, normalized size = 1.56

$$\frac{b^2x_3F_4\left(\begin{matrix}\frac{1}{2},1,1\\ \frac{3}{2},\frac{3}{2},2,2\end{matrix}\middle|\frac{b^2x^2}{4}\right)}{4}-\frac{\log\left(b^2x^2\right)}{2x}-\frac{1}{x}-\frac{\gamma}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x**2,x)
```

```
[Out] b**2*x*hyper((1/2, 1, 1), (3/2, 3/2, 2, 2), b**2*x**2/4)/4 - log(b**2*x**2)
/(2*x) - 1/x - EulerGamma/x
```

3.76 $\int \frac{\mathbf{Chi}(bx)}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{4}b^2\mathrm{Chi}(bx) - \frac{\mathrm{Chi}(bx)}{2x^2} - \frac{\cosh(bx)}{4x^2} - \frac{b\sinh(bx)}{4x}$$

```
[Out] 1/4*b^2*Chi(b*x)-1/2*Chi(b*x)/x^2-1/4*cosh(b*x)/x^2-1/4*b*sinh(b*x)/x
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6533, 12, 3297, 3301}

$$\frac{1}{4}b^2\mathrm{Chi}(bx) - \frac{\mathrm{Chi}(bx)}{2x^2} - \frac{\cosh(bx)}{4x^2} - \frac{b\sinh(bx)}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[b*x]/x^3,x]
```

```
[Out] -Cosh[b*x]/(4*x^2) + (b^2*CoshIntegral[b*x])/4 - CoshIntegral[b*x]/(2*x^2)
- (b*Sinh[b*x])/(4*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
```

```
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Chi}(bx)}{x^3}\,dx &= -\frac{\text{Chi}(bx)}{2x^2}+\frac{1}{2}b\int \frac{\cosh(bx)}{bx^3}\,dx\\
&= -\frac{\text{Chi}(bx)}{2x^2}+\frac{1}{2}\int \frac{\cosh(bx)}{x^3}\,dx\\
&= -\frac{\cosh(bx)}{4x^2}-\frac{\text{Chi}(bx)}{2x^2}+\frac{1}{4}b\int \frac{\sinh(bx)}{x^2}\,dx\\
&= -\frac{\cosh(bx)}{4x^2}-\frac{\text{Chi}(bx)}{2x^2}-\frac{b\sinh(bx)}{4x}+\frac{1}{4}b^2\int \frac{\cosh(bx)}{x}\,dx\\
&= -\frac{\cosh(bx)}{4x^2}+\frac{1}{4}b^2\text{Chi}(bx)-\frac{\text{Chi}(bx)}{2x^2}-\frac{b\sinh(bx)}{4x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{1}{4}b^2\text{Chi}(bx)-\frac{\text{Chi}(bx)}{2x^2}-\frac{\cosh(bx)}{4x^2}-\frac{b\sinh(bx)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[b*x]/x^3,x]
[Out] -1/4*Cosh[b*x]/x^2 + (b^2*CoshIntegral[b*x])/4 - CoshIntegral[b*x]/(2*x^2)
- (b*Sinh[b*x])/(4*x)
```

fricas [F] time = 2.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\left(bx\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x^3,x, algorithm="fricas")
[Out] integral(cosh_integral(b*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x)/x^3, x)
```

maple [A] time = 0.01, size = 48, normalized size = 1.04

$$b^{2}\left(-\frac{\mathrm{X}\left(bx\right)}{2b^{2}x^{2}}-\frac{\cosh\left(bx\right)}{4b^{2}x^{2}}-\frac{\sinh\left(bx\right)}{4bx}+\frac{\mathrm{X}\left(bx\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2*Chi(b*x)/b^2/x^2-1/4/b^2/x^2*cosh(b*x)-1/4*sinh(b*x)/b/x+1/4*Chi(
b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{b^{2}\,\mathrm{coshint}\left(b\,x\right)}{4}-\frac{\frac{\mathrm{coshint}(b\,x)}{2}+\frac{\cosh(b\,x)}{4}+\frac{b\,x\,\sinh(b\,x)}{4}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)/x^3,x)
```

```
[Out] (b^2*coshint(b*x))/4 - (coshint(b*x)/2 + cosh(b*x)/4 + (b*x*sinh(b*x))/4)/x
^2
```

sympy [B] time = 2.30, size = 87, normalized size = 1.89

$$-\frac{b^{2}\log\left(bx\right)}{4}+\frac{b^{2}\log\left(b^{2}x^{2}\right)}{8}+\frac{b^{2}\,\mathrm{Chi}\left(bx\right)}{4}-\frac{b\sinh\left(bx\right)}{4x}+\frac{\log\left(bx\right)}{2x^{2}}-\frac{\log\left(b^{2}x^{2}\right)}{4x^{2}}-\frac{\cosh\left(bx\right)}{4x^{2}}-\frac{\mathrm{Chi}\left(bx\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)/x**3,x)
```

```
[Out] -b**2*log(b*x)/4 + b**2*log(b**2*x**2)/8 + b**2*Chi(b*x)/4 - b*sinh(b*x)/(4
*x) + log(b*x)/(2*x**2) - log(b**2*x**2)/(4*x**2) - cosh(b*x)/(4*x**2) - Ch
i(b*x)/(2*x**2)
```

3.77 $\int x^m \mathbf{Chi}(bx)^2 \, dx$

Optimal. Leaf size=13

$$\operatorname{Int}\left(x^m \operatorname{Chi}(bx)^2, x\right)$$

```
[Out] CannotIntegrate(x^m*Chi(b*x)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{Chi}(bx)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*CoshIntegral[b*x]^2,x]
[Out] Defer[Int][x^m*CoshIntegral[b*x]^2, x]
```

Rubi steps

$$\int x^m \operatorname{Chi}(bx)^2 \, dx = \int x^m \operatorname{Chi}(bx)^2 \, dx$$

Mathematica [A] time = 0.57, size = 0, normalized size = 0.00

$$\int x^m \operatorname{Chi}(bx)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*CoshIntegral[b*x]^2,x]
[Out] Integrate[x^m*CoshIntegral[b*x]^2, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^m \operatorname{Chi}\left(bx\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh_integral(b*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Chi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x)^2,x, algorithm="giac")
[Out] integrate(x^m*Chi(b*x)^2, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int x^m \mathrm{X}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Chi(b*x)^2,x)
[Out] int(x^m*Chi(b*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Chi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^m*Chi(b*x)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int x^m \operatorname{coshint}(b\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*coshint(b*x)^2,x)
[Out] int(x^m*coshint(b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Chi}^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Chi(b*x)**2,x)
[Out] Integral(x**m*Chi(b*x)**2, x)
```

3.78 $\int x^3 \mathbf{Chi}(bx)^2 \, dx$

Optimal. Leaf size=164

$$-\frac{3\text{Chi}(2bx)}{2b^4}+\frac{3\text{Chi}(bx)\cosh(bx)}{b^4}-\frac{3\log(x)}{2b^4}+\frac{13\sinh^2(bx)}{8b^4}+\frac{3\cosh^2(bx)}{8b^4}-\frac{3x\text{Chi}(bx)\sinh(bx)}{b^3}-\frac{x\sinh(bx)\cosh(b$$

```
[Out] -1/4*x^2/b^2+1/4*x^4*Chi(b*x)^2-3/2*Chi(2*b*x)/b^4+3*Chi(b*x)*cosh(b*x)/b^4
+3/2*x^2*Chi(b*x)*cosh(b*x)/b^2+3/8*cosh(b*x)^2/b^4-3/2*ln(x)/b^4-3*x*Chi(b
*x)*sinh(b*x)/b^3-1/2*x^3*Chi(b*x)*sinh(b*x)/b-x*cosh(b*x)*sinh(b*x)/b^3+13
/8*sinh(b*x)^2/b^4+1/4*x^2*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.24, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6537, 6543, 12, 5372, 3310, 30, 6549, 2564, 6547, 3312, 3301}

$$\frac{3x^2\text{Chi}(bx)\cosh(bx)}{2b^2}-\frac{3\text{Chi}(2bx)}{2b^4}-\frac{3x\text{Chi}(bx)\sinh(bx)}{b^3}+\frac{3\text{Chi}(bx)\cosh(bx)}{b^4}-\frac{x^2}{4b^2}+\frac{x^2\sinh^2(bx)}{4b^2}-\frac{3\log(x)}{2b^4}+\frac{13\,\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*CoshIntegral[b*x]^2,x]
```

```
[Out] -x^2/(4*b^2) + (3*Cosh[b*x]^2)/(8*b^4) + (3*Cosh[b*x]*CoshIntegral[b*x])/b^
4 + (3*x^2*Cosh[b*x]*CoshIntegral[b*x])/(2*b^2) + (x^4*CoshIntegral[b*x]^2)
/4 - (3*CoshIntegral[2*b*x])/(2*b^4) - (3*Log[x])/(2*b^4) - (x*Cosh[b*x]*Si
nh[b*x])/b^3 - (3*x*CoshIntegral[b*x]*Sinh[b*x])/b^3 - (x^3*CoshIntegral[b*
x]*Sinh[b*x])/(2*b) + (13*Sinh[b*x]^2)/(8*b^4) + (x^2*Sinh[b*x]^2)/(4*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6537

```
Int[CoshIntegral[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Cos
hIntegral[b*x]^2)/(m + 1), x] - Dist[2/(m + 1), Int[x^m*Cosh[b*x]*CoshInteg
ral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3\text{Chi}(bx)^2\,dx = \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{1}{2}\int x^3\cosh(bx)\text{Chi}(bx)\,dx$$

$$= \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{x^3\text{Chi}(bx)\sinh(bx)}{2b} + \frac{1}{2}\int \frac{x^2\cosh(bx)\sinh(bx)}{b}\,dx + \frac{3\int x^2\text{Chi}(bx)\sinh(bx)}{2b}$$

$$= \frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{x^3\text{Chi}(bx)\sinh(bx)}{2b} - \frac{3\int x\cosh(bx)\text{Chi}(bx)\,dx}{b^2} + \int$$

$$= \frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{3x\text{Chi}(bx)\sinh(bx)}{b^3} - \frac{x^3\text{Chi}(bx)\sinh(bx)}{2b} + \frac{x^2\sinh}{4b}$$

$$= \frac{3\cosh^2(bx)}{8b^4} + \frac{3\cosh(bx)\text{Chi}(bx)}{b^4} + \frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{x\cosh(bx)\sinh}{b^3}$$

$$= -\frac{x^2}{4b^2} + \frac{3\cosh^2(bx)}{8b^4} + \frac{3\cosh(bx)\text{Chi}(bx)}{b^4} + \frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{x\cosh(}{}$$

$$= -\frac{x^2}{4b^2} + \frac{3\cosh^2(bx)}{8b^4} + \frac{3\cosh(bx)\text{Chi}(bx)}{b^4} + \frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{x\cosh(}{}$$

$$= -\frac{x^2}{4b^2} + \frac{3\cosh^2(bx)}{8b^4} + \frac{3\cosh(bx)\text{Chi}(bx)}{b^4} + \frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{3\log(x}{2b^4}$$

$$= -\frac{x^2}{4b^2} + \frac{3\cosh^2(bx)}{8b^4} + \frac{3\cosh(bx)\text{Chi}(bx)}{b^4} + \frac{3x^2\cosh(bx)\text{Chi}(bx)}{2b^2} + \frac{1}{4}x^4\text{Chi}(bx)^2 - \frac{3\text{Chi}(2}{2b^4}$$

Mathematica [A] time = 0.10, size = 107, normalized size = 0.65

$$\frac{2b^4x^4\text{Chi}(bx)^2 - 4\text{Chi}(bx)\left(bx\left(b^2x^2+6\right)\sinh(bx) - 3\left(b^2x^2+2\right)\cosh(bx)\right) - 3b^2x^2 + b^2x^2\cosh(2bx) - 12\text{Chi}(2}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*CoshIntegral[b*x]^2,x]
```

```
[Out] (-3*b^2*x^2 + 8*Cosh[2*b*x] + b^2*x^2*Cosh[2*b*x] + 2*b^4*x^4*CoshIntegral[
b*x]^2 - 12*CoshIntegral[2*b*x] - 12*Log[x] - 4*CoshIntegral[b*x]*(-3*(2 +
b^2*x^2)*Cosh[b*x] + b*x*(6 + b^2*x^2)*Sinh[b*x]) - 4*b*x*Sinh[2*b*x])/(8*b
^4)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\,\text{Chi}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*cosh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Chi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*Chi(b*x)^2, x)
```

maple [A] time = 0.03, size = 138, normalized size = 0.84

$$\frac{x^4\text{X}\,(bx)^2}{4}-\frac{x^3\text{X}\,(bx)\sinh\,(bx)}{2b}+\frac{3x^2\text{X}\,(bx)\cosh\,(bx)}{2b^2}-\frac{3x\text{X}\,(bx)\sinh\,(bx)}{b^3}+\frac{3\text{X}\,(bx)\cosh\,(bx)}{b^4}+\frac{x^2\left(\cosh^2(bx)\right)}{4b^2}-\frac{x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Chi(b*x)^2,x)
```

```
[Out] 1/4*x^4*Chi(b*x)^2-1/2*x^3*Chi(b*x)*sinh(b*x)/b+3/2*x^2*Chi(b*x)*cosh(b*x)/
b^2-3*x*Chi(b*x)*sinh(b*x)/b^3+3*Chi(b*x)*cosh(b*x)/b^4+1/4/b^2*x^2*cosh(b*
x)^2-x*cosh(b*x)*sinh(b*x)/b^3-1/2*x^2/b^2+2*cosh(b*x)^2/b^4-3/2/b^4*ln(b*x
)-3/2*Chi(2*b*x)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Chi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*Chi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{coshint}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coshint(b*x)^2,x)
```

```
[Out] int(x^3*coshint(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{Chi}^2(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Chi(b*x)**2,x)
```

```
[Out] Integral(x**3*Chi(b*x)**2, x)
```

3.79 $\int x^2 \mathbf{Chi}(bx)^2\,dx$

Optimal. Leaf size=112

$$-\frac{4\text{Chi}(bx)\sinh(bx)}{3b^3}+\frac{2\text{Shi}(2bx)}{3b^3}-\frac{5\sinh(bx)\cosh(bx)}{6b^3}+\frac{4x\text{Chi}(bx)\cosh(bx)}{3b^2}-\frac{x}{2b^2}+\frac{x\sinh^2(bx)}{3b^2}+\frac{1}{3}x^3\text{Chi}(bx)^2-$$

```
[Out] -1/2*x/b^2+1/3*x^3*Chi(b*x)^2+4/3*x*Chi(b*x)*cosh(b*x)/b^2+2/3*Shi(2*b*x)/b
^3-4/3*Chi(b*x)*sinh(b*x)/b^3-2/3*x^2*Chi(b*x)*sinh(b*x)/b-5/6*cosh(b*x)*si
nh(b*x)/b^3+1/3*x*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6537, 6543, 12, 5372, 2635, 8, 6549, 6541, 5448, 3298}

$$-\frac{4\text{Chi}(bx)\sinh(bx)}{3b^3}+\frac{4x\text{Chi}(bx)\cosh(bx)}{3b^2}+\frac{2\text{Shi}(2bx)}{3b^3}-\frac{x}{2b^2}+\frac{x\sinh^2(bx)}{3b^2}-\frac{5\sinh(bx)\cosh(bx)}{6b^3}+\frac{1}{3}x^3\text{Chi}(bx)^2-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*CoshIntegral[b*x]^2,x]
```

```
[Out] -x/(2*b^2) + (4*x*Cosh[b*x]*CoshIntegral[b*x])/(3*b^2) + (x^3*CoshIntegral[
b*x]^2)/3 - (5*Cosh[b*x]*Sinh[b*x])/(6*b^3) - (4*CoshIntegral[b*x]*Sinh[b*x
])/(3*b^3) - (2*x^2*CoshIntegral[b*x]*Sinh[b*x])/(3*b) + (x*Sinh[b*x]^2)/(3
*b^2) + (2*SinhIntegral[2*b*x])/(3*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6537

```
Int[CoshIntegral[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Cos
hIntegral[b*x]^2)/(m + 1), x] - Dist[2/(m + 1), Int[x^m*Cosh[b*x]*CoshInteg
ral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\mathrm{Chi}(bx)^2\,dx &= \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{2}{3}\int x^2\cosh(bx)\mathrm{Chi}(bx)\,dx \\
&= \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{2x^2\mathrm{Chi}(bx)\sinh(bx)}{3b} + \frac{2}{3}\int \frac{x\cosh(bx)\sinh(bx)}{b}\,dx + \frac{4\int x\mathrm{Chi}(bx)\sinh(b
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4x\cosh(bx)\mathrm{Chi}(bx)}{3b^2} + \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{2x^2\mathrm{Chi}(bx)\sinh(bx)}{3b} - \frac{4\int\cosh(bx)\mathrm{Chi}(bx)\,dx}{3b^2} + 2 \\
&= \frac{4x\cosh(bx)\mathrm{Chi}(bx)}{3b^2} + \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{4\mathrm{Chi}(bx)\sinh(bx)}{3b^3} - \frac{2x^2\mathrm{Chi}(bx)\sinh(bx)}{3b} + \frac{x\sinh}{3b} \\
&= \frac{4x\cosh(bx)\mathrm{Chi}(bx)}{3b^2} + \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{5\cosh(bx)\sinh(bx)}{6b^3} - \frac{4\mathrm{Chi}(bx)\sinh(bx)}{3b^3} - \frac{2x^2\mathrm{Chi}}{} \\
&= -\frac{x}{2b^2} + \frac{4x\cosh(bx)\mathrm{Chi}(bx)}{3b^2} + \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{5\cosh(bx)\sinh(bx)}{6b^3} - \frac{4\mathrm{Chi}(bx)\sinh(bx)}{3b^3} - \\
&= -\frac{x}{2b^2} + \frac{4x\cosh(bx)\mathrm{Chi}(bx)}{3b^2} + \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{5\cosh(bx)\sinh(bx)}{6b^3} - \frac{4\mathrm{Chi}(bx)\sinh(bx)}{3b^3} - \\
&= -\frac{x}{2b^2} + \frac{4x\cosh(bx)\mathrm{Chi}(bx)}{3b^2} + \frac{1}{3}x^3\mathrm{Chi}(bx)^2 - \frac{5\cosh(bx)\sinh(bx)}{6b^3} - \frac{4\mathrm{Chi}(bx)\sinh(bx)}{3b^3} -
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.70

$$
\frac{4b^3x^3\mathrm{Chi}(bx)^2 - 8\mathrm{Chi}(bx)\left(\left(b^2x^2+2\right)\sinh(bx) - 2bx\cosh(bx)\right) + 8\mathrm{Shi}(2bx) - 8bx - 5\sinh(2bx) + 2bx\cosh(2}{12b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*CoshIntegral[b*x]^2,x]
```

```
[Out] (-8*b*x + 2*b*x*Cosh[2*b*x] + 4*b^3*x^3*CoshIntegral[b*x]^2 - 8*CoshIntegra
l[b*x]*(-2*b*x*Cosh[b*x] + (2 + b^2*x^2)*Sinh[b*x]) - 5*Sinh[2*b*x] + 8*Sin
hIntegral[2*b*x])/(12*b^3)
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\,\mathrm{Chi}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}\,(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*Chi(b*x)^2, x)
```

maple [A] time = 0.03, size = 84, normalized size = 0.75

$$\frac{\frac{b^3x^3\mathrm{X}(bx)^2}{3} - 2\mathrm{X}\,(bx)\left(\frac{b^2x^2\sinh(bx)}{3} - \frac{2bx\cosh(bx)}{3} + \frac{2\sinh(bx)}{3}\right) + \frac{bx\left(\cosh^2(bx)\right)}{3} - \frac{5\sinh(bx)\cosh(bx)}{6} - \frac{5bx}{6} + \frac{2\,\mathrm{Shi}(2bx)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*Chi(b*x)^2-2*Chi(b*x)*(1/3*b^2*x^2*sinh(b*x)-2/3*b*x*cos
h(b*x)+2/3*sinh(b*x))+1/3*b*x*cosh(b*x)^2-5/6*sinh(b*x)*cosh(b*x)-5/6*b*x+2
/3*Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}\,(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*Chi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{coshint}\left(b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(b*x)^2,x)
```

```
[Out] int(x^2*coshint(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Chi}^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x)**2,x)
```

```
[Out] Integral(x**2*Chi(b*x)**2, x)
```

3.80 $\int x\mathbf{Chi}(bx)^2\,dx$

Optimal. Leaf size=74

$$-\frac{\mathrm{Chi}(2bx)}{2b^2}+\frac{\mathrm{Chi}(bx)\cosh(bx)}{b^2}-\frac{\log(x)}{2b^2}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\mathrm{Chi}(bx)^2-\frac{x\mathrm{Chi}(bx)\sinh(bx)}{b}$$

```
[Out] 1/2*x^2*Chi(b*x)^2-1/2*Chi(2*b*x)/b^2+Chi(b*x)*cosh(b*x)/b^2-1/2*ln(x)/b^2-
x*Chi(b*x)*sinh(b*x)/b+1/2*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6537, 6543, 12, 2564, 30, 6547, 3312, 3301}

$$-\frac{\mathrm{Chi}(2bx)}{2b^2}+\frac{\mathrm{Chi}(bx)\cosh(bx)}{b^2}-\frac{\log(x)}{2b^2}+\frac{\sinh^2(bx)}{2b^2}+\frac{1}{2}x^2\mathrm{Chi}(bx)^2-\frac{x\mathrm{Chi}(bx)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[b*x]^2,x]
```

```
[Out] (Cosh[b*x]*CoshIntegral[b*x])/b^2 + (x^2*CoshIntegral[b*x]^2)/2 - CoshInteg
ral[2*b*x]/(2*b^2) - Log[x]/(2*b^2) - (x*CoshIntegral[b*x]*Sinh[b*x])/b + S
inh[b*x]^2/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6537

```
Int[CoshIntegral[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Cos
hIntegral[b*x]^2)/(m + 1), x] - Dist[2/(m + 1), Int[x^m*Cosh[b*x]*CoshInteg
ral[b*x], x], x] /; FreeQ[b, x] && IGtQ[m, 0]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\text{Chi}(bx)^2\,dx &= \frac{1}{2}x^2\text{Chi}(bx)^2 - \int x\cosh(bx)\text{Chi}(bx)\,dx \\
&= \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b} + \frac{\int \text{Chi}(bx)\sinh(bx)\,dx}{b} + \int \frac{\cosh(bx)\sinh(bx)}{b}\,dx \\
&= \frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\cosh^2(bx)}{bx}\,dx}{b} + \frac{\int \cosh(bx)\sinh(bx}{b} \\
&= \frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\cosh^2(bx)}{x}\,dx}{b^2} - \frac{\text{Subst}(\int x\,dx, x, i\,\text{si}}{b^2} \\
&= \frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{x\text{Chi}(bx)\sinh(bx)}{b} + \frac{\sinh^2(bx)}{2b^2} - \frac{\int \left(\frac{1}{2x} + \frac{\cosh(2bx)}{2x}\right)dx}{b^2} \\
&= \frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{\log(x)}{2b^2} - \frac{x\text{Chi}(bx)\sinh(bx)}{b} + \frac{\sinh^2(bx)}{2b^2} - \frac{\int \frac{\cosh(2bx)}{x}\,dx}{2b^2} \\
&= \frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{1}{2}x^2\text{Chi}(bx)^2 - \frac{\text{Chi}(2bx)}{2b^2} - \frac{\log(x)}{2b^2} - \frac{x\text{Chi}(bx)\sinh(bx)}{b} + \frac{\sinh^2(bx)}{2b^2}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.77

$$\frac{2b^2x^2\text{Chi}(bx)^2 - 2\text{Chi}(2bx) + 4\text{Chi}(bx)(\cosh(bx) - bx\sinh(bx)) + \cosh(2bx) - 2\log(x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[b*x]^2,x]
```

```
[Out] (Cosh[2*b*x] + 2*b^2*x^2*CoshIntegral[b*x]^2 - 2*CoshIntegral[2*b*x] - 2*Lo
g[x] + 4*CoshIntegral[b*x]*(Cosh[b*x] - b*x*Sinh[b*x]))/(4*b^2)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Chi}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x*cosh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*Chi(b*x)^2, x)
```

maple [A] time = 0.03, size = 69, normalized size = 0.93

$$\frac{x^2 \mathrm{X}(bx)^2}{2} - \frac{x\mathrm{X}(bx)\sinh(bx)}{b} + \frac{\mathrm{X}(bx)\cosh(bx)}{b^2} + \frac{\cosh^2(bx)}{2b^2} - \frac{\ln(bx)}{2b^2} - \frac{\mathrm{X}(2bx)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x)^2,x)
```

```
[Out] 1/2*x^2*Chi(b*x)^2-x*Chi(b*x)*sinh(b*x)/b+Chi(b*x)*cosh(b*x)/b^2+1/2/b^2*co
sh(b*x)^2-1/2/b^2*ln(b*x)-1/2*Chi(2*b*x)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{coshint}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(b*x)^2,x)
```

```
[Out] int(x*coshint(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{Chi}^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)**2,x)
```

```
[Out] Integral(x*Chi(b*x)**2, x)
```

3.81 $\int \text{Chi}(bx)^2\,dx$

Optimal. Leaf size=31

$$x\text{Chi}(bx)^2-\frac{2\text{Chi}(bx)\sinh(bx)}{b}+\frac{\text{Shi}(2bx)}{b}$$

```
[Out] x*Chi(b*x)^2+Shi(2*b*x)/b-2*Chi(b*x)*sinh(b*x)/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6535, 6541, 12, 5448, 3298}

$$x\text{Chi}(bx)^2-\frac{2\text{Chi}(bx)\sinh(bx)}{b}+\frac{\text{Shi}(2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[b*x]^2,x]
```

```
[Out] x*CoshIntegral[b*x]^2 - (2*CoshIntegral[b*x]*Sinh[b*x])/b + SinhIntegral[2*
b*x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6535

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*CoshIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Cosh[a + b*x]*CoshIntegral[a + b*x],
 x], x] /; FreeQ[{a, b}, x]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Chi}(bx)^2\,dx &= x\mathrm{Chi}(bx)^2 - 2\int \cosh(bx)\mathrm{Chi}(bx)\,dx\\
&= x\mathrm{Chi}(bx)^2 - \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b} + 2\int \frac{\cosh(bx)\sinh(bx)}{bx}\,dx\\
&= x\mathrm{Chi}(bx)^2 - \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b} + \frac{2\int \frac{\cosh(bx)\sinh(bx)}{x}\,dx}{b}\\
&= x\mathrm{Chi}(bx)^2 - \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b} + \frac{2\int \frac{\sinh(2bx)}{2x}\,dx}{b}\\
&= x\mathrm{Chi}(bx)^2 - \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b} + \frac{\int \frac{\sinh(2bx)}{x}\,dx}{b}\\
&= x\mathrm{Chi}(bx)^2 - \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b} + \frac{\mathrm{Shi}(2bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$x\mathrm{Chi}(bx)^2 - \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b} + \frac{\mathrm{Shi}(2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[b*x]^2,x]
```

```
[Out] x*CoshIntegral[b*x]^2 - (2*CoshIntegral[b*x]*Sinh[b*x])/b + SinhIntegral[2*
b*x]/b
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{Chi}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2,x, algorithm="giac")
[Out] integrate(Chi(b*x)^2, x)
```

maple [A] time = 0.01, size = 30, normalized size = 0.97

$$\frac{bx\text{X}\,(bx)^2 - 2\text{X}\,(bx)\sinh\,(bx) + \text{Shi}\,(2bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)^2,x)
[Out] 1/b*(b*x*Chi(b*x)^2-2*Chi(b*x)*sinh(b*x)+Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2,x, algorithm="maxima")
[Out] integrate(Chi(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \text{coshint}\,(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)^2,x)
[Out] int(coshint(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}^2\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)**2,x)
[Out] Integral(Chi(b*x)**2, x)
```

3.82 $\int \frac{\textbf{Chi}(bx)^2}{x}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{\mathrm{Chi}(bx)^2}{x},x\right)$$

[Out] CannotIntegrate(Chi(b*x)^2/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{Chi}(bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[CoshIntegral[b*x]^2/x,x]

[Out] Defer[Int][CoshIntegral[b*x]^2/x, x]

Rubi steps

$$\int \frac{\mathrm{Chi}(bx)^2}{x}\,dx=\int \frac{\mathrm{Chi}(bx)^2}{x}\,dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}(bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[CoshIntegral[b*x]^2/x,x]

[Out] Integrate[CoshIntegral[b*x]^2/x, x]

fricas [A] time = 0.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Chi}\left(bx\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x)^2/x, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)^2/x,x)
```

```
[Out] int(Chi(b*x)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)^2/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{coshint}\left(b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)^2/x,x)
```

```
[Out] int(coshint(b*x)^2/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}^2(bx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)**2/x,x)
```

```
[Out] Integral(Chi(b*x)**2/x, x)
```

3.83 $\int \frac{\textbf{Chi}(bx)^2}{x^2} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\text{Chi}(bx)^2}{x^2}, x\right)$$

```
[Out] CannotIntegrate(Chi(b*x)^2/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{\text{Chi}(bx)^2}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[CoshIntegral[b*x]^2/x^2,x]
```

```
[Out] Defer[Int][CoshIntegral[b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\text{Chi}(bx)^2}{x^2} \, dx = \int \frac{\text{Chi}(bx)^2}{x^2} \, dx$$

Mathematica [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(bx)^2}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[CoshIntegral[b*x]^2/x^2,x]
```

```
[Out] Integrate[CoshIntegral[b*x]^2/x^2, x]
```

fricas [A] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\,(bx)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x^2,x, algorithm="fricas")
[Out] integral(cosh_integral(b*x)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x^2,x, algorithm="giac")
[Out] integrate(Chi(b*x)^2/x^2, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)^2/x^2,x)
[Out] int(Chi(b*x)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x^2,x, algorithm="maxima")
[Out] integrate(Chi(b*x)^2/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{coshint}\,(b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)^2/x^2,x)
[Out] int(coshint(b*x)^2/x^2, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)**2/x**2,x)
```

```
[Out] Integral(Chi(b*x)**2/x**2, x)
```

3.84 $\int \frac{\mathbf{Chi}(bx)^2}{x^3}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{\mathrm{Chi}(bx)^2}{x^3},x\right)$$

```
[Out] CannotIntegrate(Chi(b*x)^2/x^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[CoshIntegral[b*x]^2/x^3,x]
[Out] Defer[Int][CoshIntegral[b*x]^2/x^3, x]
```

Rubi steps

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx = \int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx$$

Mathematica [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}(bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[CoshIntegral[b*x]^2/x^3,x]
[Out] Integrate[CoshIntegral[b*x]^2/x^3, x]
```

fricas [A] time = 2.11, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Chi}\,(bx)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x)^2/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x)^2/x^3, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)^2/x^3,x)
```

```
[Out] int(Chi(b*x)^2/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)^2/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{coshint}\,(b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)^2/x^3,x)
```

```
[Out] int(coshint(b*x)^2/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}^2\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)**2/x**3,x)
```

```
[Out] Integral(Chi(b*x)**2/x**3, x)
```

3.85 $\int x^m \mathbf{Chi}(a+bx)\,dx$

Optimal. Leaf size=48

$$\frac{x^{m+1}\text{Chi}(a+bx)}{m+1}-\frac{b\text{Int}\left(\frac{x^{m+1}\cosh(a+bx)}{a+bx},x\right)}{m+1}$$

```
[Out] -b*CannotIntegrate(x^(1+m)*cosh(b*x+a)/(b*x+a),x)/(1+m)+x^(1+m)*Chi(b*x+a)/
(1+m)
```

Rubi [A] time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\text{Chi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*CoshIntegral[a + b*x],x]
```

```
[Out] (x^(1 + m)*CoshIntegral[a + b*x])/(1 + m) - (b*Defer[Int][(x^(1 + m)*Cosh[a
 + b*x])/(a + b*x), x])/(1 + m)
```

Rubi steps

$$\int x^m\text{Chi}(a+bx)\,dx=\frac{x^{1+m}\text{Chi}(a+bx)}{1+m}-\frac{b\int\frac{x^{1+m}\cosh(a+bx)}{a+bx}\,dx}{1+m}$$

Mathematica [A] time = 9.65, size = 0, normalized size = 0.00

$$\int x^m\text{Chi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*CoshIntegral[a + b*x],x]
```

```
[Out] Integrate[x^m*CoshIntegral[a + b*x], x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{Chi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh_integral(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\text{Chi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*Chi(b*x + a), x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int x^m\text{X}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Chi(b*x+a),x)
```

```
[Out] int(x^m*Chi(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\text{Chi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*Chi(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\mathrm{coshint}\left(a+b\,x\right)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*coshint(a + b*x),x)
```

```
[Out] int(x^m*coshint(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\,\text{Chi}\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Chi(b*x+a),x)
```

```
[Out] Integral(x**m*Chi(a + b*x), x)
```

3.86 $\int x^3\mathbf{Chi}(a+bx)\,dx$

Optimal. Leaf size=184

$$-\frac{a^4\text{Chi}(a+bx)}{4b^4}+\frac{a^3\sinh(a+bx)}{4b^4}+\frac{a^2\cosh(a+bx)}{4b^4}-\frac{a^2x\sinh(a+bx)}{4b^3}+\frac{a\sinh(a+bx)}{2b^4}+\frac{3\cosh(a+bx)}{2b^4}-3x\sin$$

```
[Out] -1/4*a^4*Chi(b*x+a)/b^4+1/4*x^4*Chi(b*x+a)+3/2*cosh(b*x+a)/b^4+1/4*a^2*cosh
(b*x+a)/b^4-1/2*a*x*cosh(b*x+a)/b^3+3/4*x^2*cosh(b*x+a)/b^2+1/2*a*sinh(b*x+
a)/b^4+1/4*a^3*sinh(b*x+a)/b^4-3/2*x*sinh(b*x+a)/b^3-1/4*a^2*x*sinh(b*x+a)/
b^3+1/4*a*x^2*sinh(b*x+a)/b^2-1/4*x^3*sinh(b*x+a)/b
```

Rubi [A] time = 0.39, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6533, 6742, 2637, 3296, 2638, 3301}

$$-\frac{a^4\text{Chi}(a+bx)}{4b^4}+\frac{a^3\sinh(a+bx)}{4b^4}-\frac{a^2x\sinh(a+bx)}{4b^3}+\frac{a^2\cosh(a+bx)}{4b^4}+\frac{ax^2\sinh(a+bx)}{4b^2}+\frac{3x^2\cosh(a+bx)}{4b^2}+a$$

Antiderivative was successfully verified.

```
[In] Int[x^3*CoshIntegral[a + b*x],x]
```

```
[Out] (3*Cosh[a + b*x])/(2*b^4) + (a^2*Cosh[a + b*x])/(4*b^4) - (a*x*Cosh[a + b*x
])/(2*b^3) + (3*x^2*Cosh[a + b*x])/(4*b^2) - (a^4*CoshIntegral[a + b*x])/(4
*b^4) + (x^4*CoshIntegral[a + b*x])/4 + (a*Sinh[a + b*x])/(2*b^4) + (a^3*Si
nh[a + b*x])/(4*b^4) - (3*x*Sinh[a + b*x])/(2*b^3) - (a^2*x*Sinh[a + b*x])/
(4*b^3) + (a*x^2*Sinh[a + b*x])/(4*b^2) - (x^3*Sinh[a + b*x])/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x^3\text{Chi}(a+bx)\,dx &= \frac{1}{4}x^4\text{Chi}(a+bx) - \frac{1}{4}b\int \frac{x^4\cosh(a+bx)}{a+bx}\,dx \\
&= \frac{1}{4}x^4\text{Chi}(a+bx) - \frac{1}{4}b\int \left(-\frac{a^3\cosh(a+bx)}{b^4} + \frac{a^2x\cosh(a+bx)}{b^3} - \frac{ax^2\cosh(a+bx)}{b^2} + \frac{x^3 \cdots \\
&= \frac{1}{4}x^4\text{Chi}(a+bx) - \frac{1}{4}\int x^3\cosh(a+bx)\,dx + \frac{a^3\int\cosh(a+bx)\,dx}{4b^3} - \frac{a^4\int\frac{\cosh(a+bx)}{a+bx}\,dx}{4b^3} - \cdots \\
&= -\frac{a^4\text{Chi}(a+bx)}{4b^4} + \frac{1}{4}x^4\text{Chi}(a+bx) + \frac{a^3\sinh(a+bx)}{4b^4} - \frac{a^2x\sinh(a+bx)}{4b^3} + \frac{ax^2\sinh(a+ \cdots}{4b^2} \\
&= \frac{a^2\cosh(a+bx)}{4b^4} - \frac{ax\cosh(a+bx)}{2b^3} + \frac{3x^2\cosh(a+bx)}{4b^2} - \frac{a^4\text{Chi}(a+bx)}{4b^4} + \frac{1}{4}x^4\text{Chi}(a+b \cdots \\
&= \frac{a^2\cosh(a+bx)}{4b^4} - \frac{ax\cosh(a+bx)}{2b^3} + \frac{3x^2\cosh(a+bx)}{4b^2} - \frac{a^4\text{Chi}(a+bx)}{4b^4} + \frac{1}{4}x^4\text{Chi}(a+b \cdots \\
&= \frac{3\cosh(a+bx)}{2b^4} + \frac{a^2\cosh(a+bx)}{4b^4} - \frac{ax\cosh(a+bx)}{2b^3} + \frac{3x^2\cosh(a+bx)}{4b^2} - \frac{a^4\text{Chi}(a+bx \cdots}{4b^4}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 94, normalized size = 0.51

$$\frac{\left(b^4x^4-a^4\right)\text{Chi}(a+bx)+\left(a^2-2abx+3b^2x^2+6\right)\cosh(a+bx)+\left(a^3-a^2bx+ab^2x^2+2a-b^3x^3-6bx\right)\sinh(a \cdots}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*CoshIntegral[a + b*x],x]
```

```
[Out] ((6 + a^2 - 2*a*b*x + 3*b^2*x^2)*Cosh[a + b*x] + (-a^4 + b^4*x^4)*CoshInteg
ral[a + b*x] + (2*a + a^3 - 6*b*x - a^2*b*x + a*b^2*x^2 - b^3*x^3)*Sinh[a +
 b*x])/(4*b^4)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\,\text{Chi}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^3*cosh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Chi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*Chi(b*x + a), x)
```

maple [A] time = 0.02, size = 156, normalized size = 0.85

$$\frac{\frac{X(bx+a)b^4x^4}{4}-\frac{\sinh(bx+a)(bx+a)^3}{4}+\frac{3(bx+a)^2\cosh(bx+a)}{4}-\frac{3(bx+a)\sinh(bx+a)}{2}+\frac{3\cosh(bx+a)}{2}+a\left((bx+a)^2\sinh(bx+a)-2\,(\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Chi(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*Chi(b*x+a)*b^4*x^4-1/4*sinh(b*x+a)*(b*x+a)^3+3/4*(b*x+a)^2*cosh(
b*x+a)-3/2*(b*x+a)*sinh(b*x+a)+3/2*cosh(b*x+a)+a*((b*x+a)^2*sinh(b*x+a)-2*(
b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-3/2*a^2*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))
+a^3*sinh(b*x+a)-1/4*a^4*Chi(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{Chi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*Chi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{coshint}(a + b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coshint(a + b*x),x)
[Out] int(x^3*coshint(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{Chi}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Chi(b*x+a),x)
[Out] Integral(x**3*Chi(a + b*x), x)
```

3.87 $\int x^2 \mathbf{Chi}(a+bx)\,dx$

Optimal. Leaf size=118

$$\frac{a^3\text{Chi}(a+bx)}{3b^3}-\frac{a^2\sinh(a+bx)}{3b^3}-\frac{2\sinh(a+bx)}{3b^3}-\frac{a\cosh(a+bx)}{3b^3}+\frac{ax\sinh(a+bx)}{3b^2}+\frac{2x\cosh(a+bx)}{3b^2}+\frac{1}{3}x^3\text{Chi}($$

```
[Out] 1/3*a^3*Chi(b*x+a)/b^3+1/3*x^3*Chi(b*x+a)-1/3*a*cosh(b*x+a)/b^3+2/3*x*cosh(
b*x+a)/b^2-2/3*sinh(b*x+a)/b^3-1/3*a^2*sinh(b*x+a)/b^3+1/3*a*x*sinh(b*x+a)/
b^2-1/3*x^2*sinh(b*x+a)/b
```

Rubi [A] time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6533, 6742, 2637, 3296, 2638, 3301}

$$\frac{a^3\text{Chi}(a+bx)}{3b^3}-\frac{a^2\sinh(a+bx)}{3b^3}+\frac{ax\sinh(a+bx)}{3b^2}-\frac{2\sinh(a+bx)}{3b^3}-\frac{a\cosh(a+bx)}{3b^3}+\frac{2x\cosh(a+bx)}{3b^2}+\frac{1}{3}x^3\text{Chi}($$

Antiderivative was successfully verified.

```
[In] Int[x^2*CoshIntegral[a + b*x],x]
```

```
[Out] -(a*Cosh[a + b*x])/(3*b^3) + (2*x*Cosh[a + b*x])/(3*b^2) + (a^3*CoshIntegra
l[a + b*x])/(3*b^3) + (x^3*CoshIntegral[a + b*x])/3 - (2*Sinh[a + b*x])/(3*
b^3) - (a^2*Sinh[a + b*x])/(3*b^3) + (a*x*Sinh[a + b*x])/(3*b^2) - (x^2*Sin
h[a + b*x])/(3*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\mathrm{Chi}(a+bx)\,dx &= \frac{1}{3}x^3\mathrm{Chi}(a+bx)-\frac{1}{3}b\int\frac{x^3\cosh(a+bx)}{a+bx}\,dx\\
&= \frac{1}{3}x^3\mathrm{Chi}(a+bx)-\frac{1}{3}b\int\left(\frac{a^2\cosh(a+bx)}{b^3}-\frac{ax\cosh(a+bx)}{b^2}+\frac{x^2\cosh(a+bx)}{b}-\frac{a^3\cosh\ldots}{b^3(a\ldots}\right.\\
&= \frac{1}{3}x^3\mathrm{Chi}(a+bx)-\frac{1}{3}\int x^2\cosh(a+bx)\,dx-\frac{a^2\int\cosh(a+bx)\,dx}{3b^2}+\frac{a^3\int\frac{\cosh(a+bx)}{a+bx}\,dx}{3b^2}+\ldots\\
&= \frac{a^3\mathrm{Chi}(a+bx)}{3b^3}+\frac{1}{3}x^3\mathrm{Chi}(a+bx)-\frac{a^2\sinh(a+bx)}{3b^3}+\frac{ax\sinh(a+bx)}{3b^2}-\frac{x^2\sinh(a+bx)}{3b}\ldots\\
&= -\frac{a\cosh(a+bx)}{3b^3}+\frac{2x\cosh(a+bx)}{3b^2}+\frac{a^3\mathrm{Chi}(a+bx)}{3b^3}+\frac{1}{3}x^3\mathrm{Chi}(a+bx)-\frac{a^2\sinh(a+bx)}{3b^3}\ldots\\
&= -\frac{a\cosh(a+bx)}{3b^3}+\frac{2x\cosh(a+bx)}{3b^2}+\frac{a^3\mathrm{Chi}(a+bx)}{3b^3}+\frac{1}{3}x^3\mathrm{Chi}(a+bx)-\frac{2\sinh(a+bx)}{3b^3}\ldots\end{aligned}$$

Mathematica [A] time = 0.18, size = 64, normalized size = 0.54

$$-\frac{-\left(a^3+b^3x^3\right)\mathrm{Chi}(a+bx)+\left(a^2-abx+b^2x^2+2\right)\sinh(a+bx)+(a-2bx)\cosh(a+bx)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*CoshIntegral[a + b*x],x]
```

```
[Out] -1/3*((a - 2*b*x)*Cosh[a + b*x] - (a^3 + b^3*x^3)*CoshIntegral[a + b*x] + (
2 + a^2 - a*b*x + b^2*x^2)*Sinh[a + b*x])/b^3
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\,\mathrm{Chi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a),x, algorithm="fricas")
[Out] integral(x^2*cosh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Chi}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a),x, algorithm="giac")
[Out] integrate(x^2*Chi(b*x + a), x)
```

maple [A] time = 0.02, size = 101, normalized size = 0.86

$$\frac{\frac{b^3x^3\mathrm{X}(bx+a)}{3}-\frac{(bx+a)^2\sinh(bx+a)}{3}+\frac{2(bx+a)\cosh(bx+a)}{3}-\frac{2\sinh(bx+a)}{3}+a\left(\left(bx+a\right)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)\right)-a^2\sin}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x+a),x)
[Out] 1/b^3*(1/3*b^3*x^3*Chi(b*x+a)-1/3*(b*x+a)^2*sinh(b*x+a)+2/3*(b*x+a)*cosh(b*
x+a)-2/3*sinh(b*x+a)+a*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-a^2*sinh(b*x+a)+1/
3*a^3*Chi(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Chi}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a),x, algorithm="maxima")
[Out] integrate(x^2*Chi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{coshint}\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(a + b*x),x)
```

```
[Out] int(x^2*coshint(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Chi}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x+a),x)
```

```
[Out] Integral(x**2*Chi(a + b*x), x)
```

3.88 $\int x\mathbf{Chi}(a+bx)\,dx$

Optimal. Leaf size=71

$$-\frac{a^2\text{Chi}(a+bx)}{2b^2}+\frac{a\sinh(a+bx)}{2b^2}+\frac{\cosh(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(a+bx)-\frac{x\sinh(a+bx)}{2b}$$

```
[Out] -1/2*a^2*Chi(b*x+a)/b^2+1/2*x^2*Chi(b*x+a)+1/2*cosh(b*x+a)/b^2+1/2*a*sinh(b
*x+a)/b^2-1/2*x*sinh(b*x+a)/b
```

Rubi [A] time = 0.21, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6533, 6742, 2637, 3296, 2638, 3301}

$$-\frac{a^2\text{Chi}(a+bx)}{2b^2}+\frac{a\sinh(a+bx)}{2b^2}+\frac{\cosh(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(a+bx)-\frac{x\sinh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/(2*b^2) - (a^2*CoshIntegral[a + b*x])/(2*b^2) + (x^2*CoshInte
gral[a + b*x])/2 + (a*Sinh[a + b*x])/(2*b^2) - (x*Sinh[a + b*x])/(2*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\text{Chi}(a+bx)\,dx &= \frac{1}{2}x^2\text{Chi}(a+bx)-\frac{1}{2}b\int\frac{x^2\cosh(a+bx)}{a+bx}\,dx\\ &= \frac{1}{2}x^2\text{Chi}(a+bx)-\frac{1}{2}b\int\left(-\frac{a\cosh(a+bx)}{b^2}+\frac{x\cosh(a+bx)}{b}+\frac{a^2\cosh(a+bx)}{b^2(a+bx)}\right)dx\\ &= \frac{1}{2}x^2\text{Chi}(a+bx)-\frac{1}{2}\int x\cosh(a+bx)\,dx+\frac{a\int\cosh(a+bx)\,dx}{2b}-\frac{a^2\int\frac{\cosh(a+bx)}{a+bx}\,dx}{2b}\\ &= -\frac{a^2\text{Chi}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(a+bx)+\frac{a\sinh(a+bx)}{2b^2}-\frac{x\sinh(a+bx)}{2b}+\frac{\int\sinh(a+bx)\,dx}{2b}\\ &= \frac{\cosh(a+bx)}{2b^2}-\frac{a^2\text{Chi}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{Chi}(a+bx)+\frac{a\sinh(a+bx)}{2b^2}-\frac{x\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 47, normalized size = 0.66

$$\frac{(b^2x^2-a^2)\,\text{Chi}(a+bx)+(a-bx)\sinh(a+bx)+\cosh(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[a + b*x],x]
```

```
[Out] (Cosh[a + b*x] + (-a^2 + b^2*x^2)*CoshIntegral[a + b*x] + (a - b*x)*Sinh[a
+ b*x])/(2*b^2)
```

fricas [F] time = 2.10, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Chi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*cosh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*Chi(b*x + a), x)
```

maple [A] time = 0.02, size = 60, normalized size = 0.85

$$\frac{\mathrm{X}\left(bx+a\right)\left(\frac{(bx+a)^2}{2}-a\left(bx+a\right)\right)-\frac{(bx+a)\sinh(bx+a)}{2}+\frac{\cosh(bx+a)}{2}+a\sinh\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x+a),x)
```

```
[Out] 1/b^2*(Chi(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))-1/2*(b*x+a)*sinh(b*x+a)+1/2*cos
h(b*x+a)+a*sinh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\frac{x^2\,\mathrm{coshint}\left(a+b\,x\right)}{2}+\frac{\frac{\mathrm{e}^{-a-b\,x}\,\left(\mathrm{e}^{2\,a+2\,b\,x}-a+a\,\mathrm{e}^{2\,a+2\,b\,x}-2\,a^2\,\mathrm{coshint}(a+b\,x)\,\mathrm{e}^{a+b\,x}+1\right)}{4}+\frac{b\,\mathrm{e}^{-a-b\,x}\,\left(x-x\,\mathrm{e}^{2\,a+2\,b\,x}\right)}{4}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(a + b*x),x)
```

```
[Out] (x^2*coshint(a + b*x))/2 + ((exp(- a - b*x)*(exp(2*a + 2*b*x) - a + a*exp(2
*a + 2*b*x) - 2*a^2*coshint(a + b*x)*exp(a + b*x) + 1))/4 + (b*exp(- a - b*
x)*(x - x*exp(2*a + 2*b*x)))/4)/b^2
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Chi}(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a),x)
```

```
[Out] Integral(x*Chi(a + b*x), x)
```

3.89 $\int \mathrm{Chi}(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{(a+bx)\mathrm{Chi}(a+bx)}{b}-\frac{\sinh(a+bx)}{b}$$

```
[Out] (b*x+a)*Chi(b*x+a)/b-sinh(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6529}

$$\frac{(a+bx)\mathrm{Chi}(a+bx)}{b}-\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[a + b*x],x]
```

```
[Out] ((a + b*x)*CoshIntegral[a + b*x])/b - Sinh[a + b*x]/b
```

Rule 6529

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*CoshInte
gral[a + b*x])/b, x] - Simp[Sinh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \mathrm{Chi}(a+bx)\,dx=\frac{(a+bx)\mathrm{Chi}(a+bx)}{b}-\frac{\sinh(a+bx)}{b}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.56

$$x\mathrm{Chi}(a+bx)+\frac{a\mathrm{Chi}(a+bx)}{b}-\frac{\sinh(a)\cosh(bx)}{b}-\frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[a + b*x],x]
```

```
[Out] (a*CoshIntegral[a + b*x])/b + x*CoshIntegral[a + b*x] - (Cosh[b*x]*Sinh[a])
/b - (Cosh[a]*Sinh[b*x])/b
```

fricas [F] time = 3.29, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{Chi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\,(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a), x)
```

maple [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{(bx+a)\,\text{X}\,(bx+a)-\sinh\,(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*Chi(b*x+a)-sinh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\,(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$x\,\mathrm{coshint}\,(a+b\,x)-\frac{\mathrm{e}^{a+b\,x}}{2\,b}+\frac{\mathrm{e}^{-a-b\,x}}{2\,b}+\frac{a\,\mathrm{coshint}\,(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x),x)
```

```
[Out] x*coshint(a + b*x) - exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (a*coshint
(a + b*x))/b
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a),x)
```

```
[Out] Integral(Chi(a + b*x), x)
```

3.90 $\int \frac{\text{Chi}(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\text{Chi}(a+bx)}{x},x\right)$$

[Out] CannotIntegrate(Chi(b*x+a)/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Chi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[CoshIntegral[a + b*x]/x,x]

[Out] Defer[Int][CoshIntegral[a + b*x]/x, x]

Rubi steps

$$\int \frac{\text{Chi}(a+bx)}{x}\,dx = \int \frac{\text{Chi}(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[CoshIntegral[a + b*x]/x,x]

[Out] Integrate[CoshIntegral[a + b*x]/x, x]

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\left(bx+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(Chi(b*x+a)/x,x, algorithm="fricas")

```
[Out] integral(cosh_integral(b*x + a)/x, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x,x, algorithm="giac")
[Out] integrate(Chi(b*x + a)/x, x)
```

maple [**A**] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)/x,x)
[Out] int(Chi(b*x+a)/x,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(Chi(b*x + a)/x, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\text{coshint}\,(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)/x,x)
[Out] int(coshint(a + b*x)/x, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x,x)
```

```
[Out] Integral(Chi(a + b*x)/x, x)
```

3.91 $\int \frac{\text{Chi}(a+bx)}{x^2} \, dx$

Optimal. Leaf size=46

$$-\frac{b\text{Chi}(a+bx)}{a}-\frac{\text{Chi}(a+bx)}{x}+\frac{b\cosh(a)\text{Chi}(bx)}{a}+\frac{b\sinh(a)\text{Shi}(bx)}{a}$$

```
[Out] -b*Chi(b*x+a)/a-Chi(b*x+a)/x+b*Chi(b*x)*cosh(a)/a+b*Shi(b*x)*sinh(a)/a
```

Rubi [A] time = 0.22, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6533, 6742, 3303, 3298, 3301}

$$-\frac{b\text{Chi}(a+bx)}{a}-\frac{\text{Chi}(a+bx)}{x}+\frac{b\cosh(a)\text{Chi}(bx)}{a}+\frac{b\sinh(a)\text{Shi}(bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[a + b*x]/x^2,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x])/a - (b*CoshIntegral[a + b*x])/a - CoshIntegra
l[a + b*x]/x + (b*Sinh[a]*SinhIntegral[b*x])/a
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
```

```
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\mathrm{Chi}(a+bx)}{x^2}\,dx &= -\frac{\mathrm{Chi}(a+bx)}{x} + b\int \frac{\cosh(a+bx)}{x(a+bx)}\,dx \\
&= -\frac{\mathrm{Chi}(a+bx)}{x} + b\int \left(\frac{\cosh(a+bx)}{ax} - \frac{b\cosh(a+bx)}{a(a+bx)}\right)dx \\
&= -\frac{\mathrm{Chi}(a+bx)}{x} + \frac{b\int \frac{\cosh(a+bx)}{x}\,dx}{a} - \frac{b^2\int \frac{\cosh(a+bx)}{a+bx}\,dx}{a} \\
&= -\frac{b\mathrm{Chi}(a+bx)}{a} - \frac{\mathrm{Chi}(a+bx)}{x} + \frac{(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx}{a} + \frac{(b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx}{a} \\
&= \frac{b\cosh(a)\mathrm{Chi}(bx)}{a} - \frac{b\mathrm{Chi}(a+bx)}{a} - \frac{\mathrm{Chi}(a+bx)}{x} + \frac{b\sinh(a)\mathrm{Shi}(bx)}{a}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 39, normalized size = 0.85

$$\frac{-(a+bx)\mathrm{Chi}(a+bx) + bx\cosh(a)\mathrm{Chi}(bx) + bx\sinh(a)\mathrm{Shi}(bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[a + b*x]/x^2,x]
```

```
[Out] (b*x*Cosh[a]*CoshIntegral[b*x] - (a + b*x)*CoshIntegral[a + b*x] + b*x*Sinh
[a]*SinhIntegral[b*x])/(a*x)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Chi}\,(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(bx+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)/x^2,x)
```

```
[Out] int(Chi(b*x+a)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{coshint}\left(a+b\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)/x^2,x)
```

```
[Out] int(coshint(a + b*x)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(a+bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x**2,x)
```

```
[Out] Integral(Chi(a + b*x)/x**2, x)
```

3.92 $\int \frac{\text{Chi}(a+bx)}{x^3} \, dx$

Optimal. Leaf size=111

$$\frac{b^2\text{Chi}(a+bx)}{2a^2} - \frac{b^2\cosh(a)\text{Chi}(bx)}{2a^2} - \frac{b^2\sinh(a)\text{Shi}(bx)}{2a^2} + \frac{b^2\sinh(a)\text{Chi}(bx)}{2a} + \frac{b^2\cosh(a)\text{Shi}(bx)}{2a} - \frac{\text{Chi}(a+bx)}{2x^2} -$$

```
[Out] 1/2*b^2*Chi(b*x+a)/a^2-1/2*Chi(b*x+a)/x^2-1/2*b^2*Chi(b*x)*cosh(a)/a^2-1/2*
b*cosh(b*x+a)/a/x+1/2*b^2*cosh(a)*Shi(b*x)/a+1/2*b^2*Chi(b*x)*sinh(a)/a-1/2
*b^2*Shi(b*x)*sinh(a)/a^2
```

Rubi [A] time = 0.33, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6533, 6742, 3297, 3303, 3298, 3301}

$$\frac{b^2\text{Chi}(a+bx)}{2a^2} - \frac{b^2\cosh(a)\text{Chi}(bx)}{2a^2} - \frac{b^2\sinh(a)\text{Shi}(bx)}{2a^2} + \frac{b^2\sinh(a)\text{Chi}(bx)}{2a} + \frac{b^2\cosh(a)\text{Shi}(bx)}{2a} - \frac{\text{Chi}(a+bx)}{2x^2} -$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[a + b*x]/x^3,x]
```

```
[Out] -(b*Cosh[a + b*x])/(2*a*x) - (b^2*Cosh[a]*CoshIntegral[b*x])/(2*a^2) + (b^2
*CoshIntegral[a + b*x])/(2*a^2) - CoshIntegral[a + b*x]/(2*x^2) + (b^2*Cosh
Integral[b*x]*Sinh[a])/(2*a) + (b^2*Cosh[a]*SinhIntegral[b*x])/(2*a) - (b^2
*Sinh[a]*SinhIntegral[b*x])/(2*a^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6533

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*CoshIntegral[a + b*x])/(d*(m + 1)), x] - Dist[b/
(d*(m + 1)), Int[((c + d*x)^(m + 1)*Cosh[a + b*x])/(a + b*x), x], x] /; Fre
eQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\text{Chi}(a+bx)}{x^3}\,dx = -\frac{\text{Chi}(a+bx)}{2x^2} + \frac{1}{2}b\int \frac{\cosh(a+bx)}{x^2(a+bx)}\,dx$$
$$= -\frac{\text{Chi}(a+bx)}{2x^2} + \frac{1}{2}b\int \left(\frac{\cosh(a+bx)}{ax^2} - \frac{b\cosh(a+bx)}{a^2x} + \frac{b^2\cosh(a+bx)}{a^2(a+bx)}\right)dx$$
$$= -\frac{\text{Chi}(a+bx)}{2x^2} + \frac{b\int \frac{\cosh(a+bx)}{x^2}\,dx}{2a} - \frac{b^2\int \frac{\cosh(a+bx)}{x}\,dx}{2a^2} + \frac{b^3\int \frac{\cosh(a+bx)}{a+bx}\,dx}{2a^2}$$
$$= -\frac{b\cosh(a+bx)}{2ax} + \frac{b^2\text{Chi}(a+bx)}{2a^2} - \frac{\text{Chi}(a+bx)}{2x^2} + \frac{b^2\int \frac{\sinh(a+bx)}{x}\,dx}{2a} - \frac{\left(b^2\cosh(a)\right)\int \frac{\cosh(}{x}$$
$$= -\frac{b\cosh(a+bx)}{2ax} - \frac{b^2\cosh(a)\text{Chi}(bx)}{2a^2} + \frac{b^2\text{Chi}(a+bx)}{2a^2} - \frac{\text{Chi}(a+bx)}{2x^2} - \frac{b^2\sinh(a)\text{Shi}(bx)}{2a^2}$$
$$= -\frac{b\cosh(a+bx)}{2ax} - \frac{b^2\cosh(a)\text{Chi}(bx)}{2a^2} + \frac{b^2\text{Chi}(a+bx)}{2a^2} - \frac{\text{Chi}(a+bx)}{2x^2} + \frac{b^2\text{Chi}(bx)\sinh(a)}{2a}$$

Mathematica [A] time = 0.30, size = 80, normalized size = 0.72

$$\frac{\left(b^2x^2-a^2\right)\text{Chi}(a+bx) + b^2x^2(a\sinh(a)-\cosh(a))\text{Chi}(bx) + bx(bx(a\cosh(a)-\sinh(a))\text{Shi}(bx) - a\cosh(a+b}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[a + b*x]/x^3,x]
```

```
[Out] ((-a^2 + b^2*x^2)*CoshIntegral[a + b*x] + b^2*x^2*CoshIntegral[b*x]*(-Cosh[
a] + a*Sinh[a]) + b*x*(-(a*Cosh[a + b*x]) + b*x*(a*Cosh[a] - Sinh[a])*SinhI
ntegral[b*x]))/(2*a^2*x^2)
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\,(bx+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)/x^3, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{X}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)/x^3,x)
```

```
[Out] int(Chi(b*x+a)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{coshint}\left(a+b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)/x^3,x)
```

```
[Out] int(coshint(a + b*x)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(a+bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)/x**3,x)
```

```
[Out] Integral(Chi(a + b*x)/x**3, x)
```

3.93 $\int x^m \textbf{Chi}(a+bx)^2\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m\text{Chi}(a+bx)^2,x\right)$$

```
[Out] CannotIntegrate(x^m*Chi(b*x+a)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\text{Chi}(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*CoshIntegral[a + b*x]^2,x]
[Out] Defer[Int][x^m*CoshIntegral[a + b*x]^2, x]
```

Rubi steps

$$\int x^m\text{Chi}(a+bx)^2\,dx=\int x^m\text{Chi}(a+bx)^2\,dx$$

Mathematica [A] time = 3.45, size = 0, normalized size = 0.00

$$\int x^m\text{Chi}(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*CoshIntegral[a + b*x]^2,x]
[Out] Integrate[x^m*CoshIntegral[a + b*x]^2, x]
```

fricas [A] time = 1.94, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{Chi}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh_integral(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Chi}\,(bx+a)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*Chi(b*x + a)^2, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int x^m \mathrm{X}\,(bx+a)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*Chi(b*x+a)^2,x)
[Out] int(x^m*Chi(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \mathrm{Chi}\,(bx+a)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*Chi(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*Chi(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int x^m\,\mathrm{coshint}\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*coshint(a + b*x)^2,x)
[Out] int(x^m*coshint(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\,\mathrm{Chi}^2\,(a+bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*Chi(b*x+a)**2,x)
[Out] Integral(x**m*Chi(a + b*x)**2, x)
```

3.94 $\int x^2\mathbf{Chi}(a+bx)^2\,dx$

Optimal. Leaf size=327

$$\frac{a^2(a+bx)\text{Chi}(a+bx)^2}{3b^3}-\frac{2a^2\text{Chi}(a+bx)\sinh(a+bx)}{3b^3}+\frac{a^2\text{Shi}(2a+2bx)}{b^3}+\frac{a\text{Chi}(2a+2bx)}{b^3}-\frac{4\text{Chi}(a+bx)\sinh(}{3b^3}$$

```
[Out] -2/3*x/b^2+1/3*a^2*(b*x+a)*Chi(b*x+a)^2/b^3-1/3*a*x*(b*x+a)*Chi(b*x+a)^2/b^
2+1/3*x^2*(b*x+a)*Chi(b*x+a)^2/b+a*Chi(2*b*x+2*a)/b^3-2/3*a*Chi(b*x+a)*cosh
(b*x+a)/b^3+4/3*x*Chi(b*x+a)*cosh(b*x+a)/b^2-1/3*a*cosh(2*b*x+2*a)/b^3+1/6*
x*cosh(2*b*x+2*a)/b^2+a*ln(b*x+a)/b^3+2/3*Shi(2*b*x+2*a)/b^3+a^2*Shi(2*b*x+
2*a)/b^3-4/3*Chi(b*x+a)*sinh(b*x+a)/b^3-2/3*a^2*Chi(b*x+a)*sinh(b*x+a)/b^3+
2/3*a*x*Chi(b*x+a)*sinh(b*x+a)/b^2-2/3*x^2*Chi(b*x+a)*sinh(b*x+a)/b-2/3*cos
h(b*x+a)*sinh(b*x+a)/b^3-1/12*sinh(2*b*x+2*a)/b^3
```

Rubi [A] time = 1.31, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 19, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.583, Rules used = {6539, 6543, 5617, 6741, 6742, 2638, 3296, 2637, 3298, 6549, 2635, 8, 3312, 3301, 6541, 5448, 12, 6547, 6535}

$$\frac{a^2(a+bx)\text{Chi}(a+bx)^2}{3b^3}-\frac{2a^2\text{Chi}(a+bx)\sinh(a+bx)}{3b^3}+\frac{a^2\text{Shi}(2a+2bx)}{b^3}-\frac{ax(a+bx)\text{Chi}(a+bx)^2}{3b^2}+\frac{a\text{Chi}(2a+}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*CoshIntegral[a + b*x]^2,x]
```

```
[Out] (-2*x)/(3*b^2) - (a*Cosh[2*a + 2*b*x])/(3*b^3) + (x*Cosh[2*a + 2*b*x])/(6*b
^2) - (2*a*Cosh[a + b*x]*CoshIntegral[a + b*x])/(3*b^3) + (4*x*Cosh[a + b*x
]*CoshIntegral[a + b*x])/(3*b^2) + (a^2*(a + b*x)*CoshIntegral[a + b*x]^2)/
(3*b^3) - (a*x*(a + b*x)*CoshIntegral[a + b*x]^2)/(3*b^2) + (x^2*(a + b*x)*
CoshIntegral[a + b*x]^2)/(3*b) + (a*CoshIntegral[2*a + 2*b*x])/b^3 + (a*Log
[a + b*x])/b^3 - (2*Cosh[a + b*x]*Sinh[a + b*x])/(3*b^3) - (4*CoshIntegral[
a + b*x]*Sinh[a + b*x])/(3*b^3) - (2*a^2*CoshIntegral[a + b*x]*Sinh[a + b*x
])/(3*b^3) + (2*a*x*CoshIntegral[a + b*x]*Sinh[a + b*x])/(3*b^2) - (2*x^2*C
oshIntegral[a + b*x]*Sinh[a + b*x])/(3*b) - Sinh[2*a + 2*b*x]/(12*b^3) + (2
*SinhIntegral[2*a + 2*b*x])/(3*b^3) + (a^2*SinhIntegral[2*a + 2*b*x])/b^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6535

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*CoshIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Cosh[a + b*x]*CoshIntegral[a + b*x],
 x], x] /; FreeQ[{a, b}, x]
```

Rule 6539

```
Int[CoshIntegral[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((a + b*x)*(c + d*x)^m*CoshIntegral[a + b*x]^2)/(b*(m + 1)), x] +
(-Dist[2/(m + 1), Int[(c + d*x)^m*Cosh[a + b*x]*CoshIntegral[a + b*x], x],
x] + Dist[((b*c - a*d)*m)/(b*(m + 1)), Int[(c + d*x)^(m - 1)*CoshIntegral[a
 + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\text{Chi}(a+bx)^2\,dx = \frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b} - \frac{2}{3}\int x^2\cosh(a+bx)\text{Chi}(a+bx)\,dx - \frac{(2a)\int x\text{Chi}(a+bx)^2\,dx}{3b}$$

$$= -\frac{ax(a+bx)\text{Chi}(a+bx)^2}{3b^2} + \frac{x^2(a+bx)\text{Chi}(a+bx)^2}{3b} - \frac{2x^2\text{Chi}(a+bx)\sinh(a+bx)}{3b} + \frac{2}{3}$$

$$= \frac{4x\cosh(a+bx)\text{Chi}(a+bx)}{3b^2} + \frac{a^2(a+bx)\text{Chi}(a+bx)^2}{3b^3} - \frac{ax(a+bx)\text{Chi}(a+bx)^2}{3b^2} + \frac{x^2(a}{}$$

$$= -\frac{2a\cosh(a+bx)\text{Chi}(a+bx)}{3b^3} + \frac{4x\cosh(a+bx)\text{Chi}(a+bx)}{3b^2} + \frac{a^2(a+bx)\text{Chi}(a+bx)^2}{3b^3}$$

$$= -\frac{2a\cosh(a+bx)\text{Chi}(a+bx)}{3b^3} + \frac{4x\cosh(a+bx)\text{Chi}(a+bx)}{3b^2} + \frac{a^2(a+bx)\text{Chi}(a+bx)^2}{3b^3}$$

$$= -\frac{2a\cosh(a+bx)\text{Chi}(a+bx)}{3b^3} + \frac{4x\cosh(a+bx)\text{Chi}(a+bx)}{3b^2} + \frac{a^2(a+bx)\text{Chi}(a+bx)^2}{3b^3}$$

$$= -\frac{2x}{3b^2} - \frac{a\cosh(2a+2bx)}{6b^3} + \frac{x\cosh(2a+2bx)}{6b^2} - \frac{2a\cosh(a+bx)\text{Chi}(a+bx)}{3b^3} + \frac{4x\cosh(}{}$$

$$= -\frac{2x}{3b^2} - \frac{a\cosh(2a+2bx)}{3b^3} + \frac{x\cosh(2a+2bx)}{6b^2} - \frac{2a\cosh(a+bx)\text{Chi}(a+bx)}{3b^3} + \frac{4x\cosh(}{}$$

Mathematica [A] time = 1.08, size = 158, normalized size = 0.48

$$\frac{4\left(a^3+b^3x^3\right)\text{Chi}(a+bx)^2-8\text{Chi}(a+bx)\left(\left(a^2-abx+b^2x^2+2\right)\sinh(a+bx)+(a-2bx)\cosh(a+bx)\right)+12a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*CoshIntegral[a + b*x]^2,x]
```

```
[Out] (-8*a - 8*b*x - 4*a*Cosh[2*(a + b*x)] + 2*b*x*Cosh[2*(a + b*x)] + 4*(a^3 +
b^3*x^3)*CoshIntegral[a + b*x]^2 + 12*a*CoshIntegral[2*(a + b*x)] + 12*a*Lo
g[a + b*x] - 8*CoshIntegral[a + b*x]*((a - 2*b*x)*Cosh[a + b*x] + (2 + a^2
- a*b*x + b^2*x^2)*Sinh[a + b*x]) - 5*Sinh[2*(a + b*x)] + 8*SinhIntegral[2*
(a + b*x)] + 12*a^2*SinhIntegral[2*(a + b*x)])/(12*b^3)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{Chi}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh_integral(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{Chi}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*Chi(b*x + a)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\text{X}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x+a)^2,x)
```

```
[Out] int(x^2*Chi(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{Chi}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*Chi(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{coshint}(a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(a + b*x)^2,x)
```

```
[Out] int(x^2*coshint(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Chi}^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x+a)**2,x)
```

```
[Out] Integral(x**2*Chi(a + b*x)**2, x)
```

3.95 $\int x\mathbf{Chi}(a+bx)^2\,dx$

Optimal. Leaf size=154

$$-\frac{a(a+bx)\mathrm{Chi}(a+bx)^2}{2b^2}-\frac{\mathrm{Chi}(2a+2bx)}{2b^2}+\frac{a\mathrm{Chi}(a+bx)\sinh(a+bx)}{b^2}+\frac{\mathrm{Chi}(a+bx)\cosh(a+bx)}{b^2}-\frac{a\mathrm{Shi}(2a+2b}{b^2}$$

```
[Out] -1/2*a*(b*x+a)*Chi(b*x+a)^2/b^2+1/2*x*(b*x+a)*Chi(b*x+a)^2/b-1/2*Chi(2*b*x+
2*a)/b^2+Chi(b*x+a)*cosh(b*x+a)/b^2+1/4*cosh(2*b*x+2*a)/b^2-1/2*ln(b*x+a)/b
^2-a*Shi(2*b*x+2*a)/b^2+a*Chi(b*x+a)*sinh(b*x+a)/b^2-x*Chi(b*x+a)*sinh(b*x+
a)/b
```

Rubi [A] time = 0.33, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {6539, 6543, 5617, 6741, 6742, 2638, 3298, 6547, 3312, 3301, 6535, 6541, 5448, 12}

$$-\frac{a(a+bx)\mathrm{Chi}(a+bx)^2}{2b^2}-\frac{\mathrm{Chi}(2a+2bx)}{2b^2}+\frac{a\mathrm{Chi}(a+bx)\sinh(a+bx)}{b^2}+\frac{\mathrm{Chi}(a+bx)\cosh(a+bx)}{b^2}-\frac{a\mathrm{Shi}(2a+2b}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[a + b*x]^2,x]
```

```
[Out] Cosh[2*a + 2*b*x]/(4*b^2) + (Cosh[a + b*x]*CoshIntegral[a + b*x])/b^2 - (a*
(a + b*x)*CoshIntegral[a + b*x]^2)/(2*b^2) + (x*(a + b*x)*CoshIntegral[a +
b*x]^2)/(2*b) - CoshIntegral[2*a + 2*b*x]/(2*b^2) - Log[a + b*x]/(2*b^2) +
(a*CoshIntegral[a + b*x]*Sinh[a + b*x])/b^2 - (x*CoshIntegral[a + b*x]*Sinh
[a + b*x])/b - (a*SinhIntegral[2*a + 2*b*x])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6535

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*CoshIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Cosh[a + b*x]*CoshIntegral[a + b*x],
 x], x] /; FreeQ[{a, b}, x]
```

Rule 6539

```
Int[CoshIntegral[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((a + b*x)*(c + d*x)^m*CoshIntegral[a + b*x]^2)/(b*(m + 1)), x] +
(-Dist[2/(m + 1), Int[(c + d*x)^m*Cosh[a + b*x]*CoshIntegral[a + b*x], x],
x] + Dist[((b*c - a*d)*m)/(b*(m + 1)), Int[(c + d*x)^(m - 1)*CoshIntegral[a
 + b*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\text{Chi}(a+bx)^2\,dx = \frac{x(a+bx)\text{Chi}(a+bx)^2}{2b} - \frac{a\int \text{Chi}(a+bx)^2\,dx}{2b} - \int x\cosh(a+bx)\text{Chi}(a+bx)\,dx$$

$$= -\frac{a(a+bx)\text{Chi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Chi}(a+bx)^2}{2b} - \frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b} + \frac{\int \text{Chi}$$

$$= \frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} - \frac{a(a+bx)\text{Chi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Chi}(a+bx)^2}{2b} + \frac{a\text{Chi}(a+$$

$$= \frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} - \frac{a(a+bx)\text{Chi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Chi}(a+bx)^2}{2b} + \frac{a\text{Chi}(a+$$

$$= \frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} - \frac{a(a+bx)\text{Chi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Chi}(a+bx)^2}{2b} - \frac{\log(a+b}{2b^2}$$

$$= \frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} - \frac{a(a+bx)\text{Chi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Chi}(a+bx)^2}{2b} - \frac{\text{Chi}(2a+}{2b^2}$$

$$= \frac{\cosh(2a+2bx)}{4b^2} + \frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} - \frac{a(a+bx)\text{Chi}(a+bx)^2}{2b^2} + \frac{x(a+bx)\text{Chi}(a}{2b}$$

Mathematica [A] time = 0.19, size = 95, normalized size = 0.62

$$\frac{-2\left(a^2-b^2x^2\right)\mathrm{Chi}(a+bx)^2-2\mathrm{Chi}(2(a+bx))+4\mathrm{Chi}(a+bx)((a-bx)\sinh(a+bx)+\cosh(a+bx))-4a\mathrm{Shi}(2(}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[a + b*x]^2,x]
```

```
[Out] (Cosh[2*(a + b*x)] - 2*(a^2 - b^2*x^2)*CoshIntegral[a + b*x]^2 - 2*CoshInte
gral[2*(a + b*x)] - 2*Log[a + b*x] + 4*CoshIntegral[a + b*x]*(Cosh[a + b*x]
 + (a - b*x)*Sinh[a + b*x]) - 4*a*SinhIntegral[2*(a + b*x)])/(4*b^2)
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x\,\mathrm{Chi}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*cosh_integral(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*Chi(b*x + a)^2, x)
```

maple [A] time = 0.03, size = 135, normalized size = 0.88

$$\frac{x^2\mathrm{X}\,(bx+a)^2}{2}-\frac{\mathrm{X}\,(bx+a)^2\,a^2}{2b^2}-\frac{x\mathrm{X}\,(bx+a)\sinh\,(bx+a)}{b}+\frac{a\mathrm{X}\,(bx+a)\sinh\,(bx+a)}{b^2}+\frac{\mathrm{X}\,(bx+a)\cosh\,(bx+a)}{b^2}+\frac{\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x+a)^2,x)
```

```
[Out] 1/2*x^2*Chi(b*x+a)^2-1/2/b^2*Chi(b*x+a)^2*a^2-x*Chi(b*x+a)*sinh(b*x+a)/b+a*
Chi(b*x+a)*sinh(b*x+a)/b^2+Chi(b*x+a)*cosh(b*x+a)/b^2+1/2/b^2*cosh(b*x+a)^2
-1/2*ln(b*x+a)/b^2-1/2*Chi(2*b*x+2*a)/b^2-a*Shi(2*b*x+2*a)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x*Chi(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{coshint}\left(a+b\,x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(a + b*x)^2,x)
[Out] int(x*coshint(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{Chi}^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)**2,x)
[Out] Integral(x*Chi(a + b*x)**2, x)
```

3.96 $\int \mathbf{Chi}(a+bx)^2\,dx$

Optimal. Leaf size=48

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-\frac{2\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{\text{Shi}(2a+2bx)}{b}$$

```
[Out] (b*x+a)*Chi(b*x+a)^2/b+Shi(2*b*x+2*a)/b-2*Chi(b*x+a)*sinh(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6535, 6541, 5448, 12, 3298}

$$\frac{(a+bx)\text{Chi}(a+bx)^2}{b}-\frac{2\text{Chi}(a+bx)\sinh(a+bx)}{b}+\frac{\text{Shi}(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*CoshIntegral[a + b*x]^2)/b - (2*CoshIntegral[a + b*x]*Sinh[a + b
*x])/b + SinhIntegral[2*a + 2*b*x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6535

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*CoshIn
tegral[a + b*x]^2)/b, x] - Dist[2, Int[Cosh[a + b*x]*CoshIntegral[a + b*x],
```

```
x], x] /; FreeQ[{a, b}, x]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \mathrm{Chi}(a+bx)^2\,dx &= \frac{(a+bx)\mathrm{Chi}(a+bx)^2}{b} - 2\int \cosh(a+bx)\mathrm{Chi}(a+bx)\,dx \\
&= \frac{(a+bx)\mathrm{Chi}(a+bx)^2}{b} - \frac{2\mathrm{Chi}(a+bx)\sinh(a+bx)}{b} + 2\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}\,dx \\
&= \frac{(a+bx)\mathrm{Chi}(a+bx)^2}{b} - \frac{2\mathrm{Chi}(a+bx)\sinh(a+bx)}{b} + 2\int \frac{\sinh(2a+2bx)}{2(a+bx)}\,dx \\
&= \frac{(a+bx)\mathrm{Chi}(a+bx)^2}{b} - \frac{2\mathrm{Chi}(a+bx)\sinh(a+bx)}{b} + \int \frac{\sinh(2a+2bx)}{a+bx}\,dx \\
&= \frac{(a+bx)\mathrm{Chi}(a+bx)^2}{b} - \frac{2\mathrm{Chi}(a+bx)\sinh(a+bx)}{b} + \frac{\mathrm{Shi}(2a+2bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.85

$$\frac{(a+bx)\mathrm{Chi}(a+bx)^2 - 2\mathrm{Chi}(a+bx)\sinh(a+bx) + \mathrm{Shi}(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*CoshIntegral[a + b*x]^2 - 2*CoshIntegral[a + b*x]*Sinh[a + b*x]
+ SinhIntegral[2*(a + b*x)])/b
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{Chi}\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\,(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)^2, x)
```

maple [A] time = 0.00, size = 43, normalized size = 0.90

$$\frac{(bx+a)\,\mathrm{X}\,(bx+a)^2 - 2\mathrm{X}\,(bx+a)\sinh(bx+a) + \mathrm{Shi}\,(2bx+2a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)^2,x)
```

```
[Out] 1/b*((b*x+a)*Chi(b*x+a)^2-2*Chi(b*x+a)*sinh(b*x+a)+Shi(2*b*x+2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\,(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coshint}\,(a+b\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)^2,x)
```

```
[Out] int(coshint(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}^2\,(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)**2,x)
```

```
[Out] Integral(Chi(a + b*x)**2, x)
```

3.97 $\int \frac{\text{Chi}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\text{Chi}(a+bx)^2}{x},x\right)$$

```
[Out] CannotIntegrate(Chi(b*x+a)^2/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Chi}(a+bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[CoshIntegral[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][CoshIntegral[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\text{Chi}(a+bx)^2}{x}\,dx=\int \frac{\text{Chi}(a+bx)^2}{x}\,dx$$

Mathematica [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(a+bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[CoshIntegral[a + b*x]^2/x,x]
```

```
[Out] Integrate[CoshIntegral[a + b*x]^2/x, x]
```

fricas [A] time = 1.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\left(bx+a\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)^2/x, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)^2/x,x)
```

```
[Out] int(Chi(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)^2/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\mathrm{coshint}\left(a+b\,x\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)^2/x,x)
```

```
[Out] int(coshint(a + b*x)^2/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)**2/x,x)
```

```
[Out] Integral(Chi(a + b*x)**2/x, x)
```

3.98 $\int \frac{\text{Chi}(a+bx)^2}{x^2} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\text{Chi}(a+bx)^2}{x^2},x\right)$$

```
[Out] CannotIntegrate(Chi(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Chi}(a+bx)^2}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[CoshIntegral[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][CoshIntegral[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\text{Chi}(a+bx)^2}{x^2} \, dx = \int \frac{\text{Chi}(a+bx)^2}{x^2} \, dx$$

Mathematica [A] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(a+bx)^2}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[CoshIntegral[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[CoshIntegral[a + b*x]^2/x^2, x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\left(bx+a\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)^2/x^2, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)^2/x^2,x)
```

```
[Out] int(Chi(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)^2/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\text{coshint}\,(a+b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)^2/x^2,x)
```

```
[Out] int(coshint(a + b*x)^2/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}^2\left(a+bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)**2/x**2,x)
```

```
[Out] Integral(Chi(a + b*x)**2/x**2, x)
```

3.99 $\int \frac{\mathbf{Chi}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\text{Chi}(a+bx)^2}{x^3},x\right)$$

[Out] CannotIntegrate(Chi(b*x+a)^2/x^3,x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Chi}(a+bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

[In] Int[CoshIntegral[a + b*x]^2/x^3,x]

[Out] Defer[Int][CoshIntegral[a + b*x]^2/x^3, x]

Rubi steps

$$\int \frac{\text{Chi}(a+bx)^2}{x^3}\,dx = \int \frac{\text{Chi}(a+bx)^2}{x^3}\,dx$$

Mathematica [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(a+bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[CoshIntegral[a + b*x]^2/x^3,x]

[Out] Integrate[CoshIntegral[a + b*x]^2/x^3, x]

fricas [A] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\,(bx+a)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)^2/x^3, x)
```

giac [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)^2/x^3, x)
```

maple [A]   time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)^2/x^3,x)
```

```
[Out] int(Chi(b*x+a)^2/x^3,x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\,(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)^2/x^3, x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\text{coshint}\,(a+b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)^2/x^3,x)
```

```
[Out] int(coshint(a + b*x)^2/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}^2\left(a+bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)**2/x**3,x)
```

```
[Out] Integral(Chi(a + b*x)**2/x**3, x)
```

3.100 $\int x^2 \textbf{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=128

$$\frac{1}{3}x^3\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(3-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(bdn+3)\left(\right.}{}\right.$$

```
[Out] 1/3*x^3*Chi(d*(a+b*ln(c*x^n)))-1/6*x^3*Ei((-b*d*n+3)*(a+b*ln(c*x^n))/b/n)/e
xp(3*a/b/n)/((c*x^n)^(3/n))-1/6*x^3*Ei((b*d*n+3)*(a+b*ln(c*x^n))/b/n)/exp(3
*a/b/n)/((c*x^n)^(3/n))
```

Rubi [A] time = 0.25, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6556, 12, 5540, 2310, 2178}

$$\frac{1}{3}x^3\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(3-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{6}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(bdn+3)\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*CoshIntegral[d*(a + b*Log[c*x^n])])/3 - (x^3*ExpIntegralEi[((3 - b*d*n
)*(a + b*Log[c*x^n]))/(b*n)])/(6*E^((3*a)/(b*n))*(c*x^n)^(3/n)) - (x^3*ExpI
ntegralEi[((3 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(6*E^((3*a)/(b*n))*(c*x^
n)^(3/n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5540

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Dist[(i*x)^
r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*Lo
g[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*n
)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c, d
, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6556

```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[((e*x)^(m + 1)*CoshIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Cosh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx &= \frac{1}{3}x^3\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}(bdn)\int\frac{x^2\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}\,dx\\
&= \frac{1}{3}x^3\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}(bn)\int\frac{x^2\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}\,dx\\
&= \frac{1}{3}x^3\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int\frac{x^{2-bdn}}{a+b\log\left(cx^n\right)}\,dx\\
&= \frac{1}{3}x^3\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(be^{-ad}x^3\left(cx^n\right)^{-bd-\frac{3-bdn}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(3-bdn)x}{n}}}{a+bx}\right.\\
&= \frac{1}{3}x^3\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\text{Ei}\left(\frac{(3-bdn)\left(a+b\log\left(c\right.\right.}{bn}\right.
\end{aligned}$$

Mathematica [A] time = 0.92, size = 97, normalized size = 0.76

$$\frac{1}{6}x^3\left(2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\left(\text{Ei}\left(-\frac{(bdn-3)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\text{Ei}\left(\frac{(bdn+3)\left(a+b\log\left(c\right.\right.}{bn}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(2*CoshIntegral[d*(a + b*Log[c*x^n])] - (ExpIntegralEi[-(((-3 + b*d*n)
*(a + b*Log[c*x^n]))/(b*n))] + ExpIntegralEi[((3 + b*d*n)*(a + b*Log[c*x^n]
))/(b*n)])/(E^((3*a)/(b*n))*(c*x^n)^(3/n))))/6
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\,\mathrm{Chi}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh_integral(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Chi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*Chi((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^2\mathrm{X}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*Chi(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Chi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Chi((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{coshint}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^2*coshint(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Chi}\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*Chi(a*d + b*d*log(c*x**n)), x)
```

3.101 $\int x\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=128

$$\frac{1}{2}x^2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(bdn+2)\left(\right.}{}\right.$$

```
[Out] 1/2*x^2*Chi(d*(a+b*ln(c*x^n)))-1/4*x^2*Ei((-b*d*n+2)*(a+b*ln(c*x^n))/b/n)/e
xp(2*a/b/n)/((c*x^n)^(2/n))-1/4*x^2*Ei((b*d*n+2)*(a+b*ln(c*x^n))/b/n)/exp(2
*a/b/n)/((c*x^n)^(2/n))
```

Rubi [A] time = 0.22, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6556, 12, 5540, 2310, 2178}

$$\frac{1}{2}x^2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{4}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(bdn+2)\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*CoshIntegral[d*(a + b*Log[c*x^n])])/2 - (x^2*ExpIntegralEi[((2 - b*d*n
)*(a + b*Log[c*x^n]))/(b*n)])/(4*E^((2*a)/(b*n))*(c*x^n)^(2/n)) - (x^2*ExpI
ntegralEi[((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(4*E^((2*a)/(b*n))*(c*x^
n)^(2/n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5540

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Dist[(i*x)^
r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*Lo
g[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*n
)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c, d
, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6556

```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[((e*x)^(m + 1)*CoshIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Cosh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\int x\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\frac{1}{2}x^2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int\frac{x\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}\,dx$$

$$=\frac{1}{2}x^2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bn)\int\frac{x\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}\,dx$$

$$=\frac{1}{2}x^2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int\frac{x^{1-bdn}}{a+b\log\left(cx^n\right)}\,dx$$

$$=\frac{1}{2}x^2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(be^{-ad}x^2\left(cx^n\right)^{-bd-\frac{2-bdn}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(2-bdn)x}{n}}}{a+bx}\right.$$

$$=\frac{1}{2}x^2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\text{Ei}\left(\frac{(2-bdn)\left(a+b\log\left(cx\right.\right.}{bn}\right.$$

Mathematica [A] time = 0.89, size = 97, normalized size = 0.76

$$\frac{1}{4}x^2\left(2\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\left(\text{Ei}\left(-\frac{(bdn-2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\text{Ei}\left(\frac{(bdn+2)\left(a+b\log\left(cx\right.\right.}{bn}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

[Out] (x^2*(2*CoshIntegral[d*(a + b*Log[c*x^n])] - (ExpIntegralEi[-(((-2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))] + ExpIntegralEi[((2 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(E^((2*a)/(b*n))*(c*x^n)^(2/n)))))/4

fricas [F] time = 1.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Chi}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(x*cosh_integral(b*d*log(c*x^n) + a*d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate(x*Chi((b*log(c*x^n) + a)*d), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\text{X}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*Chi(d*(a+b*ln(c*x^n))),x)

[Out] int(x*Chi(d*(a+b*ln(c*x^n))),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate(x*Chi((b*log(c*x^n) + a)*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{coshint}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(d*(a + b*log(c*x^n))),x)
[Out] int(x*coshint(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{Chi}\left(ad + bd\log\left(cx^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x*Chi(a*d + b*d*log(c*x**n)), x)
```

3.102 $\int \mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=119

$$x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\mathrm{Ei}\left(\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\mathrm{Ei}\left(\frac{(bdn+1)\left(a+b\right.}{bn}\right.$$

```
[Out] x*Chi(d*(a+b*ln(c*x^n)))-1/2*x*Ei((-b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/n
)/((c*x^n)^(1/n))-1/2*x*Ei((b*d*n+1)*(a+b*ln(c*x^n))/b/n)/exp(a/b/n)/((c*x^
n)^(1/n))
```

Rubi [A] time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6553, 12, 5538, 2310, 2178}

$$x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\mathrm{Ei}\left(\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\mathrm{Ei}\left(\frac{(bdn+1)\left(a+b\right.}{bn}\right.$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*CoshIntegral[d*(a + b*Log[c*x^n])] - (x*ExpIntegralEi[((1 - b*d*n)*(a + b
*Log[c*x^n]))/(b*n)])/(2*E^(a/(b*n))*(c*x^n)^n^(-1)) - (x*ExpIntegralEi[((1
 + b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*E^(a/(b*n))*(c*x^n)^n^(-1))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5538

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.), x_Symbol] :> Dist[1/(E^(a*d)*(c*x^n)^(b*d)*
(2/x^(b*d*n))), Int[(h*(e + f*Log[g*x^m]))^q/x^(b*d*n), x], x] + Dist[(E^(a
*d)*(c*x^n)^(b*d))/(2*x^(b*d*n)), Int[x^(b*d*n)*(h*(e + f*Log[g*x^m]))^q, x
], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, q}, x]
```

Rule 6553

```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :>
 Simp[x*CoshIntegral[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Cosh[d*(a
+ b*Log[c*x^n])]/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, n}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx &= x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-(bdn)\int\frac{\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}\,dx\\
&= x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-(bn)\int\frac{\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}\,dx\\
&= x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int\frac{x^{-bdn}}{a+b\log\left(cx^n\right)}\,dx-\frac{1}{2}\\
&= x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(be^{-ad}x\left(cx^n\right)^{-bd-\frac{1-bdn}{n}}\right)\mathrm{Subst}\left(\int\frac{e^{\frac{(1-bdn)x}{n}}}{a+bx}\,dx,x,\right.\\
&= x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\mathrm{Ei}\left(\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)-
\end{aligned}
$$

Mathematica [A] time = 0.86, size = 93, normalized size = 0.78

$$
x\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\left(\mathrm{Ei}\left(-\frac{(bdn-1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\mathrm{Ei}\left(\frac{(bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*CoshIntegral[d*(a + b*Log[c*x^n])] - (x*(ExpIntegralEi[-(((-1 + b*d*n)*(a
 + b*Log[c*x^n]))/(b*n))] + ExpIntegralEi[((1 + b*d*n)*(a + b*Log[c*x^n]))/
(b*n)]))/(2*E^(a/(b*n))*(c*x^n)^n^(-1))
```

fricas [F] time = 2.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{Chi}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(cosh_integral(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(Chi((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \mathrm{X}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*(a+b*ln(c*x^n))),x)
[Out] int(Chi(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(Chi((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coshint}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(coshint(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Chi}\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(Chi(d*(a + b*log(c*x**n))), x)
```

3.103 $\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{(a+b\log(cx^n))\,\text{Chi}(d(a+b\log(cx^n)))}{bn}-\frac{\sinh(d(a+b\log(cx^n)))}{bdn}$$

```
[Out] Chi(d*(a+b*ln(c*x^n)))*(a+b*ln(c*x^n))/b/n-sinh(d*(a+b*ln(c*x^n)))/b/d/n
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6529}

$$\frac{(a+b\log(cx^n))\,\text{Chi}(d(a+b\log(cx^n)))}{bn}-\frac{\sinh(d(a+b\log(cx^n)))}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (CoshIntegral[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sinh[d*(a +
 b*Log[c*x^n])]/(b*d*n)
```

Rule 6529

```
Int[CoshIntegral[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*CoshInte
gral[a + b*x])/b, x] - Simp[Sinh[a + b*x]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Chi}(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int \text{Chi}(d(a+bx))\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int \text{Chi}(x)\,dx, x, ad+bd\log(cx^n)\right)}{bdn}\\ &= \frac{\text{Chi}(ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn}-\frac{\sinh(ad+bd\log(cx^n))}{bdn}\end{aligned}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 1.75

$$\frac{a\text{Chi}(ad+b\log(cx^n)\,d)}{bn}+\frac{\log(cx^n)\,\text{Chi}(d(a+b\log(cx^n)))}{n}-\frac{\sinh(ad)\cosh(bd\log(cx^n))}{bdn}-\frac{\cosh(ad)\sinh(bd\log(cx^n))}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (a*CoshIntegral[a*d + b*d*Log[c*x^n]])/(b*n) + (CoshIntegral[d*(a + b*Log[c
*x^n])]*Log[c*x^n])/n - (Cosh[b*d*Log[c*x^n]]*Sinh[a*d])/(b*d*n) - (Cosh[a*
d]*Sinh[b*d*Log[c*x^n]])/(b*d*n)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\left(bd\log\left(cx^{n}\right)+ad\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*d*log(c*x^n) + a*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] integrate(Chi((b*log(c*x^n) + a)*d)/x, x)
```

maple [A] time = 0.02, size = 73, normalized size = 1.33

$$\frac{\ln\left(c\,x^{n}\right)\text{X}\left(ad+bd\ln\left(c\,x^{n}\right)\right)}{n}+\frac{\text{X}\left(ad+bd\ln\left(c\,x^{n}\right)\right)a}{nb}-\frac{\sinh\left(ad+bd\ln\left(c\,x^{n}\right)\right)}{nbd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*Chi(a*d+b*d*ln(c*x^n))+1/n/b*Chi(a*d+b*d*ln(c*x^n))*a-1/n/b/d
*sinh(a*d+b*d*ln(c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] integrate(Chi((b*log(c*x^n) + a)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{\ln(c\,x^n)\,\operatorname{coshint}(d\,(a+b\,\ln(c\,x^n)))}{n}+\frac{a\operatorname{coshint}(d\,(a+b\,\ln(c\,x^n)))}{b\,n}-\frac{e^{a\,d}\,(c\,x^n)^{b\,d}}{2\,b\,d\,n}+\frac{e^{-a\,d}}{2\,b\,d\,n\,(c\,x^n)^{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] (log(c*x^n)*coshint(d*(a + b*log(c*x^n))))/n + (a*coshint(d*(a + b*log(c*x^
n))))/(b*n) - (exp(a*d)*(c*x^n)^(b*d))/(2*b*d*n) + exp(-a*d)/(2*b*d*n*(c*x^
n)^(b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{Chi}\left(ad+bd\log{(cx^n)}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(Chi(a*d + b*d*log(c*x**n))/x, x)
```

3.104 $\int \frac{\mathrm{Chi}\left(d\left(a+b\log(cx^n)\right)\right)}{x^2}\,dx$

Optimal. Leaf size=122

$$-\frac{\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{(1-bdn)\left(a+b\log(cx^n)\right)}{bn}\right)}{2x}+\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{(bdn+1)\left(a+b\log(cx^n)\right)}{bn}\right)}{2x}$$

```
[Out] -Chi(d*(a+b*ln(c*x^n)))/x+1/2*exp(a/b/n)*(c*x^n)^(1/n)*Ei(-(-b*d*n+1)*(a+b*
ln(c*x^n))/b/n)/x+1/2*exp(a/b/n)*(c*x^n)^(1/n)*Ei(-(b*d*n+1)*(a+b*ln(c*x^n)
)/b/n)/x
```

Rubi [A] time = 0.24, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6556, 12, 5540, 2310, 2178}

$$-\frac{\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{(1-bdn)\left(a+b\log(cx^n)\right)}{bn}\right)}{2x}+\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{(bdn+1)\left(a+b\log(cx^n)\right)}{bn}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -(CoshIntegral[d*(a + b*Log[c*x^n])]/x) + (E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIn
tegralEi[-(((1 - b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(2*x) + (E^(a/(b*n))*(
c*x^n)^n^(-1)*ExpIntegralEi[-(((1 + b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(2*
x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5540

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Dist[(i*x)^
r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*Lo
g[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*n
)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c, d
, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6556

```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[((e*x)^(m + 1)*CoshIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Cosh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx &= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+(bdn)\int \frac{\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{dx^2\left(a+b\log\left(cx^n\right)\right)}\,dx\\
&= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+(bn)\int \frac{\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2\left(a+b\log\left(cx^n\right)\right)}\,dx\\
&= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+\frac{1}{2}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int \frac{x^{-2-bdn}}{a+b\log\left(cx^n\right)}\,dx+\frac{1}{2}\left(\right.\\
&= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+\frac{\left(be^{-ad}\left(cx^n\right)^{-bd-\frac{-1-bdn}{n}}\right)\text{Subst}\left(\int \frac{e^{\frac{(-1-bdn)x}{n}}}{a+bx}\,dx,x,\log\right.}{2x}\\
&= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}+\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{(1-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2x}+\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\text{E}}{}\end{aligned}$$

Mathematica [A] time = 1.10, size = 144, normalized size = 1.18

$$\frac{1}{2}\exp\left(-\frac{(bdn-1)\left(a+b\left(\log\left(cx^n\right)-n\log(x)\right)\right)}{bn}\right)\left(\text{Ei}\left(\frac{(bdn-1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\text{Ei}\left(-\frac{(bdn+1)\left(a+b\log\left(c\right.\right.}{bn}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[CoshIntegral[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -(CoshIntegral[d*(a + b*Log[c*x^n])]/x) + ((ExpIntegralEi[((-1 + b*d*n)*(a
+ b*Log[c*x^n]))/(b*n)] + ExpIntegralEi[-(((1 + b*d*n)*(a + b*Log[c*x^n]))/
(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + Sinh[d*(a + b*(-(n*L
og[x]) + Log[c*x^n]))]))/(2*E^(((-1 + b*d*n)*(a + b*(-(n*Log[x]) + Log[c*x^
n])))/(b*n)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\left(bd\log\left(cx^{n}\right)+ad\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*d*log(c*x^n) + a*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(Chi((b*log(c*x^n) + a)*d)/x^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(Chi(d*(a+b*ln(c*x^n)))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
[Out] integrate(Chi((b*log(c*x^n) + a)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{coshint}\left(d\ \left(a+b\ \ln\left(c\, x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(d*(a + b*log(c*x^n)))/x^2,x)
[Out] int(coshint(d*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Chi}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*ln(c*x**n)))/x**2,x)
[Out] Integral(Chi(a*d + b*d*log(c*x**n))/x**2, x)
```

3.105 $\int \frac{\textbf{Chi}\left(d\left(a+b\log(cx^n)\right)\right)}{x^3}\,dx$

Optimal. Leaf size=130

$$-\frac{\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\mathrm{Ei}\left(-\frac{(2-bdn)\left(a+b\log(cx^n)\right)}{bn}\right)}{4x^2}+\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\mathrm{Ei}\left(-\frac{(bdn+2)\left(a+b\log(cx^n)\right)}{bn}\right)}{4x^2}$$

```
[Out] -1/2*Chi(d*(a+b*ln(c*x^n)))/x^2+1/4*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(-b*d*n+
2)*(a+b*ln(c*x^n))/b/n)/x^2+1/4*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-(b*d*n+2)*(a
+b*ln(c*x^n))/b/n)/x^2
```

Rubi [A] time = 0.24, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6556, 12, 5540, 2310, 2178}

$$-\frac{\mathrm{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\mathrm{Ei}\left(-\frac{(2-bdn)\left(a+b\log(cx^n)\right)}{bn}\right)}{4x^2}+\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\mathrm{Ei}\left(-\frac{(bdn+2)\left(a+b\log(cx^n)\right)}{bn}\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -CoshIntegral[d*(a + b*Log[c*x^n])]/(2*x^2) + (E^((2*a)/(b*n))*(c*x^n)^(2/n
)*ExpIntegralEi[-(((2 - b*d*n)*(a + b*Log[c*x^n]))/(b*n))])/(4*x^2) + (E^((
2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[-(((2 + b*d*n)*(a + b*Log[c*x^n]))/
(b*n))])/(4*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5540

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Dist[(i*x)^
r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*Lo
g[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*n
)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c, d
, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6556

```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[((e*x)^(m + 1)*CoshIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Cosh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\int \frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx = -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{2}(bdn)\int \frac{\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{dx^3\left(a+b\log\left(cx^n\right)\right)}\,dx$$

$$= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{2}(bn)\int \frac{\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3\left(a+b\log\left(cx^n\right)\right)}\,dx$$

$$= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}\left(be^{-ad}nx^{bdn}\left(cx^n\right)^{-bd}\right)\int \frac{x^{-3-bdn}}{a+b\log\left(cx^n\right)}\,dx+\frac{1}{4}\left(\right.$$

$$= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{\left(be^{-ad}\left(cx^n\right)^{-bd-\frac{-2-bdn}{n}}\right)\text{Subst}\left(\int \frac{e^{\frac{(-2-bdn)x}{n}}}{a+bx}\,dx,x,\log\right.}{4x^2}$$

$$= -\frac{\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{Ei}\left(-\frac{(2-bdn)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{4x^2}+\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}}{}$$

Mathematica [A] time = 1.16, size = 146, normalized size = 1.12

$$\frac{1}{4}\exp\left(-\frac{(bdn-2)\left(a+b\left(\log\left(cx^n\right)-n\log(x)\right)\right)}{bn}\right)\left(\text{Ei}\left(\frac{(bdn-2)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\text{Ei}\left(-\frac{(bdn+2)\left(a+b\log\left(c\right.\right.}{bn}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[CoshIntegral[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/2*CoshIntegral[d*(a + b*Log[c*x^n])]/x^2 + ((ExpIntegralEi[((-2 + b*d*n)
*(a + b*Log[c*x^n]))/(b*n)] + ExpIntegralEi[-(((2 + b*d*n)*(a + b*Log[c*x^n
]))/(b*n))])*(Cosh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + Sinh[d*(a + b*(-
(n*Log[x]) + Log[c*x^n]))]))/(4*E^(((-2 + b*d*n)*(a + b*(-(n*Log[x]) + Log[
c*x^n])))/(b*n)))
```

fricas [F] time = 2.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Chi}\left(bd\log\left(cx^n\right)+ad\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*d*log(c*x^n) + a*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(Chi((b*log(c*x^n) + a)*d)/x^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(Chi(d*(a+b*ln(c*x^n)))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Chi((b*log(c*x^n) + a)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{coshint}\left(d\ \left(a+b\ \ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(coshint(d*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Chi}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(Chi(a*d + b*d*log(c*x**n))/x**3, x)
```

3.106 $\int (ex)^m \mathbf{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=167

$$\frac{(ex)^{m+1}\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(m+1)}-\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\text{Ei}\left(\frac{(m-bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2(m+1)}-\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\text{E}}{2(m-}$$

```
[Out] (e*x)^(1+m)*Chi(d*(a+b*ln(c*x^n)))/e/(1+m)-1/2*x*(e*x)^m*Ei((-b*d*n+m+1)*(a
+b*ln(c*x^n))/b/n)/exp(a*(1+m)/b/n)/(1+m)/((c*x^n)^((1+m)/n))-1/2*x*(e*x)^m
*Ei((b*d*n+m+1)*(a+b*ln(c*x^n))/b/n)/exp(a*(1+m)/b/n)/(1+m)/((c*x^n)^((1+m)
/n))
```

Rubi [A] time = 0.29, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6556, 12, 5540, 2310, 2178}

$$\frac{(ex)^{m+1}\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(m+1)}-\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\text{Ei}\left(\frac{(m-bdn+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2(m+1)}-\frac{x(ex)^m e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\text{E}}{2(m-}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^(1 + m)*CoshIntegral[d*(a + b*Log[c*x^n])])/(e*(1 + m)) - (x*(e*x)^m
*ExpIntegralEi[((1 + m - b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*E^((a*(1 + m
))/(b*n))*(1 + m)*(c*x^n)^((1 + m)/n)) - (x*(e*x)^m*ExpIntegralEi[((1 + m +
 b*d*n)*(a + b*Log[c*x^n]))/(b*n)])/(2*E^((a*(1 + m))/(b*n))*(1 + m)*(c*x^n
)^((1 + m)/n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 5540

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*(((e_.) + Log[(g_.)*(
x_)^(m_.)]*(f_.))*(h_.))^(q_.)*((i_.)*(x_))^(r_.), x_Symbol] :> Dist[(i*x)^
r/(E^(a*d)*(c*x^n)^(b*d)*(2*x^(r - b*d*n))), Int[x^(r - b*d*n)*(h*(e + f*Lo
g[g*x^m]))^q, x], x] + Dist[(E^(a*d)*(i*x)^r*(c*x^n)^(b*d))/(2*x^(r + b*d*n
)), Int[x^(r + b*d*n)*(h*(e + f*Log[g*x^m]))^q, x], x] /; FreeQ[{a, b, c, d
, e, f, g, h, i, m, n, q, r}, x]
```

Rule 6556

```
Int[CoshIntegral[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^
(m_.), x_Symbol] :> Simp[((e*x)^(m + 1)*CoshIntegral[d*(a + b*Log[c*x^n])])
/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[((e*x)^m*Cosh[d*(a + b*Log[c*x
^n])])/(d*(a + b*Log[c*x^n])), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] &&
 NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m \text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{(ex)^{1+m}\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bdn)\int \frac{(ex)^m\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{d\left(a+b\log\left(cx^n\right)\right)}\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bn)\int \frac{(ex)^m\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{a+b\log\left(cx^n\right)}\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(be^{-ad}nx^{-m+bdn}(ex)^m\left(cx^n\right)^{-bd}\right)\int \frac{x^{m-b}}{a+b\log}}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(be^{-ad}x(ex)^m\left(cx^n\right)^{-bd-\frac{1+m-bdn}{n}}\right)\text{Subst}\left(\int\right.}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{e^{-\frac{a(1+m)}{bn}}x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m-bdn)(a}{bn}\right.}{2(1+m)}$$

Mathematica [A] time = 1.92, size = 119, normalized size = 0.71

$$\frac{(ex)^m \left(2x\text{Chi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-x^{-m}\exp\left(-\frac{(m+1)\left(a+b\log(cx^n)-bn\log(x)\right)}{bn}\right)\left(\text{Ei}\left(\frac{(m-bdn+1)\left(a+b\log(cx^n)\right)}{bn}\right)+\text{Ei}\left(\frac{(m+bd$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*CoshIntegral[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(2*x*CoshIntegral[d*(a + b*Log[c*x^n])] - (ExpIntegralEi[((1 + m -
 b*d*n)*(a + b*Log[c*x^n]))/(b*n)] + ExpIntegralEi[((1 + m + b*d*n)*(a + b*
Log[c*x^n]))/(b*n)])/(E^(((1 + m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*x
^m)))/(2*(1 + m))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m\,\text{Chi}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*Chi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*cosh_integral(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\,\text{Chi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*Chi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*Chi((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^m\,\text{X}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*Chi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*Chi(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\,\text{Chi}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*Chi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*Chi((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{coshint}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)\,\left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(coshint(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \operatorname{Chi}\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*Chi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*Chi(a*d + b*d*log(c*x**n)), x)
```

3.107 $\int \frac{\cosh(bx)\mathbf{Chi}(bx)}{x^3}\,dx$

Optimal. Leaf size=96

$$\frac{1}{4}b^2\mathrm{Chi}(bx)^2+b^2\mathrm{Chi}(2bx)-\frac{\mathrm{Chi}(bx)\cosh(bx)}{2x^2}-\frac{b\mathrm{Chi}(bx)\sinh(bx)}{2x}-\frac{\cosh^2(bx)}{4x^2}-\frac{b\sinh(2bx)}{4x}-\frac{b\sinh(bx)\cosh(bx)}{2x}$$

```
[Out] 1/4*b^2*Chi(b*x)^2+b^2*Chi(2*b*x)-1/2*Chi(b*x)*cosh(b*x)/x^2-1/4*cosh(b*x)^
2/x^2-1/2*b*Chi(b*x)*sinh(b*x)/x-1/2*b*cosh(b*x)*sinh(b*x)/x-1/4*b*sinh(2*b
*x)/x
```

Rubi [A] time = 0.21, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6545, 6551, 6686, 12, 5448, 3297, 3301, 3314, 29, 3312}

$$\frac{1}{4}b^2\mathrm{Chi}(bx)^2+b^2\mathrm{Chi}(2bx)-\frac{\mathrm{Chi}(bx)\cosh(bx)}{2x^2}-\frac{b\mathrm{Chi}(bx)\sinh(bx)}{2x}-\frac{\cosh^2(bx)}{4x^2}-\frac{b\sinh(2bx)}{4x}-\frac{b\sinh(bx)\cosh(bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[b*x]*CoshIntegral[b*x])/x^3,x]
```

```
[Out] -Cosh[b*x]^2/(4*x^2) - (Cosh[b*x]*CoshIntegral[b*x])/(2*x^2) + (b^2*CoshInt
egral[b*x]^2)/4 + b^2*CoshIntegral[2*b*x] - (b*Cosh[b*x]*Sinh[b*x])/(2*x) -
 (b*CoshIntegral[b*x]*Sinh[b*x])/(2*x) - (b*Sinh[2*b*x])/(4*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6545

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*Cosh[a + b*x]*CoshInteg
ral[c + d*x])/(f*(m + 1)), x] + (-Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)
*Sinh[a + b*x]*CoshIntegral[c + d*x], x], x] - Dist[d/(f*(m + 1)), Int[((e
+ f*x)^(m + 1)*Cosh[a + b*x]*Cosh[c + d*x])/(c + d*x), x], x]) /; FreeQ[{a,
 b, c, d, e, f}, x] && ILtQ[m, -1]
```

Rule 6551

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_)*Sinh[(a_.) +
 (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^(m + 1)*Sinh[a + b*x]*CoshIntegr
al[c + d*x])/(f*(m + 1)), x] + (-Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)*
Cosh[a + b*x]*CoshIntegral[c + d*x], x], x] - Dist[d/(f*(m + 1)), Int[((e +
 f*x)^(m + 1)*Sinh[a + b*x]*Cosh[c + d*x])/(c + d*x), x], x]) /; FreeQ[{a,
b, c, d, e, f}, x] && ILtQ[m, -1]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(bx)\text{Chi}(bx)}{x^3}\,dx &= -\frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{2}b\int \frac{\cosh^2(bx)}{bx^3}\,dx + \frac{1}{2}b\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx\\
&= -\frac{\cosh(bx)\text{Chi}(bx)}{2x^2} - \frac{b\text{Chi}(bx)\sinh(bx)}{2x} + \frac{1}{2}\int \frac{\cosh^2(bx)}{x^3}\,dx + \frac{1}{2}b^2\int \frac{\cosh(bx)\text{Chi}}{x}\\
&= -\frac{\cosh^2(bx)}{4x^2} - \frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Chi}(bx)^2 - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{b\text{Chi}(bx)\,\text{si}}{2x}\\
&= -\frac{\cosh^2(bx)}{4x^2} - \frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Chi}(bx)^2 - \frac{1}{2}b^2\log(x) - \frac{b\cosh(bx)\sinh(bx)}{2x}\\
&= -\frac{\cosh^2(bx)}{4x^2} - \frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Chi}(bx)^2 - \frac{b\cosh(bx)\sinh(bx)}{2x} - \frac{b\text{Chi}(bx)\,\text{si}}{2x}\\
&= -\frac{\cosh^2(bx)}{4x^2} - \frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Chi}(bx)^2 + \frac{1}{2}b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(b}{2x}\\
&= -\frac{\cosh^2(bx)}{4x^2} - \frac{\cosh(bx)\text{Chi}(bx)}{2x^2} + \frac{1}{4}b^2\text{Chi}(bx)^2 + b^2\text{Chi}(2bx) - \frac{b\cosh(bx)\sinh(bx)}{2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 96, normalized size = 1.00

$$\frac{1}{4}b^2\text{Chi}(bx)^2+b^2\text{Chi}(2bx)-\frac{\text{Chi}(bx)\cosh(bx)}{2x^2}-\frac{b\text{Chi}(bx)\sinh(bx)}{2x}-\frac{\cosh^2(bx)}{4x^2}-\frac{b\sinh(2bx)}{4x}-\frac{b\sinh(bx)\cosh(b}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x^3,x]
```

```
[Out] -1/4*Cosh[b*x]^2/x^2 - (Cosh[b*x]*CoshIntegral[b*x])/(2*x^2) + (b^2*CoshInt
egral[b*x]^2)/4 + b^2*CoshIntegral[2*b*x] - (b*Cosh[b*x]*Sinh[b*x])/(2*x) -
 (b*CoshIntegral[b*x]*Sinh[b*x])/(2*x) - (b*Sinh[2*b*x])/(4*x)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx)\,\text{Chi}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x)*cosh_integral(b*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(bx)\cosh(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x^3,x, algorithm="giac")
[Out] integrate(Chi(b*x)*cosh(b*x)/x^3, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}(bx)\cosh(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)*cosh(b*x)/x^3,x)
[Out] int(Chi(b*x)*cosh(b*x)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(bx)\cosh(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x^3,x, algorithm="maxima")
[Out] integrate(Chi(b*x)*cosh(b*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{coshint}(b\,x)\,\cosh(b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(b*x)*cosh(b*x))/x^3,x)
[Out] int((coshint(b*x)*cosh(b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx)\,\text{Chi}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x**3,x)
```

```
[Out] Integral(cosh(b*x)*Chi(b*x)/x**3, x)
```

3.108 $\int \frac{\cosh(bx)\mathbf{Chi}(bx)}{x^2}\,dx$

Optimal. Leaf size=49

$$b\text{Int}\left(\frac{\text{Chi}(bx)\sinh(bx)}{x},x\right)-\frac{\text{Chi}(bx)\cosh(bx)}{x}+b\text{Shi}(2bx)-\frac{\cosh^2(bx)}{x}$$

```
[Out] b*CannotIntegrate(Chi(b*x)*sinh(b*x)/x,x)-Chi(b*x)*cosh(b*x)/x-cosh(b*x)^2/
x+b*Shi(2*b*x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[b*x]*CoshIntegral[b*x])/x^2,x]
```

```
[Out] -(Cosh[b*x]^2/x) - (Cosh[b*x]*CoshIntegral[b*x])/x + b*SinhIntegral[2*b*x]
+ b*Defer[Int][(CoshIntegral[b*x]*Sinh[b*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx &= -\frac{\cosh(bx)\text{Chi}(bx)}{x}+b\int \frac{\cosh^2(bx)}{bx^2}\,dx+b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx\\
&= -\frac{\cosh(bx)\text{Chi}(bx)}{x}+b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx+\int \frac{\cosh^2(bx)}{x^2}\,dx\\
&= -\frac{\cosh^2(bx)}{x}-\frac{\cosh(bx)\text{Chi}(bx)}{x}+(2ib)\int -\frac{i\sinh(2bx)}{2x}\,dx+b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx\\
&= -\frac{\cosh^2(bx)}{x}-\frac{\cosh(bx)\text{Chi}(bx)}{x}+b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx+b\int \frac{\sinh(2bx)}{x}\,dx\\
&= -\frac{\cosh^2(bx)}{x}-\frac{\cosh(bx)\text{Chi}(bx)}{x}+b\text{Shi}(2bx)+b\int \frac{\text{Chi}(bx)\sinh(bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x^2,x]
[Out] Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x^2, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx)\,\text{Chi}(bx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x)*cosh_integral(b*x)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(bx)\cosh(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x^2,x, algorithm="giac")
[Out] integrate(Chi(b*x)*cosh(b*x)/x^2, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{X}(bx)\cosh(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)*cosh(b*x)/x^2,x)
[Out] int(Chi(b*x)*cosh(b*x)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Chi}(bx)\cosh(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x^2,x, algorithm="maxima")
[Out] integrate(Chi(b*x)*cosh(b*x)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{coshint}\,(b\,x)\ \cosh\,(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(b*x)*cosh(b*x))/x^2,x)
```

```
[Out] int((coshint(b*x)*cosh(b*x))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\,(bx)\,\mathrm{Chi}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x**2,x)
```

```
[Out] Integral(cosh(b*x)*Chi(b*x)/x**2, x)
```

3.109 $\int \frac{\cosh(bx)\textbf{Chi}(bx)}{x}\,dx$

Optimal. Leaf size=10

$$\frac{\text{Chi}(bx)^2}{2}$$

```
[Out] 1/2*Chi(b*x)^2
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6686}

$$\frac{\text{Chi}(bx)^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[b*x]*CoshIntegral[b*x])/x,x]
```

```
[Out] CoshIntegral[b*x]^2/2
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh(bx)\text{Chi}(bx)}{x}\,dx = \frac{\text{Chi}(bx)^2}{2}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\text{Chi}(bx)^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[b*x]*CoshIntegral[b*x])/x,x]
```

```
[Out] CoshIntegral[b*x]^2/2
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx)\,\text{Chi}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x)*cosh_integral(b*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)\cosh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x)*cosh(b*x)/x, x)
```

maple [A] time = 0.01, size = 9, normalized size = 0.90

$$\frac{\mathrm{X}\,(bx)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)*cosh(b*x)/x,x)
```

```
[Out] 1/2*Chi(b*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)\cosh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)*cosh(b*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\frac{\mathrm{coshint}\,(b\,x)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(b*x)*cosh(b*x))/x,x)
```

```
[Out] coshint(b*x)^2/2
```

sympy [A] time = 0.73, size = 7, normalized size = 0.70

$$\frac{\mathrm{Chi}^2(bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x)/x,x)
```

```
[Out] Chi(b*x)**2/2
```

3.110 $\int \cosh(bx)\textbf{Chi}(bx)\,dx$

Optimal. Leaf size=25

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}$$

```
[Out] -1/2*Shi(2*b*x)/b+Chi(b*x)*sinh(b*x)/b
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6541, 12, 5448, 3298}

$$\frac{\text{Chi}(bx)\sinh(bx)}{b}-\frac{\text{Shi}(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[b*x]*CoshIntegral[b*x],x]
```

```
[Out] (CoshIntegral[b*x]*Sinh[b*x])/b - SinhIntegral[2*b*x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh(bx)\mathrm{Chi}(bx)\,dx &= \frac{\mathrm{Chi}(bx)\sinh(bx)}{b} - \int \frac{\cosh(bx)\sinh(bx)}{bx}\,dx \\
&= \frac{\mathrm{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\cosh(bx)\sinh(bx)}{x}\,dx}{b} \\
&= \frac{\mathrm{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{2x}\,dx}{b} \\
&= \frac{\mathrm{Chi}(bx)\sinh(bx)}{b} - \frac{\int \frac{\sinh(2bx)}{x}\,dx}{2b} \\
&= \frac{\mathrm{Chi}(bx)\sinh(bx)}{b} - \frac{\mathrm{Shi}(2bx)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$
\frac{\mathrm{Chi}(bx)\sinh(bx)}{b} - \frac{\mathrm{Shi}(2bx)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[b*x]*CoshIntegral[b*x],x]
[Out] (CoshIntegral[b*x]*Sinh[b*x])/b - SinhIntegral[2*b*x]/(2*b)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\cosh\left(bx\right)\mathrm{Chi}\left(bx\right), x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x),x, algorithm="fricas")
[Out] integral(cosh(b*x)*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \mathrm{Chi}\left(bx\right)\cosh\left(bx\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x),x, algorithm="giac")
[Out] integrate(Chi(b*x)*cosh(b*x), x)
```

maple [A] time = 0.02, size = 22, normalized size = 0.88

$$\frac{\mathrm{X}\left(bx\right)\sinh\left(bx\right)-\frac{\mathrm{Shi}(2bx)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)*cosh(b*x),x)
```

```
[Out] 1/b*(Chi(b*x)*sinh(b*x)-1/2*Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)*cosh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{coshint}\left(b\,x\right)\,\cosh\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(b*x)*cosh(b*x),x)
```

```
[Out] int(coshint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx\right)\mathrm{Chi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*cosh(b*x),x)
```

```
[Out] Integral(cosh(b*x)*Chi(b*x), x)
```

3.111 $\int x \cosh(bx)\mathbf{Chi}(bx)\,dx$

Optimal. Leaf size=61

$$\frac{\text{Chi}(2bx)}{2b^2}-\frac{\text{Chi}(bx)\cosh(bx)}{b^2}+\frac{\log(x)}{2b^2}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

```
[Out] 1/2*Chi(2*b*x)/b^2-Chi(b*x)*cosh(b*x)/b^2+1/2*ln(x)/b^2+x*Chi(b*x)*sinh(b*x
)/b-1/2*sinh(b*x)^2/b^2
```

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6543, 12, 2564, 30, 6547, 3312, 3301}

$$\frac{\text{Chi}(2bx)}{2b^2}-\frac{\text{Chi}(bx)\cosh(bx)}{b^2}+\frac{\log(x)}{2b^2}-\frac{\sinh^2(bx)}{2b^2}+\frac{x\text{Chi}(bx)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[b*x]*CoshIntegral[b*x],x]
```

```
[Out] -((Cosh[b*x]*CoshIntegral[b*x])/b^2) + CoshIntegral[2*b*x]/(2*b^2) + Log[x]
/(2*b^2) + (x*CoshIntegral[b*x]*Sinh[b*x])/b - Sinh[b*x]^2/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x\cosh(bx)\text{Chi}(bx)\,dx &= \frac{x\text{Chi}(bx)\sinh(bx)}{b} - \frac{\int \text{Chi}(bx)\sinh(bx)\,dx}{b} - \int \frac{\cosh(bx)\sinh(bx)}{b}\,dx \\
&= -\frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{x\text{Chi}(bx)\sinh(bx)}{b} + \frac{\int \frac{\cosh^2(bx)}{bx}\,dx}{b} - \frac{\int \cosh(bx)\sinh(bx)\,dx}{b} \\
&= -\frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{x\text{Chi}(bx)\sinh(bx)}{b} + \frac{\int \frac{\cosh^2(bx)}{x}\,dx}{b^2} + \frac{\text{Subst}(\int x\,dx, x, i\sinh(bx}{b^2} \\
&= -\frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{x\text{Chi}(bx)\sinh(bx)}{b} - \frac{\sinh^2(bx)}{2b^2} + \frac{\int \left(\frac{1}{2x} + \frac{\cosh(2bx)}{2x}\right)dx}{b^2} \\
&= -\frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{\log(x)}{2b^2} + \frac{x\text{Chi}(bx)\sinh(bx)}{b} - \frac{\sinh^2(bx)}{2b^2} + \frac{\int \frac{\cosh(2bx)}{x}\,dx}{2b^2} \\
&= -\frac{\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{\text{Chi}(2bx)}{2b^2} + \frac{\log(x)}{2b^2} + \frac{x\text{Chi}(bx)\sinh(bx)}{b} - \frac{\sinh^2(bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 0.75

$$\frac{2\text{Chi}(2bx) + 4\text{Chi}(bx)(bx\sinh(bx) - \cosh(bx)) - \cosh(2bx) + 2\log(x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[b*x]*CoshIntegral[b*x],x]
```

```
[Out] (-Cosh[2*b*x] + 2*CoshIntegral[2*b*x] + 2*Log[x] + 4*CoshIntegral[b*x]*(-Co
sh[b*x] + b*x*Sinh[b*x]))/(4*b^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cosh\left(bx\right)\text{Chi}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*cosh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*cosh(b*x)*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*cosh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*Chi(b*x)*cosh(b*x), x)
```

maple [A] time = 0.02, size = 58, normalized size = 0.95

$$\frac{x\text{X}\left(bx\right)\sinh\left(bx\right)}{b} - \frac{\text{X}\left(bx\right)\cosh\left(bx\right)}{b^2} - \frac{\cosh^2\left(bx\right)}{2b^2} + \frac{\ln\left(bx\right)}{2b^2} + \frac{\text{X}\left(2bx\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x)*cosh(b*x),x)
```

```
[Out] x*Chi(b*x)*sinh(b*x)/b-Chi(b*x)*cosh(b*x)/b^2-1/2/b^2*cosh(b*x)^2+1/2/b^2*l
n(b*x)+1/2*Chi(2*b*x)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(bx\right)\cosh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*cosh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(b*x)*cosh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{coshint}(b\,x)\,\cosh(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(b*x)*cosh(b*x),x)
```

```
[Out] int(x*coshint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx)\operatorname{Chi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*cosh(b*x),x)
```

```
[Out] Integral(x*cosh(b*x)*Chi(b*x), x)
```

3.112 $\int x^2 \cosh(bx)\textbf{Chi}(bx)\, dx$

Optimal. Leaf size=90

$$\frac{2\text{Chi}(bx)\sinh(bx)}{b^3}-\frac{\text{Shi}(2bx)}{b^3}+\frac{5\sinh(bx)\cosh(bx)}{4b^3}-\frac{2x\text{Chi}(bx)\cosh(bx)}{b^2}+\frac{3x}{4b^2}-\frac{x\sinh^2(bx)}{2b^2}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

```
[Out] 3/4*x/b^2-2*x*Chi(b*x)*cosh(b*x)/b^2-Shi(2*b*x)/b^3+2*Chi(b*x)*sinh(b*x)/b^
3+x^2*Chi(b*x)*sinh(b*x)/b+5/4*cosh(b*x)*sinh(b*x)/b^3-1/2*x*sinh(b*x)^2/b^
2
```

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6543, 12, 5372, 2635, 8, 6549, 6541, 5448, 3298}

$$\frac{2\text{Chi}(bx)\sinh(bx)}{b^3}-\frac{2x\text{Chi}(bx)\cosh(bx)}{b^2}-\frac{\text{Shi}(2bx)}{b^3}+\frac{3x}{4b^2}-\frac{x\sinh^2(bx)}{2b^2}+\frac{5\sinh(bx)\cosh(bx)}{4b^3}+\frac{x^2\text{Chi}(bx)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[b*x]*CoshIntegral[b*x],x]
```

```
[Out] (3*x)/(4*b^2) - (2*x*Cosh[b*x]*CoshIntegral[b*x])/b^2 + (5*Cosh[b*x]*Sinh[b
*x])/(4*b^3) + (2*CoshIntegral[b*x]*Sinh[b*x])/b^3 + (x^2*CoshIntegral[b*x]
*Sinh[b*x])/b - (x*Sinh[b*x]^2)/(2*b^2) - SinhIntegral[2*b*x]/b^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\cosh(bx)\mathrm{Chi}(bx)\,dx = \frac{x^2\mathrm{Chi}(bx)\sinh(bx)}{b} - \frac{2\int x\mathrm{Chi}(bx)\sinh(bx)\,dx}{b} - \int \frac{x\cosh(bx)\sinh(bx)}{b}\,dx$$

$$= -\frac{2x\cosh(bx)\mathrm{Chi}(bx)}{b^2} + \frac{x^2\mathrm{Chi}(bx)\sinh(bx)}{b} + \frac{2\int\cosh(bx)\mathrm{Chi}(bx)\,dx}{b^2} - \frac{\int x\cosh \ldots}{}$$

$$= -\frac{2x\cosh(bx)\mathrm{Chi}(bx)}{b^2} + \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b^3} + \frac{x^2\mathrm{Chi}(bx)\sinh(bx)}{b} - \frac{x\sinh^2(bx)}{2b^2} + \ldots$$

$$= -\frac{2x\cosh(bx)\mathrm{Chi}(bx)}{b^2} + \frac{5\cosh(bx)\sinh(bx)}{4b^3} + \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b^3} + \frac{x^2\mathrm{Chi}(bx)\sin \ldots}{b}$$

$$= \frac{3x}{4b^2} - \frac{2x\cosh(bx)\mathrm{Chi}(bx)}{b^2} + \frac{5\cosh(bx)\sinh(bx)}{4b^3} + \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b^3} + \frac{x^2\mathrm{Chi}(b \ldots}{}$$

$$= \frac{3x}{4b^2} - \frac{2x\cosh(bx)\mathrm{Chi}(bx)}{b^2} + \frac{5\cosh(bx)\sinh(bx)}{4b^3} + \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b^3} + \frac{x^2\mathrm{Chi}(b \ldots}{}$$

$$= \frac{3x}{4b^2} - \frac{2x\cosh(bx)\mathrm{Chi}(bx)}{b^2} + \frac{5\cosh(bx)\sinh(bx)}{4b^3} + \frac{2\mathrm{Chi}(bx)\sinh(bx)}{b^3} + \frac{x^2\mathrm{Chi}(b \ldots}{}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 0.71

$$\frac{8\mathrm{Chi}(bx)\left(\left(b^2x^2+2\right)\sinh(bx) - 2bx\cosh(bx)\right) - 8\mathrm{Shi}(2bx) + 8bx + 5\sinh(2bx) - 2bx\cosh(2bx)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[b*x]*CoshIntegral[b*x],x]
```

```
[Out] (8*b*x - 2*b*x*Cosh[2*b*x] + 8*CoshIntegral[b*x]*(-2*b*x*Cosh[b*x] + (2 + b
^2*x^2)*Sinh[b*x]) + 5*Sinh[2*b*x] - 8*SinhIntegral[2*b*x])/(8*b^3)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\cosh(bx)\,\mathrm{Chi}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)*cosh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh(b*x)*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Chi}(bx)\cosh(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)*cosh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*Chi(b*x)*cosh(b*x), x)
```

maple [A] time = 0.02, size = 68, normalized size = 0.76

$$\frac{\mathrm{X}(bx)\left(b^2x^2\sinh(bx)-2bx\cosh(bx)+2\sinh(bx)\right)-\frac{bx\left(\cosh^2(bx)\right)}{2}+\frac{5\sinh(bx)\cosh(bx)}{4}+\frac{5bx}{4}-\mathrm{Shi}(2bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x)*cosh(b*x),x)
```

```
[Out] 1/b^3*(Chi(b*x)*(b^2*x^2*sinh(b*x)-2*b*x*cosh(b*x)+2*sinh(b*x))-1/2*b*x*cos
h(b*x)^2+5/4*sinh(b*x)*cosh(b*x)+5/4*b*x-Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{Chi}(bx)\cosh(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)*cosh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Chi(b*x)*cosh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{coshint}(b\,x)\,\cosh(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(b*x)*cosh(b*x),x)
```

```
[Out] int(x^2*coshint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh(bx)\,\mathrm{Chi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x)*cosh(b*x),x)
```

```
[Out] Integral(x**2*cosh(b*x)*Chi(b*x), x)
```

3.113 $\int x^3 \cosh(bx)\mathbf{Chi}(bx)\,dx$

Optimal. Leaf size=142

$$\frac{3\text{Chi}(2bx)}{b^4}-\frac{6\text{Chi}(bx)\cosh(bx)}{b^4}+\frac{3\log(x)}{b^4}-\frac{13\sinh^2(bx)}{4b^4}-\frac{3\cosh^2(bx)}{4b^4}+\frac{6x\text{Chi}(bx)\sinh(bx)}{b^3}+\frac{2x\sinh(bx)\cosh(}{b^3}$$

```
[Out] 1/2*x^2/b^2+3*Chi(2*b*x)/b^4-6*Chi(b*x)*cosh(b*x)/b^4-3*x^2*Chi(b*x)*cosh(b
*x)/b^2-3/4*cosh(b*x)^2/b^4+3*ln(x)/b^4+6*x*Chi(b*x)*sinh(b*x)/b^3+x^3*Chi(
b*x)*sinh(b*x)/b+2*x*cosh(b*x)*sinh(b*x)/b^3-13/4*sinh(b*x)^2/b^4-1/2*x^2*s
inh(b*x)^2/b^2
```

Rubi [A] time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6543, 12, 5372, 3310, 30, 6549, 2564, 6547, 3312, 3301}

$$-\frac{3x^2\text{Chi}(bx)\cosh(bx)}{b^2}+\frac{3\text{Chi}(2bx)}{b^4}+\frac{6x\text{Chi}(bx)\sinh(bx)}{b^3}-\frac{6\text{Chi}(bx)\cosh(bx)}{b^4}+\frac{x^2}{2b^2}-\frac{x^2\sinh^2(bx)}{2b^2}+\frac{3\log(x)}{b^4}-\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[b*x]*CoshIntegral[b*x],x]
```

```
[Out] x^2/(2*b^2) - (3*Cosh[b*x]^2)/(4*b^4) - (6*Cosh[b*x]*CoshIntegral[b*x])/b^4
 - (3*x^2*Cosh[b*x]*CoshIntegral[b*x])/b^2 + (3*CoshIntegral[2*b*x])/b^4 +
(3*Log[x])/b^4 + (2*x*Cosh[b*x]*Sinh[b*x])/b^3 + (6*x*CoshIntegral[b*x]*Sin
h[b*x])/b^3 + (x^3*CoshIntegral[b*x]*Sinh[b*x])/b - (13*Sinh[b*x]^2)/(4*b^4
) - (x^2*Sinh[b*x]^2)/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
```

```
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3\cosh(bx)\text{Chi}(bx)\,dx = \frac{x^3\text{Chi}(bx)\sinh(bx)}{b} - \frac{3\int x^2\text{Chi}(bx)\sinh(bx)\,dx}{b} - \int \frac{x^2\cosh(bx)\sinh(bx)}{b}\,dx$$

$$= -\frac{3x^2\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{x^3\text{Chi}(bx)\sinh(bx)}{b} + \frac{6\int x\cosh(bx)\text{Chi}(bx)\,dx}{b^2} - \frac{\int x^2 c}{}$$

$$= -\frac{3x^2\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{6x\text{Chi}(bx)\sinh(bx)}{b^3} + \frac{x^3\text{Chi}(bx)\sinh(bx)}{b} - \frac{x^2\sinh^2(bx}{2b^2}$$

$$= -\frac{3\cosh^2(bx)}{4b^4} - \frac{6\cosh(bx)\text{Chi}(bx)}{b^4} - \frac{3x^2\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{2x\cosh(bx)\sinh(bx}{b^3}$$

$$= \frac{x^2}{2b^2} - \frac{3\cosh^2(bx)}{4b^4} - \frac{6\cosh(bx)\text{Chi}(bx)}{b^4} - \frac{3x^2\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{2x\cosh(bx)\sin}{b^3}$$

$$= \frac{x^2}{2b^2} - \frac{3\cosh^2(bx)}{4b^4} - \frac{6\cosh(bx)\text{Chi}(bx)}{b^4} - \frac{3x^2\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{2x\cosh(bx)\sin}{b^3}$$

$$= \frac{x^2}{2b^2} - \frac{3\cosh^2(bx)}{4b^4} - \frac{6\cosh(bx)\text{Chi}(bx)}{b^4} - \frac{3x^2\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{3\log(x)}{b^4} + \frac{2x\,c}{}$$

$$= \frac{x^2}{2b^2} - \frac{3\cosh^2(bx)}{4b^4} - \frac{6\cosh(bx)\text{Chi}(bx)}{b^4} - \frac{3x^2\cosh(bx)\text{Chi}(bx)}{b^2} + \frac{3\text{Chi}(2bx)}{b^4} + \frac{3}{}$$

Mathematica [A] time = 0.10, size = 94, normalized size = 0.66

$$\frac{4\text{Chi}(bx)\left(bx\left(b^2x^2+6\right)\sinh(bx) - 3\left(b^2x^2+2\right)\cosh(bx)\right) + 3b^2x^2 - b^2x^2\cosh(2bx) + 12\text{Chi}(2bx) + 4bx\sinh(}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[b*x]*CoshIntegral[b*x],x]
```

```
[Out] (3*b^2*x^2 - 8*Cosh[2*b*x] - b^2*x^2*Cosh[2*b*x] + 12*CoshIntegral[2*b*x] +
 12*Log[x] + 4*CoshIntegral[b*x]*(-3*(2 + b^2*x^2)*Cosh[b*x] + b*x*(6 + b^2
*x^2)*Sinh[b*x]) + 4*b*x*Sinh[2*b*x])/(4*b^4)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\cosh(bx)\,\text{Chi}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)*cosh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cosh(b*x)*cosh_integral(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{Chi}\,(bx)\cosh\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)*cosh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*Chi(b*x)*cosh(b*x), x)
```

maple [A] time = 0.02, size = 125, normalized size = 0.88

$$\frac{x^3 \mathrm{X}\,(bx)\sinh\,(bx)}{b}-\frac{3x^2 \mathrm{X}\,(bx)\cosh\,(bx)}{b^2}+\frac{6x \mathrm{X}\,(bx)\sinh\,(bx)}{b^3}-\frac{6 \mathrm{X}\,(bx)\cosh\,(bx)}{b^4}-\frac{x^2\left(\cosh^2(bx)\right)}{2b^2}+\frac{2x\cosh\,(bx)\,\mathrm{s}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Chi(b*x)*cosh(b*x),x)
```

```
[Out] x^3*Chi(b*x)*sinh(b*x)/b-3*x^2*Chi(b*x)*cosh(b*x)/b^2+6*x*Chi(b*x)*sinh(b*x
)/b^3-6*Chi(b*x)*cosh(b*x)/b^4-1/2/b^2*x^2*cosh(b*x)^2+2*x*cosh(b*x)*sinh(b
*x)/b^3+x^2/b^2-4*cosh(b*x)^2/b^4+3/b^4*ln(b*x)+3*Chi(2*b*x)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{Chi}\,(bx)\cosh\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)*cosh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*Chi(b*x)*cosh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{coshint}\left(b\,x\right)\,\cosh\left(b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coshint(b*x)*cosh(b*x),x)
```

```
[Out] int(x^3*coshint(b*x)*cosh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx) \operatorname{Chi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Chi(b*x)*cosh(b*x),x)
```

```
[Out] Integral(x**3*cosh(b*x)*Chi(b*x), x)
```

3.114 $\int \frac{\mathbf{Chi}(bx)\sinh(bx)}{x^3}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}b^2\text{Int}\left(\frac{\text{Chi}(bx)\sinh(bx)}{x},x\right)+b^2\text{Shi}(2bx)-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{b\text{Chi}(bx)\cosh(bx)}{2x}-\frac{\sinh(2bx)}{8x^2}-\frac{b\cosh^2(bx)}{2x}-\frac{b\,\text{co}}{}$$

[Out] 1/2*b^2*CannotIntegrate(Chi(b*x)*sinh(b*x)/x,x)-1/2*b*Chi(b*x)*cosh(b*x)/x-1/2*b*cosh(b*x)^2/x-1/4*b*cosh(2*b*x)/x+b^2*Shi(2*b*x)-1/2*Chi(b*x)*sinh(b*x)/x^2-1/8*sinh(2*b*x)/x^2

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

[In] Int[(CoshIntegral[b*x]*Sinh[b*x])/x^3,x]

[Out] -(b*Cosh[b*x]^2)/(2*x) - (b*Cosh[2*b*x])/(4*x) - (b*Cosh[b*x]*CoshIntegral[b*x])/(2*x) - (CoshIntegral[b*x]*Sinh[b*x])/(2*x^2) - Sinh[2*b*x]/(8*x^2) + b^2*SinhIntegral[2*b*x] + (b^2*Defer[Int][(CoshIntegral[b*x]*Sinh[b*x])/x, x])/2

Rubi steps

$$\begin{aligned}
\int \frac{\text{Chi}(bx)\sinh(bx)}{x^3}\,dx &= -\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}+\frac{1}{2}b\int \frac{\cosh(bx)\text{Chi}(bx)}{x^2}\,dx+\frac{1}{2}b\int \frac{\cosh(bx)\sinh(bx)}{bx^3}\,dx\\
&= -\frac{b\cosh(bx)\text{Chi}(bx)}{2x}-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}+\frac{1}{2}\int \frac{\cosh(bx)\sinh(bx)}{x^3}\,dx+\frac{1}{2}b^2\int \frac{\cosh^2}{bx}\\
&= -\frac{b\cosh(bx)\text{Chi}(bx)}{2x}-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}+\frac{1}{2}\int \frac{\sinh(2bx)}{2x^3}\,dx+\frac{1}{2}b\int \frac{\cosh^2(bx)}{x^2}\,dx+\\
&= -\frac{b\cosh^2(bx)}{2x}-\frac{b\cosh(bx)\text{Chi}(bx)}{2x}-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}+\frac{1}{4}\int \frac{\sinh(2bx)}{x^3}\,dx+\left(ib^2\right)\int\\
&= -\frac{b\cosh^2(bx)}{2x}-\frac{b\cosh(bx)\text{Chi}(bx)}{2x}-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{\sinh(2bx)}{8x^2}+\frac{1}{4}b\int \frac{\cosh(2b}{x^2}\\
&= -\frac{b\cosh^2(bx)}{2x}-\frac{b\cosh(2bx)}{4x}-\frac{b\cosh(bx)\text{Chi}(bx)}{2x}-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{\sinh(2bx)}{8x^2}+\\
&= -\frac{b\cosh^2(bx)}{2x}-\frac{b\cosh(2bx)}{4x}-\frac{b\cosh(bx)\text{Chi}(bx)}{2x}-\frac{\text{Chi}(bx)\sinh(bx)}{2x^2}-\frac{\sinh(2bx)}{8x^2}+
\end{aligned}$$

Mathematica [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x^3,x]
```

```
[Out] Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x^3, x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Chi}\left(bx\right)\sinh\left(bx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x)*sinh(b*x)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx\right)\sinh\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x)*sinh(b*x)/x^3, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(bx\right)\sinh\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)*sinh(b*x)/x^3,x)
```

```
[Out] int(Chi(b*x)*sinh(b*x)/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx\right)\sinh\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x)*sinh(b*x)/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{coshint}(b\,x)\,\sinh(b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(b*x)*sinh(b*x))/x^3,x)
```

```
[Out] int((coshint(b*x)*sinh(b*x))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx)\operatorname{Chi}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x**3,x)
```

```
[Out] Integral(sinh(b*x)*Chi(b*x)/x**3, x)
```

3.115 $\int \frac{\mathbf{Chi}(bx)\sinh(bx)}{x^2}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}b\mathrm{Chi}(bx)^2 + b\mathrm{Chi}(2bx) - \frac{\mathrm{Chi}(bx)\sinh(bx)}{x} - \frac{\sinh(2bx)}{2x}$$

```
[Out] 1/2*b*Chi(b*x)^2+b*Chi(2*b*x)-Chi(b*x)*sinh(b*x)/x-1/2*sinh(2*b*x)/x
```

Rubi [A] time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6551, 6686, 12, 5448, 3297, 3301}

$$\frac{1}{2}b\mathrm{Chi}(bx)^2 + b\mathrm{Chi}(2bx) - \frac{\mathrm{Chi}(bx)\sinh(bx)}{x} - \frac{\sinh(2bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(CoshIntegral[b*x]*Sinh[b*x])/x^2,x]
```

```
[Out] (b*CoshIntegral[b*x]^2)/2 + b*CoshIntegral[2*b*x] - (CoshIntegral[b*x]*Sinh
[b*x])/x - Sinh[2*b*x]/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 6551

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_)*Sinh[(a_.) +
 (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^(m + 1)*Sinh[a + b*x]*CoshIntegr
al[c + d*x])/(f*(m + 1)), x] + (-Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)*
Cosh[a + b*x]*CoshIntegral[c + d*x], x], x] - Dist[d/(f*(m + 1)), Int[((e +
 f*x)^(m + 1)*Sinh[a + b*x]*Cosh[c + d*x])/(c + d*x), x], x]) /; FreeQ[{a,
b, c, d, e, f}, x] && ILtQ[m, -1]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Chi}(bx)\sinh(bx)}{x^2}\,dx &= -\frac{\text{Chi}(bx)\sinh(bx)}{x} + b\int \frac{\cosh(bx)\text{Chi}(bx)}{x}\,dx + b\int \frac{\cosh(bx)\sinh(bx)}{bx^2}\,dx \\
&= \frac{1}{2}b\text{Chi}(bx)^2 - \frac{\text{Chi}(bx)\sinh(bx)}{x} + \int \frac{\cosh(bx)\sinh(bx)}{x^2}\,dx \\
&= \frac{1}{2}b\text{Chi}(bx)^2 - \frac{\text{Chi}(bx)\sinh(bx)}{x} + \int \frac{\sinh(2bx)}{2x^2}\,dx \\
&= \frac{1}{2}b\text{Chi}(bx)^2 - \frac{\text{Chi}(bx)\sinh(bx)}{x} + \frac{1}{2}\int \frac{\sinh(2bx)}{x^2}\,dx \\
&= \frac{1}{2}b\text{Chi}(bx)^2 - \frac{\text{Chi}(bx)\sinh(bx)}{x} - \frac{\sinh(2bx)}{2x} + b\int \frac{\cosh(2bx)}{x}\,dx \\
&= \frac{1}{2}b\text{Chi}(bx)^2 + b\text{Chi}(2bx) - \frac{\text{Chi}(bx)\sinh(bx)}{x} - \frac{\sinh(2bx)}{2x}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$
\frac{1}{2}b\text{Chi}(bx)^2 + b\text{Chi}(2bx) - \frac{\text{Chi}(bx)\sinh(bx)}{x} - \frac{\sinh(2bx)}{2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x^2,x]
```

```
[Out] (b*CoshIntegral[b*x]^2)/2 + b*CoshIntegral[2*b*x] - (CoshIntegral[b*x]*Sinh
[b*x])/x - Sinh[2*b*x]/(2*x)
```

fricas [F] time = 2.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Chi}\,(bx)\sinh\,(bx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x^2,x, algorithm="fricas")
[Out] integral(cosh_integral(b*x)*sinh(b*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)\sinh\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x^2,x, algorithm="giac")
[Out] integrate(Chi(b*x)*sinh(b*x)/x^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx)\sinh\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)*sinh(b*x)/x^2,x)
[Out] int(Chi(b*x)*sinh(b*x)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)\sinh\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x^2,x, algorithm="maxima")
[Out] integrate(Chi(b*x)*sinh(b*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{coshint}\,(b\,x)\;\sinh\,(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(b*x)*sinh(b*x))/x^2,x)
```

```
[Out] int((coshint(b*x)*sinh(b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx)\,\mathrm{Chi}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x**2,x)
```

```
[Out] Integral(sinh(b*x)*Chi(b*x)/x**2, x)
```

3.116 $\int \frac{\mathbf{Chi}(bx)\sinh(bx)}{x}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{\mathrm{Chi}(bx)\sinh(bx)}{x},x\right)$$

```
[Out] CannotIntegrate(Chi(b*x)*sinh(b*x)/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(CoshIntegral[b*x]*Sinh[b*x])/x,x]
[Out] Defer[Int][(CoshIntegral[b*x]*Sinh[b*x])/x, x]
```

Rubi steps

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x}\,dx = \int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x}\,dx$$

Mathematica [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}(bx)\sinh(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x,x]
[Out] Integrate[(CoshIntegral[b*x]*Sinh[b*x])/x, x]
```

fricas [A] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Chi}\,(bx)\sinh\,(bx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x)*sinh(b*x)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)\sinh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x,x, algorithm="giac")
[Out] integrate(Chi(b*x)*sinh(b*x)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\,(bx)\sinh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x)*sinh(b*x)/x,x)
[Out] int(Chi(b*x)*sinh(b*x)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\,(bx)\sinh\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x,x, algorithm="maxima")
[Out] integrate(Chi(b*x)*sinh(b*x)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\mathrm{coshint}\,(b\,x)\ \sinh\,(b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(b*x)*sinh(b*x))/x,x)
[Out] int((coshint(b*x)*sinh(b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\,(bx)\,\mathrm{Chi}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x)/x,x)
```

```
[Out] Integral(sinh(b*x)*Chi(b*x)/x, x)
```

3.117 $\int \mathbf{Chi}(bx)\sinh(bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\text{Chi}(2bx)}{2b}+\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\log(x)}{2b}$$

```
[Out] -1/2*Chi(2*b*x)/b+Chi(b*x)*cosh(b*x)/b-1/2*ln(x)/b
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6547, 12, 3312, 3301}

$$-\frac{\text{Chi}(2bx)}{2b}+\frac{\text{Chi}(bx)\cosh(bx)}{b}-\frac{\log(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[b*x]*Sinh[b*x],x]
```

```
[Out] (Cosh[b*x]*CoshIntegral[b*x])/b - CoshIntegral[2*b*x]/(2*b) - Log[x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Chi}(bx)\sinh(bx)\,dx &= \frac{\cosh(bx)\mathrm{Chi}(bx)}{b} - \int \frac{\cosh^2(bx)}{bx}\,dx \\
&= \frac{\cosh(bx)\mathrm{Chi}(bx)}{b} - \frac{\int \frac{\cosh^2(bx)}{x}\,dx}{b} \\
&= \frac{\cosh(bx)\mathrm{Chi}(bx)}{b} - \frac{\int \left(\frac{1}{2x} + \frac{\cosh(2bx)}{2x}\right)dx}{b} \\
&= \frac{\cosh(bx)\mathrm{Chi}(bx)}{b} - \frac{\log(x)}{2b} - \frac{\int \frac{\cosh(2bx)}{x}\,dx}{2b} \\
&= \frac{\cosh(bx)\mathrm{Chi}(bx)}{b} - \frac{\mathrm{Chi}(2bx)}{2b} - \frac{\log(x)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.06

$$
-\frac{\mathrm{Chi}(2bx)}{2b} + \frac{\mathrm{Chi}(bx)\cosh(bx)}{b} - \frac{\log(bx)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[b*x]*Sinh[b*x],x]
[Out] (Cosh[b*x]*CoshIntegral[b*x])/b - CoshIntegral[2*b*x]/(2*b) - Log[b*x]/(2*b
)
```

fricas [F] time = 2.28, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\mathrm{Chi}\left(bx\right)\sinh\left(bx\right), x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x),x, algorithm="fricas")
[Out] integral(cosh_integral(b*x)*sinh(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \mathrm{Chi}\left(bx\right)\sinh\left(bx\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x)*sinh(b*x),x, algorithm="giac")
```

[Out] integrate(Chi(b*x)*sinh(b*x), x)

maple [A] time = 0.02, size = 33, normalized size = 0.97

$$\frac{\mathrm{X}\left(bx\right)\cosh\left(bx\right)}{b}-\frac{\ln\left(bx\right)}{2b}-\frac{\mathrm{X}\left(2bx\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(Chi(b*x)*sinh(b*x),x)

[Out] Chi(b*x)*cosh(b*x)/b-1/2/b*ln(b*x)-1/2*Chi(2*b*x)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(Chi(b*x)*sinh(b*x),x, algorithm="maxima")

[Out] integrate(Chi(b*x)*sinh(b*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{coshint}\left(b\,x\right)\,\sinh\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coshint(b*x)*sinh(b*x),x)

[Out] int(coshint(b*x)*sinh(b*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx\right)\mathrm{Chi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(Chi(b*x)*sinh(b*x),x)

[Out] Integral(sinh(b*x)*Chi(b*x), x)

3.118 $\int x\mathbf{Chi}(bx)\sinh(bx)\,dx$

Optimal. Leaf size=62

$$-\frac{\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{\text{Shi}(2bx)}{2b^2}-\frac{\sinh(bx)\cosh(bx)}{2b^2}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{x}{2b}$$

```
[Out] -1/2*x/b+x*Chi(b*x)*cosh(b*x)/b+1/2*Shi(2*b*x)/b^2-Chi(b*x)*sinh(b*x)/b^2-1
/2*cosh(b*x)*sinh(b*x)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6549, 12, 2635, 8, 6541, 5448, 3298}

$$-\frac{\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{\text{Shi}(2bx)}{2b^2}-\frac{\sinh(bx)\cosh(bx)}{2b^2}+\frac{x\text{Chi}(bx)\cosh(bx)}{b}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[b*x]*Sinh[b*x],x]
```

```
[Out] -x/(2*b) + (x*Cosh[b*x]*CoshIntegral[b*x])/b - (Cosh[b*x]*Sinh[b*x])/(2*b^2
) - (CoshIntegral[b*x]*Sinh[b*x])/b^2 + SinhIntegral[2*b*x]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\text{Chi}(bx)\sinh(bx)\,dx &= \frac{x\cosh(bx)\text{Chi}(bx)}{b}-\frac{\int\cosh(bx)\text{Chi}(bx)\,dx}{b}-\int\frac{\cosh^2(bx)}{b}\,dx\\ &= \frac{x\cosh(bx)\text{Chi}(bx)}{b}-\frac{\text{Chi}(bx)\sinh(bx)}{b^2}-\frac{\int\cosh^2(bx)\,dx}{b}+\frac{\int\frac{\cosh(bx)\sinh(bx)}{bx}\,dx}{b}\\ &= \frac{x\cosh(bx)\text{Chi}(bx)}{b}-\frac{\cosh(bx)\sinh(bx)}{2b^2}-\frac{\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{\int\frac{\cosh(bx)\sinh(bx)}{x}\,dx}{b^2}-\\ &= -\frac{x}{2b}+\frac{x\cosh(bx)\text{Chi}(bx)}{b}-\frac{\cosh(bx)\sinh(bx)}{2b^2}-\frac{\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{\int\frac{\sinh(2bx)}{2x}\,dx}{b^2}\\ &= -\frac{x}{2b}+\frac{x\cosh(bx)\text{Chi}(bx)}{b}-\frac{\cosh(bx)\sinh(bx)}{2b^2}-\frac{\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{\int\frac{\sinh(2bx)}{x}\,dx}{2b^2}\\ &= -\frac{x}{2b}+\frac{x\cosh(bx)\text{Chi}(bx)}{b}-\frac{\cosh(bx)\sinh(bx)}{2b^2}-\frac{\text{Chi}(bx)\sinh(bx)}{b^2}+\frac{\text{Shi}(2bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 44, normalized size = 0.71

$$-\frac{\text{Chi}(bx)(4\sinh(bx)-4bx\cosh(bx))-2\text{Shi}(2bx)+2bx+\sinh(2bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[b*x]*Sinh[b*x],x]
```

```
[Out] -1/4*(2*b*x + CoshIntegral[b*x]*(-4*b*x*Cosh[b*x] + 4*Sinh[b*x]) + Sinh[2*b
*x] - 2*SinhIntegral[2*b*x])/b^2
```

fricas [F] time = 1.95, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Chi}\left(bx\right)\sinh\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*sinh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*cosh_integral(b*x)*sinh(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*sinh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*Chi(b*x)*sinh(b*x), x)
```

maple [A] time = 0.03, size = 46, normalized size = 0.74

$$\frac{\mathrm{X}\left(bx\right)\left(bx\cosh\left(bx\right)-\sinh\left(bx\right)\right)-\frac{\sinh(bx)\cosh(bx)}{2}-\frac{bx}{2}+\frac{\text{Shi}(2bx)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x)*sinh(b*x),x)
```

```
[Out] 1/b^2*(Chi(b*x)*(b*x*cosh(b*x)-sinh(b*x))-1/2*sinh(b*x)*cosh(b*x)-1/2*b*x+1
/2*Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*sinh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(b*x)*sinh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{coshint}(b x) \sinh(b x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(b*x)*sinh(b*x),x)
```

```
[Out] int(x*coshint(b*x)*sinh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh(bx) \operatorname{Chi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x)*sinh(b*x),x)
```

```
[Out] Integral(x*sinh(b*x)*Chi(b*x), x)
```

3.119 $\int x^2\mathbf{Chi}(bx)\sinh(bx)\,dx$

Optimal. Leaf size=109

$$-\frac{\mathrm{Chi}(2bx)}{b^3}+\frac{2\mathrm{Chi}(bx)\cosh(bx)}{b^3}-\frac{\log(x)}{b^3}+\frac{\sinh^2(bx)}{b^3}+\frac{\cosh^2(bx)}{4b^3}-\frac{2x\mathrm{Chi}(bx)\sinh(bx)}{b^2}-\frac{x\sinh(bx)\cosh(bx)}{2b^2}+x$$

```
[Out] -1/4*x^2/b-Chi(2*b*x)/b^3+2*Chi(b*x)*cosh(b*x)/b^3+x^2*Chi(b*x)*cosh(b*x)/b
+1/4*cosh(b*x)^2/b^3-ln(x)/b^3-2*x*Chi(b*x)*sinh(b*x)/b^2-1/2*x*cosh(b*x)*s
inh(b*x)/b^2+sinh(b*x)^2/b^3
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6549, 12, 3310, 30, 6543, 2564, 6547, 3312, 3301}

$$-\frac{\mathrm{Chi}(2bx)}{b^3}-\frac{2x\mathrm{Chi}(bx)\sinh(bx)}{b^2}+\frac{2\mathrm{Chi}(bx)\cosh(bx)}{b^3}-\frac{\log(x)}{b^3}+\frac{\sinh^2(bx)}{b^3}+\frac{\cosh^2(bx)}{4b^3}-\frac{x\sinh(bx)\cosh(bx)}{2b^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*CoshIntegral[b*x]*Sinh[b*x],x]
```

```
[Out] -x^2/(4*b) + Cosh[b*x]^2/(4*b^3) + (2*Cosh[b*x]*CoshIntegral[b*x])/b^3 + (x
^2*Cosh[b*x]*CoshIntegral[b*x])/b - CoshIntegral[2*b*x]/b^3 - Log[x]/b^3 -
(x*Cosh[b*x]*Sinh[b*x])/(2*b^2) - (2*x*CoshIntegral[b*x]*Sinh[b*x])/b^2 + S
inh[b*x]^2/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\text{Chi}(bx)\sinh(bx)\,dx = \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{2\int x\cosh(bx)\text{Chi}(bx)\,dx}{b} - \int \frac{x\cosh^2(bx)}{b}\,dx$$

$$= \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{2x\text{Chi}(bx)\sinh(bx)}{b^2} + \frac{2\int \text{Chi}(bx)\sinh(bx)\,dx}{b^2} - \frac{\int x\cosh^2}{b}$$

$$= \frac{\cosh^2(bx)}{4b^3} + \frac{2\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{x\cosh(bx)\sinh(bx)}{2b^2} - \frac{2}{}$$

$$= -\frac{x^2}{4b} + \frac{\cosh^2(bx)}{4b^3} + \frac{2\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{x\cosh(bx)\sinh(}{2b^2}$$

$$= -\frac{x^2}{4b} + \frac{\cosh^2(bx)}{4b^3} + \frac{2\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{x\cosh(bx)\sinh(}{2b^2}$$

$$= -\frac{x^2}{4b} + \frac{\cosh^2(bx)}{4b^3} + \frac{2\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{\log(x)}{b^3} - \frac{x\cosh(}{}$$

$$= -\frac{x^2}{4b} + \frac{\cosh^2(bx)}{4b^3} + \frac{2\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^2\cosh(bx)\text{Chi}(bx)}{b} - \frac{\text{Chi}(2bx)}{b^3} - \frac{\log(}{b^3}$$

Mathematica [A] time = 0.11, size = 72, normalized size = 0.66

$$-\frac{-8\text{Chi}(bx)\left(\left(b^2x^2+2\right)\cosh(bx)-2bx\sinh(bx)\right)+2b^2x^2+8\text{Chi}(2bx)+2bx\sinh(2bx)-5\cosh(2bx)+8\log}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*CoshIntegral[b*x]*Sinh[b*x],x]
```

```
[Out] -1/8*(2*b^2*x^2 - 5*Cosh[2*b*x] + 8*CoshIntegral[2*b*x] + 8*Log[x] - 8*Cosh
Integral[b*x]*((2 + b^2*x^2)*Cosh[b*x] - 2*b*x*Sinh[b*x]) + 2*b*x*Sinh[2*b*
x])/b^3
```

fricas [F] time = 1.35, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{Chi}\,(bx)\sinh\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)*sinh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh_integral(b*x)*sinh(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}(bx)\sinh(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)*sinh(b*x),x, algorithm="giac")
[Out] integrate(x^2*Chi(b*x)*sinh(b*x), x)
```

maple [A] time = 0.03, size = 96, normalized size = 0.88

$$\frac{x^2 \mathrm{X}(bx)\cosh(bx)}{b} - \frac{2x\mathrm{X}(bx)\sinh(bx)}{b^2} + \frac{2\mathrm{X}(bx)\cosh(bx)}{b^3} - \frac{x\cosh(bx)\sinh(bx)}{2b^2} - \frac{x^2}{4b} + \frac{5\left(\cosh^2(bx)\right)}{4b^3} - \frac{\ln(bx)}{b^3} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x)*sinh(b*x),x)
[Out] x^2*Chi(b*x)*cosh(b*x)/b-2*x*Chi(b*x)*sinh(b*x)/b^2+2*Chi(b*x)*cosh(b*x)/b^
3-1/2*x*cosh(b*x)*sinh(b*x)/b^2-1/4*x^2/b+5/4*cosh(b*x)^2/b^3-1/b^3*ln(b*x)
-Chi(2*b*x)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}(bx)\sinh(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x)*sinh(b*x),x, algorithm="maxima")
[Out] integrate(x^2*Chi(b*x)*sinh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{coshint}(b\,x)\,\sinh(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(b*x)*sinh(b*x),x)
[Out] int(x^2*coshint(b*x)*sinh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh(bx)\,\mathrm{Chi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x)*sinh(b*x),x)
```

```
[Out] Integral(x**2*sinh(b*x)*Chi(b*x), x)
```

3.120 $\int x^3 \textbf{Chi}(bx) \sinh(bx) \, dx$

Optimal. Leaf size=146

$$-\frac{6\text{Chi}(bx)\sinh(bx)}{b^4}+\frac{3\text{Shi}(2bx)}{b^4}-\frac{4\sinh(bx)\cosh(bx)}{b^4}+\frac{6x\text{Chi}(bx)\cosh(bx)}{b^3}-\frac{5x}{2b^3}+\frac{3x\sinh^2(bx)}{2b^3}+\frac{x\cosh^2(bx)}{2b^3}-$$

```
[Out] -5/2*x/b^3-1/6*x^3/b+6*x*Chi(b*x)*cosh(b*x)/b^3+x^3*Chi(b*x)*cosh(b*x)/b+1/
2*x*cosh(b*x)^2/b^3+3*Shi(2*b*x)/b^4-6*Chi(b*x)*sinh(b*x)/b^4-3*x^2*Chi(b*x
)*sinh(b*x)/b^2-4*cosh(b*x)*sinh(b*x)/b^4-1/2*x^2*cosh(b*x)*sinh(b*x)/b^2+3
/2*x*sinh(b*x)^2/b^3
```

Rubi [A] time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6549, 12, 3311, 30, 2635, 8, 6543, 5372, 6541, 5448, 3298}

$$-\frac{3x^2\text{Chi}(bx)\sinh(bx)}{b^2}-\frac{6\text{Chi}(bx)\sinh(bx)}{b^4}+\frac{6x\text{Chi}(bx)\cosh(bx)}{b^3}+\frac{3\text{Shi}(2bx)}{b^4}-\frac{x^2\sinh(bx)\cosh(bx)}{2b^2}-\frac{5x}{2b^3}+\frac{3x\sin}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*CoshIntegral[b*x]*Sinh[b*x],x]
```

```
[Out] (-5*x)/(2*b^3) - x^3/(6*b) + (x*Cosh[b*x]^2)/(2*b^3) + (6*x*Cosh[b*x]*CoshI
ntegral[b*x])/b^3 + (x^3*Cosh[b*x]*CoshIntegral[b*x])/b - (4*Cosh[b*x]*Sinh
[b*x])/b^4 - (x^2*Cosh[b*x]*Sinh[b*x])/(2*b^2) - (6*CoshIntegral[b*x]*Sinh[
b*x])/b^4 - (3*x^2*CoshIntegral[b*x]*Sinh[b*x])/b^2 + (3*x*Sinh[b*x]^2)/(2*
b^3) + (3*SinhIntegral[2*b*x])/b^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
```

```
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3\text{Chi}(bx)\sinh(bx)\,dx = \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{3\int x^2\cosh(bx)\text{Chi}(bx)\,dx}{b} - \int \frac{x^2\cosh^2(bx)}{b}\,dx$$

$$= \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{3x^2\text{Chi}(bx)\sinh(bx)}{b^2} + \frac{6\int x\text{Chi}(bx)\sinh(bx)\,dx}{b^2} - \frac{\int x^2\cosh}{b}$$

$$= \frac{x\cosh^2(bx)}{2b^3} + \frac{6x\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{x^2\cosh(bx)\sinh(bx)}{2b^2} -$$

$$= -\frac{x^3}{6b} + \frac{x\cosh^2(bx)}{2b^3} + \frac{6x\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{\cosh(bx)\sinh(b}{4b^4}$$

$$= -\frac{x}{4b^3} - \frac{x^3}{6b} + \frac{x\cosh^2(bx)}{2b^3} + \frac{6x\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{4\cosh(bx}{b}$$

$$= -\frac{5x}{2b^3} - \frac{x^3}{6b} + \frac{x\cosh^2(bx)}{2b^3} + \frac{6x\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{4\cosh(bx}{b}$$

$$= -\frac{5x}{2b^3} - \frac{x^3}{6b} + \frac{x\cosh^2(bx)}{2b^3} + \frac{6x\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{4\cosh(bx}{b}$$

$$= -\frac{5x}{2b^3} - \frac{x^3}{6b} + \frac{x\cosh^2(bx)}{2b^3} + \frac{6x\cosh(bx)\text{Chi}(bx)}{b^3} + \frac{x^3\cosh(bx)\text{Chi}(bx)}{b} - \frac{4\cosh(bx}{b}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.64

$$\frac{-2b^3x^3 + 12\text{Chi}(bx)\left(bx\left(b^2x^2+6\right)\cosh(bx) - 3\left(b^2x^2+2\right)\sinh(bx)\right) - 3b^2x^2\sinh(2bx) + 36\text{Shi}(2bx) - 36bx - 2}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*CoshIntegral[b*x]*Sinh[b*x],x]
```

```
[Out] (-36*b*x - 2*b^3*x^3 + 12*b*x*Cosh[2*b*x] + 12*CoshIntegral[b*x]*(b*x*(6 +
b^2*x^2)*Cosh[b*x] - 3*(2 + b^2*x^2)*Sinh[b*x]) - 24*Sinh[2*b*x] - 3*b^2*x^
2*Sinh[2*b*x] + 36*SinhIntegral[2*b*x])/(12*b^4)
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^3\,\mathrm{Chi}\left(bx\right)\sinh\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)*sinh(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cosh_integral(b*x)*sinh(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\mathrm{Chi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)*sinh(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*Chi(b*x)*sinh(b*x), x)
```

maple [A] time = 0.04, size = 104, normalized size = 0.71

$$\frac{\mathrm{X}\left(bx\right)\left(b^3x^3\cosh\left(bx\right)-3b^2x^2\sinh\left(bx\right)+6bx\cosh\left(bx\right)-6\sinh\left(bx\right)\right)-\frac{b^2x^2\cosh(bx)\sinh(bx)}{2}-\frac{b^3x^3}{6}+2bx\left(\cosh^2\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*Chi(b*x)*sinh(b*x),x)
```

```
[Out] 1/b^4*(Chi(b*x)*(b^3*x^3*cosh(b*x)-3*b^2*x^2*sinh(b*x)+6*b*x*cosh(b*x)-6*si
nh(b*x))-1/2*b^2*x^2*cosh(b*x)*sinh(b*x)-1/6*b^3*x^3+2*b*x*cosh(b*x)^2-4*si
nh(b*x)*cosh(b*x)-4*b*x+3*Shi(2*b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\mathrm{Chi}\left(bx\right)\sinh\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*Chi(b*x)*sinh(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*Chi(b*x)*sinh(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{coshint}(b\,x)\, \sinh(b\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coshint(b*x)*sinh(b*x),x)
```

```
[Out] int(x^3*coshint(b*x)*sinh(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sinh(bx) \operatorname{Chi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*Chi(b*x)*sinh(b*x),x)
```

```
[Out] Integral(x**3*sinh(b*x)*Chi(b*x), x)
```

3.121 $\int \mathrm{Chi}(2x)\sinh(5x)\,dx$

Optimal. Leaf size=29

$$-\frac{\mathrm{Chi}(3x)}{10}-\frac{\mathrm{Chi}(7x)}{10}+\frac{1}{5}\mathrm{Chi}(2x)\cosh(5x)$$

```
[Out] -1/10*Chi(3*x)-1/10*Chi(7*x)+1/5*Chi(2*x)*cosh(5*x)
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6547, 12, 5471, 3301}

$$-\frac{\mathrm{Chi}(3x)}{10}-\frac{\mathrm{Chi}(7x)}{10}+\frac{1}{5}\mathrm{Chi}(2x)\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[2*x]*Sinh[5*x],x]
```

```
[Out] (Cosh[5*x]*CoshIntegral[2*x])/5 - CoshIntegral[3*x]/10 - CoshIntegral[7*x]/
10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5471

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) +
(f_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cosh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{Chi}(2x)\sinh(5x)\,dx &= \frac{1}{5}\cosh(5x)\operatorname{Chi}(2x)-\frac{2}{5}\int\frac{\cosh(2x)\cosh(5x)}{2x}\,dx\\
&= \frac{1}{5}\cosh(5x)\operatorname{Chi}(2x)-\frac{1}{5}\int\frac{\cosh(2x)\cosh(5x)}{x}\,dx\\
&= \frac{1}{5}\cosh(5x)\operatorname{Chi}(2x)-\frac{1}{5}\int\left(\frac{\cosh(3x)}{2x}+\frac{\cosh(7x)}{2x}\right)dx\\
&= \frac{1}{5}\cosh(5x)\operatorname{Chi}(2x)-\frac{1}{10}\int\frac{\cosh(3x)}{x}\,dx-\frac{1}{10}\int\frac{\cosh(7x)}{x}\,dx\\
&= \frac{1}{5}\cosh(5x)\operatorname{Chi}(2x)-\frac{\operatorname{Chi}(3x)}{10}-\frac{\operatorname{Chi}(7x)}{10}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.93

$$\frac{1}{10}(-\operatorname{Chi}(3x)-\operatorname{Chi}(7x)+2\operatorname{Chi}(2x)\cosh(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[2*x]*Sinh[5*x],x]
```

```
[Out] (2*Cosh[5*x]*CoshIntegral[2*x] - CoshIntegral[3*x] - CoshIntegral[7*x])/10
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*sinh(5*x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> An error occurred when FriCAS evaluated '(op
erator(Chi)((x)*(2)))*(sinh((x)*(5)))':   There are 1 exposed and 1 unexpos
ed library operations named elt having      1 argument(s) but none was dete
rmined to be applicable. Use HyperDoc      Browse, or issue
                )display op elt      to learn more about the available oper
ations. Perhaps       package-calling the operation or using coercions on t
he arguments       will allow you to apply the operation.    Cannot find ap
plication of object of type BasicOperator to argument(s)       of type(s)
                             Polynomial(Integer)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Chi}(2\,x)\sinh(5\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*sinh(5*x),x, algorithm="giac")
[Out] integrate(Chi(2*x)*sinh(5*x), x)
```

maple [A] time = 0.09, size = 24, normalized size = 0.83

$$-\frac{\mathrm{X}(3x)}{10}-\frac{\mathrm{X}(7x)}{10}+\frac{\mathrm{X}(2x)\cosh(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(2*x)*sinh(5*x),x)
[Out] -1/10*Chi(3*x)-1/10*Chi(7*x)+1/5*Chi(2*x)*cosh(5*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}(2\,x)\sinh(5\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*sinh(5*x),x, algorithm="maxima")
[Out] integrate(Chi(2*x)*sinh(5*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{coshint}(2\,x)\;\sinh(5\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(2*x)*sinh(5*x),x)
[Out] int(coshint(2*x)*sinh(5*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(5x)\,\mathrm{Chi}(2x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*sinh(5*x),x)
[Out] Integral(sinh(5*x)*Chi(2*x), x)
```

3.122 $\int \cosh(5x)\mathbf{Chi}(2x)\,dx$

Optimal. Leaf size=29

$$\frac{1}{5}\text{Chi}(2x)\sinh(5x)-\frac{\text{Shi}(3x)}{10}-\frac{\text{Shi}(7x)}{10}$$

```
[Out] -1/10*Shi(3*x)-1/10*Shi(7*x)+1/5*Chi(2*x)*sinh(5*x)
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6541, 12, 5472, 3298}

$$\frac{1}{5}\text{Chi}(2x)\sinh(5x)-\frac{\text{Shi}(3x)}{10}-\frac{\text{Shi}(7x)}{10}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[5*x]*CoshIntegral[2*x],x]
```

```
[Out] (CoshIntegral[2*x]*Sinh[5*x])/5 - SinhIntegral[3*x]/10 - SinhIntegral[7*x]/
10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh(5x)\text{Chi}(2x)\,dx &= \frac{1}{5}\text{Chi}(2x)\sinh(5x) - \frac{2}{5}\int \frac{\cosh(2x)\sinh(5x)}{2x}\,dx \\
&= \frac{1}{5}\text{Chi}(2x)\sinh(5x) - \frac{1}{5}\int \frac{\cosh(2x)\sinh(5x)}{x}\,dx \\
&= \frac{1}{5}\text{Chi}(2x)\sinh(5x) - \frac{1}{5}\int \left(\frac{\sinh(3x)}{2x} + \frac{\sinh(7x)}{2x}\right)dx \\
&= \frac{1}{5}\text{Chi}(2x)\sinh(5x) - \frac{1}{10}\int \frac{\sinh(3x)}{x}\,dx - \frac{1}{10}\int \frac{\sinh(7x)}{x}\,dx \\
&= \frac{1}{5}\text{Chi}(2x)\sinh(5x) - \frac{\text{Shi}(3x)}{10} - \frac{\text{Shi}(7x)}{10}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.93

$$\frac{1}{10}(2\text{Chi}(2x)\sinh(5x) - \text{Shi}(3x) - \text{Shi}(7x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[5*x]*CoshIntegral[2*x],x]
```

```
[Out] (2*CoshIntegral[2*x]*Sinh[5*x] - SinhIntegral[3*x] - SinhIntegral[7*x])/10
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*cosh(5*x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> An error occurred when FriCAS evaluated '(op
erator(Chi)((x)*(2)))*(cosh((x)*(5)))':   There are 1 exposed and 1 unexpos
ed library operations named elt having      1 argument(s) but none was dete
rmined to be applicable. Use HyperDoc      Browse, or issue
              )display op elt      to learn more about the available oper
ations. Perhaps       package-calling the operation or using coercions on t
he arguments       will allow you to apply the operation.    Cannot find ap
plication of object of type BasicOperator to argument(s)       of type(s)
                             Polynomial(Integer)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(2\,x\right)\cosh\left(5\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*cosh(5*x),x, algorithm="giac")
[Out] integrate(Chi(2*x)*cosh(5*x), x)
```

maple [A] time = 0.07, size = 24, normalized size = 0.83

$$-\frac{\mathrm{Shi}(3x)}{10}-\frac{\mathrm{Shi}(7x)}{10}+\frac{\mathrm{X}(2x)\sinh(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(2*x)*cosh(5*x),x)
[Out] -1/10*Shi(3*x)-1/10*Shi(7*x)+1/5*Chi(2*x)*sinh(5*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}(2x)\cosh(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*cosh(5*x),x, algorithm="maxima")
[Out] integrate(Chi(2*x)*cosh(5*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{coshint}(2x)\,\cosh(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(2*x)*cosh(5*x),x)
[Out] int(coshint(2*x)*cosh(5*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(5x)\,\mathrm{Chi}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(2*x)*cosh(5*x),x)
[Out] Integral(cosh(5*x)*Chi(2*x), x)
```

3.123 $\int x^2\mathbf{Chi}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=220

$$-\frac{a^2\text{Chi}(2a+2bx)}{2b^3}-\frac{a^2\log(a+bx)}{2b^3}-\frac{\text{Chi}(2a+2bx)}{b^3}+\frac{2\text{Chi}(a+bx)\cosh(a+bx)}{b^3}-\frac{a\text{Shi}(2a+2bx)}{b^3}-\frac{\log(a+bx)}{b^3}$$

```
[Out] 1/2*a*x/b^2-1/4*x^2/b-Chi(2*b*x+2*a)/b^3-1/2*a^2*Chi(2*b*x+2*a)/b^3+2*Chi(b
*x+a)*cosh(b*x+a)/b^3+x^2*Chi(b*x+a)*cosh(b*x+a)/b+1/4*cosh(b*x+a)^2/b^3+1/
2*cosh(2*b*x+2*a)/b^3-ln(b*x+a)/b^3-1/2*a^2*ln(b*x+a)/b^3-a*Shi(2*b*x+2*a)/
b^3-2*x*Chi(b*x+a)*sinh(b*x+a)/b^2+1/2*a*cosh(b*x+a)*sinh(b*x+a)/b^3-1/2*x*
cosh(b*x+a)*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.70, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6549, 6742, 2635, 8, 3310, 30, 3312, 3301, 6543, 5617, 6741, 2638, 3298, 6547}

$$-\frac{a^2\text{Chi}(2a+2bx)}{2b^3}-\frac{a^2\log(a+bx)}{2b^3}-\frac{\text{Chi}(2a+2bx)}{b^3}-\frac{2x\text{Chi}(a+bx)\sinh(a+bx)}{b^2}+\frac{2\text{Chi}(a+bx)\cosh(a+bx)}{b^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (a*x)/(2*b^2) - x^2/(4*b) + Cosh[a + b*x]^2/(4*b^3) + Cosh[2*a + 2*b*x]/(2*
b^3) + (2*Cosh[a + b*x]*CoshIntegral[a + b*x])/b^3 + (x^2*Cosh[a + b*x]*Cos
hIntegral[a + b*x])/b - CoshIntegral[2*a + 2*b*x]/b^3 - (a^2*CoshIntegral[2
*a + 2*b*x])/(2*b^3) - Log[a + b*x]/b^3 - (a^2*Log[a + b*x])/(2*b^3) + (a*C
osh[a + b*x]*Sinh[a + b*x])/(2*b^3) - (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^
2) - (2*x*CoshIntegral[a + b*x]*Sinh[a + b*x])/b^2 - (a*SinhIntegral[2*a +
2*b*x])/b^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\text{Chi}(a+bx)\sinh(a+bx)\,dx = \frac{x^2\cosh(a+bx)\text{Chi}(a+bx)}{b} - \frac{2\int x\cosh(a+bx)\text{Chi}(a+bx)\,dx}{b} - \int \frac{x^2\cos}{a}$$

$$= \frac{x^2\cosh(a+bx)\text{Chi}(a+bx)}{b} - \frac{2x\text{Chi}(a+bx)\sinh(a+bx)}{b^2} + \frac{2\int \text{Chi}(a+bx}{}$$

$$= \frac{2\cosh(a+bx)\text{Chi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{Chi}(a+bx)}{b} - \frac{2x\text{Chi}(a+bx)\,\text{s}}{b^2}$$

$$= \frac{\cosh^2(a+bx)}{4b^3} + \frac{2\cosh(a+bx)\text{Chi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{Chi}(a+bx)}{b}$$

$$= \frac{ax}{2b^2} - \frac{x^2}{4b} + \frac{\cosh^2(a+bx)}{4b^3} + \frac{2\cosh(a+bx)\text{Chi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{C}}{b}$$

$$= \frac{ax}{2b^2} - \frac{x^2}{4b} + \frac{\cosh^2(a+bx)}{4b^3} + \frac{2\cosh(a+bx)\text{Chi}(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)\text{C}}{b}$$

$$= \frac{ax}{2b^2} - \frac{x^2}{4b} + \frac{\cosh^2(a+bx)}{4b^3} + \frac{\cosh(2a+2bx)}{2b^3} + \frac{2\cosh(a+bx)\text{Chi}(a+bx)}{b^3}$$

Mathematica [A] time = 0.38, size = 134, normalized size = 0.61

$$-\frac{4\left(a^2+2\right)\text{Chi}(2(a+bx)) + 4a^2\log(a+bx) - 8\text{Chi}(a+bx)\left(\left(b^2x^2+2\right)\cosh(a+bx) - 2bx\sinh(a+bx)\right) + 8a\text{S}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] -1/8*(-4*a*b*x + 2*b^2*x^2 - 5*Cosh[2*(a + b*x)] + 4*(2 + a^2)*CoshIntegral
[2*(a + b*x)] + 8*Log[a + b*x] + 4*a^2*Log[a + b*x] - 8*CoshIntegral[a + b*
x]*((2 + b^2*x^2)*Cosh[a + b*x] - 2*b*x*Sinh[a + b*x]) - 2*a*Sinh[2*(a + b*
x)] + 2*b*x*Sinh[2*(a + b*x)] + 8*a*SinhIntegral[2*(a + b*x)])/b^3
```

fricas [F] time = 1.98, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{Chi}\left(bx+a\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh_integral(b*x + a)*sinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}\,(bx+a)\sinh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*Chi(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.03, size = 199, normalized size = 0.90

$$\frac{x^2 \mathrm{X}\,(bx+a)\cosh\,(bx+a)}{b}-\frac{2x\mathrm{X}\,(bx+a)\sinh\,(bx+a)}{b^2}+\frac{2\mathrm{X}\,(bx+a)\cosh\,(bx+a)}{b^3}-\frac{x\cosh\,(bx+a)\sinh\,(bx+a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] x^2*Chi(b*x+a)*cosh(b*x+a)/b-2*x*Chi(b*x+a)*sinh(b*x+a)/b^2+2*Chi(b*x+a)*co
sh(b*x+a)/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*a*cosh(b*x+a)*sinh(b*x+
a)/b^3-1/4*x^2/b+1/2*a*x/b^2+3/4/b^3*a^2+5/4*cosh(b*x+a)^2/b^3-ln(b*x+a)/b^
3-Chi(2*b*x+2*a)/b^3-a*Shi(2*b*x+2*a)/b^3-1/2*a^2*ln(b*x+a)/b^3-1/2*a^2*Chi
(2*b*x+2*a)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}\,(bx+a)\sinh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Chi(b*x + a)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{coshint}\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(a + b*x)*sinh(a + b*x),x)
```

```
[Out] int(x^2*coshint(a + b*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh(a+bx)\,\mathrm{Chi}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*Chi(a + b*x), x)
```

3.124 $\int x\mathbf{Chi}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{a\text{Chi}(2a+2bx)}{2b^2}-\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b^2}+\frac{\text{Shi}(2a+2bx)}{2b^2}+\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x\text{Chi}(a$$

```
[Out] -1/2*x/b+1/2*a*Chi(2*b*x+2*a)/b^2+x*Chi(b*x+a)*cosh(b*x+a)/b+1/2*a*ln(b*x+a
)/b^2+1/2*Shi(2*b*x+2*a)/b^2-Chi(b*x+a)*sinh(b*x+a)/b^2-1/2*cosh(b*x+a)*sin
h(b*x+a)/b^2
```

Rubi [A] time = 0.22, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6549, 6742, 2635, 8, 3312, 3301, 6541, 5448, 12, 3298}

$$\frac{a\text{Chi}(2a+2bx)}{2b^2}-\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b^2}+\frac{\text{Shi}(2a+2bx)}{2b^2}+\frac{a\log(a+bx)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x\text{Chi}(a$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] -x/(2*b) + (x*Cosh[a + b*x]*CoshIntegral[a + b*x])/b + (a*CoshIntegral[2*a
+ 2*b*x])/(2*b^2) + (a*Log[a + b*x])/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x]
)/(2*b^2) - (CoshIntegral[a + b*x]*Sinh[a + b*x])/b^2 + SinhIntegral[2*a +
2*b*x]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx = \frac{x\cosh(a+bx)\text{Chi}(a+bx)}{b} - \frac{\int \cosh(a+bx)\text{Chi}(a+bx)\,dx}{b} - \int \frac{x\cosh^2(a}{a+b}$$

$$= \frac{x\cosh(a+bx)\text{Chi}(a+bx)}{b} - \frac{\text{Chi}(a+bx)\sinh(a+bx)}{b^2} + \frac{\int \frac{\cosh(a+bx)\sinh(a+}{a+bx}}{b}$$

$$= \frac{x\cosh(a+bx)\text{Chi}(a+bx)}{b} - \frac{\text{Chi}(a+bx)\sinh(a+bx)}{b^2} - \frac{\int \cosh^2(a+bx)\,d}{b}$$

$$= \frac{x\cosh(a+bx)\text{Chi}(a+bx)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{2b^2} - \frac{\text{Chi}(a+bx)\sinh}{b^2}$$

$$= -\frac{x}{2b} + \frac{x\cosh(a+bx)\text{Chi}(a+bx)}{b} + \frac{a\log(a+bx)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+}{2b^2}$$

$$= -\frac{x}{2b} + \frac{x\cosh(a+bx)\text{Chi}(a+bx)}{b} + \frac{a\text{Chi}(2a+2bx)}{2b^2} + \frac{a\log(a+bx)}{2b^2} - \text{cos}$$

Mathematica [A] time = 0.19, size = 78, normalized size = 0.72

$$\frac{2a\text{Chi}(2(a+bx)) + 4\text{Chi}(a+bx)(bx\cosh(a+bx) - \sinh(a+bx)) + 2\text{Shi}(2(a+bx)) + 2a\log(a+bx) - \sinh(2}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (-2*b*x + 2*a*CoshIntegral[2*(a + b*x)] + 2*a*Log[a + b*x] + 4*CoshIntegral
[a + b*x]*(b*x*Cosh[a + b*x] - Sinh[a + b*x]) - Sinh[2*(a + b*x)] + 2*SinhI
ntegral[2*(a + b*x)])/(4*b^2)
```

fricas [F] time = 1.53, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{Chi}\left(bx+a\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*cosh_integral(b*x + a)*sinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*Chi(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.04, size = 106, normalized size = 0.97

$$\frac{x\mathrm{X}\left(bx+a\right)\cosh\left(bx+a\right)}{b}-\frac{\mathrm{X}\left(bx+a\right)\sinh\left(bx+a\right)}{b^{2}}-\frac{\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2b^{2}}-\frac{x}{2b}-\frac{a}{2b^{2}}+\frac{\mathrm{Shi}\left(2bx+2a\right)}{2b^{2}}+\frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] x*Chi(b*x+a)*cosh(b*x+a)/b-Chi(b*x+a)*sinh(b*x+a)/b^2-1/2*cosh(b*x+a)*sinh(
b*x+a)/b^2-1/2*x/b-1/2/b^2*a+1/2*Shi(2*b*x+2*a)/b^2+1/2*a*ln(b*x+a)/b^2+1/2
*a*Chi(2*b*x+2*a)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(b*x + a)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{coshint}\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(a + b*x)*sinh(a + b*x),x)
```

```
[Out] int(x*coshint(a + b*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(a+bx\right)\mathrm{Chi}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*Chi(a + b*x), x)
```

3.125 $\int \mathbf{Chi}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}$$

```
[Out] -1/2*Chi(2*b*x+2*a)/b+Chi(b*x+a)*cosh(b*x+a)/b-1/2*ln(b*x+a)/b
```

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6547, 3312, 3301}

$$-\frac{\text{Chi}(2a+2bx)}{2b}+\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b}-\frac{\log(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a + b*x]*CoshIntegral[a + b*x])/b - CoshIntegral[2*a + 2*b*x]/(2*b) -
 Log[a + b*x]/(2*b)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{Chi}(a+bx)\sinh(a+bx)\,dx &= \frac{\cosh(a+bx)\mathrm{Chi}(a+bx)}{b} - \int \frac{\cosh^2(a+bx)}{a+bx}\,dx\\ &= \frac{\cosh(a+bx)\mathrm{Chi}(a+bx)}{b} - \int \left(\frac{1}{2(a+bx)} + \frac{\cosh(2a+2bx)}{2(a+bx)}\right)dx\\ &= \frac{\cosh(a+bx)\mathrm{Chi}(a+bx)}{b} - \frac{\log(a+bx)}{2b} - \frac{1}{2}\int \frac{\cosh(2a+2bx)}{a+bx}\,dx\\ &= \frac{\cosh(a+bx)\mathrm{Chi}(a+bx)}{b} - \frac{\mathrm{Chi}(2a+2bx)}{2b} - \frac{\log(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.98

$$-\frac{\mathrm{Chi}(2(a+bx))}{2b} + \frac{\mathrm{Chi}(a+bx)\cosh(a+bx)}{b} - \frac{\log(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a + b*x]*CoshIntegral[a + b*x])/b - CoshIntegral[2*(a + b*x)]/(2*b) -
 Log[a + b*x]/(2*b)
```

fricas [F] time = 4.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{Chi}\left(bx+a\right)\sinh\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)*sinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.93

$$-\frac{\mathrm{X}\left(2bx+2a\right)}{2b} + \frac{\mathrm{X}\left(bx+a\right)\cosh\left(bx+a\right)}{b} - \frac{\ln\left(bx+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)*sinh(b*x+a),x)
[Out] -1/2*Chi(2*b*x+2*a)/b+Chi(b*x+a)*cosh(b*x+a)/b-1/2*ln(b*x+a)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(Chi(b*x + a)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coshint}\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)*sinh(a + b*x),x)
[Out] int(coshint(a + b*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\mathrm{Chi}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*Chi(a + b*x), x)
```

3.126 $\int \frac{\textbf{Chi}(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{Chi}(a+bx)\sinh(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{Chi}(a+bx)\sinh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(CoshIntegral[a + b*x]*Sinh[a + b*x])/x,x]
```

```
[Out] Defer[Int][(CoshIntegral[a + b*x]*Sinh[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\operatorname{Chi}(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\operatorname{Chi}(a+bx)\sinh(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Chi}(a+bx)\sinh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(CoshIntegral[a + b*x]*Sinh[a + b*x])/x,x]
```

```
[Out] Integrate[(CoshIntegral[a + b*x]*Sinh[a + b*x])/x, x]
```

fricas [A] time = 2.00, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{Chi}(bx+a)\sinh(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(b*x + a)*sinh(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx+a\right)\sinh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(Chi(b*x + a)*sinh(b*x + a)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(bx+a\right)\sinh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)*sinh(b*x+a)/x,x)
[Out] int(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(bx+a\right)\sinh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(Chi(b*x + a)*sinh(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{coshint}\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(a + b*x)*sinh(a + b*x))/x,x)
[Out] int((coshint(a + b*x)*sinh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+bx\right)\mathrm{Chi}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*Chi(a + b*x)/x, x)
```

3.127 $\int x^2 \cosh(a+bx)\mathbf{Chi}(a+bx)\,dx$

Optimal. Leaf size=186

$$-\frac{a^2\text{Shi}(2a+2bx)}{2b^3}-\frac{a\text{Chi}(2a+2bx)}{b^3}+\frac{2\text{Chi}(a+bx)\sinh(a+bx)}{b^3}-\frac{\text{Shi}(2a+2bx)}{b^3}-\frac{a\log(a+bx)}{b^3}+\frac{\sinh(2a+2bx)}{8b^3}$$

```
[Out] x/b^2-a*Chi(2*b*x+2*a)/b^3-2*x*Chi(b*x+a)*cosh(b*x+a)/b^2+1/4*a*cosh(2*b*x+
2*a)/b^3-1/4*x*cosh(2*b*x+2*a)/b^2-a*ln(b*x+a)/b^3-Shi(2*b*x+2*a)/b^3-1/2*a
^2*Shi(2*b*x+2*a)/b^3+2*Chi(b*x+a)*sinh(b*x+a)/b^3+x^2*Chi(b*x+a)*sinh(b*x+
a)/b+cosh(b*x+a)*sinh(b*x+a)/b^3+1/8*sinh(2*b*x+2*a)/b^3
```

Rubi [A] time = 0.57, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6543, 5617, 6741, 6742, 2638, 3296, 2637, 3298, 6549, 2635, 8, 3312, 3301, 6541, 5448, 12}

$$-\frac{a^2\text{Shi}(2a+2bx)}{2b^3}-\frac{a\text{Chi}(2a+2bx)}{b^3}+\frac{2\text{Chi}(a+bx)\sinh(a+bx)}{b^3}-\frac{2x\text{Chi}(a+bx)\cosh(a+bx)}{b^2}-\frac{\text{Shi}(2a+2bx)}{b^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

```
[Out] x/b^2 + (a*Cosh[2*a + 2*b*x])/(4*b^3) - (x*Cosh[2*a + 2*b*x])/(4*b^2) - (2*
x*Cosh[a + b*x]*CoshIntegral[a + b*x])/b^2 - (a*CoshIntegral[2*a + 2*b*x])/
b^3 - (a*Log[a + b*x])/b^3 + (Cosh[a + b*x]*Sinh[a + b*x])/b^3 + (2*CoshInt
egral[a + b*x]*Sinh[a + b*x])/b^3 + (x^2*CoshIntegral[a + b*x]*Sinh[a + b*x
])/b + Sinh[2*a + 2*b*x]/(8*b^3) - SinhIntegral[2*a + 2*b*x]/b^3 - (a^2*Sin
hIntegral[2*a + 2*b*x])/(2*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \cosh(a+bx)\text{Chi}(a+bx)\,dx = \frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b} - \frac{2\int x\text{Chi}(a+bx)\sinh(a+bx)\,dx}{b} - \int \frac{x^2 \cos\ldots}{\ldots}$$

$$= -\frac{2x\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{x^2\text{Chi}(a+bx)\sinh(a+bx)}{b} - \frac{1}{2}\int \frac{x^2\sinh\ldots}{a\ldots}$$

$$= -\frac{2x\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{2\text{Chi}(a+bx)\sinh(a+bx)}{b^3} + \frac{x^2\text{Chi}(a+bx)\ldots}{b\ldots}$$

$$= -\frac{2x\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{2\text{Chi}(a+bx)\sinh(a+bx)}{b^3} + \frac{x^2\text{Chi}(a+bx)\ldots}{b\ldots}$$

$$= -\frac{2x\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{\cosh(a+bx)\sinh(a+bx)}{b^3} + \frac{2\text{Chi}(a+bx)\ldots}{b^3\ldots}$$

$$= \frac{x}{b^2} + \frac{a\cosh(2a+2bx)}{4b^3} - \frac{x\cosh(2a+2bx)}{4b^2} - \frac{2x\cosh(a+bx)\text{Chi}(a+bx)}{b^2} - \ldots$$

$$= \frac{x}{b^2} + \frac{a\cosh(2a+2bx)}{4b^3} - \frac{x\cosh(2a+2bx)}{4b^2} - \frac{2x\cosh(a+bx)\text{Chi}(a+bx)}{b^2} - \ldots$$

Mathematica [A] time = 0.28, size = 123, normalized size = 0.66

$$\frac{-4a^2\text{Shi}(2(a+bx)) + 8\text{Chi}(a+bx)\left(\left(b^2x^2+2\right)\sinh(a+bx) - 2bx\cosh(a+bx)\right) - 8a\text{Chi}(2(a+bx)) - 8\text{Shi}(2(a\ldots}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

```
[Out] (8*b*x + 2*a*Cosh[2*(a + b*x)] - 2*b*x*Cosh[2*(a + b*x)] - 8*a*CoshIntegral
[2*(a + b*x)] - 8*a*Log[a + b*x] + 8*CoshIntegral[a + b*x]*(-2*b*x*Cosh[a +
 b*x] + (2 + b^2*x^2)*Sinh[a + b*x]) + 5*Sinh[2*(a + b*x)] - 8*SinhIntegral
[2*(a + b*x)] - 4*a^2*SinhIntegral[2*(a + b*x)])/(8*b^3)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\cosh\left(bx+a\right)\text{Chi}\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*cosh(b*x + a)*cosh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}\,(bx+a)\cosh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*Chi(b*x + a)*cosh(b*x + a), x)
```

maple [A] time = 0.04, size = 171, normalized size = 0.92

$$\frac{x^2 \mathrm{X}\,(bx+a)\sinh\,(bx+a)}{b}-\frac{2x\mathrm{X}\,(bx+a)\cosh\,(bx+a)}{b^2}+\frac{2\mathrm{X}\,(bx+a)\sinh\,(bx+a)}{b^3}-\frac{\left(\cosh^2\,(bx+a)\right)x}{2b^2}+\frac{a\left(\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*Chi(b*x+a)*cosh(b*x+a),x)
```

```
[Out] x^2*Chi(b*x+a)*sinh(b*x+a)/b-2*x*Chi(b*x+a)*cosh(b*x+a)/b^2+2*Chi(b*x+a)*si
nh(b*x+a)/b^3-1/2/b^2*cosh(b*x+a)^2*x+1/2/b^3*a*cosh(b*x+a)^2+5/4*cosh(b*x+
a)*sinh(b*x+a)/b^3+5/4*x/b^2+5/4/b^3*a-Shi(2*b*x+2*a)/b^3-a*ln(b*x+a)/b^3-a
*Chi(2*b*x+2*a)/b^3-1/2*a^2*Shi(2*b*x+2*a)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{Chi}\,(bx+a)\cosh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*Chi(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*Chi(b*x + a)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{coshint}\left(a+b\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coshint(a + b*x)*cosh(a + b*x),x)
```

```
[Out] int(x^2*coshint(a + b*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh\,(a+bx)\,\mathrm{Chi}\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*Chi(b*x+a)*cosh(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x)*Chi(a + b*x), x)
```

3.128 $\int x \cosh(a+bx)\textbf{Chi}(a+bx)\,dx$

Optimal. Leaf size=97

$$\frac{\text{Chi}(2a+2bx)}{2b^2}-\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b^2}+\frac{a\text{Shi}(2a+2bx)}{2b^2}+\frac{\log(a+bx)}{2b^2}-\frac{\cosh(2a+2bx)}{4b^2}+\frac{x\text{Chi}(a+bx)\sinh}{b}$$

```
[Out] 1/2*Chi(2*b*x+2*a)/b^2-Chi(b*x+a)*cosh(b*x+a)/b^2-1/4*cosh(2*b*x+2*a)/b^2+1
/2*ln(b*x+a)/b^2+1/2*a*Shi(2*b*x+2*a)/b^2+x*Chi(b*x+a)*sinh(b*x+a)/b
```

Rubi [A] time = 0.25, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6543, 5617, 6741, 6742, 2638, 3298, 6547, 3312, 3301}

$$\frac{\text{Chi}(2a+2bx)}{2b^2}-\frac{\text{Chi}(a+bx)\cosh(a+bx)}{b^2}+\frac{a\text{Shi}(2a+2bx)}{2b^2}+\frac{\log(a+bx)}{2b^2}-\frac{\cosh(2a+2bx)}{4b^2}+\frac{x\text{Chi}(a+bx)\sinh}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

```
[Out] -Cosh[2*a + 2*b*x]/(4*b^2) - (Cosh[a + b*x]*CoshIntegral[a + b*x])/b^2 + Co
shIntegral[2*a + 2*b*x]/(2*b^2) + Log[a + b*x]/(2*b^2) + (x*CoshIntegral[a
+ b*x]*Sinh[a + b*x])/b + (a*SinhIntegral[2*a + 2*b*x])/(2*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\cosh(a+bx)\text{Chi}(a+bx)\,dx = \frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b} - \frac{\int \text{Chi}(a+bx)\sinh(a+bx)\,dx}{b} - \int \frac{x\cosh(a\ -$$

$$= -\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b} - \frac{1}{2}\int \frac{x\sinh(2(a\ -}{a+bx}$$

$$= -\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b} - \frac{1}{2}\int \frac{x\sinh(2a\ +}{a+bx}$$

$$= -\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{\log(a+bx)}{2b^2} + \frac{x\text{Chi}(a+bx)\sinh(a+bx)}{b} - \frac{1}{2}$$

$$= -\frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{\text{Chi}(2a+2bx)}{2b^2} + \frac{\log(a+bx)}{2b^2} + \frac{x\text{Chi}(a+bx)}{b}$$

$$= -\frac{\cosh(2a+2bx)}{4b^2} - \frac{\cosh(a+bx)\text{Chi}(a+bx)}{b^2} + \frac{\text{Chi}(2a+2bx)}{2b^2} + \frac{\log(a+b}{2b^2}$$

Mathematica [A] time = 0.16, size = 73, normalized size = 0.75

$$\frac{2\text{Chi}(2(a+bx)) + 4\text{Chi}(a+bx)(bx\sinh(a+bx) - \cosh(a+bx)) + 2a\text{Shi}(2(a+bx)) + 2\log(a+bx) - \cosh(2(}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*CoshIntegral[a + b*x],x]
```

```
[Out] (-Cosh[2*(a + b*x)] + 2*CoshIntegral[2*(a + b*x)] + 2*Log[a + b*x] + 4*Cosh
Integral[a + b*x]*(-Cosh[a + b*x] + b*x*Sinh[a + b*x]) + 2*a*SinhIntegral[2
*(a + b*x)])/(4*b^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cosh\left(bx+a\right)\text{Chi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*cosh(b*x + a)*cosh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(bx+a\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*Chi(b*x + a)*cosh(b*x + a), x)
```

maple [A] time = 0.04, size = 89, normalized size = 0.92

$$\frac{x\mathrm{X}\left(bx+a\right)\sinh\left(bx+a\right)}{b}-\frac{\mathrm{X}\left(bx+a\right)\cosh\left(bx+a\right)}{b^{2}}-\frac{\cosh^{2}\left(bx+a\right)}{2b^{2}}+\frac{\ln\left(bx+a\right)}{2b^{2}}+\frac{\mathrm{X}\left(2bx+2a\right)}{2b^{2}}+\frac{a\,\mathrm{Shi}\left(2bx+2a\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(b*x+a)*cosh(b*x+a),x)
```

```
[Out] x*Chi(b*x+a)*sinh(b*x+a)/b-Chi(b*x+a)*cosh(b*x+a)/b^2-1/2/b^2*cosh(b*x+a)^2
+1/2*ln(b*x+a)/b^2+1/2*Chi(2*b*x+2*a)/b^2+1/2*a*Shi(2*b*x+2*a)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\left(bx+a\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(b*x + a)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{coshint}\left(a+b\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(a + b*x)*cosh(a + b*x),x)
```

```
[Out] int(x*coshint(a + b*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+bx\right)\mathrm{Chi}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(b*x+a)*cosh(b*x+a),x)
```

```
[Out] Integral(x*cosh(a + b*x)*Chi(a + b*x), x)
```

3.129 $\int \cosh(a+bx)\mathbf{Chi}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\mathrm{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\mathrm{Shi}(2a+2bx)}{2b}$$

[Out] -1/2*Shi(2*b*x+2*a)/b+Chi(b*x+a)*sinh(b*x+a)/b

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6541, 5448, 12, 3298}

$$\frac{\mathrm{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\mathrm{Shi}(2a+2bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]*CoshIntegral[a + b*x],x]

[Out] (CoshIntegral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*a + 2*b*x]/(2*b)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\text{Chi}(a+bx)\,dx &= \frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\cosh(a+bx)\sinh(a+bx)}{a+bx}\,dx\\ &= \frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\int \frac{\sinh(2a+2bx)}{2(a+bx)}\,dx\\ &= \frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{1}{2}\int \frac{\sinh(2a+2bx)}{a+bx}\,dx\\ &= \frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2a+2bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.97

$$\frac{\text{Chi}(a+bx)\sinh(a+bx)}{b}-\frac{\text{Shi}(2(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*CoshIntegral[a + b*x],x]
[Out] (CoshIntegral[a + b*x]*Sinh[a + b*x])/b - SinhIntegral[2*(a + b*x)]/(2*b)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\text{Chi}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*cosh_integral(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(bx+a\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a),x, algorithm="giac")
[Out] integrate(Chi(b*x + a)*cosh(b*x + a), x)
```

maple [A] time = 0.02, size = 30, normalized size = 0.91

$$\frac{\mathrm{X}\left(bx+a\right)\sinh\left(bx+a\right)-\frac{\text{Shi}(2bx+2a)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)*cosh(b*x+a),x)
```

```
[Out] 1/b*(Chi(b*x+a)*sinh(b*x+a)-1/2*Shi(2*b*x+2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Chi}\,(bx+a)\cosh\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{coshint}\,(a+b\,x)\;\cosh\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(a + b*x)*cosh(a + b*x),x)
```

```
[Out] int(coshint(a + b*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\,(a+bx)\,\mathrm{Chi}\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)*Chi(a + b*x), x)
```

3.130 $\int \frac{\cosh(a+bx)\mathbf{Chi}(a+bx)}{x} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\text{Chi}(a+bx)\cosh(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(Chi(b*x+a)*cosh(b*x+a)/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*CoshIntegral[a + b*x])/x,x]
[Out] Defer[Int][(Cosh[a + b*x]*CoshIntegral[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x} dx = \int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x} dx$$

Mathematica [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\text{Chi}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*CoshIntegral[a + b*x])/x,x]
[Out] Integrate[(Cosh[a + b*x]*CoshIntegral[a + b*x])/x, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)\,\text{Chi}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*cosh_integral(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}(bx+a)\cosh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(Chi(b*x + a)*cosh(b*x + a)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}(bx+a)\cosh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(b*x+a)*cosh(b*x+a)/x,x)
```

```
[Out] int(Chi(b*x+a)*cosh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}(bx+a)\cosh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(Chi(b*x + a)*cosh(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{coshint}(a+b\,x)\,\cosh(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(a + b*x)*cosh(a + b*x))/x,x)
```

```
[Out] int((coshint(a + b*x)*cosh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\,\mathrm{Chi}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(b*x+a)*cosh(b*x+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x)*Chi(a + b*x)/x, x)
```

3.131 $\int x\mathbf{Chi}(c + dx)\sinh(a + bx)\,dx$

Optimal. Leaf size=371

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}-\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b^2}+\frac{\cosh\left(a\right.}{}$$

```
[Out] 1/2*c*Chi(c*(b-d)/d+(b-d)*x)*cosh(a-b*c/d)/b/d+1/2*c*Chi(c*(b+d)/d+(b+d)*x)
*cosh(a-b*c/d)/b/d+x*Chi(d*x+c)*cosh(b*x+a)/b+1/2*cosh(a-b*c/d)*Shi(c*(b-d)
/d+(b-d)*x)/b^2+1/2*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b^2+1/2*Chi(c*(b-d
)/d+(b-d)*x)*sinh(a-b*c/d)/b^2+1/2*Chi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b^2
+1/2*c*Shi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b/d+1/2*c*Shi(c*(b+d)/d+(b+d)*x
)*sinh(a-b*c/d)/b/d-Chi(d*x+c)*sinh(b*x+a)/b^2-1/2*sinh(a-c+(b-d)*x)/b/(b-d
)-1/2*sinh(a+c+(b+d)*x)/b/(b+d)
```

Rubi [A] time = 0.99, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6549, 5643, 6742, 2637, 3303, 3298, 3301, 6541, 5472}

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}-\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b^2}+\frac{\cosh\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*CoshIntegral[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (c*Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))/d + (b - d)*x])/(2*b*d) + (x*
Cosh[a + b*x]*CoshIntegral[c + d*x])/b + (c*Cosh[a - (b*c)/d]*CoshIntegral[
(c*(b + d))/d + (b + d)*x])/(2*b*d) + (CoshIntegral[(c*(b - d))/d + (b - d)
*x]*Sinh[a - (b*c)/d])/(2*b^2) + (CoshIntegral[(c*(b + d))/d + (b + d)*x]*S
inh[a - (b*c)/d])/(2*b^2) - (CoshIntegral[c + d*x]*Sinh[a + b*x])/b^2 - Sin
h[a - c + (b - d)*x]/(2*b*(b - d)) - Sinh[a + c + (b + d)*x]/(2*b*(b + d))
+ (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b^2) + (c*
Sinh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b*d) + (Cosh[
a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b^2) + (c*Sinh[a -
 (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b*d)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 5643

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(m_.)*Cosh[(c_.) + (d_.)*(x_)]^(n_.)*(u_.), x_
Symbol] :> Int[ExpandTrigReduce[u, Cosh[a + b*x]^m*Cosh[c + d*x]^n, x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6549

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.)
+ (b_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*Cosh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Cosh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Cosh[a + b*x]*CoshIn
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\mathrm{Chi}(c+dx)\sinh(a+bx)\,dx = \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\int \cosh(a+bx)\mathrm{Chi}(c+dx)\,dx}{b} - \frac{d\int \frac{x\cosh(a+bx}{c+}}{b}$$

$$= \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{b^2} + \frac{d\int \frac{\cosh(c+dx)\sinh(a}{c+dx}}{b^2}$$

$$= \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{b^2} + \frac{d\int \left(\frac{\sinh(a-c+(b-d)x}{2(c+dx)}\right.}{}$$

$$= \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{b^2} + \frac{d\int \frac{\sinh(a-c+(b-d)x)}{c+dx}}{2b^2}$$

$$= \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{b^2} - \frac{\int \cosh(a-c+(b}{2b}$$

$$= \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} + \frac{\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b^2} + \frac{\mathrm{Chi}\left(\frac{c(}{}\right.}{}$$

$$= \frac{c\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2bd} + \frac{x\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} + \frac{c\cos}{}$$

Mathematica [B] time = 8.72, size = 916, normalized size = 2.47

$$\frac{2c\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{(b+d)(c+dx)}{d}\right)b^3 + c\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3 + c\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3 + 2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*CoshIntegral[c + d*x]*Sinh[a + b*x],x]

[Out] (2*b^3*c*Cosh[a - (b*c)/d]*CoshIntegral[((b + d)*(c + d*x))/d] - 2*b*c*d^2*
Cosh[a - (b*c)/d]*CoshIntegral[((b + d)*(c + d*x))/d] + 2*b^2*d*CoshIntegra
l[((b + d)*(c + d*x))/d]*Sinh[a - (b*c)/d] - 2*d^3*CoshIntegral[((b + d)*(c
 + d*x))/d]*Sinh[a - (b*c)/d] + 2*(b^2 - d^2)*CoshIntegral[-(((b - d)*(c +
```

```
d*x))/d)]*(b*c*Cosh[a - (b*c)/d] + d*Sinh[a - (b*c)/d]) + 4*d*(b^2 - d^2)*C
oshIntegral[c + d*x]*(b*x*Cosh[a + b*x] - Sinh[a + b*x]) - 2*b^2*d*Sinh[a -
 c + b*x - d*x] - 2*b*d^2*Sinh[a - c + b*x - d*x] - 2*b^2*d*Sinh[a + c + (b
 + d)*x] + 2*b*d^2*Sinh[a + c + (b + d)*x] + b^3*c*Cosh[a - (b*c)/d]*SinhIn
tegral[((b - d)*(c + d*x))/d] + b^2*d*Cosh[a - (b*c)/d]*SinhIntegral[((b -
d)*(c + d*x))/d] - b*c*d^2*Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x
))/d] - d^3*Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b^3*c*S
inh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b^2*d*Sinh[a - (b*c)
/d]*SinhIntegral[((b - d)*(c + d*x))/d] - b*c*d^2*Sinh[a - (b*c)/d]*SinhInt
egral[((b - d)*(c + d*x))/d] - d^3*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*
(c + d*x))/d] + 2*b^2*d*Cosh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/
d] - 2*d^3*Cosh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] + 2*b^3*c*
Sinh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] - 2*b*c*d^2*Sinh[a -
(b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] + b^3*c*Cosh[a - (b*c)/d]*Sinh
Integral[c - (b*c)/d - b*x + d*x] - b^2*d*Cosh[a - (b*c)/d]*SinhIntegral[c
- (b*c)/d - b*x + d*x] - b*c*d^2*Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d
 - b*x + d*x] + d^3*Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x]
 - b^3*c*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] + b^2*d*Si
nh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] + b*c*d^2*Sinh[a - (b
*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] - d^3*Sinh[a - (b*c)/d]*SinhIn
tegral[c - (b*c)/d - b*x + d*x])/(4*b^2*(b - d)*d*(b + d))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x\,\mathrm{Chi}\left(dx+c\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*cosh_integral(d*x + c)*sinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*Chi(d*x + c)*sinh(b*x + a), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int x\mathrm{X}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(d*x+c)*sinh(b*x+a),x)
```

```
[Out] int(x*Chi(d*x+c)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*Chi(d*x + c)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{coshint}\left(c+d\,x\right)\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(c + d*x)*sinh(a + b*x),x)
```

```
[Out] int(x*coshint(c + d*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(a+bx\right)\text{Chi}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*Chi(c + d*x), x)
```

3.132 $\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=153

$$-\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}+\frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b}-\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}-\frac{\sinh\left(a\right.}{}$$

```
[Out] -1/2*Chi(c*(b-d)/d+(b-d)*x)*cosh(a-b*c/d)/b-1/2*Chi(c*(b+d)/d+(b+d)*x)*cosh
(a-b*c/d)/b+Chi(d*x+c)*cosh(b*x+a)/b-1/2*Shi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/
d)/b-1/2*Shi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b
```

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6547, 5471, 3303, 3298, 3301}

$$-\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}+\frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b}-\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}-\frac{\sinh\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[CoshIntegral[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] -(Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))/d + (b - d)*x])/(2*b) + (Cosh[
a + b*x]*CoshIntegral[c + d*x])/b - (Cosh[a - (b*c)/d]*CoshIntegral[(c*(b +
 d))/d + (b + d)*x])/(2*b) - (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d
+ (b - d)*x])/(2*b) - (Sinh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b +
d)*x])/(2*b)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5471

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) +
(f_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cosh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \mathrm{Chi}(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{d\int \frac{\cosh(a+bx)\cosh(c+dx)}{c+dx}\,dx}{b}$$

$$= \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{d\int \left(\frac{\cosh(a-c+(b-d)x)}{2(c+dx)} + \frac{\cosh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b}$$

$$= \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{d\int \frac{\cosh(a-c+(b-d)x)}{c+dx}\,dx}{2b} - \frac{d\int \frac{\cosh(a+c+(b+d)x)}{c+dx}\,dx}{2b}$$

$$= \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\left(d\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{c(b-d)}{d}+(b-d)x\right)}{c+dx}\,dx}{2b} - \frac{\left(d\cosh\right.}{}$$

$$= -\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)}{2b} + \frac{\cosh(a+bx)\mathrm{Chi}(c+dx)}{b} - \frac{\cosh\left(a\right.}{}$$

Mathematica [A] time = 0.59, size = 209, normalized size = 1.37

$$-\frac{-4\cosh(a+bx)\mathrm{Chi}(c+dx) + 2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(-\frac{(b-d)(c+dx)}{d}\right) + 2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{(b+d)(c+dx)}{d}\right) + \sinh\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[CoshIntegral[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] -1/4*(-4*Cosh[a + b*x]*CoshIntegral[c + d*x] + 2*Cosh[a - (b*c)/d]*CoshInte
gral[-(((b - d)*(c + d*x))/d)] + 2*Cosh[a - (b*c)/d]*CoshIntegral[((b + d)*
(c + d*x))/d] + Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + Sin
h[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + 2*Sinh[a - (b*c)/d]*Si
nhIntegral[((b + d)*(c + d*x))/d] + Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c
)/d - b*x + d*x] - Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x])
/b
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{Chi}\left(dx+c\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh_integral(d*x + c)*sinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(Chi(d*x + c)*sinh(b*x + a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \mathrm{X}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*x+c)*sinh(b*x+a),x)
```

```
[Out] int(Chi(d*x+c)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Chi(d*x + c)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{coshint}(c+d\,x)\,\sinh(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(c + d*x)*sinh(a + b*x),x)
```

```
[Out] int(coshint(c + d*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\,\mathrm{Chi}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*Chi(c + d*x), x)
```

3.133 $\int \frac{\mathbf{Chi}(c+dx)\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\sinh(a+bx)\mathrm{Chi}(c+dx)}{x},x\right)$$

[Out] CannotIntegrate(Chi(d*x+c)*sinh(b*x+a)/x,x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[(CoshIntegral[c + d*x]*Sinh[a + b*x])/x,x]

[Out] Defer[Int][(CoshIntegral[c + d*x]*Sinh[a + b*x])/x, x]

Rubi steps

$$\int \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{x}\,dx=\int \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{x}\,dx$$

Mathematica [A] time = 9.46, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}(c+dx)\sinh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(CoshIntegral[c + d*x]*Sinh[a + b*x])/x,x]

[Out] Integrate[(CoshIntegral[c + d*x]*Sinh[a + b*x])/x, x]

fricas [A] time = 1.36, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{Chi}\,(dx+c)\sinh\,(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(Chi(d*x+c)*sinh(b*x+a)/x,x, algorithm="fricas")

```
[Out] integral(cosh_integral(d*x + c)*sinh(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(dx+c\right)\sinh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(Chi(d*x + c)*sinh(b*x + a)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(dx+c\right)\sinh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*x+c)*sinh(b*x+a)/x,x)
[Out] int(Chi(d*x+c)*sinh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(dx+c\right)\sinh\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(Chi(d*x + c)*sinh(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{coshint}\left(c+d\,x\right)\,\sinh\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(c + d*x)*sinh(a + b*x))/x,x)
[Out] int((coshint(c + d*x)*sinh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+bx\right)\mathrm{Chi}\left(c+dx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*Chi(c + d*x)/x, x)
```

3.134 $\int x\cosh(a+bx)\mathbf{Chi}(c+dx)\,dx$

Optimal. Leaf size=371

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}-\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b^2}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}+\frac{\sinh\left(a\right.}{}$$

```
[Out] 1/2*Chi(c*(b-d)/d+(b-d)*x)*cosh(a-b*c/d)/b^2+1/2*Chi(c*(b+d)/d+(b+d)*x)*cos
h(a-b*c/d)/b^2-Chi(d*x+c)*cosh(b*x+a)/b^2-1/2*cosh(a-c+(b-d)*x)/b/(b-d)-1/2
*cosh(a+c+(b+d)*x)/b/(b+d)+1/2*c*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b/d+1
/2*c*cosh(a-b*c/d)*Shi(c*(b+d)/d+(b+d)*x)/b/d+1/2*c*Chi(c*(b-d)/d+(b-d)*x)*
sinh(a-b*c/d)/b/d+1/2*c*Chi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)/b/d+1/2*Shi(c*
(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b^2+1/2*Shi(c*(b+d)/d+(b+d)*x)*sinh(a-b*c/d)
/b^2+x*Chi(d*x+c)*sinh(b*x+a)/b
```

Rubi [A] time = 0.86, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6543, 6742, 5618, 2638, 5472, 3303, 3298, 3301, 6547, 5471}

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b^2}-\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b^2}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b^2}+\frac{\sinh\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

```
[Out] -Cosh[a - c + (b - d)*x]/(2*b*(b - d)) - Cosh[a + c + (b + d)*x]/(2*b*(b +
d)) + (Cosh[a - (b*c)/d]*CoshIntegral[(c*(b - d))/d + (b - d)*x])/(2*b^2) -
 (Cosh[a + b*x]*CoshIntegral[c + d*x])/b^2 + (Cosh[a - (b*c)/d]*CoshIntegra
l[(c*(b + d))/d + (b + d)*x])/(2*b^2) + (c*CoshIntegral[(c*(b - d))/d + (b
- d)*x]*Sinh[a - (b*c)/d])/(2*b*d) + (c*CoshIntegral[(c*(b + d))/d + (b + d
)*x]*Sinh[a - (b*c)/d])/(2*b*d) + (x*CoshIntegral[c + d*x]*Sinh[a + b*x])/b
 + (c*Cosh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b*d) +
(Sinh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d + (b - d)*x])/(2*b^2) + (c*Co
sh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b*d) + (Sinh[a
- (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b + d)*x])/(2*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5471

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) +
(f_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cosh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 6543

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)]*((e_.) + (f_.
)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^m*Sinh[a + b*x]*CoshIntegral[c
+ d*x])/b, x] + (-Dist[d/b, Int[((e + f*x)^m*Sinh[a + b*x]*Cosh[c + d*x])/(
c + d*x), x], x] - Dist[(f*m)/b, Int[(e + f*x)^(m - 1)*Sinh[a + b*x]*CoshIn
```

```
tegral[c + d*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 6547

```
Int[CoshIntegral[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :>
 Simp[(Cosh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Cosh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\cosh(a+bx)\text{Chi}(c+dx)\,dx = \frac{x\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{\int \text{Chi}(c+dx)\sinh(a+bx)\,dx}{b} - \frac{d\int \frac{x\cosh(c+dx}{c+}}{b}$$

$$= -\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b^2} + \frac{x\text{Chi}(c+dx)\sinh(a+bx)}{b} + \frac{d\int \frac{\cosh(a+bx)\cosh}{c+dx}}{b^2}$$

$$= -\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b^2} + \frac{x\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{\int \cosh(c+dx)\,\text{s}}{b}$$

$$= -\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b^2} + \frac{x\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{\int \left(\frac{1}{2}\sinh(a-c\right.}{}$$

$$= -\frac{\cosh(a+bx)\text{Chi}(c+dx)}{b^2} + \frac{x\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{\int \sinh(a-c+(}{2b}$$

$$= -\frac{\cosh(a-c+(b-d)x)}{2b(b-d)} - \frac{\cosh(a+c+(b+d)x)}{2b(b+d)} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-}{d}\right.}{2b^2}$$

$$= -\frac{\cosh(a-c+(b-d)x)}{2b(b-d)} - \frac{\cosh(a+c+(b+d)x)}{2b(b+d)} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{c(b-}{d}\right.}{2b^2}$$

Mathematica [B] time = 2.25, size = 916, normalized size = 2.47

$$\frac{2c\text{Chi}\left(\frac{(b+d)(c+dx)}{d}\right)\sinh\left(a-\frac{bc}{d}\right)b^3 + c\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3 + c\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{(b-d)(c+dx)}{d}\right)b^3 + 2c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

```
[Out] (-2*b^2*d*Cosh[a - c + b*x - d*x] - 2*b*d^2*Cosh[a - c + b*x - d*x] - 2*b^2
*d*Cosh[a + c + (b + d)*x] + 2*b*d^2*Cosh[a + c + (b + d)*x] + 2*b^2*d*Cosh
[a - (b*c)/d]*CoshIntegral[((b + d)*(c + d*x))/d] - 2*d^3*Cosh[a - (b*c)/d]
*CoshIntegral[((b + d)*(c + d*x))/d] + 2*b^3*c*CoshIntegral[((b + d)*(c + d
*x))/d]*Sinh[a - (b*c)/d] - 2*b*c*d^2*CoshIntegral[((b + d)*(c + d*x))/d]*S
inh[a - (b*c)/d] + 2*(b^2 - d^2)*CoshIntegral[-(((b - d)*(c + d*x))/d)]*(d*
Cosh[a - (b*c)/d] + b*c*Sinh[a - (b*c)/d]) + 4*d*(b^2 - d^2)*CoshIntegral[c
 + d*x]*(-Cosh[a + b*x] + b*x*Sinh[a + b*x]) + b^3*c*Cosh[a - (b*c)/d]*Sinh
Integral[((b - d)*(c + d*x))/d] + b^2*d*Cosh[a - (b*c)/d]*SinhIntegral[((b
- d)*(c + d*x))/d] - b*c*d^2*Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d
*x))/d] - d^3*Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b^3*c
*Sinh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + b^2*d*Sinh[a - (b*
c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] - b*c*d^2*Sinh[a - (b*c)/d]*SinhI
ntegral[((b - d)*(c + d*x))/d] - d^3*Sinh[a - (b*c)/d]*SinhIntegral[((b - d
)*(c + d*x))/d] + 2*b^3*c*Cosh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x)
)/d] - 2*b*c*d^2*Cosh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] + 2*
b^2*d*Sinh[a - (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] - 2*d^3*Sinh[a
- (b*c)/d]*SinhIntegral[((b + d)*(c + d*x))/d] - b^3*c*Cosh[a - (b*c)/d]*Si
nhIntegral[c - (b*c)/d - b*x + d*x] + b^2*d*Cosh[a - (b*c)/d]*SinhIntegral[
c - (b*c)/d - b*x + d*x] + b*c*d^2*Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)
/d - b*x + d*x] - d^3*Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*
x] + b^3*c*Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] - b^2*d*
Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] - b*c*d^2*Sinh[a -
(b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x] + d^3*Sinh[a - (b*c)/d]*Sinh
Integral[c - (b*c)/d - b*x + d*x])/(4*b^2*(b - d)*d*(b + d))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cosh\left(bx+a\right)\text{Chi}\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*cosh(b*x + a)*cosh_integral(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{Chi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*cosh(b*x+a),x, algorithm="giac")
[Out] integrate(x*Chi(d*x + c)*cosh(b*x + a), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int x\mathrm{X}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*Chi(d*x+c)*cosh(b*x+a),x)
[Out] int(x*Chi(d*x+c)*cosh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{Chi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*cosh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*Chi(d*x + c)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{coshint}\left(c+d\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coshint(c + d*x)*cosh(a + b*x),x)
[Out] int(x*coshint(c + d*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+bx\right)\mathrm{Chi}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*Chi(d*x+c)*cosh(b*x+a),x)
[Out] Integral(x*cosh(a + b*x)*Chi(c + d*x), x)
```

3.135 $\int \cosh(a+bx)\mathbf{Chi}(c+dx)\,dx$

Optimal. Leaf size=153

$$-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}+\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b}-\frac{\cosh\left(a\right.}{}$$

```
[Out] -1/2*cosh(a-b*c/d)*Shi(c*(b-d)/d+(b-d)*x)/b-1/2*cosh(a-b*c/d)*Shi(c*(b+d)/d
+(b+d)*x)/b-1/2*Chi(c*(b-d)/d+(b-d)*x)*sinh(a-b*c/d)/b-1/2*Chi(c*(b+d)/d+(b
+d)*x)*sinh(a-b*c/d)/b+Chi(d*x+c)*sinh(b*x+a)/b
```

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6541, 5472, 3303, 3298, 3301}

$$-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b-d)+\frac{c(b-d)}{d}\right)}{2b}-\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(x(b+d)+\frac{c(b+d)}{d}\right)}{2b}+\frac{\sinh(a+bx)\text{Chi}(c+dx)}{b}-\frac{\cosh\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

```
[Out] -(CoshIntegral[(c*(b - d))/d + (b - d)*x]*Sinh[a - (b*c)/d])/(2*b) - (CoshI
ntegral[(c*(b + d))/d + (b + d)*x]*Sinh[a - (b*c)/d])/(2*b) + (CoshIntegral
[c + d*x]*Sinh[a + b*x])/b - (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b - d))/d
+ (b - d)*x])/(2*b) - (Cosh[a - (b*c)/d]*SinhIntegral[(c*(b + d))/d + (b +
d)*x])/(2*b)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5472

```
Int[Cosh[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Cosh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p
, 0] && IGtQ[q, 0]
```

Rule 6541

```
Int[Cosh[(a_.) + (b_.)*(x_)]*CoshIntegral[(c_.) + (d_.)*(x_)], x_Symbol] :>
 Simp[(Sinh[a + b*x]*CoshIntegral[c + d*x])/b, x] - Dist[d/b, Int[(Sinh[a +
 b*x]*Cosh[c + d*x])/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh(a+bx)\text{Chi}(c+dx)\,dx &= \frac{\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{d\int \frac{\cosh(c+dx)\sinh(a+bx)}{c+dx}\,dx}{b} \\
&= \frac{\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{d\int \left(\frac{\sinh(a-c+(b-d)x)}{2(c+dx)} + \frac{\sinh(a+c+(b+d)x)}{2(c+dx)}\right)dx}{b} \\
&= \frac{\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{d\int \frac{\sinh(a-c+(b-d)x)}{c+dx}\,dx}{2b} - \frac{d\int \frac{\sinh(a+c+(b+d)x)}{c+dx}\,dx}{2b} \\
&= \frac{\text{Chi}(c+dx)\sinh(a+bx)}{b} - \frac{\left(d\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{c(b-d)}{d}+(b-d)x\right)}{c+dx}\,dx}{2b} - \frac{\left(d\cosh\right.}{} \\
&= -\frac{\text{Chi}\left(\frac{c(b-d)}{d}+(b-d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b} - \frac{\text{Chi}\left(\frac{c(b+d)}{d}+(b+d)x\right)\sinh\left(a-\frac{bc}{d}\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.55, size = 209, normalized size = 1.37

$$-\frac{2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(-\frac{(b-d)(c+dx)}{d}\right)+2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{(b+d)(c+dx)}{d}\right)-4\sinh(a+bx)\text{Chi}(c+dx)+\sinh\left(a-\frac{b}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*CoshIntegral[c + d*x],x]
```

```
[Out] -1/4*(2*CoshIntegral[-(((b - d)*(c + d*x))/d)]*Sinh[a - (b*c)/d] + 2*CoshIn
tegral[((b + d)*(c + d*x))/d]*Sinh[a - (b*c)/d] - 4*CoshIntegral[c + d*x]*S
inh[a + b*x] + Cosh[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + Sinh
[a - (b*c)/d]*SinhIntegral[((b - d)*(c + d*x))/d] + 2*Cosh[a - (b*c)/d]*Sin
hIntegral[((b + d)*(c + d*x))/d] - Cosh[a - (b*c)/d]*SinhIntegral[c - (b*c)
/d - b*x + d*x] + Sinh[a - (b*c)/d]*SinhIntegral[c - (b*c)/d - b*x + d*x])/
b
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\text{Chi}\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*cosh_integral(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(Chi(d*x + c)*cosh(b*x + a), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \mathrm{X}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*x+c)*cosh(b*x+a),x)
```

```
[Out] int(Chi(d*x+c)*cosh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{Chi}\left(dx+c\right)\cosh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(Chi(d*x + c)*cosh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coshint}\left(c+d\,x\right)\,\cosh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coshint(c + d*x)*cosh(a + b*x),x)
[Out] int(coshint(c + d*x)*cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\mathrm{Chi}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a),x)
[Out] Integral(cosh(a + b*x)*Chi(c + d*x), x)
```

3.136 $\int \frac{\cosh(a+bx)\mathbf{Chi}(c+dx)}{x}\,dx$

Optimal. Leaf size=19
$$\text{Int}\left(\frac{\cosh(a+bx)\text{Chi}(c+dx)}{x},x\right)$$

```
[Out] CannotIntegrate(Chi(d*x+c)*cosh(b*x+a)/x,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}
$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*CoshIntegral[c + d*x])/x,x]
[Out] Defer[Int][(Cosh[a + b*x]*CoshIntegral[c + d*x])/x, x]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x}\,dx = \int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x}\,dx$$

Mathematica [A] time = 8.77, size = 0, normalized size = 0.00
$$\int \frac{\cosh(a+bx)\text{Chi}(c+dx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*CoshIntegral[c + d*x])/x,x]
[Out] Integrate[(Cosh[a + b*x]*CoshIntegral[c + d*x])/x, x]
```

fricas [A] time = 1.38, size = 0, normalized size = 0.00
$$\text{integral}\left(\frac{\cosh(bx+a)\,\text{Chi}(dx+c)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*cosh_integral(d*x + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(dx + c\right)\cosh\left(bx + a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(Chi(d*x + c)*cosh(b*x + a)/x, x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{X}\left(dx + c\right)\cosh\left(bx + a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Chi(d*x+c)*cosh(b*x+a)/x,x)
[Out] int(Chi(d*x+c)*cosh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Chi}\left(dx + c\right)\cosh\left(bx + a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(Chi(d*x + c)*cosh(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{coshint}\left(c + d\,x\right)\,\cosh\left(a + b\,x\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coshint(c + d*x)*cosh(a + b*x))/x,x)
[Out] int((coshint(c + d*x)*cosh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a + bx\right)\mathrm{Chi}\left(c + dx\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(Chi(d*x+c)*cosh(b*x+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x)*Chi(c + d*x)/x, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```