Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/20-1.1.2.3-a+b-x^2-^p-c+d-x^2-^q

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [349]. This is test number [20].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (349)	0.00 (0)
Mathematica	100.00 (349)	0.00 (0)
Maple	75.64 (264)	24.36 (85)
Fricas	55.01 (192)	44.99 (157)
Giac	30.37 (106)	69.63 (243)
Sympy	28.94 (101)	71.06 (248)
Maxima	22.64 (79)	77.36 (270)
Mupad	18.91 (66)	81.09 (283)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

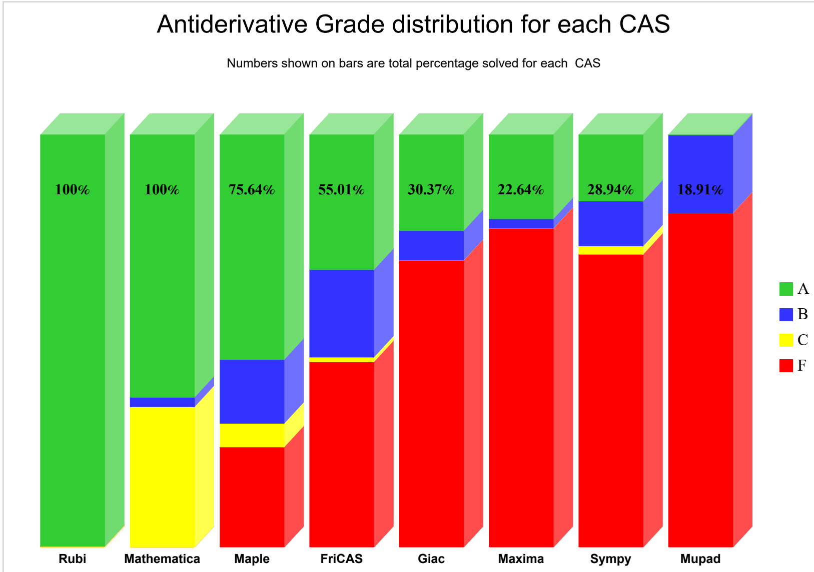
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

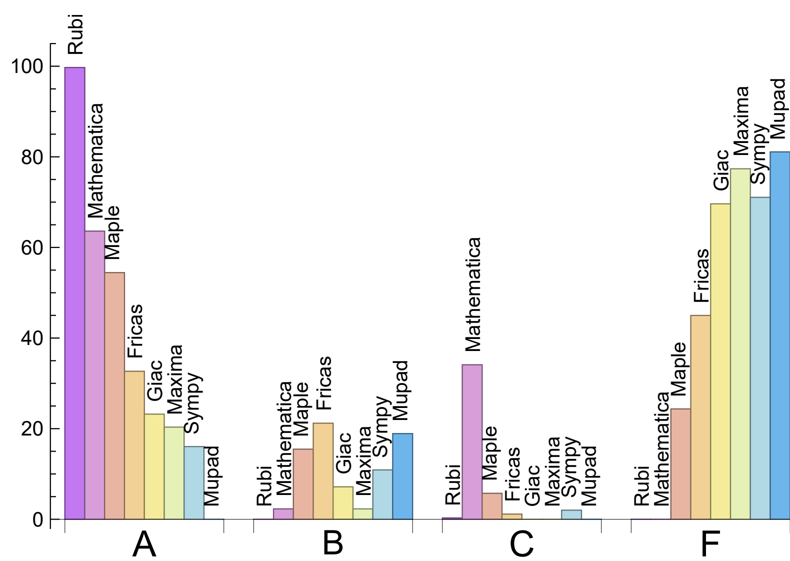
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.71	0.00	0.29	0.00
Mathematica	63.61	2.29	34.10	0.00
Maple	54.44	15.47	5.73	24.36
Fricas	32.66	21.20	1.15	44.99
Giac	23.21	7.16	0.00	69.63
Maxima	20.34	2.29	0.00	77.36
Sympy	16.05	10.89	2.01	71.06
Mupad	N/A	18.91	0.00	81.09

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	85	100.00 %	0.00 %	0.00 %
Fricas	157	24.20 %	29.30 %	46.50 %
Giac	243	97.53 %	0.00 %	2.47 %
Maxima	270	100.00 %	0.00 %	0.00 %
Sympy	248	90.73 %	9.27 %	0.00 %
Mupad	283	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

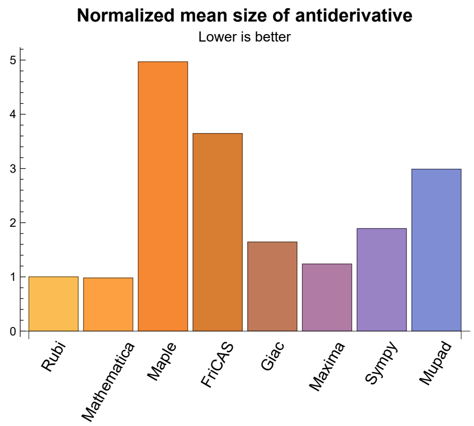
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

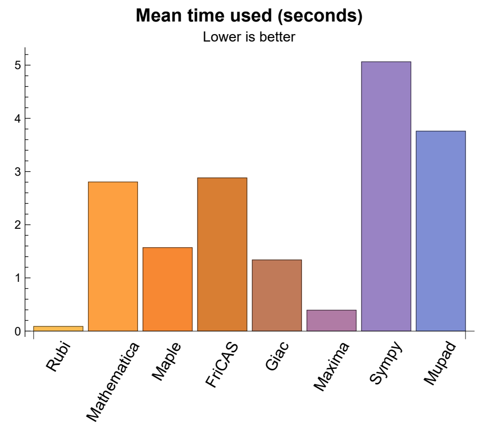
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	178.99	1.00	116.00	1.00
Mathematica	2.80	127.46	0.98	93.00	0.99
Maple	1.57	826.06	4.97	113.50	1.18
Maxima	0.39	157.72	1.24	116.00	1.16
Fricas	2.88	454.05	3.65	235.00	2.50
Sympy	5.06	149.81	1.89	94.00	1.74
Giac	1.34	229.62	1.64	129.50	1.14
Mupad	3.76	397.89	2.99	87.50	1.12

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {69, 110, 112, 114, 115, 117, 119, 120, 121, 124, 126, 127, 128, 130, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 301, 303, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 346, 347, 348}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

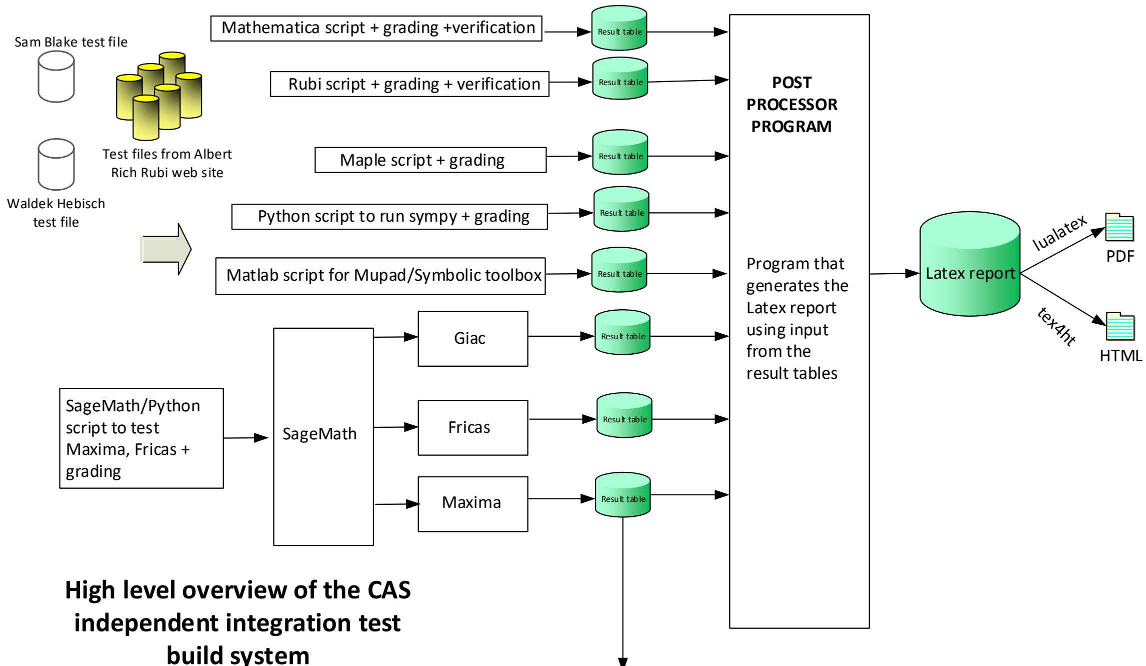
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

B grade: { }

C grade: { 301 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 137, 148, 149, 163, 164, 165, 168, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 198, 202, 203, 204, 211, 214, 215, 216, 217, 218, 220, 221, 223, 225, 226, 227, 231, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 251, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 326, 342, 343, 344, 345, 349 }

B grade: { 59, 224, 243, 293, 341, 346, 347, 348 }

C grade: { 73, 88, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 212, 213, 219, 222, 228, 229, 230, 232, 233, 241, 248, 249, 250, 252, 253, 254, 292, 301, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 93, 98, 99, 100, 105, 106, 108, 137, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 261, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 349 }

B grade: { 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 72, 73, 78, 79, 80, 86, 87, 88, 92, 94, 95, 96, 97, 101, 102, 103, 104, 107, 170, 171, 176, 177, 179, 205, 206, 213, 259, 260, 263, 264, 275, 276, 278, 281, 297, 298, 299, 300, 302, 303 }

C grade: { 146, 148, 149, 156, 157, 158, 159, 160, 162, 180, 197, 232, 241, 252, 294, 304, 305, 312, 313, 320 }

F grade: { 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 147, 150, 151, 152, 153, 154, 155, 161, 301, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 98, 99, 100, 108, 137, 163 }

B grade: { 26, 33, 34, 41, 42, 72, 73, 97 }

C grade: { }

F grade: { 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 30, 31, 32, 39, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 97, 98, 99, 100, 105, 106, 137, 163, 164, 165, 179, 188, 189, 190, 191, 192, 193, 194, 198, 203, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 244, 248, 249, 250, 253, 254, 255, 256, 257, 290, 291, 294, 296, 349 }

B grade: { 12, 13, 18, 19, 26, 27, 28, 29, 33, 34, 35, 36, 37, 38, 40, 41, 42, 50, 51, 52, 59, 60, 61, 68, 69, 70, 72, 73, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 107, 108, 146, 147, 148, 149, 156, 157, 158, 159, 160, 162, 180, 187, 224, 231, 251, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320 }

C grade: { 232, 252, 292, 313 }

F grade: { 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 155, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 185, 186, 195, 196, 197, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 238, 239, 240, 241, 242, 243, 245, 246, 247, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 293, 295, 297, 298, 299, 300, 301, 302, 303, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 14, 15, 16, 39, 43, 44, 47, 48, 56, 65, 74, 75, 76, 77, 84, 85, 109, 110, 111, 116, 117, 118, 122, 123, 124, 125, 131, 138, 180, 184, 185, 186, 188, 189, 190, 218, 220, 222, 224, 226, 231, 235, 236, 237, 246, 254, 256, 291 }

B grade: { 5, 6, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 35, 36, 37, 38, 45, 46, 53, 54, 55, 64, 92, 93, 99, 100, 108, 187, 221, 225, 227, 234 }

C grade: { 241, 245, 247, 342, 343, 344, 345 }

F grade: { 25, 26, 32, 33, 34, 40, 41, 42, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 119, 120, 121, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 223, 228, 229, 230, 232, 233, 238, 239, 240, 242, 243, 244, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 346, 347, 348, 349 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 105, 107, 108, 180 }

B grade: { 50, 51, 52, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 73, 79, 80, 87, 88, 94, 95, 96, 102, 103, 104, 106 }

C grade: { }

F grade: { 49, 57, 66, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 56, 65, 71, 72, 73, 76, 77, 84, 85, 92, 93, 97, 98, 99, 100, 105, 106, 108, 137, 345, 349 }

C grade: { }

F grade: { 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 74, 75, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 101, 102, 103, 104, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	96	96	107	98	88
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.14	1.04	0.94
time (sec)	N/A	0.048	0.017	0.105	0.276	1.103	0.016	1.693	4.775

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	76	73	65
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.032	0.012	0.104	0.278	0.809	0.014	0.727	4.753

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.020	0.010	0.089	0.280	1.394	0.011	1.045	0.047

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.010	0.005	0.084	0.295	0.850	0.006	0.919	0.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	34	99	82	34	31
N.S.	1	1.00	1.00	0.85	0.85	2.48	2.05	0.85	0.78
time (sec)	N/A	0.014	0.020	0.075	0.515	0.788	0.136	1.176	0.060

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.015	0.036	0.069	0.512	0.679	0.199	0.882	5.006

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	77	92	300	150	78	82
N.S.	1	1.00	0.89	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.023	0.047	0.083	0.516	0.538	0.283	0.970	5.061

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	124	136	131	116
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.054	0.017	0.103	0.274	0.731	0.017	1.965	4.945

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	82	97	91	75
N.S.	1	1.00	1.00	1.06	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.034	0.015	0.118	0.298	1.351	0.013	1.776	0.047

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.020	0.005	0.095	0.270	1.171	0.011	2.600	0.046

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	68	179	172	72	90
N.S.	1	1.00	0.94	1.02	1.08	2.84	2.73	1.14	1.43
time (sec)	N/A	0.031	0.036	0.076	0.516	0.952	0.220	1.767	0.088

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	89	92	96	302	236	95	124
N.S.	1	1.00	1.09	1.12	1.17	3.68	2.88	1.16	1.51
time (sec)	N/A	0.067	0.048	0.084	0.504	0.716	0.382	2.064	5.021

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	121	124	138	449	223	126	130
N.S.	1	1.00	1.04	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.050	0.070	0.094	0.508	0.776	0.541	2.418	5.030

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	161	177	167	167	189	187	152
N.S.	1	1.00	1.05	1.15	1.08	1.08	1.23	1.21	0.99
time (sec)	N/A	0.073	0.019	0.100	0.354	0.730	0.021	1.631	4.905

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	124	136	131	116
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.051	0.015	0.107	0.349	0.787	0.019	2.976	4.875

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	76	73	65
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.031	0.009	0.105	0.304	0.510	0.014	2.222	0.034

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	93	116	122	290	238	130	145
N.S.	1	1.00	0.95	1.18	1.24	2.96	2.43	1.33	1.48
time (sec)	N/A	0.045	0.044	0.089	0.519	0.532	0.314	1.678	4.870

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	138	147	444	314	152	181
N.S.	1	1.00	1.00	1.29	1.37	4.15	2.93	1.42	1.69
time (sec)	N/A	0.067	0.042	0.087	0.499	0.593	0.573	1.951	0.100

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	141	167	187	618	422	180	240
N.S.	1	1.00	1.08	1.28	1.44	4.75	3.25	1.38	1.85
time (sec)	N/A	0.117	0.057	0.094	0.520	0.470	1.001	1.228	4.956

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	136	196	187	428	326	198	216
N.S.	1	1.00	0.96	1.38	1.32	3.01	2.30	1.39	1.52
time (sec)	N/A	0.067	0.063	0.130	0.508	0.439	0.432	1.067	4.861

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	92	116	122	292	238	129	146
N.S.	1	1.00	0.94	1.18	1.24	2.98	2.43	1.32	1.49
time (sec)	N/A	0.041	0.048	0.085	0.515	1.484	0.315	2.306	0.076

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	69	181	172	72	90
N.S.	1	1.00	0.94	1.02	1.10	2.87	2.73	1.14	1.43
time (sec)	N/A	0.028	0.036	0.073	0.516	1.519	0.224	2.257	4.902

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.011	0.019	0.070	0.506	1.617	0.135	1.420	0.055

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	54	292	712	54	135
N.S.	1	1.00	0.87	0.79	0.77	4.17	10.17	0.77	1.93
time (sec)	N/A	0.019	0.032	0.108	0.508	1.696	2.729	1.836	0.320

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	93	133	711	0	122	2500
N.S.	1	1.00	0.87	0.85	1.22	6.52	0.00	1.12	22.94
time (sec)	N/A	0.060	0.129	0.150	0.504	1.064	0.000	1.430	5.688

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	158	158	277	1585	0	217	2500
N.S.	1	1.00	0.99	0.99	1.73	9.91	0.00	1.36	15.62
time (sec)	N/A	0.130	0.163	0.193	0.538	1.626	0.000	1.256	6.869

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	192	290	294	810	502	306	386
N.S.	1	1.00	1.00	1.51	1.53	4.22	2.61	1.59	2.01
time (sec)	N/A	0.108	0.067	0.094	0.502	0.787	1.113	0.845	5.024

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	206	213	612	403	220	261
N.S.	1	1.00	1.00	1.45	1.50	4.31	2.84	1.55	1.84
time (sec)	N/A	0.082	0.062	0.106	0.498	0.653	0.817	2.216	5.054

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	139	147	442	314	152	182
N.S.	1	1.00	1.00	1.31	1.39	4.17	2.96	1.43	1.72
time (sec)	N/A	0.065	0.044	0.088	0.498	0.813	0.594	1.125	0.102

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	88	94	95	297	236	94	124
N.S.	1	1.00	1.07	1.15	1.16	3.62	2.88	1.15	1.51
time (sec)	N/A	0.069	0.045	0.090	0.491	0.799	0.394	1.349	5.062

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	181	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.87	1.78	0.90	0.81
time (sec)	N/A	0.016	0.033	0.071	0.500	0.596	0.208	1.295	5.042

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	109	95	132	699	0	121	2500
N.S.	1	1.00	1.01	0.88	1.22	6.47	0.00	1.12	23.15
time (sec)	N/A	0.056	0.098	0.151	0.507	0.628	0.000	1.983	5.766

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	136	133	294	1681	0	232	2500
N.S.	1	1.00	0.81	0.80	1.76	10.07	0.00	1.39	14.97
time (sec)	N/A	0.134	0.226	0.198	0.503	1.192	0.000	1.674	6.875

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	197	198	529	3239	0	332	2500
N.S.	1	1.00	0.86	0.86	2.30	14.08	0.00	1.44	10.87
time (sec)	N/A	0.217	0.277	0.254	0.519	2.863	0.000	1.564	7.793

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	311	334	1044	615	340	409
N.S.	1	1.00	1.00	1.59	1.70	5.33	3.14	1.73	2.09
time (sec)	N/A	0.154	0.085	0.102	0.524	0.525	7.268	1.487	5.023

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	160	231	253	817	515	254	318
N.S.	1	1.00	1.00	1.44	1.58	5.11	3.22	1.59	1.99
time (sec)	N/A	0.132	0.066	0.109	0.492	0.490	1.826	1.430	0.135

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	139	170	185	606	422	178	240
N.S.	1	1.00	1.07	1.31	1.42	4.66	3.25	1.37	1.85
time (sec)	N/A	0.112	0.055	0.102	0.524	0.455	1.021	1.382	5.052

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	124	124	138	449	223	126	130
N.S.	1	1.00	1.07	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.053	0.065	0.087	0.502	0.477	0.536	3.320	5.023

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	76	92	301	150	78	81
N.S.	1	1.00	0.91	0.83	1.00	3.27	1.63	0.85	0.88
time (sec)	N/A	0.022	0.045	0.076	0.492	1.082	0.287	2.847	5.016

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	158	158	278	1587	0	218	2500
N.S.	1	1.00	0.98	0.98	1.73	9.86	0.00	1.35	15.53
time (sec)	N/A	0.128	0.190	0.205	0.496	1.044	0.000	1.450	6.892

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	197	196	530	3251	0	333	2500
N.S.	1	1.00	0.83	0.83	2.25	13.78	0.00	1.41	10.59
time (sec)	N/A	0.212	0.280	0.225	0.522	2.876	0.000	1.167	7.855

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	233	257	820	5070	0	574	2500
N.S.	1	1.00	0.74	0.82	2.60	16.10	0.00	1.82	7.94
time (sec)	N/A	0.311	0.612	0.315	0.584	9.251	0.000	1.743	8.555

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	23	33	33	31	20	31
N.S.	1	1.00	0.71	0.68	0.97	0.97	0.91	0.59	0.91
time (sec)	N/A	0.006	0.006	0.162	0.275	1.721	0.042	2.426	4.995

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	33	48	67	46	54	47
N.S.	1	1.00	0.87	0.70	1.02	1.43	0.98	1.15	1.00
time (sec)	N/A	0.010	0.010	0.096	0.529	0.915	0.061	0.914	0.042

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	180	300	281	398	484	201	-1
N.S.	1	1.00	0.78	1.30	1.22	1.72	2.10	0.87	-0.00
time (sec)	N/A	0.120	0.247	0.066	0.297	2.274	28.780	0.963	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	122	187	168	264	291	129	-1
N.S.	1	1.00	0.82	1.26	1.13	1.77	1.95	0.87	-0.01
time (sec)	N/A	0.060	0.158	0.063	0.279	3.457	8.462	0.729	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	98	81	158	144	70	-1
N.S.	1	1.00	0.85	1.13	0.93	1.82	1.66	0.80	-0.01
time (sec)	N/A	0.019	0.085	0.056	0.298	2.676	3.297	0.677	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.007	0.007	0.047	0.282	1.277	0.894	0.657	4.712

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	99	645	0	596	0	0	-1
N.S.	1	1.00	1.21	7.87	0.00	7.27	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.198	0.128	0.000	1.152	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	99	1945	0	369	0	217	-1
N.S.	1	1.00	1.21	23.72	0.00	4.50	0.00	2.65	-0.01
time (sec)	N/A	0.025	0.312	0.060	0.000	1.026	0.000	1.659	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	146	4113	0	698	0	487	-1
N.S.	1	1.00	0.98	27.60	0.00	4.68	0.00	3.27	-0.01
time (sec)	N/A	0.062	0.883	0.066	0.000	1.485	0.000	1.991	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	227	6475	0	1220	0	958	-1
N.S.	1	1.00	1.09	31.13	0.00	5.87	0.00	4.61	-0.00
time (sec)	N/A	0.148	10.774	0.078	0.000	1.334	0.000	2.210	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	225	364	364	502	665	260	-1
N.S.	1	1.00	0.83	1.34	1.34	1.85	2.44	0.96	-0.00
time (sec)	N/A	0.146	0.359	0.063	0.315	0.729	172.356	0.968	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	158	235	227	344	440	175	-1
N.S.	1	1.00	0.81	1.20	1.16	1.76	2.24	0.89	-0.01
time (sec)	N/A	0.077	0.256	0.059	0.283	0.581	35.721	0.622	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	130	116	210	253	103	-1
N.S.	1	1.00	0.84	1.10	0.98	1.78	2.14	0.87	-0.01
time (sec)	N/A	0.026	0.154	0.054	0.274	0.549	10.121	0.597	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	52	43	124	70	49	37
N.S.	1	1.00	0.92	0.80	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.010	0.014	0.000	0.280	0.512	1.533	0.506	4.709

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	126	1227	0	721	0	0	-1
N.S.	1	1.00	1.12	10.86	0.00	6.38	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.315	0.095	0.000	0.789	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	145	3349	0	907	0	317	-1
N.S.	1	1.00	1.11	25.56	0.00	6.92	0.00	2.42	-0.01
time (sec)	N/A	0.062	0.547	0.062	0.000	0.608	0.000	0.569	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	1002	6921	0	526	0	451	-1
N.S.	1	1.00	8.87	61.25	0.00	4.65	0.00	3.99	-0.01
time (sec)	N/A	0.040	3.601	0.066	0.000	0.624	0.000	1.828	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	247	12816	0	972	0	919	-1
N.S.	1	1.00	1.24	64.40	0.00	4.88	0.00	4.62	-0.01
time (sec)	N/A	0.080	10.613	0.072	0.000	0.801	0.000	1.653	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	362	22502	0	1604	0	1557	-1
N.S.	1	1.00	1.21	75.01	0.00	5.35	0.00	5.19	-0.00
time (sec)	N/A	0.253	11.025	0.086	0.000	1.763	0.000	5.114	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	269	428	447	608	0	321	-1
N.S.	1	1.00	0.77	1.23	1.28	1.74	0.00	0.92	-0.00
time (sec)	N/A	0.169	0.474	0.069	0.327	0.827	0.000	0.725	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	190	283	286	420	0	221	-1
N.S.	1	1.00	0.79	1.17	1.19	1.74	0.00	0.92	-0.00
time (sec)	N/A	0.100	0.332	0.059	0.307	0.562	0.000	0.598	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	123	162	151	260	316	135	-1
N.S.	1	1.00	0.83	1.09	1.01	1.74	2.12	0.91	-0.01
time (sec)	N/A	0.035	0.206	0.053	0.305	0.512	37.844	0.565	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	68	58	146	97	63	37
N.S.	1	1.00	0.85	0.81	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.016	0.016	0.046	0.308	0.505	2.783	0.583	4.693

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	157	2048	0	935	0	0	-1
N.S.	1	1.00	1.00	13.04	0.00	5.96	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.394	0.090	0.000	1.167	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	187	5230	0	1236	0	405	-1
N.S.	1	1.00	1.07	29.89	0.00	7.06	0.00	2.31	-0.01
time (sec)	N/A	0.147	0.665	0.130	0.000	0.969	0.000	0.570	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	266	10683	0	1517	0	659	-1
N.S.	1	1.00	1.37	55.07	0.00	7.82	0.00	3.40	-0.01
time (sec)	N/A	0.131	1.485	0.072	0.000	0.813	0.000	0.524	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	201	19519	0	706	0	846	-1
N.S.	1	1.00	1.40	135.55	0.00	4.90	0.00	5.88	-0.01
time (sec)	N/A	0.048	10.569	0.075	0.000	0.629	0.000	1.520	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	306	34027	0	1258	0	1448	-1
N.S.	1	1.00	1.23	136.65	0.00	5.05	0.00	5.82	-0.00
time (sec)	N/A	0.094	10.750	0.083	0.000	1.031	0.000	5.209	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	33	0	42	0	95	83
N.S.	1	1.00	1.53	1.10	0.00	1.40	0.00	3.17	2.77
time (sec)	N/A	0.010	0.077	0.225	0.000	0.467	0.000	0.895	0.394

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	45	84	59	67	0	70	59
N.S.	1	1.00	1.67	3.11	2.19	2.48	0.00	2.59	2.19
time (sec)	N/A	0.009	0.089	0.194	0.504	0.428	0.000	0.865	0.167

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	49	187	110	74	0	118	85
N.S.	1	1.00	1.96	7.48	4.40	2.96	0.00	4.72	3.40
time (sec)	N/A	0.009	0.049	0.225	0.496	0.472	0.000	0.828	5.347

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	138	227	199	300	400	150	-1
N.S.	1	1.00	0.82	1.34	1.18	1.78	2.37	0.89	-0.01
time (sec)	N/A	0.098	0.165	0.062	0.277	0.573	13.174	0.864	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	90	133	109	192	238	90	-1
N.S.	1	1.00	0.83	1.23	1.01	1.78	2.20	0.83	-0.01
time (sec)	N/A	0.040	0.096	0.060	0.299	0.510	4.422	1.081	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	63	47	113	126	49	86
N.S.	1	1.00	1.02	1.09	0.81	1.95	2.17	0.84	1.48
time (sec)	N/A	0.012	0.050	0.064	0.290	0.553	1.597	0.817	5.515

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.004	0.004	0.050	0.306	0.532	0.467	0.669	0.121

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	67	300	0	241	0	70	-1
N.S.	1	1.00	1.37	6.12	0.00	4.92	0.00	1.43	-0.02
time (sec)	N/A	0.014	0.100	0.061	0.000	0.548	0.000	0.706	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	118	810	0	463	0	242	-1
N.S.	1	1.00	1.17	8.02	0.00	4.58	0.00	2.40	-0.01
time (sec)	N/A	0.036	0.300	0.061	0.000	0.584	0.000	0.695	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	160	1843	0	864	0	538	-1
N.S.	1	1.00	0.98	11.31	0.00	5.30	0.00	3.30	-0.01
time (sec)	N/A	0.083	0.753	0.073	0.000	0.759	0.000	2.314	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	198	331	311	584	0	235	-1
N.S.	1	1.00	0.77	1.29	1.21	2.27	0.00	0.91	-0.00
time (sec)	N/A	0.178	0.414	0.099	0.271	0.584	0.000	0.652	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	139	215	197	416	0	157	-1
N.S.	1	1.00	0.82	1.27	1.17	2.46	0.00	0.93	-0.01
time (sec)	N/A	0.126	0.270	0.083	0.279	0.537	0.000	0.572	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	105	95	123	108	276	0	92	-1
N.S.	1	1.17	1.06	1.37	1.20	3.07	0.00	1.02	-0.01
time (sec)	N/A	0.040	0.178	0.079	0.282	0.526	0.000	0.630	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	58	55	46	167	60	50	53
N.S.	1	1.00	1.07	1.02	0.85	3.09	1.11	0.93	0.98
time (sec)	N/A	0.012	0.084	0.061	0.325	0.506	2.362	0.830	5.117

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.001	0.002	0.047	0.291	0.553	0.312	0.684	0.040

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	96	723	0	441	0	107	-1
N.S.	1	1.00	1.22	9.15	0.00	5.58	0.00	1.35	-0.01
time (sec)	N/A	0.028	0.234	0.063	0.000	0.656	0.000	0.654	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	151	1906	0	864	0	318	-1
N.S.	1	1.00	1.06	13.33	0.00	6.04	0.00	2.22	-0.01
time (sec)	N/A	0.076	0.664	0.067	0.000	0.769	0.000	1.165	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	1392	4035	0	1482	0	643	-1
N.S.	1	1.00	6.19	17.93	0.00	6.59	0.00	2.86	-0.00
time (sec)	N/A	0.173	13.591	0.088	0.000	1.169	0.000	1.658	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	202	360	392	684	0	237	-1
N.S.	1	1.00	0.79	1.41	1.54	2.68	0.00	0.93	-0.00
time (sec)	N/A	0.167	0.404	0.141	0.286	0.655	0.000	0.717	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	143	246	254	486	0	158	-1
N.S.	1	1.00	0.83	1.43	1.48	2.83	0.00	0.92	-0.01
time (sec)	N/A	0.104	0.275	0.099	0.303	0.577	0.000	0.647	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	103	156	147	318	0	103	-1
N.S.	1	1.00	0.98	1.49	1.40	3.03	0.00	0.98	-0.01
time (sec)	N/A	0.035	0.205	0.061	0.296	0.495	0.000	0.576	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	90	68	54	144	40	33
N.S.	1	1.00	0.79	1.91	1.45	1.15	3.06	0.85	0.70
time (sec)	N/A	0.007	0.081	0.055	0.287	0.525	4.499	0.579	4.785

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	32	31	47	95	27	28
N.S.	1	1.00	0.74	0.82	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.004	0.004	0.048	0.290	0.623	0.443	0.631	4.754

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	130	1378	0	764	0	320	-1
N.S.	1	1.00	1.07	11.30	0.00	6.26	0.00	2.62	-0.01
time (sec)	N/A	0.074	0.393	0.064	0.000	0.836	0.000	0.683	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	219	3449	0	1440	0	620	-1
N.S.	1	1.00	1.08	17.07	0.00	7.13	0.00	3.07	-0.00
time (sec)	N/A	0.155	1.305	0.084	0.000	1.431	0.000	1.147	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	367	7121	0	2250	0	1010	-1
N.S.	1	1.00	1.17	22.75	0.00	7.19	0.00	3.23	-0.00
time (sec)	N/A	0.288	3.781	0.088	0.000	12.309	0.000	2.132	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	163	536	465	229	0	218	326
N.S.	1	1.00	0.73	2.39	2.08	1.02	0.00	0.97	1.46
time (sec)	N/A	0.071	0.350	0.091	0.355	4.670	0.000	0.904	5.152

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	107	301	249	151	0	138	176
N.S.	1	1.00	0.61	1.73	1.43	0.87	0.00	0.79	1.01
time (sec)	N/A	0.049	0.198	0.073	0.295	2.373	0.000	0.883	4.987

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	132	103	87	566	72	87
N.S.	1	1.00	0.66	1.45	1.13	0.96	6.22	0.79	0.96
time (sec)	N/A	0.018	0.118	0.056	0.295	1.182	11.433	0.996	4.849

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	32	31	47	95	27	28
N.S.	1	1.00	0.74	0.82	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.004	0.051	0.056	0.284	1.105	0.416	0.698	4.787

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	99	733	0	442	0	107	-1
N.S.	1	1.00	1.25	9.28	0.00	5.59	0.00	1.35	-0.01
time (sec)	N/A	0.030	0.256	0.127	0.000	3.944	0.000	0.811	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	122	822	0	459	0	225	-1
N.S.	1	1.00	1.22	8.22	0.00	4.59	0.00	2.25	-0.01
time (sec)	N/A	0.036	0.326	0.065	0.000	1.710	0.000	0.942	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	151	4155	0	698	0	487	-1
N.S.	1	1.00	1.01	27.89	0.00	4.68	0.00	3.27	-0.01
time (sec)	N/A	0.055	0.969	0.076	0.000	2.099	0.000	2.766	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	179	12958	0	972	0	919	-1
N.S.	1	1.00	0.90	65.12	0.00	4.88	0.00	4.62	-0.01
time (sec)	N/A	0.075	15.217	0.092	0.000	1.981	0.000	1.620	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	27	0	18	18
N.S.	1	1.00	1.00	0.95	0.00	1.35	0.00	0.90	0.90
time (sec)	N/A	0.003	0.034	0.072	0.000	1.085	0.000	0.940	4.771

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	28	0	23	0	51	79
N.S.	1	1.00	1.00	1.12	0.00	0.92	0.00	2.04	3.16
time (sec)	N/A	0.005	0.062	0.125	0.000	1.014	0.000	0.769	0.368

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	306	0	241	0	70	-1
N.S.	1	1.00	1.43	6.24	0.00	4.92	0.00	1.43	-0.02
time (sec)	N/A	0.014	0.105	0.066	0.000	0.939	0.000	0.599	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	25	14	13	44	31	25	27
N.S.	1	1.00	1.67	0.93	0.87	2.93	2.07	1.67	1.80
time (sec)	N/A	0.003	0.040	0.098	0.490	0.868	2.339	0.578	0.039

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	99	0	0	0	136	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.21	0.00	-0.00
time (sec)	N/A	0.495	12.205	0.011	0.000	0.000	2.217	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	176	0	0	0	99	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.378	9.521	0.007	0.000	0.000	1.779	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	62	0	0	0	63	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.311	6.844	0.003	0.000	0.000	1.288	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	740	740	162	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	7.186	0.029	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	584	584	86	0	0	0	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	10.061	0.033	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	252	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	10.286	0.031	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	265	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	10.150	0.034	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	110	0	0	0	139	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.21	0.00	-0.00
time (sec)	N/A	0.498	13.802	0.010	0.000	0.000	2.902	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	637	637	173	0	0	0	131	0	-1
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.21	0.00	-0.00
time (sec)	N/A	0.438	10.635	0.007	0.000	0.000	2.587	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	68	0	0	0	100	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.368	8.119	0.004	0.000	0.000	2.038	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	231	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	8.728	0.012	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	775	775	235	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	10.111	0.028	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	815	815	252	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	10.193	0.030	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	98	0	0	0	165	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.25	0.00	-0.00
time (sec)	N/A	0.441	15.058	0.013	0.000	0.000	3.086	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	88	0	0	0	129	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.21	0.00	-0.00
time (sec)	N/A	0.384	15.047	0.008	0.000	0.000	2.464	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	158	0	0	0	94	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.306	13.135	0.007	0.000	0.000	1.911	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	62	0	0	0	60	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.254	10.027	0.005	0.000	0.000	1.146	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.021	5.941	0.024	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	234	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	10.115	0.030	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	255	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	10.129	0.030	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	623	76	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	15.054	0.011	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	592	592	168	0	0	0	0	0	-1
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	14.400	0.007	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	53	0	0	0	60	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.248	10.027	0.012	0.000	0.000	3.167	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	226	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	7.343	0.027	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	807	807	236	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	10.142	0.032	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	256	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	10.169	0.030	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	96	0	0	0	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	15.077	0.013	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	83	0	0	0	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	15.059	0.006	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	33	42	0	0	27
N.S.	1	1.00	0.55	0.55	0.75	0.95	0.00	0.00	0.61
time (sec)	N/A	0.013	15.043	0.066	0.327	1.231	0.000	0.000	4.785

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	74	0	0	0	60	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.309	10.028	0.012	0.000	0.000	5.618	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	796	796	248	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	8.699	0.027	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	827	827	259	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	10.185	0.031	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	163	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.045	6.185	0.027	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	166	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.019	6.021	0.033	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	153	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.030	6.148	0.028	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.020	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	156	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.020	6.161	0.028	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	938	0	1943	0	0	-1
N.S.	1	1.00	1.04	8.30	0.00	17.19	0.00	0.00	-0.01
time (sec)	N/A	0.010	4.628	20.491	0.000	1.794	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1685	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	15.46	0.00	0.00	-0.01
time (sec)	N/A	0.009	4.845	83.581	0.000	1.429	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	155	1033	0	285	0	0	-1
N.S.	1	1.00	1.61	10.76	0.00	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.232	8.692	0.000	10.189	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	151	1042	0	315	0	0	-1
N.S.	1	1.00	1.59	10.97	0.00	3.32	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.200	8.490	0.000	9.401	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	169	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	6.841	0.029	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	167	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	6.926	0.029	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	168	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	6.934	0.024	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	172	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	6.906	0.026	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	148	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	6.286	0.026	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	148	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	5.660	0.029	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	136	548	0	2961	0	0	-1
N.S.	1	1.00	1.11	4.46	0.00	24.07	0.00	0.00	-0.01
time (sec)	N/A	0.013	5.291	76.529	0.000	51.456	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	136	1064	0	2831	0	0	-1
N.S.	1	1.00	1.11	8.65	0.00	23.02	0.00	0.00	-0.01
time (sec)	N/A	0.011	4.856	72.683	0.000	49.516	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	136	723	0	2681	0	0	-1
N.S.	1	1.00	1.14	6.08	0.00	22.53	0.00	0.00	-0.01
time (sec)	N/A	0.012	4.861	98.377	0.000	46.323	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	136	726	0	3109	0	0	-1
N.S.	1	1.00	1.14	6.10	0.00	26.13	0.00	0.00	-0.01
time (sec)	N/A	0.012	5.394	90.711	0.000	49.087	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	124	616	0	1395	0	0	-1
N.S.	1	1.00	1.77	8.80	0.00	19.93	0.00	0.00	-0.01
time (sec)	N/A	0.006	4.725	6.441	0.000	3.999	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	137	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	5.655	0.029	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	125	365	0	269	0	0	-1
N.S.	1	1.00	1.69	4.93	0.00	3.64	0.00	0.00	-0.01
time (sec)	N/A	0.007	9.750	2.610	0.000	1.485	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	91	57	94	70	315	0	0	-1
N.S.	1	1.15	0.72	1.19	0.89	3.99	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.020	0.082	0.302	1.084	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	91	54	94	0	303	0	0	-1
N.S.	1	1.23	0.73	1.27	0.00	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.014	0.072	0.000	1.357	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	91	57	94	0	314	0	0	-1
N.S.	1	1.20	0.75	1.24	0.00	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.021	0.076	0.000	1.189	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	243	543	0	0	0	0	-1
N.S.	1	1.00	0.74	1.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	2.384	0.117	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	198	328	0	0	0	0	-1
N.S.	1	1.00	0.80	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.095	1.139	0.078	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	0	0	0	-1
N.S.	1	1.00	0.42	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.058	0.917	0.069	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	133	181	0	0	0	0	-1
N.S.	1	1.00	1.58	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.011	2.133	0.080	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	243	617	0	0	0	0	-1
N.S.	1	1.00	1.03	2.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.080	2.909	0.093	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	285	1411	0	0	0	0	-1
N.S.	1	1.00	0.92	4.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	3.118	0.094	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	302	780	0	0	0	0	-1
N.S.	1	1.00	0.74	1.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	4.393	0.103	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	246	545	0	0	0	0	-1
N.S.	1	1.00	0.73	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	2.462	0.083	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	199	330	0	0	0	0	-1
N.S.	1	1.00	0.73	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.106	2.160	0.082	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	191	332	0	0	0	0	-1
N.S.	1	1.00	0.72	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.103	3.910	0.080	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	232	607	0	0	0	0	-1
N.S.	1	1.00	1.01	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	4.730	0.089	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	285	1410	0	0	0	0	-1
N.S.	1	1.00	0.90	4.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	4.934	0.094	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	127	303	0	0	0	0	-1
N.S.	1	1.00	0.54	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	0.958	0.111	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	22	75	0	21	0	0	-1
N.S.	1	1.00	0.58	1.97	0.00	0.55	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.077	0.135	0.000	0.240	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	38	0	38	17	17	-1
N.S.	1	1.00	0.75	1.90	0.00	1.90	0.85	0.85	-0.05
time (sec)	N/A	0.003	0.002	0.062	0.000	0.630	1.430	1.135	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	0	0	0	-1
N.S.	1	1.00	0.20	0.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.728	0.072	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	78	0	0	0	0	-1
N.S.	1	1.00	0.66	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.627	0.094	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	60	53	0	0	0	0	-1
N.S.	1	1.00	0.40	0.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.642	0.080	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	0	34	0	-1
N.S.	1	1.00	1.00	1.15	0.00	0.00	1.70	0.00	-0.05
time (sec)	N/A	0.005	0.279	0.099	0.000	0.000	1.490	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	0	36	0	-1
N.S.	1	1.00	1.00	0.86	0.00	0.00	1.71	0.00	-0.05
time (sec)	N/A	0.005	0.289	0.098	0.000	0.000	1.691	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	0	0	34	0	-1
N.S.	1	1.00	1.00	1.45	0.00	0.00	1.70	0.00	-0.05
time (sec)	N/A	0.005	0.329	0.099	0.000	0.000	1.428	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	0	21	10	0	-1
N.S.	1	1.00	1.00	1.25	0.00	5.25	2.50	0.00	-0.25
time (sec)	N/A	0.004	0.275	0.095	0.000	0.240	1.313	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	21	36	0	-1
N.S.	1	1.00	1.00	0.95	0.00	1.05	1.80	0.00	-0.05
time (sec)	N/A	0.005	0.270	0.090	0.000	0.228	1.561	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	0	21	37	0	-1
N.S.	1	1.00	1.00	0.90	0.00	1.00	1.76	0.00	-0.05
time (sec)	N/A	0.005	0.303	0.092	0.000	0.270	1.622	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	23	36	0	-1
N.S.	1	1.00	1.00	0.95	0.00	1.15	1.80	0.00	-0.05
time (sec)	N/A	0.005	0.317	0.091	0.000	0.220	1.605	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	0	20	0	0	-1
N.S.	1	1.00	0.92	1.08	0.00	1.54	0.00	0.00	-0.08
time (sec)	N/A	0.010	0.281	0.080	0.000	0.310	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	27	0	23	0	0	-1
N.S.	1	1.00	0.87	0.87	0.00	0.74	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.287	0.090	0.000	0.227	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	23	0	0	-1
N.S.	1	1.00	0.77	0.89	0.00	0.66	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.305	0.087	0.000	0.346	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	23	0	0	-1
N.S.	1	1.00	0.77	0.89	0.00	0.66	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.296	0.095	0.000	0.153	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	27	30	0	0	0	0	-1
N.S.	1	1.00	0.21	0.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.276	0.076	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	27	26	0	0	0	0	-1
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.283	0.075	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	27	20	0	0	0	0	-1
N.S.	1	1.00	0.18	0.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.279	0.074	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	35	32	0	23	0	0	-1
N.S.	1	1.00	0.88	0.80	0.00	0.58	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.297	0.100	0.000	0.146	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	321	852	0	0	0	0	-1
N.S.	1	1.00	0.76	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	3.971	0.097	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	260	615	0	0	0	0	-1
N.S.	1	1.00	0.76	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	2.914	0.102	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	216	399	0	0	0	0	-1
N.S.	1	1.00	0.83	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.103	2.266	0.084	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	0	0	0	-1
N.S.	1	1.00	0.44	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.865	0.073	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	41	0	0	-1
N.S.	1	1.00	0.99	1.15	0.00	0.47	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.691	0.073	0.000	0.100	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	112	248	0	0	0	0	-1
N.S.	1	1.00	0.41	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	2.066	0.072	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	261	752	0	0	0	0	-1
N.S.	1	1.00	1.02	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.100	3.273	0.092	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	301	1607	0	0	0	0	-1
N.S.	1	1.00	0.90	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	3.550	0.083	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	318	755	0	0	0	0	-1
N.S.	1	1.00	0.71	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	5.495	0.129	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	256	539	0	0	0	0	-1
N.S.	1	1.00	0.74	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	4.680	0.106	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	196	345	0	0	0	0	-1
N.S.	1	1.00	0.76	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.098	4.086	0.077	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	136	188	0	0	0	0	-1
N.S.	1	1.00	1.62	2.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	2.058	0.079	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	112	144	0	0	0	0	-1
N.S.	1	1.00	0.58	0.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	2.102	0.074	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	224	354	0	0	0	0	-1
N.S.	1	1.00	0.93	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.082	4.506	0.094	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	337	964	0	0	0	0	-1
N.S.	1	1.00	1.04	2.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	5.726	0.098	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	41	0	0	-1
N.S.	1	1.00	0.99	1.15	0.00	0.47	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.040	0.000	0.000	0.169	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	103	0	39	0	0	-1
N.S.	1	1.00	1.00	1.18	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.731	0.079	0.000	0.158	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	103	0	39	0	0	-1
N.S.	1	1.00	1.02	1.18	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.753	0.081	0.000	0.133	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	104	0	38	0	0	-1
N.S.	1	1.00	1.00	1.18	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.751	0.077	0.000	0.142	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	8	19	0	-1
N.S.	1	1.00	1.00	1.17	0.00	0.67	1.58	0.00	-0.08
time (sec)	N/A	0.005	0.221	0.098	0.000	0.119	1.622	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	58	14	0	8	0	0	-1
N.S.	1	1.00	5.80	1.40	0.00	0.80	0.00	0.00	-0.10
time (sec)	N/A	0.006	10.040	0.110	0.000	0.142	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	8	19	0	-1
N.S.	1	1.00	1.00	1.17	0.00	0.67	1.58	0.00	-0.08
time (sec)	N/A	0.005	0.224	0.080	0.000	0.109	1.611	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	8	73	0	-1
N.S.	1	1.00	1.00	1.00	0.00	0.80	7.30	0.00	-0.10
time (sec)	N/A	0.004	10.025	0.093	0.000	0.120	10.955	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	0	8	19	0	-1
N.S.	1	1.00	1.50	1.17	0.00	0.67	1.58	0.00	-0.08
time (sec)	N/A	0.005	10.051	0.094	0.000	0.193	1.215	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	8	0	0	-1
N.S.	1	1.00	1.00	1.08	0.00	0.67	0.00	0.00	-0.08
time (sec)	N/A	0.005	0.202	0.095	0.000	0.236	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	26	8	0	68	22	0	-1
N.S.	1	1.00	3.25	1.00	0.00	8.50	2.75	0.00	-0.12
time (sec)	N/A	0.002	0.005	0.082	0.000	1.246	0.918	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	8	34	0	-1
N.S.	1	1.00	1.00	1.08	0.00	0.67	2.83	0.00	-0.08
time (sec)	N/A	0.005	0.211	0.095	0.000	0.151	1.338	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	8	39	0	-1
N.S.	1	1.00	1.00	1.10	0.00	0.80	3.90	0.00	-0.10
time (sec)	N/A	0.005	0.058	0.105	0.000	0.178	1.864	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	8	34	0	-1
N.S.	1	1.00	1.00	1.08	0.00	0.67	2.83	0.00	-0.08
time (sec)	N/A	0.005	0.215	0.097	0.000	0.206	1.337	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	10	0	0	-1
N.S.	1	1.00	0.37	0.33	0.00	0.20	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.205	0.080	0.000	0.478	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	17	15	0	10	0	0	-1
N.S.	1	1.00	0.35	0.31	0.00	0.20	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.066	0.085	0.000	0.345	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	10	0	0	-1
N.S.	1	1.00	0.37	0.33	0.00	0.20	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.193	0.066	0.000	0.239	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	0	34	8	0	-1
N.S.	1	1.00	1.00	1.00	0.00	4.25	1.00	0.00	-0.12
time (sec)	N/A	0.002	0.003	0.063	0.000	1.200	0.952	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	19	15	0	10	0	0	-1
N.S.	1	1.00	0.40	0.32	0.00	0.21	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.193	0.075	0.000	0.349	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	14	0	8	0	0	-1
N.S.	1	1.00	1.90	1.40	0.00	0.80	0.00	0.00	-0.10
time (sec)	N/A	0.005	10.038	0.089	0.000	0.177	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	8	76	0	-1
N.S.	1	1.00	1.00	1.00	0.00	0.80	7.60	0.00	-0.10
time (sec)	N/A	0.004	10.023	0.080	0.000	0.362	10.997	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	16	36	0	-1
N.S.	1	1.00	1.00	0.95	0.00	0.80	1.80	0.00	-0.05
time (sec)	N/A	0.005	0.212	0.079	0.000	0.271	1.494	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	9	41	0	-1
N.S.	1	1.00	1.00	0.94	0.00	0.56	2.56	0.00	-0.06
time (sec)	N/A	0.005	0.054	0.103	0.000	0.266	2.086	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	16	36	0	-1
N.S.	1	1.00	1.00	0.95	0.00	0.80	1.80	0.00	-0.05
time (sec)	N/A	0.006	0.207	0.094	0.000	0.154	1.501	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	0	0	0	-1
N.S.	1	1.00	1.00	1.16	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.220	0.087	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	34	0	0	0	0	-1
N.S.	1	1.00	1.00	1.13	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.069	0.099	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	0	0	0	-1
N.S.	1	1.00	1.00	1.16	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.009	0.198	0.095	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	46	30	0	0	75	0	-1
N.S.	1	1.00	1.84	1.20	0.00	0.00	3.00	0.00	-0.04
time (sec)	N/A	0.008	0.052	0.099	0.000	0.000	10.965	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	34	0	0	0	0	-1
N.S.	1	1.00	1.00	1.06	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.009	0.188	0.088	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	47	28	0	0	0	0	-1
N.S.	1	1.00	3.92	2.33	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.004	10.043	0.100	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	40	24	0	34	0	0	-1
N.S.	1	1.00	1.38	0.83	0.00	1.17	0.00	0.00	-0.03
time (sec)	N/A	0.002	0.009	0.106	0.000	0.989	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	0	37	0	-1
N.S.	1	1.00	1.25	0.91	0.00	0.00	1.16	0.00	-0.03
time (sec)	N/A	0.009	0.218	0.098	0.000	0.000	1.548	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	36	27	0	0	42	0	-1
N.S.	1	1.00	1.20	0.90	0.00	0.00	1.40	0.00	-0.03
time (sec)	N/A	0.010	0.070	0.118	0.000	0.000	2.047	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	0	37	0	-1
N.S.	1	1.00	1.25	0.91	0.00	0.00	1.16	0.00	-0.03
time (sec)	N/A	0.009	0.204	0.100	0.000	0.000	1.563	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	10	0	0	-1
N.S.	1	1.00	0.74	0.68	0.00	0.19	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.214	0.079	0.000	0.314	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	37	33	0	10	0	0	-1
N.S.	1	1.00	0.73	0.65	0.00	0.20	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.069	0.096	0.000	0.307	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	10	0	0	-1
N.S.	1	1.00	0.74	0.68	0.00	0.19	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.221	0.079	0.000	0.197	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	24	0	104	0	0	-1
N.S.	1	1.00	0.93	0.86	0.00	3.71	0.00	0.00	-0.04
time (sec)	N/A	0.003	0.009	0.077	0.000	1.066	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	33	0	16	0	0	-1
N.S.	1	1.00	1.08	0.67	0.00	0.33	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.028	0.074	0.000	0.182	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	34	0	16	0	0	-1
N.S.	1	1.00	1.26	1.10	0.00	0.52	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.211	0.091	0.000	0.264	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	65	48	30	0	8	73	0	-1
N.S.	1	1.55	1.14	0.71	0.00	0.19	1.74	0.00	-0.02
time (sec)	N/A	0.009	0.057	0.091	0.000	0.185	11.162	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	22	0	0	-1
N.S.	1	1.00	1.00	0.85	0.00	0.55	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.202	0.085	0.000	0.221	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	0	15	44	0	-1
N.S.	1	1.00	1.00	0.94	0.00	0.42	1.22	0.00	-0.03
time (sec)	N/A	0.011	0.070	0.098	0.000	0.240	1.969	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	22	0	0	-1
N.S.	1	1.00	1.00	0.85	0.00	0.55	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.209	0.090	0.000	0.262	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	104	0	0	0	0	-1
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.877	0.073	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	168	0	0	0	0	-1
N.S.	1	1.00	1.00	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.888	0.085	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	167	0	0	0	0	-1
N.S.	1	1.00	1.00	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.658	0.081	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	108	0	0	0	0	-1
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.686	0.084	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	105	0	0	0	0	-1
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.894	0.078	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	161	0	0	0	0	-1
N.S.	1	1.00	1.00	1.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.879	0.088	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	162	0	0	0	0	-1
N.S.	1	1.00	1.00	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.729	0.089	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	107	0	0	0	0	-1
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.674	0.082	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	0	0	0	-1
N.S.	1	1.00	0.44	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	89	104	0	0	0	0	-1
N.S.	1	1.00	0.44	0.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.065	0.871	0.084	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	89	108	0	0	0	0	-1
N.S.	1	1.00	0.44	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.864	0.080	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	92	165	0	0	0	0	-1
N.S.	1	1.00	0.43	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.073	0.737	0.080	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	89	161	0	0	0	0	-1
N.S.	1	1.00	0.47	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.872	0.077	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	90	107	0	0	0	0	-1
N.S.	1	1.00	0.47	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.863	0.084	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	92	109	0	0	0	0	-1
N.S.	1	1.00	0.47	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.656	0.083	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	93	166	0	0	0	0	-1
N.S.	1	1.00	0.47	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.614	0.082	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	104	0	0	0	0	-1
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.878	0.079	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	168	0	0	0	0	-1
N.S.	1	1.00	1.00	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.876	0.084	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	167	0	0	0	0	-1
N.S.	1	1.00	1.00	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.749	0.078	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	108	0	0	0	0	-1
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.632	0.082	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	160	0	0	0	0	-1
N.S.	1	1.00	1.00	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.896	0.071	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	0	0	0	0	-1
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.873	0.079	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	0	0	0	0	-1
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.807	0.078	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	163	0	0	0	0	-1
N.S.	1	1.00	1.00	1.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.707	0.081	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	0	0	0	-1
N.S.	1	1.00	0.42	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.059	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	89	161	0	0	0	0	-1
N.S.	1	1.00	0.42	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.068	0.871	0.076	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	89	162	0	0	0	0	-1
N.S.	1	1.00	0.42	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.076	0.860	0.078	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	92	111	0	0	0	0	-1
N.S.	1	1.00	0.41	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.074	0.808	0.077	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	89	161	0	0	0	0	-1
N.S.	1	1.00	0.47	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.862	0.074	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	90	107	0	0	0	0	-1
N.S.	1	1.00	0.47	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.850	0.077	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	92	109	0	0	0	0	-1
N.S.	1	1.00	0.47	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.681	0.088	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	93	166	0	0	0	0	-1
N.S.	1	1.00	0.47	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.661	0.078	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	37	38	0	34	0	0	-1
N.S.	1	1.00	0.47	0.49	0.00	0.44	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.570	0.078	0.000	0.453	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	38	0	14	20	0	-1
N.S.	1	1.00	1.03	0.97	0.00	0.36	0.51	0.00	-0.03
time (sec)	N/A	0.016	0.503	0.088	0.000	0.502	1.157	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	53	0	15	0	0	-1
N.S.	1	1.00	0.77	0.87	0.00	0.25	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.480	0.078	0.000	0.370	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	27	25	0	0	0	0	-1
N.S.	1	1.00	4.50	4.17	0.00	0.00	0.00	0.00	-0.17
time (sec)	N/A	0.005	0.230	0.097	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	30	0	0	-1
N.S.	1	1.00	1.04	1.22	0.00	1.30	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.489	0.091	0.000	0.222	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	0	0	0	-1
N.S.	1	1.00	0.20	0.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	37	0	23	0	0	-1
N.S.	1	1.00	1.84	1.95	0.00	1.21	0.00	0.00	-0.05
time (sec)	N/A	0.005	0.319	0.107	0.000	0.141	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	809	0	0	0	0	-1
N.S.	1	1.00	1.00	8.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	2.298	0.328	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	95	1388	0	0	0	0	-1
N.S.	1	1.00	1.01	14.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	2.376	0.255	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	102	1388	0	0	0	0	-1
N.S.	1	1.00	0.21	2.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	2.390	0.218	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	102	872	0	0	0	0	-1
N.S.	1	1.00	0.47	4.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	2.249	0.194	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	23	122	0	0	0	0	0	-1
N.S.	1	0.37	1.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.008	1.387	0.037	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	117	0	0	0	0	-1
N.S.	1	1.00	1.04	2.54	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.891	0.223	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	81	207	0	0	0	0	-1
N.S.	1	1.00	1.72	4.40	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	2.063	0.431	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	119	186	0	553	0	0	-1
N.S.	1	1.00	0.92	1.44	0.00	4.29	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.260	1.552	0.000	2.550	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	186	0	553	0	0	-1
N.S.	1	1.00	0.99	1.55	0.00	4.61	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.250	1.516	0.000	2.417	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	119	0	0	755	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	5.85	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.219	0.028	0.000	7.246	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	123	0	0	776	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	6.26	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.220	0.030	0.000	6.057	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	286	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.211	0.025	0.000	4.687	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	286	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.204	0.029	0.000	5.024	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	337	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.217	0.031	0.000	30.378	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	123	0	0	343	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.226	0.029	0.000	28.580	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	138	0	104	0	0	-1
N.S.	1	1.00	0.92	2.26	0.00	1.70	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.102	1.000	0.000	3.391	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	137	0	243	0	0	-1
N.S.	1	1.00	0.92	2.25	0.00	3.98	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.099	0.961	0.000	3.487	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	0	0	274	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	3.56	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.119	0.030	0.000	7.383	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	69	0	0	273	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.119	0.027	0.000	6.816	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	276	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.125	0.026	0.000	4.843	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	278	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.153	0.026	0.000	6.415	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	0	0	338	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.140	0.027	0.000	26.002	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	90	0	0	350	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.131	0.028	0.000	25.812	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	121	0	91	0	0	-1
N.S.	1	1.00	0.91	2.28	0.00	1.72	0.00	0.00	-0.02
time (sec)	N/A	0.005	0.081	1.112	0.000	3.416	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	346	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	10.333	0.011	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	348	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.142	9.823	0.013	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	161	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	8.792	0.023	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	160	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	7.409	0.027	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	160	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	6.994	0.028	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	120	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	7.597	0.023	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	327	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.113	8.533	0.024	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	331	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	9.772	0.026	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	419	0	0	0	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	10.498	0.026	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	431	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.236	10.532	0.025	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	340	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	10.240	0.030	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	341	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	10.241	0.032	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	232	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	10.154	0.031	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	232	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	10.160	0.030	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	392	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.179	10.171	0.030	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	336	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.157	10.238	0.031	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	380	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	10.380	0.029	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	387	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	10.382	0.029	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	536	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	10.763	0.031	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	550	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	10.672	0.031	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.199	0.053	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	136	0	0	0	121	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.41	0.00	-0.00
time (sec)	N/A	0.197	5.131	0.039	0.000	0.000	20.037	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	88	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.084	5.100	0.031	0.000	0.000	9.927	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	53	0	-1
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.57	0.00	-0.01
time (sec)	N/A	0.028	0.073	0.019	0.000	0.000	4.677	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	22	0	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	0.93
time (sec)	N/A	0.007	0.004	0.000	0.000	0.000	1.070	0.000	5.567

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.215	0.031	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.247	0.040	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.327	0.055	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.014	0.412	0.131	0.000	0.958	0.000	0.000	5.736

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [297] had the largest ratio of [59]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059
4	A	2	1	1.00	15	0.067
5	A	2	2	1.00	17	0.118
6	A	2	2	1.00	17	0.118
7	A	3	3	1.00	17	0.176
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	3	2	1.00	19	0.105
12	A	4	3	1.00	19	0.158
13	A	3	3	1.00	19	0.158
14	A	2	1	1.00	19	0.053
15	A	2	1	1.00	19	0.053
16	A	2	1	1.00	17	0.059
17	A	3	2	1.00	19	0.105
18	A	4	3	1.00	19	0.158
19	A	5	4	1.00	19	0.210
20	A	3	2	1.00	19	0.105
21	A	3	2	1.00	19	0.105
22	A	3	2	1.00	19	0.105
23	A	2	2	1.00	17	0.118
24	A	3	2	1.00	19	0.105
25	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	4	1.00	19	0.210
27	A	4	3	1.00	19	0.158
28	A	4	3	1.00	19	0.158
29	A	4	3	1.00	19	0.158
30	A	4	3	1.00	19	0.158
31	A	2	2	1.00	17	0.118
32	A	4	3	1.00	19	0.158
33	A	5	4	1.00	19	0.210
34	A	6	4	1.00	19	0.210
35	A	5	4	1.00	19	0.210
36	A	5	4	1.00	19	0.210
37	A	5	4	1.00	19	0.210
38	A	3	3	1.00	19	0.158
39	A	3	3	1.00	17	0.176
40	A	5	4	1.00	19	0.210
41	A	6	4	1.00	19	0.210
42	A	7	4	1.00	19	0.210
43	A	3	3	1.00	15	0.200
44	A	5	3	1.00	15	0.200
45	A	6	6	1.00	21	0.286
46	A	5	5	1.00	21	0.238
47	A	4	4	1.00	19	0.210
48	A	3	3	1.00	11	0.273
49	A	5	5	1.00	21	0.238
50	A	3	3	1.00	21	0.143
51	A	4	4	1.00	21	0.190
52	A	6	5	1.00	21	0.238
53	A	7	6	1.00	21	0.286
54	A	6	5	1.00	21	0.238
55	A	5	4	1.00	19	0.210
56	A	4	3	1.00	11	0.273
57	A	6	6	1.00	21	0.286
58	A	6	6	1.00	21	0.286
59	A	4	3	1.00	21	0.143
60	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	5	1.00	21	0.238
62	A	8	6	1.00	21	0.286
63	A	7	5	1.00	21	0.238
64	A	6	4	1.00	19	0.210
65	A	5	3	1.00	11	0.273
66	A	7	7	1.00	21	0.333
67	A	7	7	1.00	21	0.333
68	A	7	7	1.00	21	0.333
69	A	5	3	1.00	21	0.143
70	A	6	4	1.00	21	0.190
71	A	4	4	1.00	19	0.210
72	A	4	4	1.00	17	0.235
73	A	4	4	1.00	21	0.190
74	A	5	5	1.00	21	0.238
75	A	4	4	1.00	21	0.190
76	A	3	3	1.00	19	0.158
77	A	2	2	1.00	11	0.182
78	A	2	2	1.00	21	0.095
79	A	3	3	1.00	21	0.143
80	A	5	5	1.00	21	0.238
81	A	6	5	1.00	21	0.238
82	A	5	5	1.00	21	0.238
83	A	4	4	1.17	21	0.190
84	A	3	3	1.00	19	0.158
85	A	1	1	1.00	11	0.091
86	A	3	3	1.00	21	0.143
87	A	5	5	1.00	21	0.238
88	A	6	5	1.00	21	0.238
89	A	6	6	1.00	21	0.286
90	A	5	5	1.00	21	0.238
91	A	4	4	1.00	21	0.190
92	A	2	2	1.00	19	0.105
93	A	2	2	1.00	11	0.182
94	A	5	5	1.00	21	0.238
95	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	7	5	1.00	21	0.238
97	A	5	3	1.00	21	0.143
98	A	4	3	1.00	21	0.143
99	A	3	3	1.00	19	0.158
100	A	2	2	1.00	11	0.182
101	A	3	3	1.00	21	0.143
102	A	3	3	1.00	21	0.143
103	A	4	4	1.00	21	0.190
104	A	5	4	1.00	21	0.190
105	A	2	2	1.00	26	0.077
106	A	2	2	1.00	19	0.105
107	A	2	2	1.00	21	0.095
108	A	2	2	1.00	15	0.133
109	A	8	8	1.00	24	0.333
110	A	7	7	1.00	24	0.292
111	A	6	6	1.00	22	0.273
112	A	6	6	1.00	24	0.250
113	A	6	6	1.00	24	0.250
114	A	8	8	1.00	24	0.333
115	A	9	8	1.00	24	0.333
116	A	9	8	1.00	24	0.333
117	A	8	7	1.00	24	0.292
118	A	7	6	1.00	22	0.273
119	A	7	7	1.00	24	0.292
120	A	7	7	1.00	24	0.292
121	A	9	9	1.00	24	0.375
122	A	8	7	1.00	24	0.292
123	A	7	7	1.00	24	0.292
124	A	6	6	1.00	24	0.250
125	A	5	5	1.00	22	0.227
126	A	1	1	1.00	24	0.042
127	A	7	7	1.00	24	0.292
128	A	8	8	1.00	24	0.333
129	A	7	7	1.00	24	0.292
130	A	6	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	5	1.00	22	0.227
132	A	7	7	1.00	24	0.292
133	A	8	8	1.00	24	0.333
134	A	9	8	1.00	24	0.333
135	A	8	8	1.00	24	0.333
136	A	7	7	1.00	24	0.292
137	A	2	2	1.00	24	0.083
138	A	6	6	1.00	22	0.273
139	A	8	8	1.00	24	0.333
140	A	9	8	1.00	24	0.333
141	A	1	1	1.00	26	0.038
142	A	1	1	1.00	24	0.042
143	A	1	1	1.00	23	0.043
144	A	1	1	1.00	24	0.042
145	A	1	1	1.00	22	0.045
146	A	1	1	1.00	19	0.053
147	A	1	1	1.00	19	0.053
148	A	1	1	1.00	24	0.042
149	A	1	1	1.00	22	0.045
150	A	1	1	1.00	27	0.037
151	A	1	1	1.00	28	0.036
152	A	1	1	1.00	29	0.034
153	A	1	1	1.00	30	0.033
154	A	1	1	1.00	26	0.038
155	A	1	1	1.00	26	0.038
156	A	1	1	1.00	23	0.043
157	A	1	1	1.00	23	0.043
158	A	1	1	1.00	23	0.043
159	A	1	1	1.00	23	0.043
160	A	1	1	1.00	17	0.059
161	A	1	1	1.00	21	0.048
162	A	1	1	1.00	21	0.048
163	A	3	3	1.15	29	0.103
164	A	3	3	1.23	29	0.103
165	A	3	3	1.20	29	0.103

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	6	1.00	23	0.261
167	A	5	5	1.00	23	0.217
168	A	4	4	1.00	23	0.174
169	A	1	1	1.00	23	0.043
170	A	4	4	1.00	23	0.174
171	A	5	5	1.00	23	0.217
172	A	7	6	1.00	23	0.261
173	A	6	6	1.00	23	0.261
174	A	5	5	1.00	23	0.217
175	A	5	5	1.00	23	0.217
176	A	4	4	1.00	23	0.174
177	A	5	5	1.00	23	0.217
178	A	5	5	1.00	23	0.217
179	A	3	3	1.00	23	0.130
180	A	2	1	1.00	23	0.043
181	A	4	4	1.00	23	0.174
182	A	5	5	1.00	23	0.217
183	A	4	4	1.00	21	0.190
184	A	1	1	1.00	23	0.043
185	A	1	1	1.00	23	0.043
186	A	1	1	1.00	23	0.043
187	A	1	1	1.00	21	0.048
188	A	1	1	1.00	21	0.048
189	A	1	1	1.00	21	0.048
190	A	1	1	1.00	23	0.043
191	A	4	4	1.00	21	0.190
192	A	3	3	1.00	23	0.130
193	A	3	3	1.00	23	0.130
194	A	3	3	1.00	23	0.130
195	A	4	4	1.00	21	0.190
196	A	4	4	1.00	21	0.190
197	A	4	4	1.00	23	0.174
198	A	2	2	1.00	23	0.087
199	A	7	6	1.00	23	0.261
200	A	6	6	1.00	23	0.261

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	5	1.00	23	0.217
202	A	4	4	1.00	23	0.174
203	A	1	1	1.00	23	0.043
204	A	6	6	1.00	23	0.261
205	A	4	4	1.00	23	0.174
206	A	5	5	1.00	23	0.217
207	A	7	6	1.00	23	0.261
208	A	6	6	1.00	23	0.261
209	A	5	5	1.00	23	0.217
210	A	1	1	1.00	23	0.043
211	A	6	6	1.00	23	0.261
212	A	4	4	1.00	23	0.174
213	A	5	5	1.00	23	0.217
214	A	1	1	1.00	23	0.043
215	A	3	2	1.00	24	0.083
216	A	3	2	1.00	24	0.083
217	A	3	2	1.00	25	0.080
218	A	1	1	1.00	23	0.043
219	A	1	1	1.00	23	0.043
220	A	1	1	1.00	23	0.043
221	A	2	2	1.00	23	0.087
222	A	1	1	1.00	21	0.048
223	A	1	1	1.00	23	0.043
224	A	2	2	1.00	23	0.087
225	A	1	1	1.00	23	0.043
226	A	1	1	1.00	23	0.043
227	A	1	1	1.00	23	0.043
228	A	1	1	1.00	21	0.048
229	A	1	1	1.00	21	0.048
230	A	1	1	1.00	21	0.048
231	A	2	2	1.00	21	0.095
232	A	1	1	1.00	19	0.053
233	A	1	1	1.00	21	0.048
234	A	2	2	1.00	21	0.095
235	A	1	1	1.00	21	0.048

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	1	1	1.00	21	0.048
237	A	1	1	1.00	21	0.048
238	A	2	2	1.00	21	0.095
239	A	2	2	1.00	21	0.095
240	A	2	2	1.00	21	0.095
241	A	2	2	1.00	21	0.095
242	A	2	2	1.00	19	0.105
243	A	1	1	1.00	21	0.048
244	A	2	2	1.00	21	0.095
245	A	2	2	1.00	21	0.095
246	A	2	2	1.00	21	0.095
247	A	2	2	1.00	21	0.095
248	A	1	1	1.00	23	0.043
249	A	1	1	1.00	23	0.043
250	A	1	1	1.00	23	0.043
251	A	2	2	1.00	23	0.087
252	A	1	1	1.00	21	0.048
253	A	2	2	1.00	23	0.087
254	A	2	2	1.55	23	0.087
255	A	2	2	1.00	23	0.087
256	A	2	2	1.00	23	0.087
257	A	2	2	1.00	23	0.087
258	A	3	3	1.00	24	0.125
259	A	3	3	1.00	27	0.111
260	A	3	3	1.00	25	0.120
261	A	3	3	1.00	28	0.107
262	A	3	3	1.00	25	0.120
263	A	3	3	1.00	26	0.115
264	A	3	3	1.00	26	0.115
265	A	3	3	1.00	27	0.111
266	A	4	4	1.00	23	0.174
267	A	4	4	1.00	26	0.154
268	A	4	4	1.00	26	0.154
269	A	4	4	1.00	29	0.138
270	A	7	6	1.00	24	0.250

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	7	6	1.00	25	0.240
272	A	7	6	1.00	27	0.222
273	A	7	6	1.00	28	0.214
274	A	3	3	1.00	24	0.125
275	A	3	3	1.00	27	0.111
276	A	3	3	1.00	25	0.120
277	A	3	3	1.00	28	0.107
278	A	3	3	1.00	25	0.120
279	A	3	3	1.00	26	0.115
280	A	3	3	1.00	26	0.115
281	A	3	3	1.00	27	0.111
282	A	4	4	1.00	23	0.174
283	A	4	4	1.00	26	0.154
284	A	4	4	1.00	26	0.154
285	A	4	4	1.00	29	0.138
286	A	7	6	1.00	24	0.250
287	A	7	6	1.00	25	0.240
288	A	7	6	1.00	27	0.222
289	A	7	6	1.00	28	0.214
290	A	1	1	1.00	23	0.043
291	A	2	2	1.00	23	0.087
292	A	1	1	1.00	21	0.048
293	A	1	1	1.00	23	0.043
294	A	4	4	1.00	28	0.143
295	A	4	4	1.00	23	0.174
296	A	1	1	1.00	23	0.043
297	A	1	1	1.00	59	0.017
298	A	1	1	1.00	59	0.017
299	A	4	4	1.00	59	0.068
300	A	3	3	1.00	59	0.051
301	C	1	1	0.37	21	0.048
302	A	2	2	1.00	26	0.077
303	A	1	1	1.00	41	0.024
304	A	1	1	1.00	21	0.048
305	A	1	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	1	1	1.00	21	0.048
307	A	1	1	1.00	23	0.043
308	A	1	1	1.00	23	0.043
309	A	1	1	1.00	23	0.043
310	A	1	1	1.00	23	0.043
311	A	1	1	1.00	25	0.040
312	A	1	1	1.00	21	0.048
313	A	1	1	1.00	21	0.048
314	A	1	1	1.00	21	0.048
315	A	1	1	1.00	23	0.043
316	A	1	1	1.00	25	0.040
317	A	1	1	1.00	25	0.040
318	A	1	1	1.00	25	0.040
319	A	1	1	1.00	27	0.037
320	A	1	1	1.00	19	0.053
321	A	13	8	1.00	21	0.381
322	A	12	8	1.00	21	0.381
323	A	8	7	1.00	21	0.333
324	A	8	7	1.00	21	0.333
325	A	4	3	1.00	21	0.143
326	A	5	4	1.00	21	0.190
327	A	7	6	1.00	21	0.286
328	A	9	8	1.00	21	0.381
329	A	10	9	1.00	21	0.429
330	A	10	9	1.00	21	0.429
331	A	9	8	1.00	21	0.381
332	A	9	8	1.00	21	0.381
333	A	9	8	1.00	21	0.381
334	A	9	8	1.00	21	0.381
335	A	9	8	1.00	21	0.381
336	A	9	8	1.00	21	0.381
337	A	10	9	1.00	21	0.429
338	A	10	9	1.00	21	0.429
339	A	11	9	1.00	21	0.429
340	A	11	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	3	2	1.00	19	0.105
342	A	5	5	1.00	19	0.263
343	A	4	4	1.00	19	0.210
344	A	3	3	0.91	17	0.176
345	A	2	2	1.00	9	0.222
346	A	2	2	1.00	19	0.105
347	A	2	2	1.00	19	0.105
348	A	2	2	1.00	19	0.105
349	A	1	1	1.00	50	0.020

Chapter 3

Listing of integrals

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3.28 $\int \frac{(c+dx^2)^4}{(a+bx^2)^2} \, dx$ . . . 210
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3.109 $\int (a-bx^2)^{2/3}(3a+bx^2)^3\,dx$ . . . 601
3.110 $\int (a-bx^2)^{2/3}(3a+bx^2)^2\,dx$ . . . 607
3.111 $\int (a-bx^2)^{2/3}(3a+bx^2)\,dx$ . . . 612
3.112 $\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx$ . . . 617
3.113 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx$ . . . 622
3.114 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx$ . . . 627
3.115 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx$ . . . 633

3.116 $\int (a-bx^2)^{5/3}(3a+bx^2)^3\,dx$ . . . 639
3.117 $\int (a-bx^2)^{5/3}(3a+bx^2)^2\,dx$ . . . 645
3.118 $\int (a-bx^2)^{5/3}(3a+bx^2)\,dx$ . . . 651
3.119 $\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx$ . . . 656
3.120 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx$ . . . 662
3.121 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx$ . . . 668
3.122 $\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$ . . . 674
3.123 $\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx$ . . . 680
3.124 $\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$ . . . 686
3.125 $\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$ . . . 691
3.126 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$ . . . 696
3.127 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$ . . . 699
3.128 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$ . . . 705
3.129 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx$ . . . 711
3.130 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx$ . . . 717
3.131 $\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx$ . . . 722
3.132 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx$ . . . 727
3.133 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx$ . . . 733
3.134 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx$ . . . 739
3.135 $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$ . . . 745
3.136 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx$ . . . 751
3.137 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx$ . . . 756
3.138 $\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx$ . . . 759
3.139 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx$ . . . 764
3.140 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx$ . . . 770
3.141 $\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx$ . . . 776
3.142 $\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$ . . . 779
3.143 $\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$ . . . 782
3.144 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$ . . . 785
3.145 $\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$ . . . 788

3.146 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$ 791
3.147 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$ 796
3.148 $\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$ 800
3.149 $\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$ 804
3.150 $\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$ 808
3.151 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$ 811
3.152 $\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$ 814
3.153 $\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$ 817
3.154 $\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx$ 820
3.155 $\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$ 823
3.156 $\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx$ 826
3.157 $\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx$ 831
3.158 $\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx$ 836
3.159 $\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx$ 841
3.160 $\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx$ 846
3.161 $\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx$ 850
3.162 $\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx$ 853
3.163 $\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx$ 856
3.164 $\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx$ 860
3.165 $\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx$ 863
3.166 $\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx$ 866
3.167 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$ 871
3.168 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$ 875
3.169 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx$ 879
3.170 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx$ 882
3.171 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$ 886
3.172 $\int (a+bx^2)^{3/2}(c+dx^2)^{3/2}\,dx$ 891
3.173 $\int \sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,dx$ 896
3.174 $\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$ 901

3.175 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$ 906

3.176 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$ 911

3.177 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx$ 915

3.178 $\int \sqrt{2+bx^2}\,\sqrt{3+dx^2}\,dx$ 920

3.179 $\int \sqrt{3-6x^2}\,\sqrt{2+4x^2}\,dx$ 925

3.180 $\int \sqrt{2+4x^2}\,\sqrt{3+6x^2}\,dx$ 928

3.181 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$ 931

3.182 $\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$ 935

3.183 $\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx$ 939

3.184 $\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx$ 943

3.185 $\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx$ 946

3.186 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx$ 949

3.187 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx$ 952

3.188 $\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx$ 955

3.189 $\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx$ 958

3.190 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx$ 961

3.191 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$ 964

3.192 $\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx$ 967

3.193 $\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$ 970

3.194 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx$ 973

3.195 $\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx$ 976

3.196 $\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx$ 980

3.197 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx$ 984

3.198 $\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx$ 988

3.199 $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$ 991

3.200 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$ 996

3.201 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$ 1001

3.202 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$ 1006

3.203 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$ 1010

3.204 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$ 1013

3.205 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$ 1018

3.206 $\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$ 1022

3.207 $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$ 1027

3.208 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx$ 1032

3.209 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$ 1037

3.210 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx$ 1042

3.211 $\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$ 1045

3.212 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$ 1050

3.213 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$ 1054

3.214 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$ 1059

3.215 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$ 1062

3.216 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$ 1066

3.217 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$ 1070

3.218 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx$ 1074

3.219 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx$ 1077

3.220 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$ 1080

3.221 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx$ 1083

3.222 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx$ 1086

3.223 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx$ 1089

3.224 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$ 1092

3.225 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$ 1095

3.226 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx$ 1098

3.227 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx$ 1101

3.228 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx$ 1104

3.229 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx$ . . . 1107
3.230 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$ . . . 1110
3.231 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$ . . . 1113
3.232 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx$ . . . 1116
3.233 $\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx$ . . . 1119
3.234 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx$ . . . 1122
3.235 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$ . . . 1125
3.236 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx$ . . . 1128
3.237 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx$ . . . 1131
3.238 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx$ . . . 1134
3.239 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx$ . . . 1137
3.240 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx$ . . . 1140
3.241 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx$ . . . 1143
3.242 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx$ . . . 1146
3.243 $\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx$ . . . 1149
3.244 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx$ . . . 1152
3.245 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx$ . . . 1155
3.246 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx$ . . . 1158
3.247 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx$ . . . 1161
3.248 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx$ . . . 1164
3.249 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx$ . . . 1167
3.250 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx$ . . . 1170
3.251 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx$ . . . 1173
3.252 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx$ . . . 1176
3.253 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx$ . . . 1179
3.254 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx$ . . . 1182
3.255 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx$ . . . 1185
3.256 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx$ . . . 1188
3.257 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx$ . . . 1191
3.258 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$ . . . 1194

3.259 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$ 1198

3.260 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$ 1202

3.261 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx$ 1206

3.262 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$ 1210

3.263 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$ 1214

3.264 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$ 1218

3.265 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$ 1222

3.266 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$ 1226

3.267 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$ 1230

3.268 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$ 1234

3.269 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$ 1238

3.270 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$ 1242

3.271 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$ 1247

3.272 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$ 1252

3.273 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$ 1257

3.274 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$ 1262

3.275 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$ 1266

3.276 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$ 1270

3.277 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$ 1274

3.278 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$ 1278

3.279 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$ 1282

3.280 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$ 1286

3.281 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$ 1290

3.282 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$ 1294

3.283 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$ 1298

3.284 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$ 1302

3.285 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$ 1306

3.286 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$ 1310

3.287 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$ 1315

3.288 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$ 1320

3.289 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$ 1325

3.290 $\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$ 1330

3.291 $\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$ 1333

3.292 $\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$ 1336

3.293 $\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$ 1339

3.294 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$ 1342

3.295 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$ 1345

3.296 $\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$ 1349

3.297 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$ 1352

3.298 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$ 1356

3.299 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$ 1360

3.300 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$ 1366

3.301 $\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$ 1371

3.302 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx$ . . . 1374
3.303 $\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}\,x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$ . . . 1377
3.304 $\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx$ . . . 1381
3.305 $\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$ . . . 1384
3.306 $\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx$ . . . 1388
3.307 $\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx$ . . . 1392
3.308 $\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx$ . . . 1396
3.309 $\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx$ . . . 1399
3.310 $\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx$ . . . 1402
3.311 $\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx$ . . . 1405
3.312 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$ . . . 1408
3.313 $\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx$ . . . 1411
3.314 $\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx$ . . . 1414
3.315 $\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx$ . . . 1417
3.316 $\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$ . . . 1420
3.317 $\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$ . . . 1423
3.318 $\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$ . . . 1426
3.319 $\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx$ . . . 1429
3.320 $\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$ . . . 1432
3.321 $\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx$ . . . 1435
3.322 $\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx$ . . . 1440
3.323 $\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx$ . . . 1445
3.324 $\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$ . . . 1449
3.325 $\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx$ . . . 1454
3.326 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx$ . . . 1458
3.327 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx$ . . . 1462
3.328 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$ . . . 1467
3.329 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx$ . . . 1472
3.330 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx$ . . . 1477
3.331 $\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx$ . . . 1482

3.332 $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2} \, dx$ 1487

3.333 $\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2} \, dx$ 1492

3.334 $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2} \, dx$ 1497

3.335 $\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2} \, dx$ 1502

3.336 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2} \, dx$ 1507

3.337 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2} \, dx$ 1512

3.338 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2} \, dx$ 1517

3.339 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2} \, dx$ 1522

3.340 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2} \, dx$ 1527

3.341 $\int (a+bx^2)^p (c+dx^2)^q \, dx$ 1533

3.342 $\int (a+bx^2)^p (c+dx^2)^3 \, dx$ 1536

3.343 $\int (a+bx^2)^p (c+dx^2)^2 \, dx$ 1540

3.344 $\int (a+bx^2)^p (c+dx^2) \, dx$ 1544

3.345 $\int (a+bx^2)^p \, dx$ 1547

3.346 $\int \frac{(a+bx^2)^p}{c+dx^2} \, dx$ 1550

3.347 $\int \frac{(a+bx^2)^p}{(c+dx^2)^2} \, dx$ 1553

3.348 $\int \frac{(a+bx^2)^p}{(c+dx^2)^3} \, dx$ 1556

3.349 $\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}} \, dx$ 1559

3.1 $\int (a+bx^2)(c+dx^2)^4 \, dx$

Optimal. Leaf size=94

$$ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5+\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+ad)x^9+\frac{1}{11}bd^4x^{11}$$

```
[Out] a*c^4*x+1/3*c^3*(4*a*d+b*c)*x^3+2/5*c^2*d*(3*a*d+2*b*c)*x^5+2/7*c*d^2*(2*a*
d+3*b*c)*x^7+1/9*d^3*(a*d+4*b*c)*x^9+1/11*b*d^4*x^11
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$\frac{1}{3}c^3x^3(4ad+bc)+\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^2)(c+dx^2)^4 \, dx = \int (ac^4+c^3(bc+4ad)x^2+2c^2d(2bc+3ad)x^4+2cd^2(3bc+2ad)x^6+d^3(4bc+$$
$$= ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5+\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc$$

Mathematica [A]
time = 0.02, size = 94, normalized size = 1.00

$$ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5+\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+ad)x^9+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Maple [A]
time = 0.10, size = 97, normalized size = 1.03

method	result
norman	$\frac{b\,d^4x^{11}}{11}+\left(\frac{1}{9}a\,d^4+\frac{4}{9}bc\,d^3\right)x^9+\left(\frac{4}{7}ac\,d^3+\frac{6}{7}b\,c^2d^2\right)x^7+\left(\frac{6}{5}a\,c^2d^2+\frac{4}{5}b\,c^3d\right)x^5+\left(\frac{4}{3}a\,c^3d+\frac{1}{3}b\,c^4\right)x$
default	$\frac{b\,d^4x^{11}}{11}+\frac{(a\,d^4+4bc\,d^3)x^9}{9}+\frac{(4ac\,d^3+6b\,c^2d^2)x^7}{7}+\frac{(6a\,c^2d^2+4b\,c^3d)x^5}{5}+\frac{(4a\,c^3d+b\,c^4)x^3}{3}+a\,c^4x$
gosper	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+\frac{1}{3}x^3b$
risch	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+\frac{1}{3}x^3b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d^4*x^11+1/9*(a*d^4+4*b*c*d^3)*x^9+1/7*(4*a*c*d^3+6*b*c^2*d^2)*x^7+1
/5*(6*a*c^2*d^2+4*b*c^3*d)*x^5+1/3*(4*a*c^3*d+b*c^4)*x^3+a*c^4*x
```

Maxima [A]
time = 0.28, size = 96, normalized size = 1.02

$$\frac{1}{11}\,bd^4x^{11}+\frac{1}{9}\,(4\,bcd^3+ad^4)x^9+\frac{2}{7}\,(3\,bc^2d^2+2\,acd^3)x^7+ac^4x+\frac{2}{5}\,(2\,bc^3d+3\,ac^2d^2)x^5+\frac{1}{3}\,(bc^4+4\,ac^3d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d^
3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c^3
*d)*x^3
```

Fricas [A]
time = 1.10, size = 96, normalized size = 1.02

$$\frac{1}{11}\,bd^4x^{11}+\frac{1}{9}\,(4\,bcd^3+ad^4)x^9+\frac{2}{7}\,(3\,bc^2d^2+2\,acd^3)x^7+ac^4x+\frac{2}{5}\,(2\,bc^3d+3\,ac^2d^2)x^5+\frac{1}{3}\,(bc^4+4\,ac^3d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d^
3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c^3
*d)*x^3
```

Sympy [A]
time = 0.02, size = 107, normalized size = 1.14

$$ac^4x + \frac{bd^4x^{11}}{11} + x^9\left(\frac{ad^4}{9} + \frac{4bcd^3}{9}\right) + x^7\cdot\left(\frac{4acd^3}{7} + \frac{6bc^2d^2}{7}\right) + x^5\cdot\left(\frac{6ac^2d^2}{5} + \frac{4bc^3d}{5}\right) + x^3\cdot\left(\frac{4ac^3d}{3} + \frac{bc^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**11/11 + x**9*(a*d**4/9 + 4*b*c*d**3/9) + x**7*(4*a*c*d
**3/7 + 6*b*c**2*d**2/7) + x**5*(6*a*c**2*d**2/5 + 4*b*c**3*d/5) + x**3*(4*
a*c**3*d/3 + b*c**4/3)
```

Giac [A]
time = 1.69, size = 98, normalized size = 1.04

$$\frac{1}{11}\,bd^4x^{11} + \frac{4}{9}\,bcd^3x^9 + \frac{1}{9}\,ad^4x^9 + \frac{6}{7}\,bc^2d^2x^7 + \frac{4}{7}\,acd^3x^7 + \frac{4}{5}\,bc^3dx^5 + \frac{6}{5}\,ac^2d^2x^5 + \frac{1}{3}\,bc^4x^3 + \frac{4}{3}\,ac^3dx^3 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/11*b*d^4*x^11 + 4/9*b*c*d^3*x^9 + 1/9*a*d^4*x^9 + 6/7*b*c^2*d^2*x^7 + 4/7
*a*c*d^3*x^7 + 4/5*b*c^3*d*x^5 + 6/5*a*c^2*d^2*x^5 + 1/3*b*c^4*x^3 + 4/3*a*
c^3*d*x^3 + a*c^4*x
```

Mupad [B]
time = 4.77, size = 88, normalized size = 0.94

$$x^3\left(\frac{b\,c^4}{3} + \frac{4\,a\,d\,c^3}{3}\right) + x^9\left(\frac{a\,d^4}{9} + \frac{4\,b\,c\,d^3}{9}\right) + \frac{b\,d^4\,x^{11}}{11} + a\,c^4\,x + \frac{2\,c^2\,d\,x^5\,(3\,a\,d + 2\,b\,c)}{5} + \frac{2\,c\,d^2\,x^7\,(2\,a\,d + 3\,b\,c)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^4,x)
```

```
[Out] x^3*((b*c^4)/3 + (4*a*c^3*d)/3) + x^9*((a*d^4)/9 + (4*b*c*d^3)/9) + (b*d^4*
x^11)/11 + a*c^4*x + (2*c^2*d*x^5*(3*a*d + 2*b*c))/5 + (2*c*d^2*x^7*(2*a*d
+ 3*b*c))/7
```

3.2 $\int (a+bx^2)(c+dx^2)^3\,dx$

Optimal. Leaf size=70

$$ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9$$

```
[Out] a*c^3*x+1/3*c^2*(3*a*d+b*c)*x^3+3/5*c*d*(a*d+b*c)*x^5+1/7*d^2*(a*d+3*b*c)*x
^7+1/9*b*d^3*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {380}

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^2+3cd(bc+ad)x^4+d^2(3bc+ad)x^6+bd^3x^8\right)dx\\ &= ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.00

$$ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Maple [A]
time = 0.10, size = 73, normalized size = 1.04

method	result	size
norman	$\frac{b\,d^{3} x^{9}}{9}+\left(\frac{1}{7}a\,d^{3}+\frac{3}{7}b c\,d^{2}\right) x^{7}+\left(\frac{3}{5}a c\,d^{2}+\frac{3}{5}b\,c^{2} d\right) x^{5}+\left(a\,c^{2} d+\frac{1}{3}b\,c^{3}\right) x^{3}+a\,c^{3} x$	71
default	$\frac{b\,d^{3} x^{9}}{9}+\frac{\left(a\,d^{3}+3 b c\,d^{2}\right) x^{7}}{7}+\frac{\left(3 a c\,d^{2}+3 b\,c^{2} d\right) x^{5}}{5}+\frac{\left(3 a\,c^{2} d+b\,c^{3}\right) x^{3}}{3}+a\,c^{3} x$	73
gosper	$\frac{1}{9}b\,d^{3} x^{9}+\frac{1}{7}x^{7} a\,d^{3}+\frac{3}{7}x^{7} b c\,d^{2}+\frac{3}{5}x^{5} a c\,d^{2}+\frac{3}{5}x^{5} b\,c^{2} d+x^{3} a\,c^{2} d+\frac{1}{3}b\,c^{3} x^{3}+a\,c^{3} x$	74
risch	$\frac{1}{9}b\,d^{3} x^{9}+\frac{1}{7}x^{7} a\,d^{3}+\frac{3}{7}x^{7} b c\,d^{2}+\frac{3}{5}x^{5} a c\,d^{2}+\frac{3}{5}x^{5} b\,c^{2} d+x^{3} a\,c^{2} d+\frac{1}{3}b\,c^{3} x^{3}+a\,c^{3} x$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d^3*x^9+1/7*(a*d^3+3*b*c*d^2)*x^7+1/5*(3*a*c*d^2+3*b*c^2*d)*x^5+1/3*(
3*a*c^2*d+b*c^3)*x^3+a*c^3*x
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.00

$$\frac{1}{9}\,bd^{3}x^{9}+\frac{1}{7}\,\left(3\,bcd^{2}+ad^{3}\right)x^{7}+\frac{3}{5}\,\left(bc^{2}d+acd^{2}\right)x^{5}+ac^{3}x+\frac{1}{3}\,\left(bc^{3}+3\,ac^{2}d\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5 +
 a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

Fricas [A]
time = 0.81, size = 70, normalized size = 1.00

$$\frac{1}{9}\,bd^{3}x^{9}+\frac{1}{7}\,\left(3\,bcd^{2}+ad^{3}\right)x^{7}+\frac{3}{5}\,\left(bc^{2}d+acd^{2}\right)x^{5}+ac^{3}x+\frac{1}{3}\,\left(bc^{3}+3\,ac^{2}d\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5 +
 a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

Sympy [A]
time = 0.01, size = 76, normalized size = 1.09

$$ac^{3}x+\frac{bd^{3}x^{9}}{9}+x^{7}\left(\frac{ad^{3}}{7}+\frac{3bcd^{2}}{7}\right)+x^{5}\cdot\left(\frac{3acd^{2}}{5}+\frac{3bc^{2}d}{5}\right)+x^{3}\left(ac^{2}d+\frac{bc^{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**9/9 + x**7*(a*d**3/7 + 3*b*c*d**2/7) + x**5*(3*a*c*d**
2/5 + 3*b*c**2*d/5) + x**3*(a*c**2*d + b*c**3/3)
```

Giac [A]
time = 0.73, size = 73, normalized size = 1.04

$$\frac{1}{9}\,bd^3x^9+\frac{3}{7}\,bcd^2x^7+\frac{1}{7}\,ad^3x^7+\frac{3}{5}\,bc^2dx^5+\frac{3}{5}\,acd^2x^5+\frac{1}{3}\,bc^3x^3+ac^2dx^3+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b*d^3*x^9 + 3/7*b*c*d^2*x^7 + 1/7*a*d^3*x^7 + 3/5*b*c^2*d*x^5 + 3/5*a*c
*d^2*x^5 + 1/3*b*c^3*x^3 + a*c^2*d*x^3 + a*c^3*x
```

Mupad [B]
time = 4.75, size = 65, normalized size = 0.93

$$x^3\left(\frac{b\,c^3}{3}+a\,d\,c^2\right)+x^7\left(\frac{a\,d^3}{7}+\frac{3\,b\,c\,d^2}{7}\right)+\frac{b\,d^3\,x^9}{9}+a\,c^3\,x+\frac{3\,c\,d\,x^5\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^3,x)
```

```
[Out] x^3*((b*c^3)/3 + a*c^2*d) + x^7*((a*d^3)/7 + (3*b*c*d^2)/7) + (b*d^3*x^9)/9
 + a*c^3*x + (3*c*d*x^5*(a*d + b*c))/5
```

3.3 $\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=50

$$ac^2x+\frac{1}{3}c(bc+2ad)x^3+\frac{1}{5}d(2bc+ad)x^5+\frac{1}{7}bd^2x^7$$

```
[Out] a*c^2*x+1/3*c*(2*a*d+b*c)*x^3+1/5*d*(a*d+2*b*c)*x^5+1/7*b*d^2*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^2+d(2bc+ad)x^4+bd^2x^6\right)\,dx\\ &= ac^2x+\frac{1}{3}c(bc+2ad)x^3+\frac{1}{5}d(2bc+ad)x^5+\frac{1}{7}bd^2x^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$ac^2x+\frac{1}{3}c(bc+2ad)x^3+\frac{1}{5}d(2bc+ad)x^5+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Maple [A]
time = 0.09, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b\,d^2x^7}{7}+\frac{(a\,d^2+2bcd)\,x^5}{5}+\frac{(2acd+b\,c^2)\,x^3}{3}+a\,c^2x$	49
norman	$\frac{b\,d^2x^7}{7}+\left(\frac{1}{5}a\,d^2+\frac{2}{5}bcd\right)x^5+\left(\frac{2}{3}acd+\frac{1}{3}b\,c^2\right)x^3+a\,c^2x$	49
gosper	$\frac{1}{7}b\,d^2x^7+\frac{1}{5}x^5a\,d^2+\frac{2}{5}x^5bcd+\frac{2}{3}x^3acd+\frac{1}{3}x^3b\,c^2+a\,c^2x$	51
risch	$\frac{1}{7}b\,d^2x^7+\frac{1}{5}x^5a\,d^2+\frac{2}{5}x^5bcd+\frac{2}{3}x^3acd+\frac{1}{3}x^3b\,c^2+a\,c^2x$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/7*b*d^2*x^7+1/5*(a*d^2+2*b*c*d)*x^5+1/3*(2*a*c*d+b*c^2)*x^3+a*c^2*x
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.96

$$\frac{1}{7}\,bd^2x^7+\frac{1}{5}\,\left(2\,bcd+ad^2\right)x^5+ac^2x+\frac{1}{3}\,\left(bc^2+2\,acd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="maxima")
[Out] 1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d)
*x^3
```

Fricas [A]
time = 1.39, size = 48, normalized size = 0.96

$$\frac{1}{7}\,bd^2x^7+\frac{1}{5}\,\left(2\,bcd+ad^2\right)x^5+ac^2x+\frac{1}{3}\,\left(bc^2+2\,acd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="fricas")
[Out] 1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d)
*x^3
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.06

$$ac^2x+\frac{bd^2x^7}{7}+x^5\left(\frac{ad^2}{5}+\frac{2bcd}{5}\right)+x^3\cdot\left(\frac{2acd}{3}+\frac{bc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**7/7 + x**5*(a*d**2/5 + 2*b*c*d/5) + x**3*(2*a*c*d/3 +
b*c**2/3)
```

Giac [A]
time = 1.04, size = 50, normalized size = 1.00

$$\frac{1}{7}bd^2x^7 + \frac{2}{5}bcdx^5 + \frac{1}{5}ad^2x^5 + \frac{1}{3}bc^2x^3 + \frac{2}{3}acdx^3 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b*d^2*x^7 + 2/5*b*c*d*x^5 + 1/5*a*d^2*x^5 + 1/3*b*c^2*x^3 + 2/3*a*c*d*x
^3 + a*c^2*x
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.96

$$x^3\left(\frac{b\,c^2}{3}+\frac{2\,a\,d\,c}{3}\right)+x^5\left(\frac{a\,d^2}{5}+\frac{2\,b\,c\,d}{5}\right)+\frac{b\,d^2\,x^7}{7}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2)^2,x)
```

```
[Out] x^3*((b*c^2)/3 + (2*a*c*d)/3) + x^5*((a*d^2)/5 + (2*b*c*d)/5) + (b*d^2*x^7)
/7 + a*c^2*x
```

3.4 $\int (a+bx^2)(c+dx^2)\,dx$

Optimal. Leaf size=28

$$acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5$$

```
[Out] a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {380}

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)\,dx &= \int \left(ac+(bc+ad)x^2+bdx^4\right)dx\\ &= acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^{3}}{3}+\frac{bd\,x^{5}}{5}$	25
norman	$\frac{bd\,x^{5}}{5}+\left(\frac{ad}{3}+\frac{bc}{3}\right)x^{3}+acx$	26
gosper	$\frac{1}{5}bd\,x^{5}+\frac{1}{3}x^{3}ad+\frac{1}{3}bc\,x^{3}+acx$	27
risch	$\frac{1}{5}bd\,x^{5}+\frac{1}{3}x^{3}ad+\frac{1}{3}bc\,x^{3}+acx$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.86

$$\frac{1}{5}\,bdx^{5}+\frac{1}{3}\,(bc+ad)x^{3}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

Fricas [A]
time = 0.85, size = 24, normalized size = 0.86

$$\frac{1}{5}\,bdx^{5}+\frac{1}{3}\,(bc+ad)x^{3}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.93

$$acx+\frac{bdx^{5}}{5}+x^{3}\left(\frac{ad}{3}+\frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c),x)
```

```
[Out] a*c*x + b*d*x**5/5 + x**3*(a*d/3 + b*c/3)
```

Giac [A]
time = 0.92, size = 26, normalized size = 0.93

$$\frac{1}{5}\,bdx^5 + \frac{1}{3}\,bcx^3 + \frac{1}{3}\,adx^3 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/5*b*d*x^5 + 1/3*b*c*x^3 + 1/3*a*d*x^3 + a*c*x
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^5}{5} + \left(\frac{a\,d}{3} + \frac{b\,c}{3}\right)\,x^3 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x^2),x)
```

```
[Out] x^3*((a*d)/3 + (b*c)/3) + a*c*x + (b*d*x^5)/5
```

3.5 $\int \frac{a+bx^2}{c+dx^2}\,dx$

Optimal. Leaf size=40

$$\frac{bx}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}}$$

```
[Out] b*x/d-(-a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/d^(3/2)/c^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {396, 211}

$$\frac{bx}{d}-\frac{(bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{c+dx^2}\,dx &= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+dx^2}\,dx}{d}\\ &= \frac{bx}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{bx}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2),x]
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.85

method	result	size
default	$\frac{bx}{d}+\frac{(ad-bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d\sqrt{cd}}$	34
risch	$\frac{bx}{d}-\frac{\ln\left(dx+\sqrt{-cd}\right)a}{2\sqrt{-cd}}+\frac{\ln\left(dx+\sqrt{-cd}\right)bc}{2d\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a}{2\sqrt{-cd}}-\frac{\ln\left(-dx+\sqrt{-cd}\right)bc}{2d\sqrt{-cd}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] b*x/d+(a*d-b*c)/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.52, size = 34, normalized size = 0.85

$$\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

Fricas [A]
time = 0.79, size = 99, normalized size = 2.48

$$\left[\frac{2\,bcdx+(bc-ad)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{2\,cd^2},\ \frac{bcdx-(bc-ad)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{cd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(
d*x^2 + c)))/(c*d^2), (b*c*d*x - (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
))/(c*d^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.
time = 0.14, size = 82, normalized size = 2.05

$$\frac{bx}{d} - \frac{\sqrt{-\frac{1}{cd^3}}\,(ad - bc)\log\left(-cd\sqrt{-\frac{1}{cd^3}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^3}}\,(ad - bc)\log\left(cd\sqrt{-\frac{1}{cd^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c),x)
```

```
[Out] b*x/d - sqrt(-1/(c*d**3))*(a*d - b*c)*log(-c*d*sqrt(-1/(c*d**3)) + x)/2 + s
qrt(-1/(c*d**3))*(a*d - b*c)*log(c*d*sqrt(-1/(c*d**3)) + x)/2
```

Giac [A]
time = 1.18, size = 34, normalized size = 0.85

$$\frac{bx}{d} - \frac{(bc - ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.78

$$\frac{b\,x}{d} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(a\,d - b\,c)}{\sqrt{c}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2),x)
```

```
[Out] (b*x)/d + (atan((d^(1/2)*x)/c^(1/2))*(a*d - b*c))/(c^(1/2)*d^(3/2))
```

3.6 $\int \frac{a+bx^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{(bc-ad)x}{2cd\,(c+dx^2)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*x/c/d/(d*x^2+c)+1/2*(a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(
3/2)/d^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {393, 211}

$$\frac{(ad+bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x)/(c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(2*c^(3/2)*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x}{2cd\,(c+dx^2)} + \frac{(bc+ad)\int \frac{1}{c+dx^2}\,dx}{2cd}$$
$$= -\frac{(bc-ad)x}{2cd\,(c+dx^2)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 1.00

$$-\frac{(bc-ad)x}{2cd\,(c+dx^2)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^2,x]
[Out] -1/2*((b*c - a*d)*x)/(c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(2*c^(3/2)*d^(3/2))
```

Maple [A]
time = 0.07, size = 57, normalized size = 0.90

method	result	size
default	$\frac{(ad-bc)x}{2cd(d\,x^2+c)} + \frac{(ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2cd\sqrt{cd}}$	57
risch	$\frac{(ad-bc)x}{2cd(d\,x^2+c)} - \frac{\ln\left(dx+\sqrt{-cd}\right)a}{4\sqrt{-cd}\,c} - \frac{\ln\left(dx+\sqrt{-cd}\right)b}{4\sqrt{-cd}\,d} + \frac{\ln\left(-dx+\sqrt{-cd}\right)a}{4\sqrt{-cd}\,c} + \frac{\ln\left(-dx+\sqrt{-cd}\right)b}{4\sqrt{-cd}\,d}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(a*d-b*c)/c/d*x/(d*x^2+c)+1/2*(a*d+b*c)/c/d/(c*d)^(1/2)*arctan(d*x/(c*d
)^(1/2))
```

Maxima [A]
time = 0.51, size = 57, normalized size = 0.90

$$-\frac{(bc-ad)x}{2\,(cd^2x^2+c^2d)} + \frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c - a*d)*x/(c*d^2*x^2 + c^2*d) + 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*
d))/(sqrt(c*d)*c*d)
```

Fricas [A]
time = 0.68, size = 182, normalized size = 2.89

$$\left[-\frac{\left(bc^{2}+acd+\left(bcd+ad^{2}\right)x^{2}\right)\sqrt{-cd}\,\log\left(\frac{dx^{2}-2\,\sqrt{-cd}\,x-c}{dx^{2}+c}\right)+2\,\left(bc^{2}d-acd^{2}\right)x}{4\,\left(c^{2}d^{3}x^{2}+c^{3}d^{2}\right)},\frac{\left(bc^{2}+acd+\left(bcd+ad^{2}\right)x^{2}\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-\left(bc^{2}d-acd^{2}\right)x}{2\,\left(c^{2}d^{3}x^{2}+c^{3}d^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^
2), 1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x
/c) - (b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.
time = 0.20, size = 112, normalized size = 1.78

$$\frac{x(ad-bc)}{2c^{2}d+2cd^{2}x^{2}}-\frac{\sqrt{-\frac{1}{c^{3}d^{3}}}\,(ad+bc)\log\left(-c^{2}d\sqrt{-\frac{1}{c^{3}d^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^{3}d^{3}}}\,(ad+bc)\log\left(c^{2}d\sqrt{-\frac{1}{c^{3}d^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*c**2*d + 2*c*d**2*x**2) - sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(-c**2*d*sqrt(-1/(c**3*d**3)) + x)/4 + sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(c**2*d*sqrt(-1/(c**3*d**3)) + x)/4
```

Giac [A]
time = 0.88, size = 57, normalized size = 0.90

$$\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd}-\frac{bcx-adx}{2\,(dx^{2}+c)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d) - 1/2*(b*c*x - a*d*x)
/((d*x^2 + c)*c*d)
```

Mupad [B]
time = 5.01, size = 51, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)(a\,d+b\,c)}{2\,c^{3/2}\,d^{3/2}}+\frac{x\,(a\,d-b\,c)}{2\,c\,d\,\left(d\,x^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2)^2,x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(a*d + b*c))/(2*c^(3/2)*d^(3/2)) + (x*(a*d - b*c
))/(2*c*d*(c + d*x^2))
```

3.7 $\int \frac{a+bx^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)x}{8c^2d\left(c+dx^2\right)}+\frac{(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}$$

```
[Out] -1/4*(-a*d+b*c)*x/c/d/(d*x^2+c)^2+1/8*(3*a*d+b*c)*x/c^2/d/(d*x^2+c)+1/8*(3*
a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {393, 205, 211}

$$\frac{(3ad+bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x(3ad+bc)}{8c^2d\left(c+dx^2\right)}-\frac{x(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*x)/(c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*x)/(8*c^2*d*(c +
d*x^2)) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
+ p, 0])
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = -\frac{(bc-ad)x}{4cd\,(c+dx^2)^2} + \frac{(bc+3ad)\int \frac{1}{(c+dx^2)^2}\,dx}{4cd}$$
$$= -\frac{(bc-ad)x}{4cd\,(c+dx^2)^2} + \frac{(bc+3ad)x}{8c^2d\,(c+dx^2)} + \frac{(bc+3ad)\int \frac{1}{c+dx^2}\,dx}{8c^2d}$$
$$= -\frac{(bc-ad)x}{4cd\,(c+dx^2)^2} + \frac{(bc+3ad)x}{8c^2d\,(c+dx^2)} + \frac{(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}$$

Mathematica [A]
time = 0.05, size = 82, normalized size = 0.89

$$\frac{x(bc(-c+dx^2)+ad(5c+3dx^2))}{8c^2d\,(c+dx^2)^2} + \frac{(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(b*c*(-c + d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2*d*(c + d*x^2)^2) + ((b*
c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.84

method	result
default	$\frac{\frac{(3ad+bc)x^3}{8c^2}+\frac{(5ad-bc)x}{8cd}}{(dx^2+c)^2}+\frac{(3ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d\sqrt{cd}}$
risch	$\frac{\frac{(3ad+bc)x^3}{8c^2}+\frac{(5ad-bc)x}{8cd}}{(dx^2+c)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)b}{16\sqrt{-cd}\,dc}+\frac{3\ln\left(-dx+\sqrt{-cd}\right)a}{16\sqrt{-cd}\,c^2}+\frac{\ln\left(-dx+\sqrt{-cd}\right)b}{16\sqrt{-cd}\,dc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*d+b*c)/c^2*x^3+1/8*(5*a*d-b*c)/c/d*x)/(d*x^2+c)^2+1/8*(3*a*d+b*c)
/c^2/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.52, size = 92, normalized size = 1.00

$$\frac{(bcd+3\,ad^2)x^3-(bc^2-5\,acd)x}{8\,(c^2d^3x^4+2\,c^3d^2x^2+c^4d)}+\frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b*c*d + 3*a*d^2)*x^3 - (b*c^2 - 5*a*c*d)*x)/(c^2*d^3*x^4 + 2*c^3*d^2*
x^2 + c^4*d) + 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d)
```

Fricas [A]
time = 0.54, size = 300, normalized size = 3.26

$$\left[\frac{2\,(bc^2d^2+3\,acd^3)x^3-((bcd^2+3\,ad^3)x^4+bc^3+3\,ac^2d+2\,(bc^2d+3\,acd^2)x^2)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)-2\,(bc^3d-5\,ac^2d^2)x}{16\,(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2)},\frac{(bc^2d^2+3\,acd^3)x^3+((bcd^2+3\,ad^3)x^4+bc^3+3\,ac^2d+2\,(bc^2d+3\,acd^2)x^2)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-(bc^3d-5\,ac^2d^2)x}{8\,(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(b*c^2*d^2 + 3*a*c*d^3)*x^3 - ((b*c*d^2 + 3*a*d^3)*x^4 + b*c^3 + 3
*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d
)*x - c)/(d*x^2 + c)) - 2*(b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d
^3*x^2 + c^5*d^2), 1/8*((b*c^2*d^2 + 3*a*c*d^3)*x^3 + ((b*c*d^2 + 3*a*d^3)*
x^4 + b*c^3 + 3*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(c*d)*arctan(sqr
t(c*d)*x/c) - (b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d^3*x^2 + c^5
*d^2)]
```

Sympy [A]
time = 0.28, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{c^5d^3}}\cdot(3ad+bc)\log\left(-c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^3}}\cdot(3ad+bc)\log\left(c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16}+\frac{x^3\cdot(3ad^2+bcd)+x(5acd-bc^2)}{8c^4d+16c^3d^2x^2+8c^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d**3)) + x)/1
6 + sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*d**3)) + x)
/16 + (x**3*(3*a*d**2 + b*c*d) + x*(5*a*c*d - b*c**2))/(8*c**4*d + 16*c**3*
d**2*x**2 + 8*c**2*d**3*x**4)
```

Giac [A]
time = 0.97, size = 78, normalized size = 0.85

$$\frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d}+\frac{bcdx^3+3\,ad^2x^3-bc^2x+5\,acdx}{8\,(dx^2+c)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d) + 1/8*(b*c*d*x^3
+ 3*a*d^2*x^3 - b*c^2*x + 5*a*c*d*x)/((d*x^2 + c)^2*c^2*d)
```

Mupad [B]
time = 5.06, size = 82, normalized size = 0.89

$$\frac{\frac{x^3\,\left(3\,a\,d+b\,c\right)}{8\,c^2}+\frac{x\,\left(5\,a\,d-b\,c\right)}{8\,c\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a\,d+b\,c\right)}{8\,c^{5/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*d + b*c))/(8*c^2) + (x*(5*a*d - b*c))/(8*c*d))/(c^2 + d^2*x^4 +
2*c*d*x^2) + (atan((d^(1/2)*x)/c^(1/2))*(3*a*d + b*c))/(8*c^(5/2)*d^(3/2))
```

3.8 $\int (a+bx^2)^2 (c+dx^2)^3 \, dx$

Optimal. Leaf size=122

$$a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{9}bd^2(3bc+2ad)x^9+$$

```
[Out] a^2*c^3*x+1/3*a*c^2*(3*a*d+2*b*c)*x^3+1/5*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^5+1/7*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^7+1/9*b*d^2*(2*a*d+3*b*c)*x^9+1/11
*b^2*d^3*x^11
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^2 (c+dx^2)^3 \, dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^2+c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+d\left(3b^2c^2+6abcd\right.\right.\\ &= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6ab\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 122, normalized size = 1.00

$$a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{9}bd^2(3bc+2ad)x^9+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Maple [A]
time = 0.10, size = 125, normalized size = 1.02

method	result
norman	$\frac{b^2 d^3 x^{11}}{11} + \left(\frac{2}{9} ab\,d^3 + \frac{1}{3} b^2 c\,d^2\right) x^9 + \left(\frac{1}{7} a^2 d^3 + \frac{6}{7} abc\,d^2 + \frac{3}{7} b^2 c^2 d\right) x^7 + \left(\frac{3}{5} a^2 c\,d^2 + \frac{6}{5} ab\,c^2 d + \frac{1}{5} b^2 c^3\right) x^5 +$
default	$\frac{b^2 d^3 x^{11}}{11} + \frac{(2ab\,d^3 + 3b^2 c\,d^2) x^9}{9} + \frac{(a^2 d^3 + 6abc\,d^2 + 3b^2 c^2 d) x^7}{7} + \frac{(3a^2 c\,d^2 + 6ab\,c^2 d + b^2 c^3) x^5}{5} + \frac{(3a^2 c^2 d + 2ab\,c^3) x^3}{3} + a^2 c^3 x$
gosper	$\frac{1}{11} b^2 d^3 x^{11} + \frac{2}{9} x^9 ab\,d^3 + \frac{1}{3} x^9 b^2 c\,d^2 + \frac{1}{7} x^7 a^2 d^3 + \frac{6}{7} x^7 abc\,d^2 + \frac{3}{7} x^7 b^2 c^2 d + \frac{3}{5} x^5 a^2 c\,d^2 + \frac{6}{5} x^5 ab\,c^2 d + \frac{1}{5}$
risch	$\frac{1}{11} b^2 d^3 x^{11} + \frac{2}{9} x^9 ab\,d^3 + \frac{1}{3} x^9 b^2 c\,d^2 + \frac{1}{7} x^7 a^2 d^3 + \frac{6}{7} x^7 abc\,d^2 + \frac{3}{7} x^7 b^2 c^2 d + \frac{3}{5} x^5 a^2 c\,d^2 + \frac{6}{5} x^5 ab\,c^2 d + \frac{1}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*a*b*c*d^2+
3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^5+1/3*(3*a^2*c^2*d
+2*a*b*c^3)*x^3+a^2*c^3*x
```

Maxima [A]
time = 0.27, size = 124, normalized size = 1.02

$$\frac{1}{11}\,b^2d^3x^{11} + \frac{1}{9}\,(3\,b^2cd^2 + 2\,abd^3)x^9 + \frac{1}{7}\,(3\,b^2c^2d + 6\,abcd^2 + a^2d^3)x^7 + a^2c^3x + \frac{1}{5}\,(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2)x^5 + \frac{1}{3}\,(2\,abc^3 + 3\,a^2c^2d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Fricas [A]
time = 0.73, size = 124, normalized size = 1.02

$$\frac{1}{11}\,b^2d^3x^{11} + \frac{1}{9}\,(3\,b^2cd^2 + 2\,abd^3)x^9 + \frac{1}{7}\,(3\,b^2c^2d + 6\,abcd^2 + a^2d^3)x^7 + a^2c^3x + \frac{1}{5}\,(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2)x^5 + \frac{1}{3}\,(2\,abc^3 + 3\,a^2c^2d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.11

$$a^2c^3x+\frac{b^2d^3x^{11}}{11}+x^9\cdot\left(\frac{2abd^3}{9}+\frac{b^2cd^2}{3}\right)+x^7\left(\frac{a^2d^3}{7}+\frac{6abcd^2}{7}+\frac{3b^2c^2d}{7}\right)+x^5\cdot\left(\frac{3a^2cd^2}{5}+\frac{6abc^2d}{5}+\frac{b^2c^3}{5}\right)+x^3\left(a^2c^2d+\frac{2abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x*
*7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2/5
 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

Giac [A]
time = 1.96, size = 131, normalized size = 1.07

$$\frac{1}{11}b^2d^3x^{11}+\frac{1}{3}b^2cd^2x^9+\frac{2}{9}abd^3x^9+\frac{3}{7}b^2c^2dx^7+\frac{6}{7}abcd^2x^7+\frac{1}{7}a^2d^3x^7+\frac{1}{5}b^2c^3x^5+\frac{6}{5}abc^2dx^5+\frac{3}{5}a^2cd^2x^5+\frac{2}{3}abc^3x^3+a^2c^2dx^3+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^7
 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*x^
5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

Mupad [B]
time = 4.95, size = 116, normalized size = 0.95

$$x^5\left(\frac{3\,a^2\,c\,d^2}{5}+\frac{6\,a\,b\,c^2\,d}{5}+\frac{b^2\,c^3}{5}\right)+x^7\left(\frac{a^2\,d^3}{7}+\frac{6\,a\,b\,c\,d^2}{7}+\frac{3\,b^2\,c^2\,d}{7}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{11}}{11}+\frac{a\,c^2\,x^3\,(3\,a\,d+2\,b\,c)}{3}+\frac{b\,d^2\,x^9\,(2\,a\,d+3\,b\,c)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^5*((b^2*c^3)/5 + (3*a^2*c*d^2)/5 + (6*a*b*c^2*d)/5) + x^7*((a^2*d^3)/7 +
(3*b^2*c^2*d)/7 + (6*a*b*c*d^2)/7) + a^2*c^3*x + (b^2*d^3*x^11)/11 + (a*c^2
*x^3*(3*a*d + 2*b*c))/3 + (b*d^2*x^9*(2*a*d + 3*b*c))/9
```

3.9 $\int (a+bx^2)^2 (c+dx^2)^2 \, dx$

Optimal. Leaf size=82

$$a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9$$

```
[Out] a^2*c^2*x+2/3*a*c*(a*d+b*c)*x^3+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^5+2/7*b*d
*(a*d+b*c)*x^7+1/9*b^2*d^2*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx&=\int \left(a^2c^2+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^4+2bd(bc+ad)x^6+b^2d^2\right.\\&=a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Maple [A]
time = 0.12, size = 87, normalized size = 1.06

method	result	s
norman	$\frac{b^2d^2x^9}{9}+\left(\frac{2}{7}ab\,d^2+\frac{2}{7}b^2cd\right)x^7+\left(\frac{1}{5}a^2d^2+\frac{4}{5}abcd+\frac{1}{5}b^2c^2\right)x^5+\left(\frac{2}{3}a^2cd+\frac{2}{3}ab\,c^2\right)x^3+a^2c^2x$	8
default	$\frac{b^2d^2x^9}{9}+\frac{\left(2ab\,d^2+2b^2cd\right)x^7}{7}+\frac{\left(a^2d^2+4abcd+b^2c^2\right)x^5}{5}+\frac{\left(2a^2cd+2ab\,c^2\right)x^3}{3}+a^2c^2x$	8
gosper	$\frac{1}{9}b^2d^2x^9+\frac{2}{7}x^7ab\,d^2+\frac{2}{7}x^7b^2cd+\frac{1}{5}x^5a^2d^2+\frac{4}{5}x^5abcd+\frac{1}{5}x^5b^2c^2+\frac{2}{3}x^3a^2cd+\frac{2}{3}x^3ab\,c^2+a^2c^2x$	9
risch	$\frac{1}{9}b^2d^2x^9+\frac{2}{7}x^7ab\,d^2+\frac{2}{7}x^7b^2cd+\frac{1}{5}x^5a^2d^2+\frac{4}{5}x^5abcd+\frac{1}{5}x^5b^2c^2+\frac{2}{3}x^3a^2cd+\frac{2}{3}x^3ab\,c^2+a^2c^2x$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

Maxima [A]
time = 0.30, size = 82, normalized size = 1.00

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\,\left(b^2cd+abd^2\right)x^7+\frac{1}{5}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+a^2c^2x+\frac{2}{3}\,\left(abc^2+a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Fricas [A]
time = 1.35, size = 82, normalized size = 1.00

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\,\left(b^2cd+abd^2\right)x^7+\frac{1}{5}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+a^2c^2x+\frac{2}{3}\,\left(abc^2+a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Sympy [A]
time = 0.01, size = 97, normalized size = 1.18

$$a^2c^2x+\frac{b^2d^2x^9}{9}+x^7\cdot\left(\frac{2abd^2}{7}+\frac{2b^2cd}{7}\right)+x^5\left(\frac{a^2d^2}{5}+\frac{4abcd}{5}+\frac{b^2c^2}{5}\right)+x^3\cdot\left(\frac{2a^2cd}{3}+\frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5*
(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c**2
/3)
```

Giac [A]
time = 1.78, size = 91, normalized size = 1.11

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\,b^2cdx^7+\frac{2}{7}\,abd^2x^7+\frac{1}{5}\,b^2c^2x^5+\frac{4}{5}\,abcdx^5+\frac{1}{5}\,a^2d^2x^5+\frac{2}{3}\,abc^2x^3+\frac{2}{3}\,a^2cdx^3+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/5
*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*c^
2*x
```

Mupad [B]
time = 0.05, size = 75, normalized size = 0.91

$$x^5\left(\frac{a^2\,d^2}{5}+\frac{4\,a\,b\,c\,d}{5}+\frac{b^2\,c^2}{5}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^9}{9}+\frac{2\,a\,c\,x^3\,(a\,d+b\,c)}{3}+\frac{2\,b\,d\,x^7\,(a\,d+b\,c)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^2*d^2)/5 + (b^2*c^2)/5 + (4*a*b*c*d)/5) + a^2*c^2*x + (b^2*d^2*x^9)
/9 + (2*a*c*x^3*(a*d + b*c))/3 + (2*b*d*x^7*(a*d + b*c))/7
```

3.10 $\int (a+bx^2)^2 (c+dx^2)\, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7$$

```
[Out] a^2*c*x+1/3*a*(a*d+2*b*c)*x^3+1/5*b*(2*a*d+b*c)*x^5+1/7*b^2*d*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{380\}$

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)\, dx &= \int \left(a^2c+a(2bc+ad)x^2+b(bc+2ad)x^4+b^2dx^6\right)\, dx\\ &= a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Maple [A]
time = 0.10, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b^2 d x^7}{7}+\frac{(2abd+b^2c)x^5}{5}+\frac{(a^2d+2abc)x^3}{3}+a^2cx$	49
norman	$\frac{b^2 d x^7}{7}+\left(\frac{2}{5}abd+\frac{1}{5}b^2c\right)x^5+\left(\frac{1}{3}a^2d+\frac{2}{3}abc\right)x^3+a^2cx$	49
gosper	$\frac{1}{7}b^2 d x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}x^3abc+a^2cx$	51
risch	$\frac{1}{7}b^2 d x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}x^3abc+a^2cx$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.96

$$\frac{1}{7}b^2dx^7+\frac{1}{5}\left(b^2c+2\,abd\right)x^5+a^2cx+\frac{1}{3}\left(2\,abc+a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Fricas [A]
time = 1.17, size = 48, normalized size = 0.96

$$\frac{1}{7}b^2dx^7+\frac{1}{5}\left(b^2c+2\,abd\right)x^5+a^2cx+\frac{1}{3}\left(2\,abc+a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.06

$$a^2cx+\frac{b^2dx^7}{7}+x^5\cdot\left(\frac{2abd}{5}+\frac{b^2c}{5}\right)+x^3\left(\frac{a^2d}{3}+\frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 + 2
*a*b*c/3)
```

Giac [A]
time = 2.60, size = 50, normalized size = 1.00

$$\frac{1}{7}b^2dx^7 + \frac{1}{5}b^2cx^5 + \frac{2}{5}abdx^5 + \frac{2}{3}abcx^3 + \frac{1}{3}a^2dx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*x
^3 + a^2*c*x
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.96

$$x^3\left(\frac{d\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^5\left(\frac{c\,b^2}{5}+\frac{2\,a\,d\,b}{5}\right)+\frac{b^2\,d\,x^7}{7}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^3*((a^2*d)/3 + (2*a*b*c)/3) + x^5*((b^2*c)/5 + (2*a*b*d)/5) + (b^2*d*x^7)
/7 + a^2*c*x
```

3.11 $\int \frac{(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=63

$$-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/3*b^2*x^3/d+(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/
d^(5/2)/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {398, 211}

$$\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2} + \frac{b^2x^2}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2\,(c+dx^2)}\right)dx$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^3}{3d} + \frac{(bc-ad)^2 \int \frac{1}{c+dx^2}\,dx}{d^2}$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^3}{3d} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}$$

<u>**Mathematica**</u> [A]
time = 0.04, size = 59, normalized size = 0.94

$$\frac{bx(-3bc+6ad+bdx^2)}{3d^2} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2),x]
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

<u>**Maple**</u> [A]
time = 0.08, size = 64, normalized size = 1.02

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^3+2adx-bcx\right)}{d^2} + \frac{(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^2\sqrt{cd}}$
risch	$\frac{b^2x^3}{3d} + \frac{2bax}{d} - \frac{b^2cx}{d^2} - \frac{\ln\left(dx+\sqrt{-cd}\right)a^2}{2\sqrt{-cd}} + \frac{\ln\left(dx+\sqrt{-cd}\right)abc}{d\sqrt{-cd}} - \frac{\ln\left(dx+\sqrt{-cd}\right)b^2c^2}{2d^2\sqrt{-cd}} + \frac{\ln\left(-dx+\sqrt{-cd}\right)}{2\sqrt{-cd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] b/d^2*(1/3*b*d*x^3+2*a*d*x-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2/(c*d)^(1/
2)*arctan(d*x/(c*d)^(1/2))
```

<u>**Maxima**</u> [A]
time = 0.52, size = 68, normalized size = 1.08

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2} + \frac{b^2dx^3-3\,(b^2c-2\,abd)x}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)/d^2
```

Fricas [A]
time = 0.95, size = 179, normalized size = 2.84

$$\left[\frac{2\,b^{2}cd^{2}x^{3} - 3\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{-cd}\,\log\left(\frac{dx^{2} - 2\,\sqrt{-cd}\,x - c}{dx^{2} + c}\right) - 6\,(b^{2}c^{2}d - 2\,abcd^{2})x}{6\,cd^{3}}, \frac{b^{2}cd^{2}x^{3} + 3\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) - 3\,(b^{2}c^{2}d - 2\,abcd^{2})x}{3\,cd^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((d
*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c
*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.
time = 0.22, size = 172, normalized size = 2.73

$$\frac{b^{2}x^{3}}{3d} + x\left(\frac{2ab}{d} - \frac{b^{2}c}{d^{2}}\right) - \frac{\sqrt{-\frac{1}{cd^{5}}}\,(ad - bc)^{2}\log\left(-\frac{cd^{2}\sqrt{-\frac{1}{cd^{5}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^{5}}}\,(ad - bc)^{2}\log\left(\frac{cd^{2}\sqrt{-\frac{1}{cd^{5}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) + x*(2*a*b/d - b**2*c/d**2) - sqrt(-1/(c*d**5))*(a*d - b*c)
**2*log(-c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b
**2*c**2) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c*d
**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

Giac [A]
time = 1.77, size = 72, normalized size = 1.14

$$\frac{(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^{2}} + \frac{b^{2}d^{2}x^{3} - 3\,b^{2}cdx + 6\,abd^{2}x}{3\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

Mupad [B]
time = 0.09, size = 90, normalized size = 1.43

$$\frac{b^2\,x^3}{3\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2),x)
```

```
[Out] (b^2*x^3)/(3*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((d^(1/2)*x*(a*d - b*c
)^2)/(c^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(1/2)*d^(
5/2))
```

3.12 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=82

$$\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

```
[Out] b^2*x/d^2+1/2*(-a*d+b*c)^2*x/c/d^2/(d*x^2+c)-1/2*(-a*d+b*c)*(a*d+3*b*c)*arc
tan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {398, 393, 211}

$$-\frac{(bc-ad)(ad+3bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx &= \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{d^2\,(c+dx^2)^2}\right)dx\\ &= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{(c+dx^2)^2}\,dx}{d^2}\\ &= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\,(c+dx^2)}-\frac{((bc-ad)(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2cd^2}\\ &= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\,(c+dx^2)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 1.09

$$\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\,(c+dx^2)}-\frac{(3b^2c^2-2abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*b
*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Maple [A]
time = 0.08, size = 92, normalized size = 1.12

method	result
default	$\frac{b^2x}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{2c\left(d\,x^2+c\right)}+\frac{\left(a^2d^2+2abcd-3b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}}{d^2}$
risch	$\frac{b^2x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2c\,d^2(d\,x^2+c)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^2}{4\sqrt{-cd}\;c}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{2d\sqrt{-cd}}+\frac{3c\ln\left(dx+\sqrt{-cd}\right)b^2}{4d^2\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-\right.}}{4\sqrt{-cd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^2+1/d^2*(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/(d*x^2+c)+1/2*(a^2*d^2
+2*a*b*c*d-3*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.50, size = 96, normalized size = 1.17

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)x}{2\,(cd^3x^2 + c^2d^2)} + \frac{b^2x}{d^2} - \frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^2 + c^2*d^2) + b^2*x/d^2 - 1
/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^2
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(70) = 140.
time = 0.72, size = 302, normalized size = 3.68

$$\left[\frac{4\,b^2c^2d^2x^3 + (3\,b^2c^3 - 2\,abc^2d - a^2cd^2 + (3\,b^2c^2d - 2\,abcd^2 - a^2d^3)x^2)\sqrt{-cd}\,\log\left(\frac{dx^2 - 2\,\sqrt{-cd}\,x - c}{dx^2 + c}\right) + 2\,(3\,b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x}{4\,(c^2d^4x^2 + c^3d^3)}, \frac{2\,b^2c^2d^2x^3 - (3\,b^2c^3 - 2\,abc^2d - a^2cd^2 + (3\,b^2c^2d - 2\,abcd^2 - a^2d^3)x^2)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) + (3\,b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x}{2\,(c^2d^4x^2 + c^3d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c
)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^
2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*d^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(73) = 146.
time = 0.38, size = 236, normalized size = 2.88

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{2c^2d^2 + 2cd^3x^2} - \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad - bc)\,(ad + 3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad - bc)\,(ad + 3bc)\log\left(\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**3
*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sqrt
(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*
c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2
```

```
*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*
b**2*c**2) + x)/4
```

Giac [A]
time = 2.06, size = 95, normalized size = 1.16

$$\frac{b^2x}{d^2} - \frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2+c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c*d
^2)
```

Mupad [B]
time = 5.02, size = 124, normalized size = 1.51

$$\frac{b^2\,x}{d^2} + \frac{x\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}{2\,c\,(d^3\,x^2 + c\,d^2)} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{c}\,(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,c^{3/2}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2)) +
 (atan((d^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(c^(1/2)*(a^2*d^2 - 3*b^2*c^2
+ 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*c^(3/2)*d^(5/2))
```

3.13 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=116

$$-\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^2+a)/c/d/(d*x^2+c)^2+3/8*(a^2/c^2-b^2/d^2)*x/(d*x^2+
c)+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^
(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {424, 393, 211}

$$\frac{(3a^2d^2+2abcd+3b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}+\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\,(c+dx^2)}-\frac{x(a+bx^2)\,(bc-ad)}{4cd\,(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^2))/(c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^
2)*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx &= -\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{\int \frac{a(bc+3ad)+b(3bc+ad)x^2}{(c+dx^2)^2}\,dx}{4cd}\\ &= -\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)}+\frac{(3b^2c^2+2abcd+3a^2d^2)\int \frac{1}{c+dx^2}\,dx}{8c^2d^2}\\ &= -\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 121, normalized size = 1.04

$$\frac{x(-2abcd(c-dx^2)+a^2d^2(5c+3dx^2)-b^2c^2(3c+5dx^2))}{8c^2d^2\,(c+dx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3*c + 5*d*x
^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Maple [A]
time = 0.09, size = 124, normalized size = 1.07

method	result
default	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8d^2c}}{(d\,x^2+c)^2}+\frac{(3a^2d^2+2abcd+3b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d^2\sqrt{cd}}$
risch	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8d^2c}}{(d\,x^2+c)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{8\sqrt{-cd}\,dc}-\frac{3\ln\left(dx+\sqrt{-cd}\right)b^2}{16\sqrt{-cd}\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*b
^2*c^2)/d^2/c*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/c^2/d^2/(c
*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.51, size = 138, normalized size = 1.19

$$-\frac{(5\,b^2c^2d-2\,abcd^2-3\,a^2d^3)x^3+(3\,b^2c^3+2\,abc^2d-5\,a^2cd^2)x}{8\,(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2)}+\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c^3 + 2*a*b*c^2*
d - 5*a^2*c*d^2)*x)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1/8*(3*b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(102) = 204.
time = 0.78, size = 449, normalized size = 3.87

$$\left[-\frac{2\,(5\,b^2c^3d^2-2\,abc^2d^3-3\,a^2cd^4)x^3+(3\,b^2c^4+2\,abc^3d+3\,a^2c^2d^2+(3\,b^2c^2d^2+2\,abcd^3+3\,a^2d^4)x^4+2\,(3\,b^2c^3d+2\,abc^2d^2+3\,a^2cd^3)x^2)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\,(3\,b^2c^4d+2\,abc^3d^2-5\,a^2c^2d^3)x}{16\,(c^3d^5x^4+2\,c^4d^4x^2+c^5d^3)},\ -\frac{(5\,b^2c^3d^2-2\,abc^2d^3-3\,a^2cd^4)x^3-(3\,b^2c^4+2\,abc^3d+3\,a^2c^2d^2+(3\,b^2c^2d^2+2\,abcd^3+3\,a^2d^4)x^4+2\,(3\,b^2c^3d+2\,abc^2d^2+3\,a^2cd^3)x^2)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+(3\,b^2c^4d+2\,abc^3d^2-5\,a^2c^2d^3)x}{8\,(c^3d^5x^4+2\,c^4d^4x^2+c^5d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2
 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^
2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^3*d^2
 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*
d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*
b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*b^2*c^4*
d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^
3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(110) = 220.
time = 0.54, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{x^3\cdot(3a^2d^3+2abcd^2-5b^2c^2d)+x(5a^2cd^2-2abc^2d-3b^2c^3)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d**
2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3*a*
*2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d -
 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

Giac [A]
time = 2.42, size = 126, normalized size = 1.09

$$\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^2}-\frac{5\,b^2c^2dx^3-2\,abcd^2x^3-3\,a^2d^3x^3+3\,b^2c^3x+2\,abc^2dx-5\,a^2cd^2x}{8\,(dx^2+c)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^
2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c^3
*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

Mupad [B]
time = 5.03, size = 130, normalized size = 1.12

$$\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{5/2}}-\frac{\frac{x\,\left(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c\,d^2}-\frac{x^3\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2\right)}{8\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^3,x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*c^(5/2)*
d^(5/2)) - ((x*(3*b^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*c*d^2) - (x^3*(3*a^2
*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

3.14 $\int (a+bx^2)^3 (c+dx^2)^3 \, dx$

Optimal. Leaf size=154

$$a^3c^3x+a^2c^2(bc+ad)x^3+\frac{3}{5}ac\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{7}(bc+ad)\left(b^2c^2+8abcd+a^2d^2\right)x^7+\frac{1}{3}bd\left(b^2c^2+3abcd\right.$$

```
[Out] a^3*c^3*x+a^2*c^2*(a*d+b*c)*x^3+3/5*a*c*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^5+1/7
*(a*d+b*c)*(a^2*d^2+8*a*b*c*d+b^2*c^2)*x^7+1/3*b*d*(a^2*d^2+3*a*b*c*d+b^2*c
^2)*x^9+3/11*b^2*d^2*(a*d+b*c)*x^11+1/13*b^3*d^3*x^13
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$a^3c^3x+\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+\frac{3}{11}b^2d^2x^{11}(ad+bc)+\frac{1}{13}b^3d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b^2*
c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11 + (b^3*
d^3*x^13)/13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^3 \, dx = \int \left(a^3c^3+3a^2c^2(bc+ad)x^2+3ac\left(b^2c^2+3abcd+a^2d^2\right)x^4+(bc+ad)\left(b^2c^2+\right.\right.$$
$$= a^3c^3x+a^2c^2(bc+ad)x^3+\frac{3}{5}ac\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{7}(bc+ad)\left(b^2c^2+\right.$$

Mathematica [A]
time = 0.02, size = 161, normalized size = 1.05

$$a^3c^3x+a^2c^2(bc+ad)x^3+\frac{3}{5}ac\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{7}\left(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3\right)x^7+\frac{1}{3}bd\left(b^2c^2+3abcd+a^2d^2\right)x^9+\frac{3}{11}b^2d^2(bc+ad)x^{11}+\frac{1}{13}b^3d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7)/7 + (b
*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11
 + (b^3*d^3*x^13)/13
```

Maple [A]
time = 0.10, size = 177, normalized size = 1.15

method	result
norman	$\frac{b^3d^3x^{13}}{13}+\left(\frac{3}{11}a\,b^2d^3+\frac{3}{11}b^3c\,d^2\right)x^{11}+\left(\frac{1}{3}a^2b\,d^3+a\,b^2c\,d^2+\frac{1}{3}b^3c^2d\right)x^9+\left(\frac{1}{7}a^3d^3+\frac{9}{7}a^2bc\,d^2+\frac{9}{7}a\,b$
default	$\frac{b^3d^3x^{13}}{13}+\frac{(3a\,b^2d^3+3b^3c\,d^2)x^{11}}{11}+\frac{(3a^2b\,d^3+9a\,b^2c\,d^2+3b^3c^2d)x^9}{9}+\frac{(a^3d^3+9a^2bc\,d^2+9a\,b^2c^2d+b^3c^3)x^7}{7}+\frac{(3a^3c\,d^2+9a^2b\,c}{5}$
gosper	$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}x^{11}a\,b^2d^3+\frac{3}{11}x^{11}b^3c\,d^2+\frac{1}{3}x^9a^2b\,d^3+x^9a\,b^2c\,d^2+\frac{1}{3}x^9b^3c^2d+\frac{1}{7}x^7a^3d^3+\frac{9}{7}x^7a^2bc$
risch	$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}x^{11}a\,b^2d^3+\frac{3}{11}x^{11}b^3c\,d^2+\frac{1}{3}x^9a^2b\,d^3+x^9a\,b^2c\,d^2+\frac{1}{3}x^9b^3c^2d+\frac{1}{7}x^7a^3d^3+\frac{9}{7}x^7a^2bc$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^3*d^3*x^13+1/11*(3*a*b^2*d^3+3*b^3*c*d^2)*x^11+1/9*(3*a^2*b*d^3+9*a*
b^2*c*d^2+3*b^3*c^2*d)*x^9+1/7*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3
)*x^7+1/5*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^5+1/3*(3*a^3*c^2*d+3*a^
2*b*c^3)*x^3+a^3*c^3*x
```

Maxima [A]
time = 0.35, size = 167, normalized size = 1.08

$$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}(b^3cd^2+ab^2d^3)x^{11}+\frac{1}{3}(b^3c^2d+3\,ab^2cd^2+a^2bd^3)x^9+\frac{1}{7}(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)x^7+a^3c^3x+\frac{3}{5}(ab^2c^3+3\,a^2bc^2d+a^3cd^2)x^5+(a^2bc^3+a^3c^2d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2
 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x
^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

Fricas [A]
time = 0.73, size = 167, normalized size = 1.08

$$\frac{1}{13}b^3d^3x^{13}+\frac{3}{11}(b^3cd^2+ab^2d^3)x^{11}+\frac{1}{3}(b^3c^2d+3\,ab^2cd^2+a^2bd^3)x^9+\frac{1}{7}(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)x^7+a^3c^3x+\frac{3}{5}(ab^2c^3+3\,a^2bc^2d+a^3cd^2)x^5+(a^2bc^3+a^3c^2d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2
 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x
^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

Sympy [A]
time = 0.02, size = 189, normalized size = 1.23

$$a^3c^3x + \frac{b^3d^3x^{13}}{13} + x^{11} \cdot \left(\frac{3ab^2d^3}{11} + \frac{3b^3cd^2}{11}\right) + x^9\left(\frac{a^2bd^3}{3} + ab^2cd^2 + \frac{b^3c^2d}{3}\right) + x^7\left(\frac{a^3d^3}{7} + \frac{9a^2bcd^2}{7} + \frac{9ab^2c^2d}{7} + \frac{b^3c^3}{7}\right) + x^5 \cdot \left(\frac{3a^3cd^2}{5} + \frac{9a^2bc^2d}{5} + \frac{3ab^2c^3}{5}\right) + x^3\left(a^3c^2d + a^2bc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**13/13 + x**11*(3*a*b**2*d**3/11 + 3*b**3*c*d**2/
11) + x**9*(a**2*b*d**3/3 + a*b**2*c*d**2 + b**3*c**2*d/3) + x**7*(a**3*d**
3/7 + 9*a**2*b*c*d**2/7 + 9*a*b**2*c**2*d/7 + b**3*c**3/7) + x**5*(3*a**3*c
*d**2/5 + 9*a**2*b*c**2*d/5 + 3*a*b**2*c**3/5) + x**3*(a**3*c**2*d + a**2*b
*c**3)
```

Giac [A]
time = 1.63, size = 187, normalized size = 1.21

$$\frac{1}{13}b^3d^3x^{13} + \frac{3}{11}b^3cd^2x^{11} + \frac{3}{11}ab^2d^3x^{11} + \frac{1}{3}b^3c^2dx^9 + ab^2cd^2x^9 + \frac{1}{3}a^2bd^3x^9 + \frac{1}{7}b^3c^3x^7 + \frac{9}{7}ab^2c^2dx^7 + \frac{9}{7}a^2bcd^2x^7 + \frac{1}{7}a^3d^3x^7 + \frac{3}{5}ab^2c^3x^5 + \frac{9}{5}a^2bc^2dx^5 + \frac{3}{5}a^3cd^2x^5 + a^2bc^3x^3 + a^3c^2dx^3 + a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*b^3*c*d^2*x^11 + 3/11*a*b^2*d^3*x^11 + 1/3*b^3*c^2
*d*x^9 + a*b^2*c*d^2*x^9 + 1/3*a^2*b*d^3*x^9 + 1/7*b^3*c^3*x^7 + 9/7*a*b^2*
c^2*d*x^7 + 9/7*a^2*b*c*d^2*x^7 + 1/7*a^3*d^3*x^7 + 3/5*a*b^2*c^3*x^5 + 9/5
*a^2*b*c^2*d*x^5 + 3/5*a^3*c*d^2*x^5 + a^2*b*c^3*x^3 + a^3*c^2*d*x^3 + a^3*
c^3*x
```

Mupad [B]
time = 4.90, size = 152, normalized size = 0.99

$$x^7\left(\frac{a^3\,d^3}{7} + \frac{9\,a^2\,b\,c\,d^2}{7} + \frac{9\,a\,b^2\,c^2\,d}{7} + \frac{b^3\,c^3}{7}\right) + a^3\,c^3\,x + \frac{b^3\,d^3\,x^{13}}{13} + \frac{3\,a\,c\,x^5\,\left(a^2\,d^2 + 3\,a\,b\,c\,d + b^2\,c^2\right)}{5} + \frac{b\,d\,x^9\,\left(a^2\,d^2 + 3\,a\,b\,c\,d + b^2\,c^2\right)}{3} + a^2\,c^2\,x^3\,\left(a\,d + b\,c\right) + \frac{3\,b^2\,d^2\,x^{11}\,\left(a\,d + b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(c + d*x^2)^3,x)
```

```
[Out] x^7*((a^3*d^3)/7 + (b^3*c^3)/7 + (9*a*b^2*c^2*d)/7 + (9*a^2*b*c*d^2)/7) + a
^3*c^3*x + (b^3*d^3*x^13)/13 + (3*a*c*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/
5 + (b*d*x^9*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/3 + a^2*c^2*x^3*(a*d + b*c) +
 (3*b^2*d^2*x^11*(a*d + b*c))/11
```

3.15 $\int (a+bx^2)^3 (c+dx^2)^2 \, dx$

Optimal. Leaf size=122

$$a^3c^2x+\frac{1}{3}a^2c(3bc+2ad)x^3+\frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}b\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}b^2d(2bc+3ad)x^9+$$

```
[Out] a^3*c^2*x+1/3*a^2*c*(2*a*d+3*b*c)*x^3+1/5*a*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x
^5+1/7*b*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^7+1/9*b^2*d*(3*a*d+2*b*c)*x^9+1/11
*b^3*d^2*x^11
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$a^3c^2x+\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^3 (c+dx^2)^2 \, dx &= \int \left(a^3c^2+a^2c(3bc+2ad)x^2+a\left(3b^2c^2+6abcd+a^2d^2\right)x^4+b\left(b^2c^2+6abcd\right.\right.\\ &= a^3c^2x+\frac{1}{3}a^2c(3bc+2ad)x^3+\frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}b\left(b^2c^2+6abc\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 122, normalized size = 1.00

$$a^3c^2x+\frac{1}{3}a^2c(3bc+2ad)x^3+\frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}b\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}b^2d(2bc+3ad)x^9+\frac{1}{11}b^3d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Maple [A]
time = 0.11, size = 125, normalized size = 1.02

method	result
norman	$\frac{b^3d^2x^{11}}{11}+\left(\frac{1}{3}a\,b^2d^2+\frac{2}{9}b^3cd\right)x^9+\left(\frac{3}{7}a^2b\,d^2+\frac{6}{7}a\,b^2cd+\frac{1}{7}b^3c^2\right)x^7+\left(\frac{1}{5}a^3d^2+\frac{6}{5}a^2bcd+\frac{3}{5}a\,b^2c^2\right)x^5$
default	$\frac{b^3d^2x^{11}}{11}+\frac{(3a\,b^2d^2+2b^3cd)x^9}{9}+\frac{(3a^2b\,d^2+6a\,b^2cd+b^3c^2)x^7}{7}+\frac{(a^3d^2+6a^2bcd+3a\,b^2c^2)x^5}{5}+\frac{(2a^3cd+3a^2b\,c^2)x^3}{3}+a^3c^2x$
gosper	$\frac{1}{11}b^3d^2x^{11}+\frac{1}{3}x^9a\,b^2d^2+\frac{2}{9}x^9b^3cd+\frac{3}{7}x^7a^2b\,d^2+\frac{6}{7}x^7a\,b^2cd+\frac{1}{7}x^7b^3c^2+\frac{1}{5}x^5a^3d^2+\frac{6}{5}x^5a^2bcd+\frac{3}{5}x^5$
risch	$\frac{1}{11}b^3d^2x^{11}+\frac{1}{3}x^9a\,b^2d^2+\frac{2}{9}x^9b^3cd+\frac{3}{7}x^7a^2b\,d^2+\frac{6}{7}x^7a\,b^2cd+\frac{1}{7}x^7b^3c^2+\frac{1}{5}x^5a^3d^2+\frac{6}{5}x^5a^2bcd+\frac{3}{5}x^5$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^3*d^2*x^11+1/9*(3*a*b^2*d^2+2*b^3*c*d)*x^9+1/7*(3*a^2*b*d^2+6*a*b^2*
c*d+b^3*c^2)*x^7+1/5*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^5+1/3*(2*a^3*c*d+3
*a^2*b*c^2)*x^3+a^3*c^2*x
```

Maxima [A]
time = 0.35, size = 124, normalized size = 1.02

$$\frac{1}{11}b^3d^2x^{11}+\frac{1}{9}\left(2\,b^3cd+3\,ab^2d^2\right)x^9+\frac{1}{7}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^7+a^3c^2x+\frac{1}{5}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^5+\frac{1}{3}\left(3\,a^2bc^2+2\,a^3cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a*
b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

Fricas [A]
time = 0.79, size = 124, normalized size = 1.02

$$\frac{1}{11}b^3d^2x^{11}+\frac{1}{9}\left(2\,b^3cd+3\,ab^2d^2\right)x^9+\frac{1}{7}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^7+a^3c^2x+\frac{1}{5}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^5+\frac{1}{3}\left(3\,a^2bc^2+2\,a^3cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a*
b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.11

$$a^3c^2x + \frac{b^3d^2x^{11}}{11} + x^9\left(\frac{ab^2d^2}{3} + \frac{2b^3cd}{9}\right) + x^7\cdot\left(\frac{3a^2bd^2}{7} + \frac{6ab^2cd}{7} + \frac{b^3c^2}{7}\right) + x^5\left(\frac{a^3d^2}{5} + \frac{6a^2bcd}{5} + \frac{3ab^2c^2}{5}\right) + x^3\cdot\left(\frac{2a^3cd}{3} + a^2bc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**11/11 + x**9*(a*b**2*d**2/3 + 2*b**3*c*d/9) + x*
*7*(3*a**2*b*d**2/7 + 6*a*b**2*c*d/7 + b**3*c**2/7) + x**5*(a**3*d**2/5 + 6
*a**2*b*c*d/5 + 3*a*b**2*c**2/5) + x**3*(2*a**3*c*d/3 + a**2*b*c**2)
```

Giac [A]
time = 2.98, size = 131, normalized size = 1.07

$$\frac{1}{11}b^3d^2x^{11} + \frac{2}{9}b^3cdx^9 + \frac{1}{3}ab^2d^2x^9 + \frac{1}{7}b^3c^2x^7 + \frac{6}{7}ab^2cdx^7 + \frac{3}{7}a^2bd^2x^7 + \frac{3}{5}ab^2c^2x^5 + \frac{6}{5}a^2bcdx^5 + \frac{1}{5}a^3d^2x^5 + a^2bc^2x^3 + \frac{2}{3}a^3cdx^3 + a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^2*x^11 + 2/9*b^3*c*d*x^9 + 1/3*a*b^2*d^2*x^9 + 1/7*b^3*c^2*x^7 +
 6/7*a*b^2*c*d*x^7 + 3/7*a^2*b*d^2*x^7 + 3/5*a*b^2*c^2*x^5 + 6/5*a^2*b*c*d*
x^5 + 1/5*a^3*d^2*x^5 + a^2*b*c^2*x^3 + 2/3*a^3*c*d*x^3 + a^3*c^2*x
```

Mupad [B]
time = 4.87, size = 116, normalized size = 0.95

$$x^5\left(\frac{a^3\,d^2}{5} + \frac{6\,a^2\,b\,c\,d}{5} + \frac{3\,a\,b^2\,c^2}{5}\right) + x^7\left(\frac{3\,a^2\,b\,d^2}{7} + \frac{6\,a\,b^2\,c\,d}{7} + \frac{b^3\,c^2}{7}\right) + a^3\,c^2\,x + \frac{b^3\,d^2\,x^{11}}{11} + \frac{a^2\,c\,x^3\,(2\,a\,d + 3\,b\,c)}{3} + \frac{b^2\,d\,x^9\,(3\,a\,d + 2\,b\,c)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^3*d^2)/5 + (3*a*b^2*c^2)/5 + (6*a^2*b*c*d)/5) + x^7*((b^3*c^2)/7 +
(3*a^2*b*d^2)/7 + (6*a*b^2*c*d)/7) + a^3*c^2*x + (b^3*d^2*x^11)/11 + (a^2*c
*x^3*(2*a*d + 3*b*c))/3 + (b^2*d*x^9*(3*a*d + 2*b*c))/9
```

3.16 $\int (a+bx^2)^3 (c+dx^2)\, dx$

Optimal. Leaf size=70

$$a^3cx + \frac{1}{3}a^2(3bc+ad)x^3 + \frac{3}{5}ab(bc+ad)x^5 + \frac{1}{7}b^2(bc+3ad)x^7 + \frac{1}{9}b^3dx^9$$

```
[Out] a^3*c*x+1/3*a^2*(a*d+3*b*c)*x^3+3/5*a*b*(a*d+b*c)*x^5+1/7*b^2*(3*a*d+b*c)*x
^7+1/9*b^3*d*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$a^3cx + \frac{1}{3}a^2x^3(ad+3bc) + \frac{1}{7}b^2x^7(3ad+bc) + \frac{3}{5}abx^5(ad+bc) + \frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^3 (c+dx^2)\, dx &= \int \left(a^3c + a^2(3bc+ad)x^2 + 3ab(bc+ad)x^4 + b^2(bc+3ad)x^6 + b^3dx^8\right) dx\\ &= a^3cx + \frac{1}{3}a^2(3bc+ad)x^3 + \frac{3}{5}ab(bc+ad)x^5 + \frac{1}{7}b^2(bc+3ad)x^7 + \frac{1}{9}b^3dx^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.00

$$a^3cx + \frac{1}{3}a^2(3bc+ad)x^3 + \frac{3}{5}ab(bc+ad)x^5 + \frac{1}{7}b^2(bc+3ad)x^7 + \frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Maple [A]
time = 0.10, size = 73, normalized size = 1.04

method	result	size
norman	$\frac{b^3 d\,x^9}{9}+\left(\frac{3}{7}a\,b^2 d+\frac{1}{7}b^3 c\right)x^7+\left(\frac{3}{5}a^2 b d+\frac{3}{5}a\,b^2 c\right)x^5+\left(\frac{1}{3}a^3 d+a^2 b c\right)x^3+a^3 c x$	71
default	$\frac{b^3 d\,x^9}{9}+\frac{(3a\,b^2 d+b^3 c)x^7}{7}+\frac{(3a^2 b d+3a\,b^2 c)x^5}{5}+\frac{(a^3 d+3a^2 b c)x^3}{3}+a^3 c x$	73
gosper	$\frac{1}{9}b^3 d\,x^9+\frac{3}{7}x^7 a\,b^2 d+\frac{1}{7}b^3 c\,x^7+\frac{3}{5}x^5 a^2 b d+\frac{3}{5}x^5 a\,b^2 c+\frac{1}{3}x^3 a^3 d+x^3 a^2 b c+a^3 c x$	74
risch	$\frac{1}{9}b^3 d\,x^9+\frac{3}{7}x^7 a\,b^2 d+\frac{1}{7}b^3 c\,x^7+\frac{3}{5}x^5 a^2 b d+\frac{3}{5}x^5 a\,b^2 c+\frac{1}{3}x^3 a^3 d+x^3 a^2 b c+a^3 c x$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^3*d*x^9+1/7*(3*a*b^2*d+b^3*c)*x^7+1/5*(3*a^2*b*d+3*a*b^2*c)*x^5+1/3*(
a^3*d+3*a^2*b*c)*x^3+a^3*c*x
```

Maxima [A]
time = 0.30, size = 70, normalized size = 1.00

$$\frac{1}{9}\,b^3dx^9+\frac{1}{7}\,(b^3c+3\,ab^2d)x^7+\frac{3}{5}\,(ab^2c+a^2bd)x^5+a^3cx+\frac{1}{3}\,(3\,a^2bc+a^3d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5 +
 a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

Fricas [A]
time = 0.51, size = 70, normalized size = 1.00

$$\frac{1}{9}\,b^3dx^9+\frac{1}{7}\,(b^3c+3\,ab^2d)x^7+\frac{3}{5}\,(ab^2c+a^2bd)x^5+a^3cx+\frac{1}{3}\,(3\,a^2bc+a^3d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5 +
 a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

Sympy [A]
time = 0.01, size = 76, normalized size = 1.09

$$a^3cx+\frac{b^3dx^9}{9}+x^7\cdot\left(\frac{3ab^2d}{7}+\frac{b^3c}{7}\right)+x^5\cdot\left(\frac{3a^2bd}{5}+\frac{3ab^2c}{5}\right)+x^3\left(\frac{a^3d}{3}+a^2bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**9/9 + x**7*(3*a*b**2*d/7 + b**3*c/7) + x**5*(3*a**2*b*
d/5 + 3*a*b**2*c/5) + x**3*(a**3*d/3 + a**2*b*c)
```

Giac [A]
time = 2.22, size = 73, normalized size = 1.04

$$\frac{1}{9}\,b^3dx^9 + \frac{1}{7}\,b^3cx^7 + \frac{3}{7}\,ab^2dx^7 + \frac{3}{5}\,ab^2cx^5 + \frac{3}{5}\,a^2bdx^5 + a^2bcx^3 + \frac{1}{3}\,a^3dx^3 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*b^3*c*x^7 + 3/7*a*b^2*d*x^7 + 3/5*a*b^2*c*x^5 + 3/5*a^2
*b*d*x^5 + a^2*b*c*x^3 + 1/3*a^3*d*x^3 + a^3*c*x
```

Mupad [B]
time = 0.03, size = 65, normalized size = 0.93

$$x^7\left(\frac{c\,b^3}{7}+\frac{3\,a\,d\,b^2}{7}\right)+x^3\left(\frac{d\,a^3}{3}+b\,c\,a^2\right)+\frac{b^3\,d\,x^9}{9}+a^3\,c\,x+\frac{3\,a\,b\,x^5\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(c + d*x^2),x)
```

```
[Out] x^7*((b^3*c)/7 + (3*a*b^2*d)/7) + x^3*((a^3*d)/3 + a^2*b*c) + (b^3*d*x^9)/9
 + a^3*c*x + (3*a*b*x^5*(a*d + b*c))/5
```

3.17 $\int \frac{(a+bx^2)^3}{c+dx^2} \, dx$

Optimal. Leaf size=98

$$\frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3}-\frac{b^2(bc-3ad)x^3}{3d^2}+\frac{b^3x^5}{5d}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}$$

```
[Out] b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x/d^3-1/3*b^2*(-3*a*d+b*c)*x^3/d^2+1/5*b^3*
x^5/d-(-a*d+b*c)^3*arctan(x*d^(1/2)/c^(1/2))/d^(7/2)/c^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {398, 211}

$$\frac{bx(3a^2d^2-3abcd+b^2c^2)}{d^3}-\frac{(bc-ad)^3\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}-\frac{b^2x^3(bc-3ad)}{3d^2}+\frac{b^3x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*x^3)/(3*d^
2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]
*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = \int \left(\frac{b(b^2c^2-3abcd+3a^2d^2)}{d^3} - \frac{b^2(bc-3ad)x^2}{d^2} + \frac{b^3x^4}{d} + \frac{-b^3c^3+3ab^2c^2d-3a^2bcd^2+a^3}{d^3\,(c+dx^2)}\right.$$

$$= \frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3} - \frac{b^2(bc-3ad)x^3}{3d^2} + \frac{b^3x^5}{5d} - \frac{(bc-ad)^3\int\frac{1}{c+dx^2}\,dx}{d^3}$$

$$= \frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3} - \frac{b^2(bc-3ad)x^3}{3d^2} + \frac{b^3x^5}{5d} - \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}$$

Mathematica [A]
time = 0.04, size = 93, normalized size = 0.95

$$\frac{bx(45a^2d^2+15abd(-3c+dx^2)+b^2(15c^2-5cdx^2+3d^2x^4))}{15d^3} - \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*x*(45*a^2*d^2 + 15*a*b*d*(-3*c + d*x^2) + b^2*(15*c^2 - 5*c*d*x^2 + 3*d^
2*x^4)))/(15*d^3) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^
(7/2))
```

Maple [A]
time = 0.09, size = 116, normalized size = 1.18

method	result
default	$\frac{b\left(\frac{1}{5}b^2x^5d^2+ab\,d^2x^3-\frac{1}{3}b^2cd\,x^3+3a^2d^2x-3abcdx+b^2c^2x\right)}{d^3}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^3\sqrt{cd}}$
risch	$\frac{b^3x^5}{5d}+\frac{b^2a\,x^3}{d}-\frac{b^3c\,x^3}{3d^2}+\frac{3b\,a^2x}{d}-\frac{3b^2acx}{d^2}+\frac{b^3c^2x}{d^3}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^3}{2\sqrt{-cd}}+\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2bc}{2d\sqrt{-cd}}-\frac{3\ln\left(dx+\sqrt{\right.}}{2d^2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(1/5*b^2*x^5*d^2+a*b*d^2*x^3-1/3*b^2*c*d*x^3+3*a^2*d^2*x-3*a*b*c*d*x+
b^2*c^2*x)+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^3/(c*d)^(1/2)*ar
ctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.52, size = 122, normalized size = 1.24

$$-\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3} + \frac{3\,b^3d^2x^5 - 5\,(b^3cd - 3\,ab^2d^2)x^3 + 15\,(b^3c^2 - 3\,ab^2cd + 3\,a^2bd^2)x}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/
(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^2*x^5 - 5*(b^3*c*d - 3*a*b^2*d^2)*x^3 + 15*
(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*x)/d^3
```

Fricas [A]
time = 0.53, size = 290, normalized size = 2.96

$$\left[\frac{6\,b^3cd^3x^5 - 10\,(b^3c^2d^2 - 3\,ab^2cd^3)x^3 + 15\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{-cd}\,\log\left(\frac{dx^2 - 2\sqrt{-cd}\,x - c}{dx^2 + c}\right) + 30\,(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3)x}{30\,cd^4}, \frac{3\,b^3cd^3x^5 - 5\,(b^3c^2d^2 - 3\,ab^2cd^3)x^3 - 15\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) + 15\,(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3)x}{15\,cd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^3*c*d^3*x^5 - 10*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(
-c*d)*x - c)/(d*x^2 + c)) + 30*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
)*x)/(c*d^4), 1/15*(3*b^3*c*d^3*x^5 - 5*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 -
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)*arctan(sq
rt(c*d)*x/c) + 15*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*x)/(c*d^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(92) = 184$.
time = 0.31, size = 238, normalized size = 2.43

$$\frac{b^3x^5}{5d} + x^3\left(\frac{ab^2}{d} - \frac{b^3c}{3d^2}\right) + x\left(\frac{3a^2b}{d} - \frac{3ab^2c}{d^2} + \frac{b^3c^2}{d^3}\right) - \frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] b**3*x**5/(5*d) + x**3*(a*b**2/d - b**3*c/(3*d**2)) + x*(3*a**2*b/d - 3*a*b
**2*c/d**2 + b**3*c**2/d**3) - sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(-c*d**3
*sqrt(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c
**2*d - b**3*c**3) + x)/2 + sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(c*d**3*sqr
t(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*
d - b**3*c**3) + x)/2
```

Giac [A]
time = 1.68, size = 130, normalized size = 1.33

$$-\frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3} + \frac{3\,b^3d^4x^5 - 5\,b^3cd^3x^3 + 15\,ab^2d^4x^3 + 15\,b^3c^2d^2x - 45\,ab^2cd^3x + 45\,a^2bd^4x}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/
(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^4*x^5 - 5*b^3*c*d^3*x^3 + 15*a*b^2*d^4*x^3
+ 15*b^3*c^2*d^2*x - 45*a*b^2*c*d^3*x + 45*a^2*b*d^4*x)/d^5
```

Mupad [B]
time = 4.87, size = 145, normalized size = 1.48

$$x^3\left(\frac{a\,b^2}{d} - \frac{b^3\,c}{3\,d^2}\right) + x\left(\frac{3\,a^2\,b}{d} - \frac{c\left(\frac{3\,a\,b^2}{d} - \frac{b^3\,c}{d^2}\right)}{d}\right) + \frac{b^3\,x^5}{5\,d} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^3}{\sqrt{c}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,(a\,d-b\,c)^3}{\sqrt{c}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2),x)
```

```
[Out] x^3*((a*b^2)/d - (b^3*c)/(3*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^3
*c)/d^2))/d) + (b^3*x^5)/(5*d) + (atan((d^(1/2)*x*(a*d - b*c)^3)/(c^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(c^(1/2
)*d^(7/2))
```

3.18 $\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\,(c+dx^2)}+\frac{(bc-ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}$$

```
[Out] -b^2*(-3*a*d+2*b*c)*x/d^3+1/3*b^3*x^3/d^2-1/2*(-a*d+b*c)^3*x/c/d^3/(d*x^2+c
)+1/2*(-a*d+b*c)^2*(a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {398, 393, 211}

$$\frac{(ad+5bc)(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{b^2x(2bc-3ad)}{d^3}-\frac{x(bc-ad)^3}{2cd^3\,(c+dx^2)}+\frac{b^3x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x^2}{d^2}+\frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{d^3\,(c+dx^2)^2}\right)dx\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}+\frac{\int \frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{(c+dx^2)^2}\,dx}{d^3}\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\,(c+dx^2)}+\frac{((bc-ad)^2(5bc+ad))\int \frac{1}{c+dx^2}\,dx}{2cd^3}\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\,(c+dx^2)}+\frac{(bc-ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 107, normalized size = 1.00

$$-\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\,(c+dx^2)}+\frac{(bc-ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Maple [A]
time = 0.09, size = 138, normalized size = 1.29

method	result
default	$\frac{b^2\left(\frac{1}{3}bd\,x^3+3adx-2bcx\right)}{d^3}+\frac{\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2c\left(d\,x^2+c\right)}+\frac{\left(a^3d^3+3a^2bc\,d^2-9a\,b^2c^2d+5b^3c^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}}{d^3}$
risch	$\frac{b^3x^3}{3d^2}+\frac{3b^2ax}{d^2}-\frac{2b^3cx}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{2c\,d^3(d\,x^2+c)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^3}{4\sqrt{-cd}\;c}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2b}{4d\sqrt{-cd}}+\frac{9c\ln\left(da\right.}{4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/d^3*(1/3*b*d*x^3+3*a*d*x-2*b*c*x)+1/d^3*(1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/c*x/(d*x^2+c)+1/2*(a^3*d^3+3*a^2*b*c*d^2-9*a*b^2*c^2*d+
5*b^3*c^3)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 1.37

$$-\frac{\left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) x}{2\left(c d^{4} x^{2}+c^{2} d^{3}\right)}+\frac{b^{3} d x^{3}-3\left(2 b^{3} c-3 a b^{2} d\right) x}{3 d^{3}}+\frac{\left(5 b^{3} c^{3}-9 a b^{2} c^{2} d+3 a^{2} b c d^{2}+a^{3} d^{3}\right) \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{2 \sqrt{c d}\, c d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(c*d^4*x^2 + c^2
*d^3) + 1/3*(b^3*d*x^3 - 3*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 1/2*(5*b^3*c^3 -
9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c
*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(93) = 186.
time = 0.59, size = 444, normalized size = 4.15

$$\left[\frac{4 b^{3} c^{2} d^{3} x^{5}-4\left(5 b^{3} c^{3} d^{2}-9 a b^{2} c^{2} d^{3}\right) x^{3}-3\left(5 b^{3} c^{4}-9 a b^{2} c^{3} d+3 a^{2} b c^{2} d^{2}+a^{3} c d^{3}+\left(5 b^{3} c^{3} d-9 a b^{2} c^{2} d^{2}+3 a^{2} b c d^{3}+a^{3} d^{4}\right) x^{2}\right) \sqrt{-c d} \log \left(\frac{d x^{2}-2 \sqrt{-c d}\, x-c}{d x^{2}+c}\right)-6\left(5 b^{3} c^{4} d-9 a b^{2} c^{3} d^{2}+3 a^{2} b c^{2} d^{3}-a^{3} c d^{4}\right) x}{12\left(c^{2} d^{5} x^{2}+c^{3} d^{4}\right)}, \frac{2 b^{3} c^{2} d^{3} x^{5}-2\left(5 b^{3} c^{3} d^{2}-9 a b^{2} c^{2} d^{3}\right) x^{3}+3\left(5 b^{3} c^{4}-9 a b^{2} c^{3} d+3 a^{2} b c^{2} d^{2}+a^{3} c d^{3}+\left(5 b^{3} c^{3} d-9 a b^{2} c^{2} d^{2}+3 a^{2} b c d^{3}+a^{3} d^{4}\right) x^{2}\right) \sqrt{c d} \arctan \left(\frac{\sqrt{c d}\, x}{c}\right)-3\left(5 b^{3} c^{4} d-9 a b^{2} c^{3} d^{2}+3 a^{2} b c^{2} d^{3}-a^{3} c d^{4}\right) x}{6\left(c^{2} d^{5} x^{2}+c^{3} d^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^3*c^2*d^3*x^5 - 4*(5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 - 3*(5*b
^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b
^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*d^4)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-
c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d
^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4), 1/6*(2*b^3*c^2*d^3*x^5 - 2*(5*b
^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 + 3*(5*b^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*
c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*
d^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(5*b^3*c^4*d - 9*a*b^2*c^3*d^
2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(95) = 190.
time = 0.57, size = 314, normalized size = 2.93

$$\frac{b^{3} x^{3}}{3 d^{2}}+x\left(\frac{3 a b^{2}}{d^{2}}-\frac{2 b^{3} c}{d^{3}}\right)+\frac{x\left(a^{3} d^{3}-3 a^{2} b c d^{2}+3 a b^{2} c^{2} d-b^{3} c^{3}\right)}{2 c^{2} d^{3}+2 c d^{4} x^{2}}-\frac{\sqrt{-\frac{1}{c^{3} d^{7}}}\,(a d-b c)^{2}(a d+5 b c) \log \left(-\frac{c^{2} d^{3} \sqrt{-\frac{1}{c^{3} d^{7}}}\,(a d-b c)^{2}(a d+5 b c)}{a^{3} d^{3}+3 a^{2} b c d^{2}-9 a b^{2} c^{2} d+5 b^{3} c^{3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^{3} d^{7}}}\,(a d-b c)^{2}(a d+5 b c) \log \left(\frac{c^{2} d^{3} \sqrt{-\frac{1}{c^{3} d^{7}}}\,(a d-b c)^{2}(a d+5 b c)}{a^{3} d^{3}+3 a^{2} b c d^{2}-9 a b^{2} c^{2} d+5 b^{3} c^{3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] b**3*x**3/(3*d**2) + x*(3*a*b**2/d**2 - 2*b**3*c/d**3) + x*(a**3*d**3 - 3*a
**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*c**2*d**3 + 2*c*d**4*x**2) -
 sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-c**2*d**3*sqrt(-1/(
c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a
*b**2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a
*d + 5*b*c)*log(c**2*d**3*sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)
/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4
```

Giac [A]
time = 1.95, size = 152, normalized size = 1.42

$$\frac{(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^3}-\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(dx^2+c)cd^3}+\frac{b^3d^4x^3-6\,b^3cd^3x+9\,ab^2d^4x}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c
*d))/(sqrt(c*d)*c*d^3) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((d*x^2 + c)*c*d^3) + 1/3*(b^3*d^4*x^3 - 6*b^3*c*d^3*x + 9*a*
b^2*d^4*x)/d^6
```

Mupad [B]
time = 0.10, size = 181, normalized size = 1.69

$$x\left(\frac{3\,a\,b^2}{d^2}-\frac{2\,b^3\,c}{d^3}\right)+\frac{b^3\,x^3}{3\,d^2}+\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,c\,\left(d^4\,x^2+c\,d^3\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(a\,d+5\,b\,c\right)}{\sqrt{c}\,\left(a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(a\,d+5\,b\,c\right)}{2\,c^{3/2}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2)^2,x)
```

```
[Out] x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (b^3*x^3)/(3*d^2) + (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*c*(c*d^3 + d^4*x^2)) + (atan((d^(1/
2)*x*(a*d - b*c)^2*(a*d + 5*b*c))/(c^(1/2)*(a^3*d^3 + 5*b^3*c^3 - 9*a*b^2*c
^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)^2*(a*d + 5*b*c))/(2*c^(3/2)*d^(7/2))
```

3.19 $\int \frac{(a+bx^2)^3}{(c+dx^2)^3} dx$

Optimal. Leaf size=130

$$\frac{b^3x}{d^3}-\frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2}+\frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\left(c+dx^2\right)}-\frac{3(bc-ad)\left(4b^2c^2+(bc+ad)^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}$$

```
[Out] b^3*x/d^3-1/4*(-a*d+b*c)^3*x/c/d^3/(d*x^2+c)^2+3/8*(-a*d+b*c)^2*(a*d+3*b*c)
*x/c^2/d^3/(d*x^2+c)-3/8*(-a*d+b*c)*(4*b^2*c^2+(a*d+b*c)^2)*arctan(x*d^(1/2
)/c^(1/2))/c^(5/2)/d^(7/2)
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {398, 1171, 393, 211}

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*d)*(4*b^2*c^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx = \int \left(\frac{b^3}{d^3} - \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{d^3\,(c+dx^2)^3}\right) dx$$

$$= \frac{b^3x}{d^3} - \frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{(c+dx^2)^3}\,dx}{d^3}$$

$$= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\,(c+dx^2)^2} + \frac{\int \frac{-3(bc-ad)(bc+ad)^2-12b^2cd(bc-ad)x^2}{(c+dx^2)^2}\,dx}{4cd^3}$$

$$= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\,(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\,(c+dx^2)} - \frac{(3(bc-ad)\,(4b^2c^2+(bc+ad)^2))\int \frac{}{c+}}{8c^2d^3}$$

$$= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\,(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\,(c+dx^2)} - \frac{3(bc-ad)\,(4b^2c^2+(bc+ad)^2)\tan^{-1}}{8c^{5/2}d^{7/2}}$$

Mathematica [A]
time = 0.06, size = 141, normalized size = 1.08

$$\frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\,(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\,(c+dx^2)} - \frac{3(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(5*b^3*c^3 - 3*a*b^2*c^2*d -
a^2*b*c*d^2 - a^3*d^3)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Maple [A]
time = 0.09, size = 167, normalized size = 1.28

method	result
default	$\frac{b^{3} x}{d^{3}}+\frac{\frac{\frac{3 d \left(a^{3} d^{3}+a^{2} b c \,d^{2}-5 a \,b^{2} c^{2} d +3 b^{3} c^{3}\right) x^{3}}{8 c^{2}}+\frac{\left(5 a^{3} d^{3}-3 a^{2} b c \,d^{2}-9 a \,b^{2} c^{2} d +7 b^{3} c^{3}\right) x}{8 c}}{\left(d \,x^{2}+c \right)^{2}}+\frac{3 \left(a^{3} d^{3}+a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -5 b^{3} c^{3}\right) \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{8 c^{2} \sqrt{c d}}}{d^{3}}$
risch	$\frac{b^{3} x}{d^{3}}+\frac{\frac{3 d \left(a^{3} d^{3}+a^{2} b c \,d^{2}-5 a \,b^{2} c^{2} d +3 b^{3} c^{3}\right) x^{3}}{8 c^{2}}+\frac{\left(5 a^{3} d^{3}-3 a^{2} b c \,d^{2}-9 a \,b^{2} c^{2} d +7 b^{3} c^{3}\right) x}{8 c}}{d^{3} \left(d \,x^{2}+c \right)^{2}}-\frac{3 \ln \left(d x +\sqrt{-c d}\right) a^{3}}{16 \sqrt{-c d}\, c^{2}}-\frac{3 \ln \left(d x +\sqrt{-c d}\right.}{16 d \sqrt{-c d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d^3+1/d^3*((3/8*d*(a^3*d^3+a^2*b*c*d^2-5*a*b^2*c^2*d+3*b^3*c^3)/c^2*x
^3+1/8*(5*a^3*d^3-3*a^2*b*c*d^2-9*a*b^2*c^2*d+7*b^3*c^3)/c*x)/(d*x^2+c)^2+3
/8*(a^3*d^3+a^2*b*c*d^2+3*a*b^2*c^2*d-5*b^3*c^3)/c^2/(c*d)^(1/2)*arctan(d*x
/(c*d)^(1/2)))
```

Maxima [A]
time = 0.52, size = 187, normalized size = 1.44

$$\frac{b^{3} x}{d^{3}}+\frac{3 \, {\left(3 \, b^{3} c^{3} d - 5 \, a b^{2} c^{2} d^{2} + a^{2} b c d^{3} + a^{3} d^{4}\right)} x^{3} + {\left(7 \, b^{3} c^{4} - 9 \, a b^{2} c^{3} d - 3 \, a^{2} b c^{2} d^{2} + 5 \, a^{3} c d^{3}\right)} x}{8 \, {\left(c^{2} d^{5} x^{4} + 2 \, c^{3} d^{4} x^{2} + c^{4} d^{3}\right)}} - \frac{3 \, {\left(5 \, b^{3} c^{3} - 3 \, a b^{2} c^{2} d - a^{2} b c d^{2} - a^{3} d^{3}\right)} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 \, \sqrt{c d} c^{2} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 + 1/8*(3*(3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3 + a^3*d^4)*
x^3 + (7*b^3*c^4 - 9*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x)/(c^2*d
^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3) - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*
c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(116) = 232.
time = 0.47, size = 618, normalized size = 4.75

$$\left[\frac{16 \, b^{3} c^{3} d^{3} x^{5} + 2 \, {\left(25 \, b^{3} c^{4} d^{2} - 15 \, a b^{2} c^{3} d^{3} + 3 \, a^{2} b c^{2} d^{4} + 3 \, a^{3} c d^{5}\right)} x^{3} + 3 \, {\left(5 \, b^{3} c^{5} - 3 \, a b^{2} c^{4} d - a^{2} b c^{3} d^{2} - a^{3} c^{2} d^{3} + {\left(5 \, b^{3} c^{3} d^{2} - 3 \, a b^{2} c^{2} d^{3} - a^{2} b c d^{4} - a^{3} d^{5}\right)} x^{4} + 2 \, {\left(5 \, b^{3} c^{4} d - 3 \, a b^{2} c^{3} d^{2} - a^{2} b c^{2} d^{3} - a^{3} c d^{4}\right)} x^{2}\right)} \sqrt{-c d} \log\left(\frac{d x^{2} - 2 \, \sqrt{-c d} x - c}{d x^{2} + c}\right) + 2 \, {\left(15 \, b^{3} c^{5} d - 9 \, a b^{2} c^{4} d^{2} - 3 \, a^{2} b c^{3} d^{3} + 5 \, a^{3} c^{2} d^{4}\right)} x}{16 \, {\left(c^{3} d^{6} x^{4} + 2 \, c^{4} d^{5} x^{2} + c^{5} d^{4}\right)}}, \frac{8 \, b^{3} c^{3} d^{3} x^{5} + {\left(25 \, b^{3} c^{4} d^{2} - 15 \, a b^{2} c^{3} d^{3} + 3 \, a^{2} b c^{2} d^{4} + 3 \, a^{3} c d^{5}\right)} x^{3} - 3 \, {\left(5 \, b^{3} c^{5} - 3 \, a b^{2} c^{4} d - a^{2} b c^{3} d^{2} - a^{3} c^{2} d^{3} + {\left(5 \, b^{3} c^{3} d^{2} - 3 \, a b^{2} c^{2} d^{3} - a^{2} b c d^{4} - a^{3} d^{5}\right)} x^{4} + 2 \, {\left(5 \, b^{3} c^{4} d - 3 \, a b^{2} c^{3} d^{2} - a^{2} b c^{2} d^{3} - a^{3} c d^{4}\right)} x^{2}\right)} \sqrt{c d} \arctan\left(\frac{\sqrt{c d} x}{c}\right) + {\left(15 \, b^{3} c^{5} d - 9 \, a b^{2} c^{4} d^{2} - 3 \, a^{2} b c^{3} d^{3} + 5 \, a^{3} c^{2} d^{4}\right)} x}{8 \, {\left(c^{3} d^{6} x^{4} + 2 \, c^{4} d^{5} x^{2} + c^{5} d^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^3*c^3*d^3*x^5 + 2*(25*b^3*c^4*d^2 - 15*a*b^2*c^3*d^3 + 3*a^2*b*
c^2*d^4 + 3*a^3*c*d^5)*x^3 + 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^3*d^2 -
 a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4 - a^3*d^5)*x^
```

```
4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)*sqrt
(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^3*c^5*d - 9*
a*b^2*c^4*d^2 - 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^
5*x^2 + c^5*d^4), 1/8*(8*b^3*c^3*d^3*x^5 + (25*b^3*c^4*d^2 - 15*a*b^2*c^3*d
^3 + 3*a^2*b*c^2*d^4 + 3*a^3*c*d^5)*x^3 - 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^
2*b*c^3*d^2 - a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4
- a^3*d^5)*x^4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d
^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (15*b^3*c^5*d - 9*a*b^2*c^4*d^2
- 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^5*x^2 + c^5*d^
4)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(122) = 244$.
time = 1.00, size = 422, normalized size = 3.25

$$\frac{b^3x}{d^3} - \frac{3\sqrt{-\frac{1}{c^5d^7}}\,(ad-bc)\,(a^2d^2+2abcd+5b^2c^2)\log\left(-\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)(a^2d^2+2abcd+5b^2c^2)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{c^5d^7}}\,(ad-bc)\,(a^2d^2+2abcd+5b^2c^2)\log\left(\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)(a^2d^2+2abcd+5b^2c^2)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16} + \frac{x^3\cdot(3a^3d^4+3a^2bcd^3-15ab^2c^2d^2+9b^3c^3d)+x(5a^3cd^3-3a^2bc^2d^2-9ab^2c^3d+7b^3c^4)}{8c^4d^3+16c^3d^4x^2+8c^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] b**3*x/d**3 - 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a*b*c*d + 5
*b**2*c**2)*log(-3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 +
2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**2*c**2*d -
 15*b**3*c**3) + x)/16 + 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*
a*b*c*d + 5*b**2*c**2)*log(3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a*
*2*d**2 + 2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**
2*c**2*d - 15*b**3*c**3) + x)/16 + (x**3*(3*a**3*d**4 + 3*a**2*b*c*d**3 - 1
5*a*b**2*c**2*d**2 + 9*b**3*c**3*d) + x*(5*a**3*c*d**3 - 3*a**2*b*c**2*d**2
 - 9*a*b**2*c**3*d + 7*b**3*c**4))/(8*c**4*d**3 + 16*c**3*d**4*x**2 + 8*c**
2*d**5*x**4)
```

Giac [**A**]
time = 1.23, size = 180, normalized size = 1.38

$$\frac{b^3x}{d^3} - \frac{3\,(5\,b^3c^3 - 3\,ab^2c^2d - a^2bcd^2 - a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^3} + \frac{9\,b^3c^3dx^3 - 15\,ab^2c^2d^2x^3 + 3\,a^2bcd^3x^3 + 3\,a^3d^4x^3 + 7\,b^3c^4x - 9\,ab^2c^3dx - 3\,a^2bc^2d^2x + 5\,a^3cd^3x}{8\,(dx^2+c)^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*arctan(
d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3) + 1/8*(9*b^3*c^3*d*x^3 - 15*a*b^2*c^2*d^
2*x^3 + 3*a^2*b*c*d^3*x^3 + 3*a^3*d^4*x^3 + 7*b^3*c^4*x - 9*a*b^2*c^3*d*x -
 3*a^2*b*c^2*d^2*x + 5*a^3*c*d^3*x)/((d*x^2 + c)^2*c^2*d^3)
```

Mupad [B]
time = 4.96, size = 240, normalized size = 1.85

$$\frac{\frac{x\,\left(5\,a^3\,d^3-3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+7\,b^3\,c^3\right)}{8\,c}+\frac{3\,x^3\,\left(a^3\,d^4+a^2\,b\,c\,d^3-5\,a\,b^2\,c^2\,d^2+3\,b^3\,c^3\,d\right)}{8\,c^2}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4}+\frac{b^3\,x}{d^3}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{d}\,x\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{\sqrt{c}\,\left(a^3\,d^3+a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2)^3,x)
```

```
[Out] ((x*(5*a^3*d^3 + 7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(8*c) + (3*x^3
*(a^3*d^4 + 3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3))/(8*c^2))/(c^2*d^3
 + d^5*x^4 + 2*c*d^4*x^2) + (b^3*x)/d^3 + (3*atan((d^(1/2)*x*(a*d - b*c)*(a
^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(c^(1/2)*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^2*c^
2*d + a^2*b*c*d^2)))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(8*c^(5
/2)*d^(7/2))
```

3.20 $\int \frac{(c+dx^2)^4}{a+bx^2}\,dx$

Optimal. Leaf size=142

$$\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3}+\frac{d^3(4bc-ad)x^5}{5b^2}+\frac{d^4x^7}{7b}+\frac{(bc-ad)^4\tan^{-1}}{\sqrt{a}\,b^{9/}}$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/3*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^3/b^3+1/5*d^3*(-a*d+4*b*c)*x^5/b^2+1/7*d^4*x^7/b+(-a*d+b*c)
^4*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {398, 211}

$$\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^4}{\sqrt{a}\,b^{9/2}}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{d^4x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*d)*x^5)/(5*b^2) + (d
^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \int \left(\frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^2}{b^3} + \frac{d^3(4bc-ad)x}{b^2}\right.$$

$$= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,x}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^3}{3b^3} + \frac{d^3(4bc-ad)x^5}{5b^2}$$

$$= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,x}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^3}{3b^3} + \frac{d^3(4bc-ad)x^5}{5b^2}$$

Mathematica [A]
time = 0.06, size = 136, normalized size = 0.96

$$\frac{dx(-105a^3d^3+35a^2bd^2(12c+dx^2)-7ab^2d(90c^2+20cdx^2+3d^2x^4)+3b^3(140c^3+70c^2dx^2+28cd^2x^4+5d^3x^6))}{105b^4} + \frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*x*(-105*a^3*d^3 + 35*a^2*b*d^2*(12*c + d*x^2) - 7*a*b^2*d*(90*c^2 + 20*c
*d*x^2 + 3*d^2*x^4) + 3*b^3*(140*c^3 + 70*c^2*d*x^2 + 28*c*d^2*x^4 + 5*d^3*
x^6)))/(105*b^4) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(
9/2))
```

Maple [A]
time = 0.13, size = 196, normalized size = 1.38

method	result
default	$-\frac{d\left(-\frac{d^3x^7b^3}{7}+\frac{\left((ad-2bc)b^2d^2-2b^3c\,d^2\right)x^5}{5}+\frac{\left(2(ad-2bc)b^2cd-bd\left(a^2d^2-2abcd+2b^2c^2\right)\right)x^3}{3}+(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)x\right)}{b^4}+\frac{(a^4d}{}$
risch	$\frac{d^4x^7}{7b}-\frac{d^4x^5a}{5b^2}+\frac{4d^3x^5c}{5b}-\frac{4d^3x^3ac}{3b^2}+\frac{2d^2x^3c^2}{b}+\frac{d^4x^3a^2}{3b^3}-\frac{d^4a^3x}{b^4}+\frac{4d^3a^2cx}{b^3}-\frac{6d^2a\,c^2x}{b^2}+\frac{4d\,c^3x}{b}-\frac{\ln\left(bx+\sqrt{-}\right.}{2b^4\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^4*(-1/7*d^3*x^7*b^3+1/5*((a*d-2*b*c)*b^2*d^2-2*b^3*c*d^2)*x^5+1/3*(2*(
a*d-2*b*c)*b^2*c*d-b*d*(a^2*d^2-2*a*b*c*d+2*b^2*c^2))*x^3+(a*d-2*b*c)*(a^2*
d^2-2*a*b*c*d+2*b^2*c^2)*x)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^
3*c^3*d+b^4*c^4)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 187, normalized size = 1.32

$$\frac{\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{4}} + \frac{15 \, b^{3} d^{4} x^{7} + 21 \, {\left(4 \, b^{3} c d^{3} - a b^{2} d^{4}\right)} x^{5} + 35 \, {\left(6 \, b^{3} c^{2} d^{2} - 4 \, a b^{2} c d^{3} + a^{2} b d^{4}\right)} x^{3} + 105 \, {\left(4 \, b^{3} c^{3} d - 6 \, a b^{2} c^{2} d^{2} + 4 \, a^{2} b c d^{3} - a^{3} d^{4}\right)} x}{105 \, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*d^4*x^7 + 21*(4*b^3*c*d^
3 - a*b^2*d^4)*x^5 + 35*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^3 + 1
05*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4
```

Fricas [A]
time = 0.44, size = 428, normalized size = 3.01

$$\left[\frac{30 \, a b^{4} d^{4} x^{7} + 42 \, {\left(4 \, a b^{4} c d^{3} - a^{2} b^{3} d^{4}\right)} x^{5} + 70 \, {\left(6 \, a b^{4} c^{2} d^{2} - 4 \, a^{2} b^{3} c d^{3} + a^{3} b^{2} d^{4}\right)} x^{3} - 105 \, {\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{-a b} \log\left(\frac{b x^{2} - 2 \, \sqrt{-a b} x - a}{b x^{2} + a}\right) + 210 \, {\left(4 \, a b^{4} c^{3} d - 6 \, a^{2} b^{3} c^{2} d^{2} + 4 \, a^{3} b^{2} c d^{3} - a^{4} b d^{4}\right)} x}{210 \, a b^{5}}, \frac{15 \, a b^{4} d^{4} x^{7} + 21 \, {\left(4 \, a b^{4} c d^{3} - a^{2} b^{3} d^{4}\right)} x^{5} + 35 \, {\left(6 \, a b^{4} c^{2} d^{2} - 4 \, a^{2} b^{3} c d^{3} + a^{3} b^{2} d^{4}\right)} x^{3} + 105 \, {\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{a b} \arctan\left(\frac{\sqrt{a b} x}{a}\right) + 105 \, {\left(4 \, a b^{4} c^{3} d - 6 \, a^{2} b^{3} c^{2} d^{2} + 4 \, a^{3} b^{2} c d^{3} - a^{4} b d^{4}\right)} x}{105 \, a b^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*a*b^4*d^4*x^7 + 42*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 70*(6*a*b
^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3*b^2*d^4)*x^3 - 105*(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(4*a*b^4*c^3*d - 6*a^2*b^3*c^2*d^2 +
 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5), 1/105*(15*a*b^4*d^4*x^7 + 21*(4*a
*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 35*(6*a*b^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3
*b^2*d^4)*x^3 + 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(4*a*b^4*c^3*d - 6*a
^2*b^3*c^2*d^2 + 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(136) = 272$.
time = 0.43, size = 326, normalized size = 2.30

$$x^{5} \left(- \frac{a d^{4}}{5 b^{2}} + \frac{4 c d^{3}}{5 b}\right) + x^{3} \left(\frac{a^{2} d^{4}}{3 b^{3}} - \frac{4 a c d^{3}}{3 b^{2}} + \frac{2 c^{2} d^{2}}{b}\right) + x \left(- \frac{a^{3} d^{4}}{b^{4}} + \frac{4 a^{2} c d^{3}}{b^{3}} - \frac{6 a c^{2} d^{2}}{b^{2}} + \frac{4 c^{3} d}{b}\right) - \frac{\sqrt{- \frac{1}{a b^{9}}} \left(a d - b c\right)^{4} \log{\left(- \frac{a b^{4} \sqrt{- \frac{1}{a b^{9}}} \left(a d - b c\right)^{4}}{a^{4} d^{4} - 4 a^{3} b c d^{3} + 6 a^{2} b^{2} c^{2} d^{2} - 4 a b^{3} c^{3} d + b^{4} c^{4}} + x \right)}}{2} + \frac{\sqrt{- \frac{1}{a b^{9}}} \left(a d - b c\right)^{4} \log{\left(\frac{a b^{4} \sqrt{- \frac{1}{a b^{9}}} \left(a d - b c\right)^{4}}{a^{4} d^{4} - 4 a^{3} b c d^{3} + 6 a^{2} b^{2} c^{2} d^{2} - 4 a b^{3} c^{3} d + b^{4} c^{4}} + x \right)}}{2} + \frac{d^{4} x^{7}}{7 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a),x)
```

```
[Out] x**5*(-a*d**4/(5*b**2) + 4*c*d**3/(5*b)) + x**3*(a**2*d**4/(3*b**3) - 4*a*c
*d**3/(3*b**2) + 2*c**2*d**2/b) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 -
 6*a*c**2*d**2/b**2 + 4*c**3*d/b) - sqrt(-1/(a*b**9))*(a*d - b*c)**4*log(-a
*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**
```

```
2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2 + sqrt(-1/(a*b**9))*
(a*d - b*c)**4*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a
**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2
+ d**4*x**7/(7*b)
```

Giac [A]
time = 1.07, size = 198, normalized size = 1.39

$$\frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6d^4x^7+84\,b^6cd^3x^5-21\,ab^5d^4x^5+210\,b^6c^2d^2x^3-140\,ab^5cd^3x^3+35\,a^2b^4d^4x^3+420\,b^6c^3dx-630\,ab^5c^2d^2x+420\,a^2b^4cd^3x-105\,a^3b^3d^4x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^4*x^7 + 84*b^6*c*d^3*x
^5 - 21*a*b^5*d^4*x^5 + 210*b^6*c^2*d^2*x^3 - 140*a*b^5*c*d^3*x^3 + 35*a^2*
b^4*d^4*x^3 + 420*b^6*c^3*d*x - 630*a*b^5*c^2*d^2*x + 420*a^2*b^4*c*d^3*x -
 105*a^3*b^3*d^4*x)/b^7
```

Mupad [B]
time = 4.86, size = 216, normalized size = 1.52

$$x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^5\left(\frac{a\,d^4}{5\,b^2}-\frac{4\,c\,d^3}{5\,b}\right)+x^3\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{3\,b}+\frac{2\,c^2\,d^2}{b}\right)+\frac{d^4\,x^7}{7\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^4}{\sqrt{a}\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}\right)(a\,d-b\,c)^4}{\sqrt{a}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^5*((a*d^4)/(5*b^2) - (4*c*d^3)/(5*b)) + x^3*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(3*b) + (2*c^2*d^2)/b) + (d^4*x^7)/(7*b) + (atan((b^(1/2)*x*(a*d - b
*c)^4)/(a^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3)))*(a*d - b*c)^4)/(a^(1/2)*b^(9/2))
```

3.21 $\int \frac{(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=98

$$\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/3*d^2*(-a*d+3*b*c)*x^3/b^2+1/5*d^3*
x^5/b+(-a*d+b*c)^3*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {398, 211}

$$\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^3}{\sqrt{a}\,b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^4}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3}{b^3(a+bx^2)}\right.$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^2}\,dx}{b^3}$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

Mathematica [A]
time = 0.05, size = 92, normalized size = 0.94

$$\frac{dx(15a^2d^2-5abd(9c+dx^2)+3b^2(15c^2+5cdx^2+d^2x^4))}{15b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2*
x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7
/2))
```

Maple [A]
time = 0.08, size = 116, normalized size = 1.18

method	result
default	$\frac{d\left(\frac{1}{5}b^2x^5d^2-\frac{1}{3}ab\,d^2x^3+b^2cd\,x^3+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d^3x^5}{5b}-\frac{d^3a\,x^3}{3b^2}+\frac{d^2c\,x^3}{b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)a^3d^3}{2b^3\sqrt{-ab}}+\frac{3\ln\left(bx-\sqrt{-ab}\right)a^2c\,d^2}{2b^2\sqrt{-ab}}-\frac{3\ln\left(b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^3*(1/5*b^2*x^5*d^2-1/3*a*b*d^2*x^3+b^2*c*d*x^3+a^2*d^2*x-3*a*b*c*d*x+3*
b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 122, normalized size = 1.24

$$\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^2d^3x^5 + 5\,(3\,b^2cd^2 - abd^3)x^3 + 15\,(3\,b^2c^2d - 3\,abcd^2 + a^2d^3)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(
3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

Fricas [A]
time = 1.48, size = 292, normalized size = 2.98

$$\left[\frac{6\,ab^3d^3x^5 + 10\,(3\,ab^3cd^2 - a^2b^2d^3)x^3 + 15\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{-ab}\,\log\left(\frac{bx^2 + 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 30\,(3\,ab^3c^2d - 3\,a^2b^2cd^2 + a^3bd^3)x}{30\,ab^4}, \frac{3\,ab^3d^3x^5 + 5\,(3\,ab^3cd^2 - a^2b^2d^3)x^3 + 15\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 15\,(3\,ab^3c^2d - 3\,a^2b^2cd^2 + a^3bd^3)x}{15\,ab^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3
)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 +
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(92) = 184$.
time = 0.31, size = 238, normalized size = 2.43

$$x^3\left(-\frac{ad^3}{3b^2} + \frac{cd^2}{b}\right) + x\left(\frac{a^2d^3}{b^3} - \frac{3acd^2}{b^2} + \frac{3c^2d}{b}\right) + \frac{\sqrt{-\frac{1}{ab^7}}\,(ad - bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}\,(ad - bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2} + \frac{d^3x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d**3/(3*b**2) + c*d**2/b) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b**2 +
3*c**2*d/b) + sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3
) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
x)/2 + d**3*x**5/(5*b)
```

Giac [A]
time = 2.31, size = 129, normalized size = 1.32

$$\frac{\left(b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{3}} + \frac{3 \, b^{4} d^{3} x^{5} + 15 \, b^{4} c d^{2} x^{3} - 5 \, a b^{3} d^{3} x^{3} + 45 \, b^{4} c^{2} d x - 45 \, a b^{3} c d^{2} x + 15 \, a^{2} b^{2} d^{3} x}{15 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3 +
 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

Mupad [B]
time = 0.08, size = 146, normalized size = 1.49

$$x\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^3\,\left(\frac{a\,d^3}{3\,b^2}-\frac{c\,d^2}{b}\right)+\frac{d^3\,x^5}{5\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3}{\sqrt{a}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^3*((a*d^3)/(3*b^2)
- (c*d^2)/b) + (d^3*x^5)/(5*b) - (atan((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(1/2
)*b^(7/2))
```

3.22 $\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=63

$$\frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/3*d^2*x^3/b+(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/b
^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {398, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2}{\sqrt{a}\,b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^2}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2(a+bx^2)}\right)dx$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\int\frac{1}{a+bx^2}\,dx}{b^2}$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.94

$$\frac{dx(6bc-3ad+bdx^2)}{3b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2),x]
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.07, size = 64, normalized size = 1.02

method	result
default	$-\frac{d\left(-\frac{1}{3}bd\,x^3+adx-2bcx\right)}{b^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{d^2x^3}{3b}-\frac{d^2ax}{b^2}+\frac{2dcx}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right)a^2d^2}{2b^2\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)acd}{b\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)c^2}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2b^2\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -d/b^2*(-1/3*b*d*x^3+a*d*x-2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.52, size = 69, normalized size = 1.10

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{bd^2x^3+3\,(2\,bcd-ad^2)x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)/b^2
```

Fricas [A]
time = 1.52, size = 181, normalized size = 2.87

$$\left[\frac{2\,ab^{2}d^{2}x^{3} - 3\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{-ab}\,\log\left(\frac{bx^{2} - 2\,\sqrt{-ab}\,x - a}{bx^{2} + a}\right) + 6\,(2\,ab^{2}cd - a^{2}bd^{2})x}{6\,ab^{3}}, \frac{ab^{2}d^{2}x^{3} + 3\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 3\,(2\,ab^{2}cd - a^{2}bd^{2})x}{3\,ab^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a
*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.
time = 0.22, size = 172, normalized size = 2.73

$$x\left(-\frac{ad^{2}}{b^{2}} + \frac{2cd}{b}\right) - \frac{\sqrt{-\frac{1}{ab^{5}}}\,(ad - bc)^{2}\log\left(-\frac{ab^{2}\sqrt{-\frac{1}{ab^{5}}}\,(ad-bc)^{2}}{a^{2}d^{2} - 2abcd + b^{2}c^{2}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^{5}}}\,(ad - bc)^{2}\log\left(\frac{ab^{2}\sqrt{-\frac{1}{ab^{5}}}\,(ad-bc)^{2}}{a^{2}d^{2} - 2abcd + b^{2}c^{2}} + x\right)}{2} + \frac{d^{2}x^{3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) - sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*s
qrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
+ sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)
**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b)
```

Giac [A]
time = 2.26, size = 72, normalized size = 1.14

$$\frac{(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}} + \frac{b^{2}d^{2}x^{3} + 6\,b^{2}cdx - 3\,abd^{2}x}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

Mupad [B]
time = 4.90, size = 90, normalized size = 1.43

$$\frac{d^2 x^3}{3 b} - x\left(\frac{a d^2}{b^2} - \frac{2 c d}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\, x\, (a d - b c)^2}{\sqrt{a}\, (a^2 d^2 - 2 a b c d + b^2 c^2)}\right) (a d - b c)^2}{\sqrt{a}\, b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2),x)
```

```
[Out] (d^2*x^3)/(3*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((b^(1/2)*x*(a*d - b*c
)^2)/(a^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(1/2)*b^(
5/2))
```

3.23 $\int \frac{c+dx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{dx}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] d*x/b+(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {396, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)}{\sqrt{a}\,b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^2}\,dx &= \frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{dx}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.03

$$\frac{dx}{b}-\frac{(-bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2),x]
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.07, size = 34, normalized size = 0.87

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{dx}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}+\frac{\ln\left(-bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}-\frac{\ln\left(-bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] d*x/b+(-a*d+b*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 33, normalized size = 0.85

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 1.62, size = 98, normalized size = 2.51

$$\left[\frac{2\,abdx+\sqrt{-ab}\,(bc-ad)\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\frac{abdx+\sqrt{ab}\,(bc-ad)\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.
time = 0.13, size = 82, normalized size = 2.10

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

Giac [A]
time = 1.42, size = 33, normalized size = 0.85

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B]
time = 0.06, size = 32, normalized size = 0.82

$$\frac{d\,x}{b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(a\,d-b\,c\right)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2),x)
```

```
[Out] (d*x)/b - (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(1/2)*b^(3/2))
```

3.24 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-arctan(x*d^(1/2)/c^(1/
2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {400, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx &= \frac{b\int \frac{1}{a+bx^2}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^2}\,dx}{bc-ad}\\ &= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.79

method	result
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$
risch	$\frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a(ad-bc)}-\frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a(ad-bc)}+\frac{\sqrt{-cd}\,\ln\left(dx+\sqrt{-cd}\right)}{2c(ad-bc)}-\frac{\sqrt{-cd}\,\ln\left(dx-\sqrt{-cd}\right)}{2c(ad-bc)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)/(c*d)^(1/2)*ar
ctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.51, size = 54, normalized size = 0.77

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

Fricas [A]
time = 1.70, size = 292, normalized size = 4.17

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2(bc-ad)},\ -\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2(bc-ad)},\ \frac{2\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2(bc-ad)},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)}{bc-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a
)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

<u>Sympy</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(60) = 120$.
time = 2.73, size = 712, normalized size = 10.17

$$\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}+\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2
*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d -
b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(a*d
 - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqr
t(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**
2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)
**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*d - b
*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) + sqrt(-d
/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(
-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3
 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)
**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)
*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 + a
**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3
+ b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

<u>Giac</u> [**A**]
time = 1.84, size = 54, normalized size = 0.77

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

Mupad [B]
time = 0.32, size = 135, normalized size = 1.93

$$\frac{\ln\left(b\,x-\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,a^2\,d-2\,a\,b\,c}-\frac{\ln\left(d\,x+\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(b\,c^2-a\,c\,d\right)}-\frac{\ln\left(b\,x+\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(a^2\,d-a\,b\,c\right)}+\frac{\ln\left(d\,x-\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,b\,c^2-2\,a\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(b*x - (-a*b)^(1/2))*(-a*b)^(1/2))/(2*a^2*d - 2*a*b*c) - (log(d*x + (-c
*d)^(1/2))*(-c*d)^(1/2))/(2*(b*c^2 - a*c*d)) - (log(b*x + (-a*b)^(1/2))*(-a
*b)^(1/2))/(2*(a^2*d - a*b*c)) + (log(d*x - (-c*d)^(1/2))*(-c*d)^(1/2))/(2*
b*c^2 - 2*a*c*d)
```

3.25 $\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{dx}{2c(bc-ad)\left(c+dx^2\right)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c
)^2/a^(1/2)-1/2*(-a*d+3*b*c)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(3/2)/(-a*
d+b*c)^2
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x)/(c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{\int \frac{2bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}\\ &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}-\frac{(d(3bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^2}\\ &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 95, normalized size = 0.87

$$\frac{\frac{d(-bc+ad)x}{c(c+dx^2)}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}\,(-3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2)
)/(2*(b*c - a*d)^2)
```

Maple [A]
time = 0.15, size = 93, normalized size = 0.85

method	result
default	$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2\sqrt{ab}}+\frac{d\left(\frac{(ad-bc)x}{2c\left(dx^2+c\right)}+\frac{(ad-3bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}\right)}{(ad-bc)^2}$
risch	$\frac{dx}{2c(ad-bc)(dx^2+c)}+\frac{\sqrt{-ab}\,b\ln\left(\left(-4(-ab)^{\frac{3}{2}}abc^2d-4(-ab)^{\frac{3}{2}}b^2c^3-\sqrt{-ab}\,a^4d^3+6\sqrt{-ab}\,a^3bcd^2-13\sqrt{-ab}\,a^2b^2c^2d\right)\right.}{2a(ad-bc)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^2*(1/2*(a*d
-b*c)/c*x/(d*x^2+c)+1/2*(a*d-3*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.50, size = 133, normalized size = 1.22

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} - \frac{dx}{2\,(bc^3 - ac^2d + (bc^2d - acd^2)x^2)} - \frac{(3\,bcd - ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) - 1/2*(3*b*c*d - a*d^2)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
```

Fricas [A]
time = 1.06, size = 711, normalized size = 6.52

$$\left[\frac{2\,(bcdx^2 + bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - (3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 + 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right) - 2\,(bcd - ad^2)x}{4\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}, -\frac{(3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) - (bcdx^2 + bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + (bcd - ad^2)x}{2\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}, \frac{4\,(bcdx^2 + bc^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) - (3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 + 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right) - 2\,(bcd - ad^2)x}{4\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}, \frac{2\,(bcdx^2 + bc^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) - (3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) - (bcd - ad^2)x}{2\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x
^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1
/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
 - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*
arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*
log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*
x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 -
 a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d - a*
d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.43, size = 122, normalized size = 1.12

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}}-\frac{(3\,bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}}-\frac{dx}{2\,(bc^2-acd)(dx^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

Mupad [B]
time = 5.69, size = 2500, normalized size = 22.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (d*x)/(2*c*(c + d*x^2)*(a*d - b*c)) - (atan((((-c^3*d)^(1/2)*(a*d - 3*b*c)*
((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7
+ 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 -
a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3
*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a
*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a
^2*c^3*d^2 - 2*a*b*c^4*d)) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5
 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)) + (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4
*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*
a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^
7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4
*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*
(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
```

```
*b*c^4*d)))/(((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5 - a^3*c^2*d^3 + 3*a
^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*
d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*
c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 +
 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7
*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*
a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/
(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
*b*c^4*d)) - ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^
3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)) + (((4*b^7*c^
6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^
4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 -
3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6
*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 1
6*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2
*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c
^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))*(-c^3*
d)^(1/2)*(a*d - 3*b*c)*1i)/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) - (ata
n((((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 +
 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5
- a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (x*(-a*b^3)^(1/2)*(16*b
^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 4
8*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*
c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 +
a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^
4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*b^2*c^2 - 2
*a^2*b*c*d) - ((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*
b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(
2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^
(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*
(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*
b^2*c^2 - 2*a^2*b*c*d))/(((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d
^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3
*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (
x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 +
32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4
+ a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)
^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*
c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))))/(a^3*
d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - ((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5
```

```
 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-a*b^3)^(1/2)*((((4*b
^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a
^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^
3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^(1/2)*(16...
```

3.26 $\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{\sqrt{d}\,\left(15b^2c^2-10abcd+3a^2d^2\right)\tan}{8c^{5/2}(bc-ad)^3}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2-1/8*d*(-3*a*d+7*b*c)*x/c^2/(-a*d+b*c)^2/(
d*x^2+c)+b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c)^3/a^(1/2)-1/8*(3*a^2*
d^2-10*a*b*c*d+15*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(5/2)/(-a*d+
b*c)^3
```

Rubi [A]
time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$-\frac{\sqrt{d}\,\left(3a^2d^2-10abcd+15b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*
c - a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b
*c - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)(c+dx^2)^2} + \frac{\int \frac{4bc-3ad-3bdx^2}{(a+bx^2)(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{\int \frac{8b^2c^2-7abcd+3a^2d^2-bd(7bc-3}{(a+bx^2)(c+dx^2)}}{8c^2(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^3\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3} - \frac{(d(15b^2c^2}{}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3} - \frac{\sqrt{a}}{}$$

Mathematica [A]
time = 0.16, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{2dx}{c(bc-ad)(c+dx^2)^2} + \frac{d(-7bc+3ad)x}{c^2(bc-ad)^2(c+dx^2)} - \frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(-bc+ad)^3} - \frac{\sqrt{d}\,(15b^2c^2-10abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-
(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.19, size = 158, normalized size = 0.99

method	result
default	$-\frac{b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{(a d-b c)^{3} \sqrt{a b}}+\frac{d\left(\frac{\frac{d\left(3 a^{2} d^{2}-10 a b c d+7 b^{2} c^{2}\right) x^{3}}{8 c^{2}}+\frac{\left(5 a^{2} d^{2}-14 a b c d+9 b^{2} c^{2}\right) x}{8 c}}{\left(d x^{2}+c\right)^{2}}+\frac{\left(3 a^{2} d^{2}-10 a b c d+15 b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 c^{2} \sqrt{c d}}\right)}{(a d-b c)^{3}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^3*((1/8*d*
(3*a^2*d^2-10*a*b*c*d+7*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-14*a*b*c*d+9*b^2*c^
2)/c*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)/c^2/(c*d)^(1/2)*a
rctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(138) = 276$.
time = 0.54, size = 277, normalized size = 1.73

$$\frac{b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \sqrt{a b}}-\frac{\left(15 b^{2} c^{2} d-10 a b c d^{2}+3 a^{2} d^{3}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8\left(b^{3} c^{5}-3 a b^{2} c^{4} d+3 a^{2} b c^{3} d^{2}-a^{3} c^{2} d^{3}\right) \sqrt{c d}}-\frac{\left(7 b c d^{2}-3 a d^{3}\right) x^{3}+\left(9 b c^{2} d-5 a c d^{2}\right) x}{8\left(b^{2} c^{6}-2 a b c^{5} d+a^{2} c^{4} d^{2}+\left(b^{2} c^{4} d^{2}-2 a b c^{3} d^{3}+a^{2} c^{2} d^{4}\right) x^{4}+2\left(b^{2} c^{5} d-2 a b c^{4} d^{2}+a^{2} c^{3} d^{3}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*((7*b*c*d^2 - 3*a*d^3)*x^3 + (9*b*c^2*d - 5*a*c*d^2)*x)/(b^2*c^6
 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*
x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(138) = 276$.
time = 1.63, size = 1585, normalized size = 9.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*x
^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*
d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*
d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*
b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*
x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (9*b^
2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4
 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^
3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 -
16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/
a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c
^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*
c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 -
a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c
^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (
15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3
+ 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*
b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 1.26, size = 217, normalized size = 1.36

$$\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}}-\frac{\left(15\,b^{2}c^{2}d-10\,abcd^{2}+3\,a^{2}d^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)\sqrt{cd}}-\frac{7\,bcd^{2}x^{3}-3\,ad^{3}x^{3}+9\,bc^{2}dx-5\,acd^{2}x}{8\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*
c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

Mupad [B]
time = 6.87, size = 2500, normalized size = 15.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((x^3*(3*a*d^3 - 7*b*c*d^2))/(8*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(
5*a*d^2 - 9*b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 +
 2*c*d*x^2) - (atan((((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 -
 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8
+ a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-
a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^
4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 30
40*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*
d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1
280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7
*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*
(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*
d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(
a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + ((-a*b^5)^(1/2)*(
(x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
+ 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^
2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 176
0*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^
6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d
^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 1
5*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d
```

```
)) + (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b
^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6
*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^
3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d
^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*
b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d
 - 3*a^3*b*c*d^2)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 -
51*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^
4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + ((-
a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*
a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*
c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-a*b^5)^(1/2)*((256*b^10*
c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5
 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800
*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5
*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -
 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^
3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 23
04*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*
d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 -
 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^
3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2
*b^2*c^2*d - 3*a^3*b*c*d^2)) - ((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7
*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32
*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7
*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*
b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^
5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(
64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*
b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-a*b^5)^(1/2)*(256
*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c
^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8
 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3
*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*
a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))
))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))*(-a*b^5)^(
1/2)*1i)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2) - (atan(((
((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a
^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^
3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 68
16*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^
2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*
d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -...
```

3.27 $\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=192

$$\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4}+\frac{d^4(5bc-2ad)x^5}{5b^3}+\frac{d^5x^7}{7b^2}+\frac{(}{2a}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/3*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^3/b^4+1/5*d^4*(-2*a*d+5*b*c)*x^5/b^3+1/7*d
^5*x^7/b^2+1/2*(-a*d+b*c)^5*x/a/b^5/(b*x^2+a)+1/2*(-a*d+b*c)^4*(9*a*d+b*c)*
arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(11/2)
```

Rubi [A]
time = 0.11, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {398, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^4(9ad+bc)}{2a^{3/2}b^{11/2}}+\frac{d^3x^3(3a^2d^2-10abcd+10b^2c^2)}{3b^4}+\frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{x(bc-ad)^5}{2ab^5\,(a+bx^2)}+\frac{d^4x^5(5bc-2ad)}{5b^3}+\frac{d^5x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = \int \left(\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^2}{b^4} + \frac{d^4(5\right.$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc}{5}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc}{5}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc}{5}$$

Mathematica [A]
time = 0.07, size = 192, normalized size = 1.00

$$\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc-2ad)x^5}{5b^3} + \frac{d^5x^7}{7b^2} + \frac{(bc-ad)^5x}{2ab^5\,(a+bx^2)} + \frac{(bc-ad)^4(bc+9ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

Maple [A]
time = 0.09, size = 290, normalized size = 1.51

method	result
default	$-\frac{d^2\left(-\frac{1}{7}d^3x^7b^3+\frac{2}{5}a\,b^2d^3x^5-b^3c\,d^2x^5-a^2b\,d^3x^3+\frac{10}{3}a\,b^2c\,d^2x^3-\frac{10}{3}b^3c^2d\,x^3+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+\frac{-\left(a^5d\right.}{}$

risch	$\frac{d^5x^7}{7b^2} - \frac{2d^5ax^5}{5b^3} + \frac{d^4cx^5}{b^2} + \frac{d^5a^2x^3}{b^4} - \frac{10d^4acx^3}{3b^3} + \frac{10d^3c^2x^3}{3b^2} - \frac{4d^5a^3x}{b^5} + \frac{15d^4a^2cx}{b^4} - \frac{20d^3ac^2x}{b^3} + \frac{10d^2c^3x}{b^2} - \frac{(a^5d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^5*(-1/7*d^3*x^7*b^3+2/5*a*b^2*d^3*x^5-b^3*c*d^2*x^5-a^2*b*d^3*x^3+10
/3*a*b^2*c*d^2*x^3-10/3*b^3*c^2*d*x^3+4*a^3*d^3*x-15*a^2*b*c*d^2*x+20*a*b^2
*c^2*d*x-10*b^3*c^3*x)+1/b^5*(-1/2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^
3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/(b*x^2+a)+1/2*(9*a^5*d^5-35
*a^4*b*c*d^4+50*a^3*b^2*c^2*d^3-30*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+b^5*c^5)/a
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 294, normalized size = 1.53

$$\frac{(b^5c^5 - 5ab^4c^4d + 10a^2b^3c^3d^2 - 10a^3b^2c^2d^3 + 5a^4bcd^4 - a^5d^5)x}{2(ab^6x^2 + a^2b^5)} + \frac{15b^3d^5x^7 + 21(5b^3cd^4 - 2ab^2d^5)x^5 + 35(10b^3c^2d^3 - 10ab^2cd^4 + 3a^2bd^5)x^3 + 105(10b^3c^3d^2 - 20ab^2c^2d^3 + 15a^2bcd^4 - 4a^3d^5)x}{105b^5} + \frac{(b^5c^5 + 5ab^4c^4d - 30a^2b^3c^3d^2 + 50a^3b^2c^2d^3 - 35a^4bcd^4 + 9a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^2 + a^2*b^5) + 1/105*(15*b^3*d^5*x^7 + 21
*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^5 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3*
a^2*b*d^5)*x^3 + 105*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 -
4*a^3*d^5)*x)/b^5 + 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*
a^3*b^2*c^2*d^3 - 35*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(174) = 348$.
time = 0.79, size = 810, normalized size = 4.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*a^2*b^5*d^5*x^9 + 12*(35*a^2*b^5*c*d^4 - 9*a^3*b^4*d^5)*x^7 + 28
*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*a^4*b^3*d^5)*x^5 + 140*(30*a^2*
b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3*c*d^4 - 9*a^5*b^2*d^5)*x^3 -
105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 + 50*a^4*b^2*c^2*d^3
- 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a*b^5*c^4*d - 30*a^2*b^4*c^3*d^
2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9*a^5*b*d^5)*x^2)*sqrt(-a*b)*lo
```

```
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(a*b^6*c^5 - 5*a^2*b^5*c^
4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a^6*b*
d^5)*x)/(a^2*b^7*x^2 + a^3*b^6), 1/210*(30*a^2*b^5*d^5*x^9 + 6*(35*a^2*b^5*
c*d^4 - 9*a^3*b^4*d^5)*x^7 + 14*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*
a^4*b^3*d^5)*x^5 + 70*(30*a^2*b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3
*c*d^4 - 9*a^5*b^2*d^5)*x^3 + 105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3
*c^3*d^2 + 50*a^4*b^2*c^2*d^3 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a
*b^5*c^4*d - 30*a^2*b^4*c^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9
*a^5*b*d^5)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(a*b^6*c^5 - 5*a^2*b
^5*c^4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a
^6*b*d^5)*x)/(a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(185) = 370$.
time = 1.11, size = 502, normalized size = 2.61

$$x^5\left(-\frac{2ad^5}{5b^3}+\frac{cd^4}{b^2}\right)+x^3\left(\frac{a^2d^5}{b^4}-\frac{10acd^4}{3b^3}+\frac{10c^2d^3}{3b^2}\right)+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)+\frac{x(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5)}{2a^2b^5+2ab^6x^2}-\frac{\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)\log\left(-\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)\log\left(\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4}+\frac{d^5x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*d**5/(5*b**3) + c*d**4/b**2) + x**3*(a**2*d**5/b**4 - 10*a*c*d**
4/(3*b**3) + 10*c**2*d**3/(3*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a**2*c*d**4
/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5 + 5*a**4*b
*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d
 + b**5*c**5)/(2*a**2*b**5 + 2*a*b**6*x**2) - sqrt(-1/(a**3*b**11))*(a*d -
b*c)**4*(9*a*d + b*c)*log(-a**2*b**5*sqrt(-1/(a**3*b**11))*(a*d - b*c)**4*(
9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c**2*d**3 - 30*
a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + sqrt(-1/(a**3*b
**11))*(a*d - b*c)**4*(9*a*d + b*c)*log(a**2*b**5*sqrt(-1/(a**3*b**11))*(a*
d - b*c)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c*
*2*d**3 - 30*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + d*
*5*x**7/(7*b**2)
```

Giac [A]
time = 0.85, size = 306, normalized size = 1.59

$$\frac{(b^5c^5+5\,ab^4c^4d-30\,a^2b^3c^3d^2+50\,a^3b^2c^2d^3-35\,a^4bcd^4+9\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab^5}+\frac{b^5c^5x-5\,ab^4c^4dx+10\,a^2b^3c^3d^2x-10\,a^3b^2c^2d^3x+5\,a^4bcd^4x-a^5d^5x}{2\,(bx^2+a)ab^5}+\frac{15\,b^{12}d^5x^7+105\,b^{12}cd^4x^5-42\,ab^{11}d^5x^5+350\,b^{12}c^2d^3x^3-350\,ab^{11}cd^4x^3+105\,a^2b^{10}d^5x^3+1050\,b^{12}c^3d^2x-2100\,ab^{11}c^2d^3x+1575\,a^2b^{10}cd^4x-420\,a^3b^9d^5x}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*a^3*b^2*c^2*d^3 - 35
*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^5) + 1/2*(b^
5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x + 5
```

```
*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^2 + a)*a*b^5) + 1/105*(15*b^12*d^5*x^7 +
105*b^12*c*d^4*x^5 - 42*a*b^11*d^5*x^5 + 350*b^12*c^2*d^3*x^3 - 350*a*b^11*
c*d^4*x^3 + 105*a^2*b^10*d^5*x^3 + 1050*b^12*c^3*d^2*x - 2100*a*b^11*c^2*d^
3*x + 1575*a^2*b^10*c*d^4*x - 420*a^3*b^9*d^5*x)/b^14
```

Mupad [B]
time = 5.02, size = 386, normalized size = 2.01

$$x\left(\frac{10\,c^3\,d^2}{b^2}-\frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b}-\frac{a^2\,d^5}{b^4}+\frac{10\,c^2\,d^3}{b^2}\right)}{b}+\frac{a^2\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right)-x^5\left(\frac{2\,a\,d^5}{5\,b^3}-\frac{c\,d^4}{b^2}\right)+x^3\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{3\,b}-\frac{a^2\,d^5}{3\,b^4}+\frac{10\,c^2\,d^3}{3\,b^2}\right)+\frac{d^5\,x^7}{7\,b^2}-\frac{x\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{2\,a\left(b^6\,x^2+a\,b^5\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)^4\,\left(9\,a\,d+b\,c\right)}{\sqrt{a}\,\left(9\,a^5\,d^5-35\,a^4\,b\,c\,d^4+50\,a^3\,b^2\,c^2\,d^3-30\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d+b^5\,c^5\right)}\right)\,\left(a\,d-b\,c\right)^4\,\left(9\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^5/(a + b*x^2)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^5*((2*a*d^5)/(5*b^3) - (c*d^4)/b^2) + x^3*((2*a*((2*a*d^5)/b^3 - (5*c
*d^4)/b^2))/(3*b) - (a^2*d^5)/(3*b^4) + (10*c^2*d^3)/(3*b^2)) + (d^5*x^7)/(
7*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 +
5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(2*a*(a*b^5 + b^6*x^2)) + (atan((b^(1/2)*x*
(a*d - b*c)^4*(9*a*d + b*c))/(a^(1/2)*(9*a^5*d^5 + b^5*c^5 - 30*a^2*b^3*c^3
*d^2 + 50*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 35*a^4*b*c*d^4)))*(a*d - b*c)^4
*(9*a*d + b*c))/(2*a^(3/2)*b^(11/2))
```

3.28 $\int \frac{(c+dx^2)^4}{(a+bx^2)^2} dx$

Optimal. Leaf size=142

$$\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{(bc-ad)^4x}{2ab^4\,(a+bx^2)}+\frac{(bc-ad)^3(bc+7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/3*d^3*(-a*d+2*b*c)*x^3/b^3+1/5*
d^4*x^5/b^2+1/2*(-a*d+b*c)^4*x/a/b^4/(b*x^2+a)+1/2*(-a*d+b*c)^3*(7*a*d+b*c)
*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(9/2)
```

Rubi [A]
time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {398, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^3(7ad+bc)}{2a^{3/2}b^{9/2}}+\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{x(bc-ad)^4}{2ab^4\,(a+bx^2)}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{d^4x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4} + \frac{2d^3(2bc-ad)x^2}{b^3} + \frac{d^4x^4}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bd\ldots}{b^4(a+bx^2)^2}\right.$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^2}{(a+bx^2)^2}}{b^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4(a+bx^2)} + \frac{((bc-ad)^3\ldots}{\ldots}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4(a+bx^2)} + \frac{(bc-ad)^3(\ldots}{\ldots}$$

Mathematica [A]
time = 0.06, size = 142, normalized size = 1.00

$$\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4(a+bx^2)} + \frac{(bc-ad)^3(bc+7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Maple [A]
time = 0.11, size = 206, normalized size = 1.45

method	result
default	$\frac{d^2\left(\frac{1}{5}b^2x^5d^2-\frac{2}{3}ab\,d^2x^3+\frac{4}{3}b^2cd\,x^3+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4} - \frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(7a^4d^4-20a^3bc\,d^3\ldots\right.}{\ldots}}{b^4}$
risch	$\frac{d^4x^5}{5b^2} - \frac{2d^4a\,x^3}{3b^3} + \frac{4d^3c\,x^3}{3b^2} + \frac{3d^4a^2x}{b^4} - \frac{8d^3acx}{b^3} + \frac{6d^2c^2x}{b^2} + \frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{2a\,b^4\left(b\,x^2+a\right)} - \frac{7a^3\ln\left(bx\ldots\right.}{4b^4\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/b^4*(1/5*b^2*x^5*d^2-2/3*a*b*d^2*x^3+4/3*b^2*c*d*x^3+3*a^2*d^2*x-8*a*b*
c*d*x+6*b^2*c^2*x)-1/b^4*(-1/2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/a*x/(b*x^2+a)+1/2*(7*a^4*d^4-20*a^3*b*c*d^3+18*a^2*b^2*
c^2*d^2-4*a*b^3*c^3*d-b^4*c^4)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [**A**]
time = 0.50, size = 213, normalized size = 1.50

$$\frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)x}{2\,(ab^5x^2 + a^2b^4)} + \frac{3\,b^2d^4x^5 + 10\,(2\,b^2cd^3 - abd^4)x^3 + 15\,(6\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x}{15\,b^4} + \frac{(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^2 + a^2*b^4) + 1/15*(3*b^2*d^4*x^5 + 10*(2*b^2*c*d^3 - a*b*d^4)
*x^3 + 15*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/2*(b^4*c^4 +
 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*d^4)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(126) = 252.
time = 0.65, size = 612, normalized size = 4.31

$$\left[\frac{12\,a^2b^4d^4x^7 + 4\,(20\,a^2b^4cd^3 - 7\,a^3b^3d^4)x^5 + 20\,(18\,a^2b^4c^2d^2 - 20\,a^3b^3cd^3 + 7\,a^4b^2d^4)x^3 + 15\,(ab^4c^4 + 4\,a^2b^3c^3d - 18\,a^3b^2c^2d^2 + 20\,a^4bcd^3 - 7\,a^5d^4 + (b^5c^4 + 4\,ab^4c^3d - 18\,a^2b^3c^2d^2 + 20\,a^3b^2cd^3 - 7\,a^4bd^4)x^2)\sqrt{-ab}\log\left(\frac{bx^2 + 2\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 30\,(ab^5c^4 - 4\,a^2b^4c^3d + 18\,a^3b^3c^2d^2 - 20\,a^4b^2cd^3 + 7\,a^5bd^4)x}{60\,(a^2b^6x^2 + a^3b^5)}, \frac{6\,a^2b^4d^4x^7 + 2\,(20\,a^2b^4cd^3 - 7\,a^3b^3d^4)x^5 + 10\,(18\,a^2b^4c^2d^2 - 20\,a^3b^3cd^3 + 7\,a^4b^2d^4)x^3 + 15\,(ab^4c^4 + 4\,a^2b^3c^3d - 18\,a^3b^2c^2d^2 + 20\,a^4bcd^3 - 7\,a^5d^4 + (b^5c^4 + 4\,ab^4c^3d - 18\,a^2b^3c^2d^2 + 20\,a^3b^2cd^3 - 7\,a^4bd^4)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 15\,(ab^5c^4 - 4\,a^2b^4c^3d + 18\,a^3b^3c^2d^2 - 20\,a^4b^2cd^3 + 7\,a^5bd^4)x}{30\,(a^2b^6x^2 + a^3b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a^2*b^4*d^4*x^7 + 4*(20*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x^5 + 20*(
18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^3 + 15*(a*b^4*c^4
+ 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*b*c*d^3 - 7*a^5*d^4 + (b^5*
c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*d^4)*
x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(a*b^5*c
^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^4)
*x)/(a^2*b^6*x^2 + a^3*b^5), 1/30*(6*a^2*b^4*d^4*x^7 + 2*(20*a^2*b^4*c*d^3
- 7*a^3*b^3*d^4)*x^5 + 10*(18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^
2*d^4)*x^3 + 15*(a*b^4*c^4 + 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*
b*c*d^3 - 7*a^5*d^4 + (b^5*c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^
3*b^2*c*d^3 - 7*a^4*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(a*b^5
*c^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^
4)*x)/(a^2*b^6*x^2 + a^3*b^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(133) = 266.
time = 0.82, size = 403, normalized size = 2.84

$$x^3\left(-\frac{2ad^4}{3b^3} + \frac{4cd^3}{3b^2}\right) + x\left(\frac{3a^2d^4}{b^4} - \frac{8acd^3}{b^3} + \frac{6c^2d^2}{b^2}\right) + \frac{x(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)}{2a^2b^4 + 2ab^5x^2} + \frac{\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\cdot(7ad+bc)\log\left(-\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\cdot(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4} + x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\cdot(7ad+bc)\log\left(\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\cdot(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4} + x\right)}{4} + \frac{d^4x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*d**4/(3*b**3) + 4*c*d**3/(3*b**2)) + x*(3*a**2*d**4/b**4 - 8*a*c
*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b*
*2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(2*a**2*b**4 + 2*a*b**5*x**2) +
 sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(-a**2*b**4*sqrt(-1/(
a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 +
18*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 - sqrt(-1/(a**
3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(a**2*b**4*sqrt(-1/(a**3*b**9))*(a
*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 + 18*a**2*b**2*c
**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 + d**4*x**5/(5*b**2)
```

Giac [A]
time = 2.22, size = 220, normalized size = 1.55

$$\frac{\left(b^{4} c^{4} + 4 \, a b^{3} c^{3} d - 18 \, a^{2} b^{2} c^{2} d^{2} + 20 \, a^{3} b c d^{3} - 7 \, a^{4} d^{4}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a b^{4}} + \frac{b^{4} c^{4} x - 4 \, a b^{3} c^{3} d x + 6 \, a^{2} b^{2} c^{2} d^{2} x - 4 \, a^{3} b c d^{3} x + a^{4} d^{4} x}{2 \, {\left(b x^{2} + a\right)} a b^{4}} + \frac{3 \, b^{8} d^{4} x^{5} + 20 \, b^{8} c d^{3} x^{3} - 10 \, a b^{7} d^{4} x^{3} + 90 \, b^{8} c^{2} d^{2} x - 120 \, a b^{7} c d^{3} x + 45 \, a^{2} b^{6} d^{4} x}{15 \, b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*c^4 + 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*
d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/2*(b^4*c^4*x - 4*a*b^3*c^3
*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^2 + a)*a*b^
4) + 1/15*(3*b^8*d^4*x^5 + 20*b^8*c*d^3*x^3 - 10*a*b^7*d^4*x^3 + 90*b^8*c^2
*d^2*x - 120*a*b^7*c*d^3*x + 45*a^2*b^6*d^4*x)/b^10
```

Mupad [B]
time = 5.05, size = 261, normalized size = 1.84

$$x\,\left(\frac{2\,a\,\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^3\,\left(\frac{2\,a\,d^4}{3\,b^3}-\frac{4\,c\,d^3}{3\,b^2}\right)+\frac{d^4\,x^5}{5\,b^2}+\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{2\,a\,\left(b^5\,x^2+a\,b^4\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3\,\left(7\,a\,d+b\,c\right)}{\sqrt{a}\,\left(-7\,a^4\,d^4+20\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}\right)\,{\left(a\,d-b\,c\right)}^3\,\left(7\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^3*((2*a*d^4)/(3*b^3) - (4*c*d^3)/(3*b^2)) + (d^4*x^5)/(5*b^2) + (x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(2*
a*(a*b^4 + b^5*x^2)) + (atan((b^(1/2)*x*(a*d - b*c)^3*(7*a*d + b*c))/(a^(1/
2)*(b^4*c^4 - 7*a^4*d^4 - 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 20*a^3*b*c*d
^3)))*(a*d - b*c)^3*(7*a*d + b*c))/(2*a^(3/2)*b^(9/2))
```

3.29 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/3*d^3*x^3/b^2+1/2*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a)
+1/2*(-a*d+b*c)^2*(5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(7/2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$,
Rules used = {398, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5ad+bc)(bc-ad)^2}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^2}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{b^3\,(a+bx^2)^2}\right)dx\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{(a+bx^2)^2}\,dx}{b^3}\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\,(a+bx^2)}+\frac{\left((bc-ad)^2(bc+5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2ab^3}\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\,(a+bx^2)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 106, normalized size = 1.00

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\,(a+bx^2)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Maple [A]
time = 0.09, size = 139, normalized size = 1.31

method	result
default	$-\frac{d^2\left(-\frac{1}{3}bd\,x^3+2adx-3bcx\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}$
risch	$\frac{d^3x^3}{3b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2a\,b^3(b\,x^2+a)}-\frac{5a^2\ln\left(bx+\sqrt{-ab}\right)d^3}{4b^3\sqrt{-ab}}+\frac{9a\ln\left(bx+\sqrt{-ab}\right)c\,d^2}{4b^2\sqrt{-ab}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3*(-1/3*b*d*x^3+2*a*d*x-3*b*c*x)+1/b^3*(-1/2*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/a*x/(b*x^2+a)+1/2*(5*a^3*d^3-9*a^2*b*c*d^2+3*a*b^2*c
^2*d+b^3*c^3)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 1.39

$$\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x}{2\,(ab^4x^2 + a^2b^3)} + \frac{bd^3x^3 + 3\,(3\,bcd^2 - 2\,ad^3)x}{3\,b^3} + \frac{(b^3c^3 + 3\,ab^2c^2d - 9\,a^2bcd^2 + 5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^2 + a^2*
b^3) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/2*(b^3*c^3 + 3*a
*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(92) = 184$.
time = 0.81, size = 442, normalized size = 4.17

$$\left[\frac{4\,a^2b^3d^3x^5 + 4\,(9\,a^2b^3cd^2 - 5\,a^3b^2d^3)x^3 - 3\,(ab^3c^3 + 3\,a^2b^2c^2d - 9\,a^3bcd^2 + 5\,a^4d^3 + (b^4c^3 + 3\,ab^3c^2d - 9\,a^2b^2cd^2 + 5\,a^3bd^3)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 6\,(ab^4c^3 - 3\,a^2b^3c^2d + 9\,a^3b^2cd^2 - 5\,a^4bd^3)x}{12\,(a^2b^5x^2 + a^3b^4)}, \frac{2\,a^2b^3d^3x^5 + 2\,(9\,a^2b^3cd^2 - 5\,a^3b^2d^3)x^3 + 3\,(ab^3c^3 + 3\,a^2b^2c^2d - 9\,a^3bcd^2 + 5\,a^4d^3 + (b^4c^3 + 3\,ab^3c^2d - 9\,a^2b^2cd^2 + 5\,a^3bd^3)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 3\,(ab^4c^3 - 3\,a^2b^3c^2d + 9\,a^3b^2cd^2 - 5\,a^4bd^3)x}{6\,(a^2b^5x^2 + a^3b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c
^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2
 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2*(9*a
^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*
b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*
d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^3*c^2*d
+ 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.
time = 0.59, size = 314, normalized size = 2.96

$$x\left(-\frac{2ad^3}{b^3} + \frac{3cd^2}{b^2}\right) + \frac{x(-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3)}{2a^2b^3 + 2ab^4x^2} - \frac{\sqrt{-\frac{1}{a^3b^7}}\,(ad - bc)^2 \cdot (5ad + bc)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}\,(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3 - 9a^2bcd^2 + 3ab^2c^2d + b^3c^3} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^7}}\,(ad - bc)^2 \cdot (5ad + bc)\log\left(\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}\,(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3 - 9a^2bcd^2 + 3ab^2c^2d + b^3c^3} + x\right)}{4} + \frac{d^3x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7
))*(a*d - b*c)**2*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d -
b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b*
*3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*log(a**
2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a
**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + d**3*x**3/(3*b**2)
```

Giac [A]
time = 1.12, size = 152, normalized size = 1.43

$$\frac{\left(b^{3} c^{3}+3 \, a b^{2} c^{2} d-9 \, a^{2} b c d^{2}+5 \, a^{3} d^{3}\right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a b^{3}}+\frac{b^{3} c^{3} x-3 \, a b^{2} c^{2} d x+3 \, a^{2} b c d^{2} x-a^{3} d^{3} x}{2 \,\left(b x^{2}+a\right) a b^{3}}+\frac{b^{4} d^{3} x^{3}+9 \, b^{4} c d^{2} x-6 \, a b^{3} d^{3} x}{3 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6*a*
b^3*d^3*x)/b^6
```

Mupad [B]
time = 0.10, size = 182, normalized size = 1.72

$$\frac{d^3\,x^3}{3\,b^2}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)-\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,\left(b^4\,x^2+a\,b^3\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{\sqrt{a}\,\left(5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^2,x)
```

```
[Out] (d^3*x^3)/(3*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/
2)*x*(a*d - b*c)^2*(5*a*d + b*c))/(a^(1/2)*(5*a^3*d^3 + b^3*c^3 + 3*a*b^2*c
^2*d - 9*a^2*b*c*d^2)))*(a*d - b*c)^2*(5*a*d + b*c))/(2*a^(3/2)*b^(7/2))
```

3.30 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=82

$$\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

```
[Out] d^2*x/b^2+1/2*(-a*d+b*c)^2*x/a/b^2/(b*x^2+a)+1/2*(-a*d+b*c)*(3*a*d+b*c)*arc
tan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {398, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)(3ad+bc)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx &= \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{b^2(a+bx^2)^2}\right)dx \\
&= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{(a+bx^2)^2}\,dx}{b^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2(a+bx^2)}+\frac{((bc-ad)(bc+3ad))\int \frac{1}{a+bx^2}\,dx}{2ab^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2(a+bx^2)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 1.07

$$
\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2(a+bx^2)}+\frac{(b^2c^2+2abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Maple [A]
time = 0.09, size = 94, normalized size = 1.15

method	result
default	$\frac{d^2x}{b^2}-\frac{-\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{2a\left(bx^2+a\right)}+\frac{\left(3a^2d^2-2abcd-b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2ab^2(bx^2+a)}-\frac{3a\ln\left(bx-\sqrt{-ab}\right)d^2}{4b^2\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)cd}{2b\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)c^2}{4\sqrt{-ab}\,a}+\frac{3a\ln\left(-bx-\sqrt{\ }\right.}{4b^2\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2-1/b^2*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/(b*x^2+a)+1/2*(3*a^2*
d^2-2*a*b*c*d-b^2*c^2)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 95, normalized size = 1.16

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)x}{2\,(ab^3x^2 + a^2b^2)} + \frac{d^2x}{b^2} + \frac{(b^2c^2 + 2\,abcd - 3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^2 + a^2*b^2) + d^2*x/b^2 + 1
/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2
)
```

Fricas [A]
time = 0.80, size = 297, normalized size = 3.62

$$\left[\frac{4\,a^2b^2d^2x^3 + (ab^2c^2 + 2\,a^2bcd - 3\,a^3d^2 + (b^3c^2 + 2\,ab^2cd - 3\,a^2bd^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2 + 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 2\,(ab^3c^2 - 2\,a^2b^2cd + 3\,a^3bd^2)x}{4\,(a^2b^4x^2 + a^3b^3)}, \frac{2\,a^2b^2d^2x^3 + (ab^2c^2 + 2\,a^2bcd - 3\,a^3d^2 + (b^3c^2 + 2\,ab^2cd - 3\,a^2bd^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + (ab^3c^2 - 2\,a^2b^2cd + 3\,a^3bd^2)x}{2\,(a^2b^4x^2 + a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 +
 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^
2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2
 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(73) = 146$.
time = 0.39, size = 236, normalized size = 2.88

$$\frac{x(a^2d^2 - 2abcd + b^2c^2)}{2a^2b^2 + 2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad - bc)\,(3ad + bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)(3ad+bc)}{3a^2d^2 - 2abcd - b^2c^2} + x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad - bc)\,(3ad + bc)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)(3ad+bc)}{3a^2d^2 - 2abcd - b^2c^2} + x\right)}{4} + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt(
-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 -
 sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x
)/4 + d**2*x/b**2
```

Giac [A]
time = 1.35, size = 94, normalized size = 1.15

$$\frac{d^2x}{b^2} + \frac{(b^2c^2 + 2\,abcd - 3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(bx^2 + a)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a*b
^2)
```

Mupad [B]
time = 5.06, size = 124, normalized size = 1.51

$$\frac{d^2\,x}{b^2} + \frac{x\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}{2\,a\,(b^3\,x^2 + a\,b^2)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{a}\,(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d - b\,c)\,(3\,a\,d + b\,c)}{2\,a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*(a*b^2 + b^3*x^2)) +
 (atan((b^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(a^(1/2)*(b^2*c^2 - 3*a^2*d^2
+ 2*a*b*c*d))))*(a*d - b*c)*(3*a*d + b*c))/(2*a^(3/2)*b^(5/2))
```

3.31 $\int \frac{c+dx^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)x}{2ab\,(a+bx^2)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

```
[Out] 1/2*(-a*d+b*c)*x/a/b/(b*x^2+a)+1/2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3
/2)/b^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(ad+bc)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2ab}$$
$$= \frac{(bc-ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$-\frac{(-bc+ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(b*c) + a*d)*x)/(a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.07, size = 57, normalized size = 0.90

method	result	size
default	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}+\frac{(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}-\frac{\ln\left(bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2*(a*d+b*c)/a/b/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

Maxima [A]
time = 0.50, size = 57, normalized size = 0.90

$$\frac{(bc-ad)x}{2\,(ab^2x^2+a^2b)} + \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*x/(a*b^2*x^2 + a^2*b) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.60, size = 181, normalized size = 2.87

$$\left[-\frac{(abc + a^2d + (b^2c + abd)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 2\,(ab^2c - a^2bd)x}{4\,(a^2b^3x^2 + a^3b^2)}, \frac{(abc + a^2d + (b^2c + abd)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + (ab^2c - a^2bd)x}{2\,(a^2b^3x^2 + a^3b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.
time = 0.21, size = 112, normalized size = 1.78

$$\frac{x(-ad + bc)}{2a^2b + 2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad + bc)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad + bc)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Giac [A]
time = 1.30, size = 57, normalized size = 0.90

$$\frac{(bc + ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab} + \frac{bcx - adx}{2\,(bx^2 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x)
/((b*x^2 + a)*a*b)
```

Mupad [B]
time = 5.04, size = 51, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+b\,c)}{2\,a^{3/2}\,b^{3/2}}-\frac{x\,(a\,d-b\,c)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d + b*c))/(2*a^(3/2)*b^(3/2)) - (x*(a*d - b*c
))/(2*a*b*(a + b*x^2))
```

3.32 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=108

$$\frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}$$

```
[Out] 1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)+1/2*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*b
^(1/2)/a^(3/2)/(-a*d+b*c)^2+d^(3/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^2/
c^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-3ad)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx &= \frac{bx}{2a(bc-ad)(a+bx^2)} - \frac{\int \frac{-bc+2ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2a(bc-ad)} \\ &= \frac{bx}{2a(bc-ad)(a+bx^2)} + \frac{d^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^2} \\ &= \frac{bx}{2a(bc-ad)(a+bx^2)} + \frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 109, normalized size = 1.01

$$-\frac{bx}{2a(-bc+ad)(a+bx^2)} - \frac{\sqrt{b}(-bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(-bc+ad)^2} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
[Out] -1/2*(b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Maple [A]
time = 0.15, size = 95, normalized size = 0.88

method	result
default	$-\frac{b\left(\frac{(ad-bc)x}{2a(bx^2+a)}+\frac{(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^2}+\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^2\sqrt{cd}}$
risch	$-\frac{bx}{2a(ad-bc)(bx^2+a)}+\frac{3\sqrt{-ab}\,\ln\left(\left(-9(-ab)^{\frac{3}{2}}a^3d^3-3(-ab)^{\frac{3}{2}}a^2bcd^2+5(-ab)^{\frac{3}{2}}ab^2c^2d-(-ab)^{\frac{3}{2}}b^3c^3-13\sqrt{-ab}\,a^4bd^3+6\right.\right.}{4a(ad-bc)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] -b/(a*d-b*c)^2*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(3*a*d-b*c)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))

Maxima [A]
time = 0.51, size = 132, normalized size = 1.22

$$\frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{cd}} + \frac{bx}{2\,(a^2bc - a^3d + (ab^2c - a^2bd)x^2)} + \frac{(b^2c - 3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2 - 2\,a^2bcd + a^3d^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")

[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))

Fricas [A]
time = 0.63, size = 699, normalized size = 6.47

$$\left[-\frac{(abc - 3\,a^2d + (b^2c - 3\,abd)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - 2\,(abdx^2 + a^2d)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 + 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right) - 2\,(b^2c - abd)x}{4\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x^2)}, \frac{4\,(abdx^2 + a^2d)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) - (abc - 3\,a^2d + (b^2c - 3\,abd)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + 2\,(b^2c - abd)x}{4\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x^2)}, \frac{(abc - 3\,a^2d + (b^2c - 3\,abd)x^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + (abdx^2 + a^2d)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 + 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right) + (b^2c - abd)x}{2\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x^2)}, \frac{(abc - 3\,a^2d + (b^2c - 3\,abd)x^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 2\,(abdx^2 + a^2d)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) + (b^2c - abd)x}{2\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")

[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.98, size = 121, normalized size = 1.12

$$\frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{cd}} + \frac{(b^2c - 3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2 - 2\,a^2bcd + a^3d^2)\sqrt{ab}} + \frac{bx}{2\,(abc - a^2d)(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^
2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

Mupad [B]
time = 5.77, size = 2500, normalized size = 23.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*
a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7
- 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*
d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c
^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2
 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)
^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^
2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4
*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-
a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*
c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d
)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
)*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))/(((3*a*b^3*d^5)/2 - (b^4*c
```

```
*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - ((-a^3
*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))
/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7 - 2*a*b^7*c^5
*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*
b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - (x
*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b
^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))
/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d))))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)^(1/2)*(3*a*d -
b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c
*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-a^3*b)^(1/2)*(3*
a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3
*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/
2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))))/(4*(a^5*d^2 +
 a^3*b^2*c^2 - 2*a^4*b*c*d))))*(-a^3*b)^(1/2)*(3*a*d - b*c)*1i)/(2*(a^5*d^2
 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - (atan((((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 -
 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d
^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^
4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2
*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5
))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^
2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2
*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*
b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d) - ((-c*d^3)^(1/2)*((((4*a^
6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^
3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2
 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) +
 (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))/(((3*a*b^3*d^5
)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^
6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(1
6*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3
+ 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3
 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c
*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c*d^2 - 2
```

```
*a*b*c^2*d) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*
b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(
2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^
(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a...
```

3.33 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=167

$$\frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)}+\frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)}+\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}$$

```
[Out] 1/2*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)
/(d*x^2+c)+1/2*b^(3/2)*(-5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d
+b*c)^3+1/2*d^(3/2)*(-a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*
c)^3
```

Rubi [A]
time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-5ad)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{dx(ad+bc)}{2ac\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\int \frac{-bc+2ad-3bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}\\ &= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\int \frac{-2(b^2c^2-4abcd+a}{(a+bx^2}}{4ac(b}\\ &= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\left(b^2(bc-5ad)\right)\int}{2a(bc-a}\\ &= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{b^{3/2}(bc-5ad)\,\mathrm{ta}}{2a^{3/2}(bc}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 136, normalized size = 0.81

$$\frac{1}{2}\left(\frac{b^{3/2}(-bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^3}+\frac{(bc-ad)x\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d^(
3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/2
```

Maple [A]
time = 0.20, size = 133, normalized size = 0.80

method	result	size
default	$\frac{b^{2}\left(\frac{(ad-bc)x}{2a\left(b\,x^{2}+a\right)}+\frac{(5ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^{3}}+\frac{d^{2}\left(\frac{(ad-bc)x}{2c\left(d\,x^{2}+c\right)}+\frac{(ad-5bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}\right)}{(ad-bc)^{3}}$	133
risch	Expression too large to display	2124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(5*a*d-b*c)/a/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/c*x/(d*x^2+c)+1/2*(
a*d-5*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(143) = 286.
time = 0.50, size = 294, normalized size = 1.76

$$\frac{(b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}}+\frac{(5\,bcd^2-ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}}+\frac{(b^2cd+abd^2)x^3+(b^2c^2+a^2d^2)x}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*((b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d
^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(143) = 286.
time = 1.19, size = 1681, normalized size = 10.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2
*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sq
```

```
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*d -
 a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d
^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c
)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*
a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^
3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d
 + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 +
2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c
^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (a*b^2
*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2
*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^
3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*
a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2
*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d
^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 +
(b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)
) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2
*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*
x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*
c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c
^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d -
 a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5
*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2
 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.67, size = 232, normalized size = 1.39

$$\frac{(b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}}+\frac{(5\,bcd^2-ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}}+\frac{b^2cdx^3+abd^2x^3+b^2c^2x+a^2d^2x}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2)(bdx^4+bcx^2+adx^2+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^2*d^2*x)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

Mupad [B]
time = 6.87, size = 2500, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3
*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*
c) + b*d*x^4) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 -
 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*
a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2
 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7
*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 -
 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b
*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) -
(x*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3
+ 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b
^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^
3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3
*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3
)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*
a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 1
0*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^
3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 +
 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c
^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 2
0*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c
^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x
*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 +
144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4
*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*
c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b
^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^
```

(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*
d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d -
3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^6*c^2*d
^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d
- 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^
4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*
d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d
- 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c
*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220
*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^
2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*
a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(5*a*d - b*
c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*
c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 8
0*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4
*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a
^3*b^3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*d^6 +
50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a
^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c*d^10
- 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220*a^5*
b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^2*d^9
)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b
^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(5*a*d - b*c)*(-
a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*c^7*d
^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 80*a^8
*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c
^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*
b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a^6*d^
3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a^3*b^
3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(
5*a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(2*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2)) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^
3*d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d
^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*
c^9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*
a^4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6...

3.34 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=230

$$\frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)}+\frac{b^{5/2}(bc-7ad)\tan^{-1}}{2a^{3/2}(bc-ad)}$$

```
[Out] 1/4*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x/a/(-a*d+b*c)/(b*x^
2+a)/(d*x^2+c)^2+1/8*d*(-a*d+4*b*c)*(3*a*d+b*c)*x/a/c^2/(-a*d+b*c)^3/(d*x^2
+c)+1/2*b^(5/2)*(-7*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)^4
+1/8*d^(3/2)*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^
(5/2)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.22, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-7ad)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{-bc+2ad-5bdx^2}{(a+bx^2)(c+dx^2)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{-2(2b^2c^2-8abc}{(a+}}{8a}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4bc-ad)(b}{8ac^2(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4bc-ad)(b}{8ac^2(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4bc-ad)(b}{8ac^2(bc-ad)}$$

Mathematica [A]
time = 0.28, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(-\frac{4b^3x}{a(-bc+ad)^3(a+bx^2)} + \frac{2d^2x}{c(bc-ad)^2(c+dx^2)^2} + \frac{d^2(11bc-3ad)x}{c^2(bc-ad)^3(c+dx^2)} + \frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4} + \frac{d^{3/2}(35b^2c^2-14abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*(
c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2)) +
(4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^
4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

Maple [A]
time = 0.25, size = 198, normalized size = 0.86

method	result
default	$-\frac{b^3\left(\frac{(ad-bc)x}{2a\left(b\,x^2+a\right)}+\frac{(7ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{d^2\left(\frac{\frac{d\left(3a^2d^2-14abcd+11b^2c^2\right)x^3}{8c^2}+\frac{\left(5a^2d^2-18abcd+13b^2c^2\right)x}{8c}}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2-14abcd+35b^2\right.}{8c^2\sqrt{}}\right.}{(ad-bc)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^4*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(7*a*d-b*c)/a/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^4*((1/8*d*(3*a^2*d^2-14*a*b*c*d+11*
b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)/c*x)/(d*x^2+c)^2+1/8
*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(204) = 408$.
time = 0.52, size = 529, normalized size = 2.30

$$\frac{(b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}}+\frac{(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}}+\frac{(4\,b^3c^2d^2+11\,ab^2cd^3-3\,a^2bd^4)x^5+(8\,b^3c^3d+13\,ab^2c^2d^2+6\,a^2bcd^3-3\,a^3d^4)x^3+(4\,b^3c^4+13\,a^2bc^2d^2-5\,a^3cd^3)x}{8\,(a^2b^3c^7-3\,a^3b^2c^6d+3\,a^4bc^5d^2-a^5c^4d^3+(ab^4c^5d^2-3\,a^2b^3c^4d^3+3\,a^3b^2c^3d^4-a^4bc^2d^5)x^6+(2\,ab^4c^6d-5\,a^2b^3c^5d^2+3\,a^3b^2c^4d^3+a^4bc^3d^4-a^5c^2d^5)x^4+(ab^4c^7-a^2b^3c^6d-3\,a^3b^2c^5d^2+5\,a^4bc^4d^3-2\,a^5c^3d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^3*c^3*d
 + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*c^
2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*b^3
*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d)) + 1/
8*((4*b^3*c^2*d^2 + 11*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (8*b^3*c^3*d + 13*a
*b^2*c^2*d^2 + 6*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (4*b^3*c^4 + 13*a^2*b*c^2*d
^2 - 5*a^3*c*d^3)*x)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5
*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c
^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*
b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d
^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(204) = 408$.
time = 2.86, size = 3239, normalized size = 14.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4
 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3
*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 +
(b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 +
 (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (
35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 -
 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a
^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^
4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b
^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^
5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c
^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^
2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x
^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4
*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b
^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^
2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c
^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*(a*b^3*c^5 - 7*a^2*b^2*c^4*d +
(b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a
```

```
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4
*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*
b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^
3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c
*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^
3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*
c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^
2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b
/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*
d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c
^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4
*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^
3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^
6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3
+ 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7
*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2
 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.56, size = 332, normalized size = 1.44

$$\frac{b^{3} x}{2 \, {\left(a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + 3 \, a^{3} b c d^{2} - a^{4} d^{3}\right)} {\left(b x^{2} + a\right)}} + \frac{{\left(b^{4} c - 7 \, a b^{3} d\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, {\left(a b^{4} c^{4} - 4 \, a^{2} b^{3} c^{3} d + 6 \, a^{3} b^{2} c^{2} d^{2} - 4 \, a^{4} b c d^{3} + a^{5} d^{4}\right)} \sqrt{a b}} + \frac{{\left(35 \, b^{2} c^{2} d^{2} - 14 \, a b c d^{3} + 3 \, a^{2} d^{4}\right)} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 \, {\left(b^{4} c^{6} - 4 \, a b^{3} c^{5} d + 6 \, a^{2} b^{2} c^{4} d^{2} - 4 \, a^{3} b c^{3} d^{3} + a^{4} c^{2} d^{4}\right)} \sqrt{c d}} + \frac{11 \, b c d^{3} x^{3} - 3 \, a d^{4} x^{3} + 13 \, b c^{2} d^{2} x - 5 \, a c d^{3} x}{8 \, {\left(b^{3} c^{5} - 3 \, a b^{2} c^{4} d + 3 \, a^{2} b c^{3} d^{2} - a^{3} c^{2} d^{3}\right)} {\left(d x^{2} + c\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2 +
 a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35
*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 -
4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d
)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^
3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

Mupad [B]
time = 7.79, size = 2500, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^
4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7)
)/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*
a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^13*c
^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*
c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7
*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 +
(35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^
4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*
b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d
^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^
11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^
7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 +
256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a
^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9
*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2
*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^
4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d))
)*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 +
 a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) + (((x
*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2
009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*
b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8
*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + (((2*a*b^13*c^13*d^2 - 2
8*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*
a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^
```

3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a
^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^
3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9
*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(
256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 71
68*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^
8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^
2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*
d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*
b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))
*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4
*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3
)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 + a^4*c^5*d^
4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))/(((63*a^5*b^5*d^
9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)
/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 -
 a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 -
 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b
^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 2
24*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^
3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c
^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b
^2*c^6*d^4)) - (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*
c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^
8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^
10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13
)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 3
6*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^
4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*
(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^1
0*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c
^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9
 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5
*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 +
 a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*
a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^
2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b
^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b...

3.35 $\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=196

$$\frac{d^3(10b^2c^2-15abcd+6a^2d^2)\,x}{b^5}+\frac{d^4(5bc-3ad)x^3}{3b^4}+\frac{d^5x^5}{5b^3}+\frac{(bc-ad)^5x}{4ab^5\,(a+bx^2)^2}+\frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5\,(a+bx^2)}+\frac{(bc-}{}$$

```
[Out] d^3*(6*a^2*d^2-15*a*b*c*d+10*b^2*c^2)*x/b^5+1/3*d^4*(-3*a*d+5*b*c)*x^3/b^4+
1/5*d^5*x^5/b^3+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^2+a)^2+1/8*(-a*d+b*c)^4*(17*a
*d+3*b*c)*x/a^2/b^5/(b*x^2+a)+1/8*(-a*d+b*c)^3*(63*a^2*d^2+14*a*b*c*d+3*b^2
*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(11/2)
```

Rubi [A]
time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {398, 1171, 393, 211}

$$\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\,(a+bx^2)}+\frac{d^3x(6a^2d^2-15abcd+10b^2c^2)}{b^5}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(63a^2d^2+14abcd+3b^2c^2)\,(bc-ad)^3}{8a^{5/2}b^{11/2}}+\frac{x(bc-ad)^5}{4ab^5\,(a+bx^2)^2}+\frac{d^4x^3(5bc-3ad)}{3b^4}+\frac{d^5x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx = \int \left(\frac{d^3(10b^2c^2-15abcd+6a^2d^2)}{b^5} + \frac{d^4(5bc-3ad)x^2}{b^4} + \frac{d^5x^4}{b^3} + \frac{(bc-ad)^3\,(b^2c^2+3abcd \ldots}{\ldots}\right.$$

$$= \frac{d^3(10b^2c^2-15abcd+6a^2d^2)\,x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{\int \frac{(bc-ad)^3\,(b^2c^2+3abcd+6a^2d^2)+5b\ldots}{(\ldots}}{\ldots}$$

$$= \frac{d^3(10b^2c^2-15abcd+6a^2d^2)\,x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5\,(a+bx^2)^2} - \frac{\int \frac{-(bc-ad)\ldots}{\ldots}}{\ldots}$$

$$= \frac{d^3(10b^2c^2-15abcd+6a^2d^2)\,x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5\,(a+bx^2)^2} + \frac{(bc-ad)\ldots}{8a^2b^5\ldots}$$

$$= \frac{d^3(10b^2c^2-15abcd+6a^2d^2)\,x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5\,(a+bx^2)^2} + \frac{(bc-ad)\ldots}{8a^2b^5\ldots}$$

Mathematica [A]
time = 0.08, size = 196, normalized size = 1.00

$$\frac{d^3(10b^2c^2-15abcd+6a^2d^2)\,x}{b^5} + \frac{d^4(5bc-3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5\,(a+bx^2)^2} + \frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5\,(a+bx^2)} + \frac{(bc-ad)^3\,(3b^2c^2+14abcd+63a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
```

```
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Maple [A]
time = 0.10, size = 311, normalized size = 1.59

method	result
default	$\frac{d^3\left(\frac{1}{5}b^2x^5d^2-ab\,d^2x^3+\frac{5}{3}b^2cd\,x^3+6a^2d^2x-15abcdx+10b^2c^2x\right)}{b^5}-\frac{-\frac{b\left(17a^5d^5-65a^4bc\,d^4+90a^3b^2c^2d^3-50a^2b^3c^3d^2+5a\,b^4c^4d+3b^5c^5\right)x^3 \ldots}{8a^2}}{\left(b\,x^2+\ldots\right.}$
risch	$\frac{d^5x^5}{5b^3}-\frac{d^5a\,x^3}{b^4}+\frac{5d^4c\,x^3}{3b^3}+\frac{6d^5a^2x}{b^5}-\frac{15d^4acx}{b^4}+\frac{10d^3c^2x}{b^3}+\frac{\frac{b\left(17a^5d^5-65a^4bc\,d^4+90a^3b^2c^2d^3-50a^2b^3c^3d^2+5a\,b^4c^4d+3b^5c^5\right)\ldots}{8a^2}}{b^5\left(b\,\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/b^5*(1/5*b^2*x^5*d^2-a*b*d^2*x^3+5/3*b^2*c*d*x^3+6*a^2*d^2*x-15*a*b*c*d
*x+10*b^2*c^2*x)-1/b^5*((-1/8*b*(17*a^5*d^5-65*a^4*b*c*d^4+90*a^3*b^2*c^2*d
^3-50*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+3*b^5*c^5)/a^2*x^3-5/8*(3*a^5*d^5-11*a^
4*b*c*d^4+14*a^3*b^2*c^2*d^3-6*a^2*b^3*c^3*d^2-a*b^4*c^4*d+b^5*c^5)/a*x)/(b
*x^2+a)^2+1/8*(63*a^5*d^5-175*a^4*b*c*d^4+150*a^3*b^2*c^2*d^3-30*a^2*b^3*c^
3*d^2-5*a*b^4*c^4*d-3*b^5*c^5)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 334, normalized size = 1.70

$$\frac{(3\,b^6c^5+5\,ab^5c^4d-50\,a^2b^4c^3d^2+90\,a^3b^3c^2d^3-65\,a^4b^2cd^4+17\,a^5bd^5)x^3+5\,(ab^5c^5-a^2b^4c^4d-6\,a^3b^3c^3d^2+14\,a^4b^2c^2d^3-11\,a^5bcd^4+3\,a^6d^5)x}{8\,(a^2b^7x^4+2\,a^3b^6x^2+a^4b^5)}+\frac{3\,b^2d^5x^5+5\,(5\,b^2cd^4-3\,abd^5)x^3+15\,(10\,b^2c^2d^3-15\,abcd^4+6\,a^2d^5)x}{15\,b^5}+\frac{(3\,b^5c^5+5\,ab^4c^4d+30\,a^2b^3c^3d^2-150\,a^3b^2c^2d^3+175\,a^4bcd^4-63\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^6*c^5 + 5*a*b^5*c^4*d - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3 -
 65*a^4*b^2*c*d^4 + 17*a^5*b*d^5)*x^3 + 5*(a*b^5*c^5 - a^2*b^4*c^4*d - 6*a^
3*b^3*c^3*d^2 + 14*a^4*b^2*c^2*d^3 - 11*a^5*b*c*d^4 + 3*a^6*d^5)*x)/(a^2*b^
7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5) + 1/15*(3*b^2*d^5*x^5 + 5*(5*b^2*c*d^4 - 3
*a*b*d^5)*x^3 + 15*(10*b^2*c^2*d^3 - 15*a*b*c*d^4 + 6*a^2*d^5)*x)/b^5 + 1/8
*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 + 17
5*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(178) = 356$.
time = 0.53, size = 1044, normalized size = 5.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*a^3*b^5*d^5*x^9 + 16*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 16
*(150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 10*(9*a*b
^7*c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875
*a^5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d
 + 30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5
+ (3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 1
75*a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d +
 30*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^
5)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(5*a^
2*b^6*c^5 - 5*a^3*b^5*c^4*d - 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 17
5*a^6*b^2*c*d^4 + 63*a^7*b*d^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6),
 1/120*(24*a^3*b^5*d^5*x^9 + 8*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 8*(
150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 5*(9*a*b^7*
c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875*a^
5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d +
30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5 + (
3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 175*
a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d + 30
*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^5)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(5*a^2*b^6*c^5 - 5*a^3*b^5*c^4*d
- 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 175*a^6*b^2*c*d^4 + 63*a^7*b*d
^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(187) = 374$.

time = 7.27, size = 615, normalized size = 3.14

$$x^3\left(-\frac{ad^5}{b^4}+\frac{5cd^4}{3b^3}\right)+x\left(\frac{6a^2d^5}{b^5}-\frac{15acd^4}{b^4}+\frac{10c^2d^3}{b^3}\right)+\frac{\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)\log\left(-\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+x\right)}{16}-\frac{\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)\log\left(\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+x\right)}{16}+\frac{x^3\cdot(17a^5bd^5-65a^4b^2cd^4+90a^3b^3c^2d^3-50a^2b^4c^3d^2+5ab^5c^4d+3b^6c^5)+x(15a^6d^5-55a^5bcd^4+70a^4b^2c^2d^3-30a^3b^3c^3d^2-5a^2b^4c^4d+5ab^5c^5)}{8a^4b^5+16a^3b^6x^2+8a^2b^7x^4}+\frac{d^5x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**3,x)
```

```
[Out] x**3*(-a*d**5/b**4 + 5*c*d**4/(3*b**3)) + x*(6*a**2*d**5/b**5 - 15*a*c*d**4
/b**4 + 10*c**2*d**3/b**3) + sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*
d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d
- b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**
4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c*
*4*d - 3*b**5*c**5) + x)/16 - sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2
*d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d
- b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**
4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c*
*4*d - 3*b**5*c**5) + x)/16 + (x**3*(17*a**5*b*d**5 - 65*a**4*b**2*c*d**4 +
```

```
 90*a**3*b**3*c**2*d**3 - 50*a**2*b**4*c**3*d**2 + 5*a*b**5*c**4*d + 3*b**6
*c**5) + x*(15*a**6*d**5 - 55*a**5*b*c*d**4 + 70*a**4*b**2*c**2*d**3 - 30*a
**3*b**3*c**3*d**2 - 5*a**2*b**4*c**4*d + 5*a*b**5*c**5))/(8*a**4*b**5 + 16
*a**3*b**6*x**2 + 8*a**2*b**7*x**4) + d**5*x**5/(5*b**3)
```

Giac [A]
time = 1.49, size = 340, normalized size = 1.73

$$\frac{(3\,b^5c^5+5\,ab^4c^4d+30\,a^2b^3c^3d^2-150\,a^3b^2c^2d^3+175\,a^4bcd^4-63\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^5}+\frac{3\,b^6c^5x^3+5\,ab^5c^4dx^3-50\,a^2b^4c^3d^2x^3+90\,a^3b^3c^2d^3x^3-65\,a^4b^2cd^4x^3+17\,a^5bd^5x^3+5\,ab^5c^5x-5\,a^2b^4c^4dx-30\,a^3b^3c^3d^2x+70\,a^4b^2c^2d^3x-55\,a^5bcd^4x+15\,a^6d^5x}{8\,(bx^2+a)^2a^2b^5}+\frac{3\,b^{12}d^5x^5+25\,b^{12}cd^4x^3-15\,ab^{11}d^5x^3+150\,b^{12}c^2d^3x-225\,ab^{11}cd^4x+90\,a^2b^{10}d^5x}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 +
 175*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5) +
1/8*(3*b^6*c^5*x^3 + 5*a*b^5*c^4*d*x^3 - 50*a^2*b^4*c^3*d^2*x^3 + 90*a^3*b^
3*c^2*d^3*x^3 - 65*a^4*b^2*c*d^4*x^3 + 17*a^5*b*d^5*x^3 + 5*a*b^5*c^5*x - 5
*a^2*b^4*c^4*d*x - 30*a^3*b^3*c^3*d^2*x + 70*a^4*b^2*c^2*d^3*x - 55*a^5*b*c
*d^4*x + 15*a^6*d^5*x)/((b*x^2 + a)^2*a^2*b^5) + 1/15*(3*b^12*d^5*x^5 + 25*
b^12*c*d^4*x^3 - 15*a*b^11*d^5*x^3 + 150*b^12*c^2*d^3*x - 225*a*b^11*c*d^4*
x + 90*a^2*b^10*d^5*x)/b^15
```

Mupad [B]
time = 5.02, size = 409, normalized size = 2.09

$$\frac{\frac{5\,x\,\left(3\,a^5\,d^5-11\,a^4\,b\,c\,d^4+14\,a^3\,b^2\,c^2\,d^3-6\,a^2\,b^3\,c^3\,d^2-a\,b^4\,c^4\,d+b^5\,c^5\right)}{8\,a}+\frac{x^3\,\left(17\,a^5\,b\,d^5-65\,a^4\,b^2\,c\,d^4+90\,a^3\,b^3\,c^2\,d^3-50\,a^2\,b^4\,c^3\,d^2+5\,a\,b^5\,c^4\,d+3\,b^6\,c^5\right)}{8\,a^2}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}-x^3\,\left(\frac{a\,d^5}{b^4}-\frac{5\,c\,d^4}{3\,b^3}\right)+x\,\left(\frac{3\,a\,\left(\frac{3\,a\,d^5}{b^4}-\frac{5\,c\,d^4}{b^3}\right)}{b}-\frac{3\,a^2\,d^5}{b^5}+\frac{10\,c^2\,d^3}{b^3}\right)+\frac{d^5\,x^5}{5\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{\sqrt{a}\,\left(-63\,a^5\,d^5+175\,a^4\,b\,c\,d^4-150\,a^3\,b^2\,c^2\,d^3+30\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d+3\,b^5\,c^5\right)}\right)\,{\left(a\,d-b\,c\right)}^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^{5/2}\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^5/(a + b*x^2)^3,x)
```

```
[Out] ((5*x*(3*a^5*d^5 + b^5*c^5 - 6*a^2*b^3*c^3*d^2 + 14*a^3*b^2*c^2*d^3 - a*b^4
*c^4*d - 11*a^4*b*c*d^4))/(8*a) + (x^3*(3*b^6*c^5 + 17*a^5*b*d^5 - 65*a^4*b
^2*c*d^4 - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3 + 5*a*b^5*c^4*d))/(8*a^2
))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x^3*((a*d^5)/b^4 - (5*c*d^4)/(3*b^3)
) + x*((3*a*((3*a*d^5)/b^4 - (5*c*d^4)/b^3))/b - (3*a^2*d^5)/b^5 + (10*c^2*
d^3)/b^3) + (d^5*x^5)/(5*b^3) + (atan((b^(1/2)*x*(a*d - b*c)^3*(63*a^2*d^2
+ 3*b^2*c^2 + 14*a*b*c*d))/(a^(1/2)*(3*b^5*c^5 - 63*a^5*d^5 + 30*a^2*b^3*c^
3*d^2 - 150*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d + 175*a^4*b*c*d^4)))*(a*d - b*c
)^3*(63*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(8*a^(5/2)*b^(11/2))
```

3.36 $\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=160

$$\frac{d^3(4bc-3ad)x}{b^4}+\frac{d^4x^3}{3b^3}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2}+\frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)}+\frac{(bc-ad)^2\left(3b^2c^2+10abcd+35a^2d^2\right)\tan}{8a^{5/2}b^{9/2}}$$

```
[Out] d^3*(-3*a*d+4*b*c)*x/b^4+1/3*d^4*x^3/b^3+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^2+a)
^2+1/8*(-a*d+b*c)^3*(13*a*d+3*b*c)*x/a^2/b^4/(b*x^2+a)+1/8*(-a*d+b*c)^2*(35
*a^2*d^2+10*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(9/2)
```

Rubi [A]
time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {398, 1171, 393, 211}

$$\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)}{8a^{5/2}b^{9/2}}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2}+\frac{d^4x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = \int \left(\frac{d^3(4bc-3ad)}{b^4} + \frac{d^4x^2}{b^3} + \frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2\cdots}{b^4\left(a+bx^2\right)^3}\right.$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{\int \frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2d^2(bc-ad)^2x^4}{(a+bx^2)^3}\,dx}{b^4}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} - \frac{\int \frac{-(bc-ad)^2\left(3b^2c^2+10abcd+11a^2d^2\right)-24abd^2(bc-ad)^2x\cdots}{(a+bx^2)^2}}{4ab^4}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} + \frac{\left((bc-ad)^2\left(3b^2\cdots\right.\right.}{\cdots}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} + \frac{(bc-ad)^2\left(3b^2\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.07, size = 160, normalized size = 1.00

$$\frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} + \frac{(bc-ad)^2\left(3b^2c^2+10abcd+35a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Maple [A]
time = 0.11, size = 231, normalized size = 1.44

method	result
default	$-\frac{d^{3}\left(-\frac{1}{3} b d\,x^{3}+3 a d x-4 b c x\right)}{b^{4}}+\frac{\frac{-\frac{b\left(13 a^{4} d^{4}-36 a^{3} b c\,d^{3}+30 a^{2} b^{2} c^{2} d^{2}-4 a\,b^{3} c^{3} d-3 b^{4} c^{4}\right) x^{3}}{8 a^{2}}-\frac{\left(11 a^{4} d^{4}-28 a^{3} b c\,d^{3}+18 a^{2} b^{2} c^{2} d^{2}+4 a\,b^{3} c^{3} d-5 b^{4} c\right.}{8 a}}{\left(b\,x^{2}+a\right)^{2}}}{b^{4}}$
risch	$\frac{d^{4} x^{3}}{3 b^{3}}-\frac{3 d^{4} a x}{b^{4}}+\frac{4 d^{3} c x}{b^{3}}+\frac{-\frac{b\left(13 a^{4} d^{4}-36 a^{3} b c\,d^{3}+30 a^{2} b^{2} c^{2} d^{2}-4 a\,b^{3} c^{3} d-3 b^{4} c^{4}\right) x^{3}}{8 a^{2}}-\frac{\left(11 a^{4} d^{4}-28 a^{3} b c\,d^{3}+18 a^{2} b^{2} c^{2} d^{2}+4 a\,b^{3} c^{3} d-5 b^{4} c^{4}\right)}{8 a}}{b^{4}\left(b\,x^{2}+a\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3/b^4*(-1/3*b*d*x^3+3*a*d*x-4*b*c*x)+1/b^4*((-1/8*b*(13*a^4*d^4-36*a^3*b
*c*d^3+30*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d-3*b^4*c^4)/a^2*x^3-1/8*(11*a^4*d^4-
28*a^3*b*c*d^3+18*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d-5*b^4*c^4)/a*x)/(b*x^2+a)^2
+1/8*(35*a^4*d^4-60*a^3*b*c*d^3+18*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d+3*b^4*c^4)
/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 253, normalized size = 1.58

$$\frac{(3\,b^5c^4+4\,ab^4c^3d-30\,a^2b^3c^2d^2+36\,a^3b^2cd^3-13\,a^4bd^4)x^3+(5\,ab^4c^4-4\,a^2b^3c^3d-18\,a^3b^2c^2d^2+28\,a^4bcd^3-11\,a^5d^4)x}{8\,(a^2b^6x^4+2\,a^3b^5x^2+a^4b^4)}+\frac{bd^4x^3+3\,(4\,bcd^3-3\,ad^4)x}{3\,b^4}+\frac{(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^5*c^4 + 4*a*b^4*c^3*d - 30*a^2*b^3*c^2*d^2 + 36*a^3*b^2*c*d^3 - 1
3*a^4*b*d^4)*x^3 + (5*a*b^4*c^4 - 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 28
*a^4*b*c*d^3 - 11*a^5*d^4)*x)/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4) + 1/3
*(b*d^4*x^3 + 3*(4*b*c*d^3 - 3*a*d^4)*x)/b^4 + 1/8*(3*b^4*c^4 + 4*a*b^3*c^3
*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a^4*d^4)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(144) = 288$.
time = 0.49, size = 817, normalized size = 5.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*b^4*d^4*x^7 + 16*(12*a^3*b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + 2*(
9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 1
75*a^5*b^2*d^4)*x^3 - 3*(3*a^2*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d
^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*
c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b
^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(
-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*a^2*b^5*c^4 - 4*
a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d^4)*x)/(a
^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5), 1/24*(8*a^3*b^4*d^4*x^7 + 8*(12*a^3*
b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + (9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b
^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 175*a^5*b^2*d^4)*x^3 + 3*(3*a^2*b^4*c^4 +
4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6
*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d
^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^
2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*a^2*b^5
*c^4 - 4*a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d
^4)*x)/(a^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(150) = 300$.
time = 1.83, size = 515, normalized size = 3.22

$$x\left(-\frac{3ad^4}{b^4}+\frac{4cd^3}{b^3}\right)-\frac{\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)\log\left(-\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)\log\left(\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16}+\frac{x^3(-13a^4bd^4+36a^3b^2cd^3-30a^2b^3c^2d^2+4ab^4c^3d+3b^5c^4)+x(-11a^5d^4+28a^4bcd^3-18a^3b^2c^2d^2-4a^2b^3c^3d+5ab^4c^4)}{8a^4b^4+16a^3b^5x^2+8a^2b^6x^4}+\frac{d^4x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*a*d**4/b**4 + 4*c*d**3/b**3) - sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(3
5*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**4*sqrt(-1/(a**5*b**9))
*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 6
0*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) +
 x)/16 + sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3
*b**2*c**2)*log(a**3*b**4*sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2
 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 60*a**3*b*c*d**3 + 18*a**2*b**
2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) + x)/16 + (x**3*(-13*a**4*b*d*
*4 + 36*a**3*b**2*c*d**3 - 30*a**2*b**3*c**2*d**2 + 4*a*b**4*c**3*d + 3*b**
5*c**4) + x*(-11*a**5*d**4 + 28*a**4*b*c*d**3 - 18*a**3*b**2*c**2*d**2 - 4*
a**2*b**3*c**3*d + 5*a*b**4*c**4))/(8*a**4*b**4 + 16*a**3*b**5*x**2 + 8*a**
2*b**6*x**4) + d**4*x**3/(3*b**3)
```

Giac [A]
time = 1.43, size = 254, normalized size = 1.59

$$\frac{(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^4}+\frac{3\,b^5c^4x^3+4\,ab^4c^3dx^3-30\,a^2b^3c^2d^2x^3+36\,a^3b^2cd^3x^3-13\,a^4bd^4x^3+5\,ab^4c^4x-4\,a^2b^3c^3dx-18\,a^3b^2c^2d^2x+28\,a^4bcd^3x-11\,a^5d^4x}{8\,(bx^2+a)^2a^2b^4}+\frac{b^6d^4x^3+12\,b^6cd^3x-9\,ab^5d^4x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^4*c^4 + 4*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a
^4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^4) + 1/8*(3*b^5*c^4*x^3 + 4*
a*b^4*c^3*d*x^3 - 30*a^2*b^3*c^2*d^2*x^3 + 36*a^3*b^2*c*d^3*x^3 - 13*a^4*b*
d^4*x^3 + 5*a*b^4*c^4*x - 4*a^2*b^3*c^3*d*x - 18*a^3*b^2*c^2*d^2*x + 28*a^4
*b*c*d^3*x - 11*a^5*d^4*x)/((b*x^2 + a)^2*a^2*b^4) + 1/3*(b^6*d^4*x^3 + 12*
b^6*c*d^3*x - 9*a*b^5*d^4*x)/b^9
```

Mupad [B]
time = 0.14, size = 318, normalized size = 1.99

$$\frac{d^4\,x^3}{3\,b^3}-x\,\left(\frac{3\,a\,d^4}{b^4}-\frac{4\,c\,d^3}{b^3}\right)-\frac{\frac{x\,\left(11\,a^4\,d^4-28\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d-5\,b^4\,c^4\right)}{8\,a}-\frac{x^3\,\left(-13\,a^4\,b\,d^4+36\,a^3\,b^2\,c\,d^3-30\,a^2\,b^3\,c^2\,d^2+4\,a\,b^4\,c^3\,d+3\,b^5\,c^4\right)}{8\,a^2}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{\sqrt{a}\,\left(35\,a^4\,d^4-60\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+3\,b^4\,c^4\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^{5/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^3,x)
```

```
[Out] (d^4*x^3)/(3*b^3) - x*((3*a*d^4)/b^4 - (4*c*d^3)/b^3) - ((x*(11*a^4*d^4 - 5
*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 28*a^3*b*c*d^3))/(8*a) - (x
^3*(3*b^5*c^4 - 13*a^4*b*d^4 + 36*a^3*b^2*c*d^3 - 30*a^2*b^3*c^2*d^2 + 4*a*
b^4*c^3*d))/(8*a^2))/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (atan((b^(1/2)*x*(
a*d - b*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(a^(1/2)*(35*a^4*d^4 +
3*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 60*a^3*b*c*d^3)))*(a*d - b
*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(8*a^(5/2)*b^(9/2))
```

3.37 $\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=130

$$\frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\,(a+bx^2)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\,(a+bx^2)}+\frac{3(bc-ad)\,(4a^2d^2+(bc+ad)^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

```
[Out] d^3*x/b^3+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a)^2+3/8*(-a*d+b*c)^2*(3*a*d+b*c)
*x/a^2/b^3/(b*x^2+a)+3/8*(-a*d+b*c)*(4*a^2*d^2+(a*d+b*c)^2)*arctan(x*b^(1/2
)/a^(1/2))/a^(5/2)/b^(7/2)
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {398, 1171, 393, 211}

$$\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3\,(a+bx^2)}+\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)\,(4a^2d^2+(ad+bc)^2)}{8a^{5/2}b^{7/2}}+\frac{x(bc-ad)^3}{4ab^3\,(a+bx^2)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*d)*(4*a^2*d^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx &= \int \left(\frac{d^3}{b^3}+\frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{b^3(a+bx^2)^3}\right)dx\\
&= \frac{d^3x}{b^3}+\frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{(a+bx^2)^3}\,dx}{b^3}\\
&= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3(a+bx^2)^2}-\frac{\int \frac{-3(bc-ad)(bc+ad)^2-12abd^2(bc-ad)x^2}{(a+bx^2)^2}\,dx}{4ab^3}\\
&= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3(a+bx^2)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3(a+bx^2)}+\frac{(3(bc-ad)(4a^2d^2+(bc+ad)^2))\int \frac{}{a\cdots}}{8a^2b^3}\\
&= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3(a+bx^2)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3(a+bx^2)}+\frac{3(bc-ad)(4a^2d^2+(bc+ad)^2)\tan^{-1}}{8a^{5/2}b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 139, normalized size = 1.07

$$\frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3(a+bx^2)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3(a+bx^2)}+\frac{3(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Maple [A]
time = 0.10, size = 170, normalized size = 1.31

method	result
default	$\frac{d^3x}{b^3}-\frac{\frac{-\frac{3b\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{8a^2}-\frac{\left(7a^3d^3-9a^2bc\,d^2-3a\,b^2c^2d+5b^3c^3\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{3\left(5a^3d^3-3a^2bc\,d^2-a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}}{b^3}$
risch	$\frac{d^3x}{b^3}+\frac{\frac{3b\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{8a^2}+\frac{\left(7a^3d^3-9a^2bc\,d^2-3a\,b^2c^2d+5b^3c^3\right)x}{8a}}{b^3\left(b\,x^2+a\right)^2}-\frac{15a\ln\left(bx-\sqrt{-ab}\right)d^3}{16b^3\sqrt{-ab}}+\frac{9\ln\left(bx-\sqrt{-a}\right.}{16b^2\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*x/b^3-1/b^3*((-3/8*b*(3*a^3*d^3-5*a^2*b*c*d^2+a*b^2*c^2*d+b^3*c^3)/a^2*
x^3-1/8*(7*a^3*d^3-9*a^2*b*c*d^2-3*a*b^2*c^2*d+5*b^3*c^3)/a*x)/(b*x^2+a)^2+
3/8*(5*a^3*d^3-3*a^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/a^2/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 185, normalized size = 1.42

$$\frac{d^3x}{b^3}+\frac{3\,(b^4c^3+ab^3c^2d-5\,a^2b^2cd^2+3\,a^3bd^3)x^3+(5\,ab^3c^3-3\,a^2b^2c^2d-9\,a^3bcd^2+7\,a^4d^3)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}+\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 + 1/8*(3*(b^4*c^3 + a*b^3*c^2*d - 5*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*
x^3 + (5*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 7*a^4*d^3)*x)/(a^2*b
^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*
d^2 - 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(116) = 232.
time = 0.46, size = 606, normalized size = 4.66

$$\left[\frac{16\,a^3b^3d^3x^5+2\,(3\,ab^5c^3+3\,a^2b^4c^2d-15\,a^3b^3cd^2+25\,a^4b^2d^3)x^3+3\,(a^2b^3c^3+a^3b^2c^2d+3\,a^4bcd^2-5\,a^5d^3+(b^5c^3+ab^4c^2d+3\,a^2b^3cd^2-5\,a^3b^2d^3)x^4+2\,(ab^4c^3+a^2b^3c^2d+3\,a^3b^2cd^2-5\,a^4bd^3)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(5\,a^2b^4c^3-3\,a^3b^3c^2d-9\,a^4b^2cd^2+15\,a^5bd^3)x}{16\,(a^3b^6x^4+2\,a^4b^5x^2+a^5b^4)},\ \frac{8\,a^3b^3d^3x^5+(3\,ab^5c^3+3\,a^2b^4c^2d-15\,a^3b^3cd^2+25\,a^4b^2d^3)x^3+3\,(a^2b^3c^3+a^3b^2c^2d+3\,a^4bcd^2-5\,a^5d^3+(b^5c^3+ab^4c^2d+3\,a^2b^3cd^2-5\,a^3b^2d^3)x^4+2\,(ab^4c^3+a^2b^3c^2d+3\,a^3b^2cd^2-5\,a^4bd^3)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(5\,a^2b^4c^3-3\,a^3b^3c^2d-9\,a^4b^2cd^2+15\,a^5bd^3)x}{8\,(a^3b^6x^4+2\,a^4b^5x^2+a^5b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*d^3*x^5 + 2*(3*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 15*a^3*b^3*c
*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2
 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
```

```
4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt
(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^4*c^3 - 3
*a^3*b^3*c^2*d - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^
5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*d^3*x^5 + (3*a*b^5*c^3 + 3*a^2*b^4*c^2*d -
 15*a^3*b^3*c*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d +
3*a^4*b*c*d^2 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^
3*b^2*d^3)*x^4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d
^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^4*c^3 - 3*a^3*b^3*c^2*d
 - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^
4)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(122) = 244$.
time = 1.02, size = 422, normalized size = 3.25

$$\frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\,(5a^2d^2+2abcd+b^2c^2)\log\left(-\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)(5a^2d^2+2abcd+b^2c^2)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}-\frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\,(5a^2d^2+2abcd+b^2c^2)\log\left(\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)(5a^2d^2+2abcd+b^2c^2)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}+\frac{x^3\cdot(9a^3bd^3-15a^2b^2cd^2+3ab^3c^2d+3b^4c^3)+x(7a^4d^3-9a^3bcd^2-3a^2b^2c^2d+5ab^3c^3)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}+\frac{d^3x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**2*c**2)*lo
g(-3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d +
b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*b**3*c**3)
 + x)/16 - 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b*
*2*c**2)*log(3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*
a*b*c*d + b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*
b**3*c**3) + x)/16 + (x**3*(9*a**3*b*d**3 - 15*a**2*b**2*c*d**2 + 3*a*b**3*
c**2*d + 3*b**4*c**3) + x*(7*a**4*d**3 - 9*a**3*b*c*d**2 - 3*a**2*b**2*c**2
*d + 5*a*b**3*c**3))/(8*a**4*b**3 + 16*a**3*b**4*x**2 + 8*a**2*b**5*x**4) +
 d**3*x/b**3
```

Giac [**A**]
time = 1.38, size = 178, normalized size = 1.37

$$\frac{d^3x}{b^3}+\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}+\frac{3\,b^4c^3x^3+3\,ab^3c^2dx^3-15\,a^2b^2cd^2x^3+9\,a^3bd^3x^3+5\,ab^3c^3x-3\,a^2b^2c^2dx-9\,a^3bcd^2x+7\,a^4d^3x}{8\,(bx^2+a)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c^3*x^3 + 3*a*b^3*c^2*d*x^3
 - 15*a^2*b^2*c*d^2*x^3 + 9*a^3*b*d^3*x^3 + 5*a*b^3*c^3*x - 3*a^2*b^2*c^2*d
*x - 9*a^3*b*c*d^2*x + 7*a^4*d^3*x)/((b*x^2 + a)^2*a^2*b^3)
```

Mupad [B]
time = 5.05, size = 240, normalized size = 1.85

$$\frac{\frac{x\,\left(7\,a^3\,d^3-9\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}{8\,a}+\frac{3\,x^3\,\left(3\,a^3\,b\,d^3-5\,a^2\,b^2\,c\,d^2+a\,b^3\,c^2\,d+b^4\,c^3\right)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{d^3\,x}{b^3}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}{\sqrt{a}\,\left(-5\,a^3\,d^3+3\,a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}{8\,a^{5/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^3,x)
```

```
[Out] ((x*(7*a^3*d^3 + 5*b^3*c^3 - 3*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(8*a) + (3*x^3
*(b^4*c^3 + 3*a^3*b*d^3 - 5*a^2*b^2*c*d^2 + a*b^3*c^2*d))/(8*a^2))/(a^2*b^3
 + b^5*x^4 + 2*a*b^4*x^2) + (d^3*x)/b^3 + (3*atan((b^(1/2)*x*(a*d - b*c)*(5
*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(a^(1/2)*(b^3*c^3 - 5*a^3*d^3 + a*b^2*c^2*
d + 3*a^2*b*c*d^2)))*(a*d - b*c)*(5*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(8*a^(5
/2)*b^(7/2))
```

3.38 $\int \frac{(c+dx^2)^2}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=116

$$\frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\left(a+bx^2\right)}+\frac{(bc-ad)x(c+dx^2)}{4ab\left(a+bx^2\right)^2}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

```
[Out] 3/8*(c^2/a^2-d^2/b^2)*x/(b*x^2+a)+1/4*(-a*d+b*c)*x*(d*x^2+c)/a/b/(b*x^2+a)^
2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(
5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {424, 393, 211}

$$\frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8\left(a+bx^2\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^2d^2+2abcd+3b^2c^2\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (3*(c^2/a^2 - d^2/b^2)*x)/(8*(a + b*x^2)) + ((b*c - a*d)*x*(c + d*x^2))/(4*
a*b*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx &= \frac{(bc-ad)x(c+dx^2)}{4ab\,(a+bx^2)^2}+\frac{\int \frac{c(3bc+ad)+d(bc+3ad)x^2}{(a+bx^2)^2}\,dx}{4ab}\\ &= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\,(a+bx^2)}+\frac{(bc-ad)x(c+dx^2)}{4ab\,(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\int \frac{1}{a+bx^2}\,dx}{8a^2b^2}\\ &= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\,(a+bx^2)}+\frac{(bc-ad)x(c+dx^2)}{4ab\,(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 124, normalized size = 1.07

$$\frac{x(-3a^3d^2+3b^3c^2x^2+ab^2c(5c+2dx^2)-a^2bd(2c+5dx^2))}{8a^2b^2\,(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (x*(-3*a^3*d^2 + 3*b^3*c^2*x^2 + a*b^2*c*(5*c + 2*d*x^2) - a^2*b*d*(2*c + 5
*d*x^2)))/(8*a^2*b^2*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Maple [A]
time = 0.09, size = 124, normalized size = 1.07

method	result
default	$\frac{-\frac{(5a^2d^2-2abcd-3b^2c^2)x^3}{8a^2b}-\frac{(3a^2d^2+2abcd-5b^2c^2)x}{8b^2a}}{(bx^2+a)^2}+\frac{(3a^2d^2+2abcd+3b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b^2\sqrt{ab}}$
risch	$\frac{-\frac{(5a^2d^2-2abcd-3b^2c^2)x^3}{8a^2b}-\frac{(3a^2d^2+2abcd-5b^2c^2)x}{8b^2a}}{(bx^2+a)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)d^2}{16\sqrt{-ab}\,b^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)cd}{8\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)c^{}}{16\sqrt{-ab}\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] (-1/8*(5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/a^2/b*x^3-1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/b^2/a*x)/(b*x^2+a)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [A]
time = 0.50, size = 138, normalized size = 1.19

$$\frac{(3\,b^3c^2+2\,ab^2cd-5\,a^2bd^2)x^3+(5\,ab^2c^2-2\,a^2bcd-3\,a^3d^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}+\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/8*((3*b^3*c^2 + 2*a*b^2*c*d - 5*a^2*b*d^2)*x^3 + (5*a*b^2*c^2 - 2*a^2*b*c*d - 3*a^3*d^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(102) = 204.
time = 0.48, size = 449, normalized size = 3.87

$$\left[\frac{2\,(3\,ab^4c^2+2\,a^2b^3cd-5\,a^3b^2d^2)x^3-(3\,a^2b^2c^2+2\,a^3bcd+3\,a^4d^2+(3\,b^4c^2+2\,ab^3cd+3\,a^2b^2d^2)x^4+2\,(3\,ab^3c^2+2\,a^2b^2cd+3\,a^3bd^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(5\,a^2b^3c^2-2\,a^3b^2cd-3\,a^4bd^2)x}{16\,(a^3b^5x^4+2\,a^4b^4x^2+a^5b^3)},\ \frac{(3\,ab^4c^2+2\,a^2b^3cd-5\,a^3b^2d^2)x^3+(3\,a^2b^2c^2+2\,a^3bcd+3\,a^4d^2+(3\,b^4c^2+2\,ab^3cd+3\,a^2b^2d^2)x^4+2\,(3\,ab^3c^2+2\,a^2b^2cd+3\,a^3bd^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(5\,a^2b^3c^2-2\,a^3b^2cd-3\,a^4bd^2)x}{8\,(a^3b^5x^4+2\,a^4b^4x^2+a^5b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="fricas")

[Out] [1/16*(2*(3*a*b^4*c^2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 - (3*a^2*b^2*c^2 + 2*a^3*b*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^3*c^2 - 2*a^3*b^2*c*d - 3*a^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3), 1/8*((3*a*b^4*c^2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 + (3*a^2*b^2*c^2 + 2*a^3*b*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^3*c^2 - 2*a^3*b^2*c*d - 3*a^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3)
]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(110) = 220.
time = 0.54, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}+\frac{x^3(-5a^2bd^2+2ab^2cd+3b^3c^2)+x(-3a^3d^2-2a^2bcd+5ab^2c^2)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**
2*sqrt(-1/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 + (x**3*(-5*a
**2*b*d**2 + 2*a*b**2*c*d + 3*b**3*c**2) + x*(-3*a**3*d**2 - 2*a**2*b*c*d +
 5*a*b**2*c**2))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

Giac [A]
time = 3.32, size = 126, normalized size = 1.09

$$\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^2}+\frac{3\,b^3c^2x^3+2\,ab^2cdx^3-5\,a^2bd^2x^3+5\,ab^2c^2x-2\,a^2bcdx-3\,a^3d^2x}{8\,(bx^2+a)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
2*b^2) + 1/8*(3*b^3*c^2*x^3 + 2*a*b^2*c*d*x^3 - 5*a^2*b*d^2*x^3 + 5*a*b^2*c
^2*x - 2*a^2*b*c*d*x - 3*a^3*d^2*x)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [B]
time = 5.02, size = 130, normalized size = 1.12

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^{5/2}\,b^{5/2}}-\frac{\frac{x\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2\right)}{8\,a\,b^2}-\frac{x^3\,\left(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^3,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*a^(5/2)*
b^(5/2)) - ((x*(3*a^2*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*a*b^2) - (x^3*(3*b^2
*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*a^2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.39 $\int \frac{c+dx^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)x}{4ab\,(a+bx^2)^2}+\frac{(3bc+ad)x}{8a^2b\,(a+bx^2)}+\frac{(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

```
[Out] 1/4*(-a*d+b*c)*x/a/b/(b*x^2+a)^2+1/8*(a*d+3*b*c)*x/a^2/b/(b*x^2+a)+1/8*(a*d
+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {393, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(ad+3bc)}{8a^{5/2}b^{3/2}}+\frac{x(ad+3bc)}{8a^2b\,(a+bx^2)}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
+ p, 0])
```

Rubi steps

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{(bc-ad)x}{4ab(a+bx^2)^2} + \frac{(3bc+ad)\int \frac{1}{(a+bx^2)^2}\,dx}{4ab}$$
$$= \frac{(bc-ad)x}{4ab(a+bx^2)^2} + \frac{(3bc+ad)x}{8a^2b(a+bx^2)} + \frac{(3bc+ad)\int \frac{1}{a+bx^2}\,dx}{8a^2b}$$
$$= \frac{(bc-ad)x}{4ab(a+bx^2)^2} + \frac{(3bc+ad)x}{8a^2b(a+bx^2)} + \frac{(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.91

$$\frac{x(-a^2d+3b^2cx^2+ab(5c+dx^2))}{8a^2b(a+bx^2)^2} + \frac{(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^3,x]
[Out] (x*(-(a^2*d) + 3*b^2*c*x^2 + a*b*(5*c + d*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Maple [A]
time = 0.08, size = 76, normalized size = 0.83

method	result
default	$\frac{\frac{(ad+3bc)x^3}{8a^2}-\frac{(ad-5bc)x}{8ab}}{(bx^2+a)^2}+\frac{(ad+3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$
risch	$\frac{\frac{(ad+3bc)x^3}{8a^2}-\frac{(ad-5bc)x}{8ab}}{(bx^2+a)^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{16\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)c}{16\sqrt{-ab}\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{16\sqrt{-ab}\,ba}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)c}{16\sqrt{-ab}\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] (1/8*(a*d+3*b*c)/a^2*x^3-1/8*(a*d-5*b*c)/a/b*x)/(b*x^2+a)^2+1/8*(a*d+3*b*c)
/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 92, normalized size = 1.00

$$\frac{(3\,b^2c+abd)x^3+(5\,abc-a^2d)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}+\frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^2*c + a*b*d)*x^3 + (5*a*b*c - a^2*d)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*
x^2 + a^4*b) + 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Fricas [A]
time = 1.08, size = 301, normalized size = 3.27

$$\left[\frac{2\,(3\,ab^3c+a^2b^2d)x^3-((3\,b^3c+ab^2d)x^4+3\,a^2bc+a^3d+2\,(3\,ab^2c+a^2bd)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(5\,a^2b^2c-a^3bd)x}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)},\ \frac{(3\,ab^3c+a^2b^2d)x^3+((3\,b^3c+ab^2d)x^4+3\,a^2bc+a^3d+2\,(3\,ab^2c+a^2bd)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(5\,a^2b^2c-a^3bd)x}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*c + a^2*b^2*d)*x^3 - ((3*b^3*c + a*b^2*d)*x^4 + 3*a^2*b*c
 + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*c + a^2*b^2*d)*x^3 + ((3*b^3*c + a*b^2*d)*
x^4 + 3*a^2*b*c + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

Sympy [A]
time = 0.29, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad+3bc)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad+3bc)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{x^3(abd+3b^2c)+x(-a^2d+5abc)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (x**3*(a*b*d + 3*b**2*c) + x*(-a**2*d + 5*a*b*c))/(8*a**4*b + 16*a**3
*b**2*x**2 + 8*a**2*b**3*x**4)
```

Giac [A]
time = 2.85, size = 78, normalized size = 0.85

$$\frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{3\,b^2cx^3+abdx^3+5\,abcx-a^2dx}{8\,(bx^2+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(3*b^2*c*x^
3 + a*b*d*x^3 + 5*a*b*c*x - a^2*d*x)/((b*x^2 + a)^2*a^2*b)
```

Mupad [B]
time = 5.02, size = 81, normalized size = 0.88

$$\frac{\frac{x^3\,(a\,d+3\,b\,c)}{8\,a^2}-\frac{x\,(a\,d-5\,b\,c)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+3\,b\,c)}{8\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^3,x)
```

```
[Out] ((x^3*(a*d + 3*b*c))/(8*a^2) - (x*(a*d - 5*b*c))/(8*a*b))/(a^2 + b^2*x^4 +
2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(a*d + 3*b*c))/(8*a^(5/2)*b^(3/2))
```

3.40 $\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx$

Optimal. Leaf size=161

$$\frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2}+\frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)}+\frac{\sqrt{b}\left(3b^2c^2-10abcd+15a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\tan^{-1}}{\sqrt{c}\,(bc}$$

```
[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)^2/(b
*x^2+a)+1/8*(15*a^2*d^2-10*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))*b^(
1/2)/a^(5/2)/(-a*d+b*c)^3-d^(5/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^3/c^
(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$\frac{bx(3bc-7ad)}{8a^2\left(a+bx^2\right)(bc-ad)^2}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(15a^2d^2-10abcd+3b^2c^2\right)}{8a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^3}+\frac{bx}{4a\left(a+bx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(8*a^2*(b*c -
 a*d)^2*(a + b*x^2)) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^3\,(c+dx^2)}\,dx = \frac{bx}{4a(bc-ad)\,(a+bx^2)^2} - \frac{\int \frac{-3bc+4ad-3bdx^2}{(a+bx^2)^2(c+dx^2)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\,(a+bx^2)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\,(a+bx^2)} + \frac{\int \frac{3b^2c^2-7abcd+8a^2d^2+bd(3bc-7}{(a+bx^2)(c+dx^2)}}{8a^2(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\,(a+bx^2)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\,(a+bx^2)} - \frac{d^3\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^3} + \frac{(b(3b^2c^2 -}{}$$

$$= \frac{bx}{4a(bc-ad)\,(a+bx^2)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\,(a+bx^2)} + \frac{\sqrt{b}\,(3b^2c^2-10abcd+15a}{8a^{5/2}(bc-}$$

Mathematica [A]
time = 0.19, size = 158, normalized size = 0.98

$$\frac{1}{8}\left(-\frac{2bx}{a(-bc+ad)\,(a+bx^2)^2} + \frac{b(3bc-7ad)x}{a^2(bc-ad)^2\,(a+bx^2)} - \frac{\sqrt{b}\,(3b^2c^2-10abcd+15a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^3} - \frac{8d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] ((-2*b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(a^2*(b*
c - a*d)^2*(a + b*x^2)) - (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) - (8*d^(5/2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.20, size = 158, normalized size = 0.98

method	result	si
default	$-\frac{b\left(\frac{\frac{b(7a^2d^2-10abcd+3b^2c^2)x^3}{8a^2}+\frac{(9a^2d^2-14abcd+5b^2c^2)x}{8a}}{(bx^2+a)^2}+\frac{(15a^2d^2-10abcd+3b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^3\sqrt{cd}}$	1
risch	Expression too large to display	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a*d-b*c)^3*((1/8*b*(7*a^2*d^2-10*a*b*c*d+3*b^2*c^2)/a^2*x^3+1/8*(9*a^2*
d^2-14*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+1/8*(15*a^2*d^2-10*a*b*c*d+3*b^2
*c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^3/(c*d)^(1/2)*
arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.50, size = 278, normalized size = 1.73

$$-\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}}+\frac{(3\,b^3c^2-10\,ab^2cd+15\,a^2bd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}}+\frac{(3\,b^3c-7\,ab^2d)x^3+(5\,ab^2c-9\,a^2bd)x}{8\,(a^4b^2c^2-2\,a^5bcd+a^6d^2+(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)x^4+2\,(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*x/
sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(
a*b)) + 1/8*((3*b^3*c - 7*a*b^2*d)*x^3 + (5*a*b^2*c - 9*a^2*b*d)*x)/(a^4*b^
2*c^2 - 2*a^5*b*c*d + a^6*d^2 + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)
*x^4 + 2*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(139) = 278$.
time = 1.04, size = 1587, normalized size = 9.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 - (3*a^2*b^2*c^2 -
10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d^2)*x^4
```

```
 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(-b/a)*log((b*x
^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*
x^2 + a^4*d^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) +
 2*(5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2
*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b
^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*
c*d^2 - a^6*b*d^3)*x^2), 1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)
*x^3 - 16*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/c)*arctan(x*
sqrt(d/c)) - (3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a
*b^3*c*d + 15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b
*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(
5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2
*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c
*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^
2 - a^6*b*d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 +
(3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15
*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sq
rt(b/a)*arctan(x*sqrt(b/a)) - 4*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^
2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^3*c^
2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6
*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5
*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*
d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 + (3*a^2*b^2
*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d
^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(b/a)*ar
ctan(x*sqrt(b/a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/
c)*arctan(x*sqrt(d/c)) + (5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a
^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a
^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*
b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.45, size = 218, normalized size = 1.35

$$-\frac{d^{3} \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3})\sqrt{cd}} + \frac{(3\,b^{3}c^{2} - 10\,ab^{2}cd + 15\,a^{2}bd^{2})\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^{2}b^{3}c^{3} - 3\,a^{3}b^{2}c^{2}d + 3\,a^{4}bcd^{2} - a^{5}d^{3})\sqrt{ab}} + \frac{3\,b^{3}cx^{3} - 7\,ab^{2}dx^{3} + 5\,ab^{2}cx - 9\,a^{2}bdx}{8\,(a^{2}b^{2}c^{2} - 2\,a^{3}bcd + a^{4}d^{2})(bx^{2} + a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")

[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) + 1/8*(3*b^3*c*x^3 - 7*a*b^2*d*x^3 + 5*a*b^2*c*x - 9*a^2*b*d*x)/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)^2)

Mupad [B]
time = 6.89, size = 2500, normalized size = 15.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^2)^3*(c + d*x^2)),x)

[Out] ((x^3*(3*b^3*c - 7*a*b^2*d))/(8*a^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(5*b^2*c - 9*a*b*d))/(8*a*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (atan(((((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2)*1i)/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a

```
*b^2*c^3*d)))*(-c*d^5)^(1/2)*1i)/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2
- 3*a*b^2*c^3*d)))/((105*a^3*b^3*d^8 - 9*b^6*c^3*d^5 + 51*a*b^5*c^2*d^6 - 1
15*a^2*b^4*c*d^7)/(32*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^
4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (((
x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 +
 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6
*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 - 1760
*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c
^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7
 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15
*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)
) - (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10*b^3*c*d^8 + 256*a^4*b^9
*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d
^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3
 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*
d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b
*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2
*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a
*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4
*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + ((-c*d^5
)^(1/2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 80
0*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*
b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 +
a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 1
5*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 -
 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^
6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*
c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*
d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))
/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2
))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d))))*(-c*d^5)^(
1/2)*1i)/(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d) + (atan(((
((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6
 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a
^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - (((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^
9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816
*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b
^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*
d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 -...
```

3.41 $\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx$

Optimal. Leaf size=236

$$\frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3(c+dx^2)}+\frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)}+\frac{3b(bc-3ad)x}{8a^2(bc-ad)^2(a+bx^2)(c+dx^2)}+\frac{b^{3/2}(3b^2c^2-}{}$$

```
[Out] 1/8*d*(-4*a*d+b*c)*(a*d+3*b*c)*x/a^2/c/(-a*d+b*c)^3/(d*x^2+c)+1/4*b*x/a/(-a
*d+b*c)/(b*x^2+a)^2/(d*x^2+c)+3/8*b*(-3*a*d+b*c)*x/a^2/(-a*d+b*c)^2/(b*x^2+
a)/(d*x^2+c)+1/8*b^(3/2)*(35*a^2*d^2-14*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)
/a^(1/2))/a^(5/2)/(-a*d+b*c)^4-1/2*d^(5/2)*(-a*d+7*b*c)*arctan(x*d^(1/2)/c^
(1/2))/c^(3/2)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.21, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$\frac{dx(bc-4ad)(ad+3bc)}{8a^2c(c+dx^2)(bc-ad)^3}+\frac{3bx(bc-3ad)}{8a^2(a+bx^2)(c+dx^2)(bc-ad)^2}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(35a^2d^2-14abcd+3b^2c^2)}{8a^{5/2}(bc-ad)^4}-\frac{d^{5/2}(7bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4}+\frac{bx}{4a(a+bx^2)^2(c+dx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c - 4*a*d)*(3*b*c + a*d)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^2)) + (b*
x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + (3*b*(b*c - 3*a*d)*x)/(8*a
^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(3*b^2*c^2 - 14*a*b*c*
d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^4) - (d
^(5/2)*(7*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^2}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} - \frac{\int \frac{-3bc+4ad-5bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\int \frac{3b^2c}{}}{}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(b}{8a^2(bc-ad)^2}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(b}{8a^2(bc-ad)^2}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(b}{8a^2(bc-ad)^2}$$

Mathematica [A]
time = 0.28, size = 197, normalized size = 0.83

$$\frac{1}{8}\left(\frac{2b^2x}{a(bc-ad)^2\left(a+bx^2\right)^2} + \frac{b^2(-3bc+11ad)x}{a^2(-bc+ad)^3\left(a+bx^2\right)} - \frac{4d^3x}{c(bc-ad)^3\left(c+dx^2\right)} + \frac{b^{3/2}\left(3b^2c^2-14abcd+35a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4} + \frac{4d^{5/2}(-7bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] ((2*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (b^2*(-3*b*c + 11*a*d)*x)/(a^2
*(-(b*c) + a*d)^3*(a + b*x^2)) - (4*d^3*x)/(c*(b*c - a*d)^3*(c + d*x^2)) +
(b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(5/2)*(b*c - a*d)^4) + (4*d^(5/2)*(-7*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(c^(3/2)*(b*c - a*d)^4))/8
```

Maple [A]
time = 0.22, size = 196, normalized size = 0.83

method	result
default	$\frac{b^{2}\left(\frac{\frac{b\left(11a^{2}d^{2}-14abcd+3b^{2}c^{2}\right)x^{3}}{8a^{2}}+\frac{\left(13a^{2}d^{2}-18abcd+5b^{2}c^{2}\right)x}{8a}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(35a^{2}d^{2}-14abcd+3b^{2}c^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^{2}\sqrt{ab}}\right)}{\left(ad-bc\right)^{4}}+\frac{d^{3}\left(\frac{\left(ad-bc\right)x}{2c\left(d\,x^{2}+c\right)}+\frac{\left(ad-7bc\right)}{}\right.}{\left(ad-bc\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^4*((1/8*b*(11*a^2*d^2-14*a*b*c*d+3*b^2*c^2)/a^2*x^3+1/8*(13*a
^2*d^2-18*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+1/8*(35*a^2*d^2-14*a*b*c*d+3*
b^2*c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^4*(1/2*(a*d
-b*c)/c*x/(d*x^2+c)+1/2*(a*d-7*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(210) = 420.
time = 0.52, size = 530, normalized size = 2.25

$$\frac{\left(3\,b^{4}c^{2}-14\,ab^{3}cd+35\,a^{2}b^{2}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\left(a^{2}b^{4}c^{4}-4\,a^{3}b^{3}c^{3}d+6\,a^{4}b^{2}c^{2}d^{2}-4\,a^{5}bcd^{3}+a^{6}d^{4}\right)\sqrt{ab}}-\frac{\left(7\,bcd^{3}-ad^{4}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\left(b^{4}c^{5}-4\,ab^{3}c^{4}d+6\,a^{2}b^{2}c^{3}d^{2}-4\,a^{3}bc^{2}d^{3}+a^{4}cd^{4}\right)\sqrt{cd}}+\frac{\left(3\,b^{4}c^{2}d-11\,ab^{3}cd^{2}-4\,a^{2}b^{2}d^{3}\right)x^{5}+\left(3\,b^{4}c^{3}-6\,ab^{3}c^{2}d-13\,a^{2}b^{2}cd^{2}-8\,a^{3}bd^{3}\right)x^{3}+\left(5\,ab^{3}c^{3}-13\,a^{2}b^{2}c^{2}d-4\,a^{4}d^{3}\right)x}{8\,\left(a^{4}b^{3}c^{5}-3\,a^{5}b^{2}c^{4}d+3\,a^{6}bc^{3}d^{2}-a^{7}c^{2}d^{3}+\left(a^{2}b^{5}c^{4}d-3\,a^{3}b^{4}c^{3}d^{2}+3\,a^{4}b^{3}c^{2}d^{3}-a^{5}b^{2}cd^{4}\right)x^{6}+\left(a^{2}b^{5}c^{5}-a^{3}b^{4}c^{4}d-3\,a^{4}b^{3}c^{3}d^{2}+5\,a^{5}b^{2}c^{2}d^{3}-2\,a^{6}bcd^{4}\right)x^{4}+\left(2\,a^{3}b^{4}c^{5}-5\,a^{4}b^{3}c^{4}d+3\,a^{5}b^{2}c^{3}d^{2}+a^{6}bc^{2}d^{3}-a^{7}cd^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b))/((a^2
*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*s
qrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b
^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) + 1/
8*((3*b^4*c^2*d - 11*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*x^5 + (3*b^4*c^3 - 6*a*b^
3*c^2*d - 13*a^2*b^2*c*d^2 - 8*a^3*b*d^3)*x^3 + (5*a*b^3*c^3 - 13*a^2*b^2*c
^2*d - 4*a^4*d^3)*x)/(a^4*b^3*c^5 - 3*a^5*b^2*c^4*d + 3*a^6*b*c^3*d^2 - a^7
*c^2*d^3 + (a^2*b^5*c^4*d - 3*a^3*b^4*c^3*d^2 + 3*a^4*b^3*c^2*d^3 - a^5*b^2
*c*d^4)*x^6 + (a^2*b^5*c^5 - a^3*b^4*c^4*d - 3*a^4*b^3*c^3*d^2 + 5*a^5*b^2*
c^2*d^3 - 2*a^6*b*c*d^4)*x^4 + (2*a^3*b^4*c^5 - 5*a^4*b^3*c^4*d + 3*a^5*b^2
*c^3*d^2 + a^6*b*c^2*d^3 - a^7*c*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(210) = 420$.
time = 2.88, size = 3251, normalized size = 13.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4)
*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 +
 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35*a^4*b*c^2*d^2 +
(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 - 8*a*
b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 - 25*a
^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 +
 (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a
^2*b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^
3*c^2*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 - 8*(7*a^4*b*c^2*d^2 - a^5*c
*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^
2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 3
5*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
 (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (
6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*
b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2
*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3
*d + 35*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)
```

```
*x^6 + (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x
^4 + (6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)
*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 2*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a
^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2
*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(
-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^4*c^4 - 18*a
^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*
c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 +
 (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4
 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2
+ 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c
^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*
d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d
^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 +
5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35
*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
(3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6
*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a^2*b^3*
c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2*a^4*b*
d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (5*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 -
4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^
4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2
+ 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6
 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*
d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4
*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.17, size = 333, normalized size = 1.41

$$-\frac{d^3x}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)(dx^2+c)}+\frac{(3\,b^4c^2-14\,ab^3cd+35\,a^2b^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4)\sqrt{ab}}-\frac{(7\,bcd^3-ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}}+\frac{3\,b^4cx^3-11\,ab^3dx^3+5\,ab^3cx-13\,a^2b^2dx}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*x/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2
+ c)) + 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b
))/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^
6*d^4)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*
d)) + 1/8*(3*b^4*c*x^3 - 11*a*b^3*d*x^3 + 5*a*b^3*c*x - 13*a^2*b^2*d*x)/((a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(b*x^2 + a)^2)
```

Mupad [B]
time = 7.85, size = 2500, normalized size = 10.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)^2),x)
```

```
[Out] ((x^5*(4*a^2*b^2*d^3 - 3*b^4*c^2*d + 11*a*b^3*c*d^2))/(8*a^2*c*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*a^3*d^3 - 5*b^3*c^3 + 13*a
*b^2*c^2*d))/(8*a*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*x^3*(
8*a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d + 13*a^2*b*c*d^2))/(8*a^2*c*(a*d - b*
c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2*c + x^2*(a^2*d + 2*a*b*c) + x^4*(
b^2*c + 2*a*b*d) + b^2*d*x^6) - (atan(((((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3
 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6
*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b
^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a
^8*b^2*c^4*d^4)) - (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3
*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b
^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*
d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c
^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8
 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*
b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (x*(-a^5*b^3)^(1
/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5
*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7
*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4
*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5
*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^
8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 -
 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 +
3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a
^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2
- 14*a*b*c*d)*1i)/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*
c^2*d^2 - 4*a^8*b*c*d^3)) + (((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8
*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 +
```

```
2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d -
6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*
d^4)) + (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*
d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6
+ 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*
a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a
^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b
^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5
 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) + (x*(-a^5*b^3)^(1/2)*(35*a^2
*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*
d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 +
3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792
*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5*b^4*c^4 -
4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^8 + a^10*c^
2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3
*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 -
 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*
d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*
d)*1i)/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4
*a^8*b*c*d^3)))/(((35*a^5*b^4*d^10)/16 + (63*b^9*c^5*d^5)/64 - (267*a*b^8*c
^4*d^6)/32 - (651*a^4*b^5*c*d^9)/64 + (451*a^2*b^7*c^3*d^7)/16 - (1275*a^3*
b^6*c^2*d^8)/32)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b
*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 -
126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (((x*(16
*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a
^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*
c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2
 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) - (((2*a^13*b^2*c*d^13 - (3*a^
2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*
a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^
7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4
*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b
^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 12
6*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2
*c^4*d^7) - (x*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*
a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^
7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5
*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^
2*d^11))/(512*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^...
```

3.42 $\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx$

Optimal. Leaf size=315

$$\frac{d(3b^2c^2-13abcd-2a^2d^2)\,x}{8a^2c(bc-ad)^3\,(c+dx^2)^2}+\frac{bx}{4a(bc-ad)\,(a+bx^2)^2\,(c+dx^2)^2}+\frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\,(a+bx^2)\,(c+dx^2)^2}+\frac{3d(bc$$

```
[Out] 1/8*d*(-2*a^2*d^2-13*a*b*c*d+3*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(d*x^2+c)^2+1/
4*b*x/a/(-a*d+b*c)/(b*x^2+a)^2/(d*x^2+c)^2+1/8*b*(-11*a*d+3*b*c)*x/a^2/(-a*
d+b*c)^2/(b*x^2+a)/(d*x^2+c)^2+3/8*d*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*
x/a^2/c^2/(-a*d+b*c)^4/(d*x^2+c)+3/8*b^(5/2)*(21*a^2*d^2-6*a*b*c*d+b^2*c^2)
*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^5-3/8*d^(5/2)*(a^2*d^2-6*a*b*
c*d+21*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^5
```

Rubi [A]
time = 0.31, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$-\frac{3d^{5/2}(a^2d^2-6abcd+21b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}+\frac{3dx(ad+bc)\,(a^2d^2-6abcd+b^2c^2)}{8a^2c^2\,(c+dx^2)\,(bc-ad)^4}+\frac{dx(-2a^2d^2-13abcd+3b^2c^2)}{8a^2c\,(c+dx^2)^2\,(bc-ad)^3}+\frac{bx(3bc-11ad)}{8a^2\,(a+bx^2)\,(c+dx^2)^2\,(bc-ad)^2}+\frac{3b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(21a^2d^2-6abcd+b^2c^2)}{8a^{5/2}(bc-ad)^5}+\frac{bx}{4a\,(a+bx^2)^2\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 13*a*b*c*d - 2*a^2*d^2)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^
2)^2) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)^2) + (b*(3*b*c - 1
1*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)^2) + (3*d*(b*c + a*d
)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x)/(8*a^2*c^2*(b*c - a*d)^4*(c + d*x^2))
+ (3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(5/2)*(b*c - a*d)^5) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^5)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx = \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} - \frac{\int \frac{-3bc+4ad-7bdx^2}{(a+bx^2)^2(c+dx^2)^3}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2(a+bx^2)(c+dx^2)^2} + \frac{\int \frac{3b^2}{}}{}$$

$$= \frac{d(3b^2c^2-13abcd-2a^2d^2)x}{8a^2c(bc-ad)^3(c+dx^2)^2} + \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} + \frac{b}{8a^2(bc-a}$$

$$= \frac{d(3b^2c^2-13abcd-2a^2d^2)x}{8a^2c(bc-ad)^3(c+dx^2)^2} + \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} + \frac{b}{8a^2(bc-a}$$

$$= \frac{d(3b^2c^2-13abcd-2a^2d^2)x}{8a^2c(bc-ad)^3(c+dx^2)^2} + \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} + \frac{b}{8a^2(bc-a}$$

$$= \frac{d(3b^2c^2-13abcd-2a^2d^2)x}{8a^2c(bc-ad)^3(c+dx^2)^2} + \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} + \frac{b}{8a^2(bc-a}$$

Mathematica [A]

time = 0.61, size = 233, normalized size = 0.74

$$\frac{1}{8}\left(-\frac{3b^{5/2}(b^2c^2-6abcd+21a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^5}+\frac{(bc-ad)x\left(\frac{3b^4c}{a^2(a+bx^2)}+\frac{3ad^4}{c^2(c+dx^2)}+\frac{b^3(2bc-17ad-15bdx^2)}{a(a+bx^2)^2}-\frac{d^3(17bc-2ad+15bdx^2)}{c(c+dx^2)^2}\right)-\frac{3d^{5/2}(21b^2c^2-6abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}}}{(bc-ad)^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] ((-3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(a^(5/2)*(-(b*c) + a*d)^5) + ((b*c - a*d)*x*((3*b^4*c)/(a^2*(a + b*x^2))
+ (3*a*d^4)/(c^2*(c + d*x^2)) + (b^3*(2*b*c - 17*a*d - 15*b*d*x^2))/(a*(a +
 b*x^2)^2) - (d^3*(17*b*c - 2*a*d + 15*b*d*x^2))/(c*(c + d*x^2)^2)) - (3*d^
(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(5/
2))/(b*c - a*d)^5)/8
```

Maple [A]
time = 0.32, size = 257, normalized size = 0.82

method	result
default	$-\frac{b^3\left(\frac{\frac{3b(5a^2d^2-6abcd+b^2c^2)x^3}{8a^2}+\frac{(17a^2d^2-22abcd+5b^2c^2)x}{8a}}{(bx^2+a)^2}+\frac{3(21a^2d^2-6abcd+b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(ad-bc)^5}+\frac{d^3\left(\frac{3d(a^2d^2-6abcd+5b^2 \ldots}{8c^2}\right.}{}$ (truncated at page edge)
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^5*((3/8*b*(5*a^2*d^2-6*a*b*c*d+b^2*c^2)/a^2*x^3+1/8*(17*a^2*
d^2-22*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+3/8*(21*a^2*d^2-6*a*b*c*d+b^2*c^
2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^5*((3/8*d*(a^2*d^
2-6*a*b*c*d+5*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-22*a*b*c*d+17*b^2*c^2)/c*x)/(
d*x^2+c)^2+3/8*(a^2*d^2-6*a*b*c*d+21*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c
*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(287) = 574$.
time = 0.58, size = 820, normalized size = 2.60

$$\frac{3\,(b^5c^2-6\,ab^4cd+21\,a^2b^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^5c^5-5\,a^3b^4c^4d+10\,a^4b^3c^3d^2-10\,a^5b^2c^2d^3+5\,a^6bcd^4-a^7d^5)\sqrt{ab}}-\frac{3\,(21\,b^2c^2d^3-6\,abcd^4+a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^5c^7-5\,ab^4c^6d+10\,a^2b^3c^5d^2-10\,a^3b^2c^4d^3+5\,a^4bc^3d^4-a^5c^2d^5)\sqrt{cd}}+\frac{3\,(b^5c^3d^2-5\,ab^4c^2d^3-5\,a^2b^3cd^4+a^3b^2d^5)x^7+(6\,b^5c^4d-25\,ab^4c^3d^2-34\,a^2b^3c^2d^3-25\,a^3b^2cd^4+6\,a^4bd^5)x^5+(3\,b^5c^5-5\,ab^4c^4d-34\,a^2b^3c^3d^2-34\,a^3b^2c^2d^3-5\,a^4bcd^4+3\,a^5d^5)x^3+(5\,ab^4c^5-17\,a^2b^3c^4d-17\,a^4bc^2d^3+5\,a^5cd^4)x}{8\,(a^4b^4c^8-4\,a^5b^3c^7d+6\,a^6b^2c^6d^2-4\,a^7bc^5d^3+a^8c^4d^4+(a^2b^6c^6d^2-4\,a^3b^5c^5d^3+6\,a^4b^4c^4d^4-4\,a^5b^3c^3d^5+a^6b^2c^2d^6)x^8+2\,(a^2b^6c^7d-3\,a^3b^5c^6d^2+2\,a^4b^4c^5d^3+2\,a^5b^3c^4d^4-3\,a^6b^2c^3d^5+a^7bc^2d^6)x^6+(a^2b^6c^8-9\,a^4b^4c^6d^2+16\,a^5b^3c^5d^3-9\,a^6b^2c^4d^4+a^8c^2d^6)x^4+2\,(a^3b^5c^8-3\,a^4b^4c^7d+2\,a^5b^3c^6d^2+2\,a^6b^2c^5d^3-3\,a^7bc^4d^4+a^8c^3d^5)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^
5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6*b
*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d^5)
*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 - 10*
a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*(b^5*c
^3*d^2 - 5*a*b^4*c^2*d^3 - 5*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^7 + (6*b^5*c^4*
d - 25*a*b^4*c^3*d^2 - 34*a^2*b^3*c^2*d^3 - 25*a^3*b^2*c*d^4 + 6*a^4*b*d^5)
*x^5 + (3*b^5*c^5 - 5*a*b^4*c^4*d - 34*a^2*b^3*c^3*d^2 - 34*a^3*b^2*c^2*d^3
 - 5*a^4*b*c*d^4 + 3*a^5*d^5)*x^3 + (5*a*b^4*c^5 - 17*a^2*b^3*c^4*d - 17*a^
4*b*c^2*d^3 + 5*a^5*c*d^4)*x)/(a^4*b^4*c^8 - 4*a^5*b^3*c^7*d + 6*a^6*b^2*c^
6*d^2 - 4*a^7*b*c^5*d^3 + a^8*c^4*d^4 + (a^2*b^6*c^6*d^2 - 4*a^3*b^5*c^5*d^
3 + 6*a^4*b^4*c^4*d^4 - 4*a^5*b^3*c^3*d^5 + a^6*b^2*c^2*d^6)*x^8 + 2*(a^2*b
^6*c^7*d - 3*a^3*b^5*c^6*d^2 + 2*a^4*b^4*c^5*d^3 + 2*a^5*b^3*c^4*d^4 - 3*a^
6*b^2*c^3*d^5 + a^7*b*c^2*d^6)*x^6 + (a^2*b^6*c^8 - 9*a^4*b^4*c^6*d^2 + 16*
a^5*b^3*c^5*d^3 - 9*a^6*b^2*c^4*d^4 + a^8*c^2*d^6)*x^4 + 2*(a^3*b^5*c^8 - 3
*a^4*b^4*c^7*d + 2*a^5*b^3*c^6*d^2 + 2*a^6*b^2*c^5*d^3 - 3*a^7*b*c^4*d^4 +
a^8*c^3*d^5)*x^2)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(287) = 574$.
time = 9.25, size = 5070, normalized size = 16.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^
7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*c^2*d
^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*d - 2
9*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 - 3
*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b
^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*
a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d - 2*a^
2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6
 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*(21*a^4*b^2*c^4*d^2
- 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 + a
^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*
d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 - 2*a^4*b^2
*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*c^4*d^2 + 15*a^4*b^
2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x
*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*
b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*
b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8
```

```
*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^
5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 +
 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d
^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d
- 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c
^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*
d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*
x^2), 1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^
6)*x^7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*
c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*
d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^
3 - 6*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2
*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*
b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a
^3*b^3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a
^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) - 3*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*
c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6
*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (
b^6*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b
^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*
a^4*b^2*c^3*d^3)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
+ a)) + 2*(5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2
*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*
d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5
 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c
^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^
3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 -
 a^8*b*c^2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25
*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 -
 a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 -
5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*x^2), 1/16*(6*(b^6*c^4*d
^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^7 + 2*(6*b^6*c^5*d
- 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d
^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 +
 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 + 6*(a^2*b^4*c^6 - 6*a
^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2
*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 2
1*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*
a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d
+ 15*a^3*b^3*c^4*d^2 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a
)) - 3*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^
2*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3
*b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*
a^3*b^3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*
```

```
a^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sq
rt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^5*c^6 -
 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 + 22*a^5*b*c^2*
d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.74, size = 574, normalized size = 1.82

$$\frac{3\,(b^5c^2-6\,ab^4cd+21\,a^2b^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^5c^5-5\,a^3b^4c^4d+10\,a^4b^3c^3d^2-10\,a^5b^2c^2d^3+5\,a^6bcd^4-a^7d^5)\sqrt{ab}}-\frac{3\,(21\,b^2c^2d^3-6\,abcd^4+a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^5c^7-5\,ab^4c^6d+10\,a^2b^3c^5d^2-10\,a^3b^2c^4d^3+5\,a^4bc^3d^4-a^5c^2d^5)\sqrt{cd}}+\frac{3\,b^5c^3d^2x^7-15\,ab^4c^2d^3x^7-15\,a^2b^3cd^4x^7+3\,a^3b^2d^5x^7+6\,b^5c^4dx^5-25\,ab^4c^3d^2x^5-34\,a^2b^3c^2d^3x^5-25\,a^3b^2cd^4x^5+6\,a^4bd^5x^5+3\,b^5c^5x^3-5\,ab^4c^4dx^3-34\,a^2b^3c^3d^2x^3-34\,a^3b^2c^2d^3x^3-5\,a^4bcd^4x^3+3\,a^5d^5x^3+5\,ab^4c^5x-17\,a^2b^3c^4dx-17\,a^4bc^2d^3x+5\,a^5cd^4x}{8\,(a^2b^4c^6-4\,a^3b^3c^5d+6\,a^4b^2c^4d^2-4\,a^5bc^3d^3+a^6c^2d^4)(bdx^4+bcx^2+adx^2+ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^
5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6*b
*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d^5)
*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 - 10*
a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*b^5*c^
3*d^2*x^7 - 15*a*b^4*c^2*d^3*x^7 - 15*a^2*b^3*c*d^4*x^7 + 3*a^3*b^2*d^5*x^7
 + 6*b^5*c^4*d*x^5 - 25*a*b^4*c^3*d^2*x^5 - 34*a^2*b^3*c^2*d^3*x^5 - 25*a^3
*b^2*c*d^4*x^5 + 6*a^4*b*d^5*x^5 + 3*b^5*c^5*x^3 - 5*a*b^4*c^4*d*x^3 - 34*a
^2*b^3*c^3*d^2*x^3 - 34*a^3*b^2*c^2*d^3*x^3 - 5*a^4*b*c*d^4*x^3 + 3*a^5*d^5
*x^3 + 5*a*b^4*c^5*x - 17*a^2*b^3*c^4*d*x - 17*a^4*b*c^2*d^3*x + 5*a^5*c*d^
4*x)/((a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3
+ a^6*c^2*d^4)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)^2)
```

Mupad [**B**]
time = 8.55, size = 2500, normalized size = 7.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)^3),x)
```

```
[Out] ((x*(5*a^4*d^4 + 5*b^4*c^4 - 17*a*b^3*c^3*d - 17*a^3*b*c*d^3))/(8*a*c*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^3*
```

(34*a^2*b^3*c^3*d^2 - 3*b^5*c^5 - 3*a^5*d^5 + 34*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d + 5*a^4*b*c*d^4))/(8*a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^5*(25*a*b^4*c^3*d^2 - 6*b^5*c^4*d - 6*a^4*b*d^5 + 25*a^3*b^2*c*d^4 + 34*a^2*b^3*c^2*d^3))/(8*a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*b*d*x^7*(a^3*b*d^4 + b^4*c^3*d - 5*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3))/(8*a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))/(x^4*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^2*(2*a*b*c^2 + 2*a^2*c*d) + x^6*(2*a*b*d^2 + 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^8) - (atan(((((x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) - (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 - 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 - 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*3i)/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)) + (((x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (3*(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12

```
)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*
d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*
b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*
d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 -
 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^
14*b^2*c^6*d^10) + (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)
*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 -
 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7
- 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10
 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13)
)/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*
a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c
^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^
8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-a^5*b^5)^(1/2
)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b
^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*(-a^5
*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*3i)/(16*(a^10*d^5 - a^5*b^5*
c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c
*d^4)))/(((567*a^7*b^5*d^12)/256 + (567*b^12*c^7*d^5)/256 - (6399*a*b^11*c^
6*d^6)/256 - (6399*a^6*b^6*c*d^11)/256 + (27891...
```

3.43 $\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx$

Optimal. Leaf size=34

$$-\frac{x(1-x^2)^2}{3\,(1+x^2)^3}-\frac{2x}{3\,(1+x^2)}$$

```
[Out] -1/3*x*(-x^2+1)^2/(x^2+1)^3-2/3*x/(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {424, 21, 391}

$$-\frac{x(1-x^2)^2}{3\,(x^2+1)^3}-\frac{2x}{3\,(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^3/(1 + x^2)^4,x]
```

```
[Out] -1/3*(x*(1 - x^2)^2)/(1 + x^2)^3 - (2*x)/(3*(1 + x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = -\frac{x(1-x^2)^2}{3(1+x^2)^3} + \frac{1}{6}\int \frac{(-1+x^2)(4+4x^2)}{(1+x^2)^3}\,dx$$
$$= -\frac{x(1-x^2)^2}{3(1+x^2)^3} + \frac{2}{3}\int \frac{-1+x^2}{(1+x^2)^2}\,dx$$
$$= -\frac{x(1-x^2)^2}{3(1+x^2)^3} - \frac{2x}{3(1+x^2)}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.71

$$-\frac{x(3+2x^2+3x^4)}{3(1+x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^3/(1 + x^2)^4,x]
```

```
[Out] -1/3*(x*(3 + 2*x^2 + 3*x^4))/(1 + x^2)^3
```

Maple [A]
time = 0.16, size = 23, normalized size = 0.68

method	result	size
gosper	$-\frac{x(3x^4+2x^2+3)}{3(x^2+1)^3}$	23
default	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
norman	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
risch	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
meijerg	$-\frac{x(15x^4+40x^2+33)}{48(x^2+1)^3}-\frac{x(231x^4+280x^2+105)}{336(x^2+1)^3}+\frac{x(-15x^4+40x^2+15)}{80(x^2+1)^3}-\frac{x(-3x^4-8x^2+3)}{16(x^2+1)^3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^3/(x^2+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-x^5-2/3*x^3-x)/(x^2+1)^3
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.97

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\,(x^6+3\,x^4+3\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Fricas [A]
time = 1.72, size = 33, normalized size = 0.97

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\,(x^6+3\,x^4+3\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Sympy [A]
time = 0.04, size = 31, normalized size = 0.91

$$\frac{-3x^5-2x^3-3x}{3x^6+9x^4+9x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3/(x**2+1)**4,x)
```

```
[Out] (-3*x**5 - 2*x**3 - 3*x)/(3*x**6 + 9*x**4 + 9*x**2 + 3)
```

Giac [A]
time = 2.43, size = 20, normalized size = 0.59

$$-\frac{3\left(x+\frac{1}{x}\right)^2-4}{3\left(x+\frac{1}{x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(x + 1/x)^2 - 4)/(x + 1/x)^3
```

Mupad [B]
time = 5.00, size = 31, normalized size = 0.91

$$\frac{4\,x}{3\,(x^2+1)^2}-\frac{x}{x^2+1}-\frac{4\,x}{3\,(x^2+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^3/(x^2 + 1)^4,x)
```

```
[Out] (4*x)/(3*(x^2 + 1)^2) - x/(x^2 + 1) - (4*x)/(3*(x^2 + 1)^3)
```

3.44 $\int \frac{\left(-1+x^2\right)^4}{\left(1+x^2\right)^5} dx$

Optimal. Leaf size=47

$$\frac{x(1-x^2)^3}{4\left(1+x^2\right)^4}+\frac{3x(1-x^2)}{8\left(1+x^2\right)^2}+\frac{3}{8}\tan^{-1}(x)$$

```
[Out] 1/4*x*(-x^2+1)^3/(x^2+1)^4+3/8*x*(-x^2+1)/(x^2+1)^2+3/8*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {424, 21, 209}

$$\frac{3\text{ArcTan}(x)}{8}+\frac{x(1-x^2)^3}{4\left(x^2+1\right)^4}+\frac{3x(1-x^2)}{8\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^4/(1 + x^2)^5,x]
```

```
[Out] (x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*x*(1 - x^2))/(8*(1 + x^2)^2) + (3*ArcT
an[x])/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx &= \frac{x(1-x^2)^3}{4(1+x^2)^4} + \frac{1}{8}\int \frac{(-1+x^2)^2(6+6x^2)}{(1+x^2)^4}\,dx \\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4} + \frac{3}{4}\int \frac{(-1+x^2)^2}{(1+x^2)^3}\,dx \\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4} + \frac{3x(1-x^2)}{8(1+x^2)^2} + \frac{3}{16}\int \frac{2+2x^2}{(1+x^2)^2}\,dx \\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4} + \frac{3x(1-x^2)}{8(1+x^2)^2} + \frac{3}{8}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4} + \frac{3x(1-x^2)}{8(1+x^2)^2} + \frac{3}{8}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.87

$$\frac{5x-3x^3+3x^5-5x^7+3(1+x^2)^4\tan^{-1}(x)}{8(1+x^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^4/(1 + x^2)^5,x]
[Out] (5*x - 3*x^3 + 3*x^5 - 5*x^7 + 3*(1 + x^2)^4*ArcTan[x])/(8*(1 + x^2)^4)
```

Maple [A]
time = 0.10, size = 33, normalized size = 0.70

method	result
default	$\frac{-\frac{5}{8}x^7+\frac{3}{8}x^5-\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^4}+\frac{3\arctan(x)}{8}$
risch	$\frac{-\frac{5}{8}x^7+\frac{3}{8}x^5-\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^4}+\frac{3\arctan(x)}{8}$
meijerg	$\frac{x(105x^6+385x^4+511x^2+279)}{384(x^2+1)^4}+\frac{3\arctan(x)}{8}-\frac{x(837x^6+1533x^4+1155x^2+315)}{1152(x^2+1)^4}+\frac{x(-105x^6+511x^4+385x^2+105)}{672(x^2+1)^4}-\frac{3x(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^4/(x^2+1)^5,x,method=_RETURNVERBOSE)
[Out] (-5/8*x^7+3/8*x^5-3/8*x^3+5/8*x)/(x^2+1)^4+3/8*arctan(x)
```

Maxima [A]
time = 0.53, size = 48, normalized size = 1.02

$$-\frac{5\,x^{7}-3\,x^{5}+3\,x^{3}-5\,x}{8\,\left(x^{8}+4\,x^{6}+6\,x^{4}+4\,x^{2}+1\right)}+\frac{3}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1) + 3/8*
arctan(x)
```

Fricas [A]
time = 0.91, size = 67, normalized size = 1.43

$$-\frac{5\,x^{7}-3\,x^{5}+3\,x^{3}-3\,\left(x^{8}+4\,x^{6}+6\,x^{4}+4\,x^{2}+1\right)\arctan\left(x\right)-5\,x}{8\,\left(x^{8}+4\,x^{6}+6\,x^{4}+4\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="fricas")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 3*(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)*arctan(x)
 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)
```

Sympy [A]
time = 0.06, size = 46, normalized size = 0.98

$$\frac{-5x^{7}+3x^{5}-3x^{3}+5x}{8x^{8}+32x^{6}+48x^{4}+32x^{2}+8}+\frac{3\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**4/(x**2+1)**5,x)
```

```
[Out] (-5*x**7 + 3*x**5 - 3*x**3 + 5*x)/(8*x**8 + 32*x**6 + 48*x**4 + 32*x**2 + 8
) + 3*atan(x)/8
```

Giac [A]
time = 0.91, size = 54, normalized size = 1.15

$$\frac{3}{32}\,\pi\mathrm{sgn}(x)-\frac{5\,\left(x-\frac{1}{x}\right)^{3}+12\,x-\frac{12}{x}}{8\,\left(\left(x-\frac{1}{x}\right)^{2}+4\right)^{2}}+\frac{3}{16}\,\arctan\left(\frac{x^{2}-1}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="giac")
```

```
[Out] 3/32*pi*sgn(x) - 1/8*(5*(x - 1/x)^3 + 12*x - 12/x)/((x - 1/x)^2 + 4)^2 + 3/
16*arctan(1/2*(x^2 - 1)/x)
```

Mupad [B]
time = 0.04, size = 47, normalized size = 1.00

$$\frac{3\,\mathrm{atan}(x)}{8}+\frac{-\frac{5\,x^7}{8}+\frac{3\,x^5}{8}-\frac{3\,x^3}{8}+\frac{5\,x}{8}}{x^8+4\,x^6+6\,x^4+4\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^4/(x^2 + 1)^5,x)
```

```
[Out] (3*atan(x))/8 + ((5*x)/8 - (3*x^3)/8 + (3*x^5)/8 - (5*x^7)/8)/(4*x^2 + 6*x^
4 + 4*x^6 + x^8 + 1)
```

3.45 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=231

$$\frac{(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}}{128b^3}+\frac{d(72b^2c^2-52abcd+15a^2d^2)\,x(a+bx^2)^{3/2}}{192b^3}+\frac{d(12bc-$$

```
[Out] 1/192*d*(15*a^2*d^2-52*a*b*c*d+72*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^3+1/48*d*(-5
*a*d+12*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)/b^2+1/8*d*x*(b*x^2+a)^(3/2)*(d*x^2
+c)^2/b+1/128*a*(-5*a^3*d^3+24*a^2*b*c*d^2-48*a*b^2*c^2*d+64*b^3*c^3)*arcta
nh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/128*(-5*a^3*d^3+24*a^2*b*c*d^2-48*a
*b^2*c^2*d+64*b^3*c^3)*x*(b*x^2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {427, 542, 396, 201, 223, 212}

$$\frac{dx(a+bx^2)^{3/2}(15a^2d^2-52abcd+72b^2c^2)}{192b^3}+\frac{x\sqrt{a+bx^2}(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3)}{128b^3}+\frac{a(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)(12bc-5ad)}{48b^2}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^
2])/(128*b^3) + (d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(3/
2))/(192*b^3) + (d*(12*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(48*b^
2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + (a*(64*b^3*c^3 - 48*a*b^
2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(128*b^(7/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx = \frac{dx(a+bx^2)^{3/2}\left(c+dx^2\right)^2}{8b} + \frac{\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(c(8bc-ad)+d(12bc-5a\right.}{8b}$$

$$= \frac{d(12bc-5ad)x(a+bx^2)^{3/2}\left(c+dx^2\right)}{48b^2} + \frac{dx(a+bx^2)^{3/2}\left(c+dx^2\right)^2}{8b} + \frac{\int \sqrt{a+b}}{}$$

$$= \frac{d\left(72b^2c^2-52abcd+15a^2d^2\right)x(a+bx^2)^{3/2}}{192b^3} + \frac{d(12bc-5ad)x(a+bx^2)^{3/2}(c+}{48b^2}$$

$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d(72b^2c^2-52abcd+1}{192b}$$

$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d(72b^2c^2-52abcd+1}{192b}$$

$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d(72b^2c^2-52abcd+1}{192b}$$

Mathematica [A]
time = 0.25, size = 180, normalized size = 0.78

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(15a^3d^3-2a^2bd^2(36c+5dx^2)+8ab^2d(18c^2+6cdx^2+d^2x^4)+48b^3(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6)\right)+3a\left(-64b^3c^3+48ab^2c^2d-24a^2bcd^2+5a^3d^3\right)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{384b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d^3 - 2*a^2*b*d^2*(36*c + 5*d*x^2) + 8*a
*b^2*d*(18*c^2 + 6*c*d*x^2 + d^2*x^4) + 48*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d
^2*x^4 + d^3*x^6)) + 3*a*(-64*b^3*c^3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5
*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(7/2))
```

Maple [A]
time = 0.07, size = 300, normalized size = 1.30

method	result
risch	$\frac{x\left(48b^3d^3x^6+8a\,b^2d^3x^4+192b^3c\,d^2x^4-10a^2b\,d^3x^2+48a\,b^2c\,d^2x^2+288b^3c^2d\,x^2+15a^3d^3-72a^2bc\,d^2+144a\,b^2c^2d+192b^3c^3\right)\sqrt{b\,x^2+}}{384b^3}$

default	$d^3 \left(\frac{x^5 (b x^2+a)^{\frac{3}{2}}}{8b}-\frac{5a \left(\frac{x^3 (b x^2+a)^{\frac{3}{2}}}{6b}-\frac{a \left(\frac{x (b x^2+a)^{\frac{3}{2}}}{4b}-\frac{a \left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a \ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)}{8b}\right)+3c\, d^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/8*x^5*(b*x^2+a)^(3/2)/b-5/8*a/b*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(
1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))))))+3*c*d^2*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4
*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2)))))+3*c^2*d*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+c^3*(1/2*x*(b*x^
2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.30, size = 281, normalized size = 1.22

$$\frac{(bx^2+a)^{\frac{3}{2}} d^3 x^5}{8\,b}+\frac{(bx^2+a)^{\frac{3}{2}} c d^2 x^3}{2\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}} a d^3 x^3}{48\,b^2}+\frac{1}{2}\,\sqrt{bx^2+a}\,c^3 x+\frac{3\,(bx^2+a)^{\frac{3}{2}} c^2 d x}{4\,b}-\frac{3\,\sqrt{bx^2+a}\,a c^2 d x}{8\,b}-\frac{3\,(bx^2+a)^{\frac{3}{2}} a c d^2 x}{8\,b^2}+\frac{3\,\sqrt{bx^2+a}\,a^2 c d^2 x}{16\,b^2}+\frac{5\,(bx^2+a)^{\frac{3}{2}} a^2 d^3 x}{64\,b^3}-\frac{5\,\sqrt{bx^2+a}\,a^3 d^3 x}{128\,b^3}+\frac{a c^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-\frac{3\,a^2 c^2 d \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{3\,a^3 c d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{5\,a^4 d^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(3/2)*d^3*x^5/b + 1/2*(b*x^2 + a)^(3/2)*c*d^2*x^3/b - 5/48*
(b*x^2 + a)^(3/2)*a*d^3*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*c^3*x + 3/4*(b*x^2 +
a)^(3/2)*c^2*d*x/b - 3/8*sqrt(b*x^2 + a)*a*c^2*d*x/b - 3/8*(b*x^2 + a)^(3/2
)*a*c*d^2*x/b^2 + 3/16*sqrt(b*x^2 + a)*a^2*c*d^2*x/b^2 + 5/64*(b*x^2 + a)^(
```

```
3/2)*a^2*d^3*x/b^3 - 5/128*sqrt(b*x^2 + a)*a^3*d^3*x/b^3 + 1/2*a*c^3*arcsin
h(b*x/sqrt(a*b))/sqrt(b) - 3/8*a^2*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3
/16*a^3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*a^4*d^3*arcsinh(b*x/sq
rt(a*b))/b^(7/2)
```

Fricas [A]
time = 2.27, size = 398, normalized size = 1.72

$$\left[-\frac{3 \, {\left(64 \, a b^{3} c^{3} - 48 \, a^{2} b^{2} c^{2} d + 24 \, a^{3} b c d^{2} - 5 \, a^{4} d^{3}\right)} \sqrt{b} \log\left(-2 \, b x^{2} + 2 \, \sqrt{b x^{2} + a} \sqrt{b} x - a\right) - 2 \, {\left(48 \, b^{4} d^{3} x^{7} + 8 \, {\left(24 \, b^{4} c d^{2} + a b^{3} d^{3}\right)} x^{5} + 2 \, {\left(144 \, b^{4} c^{2} d + 24 \, a b^{3} c d^{2} - 5 \, a^{2} b^{2} d^{3}\right)} x^{3} + 3 \, {\left(64 \, b^{4} c^{3} + 48 \, a b^{3} c^{2} d - 24 \, a^{2} b^{2} c d^{2} + 5 \, a^{3} b d^{3}\right)} x\right)} \sqrt{b x^{2} + a}}{768 \, b^{4}}, -\frac{3 \, {\left(64 \, a b^{3} c^{3} - 48 \, a^{2} b^{2} c^{2} d + 24 \, a^{3} b c d^{2} - 5 \, a^{4} d^{3}\right)} \sqrt{-b} \arctan\left(\frac{\sqrt{-b} x}{\sqrt{b x^{2} + a}}\right) - {\left(48 \, b^{4} d^{3} x^{7} + 8 \, {\left(24 \, b^{4} c d^{2} + a b^{3} d^{3}\right)} x^{5} + 2 \, {\left(144 \, b^{4} c^{2} d + 24 \, a b^{3} c d^{2} - 5 \, a^{2} b^{2} d^{3}\right)} x^{3} + 3 \, {\left(64 \, b^{4} c^{3} + 48 \, a b^{3} c^{2} d - 24 \, a^{2} b^{2} c d^{2} + 5 \, a^{3} b d^{3}\right)} x\right)} \sqrt{b x^{2} + a}}{384 \, b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*s
qrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*d^3*x^7
+ 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d + 24*a*b^3*c*d^2 - 5*
a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a^2*b^2*c*d^2 + 5*a^
3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/384*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*
d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (48*b^4*d^3*x^7 + 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d +
24*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a
^2*b^2*c*d^2 + 5*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(233) = 466.
time = 28.78, size = 484, normalized size = 2.10

$$\frac{5a^{\frac{7}{2}}d^{3}x}{128b^{3}\sqrt{1+\frac{bx^{2}}{a}}} - \frac{3a^{\frac{5}{2}}cd^{2}x}{16b^{2}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{5a^{\frac{5}{2}}d^{3}x^{3}}{384b^{2}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{3a^{\frac{3}{2}}c^{2}dx}{8b\sqrt{1+\frac{bx^{2}}{a}}} - \frac{a^{\frac{3}{2}}cd^{2}x^{3}}{16b\sqrt{1+\frac{bx^{2}}{a}}} - \frac{a^{\frac{3}{2}}d^{3}x^{5}}{192b\sqrt{1+\frac{bx^{2}}{a}}} + \frac{\sqrt{a}c^{3}x\sqrt{1+\frac{bx^{2}}{a}}}{2} + \frac{9\sqrt{a}c^{2}dx^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}} + \frac{5\sqrt{a}cd^{2}x^{5}}{8\sqrt{1+\frac{bx^{2}}{a}}} + \frac{7\sqrt{a}d^{3}x^{7}}{48\sqrt{1+\frac{bx^{2}}{a}}} - \frac{5a^{4}d^{3}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}} + \frac{3a^{3}cd^{2}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}} - \frac{3a^{2}c^{2}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{ac^{3}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{3bc^{2}dx^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{bcd^{2}x^{7}}{2\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{bd^{3}x^{9}}{8\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3,x)
```

```
[Out] 5*a**(7/2)*d**3*x/(128*b**3*sqrt(1 + b*x**2/a)) - 3*a**(5/2)*c*d**2*x/(16*b
**2*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*d**3*x**3/(384*b**2*sqrt(1 + b*x**2/a)
) + 3*a**(3/2)*c**2*d*x/(8*b*sqrt(1 + b*x**2/a)) - a**(3/2)*c*d**2*x**3/(16
*b*sqrt(1 + b*x**2/a)) - a**(3/2)*d**3*x**5/(192*b*sqrt(1 + b*x**2/a)) + sq
rt(a)*c**3*x*sqrt(1 + b*x**2/a)/2 + 9*sqrt(a)*c**2*d*x**3/(8*sqrt(1 + b*x**
2/a)) + 5*sqrt(a)*c*d**2*x**5/(8*sqrt(1 + b*x**2/a)) + 7*sqrt(a)*d**3*x**7/
(48*sqrt(1 + b*x**2/a)) - 5*a**4*d**3*asinh(sqrt(b)*x/sqrt(a))/(128*b**(7/2
)) + 3*a**3*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) - 3*a**2*c**2*d*a
sinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + a*c**3*asinh(sqrt(b)*x/sqrt(a))/(2*s
qrt(b)) + 3*b*c**2*d*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b*c*d**2*x**7/(2
*sqrt(a)*sqrt(1 + b*x**2/a)) + b*d**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.96, size = 201, normalized size = 0.87

$$\frac{1}{384} \, {\left(2 \, {\left(4 \, {\left(6 \, d^{3} x^{2} + \frac{24 \, b^{6} c d^{2} + a b^{5} d^{3}}{b^{6}}\right)} x^{2} + \frac{144 \, b^{6} c^{2} d + 24 \, a b^{5} c d^{2} - 5 \, a^{2} b^{4} d^{3}}{b^{6}}\right)} x^{2} + \frac{3 \, {\left(64 \, b^{6} c^{3} + 48 \, a b^{5} c^{2} d - 24 \, a^{2} b^{4} c d^{2} + 5 \, a^{3} b^{3} d^{3}\right)}}{b^{6}}\right)} \sqrt{b x^{2} + a} x - \frac{{\left(64 \, a b^{3} c^{3} - 48 \, a^{2} b^{2} c^{2} d + 24 \, a^{3} b c d^{2} - 5 \, a^{4} d^{3}\right)} \log\left({\left| -\sqrt{b} x + \sqrt{b x^{2} + a} \right|}\right)}{128 \, b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*d^3*x^2 + (24*b^6*c*d^2 + a*b^5*d^3)/b^6)*x^2 + (144*b^6*c^2
*d + 24*a*b^5*c*d^2 - 5*a^2*b^4*d^3)/b^6)*x^2 + 3*(64*b^6*c^3 + 48*a*b^5*c^
2*d - 24*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(64*
a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^3, x)
```

3.46 $\int \sqrt{a+bx^2}\,(c+dx^2)^2\,dx$

Optimal. Leaf size=149

$$\frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2}+\frac{dx(a+bx^2)^{3/2}\,(c+dx^2)}{6b}+\frac{a(8b^2c^2-4abcd$$

```
[Out] 1/24*d*(-3*a*d+8*b*c)*x*(b*x^2+a)^(3/2)/b^2+1/6*d*x*(b*x^2+a)^(3/2)*(d*x^2+
c)/b+1/16*a*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2)
)/b^(5/2)+1/16*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {427, 396, 201, 223, 212}

$$\frac{x\sqrt{a+bx^2}\,(a^2d^2-4abcd+8b^2c^2)}{16b^2}+\frac{a(a^2d^2-4abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{dx(a+bx^2)^{3/2}\,(8bc-3ad)}{24b^2}+\frac{dx(a+bx^2)^{3/2}\,(c+dx^2)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(16*b^2) + (d*(8*b*c
- 3*a*d)*x*(a + b*x^2)^(3/2))/(24*b^2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)
)/(6*b) + (a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(16*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx = \frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b} + \frac{\int \sqrt{a+bx^2}\,(c(6bc-ad)+d(8bc-3ad)x^2)\,dx}{6b}$$

$$= \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b} + \frac{(8b^2c^2-4abcd+a^2}{8b}$$

$$= \frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2} + \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+b}{}$$

$$= \frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2} + \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+b}{}$$

$$= \frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2} + \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+b}{}$$

Mathematica [A]
time = 0.16, size = 122, normalized size = 0.82

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\,(-3a^2d^2+2abd(6c+dx^2)+8b^2(3c^2+3cdx^2+d^2x^4))-3a(8b^2c^2-4abcd+a^2d^2)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{48b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a^2*d^2 + 2*a*b*d*(6*c + d*x^2) + 8*b^2*(3*c
^2 + 3*c*d*x^2 + d^2*x^4)) - 3*a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(5/2))

Maple [A]
time = 0.06, size = 187, normalized size = 1.26

method	result
risch	$-\frac{x\left(-8b^2d^2x^4-2ab\,d^2x^2-24b^2cd\,x^2+3a^2d^2-12abcd-24b^2c^2\right)\sqrt{b\,x^2+a}}{48b^2}+\frac{a^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)d^2}{16b^{\frac{5}{2}}}-\frac{a^2\ln\left(x\sqrt{b}\right.}{}$
default	$d^2\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+2cd\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] d^2*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/
2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+2*c*d*(1
/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))))+c^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2)))

Maxima [A]
time = 0.28, size = 168, normalized size = 1.13

$$\frac{(bx^2+a)^{\frac{3}{2}}d^2x^3}{6\,b}+\frac{1}{2}\sqrt{bx^2+a}\,c^2x+\frac{(bx^2+a)^{\frac{3}{2}}cdx}{2\,b}-\frac{\sqrt{bx^2+a}\,acdx}{4\,b}-\frac{(bx^2+a)^{\frac{3}{2}}ad^2x}{8\,b^2}+\frac{\sqrt{bx^2+a}\,a^2d^2x}{16\,b^2}+\frac{ac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}-\frac{a^2cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\,b^{\frac{3}{2}}}+\frac{a^3d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="maxima")

[Out] 1/6*(b*x^2 + a)^(3/2)*d^2*x^3/b + 1/2*sqrt(b*x^2 + a)*c^2*x + 1/2*(b*x^2 +
a)^(3/2)*c*d*x/b - 1/4*sqrt(b*x^2 + a)*a*c*d*x/b - 1/8*(b*x^2 + a)^(3/2)*a*
d^2*x/b^2 + 1/16*sqrt(b*x^2 + a)*a^2*d^2*x/b^2 + 1/2*a*c^2*arcsinh(b*x/sqrt
(a*b))/sqrt(b) - 1/4*a^2*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/16*a^3*d^2*
arcsinh(b*x/sqrt(a*b))/b^(5/2)

Fricas [A]
time = 3.46, size = 264, normalized size = 1.77

$$\left[\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(8\,b^3d^2x^5+2\,(12\,b^3cd+ab^2d^2)x^3+3\,(8\,b^3c^2+4\,ab^2cd-a^2bd^2)x)\sqrt{bx^2+a}}{96\,b^3},-\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(8\,b^3d^2x^5+2\,(12\,b^3cd+ab^2d^2)x^3+3\,(8\,b^3c^2+4\,ab^2cd-a^2bd^2)x)\sqrt{bx^2+a}}{48\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)
*x^3 + 3*(8*b^3*c^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/
48*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)*x^3 + 3*(8*b^3*c
^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(143) = 286.
time = 8.46, size = 291, normalized size = 1.95

$$-\frac{a^{\frac{5}{2}}d^2x}{16b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}cdx}{4b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}d^2x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3\sqrt{a}\,cdx^3}{4\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,d^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{a^3d^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}-\frac{a^2cd\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{bcdx^5}{2\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{bd^2x^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2,x)
```

```
[Out] -a**(5/2)*d**2*x/(16*b**2*sqrt(1 + b*x**2/a)) + a**(3/2)*c*d*x/(4*b*sqrt(1
+ b*x**2/a)) - a**(3/2)*d**2*x**3/(48*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c**2*
x*sqrt(1 + b*x**2/a)/2 + 3*sqrt(a)*c*d*x**3/(4*sqrt(1 + b*x**2/a)) + 5*sqrt
(a)*d**2*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*d**2*asinh(sqrt(b)*x/sqrt(a))/
(16*b**(5/2)) - a**2*c*d*asinh(sqrt(b)*x/sqrt(a))/(4*b**(3/2)) + a*c**2*asi
nh(sqrt(b)*x/sqrt(a))/(2*sqrt(b)) + b*c*d*x**5/(2*sqrt(a)*sqrt(1 + b*x**2/a
)) + b*d**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.73, size = 129, normalized size = 0.87

$$\frac{1}{48}\left(2\left(4\,d^2x^2+\frac{12\,b^4cd+ab^3d^2}{b^4}\right)x^2+\frac{3\,(8\,b^4c^2+4\,ab^3cd-a^2b^2d^2)}{b^4}\right)\sqrt{bx^2+a}\,x-\frac{(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2 + (12*b^4*c*d + a*b^3*d^2)/b^4)*x^2 + 3*(8*b^4*c^2 + 4*a
*b^3*c*d - a^2*b^2*d^2)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(8*a*b^2*c^2 - 4*a^2*
b*c*d + a^3*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^2, x)
```

3.47 $\int \sqrt{a+bx^2}\,(c+dx^2)\,dx$

Optimal. Leaf size=87

$$\frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx(a+bx^2)^{3/2}}{4b}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] 1/4*d*x*(b*x^2+a)^(3/2)/b+1/8*a*(-a*d+4*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(3/2)+1/8*(-a*d+4*b*c)*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {396, 201, 223, 212}

$$\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}\,(4bc-ad)}{8b}+\frac{dx(a+bx^2)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] ((4*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b) + (d*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\,(c+dx^2)\,dx &= \frac{dx(a+bx^2)^{3/2}}{4b}-\frac{(-4bc+ad)\int \sqrt{a+bx^2}\,dx}{4b}\\ &= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx(a+bx^2)^{3/2}}{4b}+\frac{(a(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx(a+bx^2)^{3/2}}{4b}+\frac{(a(4bc-ad))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\right.}{8b}\\ &= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx(a+bx^2)^{3/2}}{4b}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 74, normalized size = 0.85

$$\frac{x\sqrt{a+bx^2}\,(4bc+ad+2bdx^2)}{8b}+\frac{a(-4bc+ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2),x]
[Out] (x*Sqrt[a + b*x^2]*(4*b*c + a*d + 2*b*d*x^2))/(8*b) + (a*(-4*b*c + a*d)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Maple [A]
time = 0.06, size = 98, normalized size = 1.13

method	result
risch	$\frac{x(2bd\,x^2+ad+4bc)\sqrt{b\,x^2+a}}{8b}-\frac{a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)d}{8b^{\frac{3}{2}}}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)c}{2\sqrt{b}}$

default	$d\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+c\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+\right.}}{2\sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))))+c*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.30, size = 81, normalized size = 0.93

$$\frac{1}{2}\sqrt{bx^2+a}\,cx+\frac{(bx^2+a)^{\frac{3}{2}}dx}{4\,b}-\frac{\sqrt{bx^2+a}\,adx}{8\,b}+\frac{ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-\frac{a^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*c*x + 1/4*(b*x^2 + a)^(3/2)*d*x/b - 1/8*sqrt(b*x^2 + a)
*a*d*x/b + 1/2*a*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^2*d*arcsinh(b*x/s
qrt(a*b))/b^(3/2)
```

Fricas [A]
time = 2.68, size = 158, normalized size = 1.82

$$\left[-\frac{(4\,abc-a^2d)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(2\,b^2dx^3+(4\,b^2c+abd)x)\sqrt{bx^2+a}}{16\,b^2},-\frac{(4\,abc-a^2d)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,b^2dx^3+(4\,b^2c+abd)x)\sqrt{bx^2+a}}{8\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*a*b*c - a^2*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(2*b^2*d*x^3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/8*(
(4*a*b*c - a^2*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d*x^
3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 3.30, size = 144, normalized size = 1.66

$$\frac{a^{\frac{3}{2}}dx}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3\sqrt{a}\,dx^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{a^2d\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{ac\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{bdx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c),x)
```

```
[Out] a**(3/2)*d*x/(8*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c*x*sqrt(1 + b*x**2/a)/2 +
3*sqrt(a)*d*x**3/(8*sqrt(1 + b*x**2/a)) - a**2*d*asinh(sqrt(b)*x/sqrt(a))/(
8*b**(3/2)) + a*c*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b)) + b*d*x**5/(4*sqrt(a
)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.68, size = 70, normalized size = 0.80

$$\frac{1}{8}\sqrt{bx^2+a}\left(2\,dx^2+\frac{4\,b^2c+abd}{b^2}\right)x-\frac{(4\,abc-a^2d)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d*x^2 + (4*b^2*c + a*b*d)/b^2)*x - 1/8*(4*a*b*c - a^
2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2), x)
```

3.48 $\int \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/2*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {201, 223, 212}

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\,dx &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.04

$$\frac{1}{2}x\sqrt{a+bx^2}-\frac{a\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.78

method	result	size
default	$\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$	36
risch	$\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.61

$$\frac{1}{2}\,\sqrt{bx^2+a}\,x+\frac{a\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 1.28, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{bx^2+a}\,bx+a\sqrt{b}\,\log\left(-2\,bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b},\ \frac{\sqrt{bx^2+a}\,bx-a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

Sympy [A]
time = 0.89, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A]
time = 0.66, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{bx^2+a}\,x-\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [B]
time = 4.71, size = 35, normalized size = 0.76

$$\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2),x)
```

```
[Out] (x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2)
)
```

3.49 $\int \frac{\sqrt{a+bx^2}}{c+dx^2} \, dx$

Optimal. Leaf size=82

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d-arctanh(x*(-a*d+b*c)^(1/2)/c^(
1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/d/c^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {399, 223, 212, 385, 214}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh
[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \frac{b\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{d}$$
$$= \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d}$$
$$= \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d}$$

Mathematica [A]
time = 0.20, size = 99, normalized size = 1.21

$$-\frac{\frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{\sqrt{c}} + \sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2),x]
[Out] -(((Sqrt[-(b*c) + a*d]*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2)
)/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[c] + Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(66) = 132.
time = 0.13, size = 645, normalized size = 7.87

method	result

default	$\frac{\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^{2} b+\frac{2 b \sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}+\frac{\sqrt{b}\,\sqrt{-cd}\,\ln\left(\frac{\frac{b\sqrt{-cd}}{d}+b\left(x-\frac{\sqrt{-cd}}{d}\right)}{\sqrt{b}}+\sqrt{\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-c*d)^(1/2)*(((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-
c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*
c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)
^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-
(-c*d)^(1/2)/d)))-1/2/(-c*d)^(1/2)*(((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2
)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)^(1/2)/d*ln((-b*(-c
*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d
)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^
(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/
d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c), x)
```

Fricas [A]
time = 1.15, size = 596, normalized size = 7.27

$$\left[\frac{2\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{4d}, -\frac{4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{4d}, \frac{\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)+\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2d}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)}{2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + sqrt((b*c
 - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*
c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*
sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d, -1/4*(4*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x
 + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2
*c*d*x^2 + c^2)))/d, 1/2*(sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^
2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
 - a^2*d)*x)) + sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/d,
 -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(-(b*c - a*d)/c)
*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/
((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2}}{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{\sqrt{-b}\,\mathrm{asin}\left(x\,\sqrt{-\frac{b}{a}}\right)}{c} & \text{if } ((a+b\,c=0 \wedge d=-1) \vee a\,d=b\,c) \wedge b<0 \\ \frac{\sqrt{b}\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{d}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)\,\sqrt{a\,d-b\,c}}{\sqrt{c}\,d} & \text{if } a\neq 0 \wedge (((a+b\,c\neq 0 \vee d\neq -1) \wedge a\,d\neq b\,c) \vee \neg b<0) \\ \int \frac{\sqrt{b\,x^2+a}}{d\,x^2+c}\,dx & \text{if } ((((a+b\,c=0 \wedge d=-1) \vee a\,d=b\,c) \wedge b<0) \vee a=0) \wedge (((a+b\,c\neq 0 \vee d\neq -1) \wedge a\,d\neq b\,c) \vee \neg b<0) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2),x)
```

```
[Out] piecewise((a + b*c == 0 & d == -1 | a*d == b*c) & b < 0, ((-b)^(1/2)*asin(x
*(-b/a)^(1/2)))/c, a ~= 0 & ((a + b*c ~= 0 | d ~= -1) & a*d ~= b*c | ~b < 0
), (b^(1/2)*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/d + (atan((x*(a*d - b*c
)^(1/2))/(c^(1/2)*(a + b*x^2)^(1/2)))*(a*d - b*c)^(1/2))/(c^(1/2)*d), ((a +
 b*c == 0 & d == -1 | a*d == b*c) & b < 0 | a == 0) & ((a + b*c ~= 0 | d ~=
 -1) & a*d ~= b*c | ~b < 0), int((a + b*x^2)^(1/2)/(c + d*x^2), x))
```

3.50 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=82

$$\frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*a*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c
)^(1/2)+1/2*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {386, 385, 214}

$$\frac{a\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = \frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)} + \frac{a\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{2c}$$

$$= \frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)} + \frac{a\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2c}$$

$$= \frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.31, size = 99, normalized size = 1.21

$$\frac{x\sqrt{a+bx^2}}{2c^2+2cdx^2} - \frac{a\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{2c^{3/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c^2 + 2*c*d*x^2) - (a*ArcTan[(-(d*x*Sqrt[a + b*x^2])
 + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(2*c^(3/2)*Sqrt[-(b*
c) + a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1944 vs. $2(66) = 132$.
time = 0.06, size = 1945, normalized size = 23.72

method	result	size
default	Expression too large to display	1945

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/(-c*d)^(1/2)*(((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-
(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-
b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*
d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(
```

```
x-(-c*d)^(1/2)/d))-1/4/c/(-c*d)^(1/2)*(((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^
(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)^(1/2)/d*ln((-b
*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)
/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b
*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))-1/4/d/c*(-1/(a*d-b*c)*d/(x+(-c*d)^
(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(3/2)-b*(-c*d)^(1/2)/(a*d-b*c)*(((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)^(1/2)/d*ln((-
b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*
(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c
)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-
b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))+2*b/(a*d-b*c)*d*(1/4*(2*b*(x+(-c*
d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/b*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/
d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(
3/2)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d
)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))))-1/4/d/c*(
-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/(a*d-b*c)*(((x-(-c*d
)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(
1/2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-
(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))+2*b/(a*d-
b*c)*d*(1/4*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/
d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a
*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(
1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(66) = 132$.

time = 1.03, size = 369, normalized size = 4.50

$$\left[\frac{4\,(bc^2-acd)\sqrt{bx^2+a}\,x+(adx^2+ac)\sqrt{bc^2-acd}\,\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ad)x^3+acx)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{8\,(bc^4-ac^3d+(bc^3d-ac^2d^2)x^2)}, \frac{2\,(bc^2-acd)\sqrt{bx^2+a}\,x-(adx^2+ac)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\,((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)}{4\,(bc^4-ac^3d+(bc^3d-ac^2d^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x + (a*d*x^2 + a*c)*sqrt(b*c^2 - a*
c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b*c^4 - a*c^3*d + (b*c^3*d - a*c^2*
d^2)*x^2), 1/4*(2*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x - (a*d*x^2 + a*c)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(b*c^4 -
 a*c^3*d + (b*c^3*d - a*c^2*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(66) = 132.
time = 1.66, size = 217, normalized size = 2.65

$$-\frac{a\sqrt{b}\,\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}\,c}+\frac{2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a\sqrt{b}\,d+a^2\sqrt{b}\,d}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*a*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)
/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c) + (2*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*b^(3/2)*c - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d +
 a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)*c*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^2, x)
```

3.51 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\left(c+dx^2\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*d*x*(b*x^2+a)^(3/2)/c/(-a*d+b*c)/(d*x^2+c)^2+1/8*a*(-3*a*d+4*b*c)*arct
anh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(3/2)+1/
8*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {390, 386, 385, 214}

$$\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{a+bx^2}\,(4bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)}-\frac{dx(a+bx^2)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] -1/4*(d*x*(a + b*x^2)^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*c - 3*a*
d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)*(c + d*x^2)) + (a*(4*b*c - 3*a*d)*
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(b*c - a
*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx = -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2} + \frac{(4bc-3ad)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)(c+dx^2)} + \frac{(a(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}}{8c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)(c+dx^2)} + \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\right.}{8c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)(c+dx^2)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right.}{8c^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.88, size = 146, normalized size = 0.98

$$\frac{\frac{\sqrt{c}\,x\sqrt{a+bx^2}\,(2bc(2c+dx^2)-ad(5c+3dx^2))}{(bc-ad)(c+dx^2)^2} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] ((Sqrt[c]*x*Sqrt[a + b*x^2]*(2*b*c*(2*c + d*x^2) - a*d*(5*c + 3*d*x^2)))/((
b*c - a*d)*(c + d*x^2)^2) + (a*(4*b*c - 3*a*d)*ArcTanh[(-(d*x*Sqrt[a + b*x^
2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2))/(
8*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4112 vs. $2(129) = 258$.
time = 0.07, size = 4113, normalized size = 27.60

method	result	size
default	Expression too large to display	4113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16/(-c*d)^(1/2)/c^2*(((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*
(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a
*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(
-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)
)/(x-(-c*d)^(1/2)/d)))-3/16/(-c*d)^(1/2)/c^2*(((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)^(1/2)/d*
ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b
-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*
d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(
(a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))-1/8/d/c/(-c*d)^(1/2)*(-1/2/(
a*d-b*c)*d/(x-(-c*d)^(1/2)/d)^2*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/2*b*(-c*d)^(1/2)/(a*d-b*c)*(-1/(a*d
-b*c)*d/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/(a*d-b*c)*(((x-(-c*d)^(1/2)
/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-
c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^
(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d
-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))+2*b/(a*d-b*c)*d*
(1/4*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)
/d+4*b^2*c/d)/b^(3/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))))+1/2*b/(a*d-b*c)*d*(((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d
+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2
*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))/(x-(-c*d)^(1/2)/d)))-3/16/d/c^2*(-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)*((
```

```
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3
/2)+b*(-c*d)^(1/2)/(a*d-b*c)*(((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/
2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*l
n((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2
)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(1/2))/(x-(-c*d)^(1/2)/d)))+2*b/(a*d-b*c)*d*(1/4*(2*b*(x-(-c*d)^(1/2)/d)+
2*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((b*(
-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c
*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))))-3/16/d/c^2*(-1/(a*d-b*
c)*d/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-b*(-c*d)^(1/2)/(a*d-b*c)*(((x+(-c*d)^(1/2)/d)
^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d
)^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b
*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))+2*b/(a*d-b*c)*d*(1
/4*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/b*((x+(-c*d)^(1/2)/d)^2*b-2*
b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d
+4*b^2*c/d)/b^(3/2)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x
+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2))))+1/8/d/c/(-c*d)^(1/2)*(-1/2/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)^2*((x+(-c*d
)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/2
*b*(-c*d)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-b*(-c*d)
^(1/2)/(a*d-b*c)*(((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+b*(x+(
-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b
*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(129) = 258$.

time = 1.49, size = 698, normalized size = 4.68

$$\left[\frac{\left(4abc^3 - 3a^2c^2d + (4abcd^2 - 3a^2d^3)x^4 + 2(4abc^2d - 3a^2cd^2)x^2\right)\sqrt{bc^2 - acd}\log\left(\frac{(8b^2c^2 - 8abcd + a^2d^2)x^4 + a^2c^2 + 2(4abc^2 - 3a^2cd)x^2 + 4((2bc - ad)x^3 + acx)\sqrt{bc^2 - acd}\sqrt{bx^2 + a}}{d^2x^4 + 2cdx^2 + c^2}\right) + 4\left((2b^2c^3d - 5abc^2d^2 + 3a^2cd^3)x^3 + (4b^2c^4 - 9abc^3d + 5a^2c^2d^2)x\right)\sqrt{bx^2 + a}}{32\left(b^2c^7 - 2abc^6d + a^2c^5d^2 + (b^2c^5d^2 - 2abc^4d^3 + a^2c^3d^4)x^4 + 2(b^2c^6d - 2abc^5d^2 + a^2c^4d^3)x^2\right)}, -\frac{\left(4abc^3 - 3a^2c^2d + (4abcd^2 - 3a^2d^3)x^4 + 2(4abc^2d - 3a^2cd^2)x^2\right)\sqrt{-bc^2 + acd}\arctan\left(\frac{\sqrt{-bc^2 + acd}((2bc - ad)x^2 + ac)\sqrt{bx^2 + a}}{2((b^2c^2 - abcd)x^3 + (abc^2 - a^2cd)x)}\right) - 2\left((2b^2c^3d - 5abc^2d^2 + 3a^2cd^3)x^3 + (4b^2c^4 - 9abc^3d + 5a^2c^2d^2)x\right)\sqrt{bx^2 + a}}{16\left(b^2c^7 - 2abc^6d + a^2c^5d^2 + (b^2c^5d^2 - 2abc^4d^3 + a^2c^3d^4)x^4 + 2(b^2c^6d - 2abc^5d^2 + a^2c^4d^3)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*
c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*
x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^
2)) + 4*((2*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a
*b*c^3*d + 5*a^2*c^2*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*
c^5*d^2 + (b^2*c^5*d^2 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d -
2*a*b*c^5*d^2 + a^2*c^4*d^3)*x^2), -1/16*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b
*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-b*c^2 +
a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2
 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3*d
- 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a*b*c^3*d + 5*a^2*c^2*d
^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2 + (b^2*c^5*d^2
 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^
4*d^3)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(129) = 258.
time = 1.99, size = 487, normalized size = 3.27

$$-\frac{\left(4ab^{\frac{3}{2}}c - 3a^2\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 d + 2bc - ad}{2\sqrt{-b^2c^2 + abcd}}\right)}{8\sqrt{-b^2c^2 + abcd}(bc^3 - ac^2d)} - \frac{4\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 ab^{\frac{3}{2}}cd^2 - 3\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 a^2\sqrt{b}d^3 - 16\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 b^{\frac{7}{2}}c^3 + 40\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 ab^{\frac{5}{2}}c^2d - 30\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 a^2b^{\frac{3}{2}}cd^2 + 9\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 a^3\sqrt{b}d^3 - 16\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 a^2b^{\frac{5}{2}}c^2d + 28\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 a^3b^{\frac{3}{2}}cd^2 - 9\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 a^4\sqrt{b}d^3 - 2a^4b^{\frac{3}{2}}cd^2 + 3a^5\sqrt{b}d^3}{4\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 d + 4\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 bc - 2\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 ad + a^2d\right)^2(bc^3d - ac^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*a*b^(3/2)*c - 3*a^2*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*(b*c^3 - a*c^2*d)) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d
```

```
^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^3 - 16*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*b^(7/2)*c^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)
*c^2*d - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2
*a^2*b^(5/2)*c^2*d + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 -
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^3 - 2*a^4*b^(3/2)*c*d^2 +
3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2*(b*c
^3*d - a*c^2*d^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^3, x)
```

3.52 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4} \, dx$

Optimal. Leaf size=208

$$\frac{x\sqrt{a+bx^2}}{6c\,(c+dx^2)^3}+\frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\,(c+dx^2)^2}+\frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\,(c+dx^2)}+\frac{a(8b^2c^2-12abcd+5a^2d^2)\,\mathrm{ta}}{16c^{7/2}(bc-}$$

```
[Out] 1/16*a*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/
(b*x^2+a)^(1/2))/c^(7/2)/(-a*d+b*c)^(5/2)+1/6*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)
^3+1/24*(-5*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/48*(-
5*a*d+2*b*c)*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^3/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A]
time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {423, 541, 12, 385, 214}

$$\frac{a(5a^2d^2-12abcd+8b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\,(2bc-5ad)(4bc-3ad)}{48c^3\,(c+dx^2)\,(bc-ad)^2}+\frac{x\sqrt{a+bx^2}\,(4bc-5ad)}{24c^2\,(c+dx^2)^2\,(bc-ad)}+\frac{x\sqrt{a+bx^2}}{6c\,(c+dx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + ((4*b*c - 5*a*d)*x*Sqrt[a + b*x^2
])/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + ((2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*
Sqrt[a + b*x^2])/(48*c^3*(b*c - a*d)^2*(c + d*x^2)) + (a*(8*b^2*c^2 - 12*a*
b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(16*c^(7/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^4}\,dx = \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} - \frac{\int \frac{-5a-4bx^2}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx}{6c}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} - \frac{\int \frac{-a(16bc-15ad)-2b(4bc-5ad)x^2}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx}{24c^2(bc-ad)}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} - \frac{\int -\frac{3a(}{}}{}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{(a(8b^2}{}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{(a(8b^2}{}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{a(8b^2c}{}$$

Mathematica [A]

time = 10.77, size = 227, normalized size = 1.09

$$\frac{x\sqrt{a+bx^2}\left((bc-ad)\left(8b^2c^2(3c^2+3cdx^2+d^2x^4)-2abcd(30c^2+35cdx^2+13d^2x^4)+a^2d^2(33c^2+40cdx^2+15d^2x^4)\right)+\frac{3a(8b^2c^2-12abcd+5a^2d^2)\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}(c+dx^2)^3\tanh^{-1}\left(\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}\right)}{x^2}\right)}{48c^3(bc-ad)^3(c+dx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((b*c - a*d)*(8*b^2*c^2*(3*c^2 + 3*c*d*x^2 + d^2*x^4) -
2*a*b*c*d*(30*c^2 + 35*c*d*x^2 + 13*d^2*x^4) + a^2*d^2*(33*c^2 + 40*c*d*x^2
 + 15*d^2*x^4)) + (3*a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[((b*c - a*
d)*x^2)/(c*(a + b*x^2))]*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a
 + b*x^2))]])/x^2))/(48*c^3*(b*c - a*d)^3*(c + d*x^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6474 vs. $2(184) = 368$.
time = 0.08, size = 6475, normalized size = 31.13

method	result	size
default	Expression too large to display	6475

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(184) = 368$.
time = 1.33, size = 1220, normalized size = 5.87

$$\left[\frac{3\,(8\,ab^2c^5-12\,a^2bc^4d+5\,a^3c^3d^2+(8\,ab^2c^2d^3-12\,a^2bcd^4+5\,a^3d^5)x^6+3\,(8\,ab^2c^3d^2-12\,a^2bc^2d^3+5\,a^3cd^4)x^4+3\,(8\,ab^2c^4d-12\,a^2bc^3d^2+5\,a^3c^2d^3)x^2)\sqrt{bc^2-acd}\,\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2+4((2bc-ad)x^3+acx)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2cdx^2+c^2}\right)+4\,((8\,b^3c^4d^2-34\,ab^2c^3d^3+41\,a^2bc^2d^4-15\,a^3cd^5)x^5+2\,(12\,b^3c^5d-47\,ab^2c^4d^2+55\,a^2bc^3d^3-20\,a^3c^2d^4)x^3+3\,(8\,b^3c^6-28\,ab^2c^5d+31\,a^2bc^4d^2-11\,a^3c^3d^3)x)\sqrt{bx^2+a}}{192\,(b^3c^{10}-3\,ab^2c^9d+3\,a^2bc^8d^2-a^3c^7d^3+(b^3c^7d^3-3\,ab^2c^6d^4+3\,a^2bc^5d^5-a^3c^4d^6)x^6+3\,(b^3c^8d^2-3\,ab^2c^7d^3+3\,a^2bc^6d^4-a^3c^5d^5)x^4+3\,(b^3c^9d-3\,ab^2c^8d^2+3\,a^2bc^7d^3-a^3c^6d^4)x^2)},\,-\frac{3\,(8\,ab^2c^5-12\,a^2bc^4d+5\,a^3c^3d^2+(8\,ab^2c^2d^3-12\,a^2bcd^4+5\,a^3d^5)x^6+3\,(8\,ab^2c^3d^2-12\,a^2bc^2d^3+5\,a^3cd^4)x^4+3\,(8\,ab^2c^4d-12\,a^2bc^3d^2+5\,a^3c^2d^3)x^2)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\,((2bc-ad)x^2+ac)\sqrt{bx^2+a}}{2((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)-2\,((8\,b^3c^4d^2-34\,ab^2c^3d^3+41\,a^2bc^2d^4-15\,a^3cd^5)x^5+2\,(12\,b^3c^5d-47\,ab^2c^4d^2+55\,a^2bc^3d^3-20\,a^3c^2d^4)x^3+3\,(8\,b^3c^6-28\,ab^2c^5d+31\,a^2bc^4d^2-11\,a^3c^3d^3)x)\sqrt{bx^2+a}}{96\,(b^3c^{10}-3\,ab^2c^9d+3\,a^2bc^8d^2-a^3c^7d^3+(b^3c^7d^3-3\,ab^2c^6d^4+3\,a^2bc^5d^5-a^3c^4d^6)x^6+3\,(b^3c^8d^2-3\,ab^2c^7d^3+3\,a^2bc^6d^4-a^3c^5d^5)x^4+3\,(b^3c^9d-3\,ab^2c^8d^2+3\,a^2bc^7d^3-a^3c^6d^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a*b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3
- 12*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 +
 5*a^3*c*d^4)*x^4 + 3*(8*a*b^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^
2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4*d^2
 - 34*a*b^2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*
d - 47*a*b^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^
6 - 28*a*b^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))
/(b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 -
 3*a*b^2*c^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*
a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2
*c^8*d^2 + 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2), -1/96*(3*(8*a*b^2*c^5 - 12*
a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3 - 12*a^2*b*c*d^4 + 5*a^3*d^5
)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 3*(8*a*b
^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arct
an(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2
*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4*d^2 - 34*a*b^
2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*d - 47*a*b
^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^6 - 28*a*b
^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^10
 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 - 3*a*b^2*c
^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*a*b^2*c^7*
d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2*c^8*d^2 +
 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 958 vs. 2(184) = 368.
time = 2.21, size = 958, normalized size = 4.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/16*(8*a*b^(5/2)*c^2 - 12*a^2*b^(3/2)*c*d + 5*a^3*sqrt(b)*d^2)*arctan(1/2
*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d)
)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/24*(
24*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c^2*d^3 - 36*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^2*b^(3/2)*c*d^4 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
3*sqrt(b)*d^5 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 480
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*d^3 + 330*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*
sqrt(b)*d^5 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 + 1216*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d - 2016*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*c^3*d^2 + 1736*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/
2)*c^2*d^3 - 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 + 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 384*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(9/2)*c^4*d + 1392*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/
2)*c^3*d^2 - 1608*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 + 780
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7
/2)*c^3*d^2 + 336*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 - 300
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^7*sqrt(b)*d^5 - 8*a^6*b^(5/2)*c^2*d^3 + 26*a^7*b^(3/2)*c*d^4
 - 15*a^8*sqrt(b)*d^5)/((b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*((sqrt(b)
*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^4,x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^4, x)
```

3.53 $\int (a+bx^2)^{3/2} (c+dx^2)^3 \, dx$

Optimal. Leaf size=272

$$\frac{3a(4bc-ad)(8b^2c^2-2abcd+a^2d^2)x\sqrt{a+bx^2}}{256b^3}+\frac{(4bc-ad)(8b^2c^2-2abcd+a^2d^2)x(a+bx^2)^{3/2}}{128b^3}+\frac{d(36b^2c^2$$

```
[Out] 1/128*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^3+1/16
0*d*(5*a^2*d^2-20*a*b*c*d+36*b^2*c^2)*x*(b*x^2+a)^(5/2)/b^3+1/80*d*(-5*a*d+
14*b*c)*x*(b*x^2+a)^(5/2)*(d*x^2+c)/b^2+1/10*d*x*(b*x^2+a)^(5/2)*(d*x^2+c)^
2/b+3/256*a^2*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/
(b*x^2+a)^(1/2))/b^(7/2)+3/256*a*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)
*x*(b*x^2+a)^(1/2)/b^3
```

Rubi [A]

time = 0.15, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {427, 542, 396, 201, 223, 212}

$$\frac{3a^2(4bc-ad)(a^2d^2-2abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{dx(a+bx^2)^{5/2}(5a^2d^2-20abcd+36b^2c^2)}{160b^3}+\frac{x(a+bx^2)^{3/2}(4bc-ad)(a^2d^2-2abcd+8b^2c^2)}{128b^3}+\frac{3ax\sqrt{a+bx^2}(4bc-ad)(a^2d^2-2abcd+8b^2c^2)}{256b^3}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)(14bc-5ad)}{80b^2}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)^2}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (3*a*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(25
6*b^3) + ((4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*(a + b*x^2)^(3/
2))/(128*b^3) + (d*(36*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*x*(a + b*x^2)^(5/2
))/(160*b^3) + (d*(14*b*c - 5*a*d)*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(80*b^2
) + (d*x*(a + b*x^2)^(5/2)*(c + d*x^2)^2)/(10*b) + (3*a^2*(4*b*c - a*d)*(8*
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b
^(7/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx = \frac{dx(a+bx^2)^{5/2} \left(c+dx^2\right)^2}{10b} + \frac{\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \left(c(10bc-ad)+d(14bc\right.}{10b}$$

$$= \frac{d(14bc-5ad)x(a+bx^2)^{5/2} \left(c+dx^2\right)}{80b^2} + \frac{dx(a+bx^2)^{5/2} \left(c+dx^2\right)^2}{10b} + \frac{\int (a+}{}$$

$$= \frac{d\left(36b^2c^2-20abcd+5a^2d^2\right) x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2} (c+}{80b^2}$$

$$= \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right) x(a+bx^2)^{3/2}}{128b^3} + \frac{d(36b^2c^2-20abcd+5a^2d}{160b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right) x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abc\right.}{128}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right) x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abc\right.}{128}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right) x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abc\right.}{128}$$

Mathematica [A]
time = 0.36, size = 225, normalized size = 0.83

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\,(15a^4d^3-10a^3bd^2(9c+dx^2)+4a^2b^2d(60c^2+15cdx^2+2d^2x^4)+32b^4x^2(10c^3+20c^2dx^2+15cd^2x^4+4d^3x^6)+16ab^3(50c^3+70c^2dx^2+45cd^2x^4+11d^3x^6))+15a^2(-32b^3c^3+16ab^2c^2d-6a^2bcd^2+a^3d^3)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{1280b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*d^3 - 10*a^3*b*d^2*(9*c + d*x^2) + 4*a^2
*b^2*d*(60*c^2 + 15*c*d*x^2 + 2*d^2*x^4) + 32*b^4*x^2*(10*c^3 + 20*c^2*d*x^
2 + 15*c*d^2*x^4 + 4*d^3*x^6) + 16*a*b^3*(50*c^3 + 70*c^2*d*x^2 + 45*c*d^2*
x^4 + 11*d^3*x^6)) + 15*a^2*(-32*b^3*c^3 + 16*a*b^2*c^2*d - 6*a^2*b*c*d^2 +
 a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1280*b^(7/2))
```

Maple [A]
time = 0.06, size = 364, normalized size = 1.34

method	result
risch	$\frac{x\left(128b^4d^3x^8+176a\,b^3d^3x^6+480b^4c\,d^2x^6+8a^2b^2d^3x^4+720a\,b^3c\,d^2x^4+640b^4c^2d\,x^4-10a^3b\,d^3x^2+60a^2b^2c\,d^2x^2+1120a\,b^3c^2d\,x^2+320b^4\right.}{1280b^3}$

default	d^3	$\left(\frac{x^5\left(b\,x^2+a\right)^{\frac{5}{2}}}{10b}-\frac{a\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)}{2b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/10*x^5*(b*x^2+a)^(5/2)/b-1/2*a/b*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*
(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2
+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+3*c*d^2*(1/8*x^3
*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a
)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))))))+3*c^2*d*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3
/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+
c^3*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.31, size = 364, normalized size = 1.34

$$\frac{(bx^2+a)^{\frac{5}{2}}d^3x^5}{10\,b}+\frac{3\,(bx^2+a)^{\frac{5}{2}}cd^2x^3}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}ad^3x^3}{16\,b^2}+\frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}c^3x+\frac{3}{8}\,\sqrt{bx^2+a}\,ac^3x+\frac{(bx^2+a)^{\frac{5}{2}}c^2dx}{2\,b}-\frac{(bx^2+a)^{\frac{3}{2}}ac^2dx}{8\,b}-\frac{3\,\sqrt{bx^2+a}\,a^2c^2dx}{16\,b}-\frac{3\,(bx^2+a)^{\frac{5}{2}}acd^2x}{16\,b^2}+\frac{3\,(bx^2+a)^{\frac{3}{2}}a^2cd^2x}{64\,b^2}+\frac{9\,\sqrt{bx^2+a}\,a^3cd^2x}{128\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}a^2d^3x}{32\,b^3}-\frac{(bx^2+a)^{\frac{3}{2}}a^3d^3x}{128\,b^3}-\frac{3\,\sqrt{bx^2+a}\,a^4d^3x}{256\,b^3}+\frac{3\,a^2c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{3\,a^3c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{9\,a^4cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{3\,a^5d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(5/2)*d^3*x^5/b + 3/8*(b*x^2 + a)^(5/2)*c*d^2*x^3/b - 1/16
*(b*x^2 + a)^(5/2)*a*d^3*x^3/b^2 + 1/4*(b*x^2 + a)^(3/2)*c^3*x + 3/8*sqrt(b
*x^2 + a)*a*c^3*x + 1/2*(b*x^2 + a)^(5/2)*c^2*d*x/b - 1/8*(b*x^2 + a)^(3/2)
*a*c^2*d*x/b - 3/16*sqrt(b*x^2 + a)*a^2*c^2*d*x/b - 3/16*(b*x^2 + a)^(5/2)*
a*c*d^2*x/b^2 + 3/64*(b*x^2 + a)^(3/2)*a^2*c*d^2*x/b^2 + 9/128*sqrt(b*x^2 +
 a)*a^3*c*d^2*x/b^2 + 1/32*(b*x^2 + a)^(5/2)*a^2*d^3*x/b^3 - 1/128*(b*x^2 +
 a)^(3/2)*a^3*d^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*a^4*d^3*x/b^3 + 3/8*a^2*c^3
*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/16*a^3*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(
3/2) + 9/128*a^4*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 3/256*a^5*d^3*arcsi
nh(b*x/sqrt(a*b))/b^(7/2)
```

Fricas [A]
time = 0.73, size = 502, normalized size = 1.85

$$\left[-\frac{15\,(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(128\,b^5d^3x^9+16\,(30\,b^5cd^2+11\,ab^4d^3)x^7+8\,(80\,b^5c^2d+90\,ab^4cd^2+a^2b^3d^3)x^5+10\,(32\,b^5c^3+112\,ab^4c^2d+6\,a^2b^3cd^2-a^3b^2d^3)x^3+5\,(160\,ab^4c^3+48\,a^2b^3c^2d-18\,a^3b^2cd^2+3\,a^4bd^3)x)\sqrt{bx^2+a}}{2560\,b^4},\ -\frac{15\,(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(128\,b^5d^3x^9+16\,(30\,b^5cd^2+11\,ab^4d^3)x^7+8\,(80\,b^5c^2d+90\,ab^4cd^2+a^2b^3d^3)x^5+10\,(32\,b^5c^3+112\,ab^4c^2d+6\,a^2b^3cd^2-a^3b^2d^3)x^3+5\,(160\,ab^4c^3+48\,a^2b^3c^2d-18\,a^3b^2cd^2+3\,a^4bd^3)x)\sqrt{bx^2+a}}{1280\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*
sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(128*b^5*d^3*x^
9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^5*c^2*d + 90*a*b^4*c*d^2
 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 -
a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^2*d - 18*a^3*b^2*c*d^2 +
 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/1280*(15*(32*a^2*b^3*c^3 - 16*a^3
*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (128*b^5*d^3*x^9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^
5*c^2*d + 90*a*b^4*c*d^2 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^
```

```
2*d + 6*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^
2*d - 18*a^3*b^2*c*d^2 + 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(269) = 538$.
time = 172.36, size = 665, normalized size = 2.44

$$\frac{3a^{\frac{9}{2}}d^3x}{256b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{9a^{\frac{7}{2}}cd^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{7}{2}}d^3x^3}{256b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{5}{2}}c^2dx}{16b\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{5}{2}}cd^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{5}{2}}d^3x^5}{640b\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{3}{2}}c^3x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a^{\frac{3}{2}}c^3x}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{17a^{\frac{3}{2}}c^2dx^3}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{39a^{\frac{3}{2}}cd^2x^5}{64\sqrt{1+\frac{bx^2}{a}}} + \frac{23a^{\frac{3}{2}}d^3x^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{3\sqrt{a}\,bc^3x^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{11\sqrt{a}\,bc^2dx^5}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{15\sqrt{a}\,bcd^2x^7}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{19\sqrt{a}\,bd^3x^9}{80\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^5d^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{7}{2}}} + \frac{9a^4cd^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}} - \frac{3a^3c^2d\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + \frac{3a^2c^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}} + \frac{b^2c^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^2c^2dx^7}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{3b^2cd^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^2d^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3,x)
```

```
[Out] 3*a**(9/2)*d**3*x/(256*b**3*sqrt(1 + b*x**2/a)) - 9*a**(7/2)*c*d**2*x/(128*
b**2*sqrt(1 + b*x**2/a)) + a**(7/2)*d**3*x**3/(256*b**2*sqrt(1 + b*x**2/a))
 + 3*a**(5/2)*c**2*d*x/(16*b*sqrt(1 + b*x**2/a)) - 3*a**(5/2)*c*d**2*x**3/(
128*b*sqrt(1 + b*x**2/a)) - a**(5/2)*d**3*x**5/(640*b*sqrt(1 + b*x**2/a)) +
 a**(3/2)*c**3*x*sqrt(1 + b*x**2/a)/2 + a**(3/2)*c**3*x/(8*sqrt(1 + b*x**2/
a)) + 17*a**(3/2)*c**2*d*x**3/(16*sqrt(1 + b*x**2/a)) + 39*a**(3/2)*c*d**2*
x**5/(64*sqrt(1 + b*x**2/a)) + 23*a**(3/2)*d**3*x**7/(160*sqrt(1 + b*x**2/a
)) + 3*sqrt(a)*b*c**3*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b*c**2*d*x**
5/(8*sqrt(1 + b*x**2/a)) + 15*sqrt(a)*b*c*d**2*x**7/(16*sqrt(1 + b*x**2/a))
 + 19*sqrt(a)*b*d**3*x**9/(80*sqrt(1 + b*x**2/a)) - 3*a**5*d**3*asinh(sqrt(
b)*x/sqrt(a))/(256*b**(7/2)) + 9*a**4*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*
b**(5/2)) - 3*a**3*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*a**2*c
**3*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c**3*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + b**2*c**2*d*x**7/(2*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*b**2*c*
d**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d**3*x**11/(10*sqrt(a)*sqrt
(1 + b*x**2/a))
```

Giac [A]
time = 0.97, size = 260, normalized size = 0.96

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,bd^3x^2 + \frac{30\,b^9cd^2 + 11\,ab^8d^3}{b^8}\right)x^2 + \frac{80\,b^9c^2d + 90\,ab^8cd^2 + a^2b^7d^3}{b^8}\right)x^2 + \frac{5\left(32\,b^9c^3 + 112\,ab^8c^2d + 6\,a^2b^7cd^2 - a^3b^6d^3\right)}{b^8}\right)x^2 + \frac{5\left(160\,ab^8c^3 + 48\,a^2b^7c^2d - 18\,a^3b^6cd^2 + 3\,a^4b^5d^3\right)}{b^8}\right)\sqrt{bx^2+a}\,x - \frac{3\left(32\,a^2b^3c^3 - 16\,a^3b^2c^2d + 6\,a^4bcd^2 - a^5d^3\right)\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*b*d^3*x^2 + (30*b^9*c*d^2 + 11*a*b^8*d^3)/b^8)*x^2 + (80
*b^9*c^2*d + 90*a*b^8*c*d^2 + a^2*b^7*d^3)/b^8)*x^2 + 5*(32*b^9*c^3 + 112*a
*b^8*c^2*d + 6*a^2*b^7*c*d^2 - a^3*b^6*d^3)/b^8)*x^2 + 5*(160*a*b^8*c^3 + 4
8*a^2*b^7*c^2*d - 18*a^3*b^6*c*d^2 + 3*a^4*b^5*d^3)/b^8)*sqrt(b*x^2 + a)*x
- 3/256*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^3, x)
```

3.54 $\int (a+bx^2)^{3/2} (c+dx^2)^2 \, dx$

Optimal. Leaf size=196

$$\frac{a(48b^2c^2-16abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{128b^2}+\frac{(48b^2c^2-16abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{192b^2}+\frac{d(10bc-3ad)x(a+bx}{48b^2}$$

```
[Out] 1/192*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^2+1/48*d*(-3*a*
d+10*b*c)*x*(b*x^2+a)^(5/2)/b^2+1/8*d*x*(b*x^2+a)^(5/2)*(d*x^2+c)/b+1/128*a
^2*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
5/2)+1/128*a*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {427, 396, 201, 223, 212}

$$\frac{x(a+bx^2)^{3/2}(3a^2d^2-16abcd+48b^2c^2)}{192b^2}+\frac{ax\sqrt{a+bx^2}(3a^2d^2-16abcd+48b^2c^2)}{128b^2}+\frac{a^2(3a^2d^2-16abcd+48b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{dx(a+bx^2)^{5/2}(10bc-3ad)}{48b^2}+\frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (a*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(128*b^2) + ((4
8*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(192*b^2) + (d*(10
*b*c - 3*a*d)*x*(a + b*x^2)^(5/2))/(48*b^2) + (d*x*(a + b*x^2)^(5/2)*(c + d
*x^2))/(8*b) + (a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\,dx = \frac{dx(a+bx^2)^{5/2}\left(c+dx^2\right)}{8b}+\frac{\int \left(a+bx^2\right)^{3/2}\left(c(8bc-ad)+d(10bc-3ad)x^2\right)\,d}{8b}$$

$$= \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2}+\frac{dx(a+bx^2)^{5/2}\left(c+dx^2\right)}{8b}-\frac{(ad(10bc-3ad)-}{}$$

$$= \frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x(a+bx^2)^{3/2}}{192b^2}+\frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2}+\frac{dx}{}$$

$$= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2}+\frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x(a+}{192b^2}$$

$$= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2}+\frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x(a+}{192b^2}$$

$$= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2}+\frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x(a+}{192b^2}$$

Mathematica [A]
time = 0.26, size = 158, normalized size = 0.81

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(-9a^3d^2+6a^2bd(8c+dx^2)+16b^3x^2(6c^2+8cdx^2+3d^2x^4)+8ab^2(30c^2+28cdx^2+9d^2x^4)\right)-3a^2\left(48b^2c^2-16abcd+3a^2d^2\right)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{384b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*d^2 + 6*a^2*b*d*(8*c + d*x^2) + 16*b^3*x
^2*(6*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + 8*a*b^2*(30*c^2 + 28*c*d*x^2 + 9*d^2*x
^4)) - 3*a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/(384*b^(5/2))
```

Maple [A]
time = 0.06, size = 235, normalized size = 1.20

method	result
risch	$-\frac{x\left(-48b^3d^2x^6-72a\,b^2d^2x^4-128b^3cd\,x^4-6a^2b\,d^2x^2-224a\,b^2cd\,x^2-96b^3c^2x^2+9a^3d^2-48a^2bcd-240a\,b^2c^2\right)\sqrt{b\,x^2+a}}{384b^2}+\frac{3a^4}{}$
default	$d^2\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)+2cd$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/
4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))))))+2*c*d*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+
a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2)))))+c^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(
1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 227, normalized size = 1.16

$$\frac{(bx^2+a)^{\frac{5}{2}}d^2x^3}{8\,b}+\frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}c^2x+\frac{3}{8}\,\sqrt{bx^2+a}\,ac^2x+\frac{(bx^2+a)^{\frac{5}{2}}cdx}{3\,b}-\frac{(bx^2+a)^{\frac{3}{2}}acdx}{12\,b}-\frac{\sqrt{bx^2+a}\,a^2cdx}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}ad^2x}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}a^2d^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}\,a^3d^2x}{128\,b^2}+\frac{3\,a^2c^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{a^3cd\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{3\,a^4d^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*d^2*x^3/b + 1/4*(b*x^2 + a)^(3/2)*c^2*x + 3/8*sqrt(b*
x^2 + a)*a*c^2*x + 1/3*(b*x^2 + a)^(5/2)*c*d*x/b - 1/12*(b*x^2 + a)^(3/2)*a
*c*d*x/b - 1/8*sqrt(b*x^2 + a)*a^2*c*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*d^2*x
/b^2 + 1/64*(b*x^2 + a)^(3/2)*a^2*d^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*d^2
*x/b^2 + 3/8*a^2*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^3*c*d*arcsinh(b
*x/sqrt(a*b))/b^(3/2) + 3/128*a^4*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.58, size = 344, normalized size = 1.76

$$\left[\frac{3\,(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(48\,b^4d^2x^7+8\,(16\,b^4cd+9\,ab^3d^2)x^5+2\,(48\,b^4c^2+112\,ab^3cd+3\,a^2b^2d^2)x^3+3\,(80\,ab^3c^2+16\,a^2b^2cd-3\,a^3bd^2)x)\sqrt{bx^2+a}}{768\,b^3},\ -\frac{3\,(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(48\,b^4d^2x^7+8\,(16\,b^4cd+9\,ab^3d^2)x^5+2\,(48\,b^4c^2+112\,ab^3cd+3\,a^2b^2d^2)x^3+3\,(80\,ab^3c^2+16\,a^2b^2cd-3\,a^3bd^2)x)\sqrt{bx^2+a}}{384\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*
a*b^3*d^2)*x^5 + 2*(48*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80
*a*b^3*c^2 + 16*a^2*b^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/384*
(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - (48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*a*b^3*d^2)*x^5 + 2*(4
8*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80*a*b^3*c^2 + 16*a^2*b
^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(190) = 380.
time = 35.72, size = 440, normalized size = 2.24

$$-\frac{3a^{\frac{7}{2}}d^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}cdx}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}d^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}c^2x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}cdx^3}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}d^2x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,bc^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,bcdx^5}{12\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,bd^2x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^4d^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{a^3cd\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{b^2c^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2cdx^7}{3\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2d^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2,x)
```

```
[Out] -3*a**(7/2)*d**2*x/(128*b**2*sqrt(1 + b*x**2/a)) + a**(5/2)*c*d*x/(8*b*sqrt
(1 + b*x**2/a)) - a**(5/2)*d**2*x**3/(128*b*sqrt(1 + b*x**2/a)) + a**(3/2)*
c**2*x*sqrt(1 + b*x**2/a)/2 + a**(3/2)*c**2*x/(8*sqrt(1 + b*x**2/a)) + 17*a
**(3/2)*c*d*x**3/(24*sqrt(1 + b*x**2/a)) + 13*a**(3/2)*d**2*x**5/(64*sqrt(1
```

```
 + b*x**2/a)) + 3*sqrt(a)*b*c**2*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b
*c*d*x**5/(12*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*d**2*x**7/(16*sqrt(1 + b*x*
*2/a)) + 3*a**4*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) - a**3*c*d*asi
nh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + 3*a**2*c**2*asinh(sqrt(b)*x/sqrt(a))/(
8*sqrt(b)) + b**2*c**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*c*d*x**7/
(3*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/
a))
```

Giac [A]
time = 0.62, size = 175, normalized size = 0.89

$$\frac{1}{384}\left(2\left(4\left(6\,bd^2x^2+\frac{16\,b^7cd+9\,ab^6d^2}{b^6}\right)x^2+\frac{48\,b^7c^2+112\,ab^6cd+3\,a^2b^5d^2}{b^6}\right)x^2+\frac{3\,(80\,ab^6c^2+16\,a^2b^5cd-3\,a^3b^4d^2)}{b^6}\right)\sqrt{bx^2+a}\,x-\frac{(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b*d^2*x^2 + (16*b^7*c*d + 9*a*b^6*d^2)/b^6)*x^2 + (48*b^7*c^
2 + 112*a*b^6*c*d + 3*a^2*b^5*d^2)/b^6)*x^2 + 3*(80*a*b^6*c^2 + 16*a^2*b^5*
c*d - 3*a^3*b^4*d^2)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(48*a^2*b^2*c^2 - 16*a^
3*b*c*d + 3*a^4*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^2, x)
```

3.55 $\int (a+bx^2)^{3/2} (c+dx^2)\, dx$

Optimal. Leaf size=118

$$\frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b}+\frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b}+\frac{dx(a+bx^2)^{5/2}}{6b}+\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

```
[Out] 1/24*(-a*d+6*b*c)*x*(b*x^2+a)^(3/2)/b+1/6*d*x*(b*x^2+a)^(5/2)/b+1/16*a^2*(-
a*d+6*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/16*a*(-a*d+6*b*c)*x
*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {396, 201, 223, 212}

$$\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x(a+bx^2)^{3/2}(6bc-ad)}{24b}+\frac{ax\sqrt{a+bx^2}\,(6bc-ad)}{16b}+\frac{dx(a+bx^2)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (a*(6*b*c - a*d)*x*Sqrt[a + b*x^2])/(16*b) + ((6*b*c - a*d)*x*(a + b*x^2)^(
3/2))/(24*b) + (d*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*b*c - a*d)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\,dx=\frac{dx(a+bx^2)^{5/2}}{6b}-\frac{(-6bc+ad)\int (a+bx^2)^{3/2}\,dx}{6b}$$
$$=\frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b}+\frac{dx(a+bx^2)^{5/2}}{6b}+\frac{(a(6bc-ad))\int \sqrt{a+bx^2}\,dx}{8b}$$
$$=\frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b}+\frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b}+\frac{dx(a+bx^2)^{5/2}}{6b}+\frac{(a^2(}{}$$
$$=\frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b}+\frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b}+\frac{dx(a+bx^2)^{5/2}}{6b}+\frac{(a^2(}{}$$
$$=\frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b}+\frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b}+\frac{dx(a+bx^2)^{5/2}}{6b}+\frac{a^2(6}{}$$

Mathematica [A]
time = 0.15, size = 99, normalized size = 0.84

$$\frac{x\sqrt{a+bx^2}\,(30abc+3a^2d+12b^2cx^2+14abdx^2+8b^2dx^4)}{48b}+\frac{a^2(-6bc+ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(30*a*b*c + 3*a^2*d + 12*b^2*c*x^2 + 14*a*b*d*x^2 + 8*b^
2*d*x^4))/(48*b) + (a^2*(-6*b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/(16*b^(3/2))
```

Maple [A]
time = 0.05, size = 130, normalized size = 1.10

method	result
risch	$\frac{x(8b^2dx^4+14x^2abd+12b^2cx^2+3a^2d+30abc)\sqrt{bx^2+a}}{48b}-\frac{a^3\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)d}{16b^{\frac{3}{2}}}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right.}{8\sqrt{b}}$

default	$d\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+c\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b}\ldots}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x
^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+c*(1/4*x*(b*x^2+
a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))))
```

Maxima [A]
time = 0.27, size = 116, normalized size = 0.98

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}cx+\frac{3}{8}\sqrt{bx^2+a}\,acx+\frac{\left(bx^2+a\right)^{\frac{5}{2}}dx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}adx}{24\,b}-\frac{\sqrt{bx^2+a}\,a^2dx}{16\,b}+\frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{a^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*c*x + 3/8*sqrt(b*x^2 + a)*a*c*x + 1/6*(b*x^2 + a)^(5/
2)*d*x/b - 1/24*(b*x^2 + a)^(3/2)*a*d*x/b - 1/16*sqrt(b*x^2 + a)*a^2*d*x/b
+ 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/16*a^3*d*arcsinh(b*x/sqrt(a*
b))/b^(3/2)
```

Fricas [A]
time = 0.55, size = 210, normalized size = 1.78

$$\left[-\frac{3\left(6\,a^2bc-a^3d\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(8\,b^3dx^5+2\left(6\,b^3c+7\,ab^2d\right)x^3+3\left(10\,ab^2c+a^2bd\right)x\right)\sqrt{bx^2+a}}{96\,b^2},-\frac{3\left(6\,a^2bc-a^3d\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,b^3dx^5+2\left(6\,b^3c+7\,ab^2d\right)x^3+3\left(10\,ab^2c+a^2bd\right)x\right)\sqrt{bx^2+a}}{48\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*a^2*b*c - a^3*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x^3 + 3*(10*a*b^2*c +
 a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/48*(3*(6*a^2*b*c - a^3*d)*sqrt(-b)*ar
```

```
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x
^3 + 3*(10*a*b^2*c + a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.
time = 10.12, size = 253, normalized size = 2.14

$$\frac{a^{\frac{5}{2}}dx}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}cx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,bcx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,bdx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{a^3d\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{3a^2c\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{b^2cx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2dx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c),x)
```

```
[Out] a**(5/2)*d*x/(16*b*sqrt(1 + b*x**2/a)) + a**(3/2)*c*x*sqrt(1 + b*x**2/a)/2
+ a**(3/2)*c*x/(8*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*d*x**3/(48*sqrt(1 + b*x
**2/a)) + 3*sqrt(a)*b*c*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b*d*x**5/(
24*sqrt(1 + b*x**2/a)) - a**3*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*
a**2*c*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + b**2*d*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.60, size = 103, normalized size = 0.87

$$\frac{1}{48}\left(2\left(4\,bdx^2+\frac{6\,b^5c+7\,ab^4d}{b^4}\right)x^2+\frac{3\,(10\,ab^4c+a^2b^3d)}{b^4}\right)\sqrt{bx^2+a}\;x-\frac{(6\,a^2bc-a^3d)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*d*x^2 + (6*b^5*c + 7*a*b^4*d)/b^4)*x^2 + 3*(10*a*b^4*c + a^2*b
^3*d)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(6*a^2*b*c - a^3*d)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2), x)
```

3.56 $\int (a+bx^2)^{3/2}\, dx$

Optimal. Leaf size=65

$$\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+3/8*a*x*(b*x^2+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(3/2),x]

[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^{3/2}\,dx &= \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+bx^2}\,dx\\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 0.92

$$\frac{1}{8}x\sqrt{a+bx^2}\left(5a+2bx^2\right)-\frac{3a^2\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(8*Sqrt[b])
```

Maple [A]
time = 0.00, size = 52, normalized size = 0.80

method	result	size
risch	$\frac{x\left(2b\,x^2+5a\right)\sqrt{b\,x^2+a}}{8}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}$	48
default	$\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/
2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.66

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{bx^2+a}\,ax+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sqr
t(a*b))/sqrt(b)
```

Fricas [A]
time = 0.51, size = 124, normalized size = 1.91

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{16\,b},\,-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 1.53, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}\,bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

Giac [A]
time = 0.51, size = 49, normalized size = 0.75

$$\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}\,x-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

Mupad [B]
time = 4.71, size = 37, normalized size = 0.57

$$\frac{x\,\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2),x)
```

```
[Out] (x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/2)
```

3.57 $\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx$

Optimal. Leaf size=113

$$\frac{bx\sqrt{a+bx^2}}{2d}-\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^2}$$

```
[Out] -1/2*(-3*a*d+2*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d^2+(-a*d+b*
c)^(3/2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/d^2/c^(1/2)+1/
2*b*x*(b*x^2+a)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {427, 537, 223, 212, 385, 214}

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^2}-\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) - (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*d^2) + ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/
(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \frac{bx\sqrt{a+bx^2}}{2d} + \frac{\int \frac{-a(bc-2ad)-b(2bc-3ad)x^2}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{2d}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{(b(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^2\text{Subst}\left(\right.}{}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{bc-}}{\sqrt{c}\,\sqrt{a}}\right.}{\sqrt{c}\,d^2}$$

Mathematica [A]
time = 0.31, size = 126, normalized size = 1.12

$$\frac{bdx\sqrt{a+bx^2} - \frac{2(-bc+ad)^{3/2}\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{\sqrt{c}} + \sqrt{b}\,(2bc-3ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2] - (2*(-(b*c) + a*d)^(3/2)*ArcTan[(-(d*x*Sqrt[a + b*x
^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[c] + Sqrt[b
]*(2*b*c - 3*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. $2(91) = 182$.
time = 0.10, size = 1227, normalized size = 10.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-c*d)^(1/2)*(1/3*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/d*(1/4*(2*b*(x-(-c*d)^(1/2)/d)+2
*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((b*(-
c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x-(-c*d)^
(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/
2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-
c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))
-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2
)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))))-1/2/(-c*d)
^(1/2)*(1/3*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(3/2)-b*(-c*d)^(1/2)/d*(1/4*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^
(1/2)/d)/b*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((-b*(-c*d)^(1/2
)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/
d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x+(-c*d)^(1/2)/d)^
2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)
^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*
c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c), x)
```

Fricas [A]
time = 0.79, size = 721, normalized size = 6.38

$$\left[\frac{2\sqrt{bx^2+a}\,bdx-(2bc-3ad)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)-(bc-ad)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{4d^2},\ \frac{2\sqrt{bx^2+a}\,bdx+2(2bc-3ad)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(bc-ad)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{4d^2},\ \frac{2\sqrt{bx^2+a}\,bdx-2(bc-ad)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)-(2bc-3ad)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4d^2},\ \frac{\sqrt{bx^2+a}\,bdx+(2bc-3ad)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(bc-ad)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)}{2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) - (b*c - a*d)*sqrt((b*c - a*d)/c)*log(((8*b^2*
c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 -
4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d
^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 + a)*b*d*x + 2*(2*b*c -
3*a*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt((b*c
- a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c
^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*s
qrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 +
a)*b*d*x - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
- a^2*d)*x)) - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a))/d^2, 1/2*(sqrt(b*x^2 + a)*b*d*x + (2*b*c - 3*a*d)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1
/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c -
 a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2), x)
```

3.58 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}$$

```
[Out] b^(3/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/d^2-1/2*(a*d+2*b*c)*arctanh(x*(-
a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/c^(3/2)/d^2-1/2*(-
a*d+b*c)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {424, 537, 223, 212, 385, 214}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}-\frac{x\sqrt{a+bx^2}\,(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)) + (b^(3/2)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^2 - (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(
Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{\int \frac{a(bc+ad)+2b^2cx^2}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{2cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{b^2\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^2} - \frac{((bc-ad)(2bc+ad))\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}}{2cd^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{((bc-ad)(2bc+ad))}{}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\right.}{2c^{3/2}d^2}$$

Mathematica [A]
time = 0.55, size = 145, normalized size = 1.11

$$\frac{\frac{d(-bc+ad)x\sqrt{a+bx^2}}{c(c+dx^2)} - \frac{\sqrt{-bc+ad}\,(2bc+ad)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{c^{3/2}} - 2b^{3/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] ((d*(-(b*c) + a*d)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2)) - (Sqrt[-(b*c) + a*d]
*(2*b*c + a*d)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[
c]*Sqrt[-(b*c) + a*d])])/c^(3/2) - 2*b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*
x^2]])/(2*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3348 vs. $2(109) = 218$.
time = 0.06, size = 3349, normalized size = 25.56

method	result	size
default	Expression too large to display	3349

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/(-c*d)^(1/2)*(1/3*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/d*(1/4*(2*b*(x-(-c*d)^(1/2)/d)
+2*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((b*
(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x-(-c*d
)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(
1/2)*(-c*d)^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-
(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
))-(a*d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))))-1/4/c/(-
c*d)^(1/2)*(1/3*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(3/2)-b*(-c*d)^(1/2)/d*(1/4*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c
*d)^(1/2)/d)/b*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((-b*(-c*d)^
(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1
/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x+(-c*d)^(1/2)
/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-
c*d)^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)
^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*
d-b*c)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)
^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))))-1/4/d/c*(-1/(a
*d-b*c)*d/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(5/2)-3*b*(-c*d)^(1/2)/(a*d-b*c)*(1/3*((x+(-c*
d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-b*
```

```
(-c*d)^(1/2)/d*(1/4*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/b*((x+(-c*d
)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8
*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2
)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)^(1/2)/d*ln((-b*(-c*
d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(
1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))))+4*b/(a*d-b*c)*d*(1/8*(2*b*(x+(-c*d)^(1
/2)/d)-2*b*(-c*d)^(1/2)/d)/b*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+3/16*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b*(1/4*
(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/b*((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*
b^2*c/d)/b^(3/2)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))
)))-1/4/d/c*(-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(5/2)+3*b*(-c*d)^(1/2)/(a*d-
b*c)*(1/3*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/d*(1/4*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1
/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((b*(-c*d)^(1/2)/d
+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x-(-c*d)^(1/2)/d)^2*b
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-c*d)^(1
/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d
)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d
/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d
)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))))+4*b/(a*d-b*c)*d*(1/8*(
2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+3/16*(4*b*(a*d-b*c)/d+4*
b^2*c/d)/b*(1/4*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*
b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))
/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2, x)

Fricas [A]
time = 0.61, size = 907, normalized size = 6.92

$$\left[-\frac{4(bcd-ad^2)\sqrt{bx^2+a}\,x-4(bcdx^2+bc^2)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}\,x-a\right)-(2bc^2+acd+(2bcd+ad^2)x^2)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{8(cd^3x^2+c^2d^2)}, -\frac{4(bcd-ad^2)\sqrt{bx^2+a}\,x+8(bcdx^2+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2bc^2+acd+(2bcd+ad^2)x^2)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(4abc^2-3a^2cd)x^2-4(ac^2x+(2bc^2-acd)x^3)\sqrt{bx^2+a}\sqrt{\frac{bc-ad}{c}}}{d^2x^4+2cdx^2+c^2}\right)}{8(cd^3x^2+c^2d^2)}, -\frac{2(bcd-ad^2)\sqrt{bx^2+a}\,x-(2bc^2+acd+(2bcd+ad^2)x^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)-2(bcdx^2+bc^2)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}\,x-a\right)}{4(cd^3x^2+c^2d^2)}, -\frac{2(bcd-ad^2)\sqrt{bx^2+a}\,x+4(bcdx^2+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2bc^2+acd+(2bcd+ad^2)x^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2((b^2c-abd)x^3+(abc-a^2d)x)}\right)}{4(cd^3x^2+c^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")

[Out] [-1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - 4*(b*c*d*x^2 + b*c^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(c*d^3*x^2 + c^2*d^2), -1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 8*(b*c*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) - 2*(b*c*d*x^2 + b*c^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 4*(b*c*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/(c*d^3*x^2 + c^2*d^2)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2,x)

[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(109) = 218.

time = 0.57, size = 317, normalized size = 2.42

$$-\frac{b^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^2}+\frac{\left(2\,b^{\frac{5}{2}}c^2-ab^{\frac{3}{2}}cd-a^2\sqrt{b}\,d^2\right)\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}\,cd^2}-\frac{2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}c^2-3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}cd+\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2\sqrt{b}\,d^2+a^2b^{\frac{3}{2}}cd-a^3\sqrt{b}\,d^2}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^2 + 1/2*(2*b^(5/2)*c^2
- a*b^(3/2)*c*d - a^2*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a)
)^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c*
d^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^2 - 3*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a*b^(3/2)*c*d + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt(b)*
d^2 + a^2*b^(3/2)*c*d - a^3*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*
d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a*d + a^2*d)*c*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^2, x)
```

3.59 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=113

$$\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}+\frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/4*x*(b*x^2+a)^(3/2)/c/(d*x^2+c)^2+3/8*a^2*arctanh(x*(-a*d+b*c)^(1/2)/c^(1
/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(1/2)+3/8*a*x*(b*x^2+a)^(1/2)/c^2/(
d*x^2+c)
```

Rubi [A]

time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {386, 385, 214}

$$\frac{3a^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}+\frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*x*Sqrt[a + b*x^2])/(8*c^2*
(c + d*x^2)) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(8*c^(5/2)*Sqrt[b*c - a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
```

```
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx &= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{(3a)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{4c} \\
&= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\,(c+dx^2)} + \frac{(3a^2)\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{8c^2} \\
&= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\,(c+dx^2)} + \frac{(3a^2)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8c^2} \\
&= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\,(c+dx^2)} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1002 vs. $2(113) = 226$.
time = 3.60, size = 1002, normalized size = 8.87

$$
\frac{1}{8}\left(\frac{3a^2}{c^2dx\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}+\frac{a^2\left(a^2+11abx^2+12b^2x^4-5a\sqrt{b}\,x\sqrt{a+bx^2}-12b^{3/2}x^3\sqrt{a+bx^2}\right)}{cd^2x^3\left(\sqrt{b}\,x-\sqrt{a+bx^2}\right)\left(a^2+8abx^2+8b^2x^4-4a\sqrt{b}\,x\sqrt{a+bx^2}-8b^{3/2}x^3\sqrt{a+bx^2}\right)}+\frac{2(a+bx^2)\left(a^3+13a^2bx^2+28ab^2x^4+16b^3x^6-5a^2\sqrt{b}\,x\sqrt{a+bx^2}-20ab^{3/2}x^3\sqrt{a+bx^2}-16b^{5/2}x^5\sqrt{a+bx^2}\right)}{dx(c+dx^2)^2\left(\sqrt{b}\,x-\sqrt{a+bx^2}\right)\left(a+2bx^2-2\sqrt{b}\,x\sqrt{a+bx^2}\right)^2}-\frac{(a-2bx^2)\left(a^4+64b^{7/2}x^7\left(\sqrt{b}\,x-\sqrt{a+bx^2}\right)+a^3\left(25bx^2-7\sqrt{b}\,x\sqrt{a+bx^2}\right)-8a^2\left(-13b^2x^4+7b^{3/2}x^3\sqrt{a+bx^2}\right)-16a\left(-9b^3x^6+7b^{5/2}x^5\sqrt{a+bx^2}\right)\right)}{d^2x^3(c+dx^2)\left(\sqrt{b}\,x-\sqrt{a+bx^2}\right)\left(a+2bx^2-2\sqrt{b}\,x\sqrt{a+bx^2}\right)\left(a^2+8abx^2+8b^2x^4-4a\sqrt{b}\,x\sqrt{a+bx^2}-8b^{3/2}x^3\sqrt{a+bx^2}\right)}-\frac{3a^2\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{c^{5/2}\sqrt{-bc+ad}}-\frac{32b^2\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{\sqrt{c}\,d^2\sqrt{-bc+ad}}+\frac{24ab\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{c^{3/2}d\sqrt{-bc+ad}}-\frac{32b^2\tanh^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{bc-ad}}\right)}{\sqrt{c}\,d^2\sqrt{bc-ad}}+\frac{24ab\tanh^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{bc-ad}}\right)}{c^{3/2}d\sqrt{bc-ad}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] ((3*a^2)/(c^2*d*x*(-(Sqrt[b]*x) + Sqrt[a + b*x^2])) + (a^2*(a^2 + 11*a*b*x^
2 + 12*b^2*x^4 - 5*a*Sqrt[b]*x*Sqrt[a + b*x^2] - 12*b^(3/2)*x^3*Sqrt[a + b*
x^2]))/(c*d^2*x^3*(Sqrt[b]*x - Sqrt[a + b*x^2])*(a^2 + 8*a*b*x^2 + 8*b^2*x^
4 - 4*a*Sqrt[b]*x*Sqrt[a + b*x^2] - 8*b^(3/2)*x^3*Sqrt[a + b*x^2])) + (2*(a
 + b*x^2)*(a^3 + 13*a^2*b*x^2 + 28*a*b^2*x^4 + 16*b^3*x^6 - 5*a^2*Sqrt[b]*x
*Sqrt[a + b*x^2] - 20*a*b^(3/2)*x^3*Sqrt[a + b*x^2] - 16*b^(5/2)*x^5*Sqrt[a
 + b*x^2]))/(d*x*(c + d*x^2)^2*(Sqrt[b]*x - Sqrt[a + b*x^2])*(a + 2*b*x^2 -
 2*Sqrt[b]*x*Sqrt[a + b*x^2])^2) - ((a - 2*b*x^2)*(a^4 + 64*b^(7/2)*x^7*(Sq
rt[b]*x - Sqrt[a + b*x^2]) + a^3*(25*b*x^2 - 7*Sqrt[b]*x*Sqrt[a + b*x^2]) -
 8*a^2*(-13*b^2*x^4 + 7*b^(3/2)*x^3*Sqrt[a + b*x^2]) - 16*a*(-9*b^3*x^6 + 7
*b^(5/2)*x^5*Sqrt[a + b*x^2])))/(d^2*x^3*(c + d*x^2)*(Sqrt[b]*x - Sqrt[a +
b*x^2])*(a + 2*b*x^2 - 2*Sqrt[b]*x*Sqrt[a + b*x^2])*(a^2 + 8*a*b*x^2 + 8*b^
2*x^4 - 4*a*Sqrt[b]*x*Sqrt[a + b*x^2] - 8*b^(3/2)*x^3*Sqrt[a + b*x^2])) - (
3*a^2*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-
(b*c) + a*d])])/(c^(5/2)*Sqrt[-(b*c) + a*d]) - (32*b^2*ArcTan[(-(d*x*Sqrt[a
```

```
 + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*d
^2*Sqrt[-(b*c) + a*d]) + (24*a*b*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(
c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(c^(3/2)*d*Sqrt[-(b*c) + a*d]) -
 (32*b^2*ArcTanh[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sq
rt[b*c - a*d])])/(Sqrt[c]*d^2*Sqrt[b*c - a*d]) + (24*a*b*ArcTanh[(-(d*x*Sqr
t[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/(c^(3/2)*d
*Sqrt[b*c - a*d]))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6920 vs. $2(93) = 186$.
time = 0.07, size = 6921, normalized size = 61.25

method	result	size
default	Expression too large to display	6921

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.
time = 0.62, size = 526, normalized size = 4.65

$$\left[\frac{3\,(a^2d^2x^4 + 2\,a^2cdx^2 + a^2c^2)\sqrt{bc^2 - acd}\,\log\left(\frac{(8\,b^2c^2 - 8\,abcd + a^2d^2)x^4 + a^2c^2 + 2\,(4\,abc^2 - 3\,a^2cd)x^2 + 4\,((2\,bc - ad)x^3 + acx)\sqrt{bc^2 - acd}\,\sqrt{bx^2 + a}}{d^2x^4 + 2\,cdx^2 + c^2}\right) + 4\,((2\,b^2c^3 + abc^2d - 3\,a^2cd^2)x^3 + 5\,(abc^3 - a^2c^2d)x)\sqrt{bx^2 + a}}{32\,(bc^6 - ac^5d + (bc^4d^2 - ac^3d^3)x^4 + 2\,(bc^5d - ac^4d^2)x^2)}, -\frac{3\,(a^2d^2x^4 + 2\,a^2cdx^2 + a^2c^2)\sqrt{-bc^2 + acd}\,\arctan\left(\frac{\sqrt{-bc^2 + acd}\,((2\,bc - ad)x^2 + ac)\sqrt{bx^2 + a}}{2\,((b^2c^2 - abcd)x^3 + (abc^2 - a^2cd)x)}\right) - 2\,((2\,b^2c^3 + abc^2d - 3\,a^2cd^2)x^3 + 5\,(abc^3 - a^2c^2d)x)\sqrt{bx^2 + a}}{16\,(bc^6 - ac^5d + (bc^4d^2 - ac^3d^3)x^4 + 2\,(bc^5d - ac^4d^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2*d^2*x^4 + 2*a^2*c*d*x^2 + a^2*c^2)*sqrt(b*c^2 - a*c*d)*log(((
8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*
x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d
^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 +
 5*(a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2
- a*c^3*d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2), -1/16*(3*(a^2*d^2*x^4 + 2*
```

```
a^2*c*d*x^2 + a^2*c^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)
*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*
c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 + 5*(a*b*
c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2 - a*c^3*
d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^{\frac{3}{2}}}{(c + dx^2)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(93) = 186.
time = 1.83, size = 451, normalized size = 3.99

$$-\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{8\sqrt{-b^2c^2+abcd}\,c^2}+\frac{8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}}c^2d-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 a^2\sqrt{b}d^3+16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{7}{2}}c^3+8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 ab^{\frac{5}{2}}c^2d-18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 a^2b^{\frac{3}{2}}cd^2+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 a^3\sqrt{b}d^3+8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^2b^{\frac{5}{2}}c^2d+16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^3b^{\frac{3}{2}}cd^2-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^4\sqrt{b}d^3+2\,a^4b^{\frac{3}{2}}cd^2+3\,a^5\sqrt{b}d^3}{4\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ad+a^2d\right)^2 c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
[Out] -3/8*a^2*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*
d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^2) + 1/4*(8*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^2*sqrt(b)*d^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3 + 8*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d - 18*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3
+ 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d + 16*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*s
qrt(b)*d^3 + 2*a^4*b^(3/2)*c*d^2 + 3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b
*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a*d + a^2*d)^2*c^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(d\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^3, x)
```

3.60 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx$

Optimal. Leaf size=199

$$-\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)(c+dx^2)^3}+\frac{(6bc-5ad)x(a+bx^2)^{3/2}}{24c^2(bc-ad)(c+dx^2)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)(c+dx^2)}+\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{c}\cdots}\right)}{16c^{7/2}(bc-ad)^{3\cdots}}$$

```
[Out] -1/6*d*x*(b*x^2+a)^(5/2)/c/(-a*d+b*c)/(d*x^2+c)^3+1/24*(-5*a*d+6*b*c)*x*(b*
x^2+a)^(3/2)/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/16*a^2*(-5*a*d+6*b*c)*arctanh(x*(
-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)+1/16*a*(-
5*a*d+6*b*c)*x*(b*x^2+a)^(1/2)/c^3/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A]
time = 0.08, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {390, 386, 385, 214}

$$\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3(c+dx^2)(bc-ad)}+\frac{x(a+bx^2)^{3/2}(6bc-5ad)}{24c^2(c+dx^2)^2(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c(c+dx^2)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] -1/6*(d*x*(a + b*x^2)^(5/2))/(c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*c - 5*a*
d)*x*(a + b*x^2)^(3/2))/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + (a*(6*b*c - 5*
a*d)*x*Sqrt[a + b*x^2])/(16*c^3*(b*c - a*d)*(c + d*x^2)) + (a^2*(6*b*c - 5*
a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b
*c - a*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^4}\,dx = -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx}{6c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{(a(6bc-5ad))\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{8c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)}+\frac{(a^2(}{}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)}+\frac{(a^2(}{}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)}+\frac{a^2(6}{}$$

Mathematica [A]
time = 10.61, size = 247, normalized size = 1.24

$$\frac{ax\left(1+\frac{bx^2}{a}\right)\left(c\left(-4b^3c^2x^4\left(3c+dx^2\right)-2ab^2cx^2\left(21c^2+13cdx^2+4d^2x^4\right)+a^3d\left(33c^2+40cdx^2+15d^2x^4\right)+a^2b\left(-30c^3+11c^2dx^2+32cd^2x^4+15d^3x^6\right)\right)+\frac{3a^2(-6bc+5ad)\left(c+dx^2\right)^3\tanh^{-1}\left(\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}}\right)}{48c^4(-bc+ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(-4*b^3*c^2*x^4*(3*c + d*x^2) - 2*a*b^2*c*x^2*(21*c
^2 + 13*c*d*x^2 + 4*d^2*x^4) + a^3*d*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4) + a
^2*b*(-30*c^3 + 11*c^2*d*x^2 + 32*c*d^2*x^4 + 15*d^3*x^6)) + (3*a^2*(-6*b*c
 + 5*a*d)*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/S
qrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(48*c^4*(-(b*c) + a*d)*(a + b*x^2)
^(3/2)*(c + d*x^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 12815 vs. $2(175) = 350$.
time = 0.07, size = 12816, normalized size = 64.40

method	result	size
default	Expression too large to display	12816

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(175) = 350$.
time = 0.80, size = 972, normalized size = 4.88

$$\left[\frac{3\,{\left(6\,a^{2} b c^{4} - 5\,a^{3} c^{3} d + {\left(6\,a^{2} b c d^{3} - 5\,a^{3} d^{4}\right)} x^{6} + 3\,{\left(6\,a^{2} b c^{2} d^{2} - 5\,a^{3} c d^{3}\right)} x^{4} + 3\,{\left(6\,a^{2} b c^{3} d - 5\,a^{3} c^{2} d^{2}\right)} x^{2}\right)} \sqrt{b c^{2} - a c d} \log\left(\frac{{\left(8\,b^{2} c^{2} - 8\,a b c d + a^{2} d^{2}\right)} x^{4} + a^{2} c^{2} + 2\,{\left(4\,a b c^{2} - 3\,a^{2} c d\right)} x^{2} + 4\,{\left({\left(2\,b c - a d\right)} x^{3} + a c x\right)} \sqrt{b c^{2} - a c d} \sqrt{b x^{2} + a}}{d^{2} x^{4} + 2\,c d x^{2} + c^{2}}\right) + 4\,{\left({\left(4\,b^{3} c^{4} d + 4\,a b^{2} c^{3} d^{2} - 23\,a^{2} b c^{2} d^{3} + 15\,a^{3} c d^{4}\right)} x^{5} + 2\,{\left(6\,b^{3} c^{5} + 5\,a b^{2} c^{4} d - 31\,a^{2} b c^{3} d^{2} + 20\,a^{3} c^{2} d^{3}\right)} x^{3} + 3\,{\left(10\,a b^{2} c^{5} - 21\,a^{2} b c^{4} d + 11\,a^{3} c^{3} d^{2}\right)} x\right)} \sqrt{b x^{2} + a}}{192\,{\left(b^{2} c^{9} - 2\,a b c^{8} d + a^{2} c^{7} d^{2} + {\left(b^{2} c^{6} d^{3} - 2\,a b c^{5} d^{4} + a^{2} c^{4} d^{5}\right)} x^{6} + 3\,{\left(b^{2} c^{7} d^{2} - 2\,a b c^{6} d^{3} + a^{2} c^{5} d^{4}\right)} x^{4} + 3\,{\left(b^{2} c^{8} d - 2\,a b c^{7} d^{2} + a^{2} c^{6} d^{3}\right)} x^{2}\right)}}, -\frac{3\,{\left(6\,a^{2} b c^{4} - 5\,a^{3} c^{3} d + {\left(6\,a^{2} b c d^{3} - 5\,a^{3} d^{4}\right)} x^{6} + 3\,{\left(6\,a^{2} b c^{2} d^{2} - 5\,a^{3} c d^{3}\right)} x^{4} + 3\,{\left(6\,a^{2} b c^{3} d - 5\,a^{3} c^{2} d^{2}\right)} x^{2}\right)} \sqrt{-b c^{2} + a c d} \arctan\left(\frac{\sqrt{-b c^{2} + a c d} {\left({\left(2\,b c - a d\right)} x^{3} + a c x\right)} \sqrt{b x^{2} + a}}{2\,{\left({\left(b^{2} c^{2} - a b c d\right)} x^{4} + {\left(a b c^{2} - a^{2} c d\right)} x^{2}\right)}}\right) - 2\,{\left({\left(4\,b^{3} c^{4} d + 4\,a b^{2} c^{3} d^{2} - 23\,a^{2} b c^{2} d^{3} + 15\,a^{3} c d^{4}\right)} x^{5} + 2\,{\left(6\,b^{3} c^{5} + 5\,a b^{2} c^{4} d - 31\,a^{2} b c^{3} d^{2} + 20\,a^{3} c^{2} d^{3}\right)} x^{3} + 3\,{\left(10\,a b^{2} c^{5} - 21\,a^{2} b c^{4} d + 11\,a^{3} c^{3} d^{2}\right)} x\right)} \sqrt{b x^{2} + a}}{96\,{\left(b^{2} c^{9} - 2\,a b c^{8} d + a^{2} c^{7} d^{2} + {\left(b^{2} c^{6} d^{3} - 2\,a b c^{5} d^{4} + a^{2} c^{4} d^{5}\right)} x^{6} + 3\,{\left(b^{2} c^{7} d^{2} - 2\,a b c^{6} d^{3} + a^{2} c^{5} d^{4}\right)} x^{4} + 3\,{\left(b^{2} c^{8} d - 2\,a b c^{7} d^{2} + a^{2} c^{6} d^{3}\right)} x^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^3*d^4)*x^6 + 3*
(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a^3*c^2*d^2)*x^2
)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
+ 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
- a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((4*b^3*c^4*d +
4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a
*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*
a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^9 - 2*a*b*c^8*d +
```

```
a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*
d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2
*c^6*d^3)*x^2), -1/96*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^
3*d^4)*x^6 + 3*(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a
^3*c^2*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b
*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) - 2*((4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^
3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2
*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^
2 + a))/(b^2*c^9 - 2*a*b*c^8*d + a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4
 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3
*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2*c^6*d^3)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(175) = 350.
time = 1.65, size = 919, normalized size = 4.62

$$-\frac{\left(6a^2b^{\frac{3}{2}}c-5a^3\sqrt{b}\,d\right)\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{16\left(bc^4-ac^3d\right)\sqrt{-b^2c^2+abcd}}-\frac{18\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{3}{2}}cd^4-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}\,d^5-\cdots}{\cdots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/16*(6*a^2*b^(3/2)*c - 5*a^3*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x
^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^4 - a*c^3*d)*sq
rt(-b^2*c^2 + a*b*c*d)) - 1/24*(18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(
3/2)*c*d^4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^5 - 96*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*d + 96*(sqrt(b)*x - sqrt(b*x^2 + a))
^8*a*b^(7/2)*c^3*d^2 + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*
d^3 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d^5 - 128*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*b^(11/2)*c^5 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 720*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 - 968*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^3*b^(5/2)*c^2*d^3 + 620*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*
b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 96*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d - 288*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^3*b^(7/2)*c^3*d^2 + 864*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^
```

```
(5/2)*c^2*d^3 - 600*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 48*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^4*b^(7/2)*c^3*d^2 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
5/2)*c^2*d^3 + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 - 4*a^6*b^(5/2)*c^2*d^3 - 8*
a^7*b^(3/2)*c*d^4 + 15*a^8*sqrt(b)*d^5)/((b*c^4*d^2 - a*c^3*d^3)*((sqrt(b)*
x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(d\,x^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^4,x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^4, x)
```

3.61 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5} \, dx$

Optimal. Leaf size=300

$$-\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}+\frac{\left(16b^3c^3+40ab^2c\right.}{38}$$

```
[Out] 1/128*a^2*(35*a^2*d^2-80*a*b*c*d+48*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(
1/2)/(b*x^2+a)^(1/2))/c^(9/2)/(-a*d+b*c)^(5/2)-1/8*(-a*d+b*c)*x*(b*x^2+a)^(
1/2)/c/d/(d*x^2+c)^4+1/48*(7*a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/d/(d*x^2+c)^3
+1/192*(-35*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/c^3/d/(-a*d+b*c
)/(d*x^2+c)^2+1/384*(105*a^3*d^3-170*a^2*b*c*d^2+40*a*b^2*c^2*d+16*b^3*c^3)
*x*(b*x^2+a)^(1/2)/c^4/d/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A]
time = 0.25, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 541, 12, 385, 214}

$$\frac{a^2\left(35a^2d^2-80abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{192c^3d\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{384c^4d\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{48c^2d\left(c+dx^2\right)^3}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] -1/8*((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)^4) + ((2*b*c + 7*a*d)
*x*Sqrt[a + b*x^2])/(48*c^2*d*(c + d*x^2)^3) + ((8*b^2*c^2 + 24*a*b*c*d - 3
5*a^2*d^2)*x*Sqrt[a + b*x^2])/(192*c^3*d*(b*c - a*d)*(c + d*x^2)^2) + ((16*
b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2]
)/(384*c^4*d*(b*c - a*d)^2*(c + d*x^2)) + (a^2*(48*b^2*c^2 - 80*a*b*c*d + 3
5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(
9/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} + \frac{\int \frac{a(bc+7ad)+2b(bc+3ad)x^2}{\sqrt{a+bx^2}\,(c+dx^2)^4}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{\int \frac{a(bc-ad)(4bc+35ad)+4b(bc-ad)(2bc+7ad)}{\sqrt{a+bx^2}\,(c+dx^2)^3}}{48c^2d(bc-ad)}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+}}{192c^3d(bc-ad)\,(c+dx^2)^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+}}{192c^3d(bc-ad)\,(c+dx^2)^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+}}{192c^3d(bc-ad)\,(c+dx^2)^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+}}{192c^3d(bc-ad)\,(c+dx^2)^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+}}{192c^3d(bc-ad)\,(c+dx^2)^2}$$

Mathematica [A]
time = 11.02, size = 362, normalized size = 1.21

$$\frac{ax\left(1+\frac{bx^2}{a}\right)\left(c\left(16b^4c^3x^4(6c^2+4cdx^2+d^2x^4)+8ab^3c^2x^2(42c^3+34c^2dx^2+21cd^2x^4+5d^3x^6)+a^4d^2(279c^3+511c^2dx^2+385cd^2x^4+105d^3x^6)+2a^2b^2c(120c^4-160c^3dx^2-345c^2d^2x^4-294cd^3x^6-85d^4x^8)+a^3bd(-528c^4-563c^3dx^2-117c^2d^2x^4+215cd^3x^6+105d^4x^8)\right)+\frac{3a^2(48b^2c^2-80abcd+35a^2d^2)(c+dx^2)^4\tanh^{-1}\left(\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}\right)}{\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}}\right)}{384c^5(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(16*b^4*c^3*x^4*(6*c^2 + 4*c*d*x^2 + d^2*x^4) + 8*a
*b^3*c^2*x^2*(42*c^3 + 34*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6) + a^4*d^2*(
279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6) + 2*a^2*b^2*c*(120*c
^4 - 160*c^3*d*x^2 - 345*c^2*d^2*x^4 - 294*c*d^3*x^6 - 85*d^4*x^8) + a^3*b*
d*(-528*c^4 - 563*c^3*d*x^2 - 117*c^2*d^2*x^4 + 215*c*d^3*x^6 + 105*d^4*x^8
)) + (3*a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*(c + d*x^2)^4*ArcTanh[Sq
rt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^
2))]))/(384*c^5*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 22501 vs. 2(272) = 544.
time = 0.09, size = 22502, normalized size = 75.01

method	result	size
default	Expression too large to display	22502

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(272) = 544$.
time = 1.76, size = 1604, normalized size = 5.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*
c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3
*b*c^2*d^4 + 35*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 +
 35*a^4*c^2*d^4)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*
d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 +
a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4
*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 - 105
*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3 +
1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*d -
 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(80*a
*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sqrt(
b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3 + (
b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(b^3
*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3*c^
10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c^11
*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2), -1/768*(3*(48*
```

```
a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*c^2*d^4 - 80*a^
3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3*b*c^2*d^4 + 35
*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4
)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*d^3)*x^2)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*
b^4*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 -
105*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3
 + 1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*
d - 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(8
0*a*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sq
rt(b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3
+ (b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(
b^3*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3
*c^10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c
^11*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1557 vs. $2(272) = 544$.
time = 5.11, size = 1557, normalized size = 5.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/128*(48*a^2*b^(5/2)*c^2 - 80*a^3*b^(3/2)*c*d + 35*a^4*sqrt(b)*d^2)*arcta
n(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b
*c*d))/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1
/192*(144*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*b^(5/2)*c^2*d^5 - 240*(sqrt(
b)*x - sqrt(b*x^2 + a))^14*a^3*b^(3/2)*c*d^6 + 105*(sqrt(b)*x - sqrt(b*x^2
+ a))^14*a^4*sqrt(b)*d^7 + 2016*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2
)*c^3*d^4 - 4368*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(5/2)*c^2*d^5 + 315
0*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6 - 735*(sqrt(b)*x - sqr
t(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^
(13/2)*c^6*d + 4096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(11/2)*c^5*d^2 + 7
```

```
936*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)*c^4*d^3 - 26624*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3*d^4 + 26944*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^4*b^(5/2)*c^2*d^5 - 12320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b
^(3/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/2)*c^7 - 1024*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a*b^(13/2)*c^6*d + 27392*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(1
1/2)*c^5*d^2 - 65920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 +
81680*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(7/2)*c^3*d^4 - 58840*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d^5 + 22750*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^6*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b
)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(13/2)*c^6*d - 8192*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(11/2)*c^5*d^2 + 47104*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*a^4*b^(9/2)*c^4*d^3 - 74240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a
^5*b^(7/2)*c^3*d^4 + 56416*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*
d^5 - 22400*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 + 3675*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*sqrt(b)*d^7 - 1536*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*a^4*b^(11/2)*c^5*d^2 - 2304*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/
2)*c^4*d^3 + 17696*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 - 23
152*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(5/2)*c^2*d^5 + 11690*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^8*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*a^9*sqrt(b)*d^7 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^
3 - 512*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*b^(7/2)*c^3*d^4 + 2896*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*a^8*b^(5/2)*c^2*d^5 - 2800*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^9*b^(3/2)*c*d^6 + 735*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b
)*d^7 - 16*a^8*b^(7/2)*c^3*d^4 - 40*a^9*b^(5/2)*c^2*d^5 + 170*a^10*b^(3/2)*
c*d^6 - 105*a^11*sqrt(b)*d^7)/((b^2*c^6*d^2 - 2*a*b*c^5*d^3 + a^2*c^4*d^4)*
((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c
- 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(d\,x^2+c\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^5,x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^5, x)
```

3.62 $\int (a+bx^2)^{5/2} (c+dx^2)^3 \, dx$

Optimal. Leaf size=349

$$\frac{a^2(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}}{1024b^3}+\frac{a(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x(a+$$

```
[Out] 1/1536*a*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*x*(b*x^2+a
)^(3/2)/b^3+1/1920*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*
x*(b*x^2+a)^(5/2)/b^3+1/960*d*(15*a^2*d^2-68*a*b*c*d+152*b^2*c^2)*x*(b*x^2+
a)^(7/2)/b^3+1/120*d*(-5*a*d+16*b*c)*x*(b*x^2+a)^(7/2)*(d*x^2+c)/b^2+1/12*d
*x*(b*x^2+a)^(7/2)*(d*x^2+c)^2/b+1/1024*a^3*(-5*a^3*d^3+36*a^2*b*c*d^2-120*
a*b^2*c^2*d+320*b^3*c^3)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/1024*
a^2*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*x*(b*x^2+a)^(1/
2)/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {427, 542, 396, 201, 223, 212}

$$\frac{dx(a+bx^2)^{7/2}(15a^2d^2-68abcd+152b^2c^2)}{960b^3}+\frac{x(a+bx^2)^{5/2}(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3)}{1920b^3}+\frac{ax(a+bx^2)^{3/2}(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3)}{1536b^3}+\frac{a^2x\sqrt{a+bx^2}(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3)}{1024b^3}+\frac{a^3(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}+\frac{dx(a+bx^2)^{7/2}(c+dx^2)(16bc-5ad)}{120b^2}+\frac{dx(a+bx^2)^{7/2}(c+dx^2)^2}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(1024*b^3) + (a*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 -
 5*a^3*d^3)*x*(a + b*x^2)^(3/2))/(1536*b^3) + ((320*b^3*c^3 - 120*a*b^2*c^2
*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*(a + b*x^2)^(5/2))/(1920*b^3) + (d*(152*
b^2*c^2 - 68*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(7/2))/(960*b^3) + (d*(16*
b*c - 5*a*d)*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(120*b^2) + (d*x*(a + b*x^2)^
(7/2)*(c + d*x^2)^2)/(12*b) + (a^3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*
b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx = \frac{dx(a+bx^2)^{7/2}\left(c+dx^2\right)^2}{12b} + \frac{\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\left(c(12bc-ad)+d(16bc\right.}{12b}$$

$$= \frac{d(16bc-5ad)x(a+bx^2)^{7/2}\left(c+dx^2\right)}{120b^2} + \frac{dx(a+bx^2)^{7/2}\left(c+dx^2\right)^2}{12b} + \frac{\int (a+}{}$$

$$= \frac{d\left(152b^2c^2-68abcd+15a^2d^2\right)x(a+bx^2)^{7/2}}{960b^3} + \frac{d(16bc-5ad)x(a+bx^2)^{7/2}(}{120b^2}$$

$$= \frac{\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x(a+bx^2)^{5/2}}{1920b^3} + \frac{d(152b^2c^2-68ab}{}$$

$$= \frac{a\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x(a+bx^2)^{3/2}}{1536b^3} + \frac{(320b^3c^3-120a}{}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a(320b^3c^3-120}{}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a(320b^3c^3-120}{}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a(320b^3c^3-120}{}$$

Mathematica [A]
time = 0.47, size = 269, normalized size = 0.77

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(75a^5d^3-10a^4bd^2(54c+5dx^2)+40a^3b^2d(45c^2+9cdx^2+d^2x^4)+128b^5x^4(20c^3+45c^2dx^2+36cd^2x^4+10d^3x^6)+48a^2b^3(220c^3+295c^2dx^2+186cd^2x^4+45d^3x^6)+64ab^4x^2(130c^3+255c^2dx^2+189cd^2x^4+50d^3x^6)\right)+15a^3\left(-320b^3c^3+120ab^2c^2d-36a^2bcd^2+5a^3d^3\right)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{15360b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*d^3 - 10*a^4*b*d^2*(54*c + 5*d*x^2) + 40
*a^3*b^2*d*(45*c^2 + 9*c*d*x^2 + d^2*x^4) + 128*b^5*x^4*(20*c^3 + 45*c^2*d*
x^2 + 36*c*d^2*x^4 + 10*d^3*x^6) + 48*a^2*b^3*(220*c^3 + 295*c^2*d*x^2 + 18
6*c*d^2*x^4 + 45*d^3*x^6) + 64*a*b^4*x^2*(130*c^3 + 255*c^2*d*x^2 + 189*c*d
^2*x^4 + 50*d^3*x^6)) + 15*a^3*(-320*b^3*c^3 + 120*a*b^2*c^2*d - 36*a^2*b*c
*d^2 + 5*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(7/2))
```

Maple [A]
time = 0.07, size = 428, normalized size = 1.23

method	result

risch	$\frac{x(1280b^5d^3x^{10}+3200a\,b^4d^3x^8+4608b^5c\,d^2x^8+2160a^2b^3d^3x^6+12096a\,b^4c\,d^2x^6+5760b^5c^2d\,x^6+40a^3b^2d^3x^4+8928a^2b^3c\,d^2x^4+16320}{}$

default	d^3	$\left(\frac{x^5 (b x^2+a)^{\frac{7}{2}}}{12b} - \frac{5a\left(\frac{x^3 (b x^2+a)^{\frac{7}{2}}}{10b} - \frac{3a\left(\frac{x (b x^2+a)^{\frac{7}{2}}}{8b} - \frac{a\left(\frac{x (b x^2+a)^{\frac{5}{2}}}{6} + \frac{5a\left(\frac{x (b x^2+a)^{\frac{3}{2}}}{4} + \frac{3a\left(\frac{x\sqrt{b x^2+a}}{2} + \frac{a \ln\left(x\sqrt{b} + \sqrt{\right.}}{2\sqrt{}}\right.}{4}\right.}{6}\right.}{8b}\right.}{10b}\right.}{12b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/12*x^5*(b*x^2+a)^(7/2)/b-5/12*a/b*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a
/b*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*
x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))))))))+3*c*d^2*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2
+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4
*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+
3*c^2*d*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*
x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))))))+c^3*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)
+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
)
```

Maxima [A]
time = 0.33, size = 447, normalized size = 1.28

$$\frac{(bx^2+a)^{\frac{7}{2}}d^3x^5}{12\,b}+\frac{3\,(bx^2+a)^{\frac{7}{2}}cd^2x^3}{10\,b}-\frac{(bx^2+a)^{\frac{7}{2}}ad^3x^3}{24\,b^2}+\frac{1}{6}\,(bx^2+a)^{\frac{5}{2}}c^3x+\frac{5}{24}\,(bx^2+a)^{\frac{3}{2}}ac^3x+\frac{5}{16}\,\sqrt{bx^2+a}\,a^2c^3x+\frac{3\,(bx^2+a)^{\frac{7}{2}}c^2dx}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}ac^2dx}{16\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}a^2c^2dx}{64\,b}-\frac{15\,\sqrt{bx^2+a}\,a^3c^2dx}{128\,b}-\frac{9\,(bx^2+a)^{\frac{7}{2}}acd^2x}{80\,b^2}+\frac{3\,(bx^2+a)^{\frac{5}{2}}a^2cd^2x}{160\,b^2}+\frac{3\,(bx^2+a)^{\frac{3}{2}}a^3cd^2x}{128\,b^2}+\frac{9\,\sqrt{bx^2+a}\,a^4cd^2x}{256\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}a^2d^3x}{64\,b^3}-\frac{(bx^2+a)^{\frac{5}{2}}a^3d^3x}{384\,b^3}-\frac{5\,(bx^2+a)^{\frac{3}{2}}a^4d^3x}{1536\,b^3}-\frac{5\,\sqrt{bx^2+a}\,a^5d^3x}{1024\,b^3}+\frac{5\,a^3c^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}-\frac{15\,a^4c^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{9\,a^5cd^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}-\frac{5\,a^6d^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(7/2)*d^3*x^5/b + 3/10*(b*x^2 + a)^(7/2)*c*d^2*x^3/b - 1/2
4*(b*x^2 + a)^(7/2)*a*d^3*x^3/b^2 + 1/6*(b*x^2 + a)^(5/2)*c^3*x + 5/24*(b*x
^2 + a)^(3/2)*a*c^3*x + 5/16*sqrt(b*x^2 + a)*a^2*c^3*x + 3/8*(b*x^2 + a)^(7
/2)*c^2*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*c^2*d*x/b - 5/64*(b*x^2 + a)^(3/2)
*a^2*c^2*d*x/b - 15/128*sqrt(b*x^2 + a)*a^3*c^2*d*x/b - 9/80*(b*x^2 + a)^(7
/2)*a*c*d^2*x/b^2 + 3/160*(b*x^2 + a)^(5/2)*a^2*c*d^2*x/b^2 + 3/128*(b*x^2
+ a)^(3/2)*a^3*c*d^2*x/b^2 + 9/256*sqrt(b*x^2 + a)*a^4*c*d^2*x/b^2 + 1/64*(
b*x^2 + a)^(7/2)*a^2*d^3*x/b^3 - 1/384*(b*x^2 + a)^(5/2)*a^3*d^3*x/b^3 - 5/
1536*(b*x^2 + a)^(3/2)*a^4*d^3*x/b^3 - 5/1024*sqrt(b*x^2 + a)*a^5*d^3*x/b^3
 + 5/16*a^3*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 15/128*a^4*c^2*d*arcsinh(b
*x/sqrt(a*b))/b^(3/2) + 9/256*a^5*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/
1024*a^6*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Fricas [A]
time = 0.83, size = 608, normalized size = 1.74

$$\left[-\frac{15\,(320\,a^3b^3c^3-120\,a^4b^2c^2d+36\,a^5bcd^2-5\,a^6d^3)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(1280\,b^6d^3x^{11}+128\,(36\,b^6cd^2+25\,ab^5d^3)x^9+144\,(40\,b^6c^2d+84\,ab^5cd^2+15\,a^2b^4d^3)x^7+8\,(320\,b^6c^3+2040\,ab^5c^2d+1116\,a^2b^4cd^2+5\,a^3b^3d^3)x^5+10\,(832\,ab^5c^3+1416\,a^2b^4c^2d+36\,a^3b^3cd^2-5\,a^4b^2d^3)x^3+15\,(704\,a^2b^4c^3+120\,a^3b^3c^2d-36\,a^4b^2cd^2+5\,a^5bd^3)x)\sqrt{bx^2+a}}{30720\,b^4},\ -\frac{15\,(320\,a^3b^3c^3-120\,a^4b^2c^2d+36\,a^5bcd^2-5\,a^6d^3)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(1280\,b^6d^3x^{11}+128\,(36\,b^6cd^2+25\,ab^5d^3)x^9+144\,(40\,b^6c^2d+84\,ab^5cd^2+15\,a^2b^4d^3)x^7+8\,(320\,b^6c^3+2040\,ab^5c^2d+1116\,a^2b^4cd^2+5\,a^3b^3d^3)x^5+10\,(832\,ab^5c^3+1416\,a^2b^4c^2d+36\,a^3b^3cd^2-5\,a^4b^2d^3)x^3+15\,(704\,a^2b^4c^3+120\,a^3b^3c^2d-36\,a^4b^2cd^2+5\,a^5bd^3)x)\sqrt{bx^2+a}}{15360\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6
*d^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1280*b^6
*d^3*x^11 + 128*(36*b^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*
a*b^5*c*d^2 + 15*a^2*b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 111
6*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2
*d + 36*a^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*
b^3*c^2*d - 36*a^4*b^2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/153
60*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*s
qrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280*b^6*d^3*x^11 + 128*(36*b
^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*a*b^5*c*d^2 + 15*a^2*
b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 1116*a^2*b^4*c*d^2 + 5*a
^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2*d + 36*a^3*b^3*c*d^2
 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*b^3*c^2*d - 36*a^4*b^
2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.73, size = 321, normalized size = 0.92

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^{2}d^{3}x^{2}+\frac{36\,b^{12}cd^{2}+25\,ab^{11}d^{3}}{b^{10}}\right)x^{2}+\frac{9\,(40\,b^{12}c^{2}d+84\,ab^{11}cd^{2}+15\,a^{2}b^{10}d^{3})}{b^{10}}\right)x^{2}+\frac{320\,b^{12}c^{3}+2040\,ab^{11}c^{2}d+1116\,a^{2}b^{10}cd^{2}+5\,a^{3}b^{9}d^{3}}{b^{10}}\right)x^{2}+\frac{5\,(832\,ab^{11}c^{3}+1416\,a^{2}b^{10}c^{2}d+36\,a^{3}b^{9}cd^{2}-5\,a^{4}b^{8}d^{3})}{b^{10}}\right)x^{2}+\frac{15\,(704\,a^{2}b^{10}c^{3}+120\,a^{3}b^{9}c^{2}d-36\,a^{4}b^{8}cd^{2}+5\,a^{5}b^{7}d^{3})}{b^{10}}\right)\sqrt{bx^{2}+a}\,x-\frac{(320\,a^{3}b^{3}c^{3}-120\,a^{4}b^{2}c^{2}d+36\,a^{5}bcd^{2}-5\,a^{6}d^{3})\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{1024\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^3*x^2 + (36*b^12*c*d^2 + 25*a*b^11*d^3)/b^10)
*x^2 + 9*(40*b^12*c^2*d + 84*a*b^11*c*d^2 + 15*a^2*b^10*d^3)/b^10)*x^2 + (3
20*b^12*c^3 + 2040*a*b^11*c^2*d + 1116*a^2*b^10*c*d^2 + 5*a^3*b^9*d^3)/b^10
)*x^2 + 5*(832*a*b^11*c^3 + 1416*a^2*b^10*c^2*d + 36*a^3*b^9*c*d^2 - 5*a^4*
b^8*d^3)/b^10)*x^2 + 15*(704*a^2*b^10*c^3 + 120*a^3*b^9*c^2*d - 36*a^4*b^8*
c*d^2 + 5*a^5*b^7*d^3)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(320*a^3*b^3*c^3 -
120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(7/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^2+a\right)}^{5/2}\,{\left(d\,x^2+c\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2)^3, x)
```

3.63 $\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx$

Optimal. Leaf size=241

$$\frac{a^2(80b^2c^2-20abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{256b^2}+\frac{a(80b^2c^2-20abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{384b^2}+\frac{(80b^2c^2-20abcd+3a}{480b}$$

```
[Out] 1/384*a*(3*a^2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^2+1/480*(3*a^
2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(5/2)/b^2+3/80*d*(-a*d+4*b*c)*x*(b
*x^2+a)^(7/2)/b^2+1/10*d*x*(b*x^2+a)^(7/2)*(d*x^2+c)/b+1/256*a^3*(3*a^2*d^2
-20*a*b*c*d+80*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/256*a^
2*(3*a^2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.10, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {427, 396, 201, 223, 212}

$$\frac{x(a+bx^2)^{5/2}(3a^2d^2-20abcd+80b^2c^2)}{480b^2}+\frac{ax(a+bx^2)^{3/2}(3a^2d^2-20abcd+80b^2c^2)}{384b^2}+\frac{a^2x\sqrt{a+bx^2}(3a^2d^2-20abcd+80b^2c^2)}{256b^2}+\frac{a^3(3a^2d^2-20abcd+80b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3dx(a+bx^2)^{7/2}(4bc-ad)}{80b^2}+\frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(256*b^2) + (
a*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(384*b^2) + ((
80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(5/2))/(480*b^2) + (3*d*
(4*b*c - a*d)*x*(a + b*x^2)^(7/2))/(80*b^2) + (d*x*(a + b*x^2)^(7/2)*(c + d
*x^2))/(10*b) + (a^3*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx = \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}+\frac{\int \left(a+bx^2\right)^{5/2}\left(c(10bc-ad)+3d(4bc-ad)x^2\right)}{10b}$$

$$= \frac{3d(4bc-ad)x\left(a+bx^2\right)^{7/2}}{80b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}-\frac{(3ad(4bc-ad)-}{}$$

$$= \frac{\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{5/2}}{480b^2}+\frac{3d(4bc-ad)x\left(a+bx^2\right)^{7/2}}{80b^2}+$$

$$= \frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{384b^2}+\frac{\left(80b^2c^2-20abcd+3a^2d^2\right)x(}{480b^2}$$

$$= \frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x(}{384b^2}$$

$$= \frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x(}{384b^2}$$

$$= \frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x(}{384b^2}$$

Mathematica [A]

time = 0.33, size = 190, normalized size = 0.79

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\,(-45a^4d^2+30a^3bd(10c+dx^2)+64b^4x^4(10c^2+15cdx^2+6d^2x^4)+16ab^3x^2(130c^2+170cdx^2+63d^2x^4)+8a^2b^2(330c^2+295cdx^2+93d^2x^4))-15a^3(80b^2c^2-20abcd+3a^2d^2)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{3840b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-45*a^4*d^2 + 30*a^3*b*d*(10*c + d*x^2) + 64*b^
4*x^4*(10*c^2 + 15*c*d*x^2 + 6*d^2*x^4) + 16*a*b^3*x^2*(130*c^2 + 170*c*d*x
^2 + 63*d^2*x^4) + 8*a^2*b^2*(330*c^2 + 295*c*d*x^2 + 93*d^2*x^4)) - 15*a^3
*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/(3840*b^(5/2))
```

Maple [A]
time = 0.06, size = 283, normalized size = 1.17

method	result
risch	$-\frac{x\left(-384b^4d^2x^8-1008a\,b^3d^2x^6-960b^4cd\,x^6-744a^2b^2d^2x^4-2720a\,b^3cd\,x^4-640b^4c^2x^4-30a^3b\,d^2x^2-2360a^2b^2cd\,x^2-2080a\,b^3c^2x^2+\right.}{3840b^2}$

default	d^2	$\left(\frac{x^3(bx^2+a)^{\frac{7}{2}}}{10b} - \frac{3a\left(\frac{x(bx^2+a)^{\frac{7}{2}}}{8b} - \frac{a\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6} + \frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4} + \frac{3a\left(\frac{x\sqrt{bx^2+a}}{2} + \frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)}{10b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(
1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(
1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+2*c*d*(1/8*x*(b*x^2+a
)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a
*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))+c^2
*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)
^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.31, size = 286, normalized size = 1.19

$$\frac{(bx^2+a)^{\frac{7}{2}}d^2x^3}{10\,b}+\frac{1}{6}\,(bx^2+a)^{\frac{5}{2}}c^2x+\frac{5}{24}\,(bx^2+a)^{\frac{3}{2}}ac^2x+\frac{5}{16}\,\sqrt{bx^2+a}\,a^2c^2x+\frac{(bx^2+a)^{\frac{7}{2}}cdx}{4\,b}-\frac{(bx^2+a)^{\frac{5}{2}}acdx}{24\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}a^2cdx}{96\,b}-\frac{5\,\sqrt{bx^2+a}\,a^3cdx}{64\,b}-\frac{3\,(bx^2+a)^{\frac{7}{2}}ad^2x}{80\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}a^2d^2x}{160\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}a^3d^2x}{128\,b^2}+\frac{3\,\sqrt{bx^2+a}\,a^4d^2x}{256\,b^2}+\frac{5\,a^3c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}-\frac{5\,a^4cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{64\,b^{\frac{3}{2}}}+\frac{3\,a^5d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*d^2*x^3/b + 1/6*(b*x^2 + a)^(5/2)*c^2*x + 5/24*(b*x^
2 + a)^(3/2)*a*c^2*x + 5/16*sqrt(b*x^2 + a)*a^2*c^2*x + 1/4*(b*x^2 + a)^(7/
2)*c*d*x/b - 1/24*(b*x^2 + a)^(5/2)*a*c*d*x/b - 5/96*(b*x^2 + a)^(3/2)*a^2*
c*d*x/b - 5/64*sqrt(b*x^2 + a)*a^3*c*d*x/b - 3/80*(b*x^2 + a)^(7/2)*a*d^2*x
/b^2 + 1/160*(b*x^2 + a)^(5/2)*a^2*d^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*
d^2*x/b^2 + 3/256*sqrt(b*x^2 + a)*a^4*d^2*x/b^2 + 5/16*a^3*c^2*arcsinh(b*x/
sqrt(a*b))/sqrt(b) - 5/64*a^4*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/256*a^
5*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.56, size = 420, normalized size = 1.74

$$\left[\frac{15\,(80\,a^3b^2c^2-20\,a^4bcd+3\,a^5d^2)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(384\,b^5d^2x^9+48\,(20\,b^5cd+21\,ab^4d^2)x^7+8\,(80\,b^5c^2+340\,ab^4cd+93\,a^2b^3d^2)x^5+10\,(208\,ab^4c^2+236\,a^2b^3cd+3\,a^3b^2d^2)x^3+15\,(176\,a^2b^3c^2+20\,a^3b^2cd-3\,a^4bd^2)x)\sqrt{bx^2+a}}{7680\,b^3},\,-\frac{15\,(80\,a^3b^2c^2-20\,a^4bcd+3\,a^5d^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(384\,b^5d^2x^9+48\,(20\,b^5cd+21\,ab^4d^2)x^7+8\,(80\,b^5c^2+340\,ab^4cd+93\,a^2b^3d^2)x^5+10\,(208\,ab^4c^2+236\,a^2b^3cd+3\,a^3b^2d^2)x^3+15\,(176\,a^2b^3c^2+20\,a^3b^2cd-3\,a^4bd^2)x)\sqrt{bx^2+a}}{3840\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5*d^2*x^9 + 48*(20*b^5*c*d
+ 21*a*b^4*d^2)*x^7 + 8*(80*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 +
 10*(208*a*b^4*c^2 + 236*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3
*c^2 + 20*a^3*b^2*c*d - 3*a^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(
80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - (384*b^5*d^2*x^9 + 48*(20*b^5*c*d + 21*a*b^4*d^2)*x^7 + 8*(80
*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 + 10*(208*a*b^4*c^2 + 236*a^
2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3*c^2 + 20*a^3*b^2*c*d - 3*a
^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 221, normalized size = 0.92

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^{2}d^{2}x^{2}+\frac{20\,b^{10}cd+21\,ab^{9}d^{2}}{b^{8}}\right)x^{2}+\frac{80\,b^{10}c^{2}+340\,ab^{9}cd+93\,a^{2}b^{8}d^{2}}{b^{8}}\right)x^{2}+\frac{5\,(208\,ab^{9}c^{2}+236\,a^{2}b^{8}cd+3\,a^{3}b^{7}d^{2})}{b^{8}}\right)x^{2}+\frac{15\,(176\,a^{2}b^{8}c^{2}+20\,a^{3}b^{7}cd-3\,a^{4}b^{6}d^{2})}{b^{8}}\right)\sqrt{bx^{2}+a}\,x-\frac{(80\,a^{3}b^{2}c^{2}-20\,a^{4}bcd+3\,a^{5}d^{2})\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (20*b^10*c*d + 21*a*b^9*d^2)/b^8)*x^2 + (8
0*b^10*c^2 + 340*a*b^9*c*d + 93*a^2*b^8*d^2)/b^8)*x^2 + 5*(208*a*b^9*c^2 +
236*a^2*b^8*c*d + 3*a^3*b^7*d^2)/b^8)*x^2 + 15*(176*a^2*b^8*c^2 + 20*a^3*b^
7*c*d - 3*a^4*b^6*d^2)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(80*a^3*b^2*c^2 - 20*
a^4*b*c*d + 3*a^5*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^2+a\right)}^{5/2}\,{\left(d\,x^2+c\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2)^2, x)
```

3.64 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) dx$

Optimal. Leaf size=149

$$\frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b}+\frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b}+\frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b}+\frac{dx(a+bx^2)^{7/2}}{8b}+\frac{5a^3(8bc-$$

```
[Out] 5/192*a*(-a*d+8*b*c)*x*(b*x^2+a)^(3/2)/b+1/48*(-a*d+8*b*c)*x*(b*x^2+a)^(5/2
)/b+1/8*d*x*(b*x^2+a)^(7/2)/b+5/128*a^3*(-a*d+8*b*c)*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(3/2)+5/128*a^2*(-a*d+8*b*c)*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {396, 201, 223, 212}

$$\frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}\,(8bc-ad)}{128b}+\frac{x(a+bx^2)^{5/2}(8bc-ad)}{48b}+\frac{5ax(a+bx^2)^{3/2}(8bc-ad)}{192b}+\frac{dx(a+bx^2)^{7/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (5*a^2*(8*b*c - a*d)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*b*c - a*d)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*b*c - a*d)*x*(a + b*x^2)^(5/2))/(48*b) + (d*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\,dx = \frac{dx(a+bx^2)^{7/2}}{8b} - \frac{(-8bc+ad)\int (a+bx^2)^{5/2}\,dx}{8b}$$
$$= \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{(5a(8bc-ad))\int (a+bx^2)^{3/2}\,d}{48b}$$
$$= \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{(}{}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+}{48b}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+}{48b}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+}{48b}$$

Mathematica [A]
time = 0.21, size = 123, normalized size = 0.83

$$\frac{x\sqrt{a+bx^2}\,(264a^2bc+15a^3d+208ab^2cx^2+118a^2bdx^2+64b^3cx^4+136ab^2dx^4+48b^3dx^6)}{384b} + \frac{5a^3(-8bc+ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{128b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2),x]
[Out] (x*Sqrt[a + b*x^2]*(264*a^2*b*c + 15*a^3*d + 208*a*b^2*c*x^2 + 118*a^2*b*d*
x^2 + 64*b^3*c*x^4 + 136*a*b^2*d*x^4 + 48*b^3*d*x^6))/(384*b) + (5*a^3*(-8*
b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Maple [A]
time = 0.05, size = 162, normalized size = 1.09

method	result

risch	$\frac{x\left(48b^{3}d\,x^{6}+136a\,b^{2}d\,x^{4}+64b^{3}c\,x^{4}+118a^{2}bd\,x^{2}+208a\,b^{2}c\,x^{2}+15a^{3}d+264a^{2}bc\right)\sqrt{b\,x^{2}+a}}{384b}-\frac{5a^{4}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)d}{128b^{\frac{3}{2}}}$
default	$d\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)+c\left(\frac{x\left(b\,x^{2}\right.}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(5/2)*(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] d*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))+c*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))

Maxima [A]

time = 0.30, size = 151, normalized size = 1.01

$$\frac{1}{6}\left(bx^{2}+a\right)^{\frac{5}{2}}cx+\frac{5}{24}\left(bx^{2}+a\right)^{\frac{3}{2}}acx+\frac{5}{16}\sqrt{bx^{2}+a}\,a^{2}cx+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}dx}{8\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}adx}{48\,b}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}dx}{192\,b}-\frac{5\sqrt{bx^{2}+a}\,a^{3}dx}{128\,b}+\frac{5\,a^{3}c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}-\frac{5\,a^{4}d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="maxima")

[Out] 1/6*(b*x^2 + a)^(5/2)*c*x + 5/24*(b*x^2 + a)^(3/2)*a*c*x + 5/16*sqrt(b*x^2 + a)*a^2*c*x + 1/8*(b*x^2 + a)^(7/2)*d*x/b - 1/48*(b*x^2 + a)^(5/2)*a*d*x/b - 5/192*(b*x^2 + a)^(3/2)*a^2*d*x/b - 5/128*sqrt(b*x^2 + a)*a^3*d*x/b + 5/

```
16*a^3*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*a^4*d*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

Fricas [A]
time = 0.51, size = 260, normalized size = 1.74

$$\left[-\frac{15\,(8\,a^3bc-a^4d)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(48\,b^4dx^7+8\,(8\,b^4c+17\,ab^3d)x^5+2\,(104\,ab^3c+59\,a^2b^2d)x^3+3\,(88\,a^2b^2c+5\,a^3bd)x)\sqrt{bx^2+a}}{768\,b^2}, -\frac{15\,(8\,a^3bc-a^4d)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(48\,b^4dx^7+8\,(8\,b^4c+17\,ab^3d)x^5+2\,(104\,ab^3c+59\,a^2b^2d)x^3+3\,(88\,a^2b^2c+5\,a^3bd)x)\sqrt{bx^2+a}}{384\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(8*a^3*b*c - a^4*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^
3*c + 59*a^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/
b^2, -1/384*(15*(8*a^3*b*c - a^4*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) - (48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^3*c + 59*a
^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(134) = 268$.
time = 37.84, size = 316, normalized size = 2.12

$$\frac{5a^{\frac{7}{2}}dx}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}cx}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}dx^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}bcx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bdx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{17\sqrt{a}\,b^2cx^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}\,b^2dx^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^4d\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{5a^3c\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{b^3cx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{b^3dx^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c),x)
```

```
[Out] 5*a**(7/2)*d*x/(128*b*sqrt(1 + b*x**2/a)) + a**(5/2)*c*x*sqrt(1 + b*x**2/a)
/2 + 3*a**(5/2)*c*x/(16*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*d*x**3/(384*sqrt
(1 + b*x**2/a)) + 35*a**(3/2)*b*c*x**3/(48*sqrt(1 + b*x**2/a)) + 127*a**(3/
2)*b*d*x**5/(192*sqrt(1 + b*x**2/a)) + 17*sqrt(a)*b**2*c*x**5/(24*sqrt(1 +
b*x**2/a)) + 23*sqrt(a)*b**2*d*x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*d*asin
h(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + 5*a**3*c*asinh(sqrt(b)*x/sqrt(a))/(16
*sqrt(b)) + b**3*c*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d*x**9/(8*sqr
t(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.56, size = 135, normalized size = 0.91

$$\frac{1}{384}\left(2\left(4\left(6\,b^2dx^2+\frac{8\,b^8c+17\,ab^7d}{b^6}\right)x^2+\frac{104\,ab^7c+59\,a^2b^6d}{b^6}\right)x^2+\frac{3\,(88\,a^2b^6c+5\,a^3b^5d)}{b^6}\right)\sqrt{bx^2+a}\,x-\frac{5\,(8\,a^3bc-a^4d)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (8*b^8*c + 17*a*b^7*d)/b^6)*x^2 + (104*a*b^7*c +
 59*a^2*b^6*d)/b^6)*x^2 + 3*(88*a^2*b^6*c + 5*a^3*b^5*d)/b^6)*sqrt(b*x^2 +
a)*x - 5/128*(8*a^3*b*c - a^4*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2), x)
```

3.65 $\int (a+bx^2)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

```
[Out] 5/24*a*x*(b*x^2+a)^(3/2)+1/6*x*(b*x^2+a)^(5/2)+5/16*a^3*arctanh(x*b^(1/2)/(
b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2} dx = \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2} dx$$
$$= \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2}\, dx$$
$$= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}} dx$$
$$= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2} d\right.$$
$$= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.85

$$\frac{1}{48}\sqrt{a+bx^2}\left(33a^2x+26abx^3+8b^2x^5\right) - \frac{5a^3\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Maple [A]
time = 0.05, size = 68, normalized size = 0.81

method	result	size
risch	$\frac{x\left(8b^2x^4+26ab\,x^2+33a^2\right)\sqrt{b\,x^2+a}}{48}+\frac{5a^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}$	59
default	$\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(
1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.31, size = 58, normalized size = 0.69

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{bx^2+a}\,a^2x+\frac{5\,a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 0.50, size = 146, normalized size = 1.74

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{96\,b},-\frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{48\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 2.78, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}\,b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

Giac [A]
time = 0.58, size = 63, normalized size = 0.75

$$-\frac{5\,a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

Mupad [B]
time = 4.69, size = 37, normalized size = 0.44

$$\frac{x\,{\left(b\,x^2+a\right)}^{5/2}\,{{}}_2{\mathrm{F}}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2),x)
```

```
[Out] (x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(5/2)
```

3.66 $\int \frac{(a+bx^2)^{5/2}}{c+dx^2} \, dx$

Optimal. Leaf size=157

$$-\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2}+\frac{bx(a+bx^2)^{3/2}}{4d}+\frac{\sqrt{b}\,(8b^2c^2-20abcd+15a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{(bc-a$$

```
[Out] 1/4*b*x*(b*x^2+a)^(3/2)/d+1/8*(15*a^2*d^2-20*a*b*c*d+8*b^2*c^2)*arctanh(x*b
^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d^3-(-a*d+b*c)^(5/2)*arctanh(x*(-a*d+b*c)^(
1/2)/c^(1/2)/(b*x^2+a)^(1/2))/d^3/c^(1/2)-1/8*b*(-7*a*d+4*b*c)*x*(b*x^2+a)^
(1/2)/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {427, 542, 537, 223, 212, 385, 214}

$$\frac{\sqrt{b}\,(15a^2d^2-20abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,d^3}-\frac{bx\sqrt{a+bx^2}\,(4bc-7ad)}{8d^2}+\frac{bx(a+bx^2)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] -1/8*(b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/d^2 + (b*x*(a + b*x^2)^(3/2))/(4
*d) + (Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(8*d^3) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(S
qrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \frac{bx(a+bx^2)^{3/2}}{4d} + \frac{\int \frac{\sqrt{a+bx^2}\,(-a(bc-4ad)-b(4bc-7ad)x^2)}{c+dx^2}\,dx}{4d}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx(a+bx^2)^{3/2}}{4d} + \frac{\int \frac{a(4b^2c^2-9abcd+8a^2d^2)+b(8b^2c^2-20abcd+15a^2d}{\sqrt{a+bx^2}\,(c+dx^2)}}{8d^2}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx(a+bx^2)^{3/2}}{4d} - \frac{(bc-ad)^3 \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{d^3} + \frac{(b(}{}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx(a+bx^2)^{3/2}}{4d} - \frac{(bc-ad)^3 \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, -\right.}{d^3}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx(a+bx^2)^{3/2}}{4d} + \frac{\sqrt{b}\,(8b^2c^2-20abcd+15a^2d^2)\tanh^{-1}}{8d^3}$$

Mathematica [A]
time = 0.39, size = 157, normalized size = 1.00

$$\frac{bdx\sqrt{a+bx^2}\,(-4bc+9ad+2bdx^2) - \frac{8(-bc+ad)^{5/2}\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{\sqrt{c}} - \sqrt{b}\,(8b^2c^2-20abcd+15a^2d^2)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2]*(-4*b*c + 9*a*d + 2*b*d*x^2) - (8*(-(b*c) + a*d)^(5/
2)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*
c) + a*d])])/Sqrt[c] - Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2047 vs. $2(131) = 262$.
time = 0.09, size = 2048, normalized size = 13.04

method	result

risch	$\frac{bx\left(2bd\,x^{2}+9ad-4bc\right)\sqrt{b\,x^{2}+a}}{8d^{2}}+\frac{15\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)a^{2}}{8d}-\frac{5b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)ac}{2d^{2}}+\frac{b^{\frac{5}{2}}\ln\left(x\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-c*d)^(1/2)*(1/5*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(5/2)+b*(-c*d)^(1/2)/d*(1/8*(2*b*(x-(-c*d)^(1/2)/d)+2
*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(3/2)+3/16*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b*(1/4*(2*b*(x-
(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)
/b^(3/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2
)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))))+(a*d-b
*c)/d*(1/3*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/d*(1/4*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(
1/2)/d)/b*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((b*(-c*d)^(1/2)/
d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x-(-c*d)^(1/2)/d)^2*
b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b^(1/2)*(-c*d)^(
1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/
d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/
d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))))-1/2/(-c*d)^(1/2)*(1/
5*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(5/2)-b*(-c*d)^(1/2)/d*(1/8*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/b
*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(3/2)+3/16*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b*(1/4*(2*b*(x+(-c*d)^(1/2)/d)-2*b*
(-c*d)^(1/2)/d)/b*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2)+1/8*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((-b*(-c*
d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))))+(a*d-b*c)/d*(1/3*((x+(-c*
d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-b*
(-c*d)^(1/2)/d*(1/4*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/b*((x+(-c*d
```

```
)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/8
*(4*b*(a*d-b*c)/d+4*b^2*c/d)/b^(3/2)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2
)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2)))+(a*d-b*c)/d*(((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b^(1/2)*(-c*d)^(1/2)/d*ln((-b*(-c*
d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-(a*d-b*c)/d/((a*d-b*c)/d)^(
1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c), x)
```

Fricas [A]
time = 1.17, size = 935, normalized size = 5.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
[Out] [1/16*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a
*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2
- 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt
((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 2*(2*b^2*d^2*x^3 - (4*b^2*c
*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15
*a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 2*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*
d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
 (2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, 1/16*(8*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a
*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 +
(a*b*c - a^2*d)*x)) + (8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 - (4*b^2*c*d -
9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*
d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 4*(b^2*c^2 - 2*a*b*c*d +
```

```
 a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) -
(2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^{5/2}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2), x)
```

3.67 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=175

$$\frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2}-\frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)}-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(4bc \cdots}{\cdots}$$

```
[Out] -1/2*(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)-1/2*b^(3/2)*(-5*a*d+4*b*c)*
arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/d^3+1/2*(-a*d+b*c)^(3/2)*(a*d+4*b*c)*arc
tanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/d^3+1/2*b*(-a*d+2*
b*c)*x*(b*x^2+a)^(1/2)/c/d^2
```

Rubi [A]

time = 0.15, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {424, 542, 537, 223, 212, 385, 214}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}+\frac{bx\sqrt{a+bx^2}\,(2bc-ad)}{2cd^2}-\frac{x(a+bx^2)^{3/2}\,(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d^2) - ((b*c - a*d)*x*(a + b*x^2)^
(3/2))/(2*c*d*(c + d*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*d^3) + ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b
*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} + \frac{\int \frac{\sqrt{a+bx^2}\,(a(bc+ad)+2b(2bc-ad)x^2)}{c+dx^2}\,dx}{2cd}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} + \frac{\int \frac{-2a(2b^2c^2-2abcd-a^2d^2)-2b^2c(4bc-5ad)}{\sqrt{a+bx^2}\,(c+dx^2)}}{4cd^2}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} - \frac{(b^2(4bc-5ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^3} -$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} - \frac{(b^2(4bc-5ad))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.}{2d^3}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} - \frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx}}\right.}{2d^3}$$

Mathematica [A]
time = 0.66, size = 187, normalized size = 1.07

$$\frac{\frac{dx\sqrt{a+bx^2}\,(-2abcd+a^2d^2+b^2c(2c+dx^2))}{c(c+dx^2)} + \frac{\sqrt{-bc+ad}\,(4b^2c^2-3abcd-a^2d^2)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{c^{3/2}} + b^{3/2}(4bc-5ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] ((d*x*Sqrt[a + b*x^2]*(-2*a*b*c*d + a^2*d^2 + b^2*c*(2*c + d*x^2)))/(c*(c +
 d*x^2)) + (Sqrt[-(b*c) + a*d]*(4*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(-(
d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])]])/
c^(3/2) + b^(3/2)*(4*b*c - 5*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*d
^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5229 vs. 2(147) = 294.
time = 0.13, size = 5230, normalized size = 29.89

method	result	size
risch	Expression too large to display	3386
default	Expression too large to display	5230

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2, x)
```

Fricas [A]
time = 0.97, size = 1236, normalized size = 7.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b
)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (4*b^2*c^3 - 3*a*b*c^2*
d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)
/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3
*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b
*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/8*(
4*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*
b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(
a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) + 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), -1/4*((4*b^2*c^3 - 3*a*b
*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-(b*c
- a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c -
 a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + (4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/4*(2*(4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2
 - a^2*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)
*x)) + 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x
^2 + a))/(c*d^4*x^2 + c^2*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(147) = 294.
time = 0.57, size = 405, normalized size = 2.31

$$\frac{\sqrt{bx^2+a}\,b^2x}{2\,d^2}+\frac{\left(4\,b^{\frac{5}{2}}c-5\,ab^{\frac{3}{2}}d\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{4\,d^3}-\frac{\left(4\,b^{\frac{7}{2}}c^3-7\,ab^{\frac{5}{2}}c^2d+2\,a^2b^{\frac{3}{2}}cd^2+a^3\sqrt{b}\,d^3\right)\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}\,cd^3}+\frac{2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{7}{2}}c^3-5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{5}{2}}c^2d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}}cd^2-\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3\sqrt{b}\,d^3+a^2b^{\frac{5}{2}}c^2d-2\,a^3b^{\frac{3}{2}}cd^2+a^4\sqrt{b}\,d^3}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x/d^2 + 1/4*(4*b^(5/2)*c - 5*a*b^(3/2)*d)*log((sqrt
(b)*x - sqrt(b*x^2 + a))^2)/d^3 - 1/2*(4*b^(7/2)*c^3 - 7*a*b^(5/2)*c^2*d +
2*a^2*b^(3/2)*c*d^2 + a^3*sqrt(b)*d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*c*d^3) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(7/2)*c^3 - 5*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*b^(5/2)*c^2*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*
b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*d^3 + a^2*b^(5/
2)*c^2*d - 2*a^3*b^(3/2)*c*d^2 + a^4*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)*c*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^2, x)
```

3.68 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=194

$$-\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd\,(c+dx^2)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\,(c+dx^2)}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{\sqrt{bc-ad}\,(8b^2c^2+$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^2+b^(5/2)*arctanh(x*b^(1/2)
/(b*x^2+a)^(1/2))/d^3-1/8*(3*a^2*d^2+4*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b
*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/c^(5/2)/d^3-1/8*(-a*d+b
*c)*(3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/d^2/(d*x^2+c)
```

Rubi [A]
time = 0.13, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {424, 540, 537, 223, 212, 385, 214}

$$-\frac{\sqrt{bc-ad}\,(3a^2d^2+4abcd+8b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{x\sqrt{a+bx^2}\,(bc-ad)(3ad+4bc)}{8c^2d^2\,(c+dx^2)}-\frac{x(a+bx^2)^{3/2}\,(bc-ad)}{4cd\,(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*(c + d*x^2)^2) - ((b*c - a*d)*(
4*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*d^2*(c + d*x^2)) + (b^(5/2)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^3 - (Sqrt[b*c - a*d]*(8*b^2*c^2 + 4*a*b*c
*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*
c^(5/2)*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = -\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd\,(c+dx^2)^2} + \frac{\int \frac{\sqrt{a+bx^2}\,(a(bc+3ad)+4b^2cx^2)}{(c+dx^2)^2}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd\,(c+dx^2)^2} - \frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\,(c+dx^2)} - \frac{\int \frac{-a(4b^2c^2+ad(bc+3ad))-8b^3\cdots}{\sqrt{a+bx^2}\,(c+dx^2)}\cdots}{8c^2d^2}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd\,(c+dx^2)^2} - \frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\,(c+dx^2)} + \frac{b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^3} - \frac{((\cdots}{\cdots}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd\,(c+dx^2)^2} - \frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\,(c+dx^2)} + \frac{b^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \cdots\right.}{d^3}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd\,(c+dx^2)^2} - \frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\,(c+dx^2)} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\cdots\right.}{d^3}$$

Mathematica [A]
time = 1.48, size = 266, normalized size = 1.37

$$\frac{\frac{dx\sqrt{a+bx^2}\,(-abcd(c-3dx^2)-2b^2c^2(2c+3dx^2)+a^2d^2(5c+3dx^2))}{c^2(c+dx^2)^2} - \frac{3(-4bc+ad)^2\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{c^{5/2}} + \frac{4b(10bc-7ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{bc-ad}}\right)}{c^{3/2}} - 8b^{5/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] ((d*x*Sqrt[a + b*x^2]*(-(a*b*c*d*(c - 3*d*x^2)) - 2*b^2*c^2*(2*c + 3*d*x^2)
 + a^2*d^2*(5*c + 3*d*x^2)))/(c^2*(c + d*x^2)^2) - (3*(-4*b*c + a*d)^2*Sqrt
[-(b*c) + a*d]*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[
c]*Sqrt[-(b*c) + a*d])])/c^(5/2) + (4*b*(10*b*c - 7*a*d)*Sqrt[b*c - a*d]*Ar
cTanh[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*
d])])/c^(3/2) - 8*b^(5/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10682 vs. 2(168) = 336.
time = 0.07, size = 10683, normalized size = 55.07

method	result	size
default	Expression too large to display	10683

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(168) = 336$.
time = 0.81, size = 1517, normalized size = 7.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2
*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a
*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x +
 (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c
*d*x^2 + c^2)) - 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^
3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d
^4*x^2 + c^4*d^3), -1/32*(32*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^2*c^4 + 4*a*b*c^3*d + 3*
a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*
d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*
c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^
4 + 2*c*d*x^2 + c^2)) + 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4
*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 +
2*c^3*d^4*x^2 + c^4*d^3), 1/16*((8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 +
(8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*
d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
+ a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c -
 a^2*d)*x)) + 8*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*(2*b^2*c^2*d^2 - a*b*c*d^
3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2
+ a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), -1/16*(16*(b^2*c^2*d^2*x^4 +
 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (
```

```
8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*
a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*
c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c
 - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + 2*(3*(2*b^2*c^2*d^2
 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*
sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(168) = 336.
time = 0.52, size = 659, normalized size = 3.40

$$-\frac{b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2d^3}+\frac{\left(8b^{\frac{7}{2}}c^3-4ab^{\frac{5}{2}}c^2d-a^2b^{\frac{3}{2}}cd^2-3a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{8\sqrt{-b^2c^2+abcd}\,c^2d^3}-\frac{16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{7}{2}}c^3d-20\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}c^2d^2+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{3}{2}}cd^3+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^3\sqrt{b}d^4+48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{9}{2}}c^4-72\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4ab^{\frac{7}{2}}c^3d+18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{5}{2}}c^2d^2+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^3b^{\frac{3}{2}}cd^3-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^4\sqrt{b}d^4+32\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{7}{2}}c^3d-28\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{5}{2}}c^2d^2-13\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^4b^{\frac{3}{2}}cd^3+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^5\sqrt{b}d^4+6a^4b^{\frac{5}{2}}c^2d^2-3a^5b^{\frac{3}{2}}cd^3-3a^6\sqrt{b}d^4}{4\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^3 + 1/8*(8*b^(7/2)*c^3
- 4*a*b^(5/2)*c^2*d - a^2*b^(3/2)*c*d^2 - 3*a^3*sqrt(b)*d^3)*arctan(1/2*((s
qrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(s
qrt(-b^2*c^2 + a*b*c*d)*c^2*d^3) - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
b^(7/2)*c^3*d - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2)*c^2*d^2 + (sqr
t(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^3*sqrt(b)*d^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(9/2)*c^4 - 7
2*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c^3*d + 18*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2)*c^2*d^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*d^4 + 32*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^2*b^(7/2)*c^3*d - 28*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^3*b^(5/2)*c^2*d^2 - 13*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(3/2)*
c*d^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*d^4 + 6*a^4*b^(5/2)*c
^2*d^2 - 3*a^5*b^(3/2)*c*d^3 - 3*a^6*sqrt(b)*d^4)/((((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)^2*c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^3, x)
```

3.69 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx$

Optimal. Leaf size=144

$$\frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3}+\frac{5ax(a+bx^2)^{3/2}}{24c^2\,(c+dx^2)^2}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\,(c+dx^2)}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}$$

```
[Out] 1/6*x*(b*x^2+a)^(5/2)/c/(d*x^2+c)^3+5/24*a*x*(b*x^2+a)^(3/2)/c^2/(d*x^2+c)^
2+5/16*a^3*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(7/2)/(-a*
d+b*c)^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)/c^3/(d*x^2+c)
```

Rubi [A]
time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {386, 385, 214}

$$\frac{5a^3\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\,(c+dx^2)}+\frac{5ax(a+bx^2)^{3/2}}{24c^2\,(c+dx^2)^2}+\frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*x*(a + b*x^2)^(3/2))/(24*c
^2*(c + d*x^2)^2) + (5*a^2*x*Sqrt[a + b*x^2])/(16*c^3*(c + d*x^2)) + (5*a^3
*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*Sqrt[b
*c - a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3} + \frac{(5a)\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx}{6c}$$

$$= \frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3} + \frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2} + \frac{(5a^2)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{8c^2}$$

$$= \frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3} + \frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3(c+dx^2)} + \frac{(5a^3)\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{16c^3}$$

$$= \frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3} + \frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3(c+dx^2)} + \frac{(5a^3)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x,\right.}{16c^3}$$

$$= \frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3} + \frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3(c+dx^2)} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 10.57, size = 201, normalized size = 1.40

$$\frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(8b^2c^2x^4+2abcx^2(13c+5dx^2)+a^2(33c^2+40cdx^2+15d^2x^4)\right)}{(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}} + \frac{15a^2\sin^{-1}\left(\frac{\sqrt{\left(-\frac{b}{a}+\frac{d}{c}\right)x^2}}{\sqrt{1+\frac{dx^2}{c}}}\right)}{\sqrt{\frac{(-bc+ad)x^2}{ac}}}\right)}{48c^4\sqrt{1+\frac{bx^2}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(8*b^2*c^2*x^4 +
 2*a*b*c*x^2*(13*c + 5*d*x^2) + a^2*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4)))/((
c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]) + (15*a^2*ArcSin[Sqrt[(-(b/a) + d/c)*x^2]
```

/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(48*c^4*Sqrt[1 + (b*x^2)/a])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19518 vs. $2(120) = 240$.
time = 0.08, size = 19519, normalized size = 135.55

method	result	size
default	Expression too large to display	19519

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(120) = 240$.
time = 0.63, size = 706, normalized size = 4.90

$$\left[\frac{15\,{\left(a^{3} d^{3} x^{6} + 3\,a^{3} c d^{2} x^{4} + 3\,a^{3} c^{2} d x^{2} + a^{3} c^{3}\right)} \sqrt{b c^{2} - a c d} \log\left(\frac{{\left(8\,b^{2} c^{2} - 8\,a b c d + a^{2} d^{2}\right)} x^{4} + a^{2} c^{2} + 2\,{\left(4\,a b c^{2} - 3\,a^{2} c d\right)} x^{2} + 4\,{\left({\left(2\,b c - a d\right)} x^{3} + a c x\right)} \sqrt{b c^{2} - a c d} \sqrt{b x^{2} + a}}{d^{2} x^{4} + 2\,c d x^{2} + c^{2}}\right) + 4\,{\left({\left(8\,b^{3} c^{4} + 2\,a b^{2} c^{3} d + 5\,a^{2} b c^{2} d^{2} - 15\,a^{3} c d^{3}\right)} x^{5} + 2\,{\left(13\,a b^{2} c^{4} + 7\,a^{2} b c^{3} d - 20\,a^{3} c^{2} d^{2}\right)} x^{3} + 33\,{\left(a^{2} b c^{4} - a^{3} c^{3} d\right)} x\right)} \sqrt{b x^{2} + a}}{192\,{\left(b c^{8} - a c^{7} d + {\left(b c^{5} d^{3} - a c^{4} d^{4}\right)} x^{6} + 3\,{\left(b c^{6} d^{2} - a c^{5} d^{3}\right)} x^{4} + 3\,{\left(b c^{7} d - a c^{6} d^{2}\right)} x^{2}\right)}}, -\frac{15\,{\left(a^{3} d^{3} x^{6} + 3\,a^{3} c d^{2} x^{4} + 3\,a^{3} c^{2} d x^{2} + a^{3} c^{3}\right)} \sqrt{-b c^{2} + a c d} \arctan\left(\frac{\sqrt{-b c^{2} + a c d} {\left({\left(2\,b c - a d\right)} x^{2} + a c\right)} \sqrt{b x^{2} + a}}{2\,{\left({\left(b^{2} c^{2} - a b c d\right)} x^{3} + {\left(a b c^{2} - a^{2} c d\right)} x\right)}}\right) - 2\,{\left({\left(8\,b^{3} c^{4} + 2\,a b^{2} c^{3} d + 5\,a^{2} b c^{2} d^{2} - 15\,a^{3} c d^{3}\right)} x^{5} + 2\,{\left(13\,a b^{2} c^{4} + 7\,a^{2} b c^{3} d - 20\,a^{3} c^{2} d^{2}\right)} x^{3} + 33\,{\left(a^{2} b c^{4} - a^{3} c^{3} d\right)} x\right)} \sqrt{b x^{2} + a}}{96\,{\left(b c^{8} - a c^{7} d + {\left(b c^{5} d^{3} - a c^{4} d^{4}\right)} x^{6} + 3\,{\left(b c^{6} d^{2} - a c^{5} d^{3}\right)} x^{4} + 3\,{\left(b c^{7} d - a c^{6} d^{2}\right)} x^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt
(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4
*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4 + 2*a*b^2*
c^3*d + 5*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3
*d - 20*a^3*c^2*d^2)*x^3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(
b*c^8 - a*c^7*d + (b*c^5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x
^4 + 3*(b*c^7*d - a*c^6*d^2)*x^2), -1/96*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4
 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 +
 a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4 + 2*a*b^2*c^3*d + 5*a^2*b*c^2*d^2
 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3*d - 20*a^3*c^2*d^2)*x^
```

```
3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(b*c^8 - a*c^7*d + (b*c^
5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x^4 + 3*(b*c^7*d - a*c^6
*d^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(120) = 240.
time = 1.52, size = 846, normalized size = 5.88

$$-\frac{5a^3\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{16\sqrt{-b^2c^2+abcd}\,c^3}+\frac{48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}c^3d^2-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}d^5+192\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{9}{2}}c^4d+48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8ab^{\frac{7}{2}}c^3d^2-150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8a^3b^{\frac{3}{2}}cd^4+75\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8a^4\sqrt{b}d^5+256\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{11}{2}}c^5-64\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{9}{2}}c^4d+288\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{7}{2}}c^3d^2-440\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^3b^{\frac{5}{2}}c^2d^3+440\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^4b^{\frac{3}{2}}cd^4-150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^5\sqrt{b}d^5+192\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{9}{2}}c^4d+48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^3b^{\frac{7}{2}}c^3d^2+360\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^4b^{\frac{5}{2}}c^2d^3-420\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^5b^{\frac{3}{2}}cd^4+150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^6\sqrt{b}d^5+48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^4b^{\frac{7}{2}}c^3d^2+72\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^5b^{\frac{5}{2}}c^2d^3+120\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^6b^{\frac{3}{2}}cd^4-75\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^7\sqrt{b}d^5+8a^6b^{\frac{5}{2}}c^2d^3+10a^7b^{\frac{3}{2}}cd^4+15a^8\sqrt{b}d^5}{24\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)^3c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -5/16*a^3*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a
*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^3) + 1/24*(48*(sq
rt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c^3*d^2 - 15*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^3*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*
d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 150*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^4*sqrt(b)*d^5 + 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 - 64*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 288*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^2*b^(7/2)*c^3*d^2 - 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/
2)*c^2*d^3 + 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 - 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(9/2)*c^4*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2)
*c^3*d^2 + 360*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 - 420*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^6*sqrt(b)*d^5 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)
*c^3*d^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 + 120*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^7*sqrt(b)*d^5 + 8*a^6*b^(5/2)*c^2*d^3 + 10*a^7*b^(3/2)*c*d^4 + 1
5*a^8*sqrt(b)*d^5)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3*c^3*
d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{\left(d\,x^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^4,x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^4, x)
```

3.70 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx$

Optimal. Leaf size=249

$$-\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4}+\frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3}+\frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)(c+dx^2)^2}+\frac{5a^2(8bc-7ad)x\sqrt{a+b}}{128c^4(bc-ad)(c+dx}$$

```
[Out] -1/8*d*x*(b*x^2+a)^(7/2)/c/(-a*d+b*c)/(d*x^2+c)^4+1/48*(-7*a*d+8*b*c)*x*(b*
x^2+a)^(5/2)/c^2/(-a*d+b*c)/(d*x^2+c)^3+5/192*a*(-7*a*d+8*b*c)*x*(b*x^2+a)^
(3/2)/c^3/(-a*d+b*c)/(d*x^2+c)^2+5/128*a^3*(-7*a*d+8*b*c)*arctanh(x*(-a*d+b
*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(9/2)/(-a*d+b*c)^(3/2)+5/128*a^2*(-7*a
*d+8*b*c)*x*(b*x^2+a)^(1/2)/c^4/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A]
time = 0.09, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {390, 386, 385, 214}

$$\frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4(c+dx^2)(bc-ad)}+\frac{5ax(a+bx^2)^{3/2}(8bc-7ad)}{192c^3(c+dx^2)^2(bc-ad)}+\frac{x(a+bx^2)^{5/2}(8bc-7ad)}{48c^2(c+dx^2)^3(bc-ad)}-\frac{dx(a+bx^2)^{7/2}}{8c(c+dx^2)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] -1/8*(d*x*(a + b*x^2)^(7/2))/(c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*c - 7*a*
d)*x*(a + b*x^2)^(5/2))/(48*c^2*(b*c - a*d)*(c + d*x^2)^3) + (5*a*(8*b*c -
7*a*d)*x*(a + b*x^2)^(3/2))/(192*c^3*(b*c - a*d)*(c + d*x^2)^2) + (5*a^2*(8
*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(128*c^4*(b*c - a*d)*(c + d*x^2)) + (5*a^3
*(8*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(1
28*c^(9/2)*(b*c - a*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx}{8c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3} + \frac{(5a(8bc-7ad))\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx}{48c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)(c+dx^2)^2} +$$

$$= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)(c+dx^2)^2} +$$

$$= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)(c+dx^2)^2} +$$

$$= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)(c+dx^2)^2} +$$

Mathematica [A]
time = 10.75, size = 306, normalized size = 1.23

$$\frac{x\left(-c(16b^4c^3x^6(4c+dx^2)+8ab^3c^2x^4(34c^2+13cdx^2+3d^2x^4)+2a^2b^2cx^2(236c^3+173c^2dx^2+106cd^2x^4+25d^3x^6)-a^4d(279c^3+511c^2dx^2+385cd^2x^4+105d^3x^6)+a^3b(264c^4-21c^3dx^2-323c^2d^2x^4-335cd^3x^6-105d^4x^8))+\frac{15a^3(-8bc+7ad)(c+dx^2)^4\tanh^{-1}\left(\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}\right)}{\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}}\right)}{384c^5(-bc+ad)\sqrt{a+bx^2}\,(c+dx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] (x*(-(c*(16*b^4*c^3*x^6*(4*c + d*x^2) + 8*a*b^3*c^2*x^4*(34*c^2 + 13*c*d*x^
2 + 3*d^2*x^4) + 2*a^2*b^2*c*x^2*(236*c^3 + 173*c^2*d*x^2 + 106*c*d^2*x^4 +
 25*d^3*x^6) - a^4*d*(279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6
) + a^3*b*(264*c^4 - 21*c^3*d*x^2 - 323*c^2*d^2*x^4 - 335*c*d^3*x^6 - 105*d
^4*x^8))) + (15*a^3*(-8*b*c + 7*a*d)*(c + d*x^2)^4*ArcTanh[Sqrt[((b*c - a*d
)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(384*c^
5*(-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 34026 vs. $2(221) = 442$.
time = 0.08, size = 34027, normalized size = 136.65

method	result	size
default	Expression too large to display	34027

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

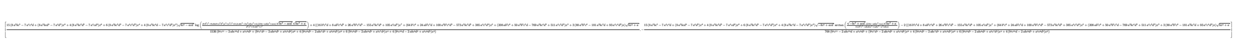
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(221) = 442$.
time = 1.03, size = 1258, normalized size = 5.05



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4 - 7*a^4*d^5)*x^8 +
4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^3)
*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b
```

```
^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2
 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c
^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^
5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208
*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 +
3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d + 93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(
b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2 + (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*
c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c
^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 +
 a^2*c^8*d^3)*x^2), -1/768*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4
- 7*a^4*d^5)*x^8 + 4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d
^2 - 7*a^4*c^2*d^3)*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(-b*c^
2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b
*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^4*c
^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c
*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*
c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^
3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 + 3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d +
93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2
+ (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*
c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*
x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 + a^2*c^8*d^3)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(221) = 442$.
time = 5.21, size = 1448, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -5/128*(8*a^3*b^(3/2)*c - 7*a^4*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*
x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^5 - a*c^4*d)*s
qrt(-b^2*c^2 + a*b*c*d)) - 1/192*(120*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*
```

```
b^(3/2)*c*d^6 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*sqrt(b)*d^7 - 768*
(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(11/2)*c^5*d^2 + 768*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2)*c^4*d^3 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3
*b^(5/2)*c^2*d^5 - 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6
+ 735*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*b^(13/2)*c^6*d + 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
a^2*b^(9/2)*c^4*d^3 + 8320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3
*d^4 - 15600*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(5/2)*c^2*d^5 + 9800*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/
2)*c^7 + 1024*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(13/2)*c^6*d - 4864*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(11/2)*c^5*d^2 + 21888*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 - 38000*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^
4*b^(7/2)*c^3*d^4 + 37400*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d
^5 - 18550*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(3/2)*c*d^6 + 3675*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*a^2*b^(13/2)*c^6*d - 9472*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(9/2)*
c^4*d^3 + 32896*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2)*c^3*d^4 - 35376
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*d^5 + 18200*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^8*sqrt(b)*d^7 - 768*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(11/2)*c^5*d^2
- 1536*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2)*c^4*d^3 - 2944*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 + 12528*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^7*b^(5/2)*c^2*d^5 - 9170*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^8*b^(3
/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*sqrt(b)*d^7 - 256*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^3 - 256*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^7*b^(7/2)*c^3*d^4 - 608*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^8*b^
(5/2)*c^2*d^5 + 1960*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b^(3/2)*c*d^6 - 73
5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b)*d^7 - 16*a^8*b^(7/2)*c^3*d^4
 - 24*a^9*b^(5/2)*c^2*d^5 - 50*a^10*b^(3/2)*c*d^6 + 105*a^11*sqrt(b)*d^7)/(
(b*c^5*d^3 - a*c^4*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^5,x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^5, x)
```

3.71 $\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx$

Optimal. Leaf size=30

$$-\sin^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$$

[Out] -arcsin(x)+arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {399, 222, 385, 209}

$$\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)-\text{ArcSin}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/(1 + x^2),x]

[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx &= 2\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx - \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\sin^{-1}(x) + 2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\ &= -\sin^{-1}(x) + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 1.53

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right) + 2\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(1 + x^2),x]
[Out] Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]] + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.22, size = 33, normalized size = 1.10

method	result
default	$-\arcsin(x) - \sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^2+1}\,x}{x^2-1}\right)$
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right) - \frac{\text{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{3\,\text{RootOf}\left(\_Z^2+2\right)x}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -arcsin(x)-2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/(x^2 + 1), x)
```

Fricas [A]
time = 0.47, size = 42, normalized size = 1.40

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right)+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) + 2*arctan((sqrt(-x^2 + 1) -
1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+1)}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(x**2+1),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(24) = 48.
time = 0.90, size = 95, normalized size = 3.17

$$-\frac{1}{2}\pi\mathrm{sgn}(x)+\frac{1}{2}\sqrt{2}\left(\pi\mathrm{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}\,x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
[Out] -1/2*pi*sgn(x) + 1/2*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x
^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - arctan(-1/2*x*((sqrt(-x^2
+ 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

Mupad [B]
time = 0.39, size = 83, normalized size = 2.77

$$-\mathrm{asin}(x)+\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)\,1\mathrm{i}}{2}-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^2 + 1),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i)
/2 - asin(x) - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i
)/(x + 1i))*1i)/2
```

3.72 $\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx$

Optimal. Leaf size=27

$$\sinh^{-1}(x)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^2}}\right)$$

[Out] arcsinh(x)-arctanh(x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {399, 221, 385, 213}

$$\sinh^{-1}(x)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2]/(-1 + x^2),x]

[Out] ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Dist[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx &= 2\int \frac{1}{(-1+x^2)\sqrt{1+x^2}}\,dx+\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \sinh^{-1}(x)+2\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\frac{x}{\sqrt{1+x^2}}\right)\\ &= \sinh^{-1}(x)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 1.67

$$\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1-x^2+x\sqrt{1+x^2}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] ArcTanh[x/Sqrt[1 + x^2]] - Sqrt[2]*ArcTanh[(1 - x^2 + x*Sqrt[1 + x^2])/Sqrt
[2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(21) = 42$.
time = 0.19, size = 84, normalized size = 3.11

method	result
trager	$-\ln\left(-\sqrt{x^2+1}+x\right)-\frac{\text{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{3\,\text{RootOf}\left(\_Z^2-2\right)x^2+4\sqrt{x^2+1}\,x+\text{RootOf}\left(\_Z^2-2\right)}{(x-1)(x+1)}\right)}{2}$
default	$-\frac{\sqrt{(x+1)^2-2x}}{2}+\text{arcsinh}(x)+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{(2-2x)\sqrt{2}}{4\sqrt{(x+1)^2-2x}}\right)}{2}+\frac{\sqrt{(x-1)^2+2x}}{2}-\frac{\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((x+1)^2-2*x)^(1/2)+arcsinh(x)+1/2*2^(1/2)*arctanh(1/4*(2-2*x)*2^(1/2)
/((x+1)^2-2*x)^(1/2))+1/2*((x-1)^2+2*x)^(1/2)-1/2*2^(1/2)*arctanh(1/4*(2+2*
x)*2^(1/2)/((x-1)^2+2*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(21) = 42$.
time = 0.50, size = 59, normalized size = 2.19

$$-\frac{1}{2}\sqrt{2}\,\operatorname{arsinh}\left(\frac{2\,x}{|2\,x+2|}-\frac{2}{|2\,x+2|}\right)-\frac{1}{2}\sqrt{2}\,\operatorname{arsinh}\left(\frac{2\,x}{|2\,x-2|}+\frac{2}{|2\,x-2|}\right)+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arcsinh(2*x/abs(2*x + 2) - 2/abs(2*x + 2)) - 1/2*sqrt(2)*arcsi
nh(2*x/abs(2*x - 2) + 2/abs(2*x - 2)) + arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(21) = 42$.
time = 0.43, size = 67, normalized size = 2.48

$$\frac{1}{2}\sqrt{2}\,\log\left(\frac{9\,x^2-2\,\sqrt{2}\,(3\,x^2+1)-2\,\sqrt{x^2+1}\,\left(3\,\sqrt{2}\,x-4\,x\right)+3}{x^2-1}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 1) - 2*sqrt(x^2 + 1)*(3*sqrt(2)
*x - 4*x) + 3)/(x^2 - 1)) - log(-x + sqrt(x^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**2-1),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/((x - 1)*(x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(21) = 42$.
time = 0.87, size = 70, normalized size = 2.59

$$-\frac{1}{2}\sqrt{2}\,\log\left(\frac{\left|2\left(x-\sqrt{x^2+1}\right)^2-4\,\sqrt{2}-6\right|}{\left|2\left(x-\sqrt{x^2+1}\right)^2+4\,\sqrt{2}-6\right|}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(2*(x - sqrt(x^2 + 1))^2 - 4*sqrt(2) - 6)/abs(2*(x - sq
rt(x^2 + 1))^2 + 4*sqrt(2) - 6)) - log(-x + sqrt(x^2 + 1))
```

Mupad [B]
time = 0.17, size = 59, normalized size = 2.19

$$\operatorname{asinh}(x)+\frac{\sqrt{2}\,\left(\ln\left(x-1\right)-\ln\left(x+\sqrt{2}\,\sqrt{x^2+1}+1\right)\right)}{2}-\frac{\sqrt{2}\,\left(\ln\left(x+1\right)-\ln\left(\sqrt{2}\,\sqrt{x^2+1}-x+1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(x^2 - 1),x)
```

```
[Out] asinh(x) + (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/2
- (2^(1/2)*(log(x + 1) - log(2^(1/2)*(x^2 + 1)^(1/2) - x + 1)))/2
```

3.73 $\int \frac{\sqrt{1-x^2}}{-1+2x^2} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

```
[Out] -1/2*arcsin(x)-1/2*arctanh(x/(-x^2+1)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {399, 222, 385, 213}

$$-\frac{\text{ArcSin}(x)}{2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\,(-1+2x^2)}\,dx\\
&= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\
&= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 49, normalized size = 1.96

$$
-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}i\tan^{-1}\left(1-2x^2-2ix\sqrt{1-x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -1/2*ArcTan[x/Sqrt[1 - x^2]] - (I/2)*ArcTan[1 - 2*x^2 - (2*I)*x*Sqrt[1 - x^
2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(19)=38$.
time = 0.22, size = 187, normalized size = 7.48

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}+\frac{\ln\left(-\frac{2\sqrt{-x^2+1}\,x-1}{2x^2-1}\right)}{4}$
default	$\frac{\sqrt{2}\left(\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4}-\frac{\sqrt{2}\arcsin(x)}{4}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(1-\left(x-\right.\right.}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2}}\right.}{4}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(2*x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(1/4*(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1
/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh((1-(x-1/2*2^(1/2))*2^(1/2))*2^(1/2
)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)))-1/2*2^(1/2)*(1
/4*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)+1/4*2^(1/2)*arc
sin(x)-1/4*2^(1/2)*arctanh((1+(x+1/2*2^(1/2))*2^(1/2))*2^(1/2)/(-4*(x+1/2*2
^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(19) = 38$.
time = 0.50, size = 110, normalized size = 4.40

$$-\frac{1}{8}\sqrt{2}\left(2\sqrt{2}\arcsin(x)-\sqrt{2}\log\left(\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^2+1}}{|4x+2\sqrt{2}|}+\frac{1}{|4x+2\sqrt{2}|}\right)+\sqrt{2}\log\left(-\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^2+1}}{|4x-2\sqrt{2}|}+\frac{1}{|4x-2\sqrt{2}|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(2*sqrt(2)*arcsin(x) - sqrt(2)*log(1/4*sqrt(2) + sqrt(2)*sqrt(
-x^2 + 1)/abs(4*x + 2*sqrt(2)) + 1/abs(4*x + 2*sqrt(2))) + sqrt(2)*log(-1/4
*sqrt(2) + sqrt(2)*sqrt(-x^2 + 1)/abs(4*x - 2*sqrt(2)) + 1/abs(4*x - 2*sqrt
(2))))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(19) = 38$.
time = 0.47, size = 74, normalized size = 2.96

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(-(x^2 + sqrt(-x^2 + 1)*(x + 1) - x
 - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) + x - 1)/x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+1)}}{2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(2*x**2-1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(2*x**2 - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(19) = 38.

time = 0.83, size = 118, normalized size = 4.72

$$-\frac{1}{4}\pi\mathrm{sgn}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)-\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)
/x + 2)) + 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2
))
```

Mupad [B]

time = 5.35, size = 85, normalized size = 3.40

$$-\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4}+\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(2*x^2 - 1),x)
```

```
[Out] log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/
4 - log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/
2))/4 - asin(x)/2
```

3.74 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=169

$$\frac{d(44b^2c^2-44abcd+15a^2d^2)\,x\sqrt{a+bx^2}}{48b^3}+\frac{5d(2bc-ad)x\sqrt{a+bx^2}\,(c+dx^2)}{24b^2}+\frac{dx\sqrt{a+bx^2}\,(c+dx^2)^2}{6b}+\frac{(2bc}{}$$

```
[Out] 1/16*(-a*d+2*b*c)*(5*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))/b^(7/2)+1/48*d*(15*a^2*d^2-44*a*b*c*d+44*b^2*c^2)*x*(b*x^2+a)^(1/
2)/b^3+5/24*d*(-a*d+2*b*c)*x*(d*x^2+c)*(b*x^2+a)^(1/2)/b^2+1/6*d*x*(d*x^2+c
)^2*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {427, 542, 396, 223, 212}

$$\frac{(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}+\frac{5dx\sqrt{a+bx^2}\,(c+dx^2)(2bc-ad)}{24b^2}+\frac{dx\sqrt{a+bx^2}\,(c+dx^2)^2}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(48*b^3) + (5*
d*(2*b*c - a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*b^2) + (d*x*Sqrt[a + b*x
^2]*(c + d*x^2)^2)/(6*b) + ((2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\,(c+dx^2)^2}{6b} + \frac{\int \frac{(c+dx^2)\left(c(6bc-ad)+5d(2bc-ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{5d(2bc-ad)x\sqrt{a+bx^2}\,(c+dx^2)}{24b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)^2}{6b} + \frac{\int \frac{c(24b^2c^2-14abcd+5a^2d^2)+d}{\sqrt{a+}}}{24}$$

$$= \frac{d(44b^2c^2-44abcd+15a^2d^2)\,x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\,(c+dx^2)}{24b^2} + \frac{dx\sqrt{a}}{}$$

$$= \frac{d(44b^2c^2-44abcd+15a^2d^2)\,x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\,(c+dx^2)}{24b^2} + \frac{dx\sqrt{a}}{}$$

$$= \frac{d(44b^2c^2-44abcd+15a^2d^2)\,x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\,(c+dx^2)}{24b^2} + \frac{dx\sqrt{a}}{}$$

Mathematica [A]
time = 0.16, size = 138, normalized size = 0.82

$$\frac{\sqrt{b}\,dx\sqrt{a+bx^2}\,(15a^2d^2-2abd(27c+5dx^2)+4b^2(18c^2+9cdx^2+2d^2x^4))+(-48b^3c^3+72ab^2c^2d-54a^2bcd^2+15a^3d^3)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[a + b*x^2]*(15*a^2*d^2 - 2*a*b*d*(27*c + 5*d*x^2) + 4*b^2
*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4)) + (-48*b^3*c^3 + 72*a*b^2*c^2*d - 54*a^2
*b*c*d^2 + 15*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(7/2))
```

Maple [A]
time = 0.06, size = 227, normalized size = 1.34

method	result
risch	$\frac{dx\left(8b^2d^2x^4-10ab\,d^2x^2+36b^2cd\,x^2+15a^2d^2-54abcd+72b^2c^2\right)\sqrt{b\,x^2+a}}{48b^3}-\frac{5\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a^3d^3}{16b^{\frac{7}{2}}}+\frac{9\ln\left(x\sqrt{b}\right.}{}$
default	$d^3\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+3c\,d^2\left(\frac{x^3\sqrt{b\,x}}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(
1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+3*c*
d^2*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+3*c^2*d*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b
^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+c^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(
1/2)
```

Maxima [A]
time = 0.28, size = 199, normalized size = 1.18

$$\frac{\sqrt{bx^2+a}\,d^3x^5}{6\,b}+\frac{3\,\sqrt{bx^2+a}\,cd^2x^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,ad^3x^3}{24\,b^2}+\frac{3\,\sqrt{bx^2+a}\,c^2dx}{2\,b}-\frac{9\,\sqrt{bx^2+a}\,acd^2x}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}\,a^2d^3x}{16\,b^3}+\frac{c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{3\,ac^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{9\,a^2cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*d^3*x^5/b + 3/4*sqrt(b*x^2 + a)*c*d^2*x^3/b - 5/24*sqrt
(b*x^2 + a)*a*d^3*x^3/b^2 + 3/2*sqrt(b*x^2 + a)*c^2*d*x/b - 9/8*sqrt(b*x^2
```

```
+ a)*a*c*d^2*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*d^3*x/b^3 + c^3*arcsinh(b*x/s
qrt(a*b))/sqrt(b) - 3/2*a*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 9/8*a^2*c*
d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*d^3*arcsinh(b*x/sqrt(a*b))/b^
(7/2)
```

Fricas [A]
time = 0.57, size = 300, normalized size = 1.78

$$\left[-\frac{3\,(16\,b^{3}c^{3}-24\,ab^{2}c^{2}d+18\,a^{2}bcd^{2}-5\,a^{3}d^{3})\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(8\,b^{3}d^{3}x^{5}+2\,(18\,b^{3}cd^{2}-5\,ab^{2}d^{3})x^{3}+3\,(24\,b^{3}c^{2}d-18\,ab^{2}cd^{2}+5\,a^{2}bd^{3})x)\sqrt{bx^{2}+a}}{96\,b^{4}},-\frac{3\,(16\,b^{3}c^{3}-24\,ab^{2}c^{2}d+18\,a^{2}bcd^{2}-5\,a^{3}d^{3})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(8\,b^{3}d^{3}x^{5}+2\,(18\,b^{3}cd^{2}-5\,ab^{2}d^{3})x^{3}+3\,(24\,b^{3}c^{2}d-18\,ab^{2}cd^{2}+5\,a^{2}bd^{3})x)\sqrt{bx^{2}+a}}{48\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b
)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*d^3*x^5 + 2*(1
8*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2*c*d^2 + 5*a^2*b
*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/48*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a
^2*b*c*d^2 - 5*a^3*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^
3*d^3*x^5 + 2*(18*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2
*c*d^2 + 5*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A]
time = 13.17, size = 400, normalized size = 2.37

$$\frac{5a^{\frac{5}{2}}d^{3}x}{16b^{3}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{9a^{\frac{3}{2}}cd^{2}x}{8b^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5a^{\frac{3}{2}}d^{3}x^{3}}{48b^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3\sqrt{a}\,c^{2}dx\sqrt{1+\frac{bx^{2}}{a}}}{2b}-\frac{3\sqrt{a}\,cd^{2}x^{3}}{8b\sqrt{1+\frac{bx^{2}}{a}}}-\frac{\sqrt{a}\,d^{3}x^{5}}{24b\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5a^{3}d^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{9a^{2}cd^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{3ac^{2}d\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c^{3}\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b<0\\\frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b>0\\\frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b>0\wedge a<0\end{cases}\right)+\frac{3cd^{2}x^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{d^{3}x^{7}}{6\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*d**3*x/(16*b**3*sqrt(1 + b*x**2/a)) - 9*a**(3/2)*c*d**2*x/(8*b**
2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*d**3*x**3/(48*b**2*sqrt(1 + b*x**2/a)) +
 3*sqrt(a)*c**2*d*x*sqrt(1 + b*x**2/a)/(2*b) - 3*sqrt(a)*c*d**2*x**3/(8*b*s
qrt(1 + b*x**2/a)) - sqrt(a)*d**3*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*d
**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 9*a**2*c*d**2*asinh(sqrt(b)*x/
sqrt(a))/(8*b**(5/2)) - 3*a*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) +
c**3*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)),
(sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acos
h(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + 3*c*d**2*x**5/(4*sqrt(a)*sq
rt(1 + b*x**2/a)) + d**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.86, size = 150, normalized size = 0.89

$$\frac{1}{48}\left(2\left(\frac{4\,d^{3}x^{2}}{b}+\frac{18\,b^{4}cd^{2}-5\,ab^{3}d^{3}}{b^{5}}\right)x^{2}+\frac{3\,(24\,b^{4}c^{2}d-18\,ab^{3}cd^{2}+5\,a^{2}b^{2}d^{3})}{b^{5}}\right)\sqrt{bx^{2}+a}\,x-\frac{(16\,b^{3}c^{3}-24\,ab^{2}c^{2}d+18\,a^{2}bcd^{2}-5\,a^{3}d^{3})\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^3*x^2/b + (18*b^4*c*d^2 - 5*a*b^3*d^3)/b^5)*x^2 + 3*(24*b^4*c^
2*d - 18*a*b^3*c*d^2 + 5*a^2*b^2*d^3)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3
*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(7/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^3}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(1/2), x)
```

3.75 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=108

$$\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2}+\frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b}+\frac{(8b^2c^2-8abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5
/2)+3/8*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b^2+1/4*d*x*(d*x^2+c)*(b*x^2+a)^(1
/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 223, 212}

$$\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{3dx\sqrt{a+bx^2}\,(2bc-ad)}{8b^2}+\frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (3*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b^2) + (d*x*Sqrt[a + b*x^2]*(c + d
*x^2))/(4*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} + \frac{\int \frac{c(4bc-ad)+3d(2bc-ad)x^2}{\sqrt{a+bx^2}}\,dx}{4b}$$
$$= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} - \frac{(3ad(2bc-ad)-2bc(4bc-ad))\int}{8b^2}$$
$$= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} - \frac{(3ad(2bc-ad)-2bc(4bc-ad))\mathrm{Su}}{8b^2}$$
$$= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}\,(c+dx^2)}{4b} + \frac{(8b^2c^2-8abcd+3a^2d^2)\tanh^{-1}\Big(}{8b^{5/2}}$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.83

$$\frac{dx\sqrt{a+bx^2}\,(8bc-3ad+2bdx^2)}{8b^2} + \frac{(-8b^2c^2+8abcd-3a^2d^2)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
[Out] (d*x*Sqrt[a + b*x^2]*(8*b*c - 3*a*d + 2*b*d*x^2))/(8*b^2) + ((-8*b^2*c^2 +
8*a*b*c*d - 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A]
time = 0.06, size = 133, normalized size = 1.23

method	result

risch	$-\frac{dx(-2bd\,x^2+3ad-8bc)\sqrt{b\,x^2+a}}{8b^2}+\frac{3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a^2d^2}{8b^{\frac{5}{2}}}-\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)acd}{b^{\frac{3}{2}}}+\frac{c^2\ln\left(x\sqrt{\right.}\right.}{}$
default	$d^2\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+2cd\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+2*c*d*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(
3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+c^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/
2)
```

Maxima [A]
time = 0.30, size = 109, normalized size = 1.01

$$\frac{\sqrt{bx^2+a}\,d^2x^3}{4\,b}+\frac{\sqrt{bx^2+a}\,cdx}{b}-\frac{3\,\sqrt{bx^2+a}\,ad^2x}{8\,b^2}+\frac{c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{acd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{3\,a^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*d^2*x^3/b + sqrt(b*x^2 + a)*c*d*x/b - 3/8*sqrt(b*x^2 +
a)*a*d^2*x/b^2 + c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - a*c*d*arcsinh(b*x/sqr
t(a*b))/b^(3/2) + 3/8*a^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.51, size = 192, normalized size = 1.78

$$\left[\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,b^2d^2x^3+(8\,b^2cd-3\,abd^2)x)\sqrt{bx^2+a}}{16\,b^3},-\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,b^2d^2x^3+(8\,b^2cd-3\,abd^2)x)\sqrt{bx^2+a}}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)*sqr
t(b*x^2 + a))/b^3, -1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)
*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 4.42, size = 238, normalized size = 2.20

$$-\frac{3a^{\frac{3}{2}}d^2x}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,cdx\sqrt{1+\frac{bx^2}{a}}}{b}-\frac{\sqrt{a}\,d^2x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{acd\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+c^2\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+\frac{d^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*d**2*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*d*x*sqrt(1 + b*x
**2/a)/b - sqrt(a)*d**2*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a**2*d**2*asinh(s
qrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*c*d*asinh(sqrt(b)*x/sqrt(a))/b**(3/2) +
c**2*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)),
(sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acos
h(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + d**2*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a))
```

Giac [A]
time = 1.08, size = 90, normalized size = 0.83

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,d^2x^2}{b}+\frac{8\,b^2cd-3\,abd^2}{b^3}\right)x-\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d^2*x^2/b + (8*b^2*c*d - 3*a*b*d^2)/b^3)*x - 1/8*(8*
b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^2}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(1/2), x)
```

3.76 $\int \frac{c+dx^2}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=58

$$\frac{dx\sqrt{a+bx^2}}{2b} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*d*x*(b*x^2+
a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 223, 212}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}} + \frac{dx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{2b} - \frac{(-2bc+ad)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{dx\sqrt{a+bx^2}}{2b} - \frac{(-2bc+ad)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{dx\sqrt{a+bx^2}}{2b} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.02

$$\frac{dx\sqrt{a+bx^2}}{2b} + \frac{(-2bc+ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((-2*b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Maple [A]
time = 0.06, size = 63, normalized size = 1.09

method	result	size
risch	$\frac{dx\sqrt{b\,x^2+a}}{2b} - \frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)ad}{2b^{\frac{3}{2}}} + \frac{c\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	62
default	$d\left(\frac{x\sqrt{b\,x^2+a}}{2b} - \frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right) + \frac{c\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+c*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.81

$$\frac{\sqrt{bx^2+a}\,dx}{2\,b} + \frac{c\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{ad\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*d*arcs
inh(b*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 0.55, size = 113, normalized size = 1.95

$$\left[\frac{2\,\sqrt{bx^2+a}\,bdx-(2\,bc-ad)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}\,bdx-(2\,bc-ad)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A]
time = 1.60, size = 126, normalized size = 2.17

$$\frac{\sqrt{a}\,dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*sqrt(1 + b*x**2/a)/(2*b) - a*d*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)
*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
```

Giac [A]
time = 0.82, size = 49, normalized size = 0.84

$$\frac{\sqrt{bx^2+a}\,dx}{2\,b}-\frac{(2\,bc-ad)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b - 1/2*(2*b*c - a*d)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

Mupad [B]
time = 5.51, size = 86, normalized size = 1.48

$$\begin{cases} \frac{d\,x^3+3\,c\,x}{3\,\sqrt{a}} & \text{if } b=0 \\ \frac{c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{a\,d\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{d\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*c*x + d*x^3)/(3*a^(1/2)), b ~= 0, (c*log(b^(1/2)*x + (
a + b*x^2)^(1/2)))/b^(1/2) - (a*d*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(
2*b^(3/2)) + (d*x*(a + b*x^2)^(1/2))/(2*b))
```

3.77 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.31, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 0.53, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 0.47, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A]
time = 0.67, size = 37, normalized size = 1.48

$$\frac{1}{2}\sqrt{bx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [B]
time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

3.78 $\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,\sqrt{bc-ad}}$$

```
[Out] arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {385, 214}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c -
a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 1.37

$$-\frac{\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{\sqrt{c}\,\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] -(ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c
) + a*d])]/(Sqrt[c]*Sqrt[-(b*c) + a*d]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(39) = 78$.
time = 0.06, size = 300, normalized size = 6.12

method	result
default	$-\frac{\ln\left(\frac{\frac{2ad-2bc}{d}+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-b}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,\sqrt{\frac{ad-bc}{d}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2/(-c
*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(39) = 78$.
time = 0.55, size = 241, normalized size = 4.92

$$\left[\frac{\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ad)x^3+acx)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,\sqrt{bc^2-acd}}, -\frac{\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\,((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)}{2\,(bc^2-acd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(b*c^2 - a*c*d), -1/2*sqrt(-b*c^2
 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x))/(b*c^2 - a*c*d)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)), x)
```

Giac [A]
time = 0.71, size = 70, normalized size = 1.43

$$-\frac{\sqrt{b}\,\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(
-b^2*c^2 + a*b*c*d))/sqrt(-b^2*c^2 + a*b*c*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)}{\sqrt{c\,(a\,d-b\,c)}} & \text{if } 0<a\,d-b\,c \\ \frac{\ln\left(\frac{\sqrt{c\,(b\,x^2+a)}+x\,\sqrt{b\,c-a\,d}}{\sqrt{c\,(b\,x^2+a)}-x\,\sqrt{b\,c-a\,d}}\right)}{2\,\sqrt{-c\,(a\,d-b\,c)}} & \text{if } a\,d-b\,c<0 \\ \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)}\,dx & \text{if } a\,d-b\,c\notin\mathbb{R}\vee a\,d=b\,c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)),x)
```

```
[Out] piecewise(0 < a*d - b*c, atan((x*(a*d - b*c)^(1/2))/(c^(1/2)*(a + b*x^2)^(1
/2)))/(c*(a*d - b*c))^(1/2), a*d - b*c < 0, log(((c*(a + b*x^2))^(1/2) + x*
(- a*d + b*c)^(1/2))/((c*(a + b*x^2))^(1/2) - x*(- a*d + b*c)^(1/2)))/(2*(-
c*(a*d - b*c))^(1/2)), ~in(a*d - b*c, 'real') | a*d == b*c, int(1/((a + b*x
^2)^(1/2)*(c + d*x^2)), x))
```

3.79 $\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\,(c+dx^2)}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2
)/(-a*d+b*c)^(3/2)-1/2*d*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 385, 214}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*Arc
Tanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx &= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\,(c+dx^2)}+\frac{(2bc-ad)\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{2c(bc-ad)}\\ &= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\,(c+dx^2)}+\frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2c(bc-ad)}\\ &= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\,(c+dx^2)}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 118, normalized size = 1.17

$$-\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\,(c+dx^2)}+\frac{(2bc-ad)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{2c^{3/2}(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*Arc
Tan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a
*d])])/(2*c^(3/2)*(-(b*c) + a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(85) = 170$.
time = 0.06, size = 810, normalized size = 8.02

method	result

default	$-\frac{\ln\left(\frac{\frac{2ad-2bc}{d}+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{4c\sqrt{-cd}\sqrt{\frac{ad-bc}{d}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/4/d
/c*(-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-
b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a
*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))+1/4/c/(-c*d)^(1/2)/((a*d-b*c)/
d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*
c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-1/4/d/c*(-1/(a*d-b*c)*d/(x+(-c*d)^(1
/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(1/2)-b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d
-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*
d)^(1/2)/d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(85) = 170$.
time = 0.58, size = 463, normalized size = 4.58

$$\left[-\frac{4\,(bc^2d-acd^2)\sqrt{bx^2+a}\,x-(2\,bc^2-acd+(2\,bcd-ad^2)x^2)\sqrt{bc^2-acd}\,\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ad)x^3+acx)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{8\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x^2)},\ -\frac{2\,(bc^2d-acd^2)\sqrt{bx^2+a}\,x+(2\,bc^2-acd+(2\,bcd-ad^2)x^2)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\,((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)}{4\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x - (2*b*c^2 - a*c*d + (2*b*c*
d - a*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*
x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b^2*
c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)
*x^2), -1/4*(2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x + (2*b*c^2 - a*c*d + (
2*b*c*d - a*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*
((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c
^2 - a^2*c*d)*x)))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*
b*c^3*d^2 + a^2*c^2*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^2}\,\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(85) = 170.
time = 0.70, size = 242, normalized size = 2.40

$$\frac{1}{2}\,b^{\frac{3}{2}}\left(\frac{(2\,bc-ad)\arctan\left(-\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{(b^2c^2-abcd)\sqrt{-b^2c^2+abcd}}-\frac{2\left(2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 bc-\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 ad+a^2d\right)}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 ad+a^2d\right)(b^2c^2-abcd)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^(3/2)*((2*b*c - a*d)*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d +
 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - a*b*c*d)*sqrt(-b^2*c^2
+ a*b*c*d)) - 2*(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - (sqrt(b)*x - sqrt(
b*x^2 + a))^2*a*d + a^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d
)*(b^2*c^2 - a*b*c*d)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2), x)
```

3.80 $\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\,(c+dx^2)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\,(c+dx^2)}+\frac{(8b^2c^2-8abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}$$

```
[Out] 1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x
^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(5/2)-1/4*d*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/
(d*x^2+c)^2-3/8*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A]
time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {425, 541, 12, 385, 214}

$$\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}-\frac{3dx\sqrt{a+bx^2}\,(2bc-ad)}{8c^2\,(c+dx^2)\,(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{4c\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^2) - (3*d*(2*b*c - a*
d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)^2*(c + d*x^2)) + ((8*b^2*c^2 - 8*a
*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])
/(8*c^(5/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx = -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\,(c+dx^2)^2} + \frac{\int \frac{4bc-3ad-2bdx^2}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\,(c+dx^2)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\,(c+dx^2)} + \frac{\int \frac{8b^2c^2-8abcd+3a^2d^2}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{8c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\,(c+dx^2)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\,(c+dx^2)} + \frac{(8b^2c^2-8abcd+3a^2d^2 \ldots}{8c^2(bc \ldots}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\,(c+dx^2)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\,(c+dx^2)} + \frac{(8b^2c^2-8abcd+3a^2d^2 \ldots}{\ldots}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\,(c+dx^2)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\,(c+dx^2)} + \frac{(8b^2c^2-8abcd+3a^2d^2 \ldots}{8c^{5/2}(\ldots}$$

Mathematica [A]
time = 0.75, size = 160, normalized size = 0.98

$$\frac{dx\sqrt{a+bx^2}\,(-2bc(4c+3dx^2)+ad(5c+3dx^2))}{8c^2(bc-ad)^2\,(c+dx^2)^2} - \frac{(8b^2c^2-8abcd+3a^2d^2)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{8c^{5/2}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*(-2*b*c*(4*c + 3*d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2
*(b*c - a*d)^2*(c + d*x^2)^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTan
[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d]
)])/(8*c^(5/2)*(-(b*c) + a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1842 vs. $2(143) = 286$.
time = 0.07, size = 1843, normalized size = 11.31

method	result	size
default	Expression too large to display	1843

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d/c/(-c*d)^(1/2)*(-1/2/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)^2*((x-(-c*d)^(1/
2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/2*b*(-
c*d)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+b*(-c*d)^(1/2
)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))+1/2*b/(a*d-b
*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1
/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))-3/16/(-c*d)^(1/2)/
c^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/16/(-c*d)^(1/2)/c^2
/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-3/16/d/c^2*(-1/(a*d-b*c)
*d/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2)+b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln
((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)
*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(1/2))/(x-(-c*d)^(1/2)/d)))-3/16/d/c^2*(-1/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)*
((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(1/2)-b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2
*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2
)/d)))+1/8/d/c/(-c*d)^(1/2)*(-1/2/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)^2*((x+(-c*
d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/
2*b*(-c*d)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-b*(-c*d
```

```
)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))+1/2*b/
(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c
*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(143) = 286$.
time = 0.76, size = 864, normalized size = 5.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
[Out] [1/32*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2
*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a
*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^3*d^2 -
 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c^3*d^2 + 5*a^2*c^2
*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*
c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x
^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x^2),
-1/16*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*
c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(
3*(2*b^2*c^3*d^2 - 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c
^3*d^2 + 5*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^
2*b*c^6*d^2 - a^3*c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^
4 - a^3*c^3*d^5)*x^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a
^3*c^4*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\,\left(c+dx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(143) = 286.
time = 2.31, size = 538, normalized size = 3.30

$$-\frac{1}{8}\,b^{\frac{5}{2}}\left(\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{\left(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2\right)\sqrt{-b^2c^2+abcd}}+\frac{2\left(8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^2c^2d-8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6abcd^2+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2d^3+48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^3c^3-72\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4ab^2c^2d+42\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2bcd^2-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^3d^3+40\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2b^2c^2d-40\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^3bcd^2+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^4d^3+6\,a^4bcd^2-3\,a^5d^3\right)}{\left(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2\right)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*b^(5/2)*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*arctan(1/2*((sqrt(b)*x -
sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^4 - 2
*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) + 2*(8*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*b^2*c^2*d - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c*d
^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*d^3 + 48*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*b^3*c^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c^2*d + 42*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^2*b*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^3*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^2*c^2*d - 40*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*a^3*b*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*d
^3 + 6*a^4*b*c*d^2 - 3*a^5*d^3)/((b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2
)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*
c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3),x)
```

```
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3), x)
```

3.81 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4}-\frac{d(24b^2c^2-64abcd+35a^2d^2)\,x\sqrt{a+bx^2}\,(c+}{24ab^3}$$

```
[Out] 1/16*d*(-35*a^3*d^3+120*a^2*b*c*d^2-144*a*b^2*c^2*d+64*b^3*c^3)*arctanh(x*b
^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)+(-a*d+b*c)*x*(d*x^2+c)^3/a/b/(b*x^2+a)^(1/2
)-1/48*d*(-105*a^3*d^3+290*a^2*b*c*d^2-248*a*b^2*c^2*d+48*b^3*c^3)*x*(b*x^2
+a)^(1/2)/a/b^4-1/24*d*(35*a^2*d^2-64*a*b*c*d+24*b^2*c^2)*x*(d*x^2+c)*(b*x^
2+a)^(1/2)/a/b^3-1/6*d*(-7*a*d+6*b*c)*x*(d*x^2+c)^2*(b*x^2+a)^(1/2)/a/b^2
```

Rubi [A]

time = 0.18, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 542, 396, 223, 212}

$$-\frac{dx\sqrt{a+bx^2}\,(c+dx^2)\,(35a^2d^2-64abcd+24b^2c^2)}{24ab^3}+\frac{d(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{dx\sqrt{a+bx^2}\,(-105a^3d^3+290a^2bcd^2-248ab^2c^2d+48b^3c^3)}{48ab^4}-\frac{dx\sqrt{a+bx^2}\,(c+dx^2)^2\,(6bc-7ad)}{6ab^2}+\frac{x(c+dx^2)^3\,(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/48*(d*(48*b^3*c^3 - 248*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3)*x*S
qrt[a + b*x^2])/(a*b^4) - (d*(24*b^2*c^2 - 64*a*b*c*d + 35*a^2*d^2)*x*Sqrt[
a + b*x^2]*(c + d*x^2))/(24*a*b^3) - (d*(6*b*c - 7*a*d)*x*Sqrt[a + b*x^2]*(
c + d*x^2)^2)/(6*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2
]) + (d*(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{(c+dx^2)^2(acd-d(6bc-7ad)x^2)}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= -\frac{d(6bc-7ad)x\sqrt{a+bx^2}\,(c+dx^2)^2}{6ab^2} + \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{(c+dx^2)(acd(12bc-7ad)-}{\sqrt{a}}$$

$$= -\frac{d(24b^2c^2-64abcd+35a^2d^2)\,x\sqrt{a+bx^2}\,(c+dx^2)}{24ab^3} - \frac{d(6bc-7ad)x\sqrt{a+bx^2}\,(c+d}{6ab^2}$$

$$= -\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4} - \frac{d(24b^2c^2-64abcd+3}{}$$

$$= -\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4} - \frac{d(24b^2c^2-64abcd+3}{}$$

$$= -\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4} - \frac{d(24b^2c^2-64abcd+3}{}$$

Mathematica [A]
time = 0.41, size = 198, normalized size = 0.77

$$\frac{\frac{\sqrt{b}\,x(48b^4c^4+105a^4d^4+5a^3bd^3(-72c+7dx^2)-2a^2b^2d^2(-216c^2+60cdx^2+7d^2x^4)+8ab^3d(-24c^3+18c^2dx^2+6cd^2x^4+d^3x^6))}{a\sqrt{a+bx^2}}+3d(-64b^3c^3+144ab^2c^2d-120a^2bcd^2+35a^3d^3)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{48b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[b]*x*(48*b^4*c^4 + 105*a^4*d^4 + 5*a^3*b*d^3*(-72*c + 7*d*x^2) - 2*a
^2*b^2*d^2*(-216*c^2 + 60*c*d*x^2 + 7*d^2*x^4) + 8*a*b^3*d*(-24*c^3 + 18*c^
2*d*x^2 + 6*c*d^2*x^4 + d^3*x^6)))/(a*Sqrt[a + b*x^2]) + 3*d*(-64*b^3*c^3 +
 144*a*b^2*c^2*d - 120*a^2*b*c*d^2 + 35*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/(48*b^(9/2))
```

Maple [A]
time = 0.10, size = 331, normalized size = 1.29

method	result
risch	$\frac{d^2x\left(8b^2d^2x^4-22ab\,d^2x^2+48b^2cd\,x^2+57a^2d^2-168abcd+144b^2c^2\right)\sqrt{b\,x^2+a}}{48b^4}+\frac{x\,a^3d^4}{b^4\sqrt{b\,x^2+a}}-\frac{4x\,a^2c\,d^3}{b^3\sqrt{b\,x^2+a}}+\frac{6\ldots}{b^2\sqrt{\ldots}}$
default	$d^4\left(\frac{x^7}{6b\sqrt{b\,x^2+a}}-\frac{7a\left(\frac{x^5}{4b\sqrt{b\,x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)}{6b}\right)\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(1/6*x^7/b/(b*x^2+a)^(1/2)-7/6*a/b*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(
1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2)))))+4*c*d^3*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(1/2*x^
3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b
*x^2+a)^(1/2)))))+6*c^2*d^2*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2
+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+4*c^3*d*(-x/b/(b*x^2+a)
^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+c^4*x/a/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 311, normalized size = 1.21

$$\frac{d^4x^7}{6\sqrt{bx^2+a}\,b}+\frac{cd^3x^5}{\sqrt{bx^2+a}\,b}-\frac{7\,ad^4x^5}{24\sqrt{bx^2+a}\,b^2}+\frac{3\,c^2d^2x^3}{\sqrt{bx^2+a}\,b}-\frac{5\,acd^3x^3}{2\sqrt{bx^2+a}\,b^2}+\frac{35\,a^2d^4x^3}{48\sqrt{bx^2+a}\,b^3}+\frac{c^4x}{\sqrt{bx^2+a}\,a}-\frac{4\,c^3dx}{\sqrt{bx^2+a}\,b}+\frac{9\,ac^2d^2x}{\sqrt{bx^2+a}\,b^2}-\frac{15\,a^2cd^3x}{2\sqrt{bx^2+a}\,b^3}+\frac{35\,a^3d^4x}{16\sqrt{bx^2+a}\,b^4}+\frac{4\,c^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{9\,ac^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}+\frac{15\,a^2cd^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}-\frac{35\,a^3d^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*d^4*x^7/(sqrt(b*x^2 + a)*b) + c*d^3*x^5/(sqrt(b*x^2 + a)*b) - 7/24*a*d^
4*x^5/(sqrt(b*x^2 + a)*b^2) + 3*c^2*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/2*a*c*d
^3*x^3/(sqrt(b*x^2 + a)*b^2) + 35/48*a^2*d^4*x^3/(sqrt(b*x^2 + a)*b^3) + c^
4*x/(sqrt(b*x^2 + a)*a) - 4*c^3*d*x/(sqrt(b*x^2 + a)*b) + 9*a*c^2*d^2*x/(sq
rt(b*x^2 + a)*b^2) - 15/2*a^2*c*d^3*x/(sqrt(b*x^2 + a)*b^3) + 35/16*a^3*d^4
*x/(sqrt(b*x^2 + a)*b^4) + 4*c^3*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9*a*c^2
*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 15/2*a^2*c*d^3*arcsinh(b*x/sqrt(a*b))
/b^(7/2) - 35/16*a^3*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Fricas [A]
time = 0.58, size = 584, normalized size = 2.27

$$\left[-\frac{3\,(64\,a^2b^3c^3d-144\,a^3b^2c^2d^2+120\,a^4bcd^3-35\,a^5d^4+(64\,ab^4c^3d-144\,a^2b^3c^2d^2+120\,a^3b^2cd^3-35\,a^4bd^4)x^2)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(8\,ab^4d^4x^7+2\,(24\,ab^4cd^3-7\,a^2b^3d^4)x^5+(144\,ab^4c^2d^2-120\,a^2b^3cd^3+35\,a^3b^2d^4)x^3+3\,(16\,b^5c^4-64\,ab^4c^3d+144\,a^2b^3c^2d^2-120\,a^3b^2cd^3+35\,a^4bd^4)x)\sqrt{bx^2+a}}{96\,(ab^6x^2+a^2b^5)},\ -\frac{3\,(64\,a^2b^3c^3d-144\,a^3b^2c^2d^2+120\,a^4bcd^3-35\,a^5d^4+(64\,ab^4c^3d-144\,a^2b^3c^2d^2+120\,a^3b^2cd^3-35\,a^4bd^4)x^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(8\,ab^4d^4x^7+2\,(24\,ab^4cd^3-7\,a^2b^3d^4)x^5+(144\,ab^4c^2d^2-120\,a^2b^3cd^3+35\,a^3b^2d^4)x^3+3\,(16\,b^5c^4-64\,ab^4c^3d+144\,a^2b^3c^2d^2-120\,a^3b^2cd^3+35\,a^4bd^4)x)\sqrt{bx^2+a}}{48\,(ab^6x^2+a^2b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2 + 120*a^4*b*c*d^3 - 35*a^
5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 120*a^3*b^2*c*d^3 - 35*a^4*
b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*
a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a*b^4*c^2*d^2
 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64*a*b^4*c^3*d
 + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*sqrt(b*x^2 +
a))/(a*b^6*x^2 + a^2*b^5), -1/48*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2
 + 120*a^4*b*c*d^3 - 35*a^5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 1
20*a^3*b^2*c*d^3 - 35*a^4*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (8*a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a
*b^4*c^2*d^2 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64
*a*b^4*c^3*d + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*s
qrt(b*x^2 + a))/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{2}\right)^{4}}{\left(a + bx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(3/2), x)
```

Giac [A]
time = 0.65, size = 235, normalized size = 0.91

$$\frac{\left(\left(2\left(\frac{4\,d^{4}x^{2}}{b} + \frac{24\,ab^{6}cd^{3} - 7\,a^{2}b^{5}d^{4}}{ab^{7}}\right)x^{2} + \frac{144\,ab^{6}c^{2}d^{2} - 120\,a^{2}b^{5}cd^{3} + 35\,a^{3}b^{4}d^{4}}{ab^{7}}\right)x^{2} + \frac{3\,(16\,b^{7}c^{4} - 64\,ab^{6}c^{3}d + 144\,a^{2}b^{5}c^{2}d^{2} - 120\,a^{3}b^{4}cd^{3} + 35\,a^{4}b^{3}d^{4})}{ab^{7}}\right)x}{48\,\sqrt{bx^{2} + a}} - \frac{(64\,b^{3}c^{3}d - 144\,ab^{2}c^{2}d^{2} + 120\,a^{2}bcd^{3} - 35\,a^{3}d^{4})\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^{2} + a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*d^4*x^2/b + (24*a*b^6*c*d^3 - 7*a^2*b^5*d^4)/(a*b^7))*x^2 + (14
4*a*b^6*c^2*d^2 - 120*a^2*b^5*c*d^3 + 35*a^3*b^4*d^4)/(a*b^7))*x^2 + 3*(16*
b^7*c^4 - 64*a*b^6*c^3*d + 144*a^2*b^5*c^2*d^2 - 120*a^3*b^4*c*d^3 + 35*a^4
*b^3*d^4)/(a*b^7))*x/sqrt(b*x^2 + a) - 1/16*(64*b^3*c^3*d - 144*a*b^2*c^2*d
^2 + 120*a^2*b*c*d^3 - 35*a^3*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^4}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^4/(a + b*x^2)^(3/2), x)
```

3.82 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3}-\frac{d(4bc-5ad)x\sqrt{a+bx^2}\,(c+dx^2)}{4ab^2}+\frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}}+\frac{3d(8b^2c}{}$$

```
[Out] 3/8*d*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(7/2)+(-a*d+b*c)*x*(d*x^2+c)^2/a/b/(b*x^2+a)^(1/2)-1/8*d*(-5*a*d+2*b*c)*(-
3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/a/b^3-1/4*d*(-5*a*d+4*b*c)*x*(d*x^2+c)*(b*x^
2+a)^(1/2)/a/b^2
```

Rubi [A]
time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 542, 396, 223, 212}

$$\frac{3d(5a^2d^2-12abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}-\frac{dx\sqrt{a+bx^2}\,(2bc-5ad)(4bc-3ad)}{8ab^3}-\frac{dx\sqrt{a+bx^2}\,(c+dx^2)\,(4bc-5ad)}{4ab^2}+\frac{x(c+dx^2)^2\,(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/8*(d*(2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(a*b^3) - (d*(4*
b*c - 5*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*a*b^2) + ((b*c - a*d)*x*(c +
 d*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (3*d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}} + \frac{\int \frac{(c+dx^2)(acd-d(4bc-5ad)x^2)}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= -\frac{d(4bc-5ad)x\sqrt{a+bx^2}\,(c+dx^2)}{4ab^2} + \frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd(8bc-5ad)-d(2bc-5ad)(4}{\sqrt{a+bx^2}}}{4ab^2}$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\,(c+dx^2)}{4ab^2} + \frac{(bc-ad}{ab\sqrt{}}$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\,(c+dx^2)}{4ab^2} + \frac{(bc-ad}{ab\sqrt{}}$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\,(c+dx^2)}{4ab^2} + \frac{(bc-ad}{ab\sqrt{}}$$

Mathematica [A]
time = 0.27, size = 139, normalized size = 0.82

$$\frac{x(8b^3c^3-15a^3d^3+a^2bd^2(36c-5dx^2)+2ab^2d(-12c^2+6cdx^2+d^2x^4))}{8ab^3\sqrt{a+bx^2}} - \frac{3d(8b^2c^2-12abcd+5a^2d^2)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(8*b^3*c^3 - 15*a^3*d^3 + a^2*b*d^2*(36*c - 5*d*x^2) + 2*a*b^2*d*(-12*c^
2 + 6*c*d*x^2 + d^2*x^4)))/(8*a*b^3*Sqrt[a + b*x^2]) - (3*d*(8*b^2*c^2 - 12
*a*b*c*d + 5*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Maple [A]
time = 0.08, size = 215, normalized size = 1.27

method	result
risch	$-\frac{d^{2} x \left(-2 b d \,x^{2}+7 a d -12 b c \right) \sqrt{b \,x^{2}+a}}{8 b^{3}}-\frac{x \,a^{2} d^{3}}{b^{3} \sqrt{b \,x^{2}+a}}+\frac{3 x a c \,d^{2}}{b^{2} \sqrt{b \,x^{2}+a}}-\frac{3 x \,c^{2} d}{b \sqrt{b \,x^{2}+a}}+\frac{15 \ln \left(x \sqrt{b}+\sqrt{b}\right.}{8 b^{\frac{7}{2}}}$
default	$d^{3} \left(\frac{x^{5}}{4 b \sqrt{b \,x^{2}+a}}-\frac{5 a \left(\frac{x^{3}}{2 b \sqrt{b \,x^{2}+a}}-\frac{3 a \left(-\frac{x}{b \sqrt{b \,x^{2}+a}}+\frac{\ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)}{4 b}\right)+3 c \,d^{2} \left(\frac{}{2 b \sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(
-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+3*c*d^2*(1/
2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))))+3*c^2*d*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(
b*x^2+a)^(1/2)))+c^3*x/a/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 197, normalized size = 1.17

$$\frac{d^{3} x^{5}}{4 \sqrt{b x^{2}+a}\, b}+\frac{3 \, c d^{2} x^{3}}{2 \sqrt{b x^{2}+a}\, b}-\frac{5 \, a d^{3} x^{3}}{8 \sqrt{b x^{2}+a}\, b^{2}}+\frac{c^{3} x}{\sqrt{b x^{2}+a}\, a}-\frac{3 \, c^{2} d x}{\sqrt{b x^{2}+a}\, b}+\frac{9 \, a c d^{2} x}{2 \sqrt{b x^{2}+a}\, b^{2}}-\frac{15 \, a^{2} d^{3} x}{8 \sqrt{b x^{2}+a}\, b^{3}}+\frac{3 \, c^{2} d \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{b^{\frac{3}{2}}}-\frac{9 \, a c d^{2} \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, b^{\frac{5}{2}}}+\frac{15 \, a^{2} d^{3} \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{8 \, b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^5/(sqrt(b*x^2 + a)*b) + 3/2*c*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/8*a
*d^3*x^3/(sqrt(b*x^2 + a)*b^2) + c^3*x/(sqrt(b*x^2 + a)*a) - 3*c^2*d*x/(sqr
```

```
t(b*x^2 + a)*b) + 9/2*a*c*d^2*x/(sqrt(b*x^2 + a)*b^2) - 15/8*a^2*d^3*x/(sqr
t(b*x^2 + a)*b^3) + 3*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9/2*a*c*d^2*ar
csinh(b*x/sqrt(a*b))/b^(5/2) + 15/8*a^2*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Fricas [A]
time = 0.54, size = 416, normalized size = 2.46

$$\left[\frac{3\,{\left(8\,a^{2}b^{2}c^{2}d - 12\,a^{3}bcd^{2} + 5\,a^{4}d^{3} + {\left(8\,ab^{3}c^{2}d - 12\,a^{2}b^{2}cd^{2} + 5\,a^{3}bd^{3}\right)}x^{2}\right)}\sqrt{b}\log\left(-2\,bx^{2} - 2\,\sqrt{bx^{2} + a}\sqrt{b}x - a\right) + 2\,{\left(2\,ab^{3}d^{3}x^{5} + {\left(12\,ab^{3}cd^{2} - 5\,a^{2}b^{2}d^{3}\right)}x^{3} + {\left(8\,b^{4}c^{3} - 24\,ab^{3}c^{2}d + 36\,a^{2}b^{2}cd^{2} - 15\,a^{3}bd^{3}\right)}x\right)}\sqrt{bx^{2} + a}}{16\,{\left(ab^{5}x^{2} + a^{2}b^{4}\right)}}, -\frac{3\,{\left(8\,a^{2}b^{2}c^{2}d - 12\,a^{3}bcd^{2} + 5\,a^{4}d^{3} + {\left(8\,ab^{3}c^{2}d - 12\,a^{2}b^{2}cd^{2} + 5\,a^{3}bd^{3}\right)}x^{2}\right)}\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2} + a}}\right) - {\left(2\,ab^{3}d^{3}x^{5} + {\left(12\,ab^{3}cd^{2} - 5\,a^{2}b^{2}d^{3}\right)}x^{3} + {\left(8\,b^{4}c^{3} - 24\,ab^{3}c^{2}d + 36\,a^{2}b^{2}cd^{2} - 15\,a^{3}bd^{3}\right)}x\right)}\sqrt{bx^{2} + a}}{8\,{\left(ab^{5}x^{2} + a^{2}b^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^
3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15*a^3*b*d^3)*x)*sqrt(
b*x^2 + a))/(a*b^5*x^2 + a^2*b^4), -1/8*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^
2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2
 - 5*a^2*b^2*d^3)*x^3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15
*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{2}\right)^{3}}{\left(a + bx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(3/2), x)
```

Giac [A]
time = 0.57, size = 157, normalized size = 0.93

$$\frac{{\left({\left(\frac{2\,d^{3}x^{2}}{b} + \frac{12\,ab^{4}cd^{2} - 5\,a^{2}b^{3}d^{3}}{ab^{5}}\right)}x^{2} + \frac{8\,b^{5}c^{3} - 24\,ab^{4}c^{2}d + 36\,a^{2}b^{3}cd^{2} - 15\,a^{3}b^{2}d^{3}}{ab^{5}}\right)}x}{8\,\sqrt{bx^{2} + a}} - \frac{3\,{\left(8\,b^{2}c^{2}d - 12\,abcd^{2} + 5\,a^{2}d^{3}\right)}\log\left({\left| -\sqrt{b}x + \sqrt{bx^{2} + a} \right|}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*d^3*x^2/b + (12*a*b^4*c*d^2 - 5*a^2*b^3*d^3)/(a*b^5))*x^2 + (8*b^5*
c^3 - 24*a*b^4*c^2*d + 36*a^2*b^3*c*d^2 - 15*a^3*b^2*d^3)/(a*b^5))*x/sqrt(b
*x^2 + a) - 3/8*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*log(abs(-sqrt(b)*x
 + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^3}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(3/2),x)
[Out] int((c + d*x^2)^3/(a + b*x^2)^(3/2), x)
```

3.83 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=90

$$\frac{(bc-ad)^2x}{ab^2\sqrt{a+bx^2}}+\frac{d^2x\sqrt{a+bx^2}}{2b^2}+\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] 1/2*d*(-3*a*d+4*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+(-a*d+b*c)^
2*x/a/b^2/(b*x^2+a)^(1/2)+1/2*d^2*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {424, 396, 223, 212}

$$\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{dx\sqrt{a+bx^2}\,(2bc-3ad)}{2ab^2}+\frac{x(c+dx^2)\,(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/2*(d*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(a*b^2) + ((b*c - a*d)*x*(c + d*
x^2))/(a*b*Sqrt[a + b*x^2]) + (d*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd-d(2bc-3ad)x^2}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}} + \frac{(d(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}$$

$$= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}} + \frac{(d(4bc-3ad))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.}{2b^2}$$

$$= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}} + \frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right.}{2b^{5/2}}$$

Mathematica [A]
time = 0.18, size = 95, normalized size = 1.06

$$\frac{x(2b^2c^2-4abcd+3a^2d^2+abd^2x^2)}{2ab^2\sqrt{a+bx^2}} - \frac{d(4bc-3ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 + a*b*d^2*x^2))/(2*a*b^2*Sqrt[a + b*x
^2]) - (d*(4*b*c - 3*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Maple [A]
time = 0.08, size = 123, normalized size = 1.37

method	result

risch	$\frac{d^{2} x \sqrt{b\,x^{2}+a}}{2 b^{2}}+\frac{x a\,d^{2}}{b^{2} \sqrt{b\,x^{2}+a}}-\frac{2 x c d}{b \sqrt{b\,x^{2}+a}}-\frac{3 \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right) a\,d^{2}}{2 b^{\frac{5}{2}}}+\frac{2 \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right.}{b^{\frac{3}{2}}}$
default	$d^{2}\left(\frac{x^{3}}{2 b \sqrt{b\,x^{2}+a}}-\frac{3 a\left(-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)+2 c d\left(-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln \left(x \sqrt{b}+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x
*b^(1/2)+(b*x^2+a)^(1/2))))+2*c*d*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2)))+c^2*x/a/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.20

$$\frac{d^{2} x^{3}}{2 \sqrt{b x^{2}+a}\, b}+\frac{c^{2} x}{\sqrt{b x^{2}+a}\, a}-\frac{2\, c d x}{\sqrt{b x^{2}+a}\, b}+\frac{3\, a d^{2} x}{2 \sqrt{b x^{2}+a}\, b^{2}}+\frac{2\, c d \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{b^{\frac{3}{2}}}-\frac{3\, a d^{2} \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{2\, b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^3/(sqrt(b*x^2 + a)*b) + c^2*x/(sqrt(b*x^2 + a)*a) - 2*c*d*x/(sqrt
(b*x^2 + a)*b) + 3/2*a*d^2*x/(sqrt(b*x^2 + a)*b^2) + 2*c*d*arcsinh(b*x/sqrt
(a*b))/b^(3/2) - 3/2*a*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.53, size = 276, normalized size = 3.07

$$\left[-\frac{\left(4 a^{2} b c d-3 a^{3} d^{2}+\left(4 a b^{2} c d-3 a^{2} b d^{2}\right) x^{2}\right) \sqrt{b} \log\left(-2 b x^{2}+2 \sqrt{b x^{2}+a} \sqrt{b} x-a\right)-2\left(a b^{2} d^{2} x^{3}+\left(2 b^{3} c^{2}-4 a b^{2} c d+3 a^{2} b d^{2}\right) x\right) \sqrt{b x^{2}+a}}{4\left(a b^{4} x^{2}+a^{2} b^{3}\right)}, -\frac{\left(4 a^{2} b c d-3 a^{3} d^{2}+\left(4 a b^{2} c d-3 a^{2} b d^{2}\right) x^{2}\right) \sqrt{-b} \arctan\left(\frac{\sqrt{-b} x}{\sqrt{b x^{2}+a}}\right)-\left(a b^{2} d^{2} x^{3}+\left(2 b^{3} c^{2}-4 a b^{2} c d+3 a^{2} b d^{2}\right) x\right) \sqrt{b x^{2}+a}}{2\left(a b^{4} x^{2}+a^{2} b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(a*b^2*d^2*x^3 + (2*b^3
*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)
, -1/2*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b^2*d^2*x^3 + (2*b^3*c^2 - 4*a*b^
2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(3/2), x)
```

Giac [A]
time = 0.63, size = 92, normalized size = 1.02

$$\frac{\left(\frac{d^2x^2}{b}+\frac{2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2}{ab^3}\right)x}{2\,\sqrt{bx^2+a}}-\frac{(4\,bcd-3\,ad^2)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*x^2/b + (2*b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)/(a*b^3))*x/sqrt(b*
x^2 + a) - 1/2*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^2}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(3/2), x)
```

3.84 $\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] d*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(-a*d+b*c)*x/a/b/(b*x^2+a)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {393, 223, 212}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)x}{ab\sqrt{a+bx^2}} + \frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}} + \frac{d\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.08, size = 58, normalized size = 1.07

$$\frac{bcx-adx}{ab\sqrt{a+bx^2}} - \frac{d\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (b*c*x - a*d*x)/(a*b*Sqrt[a + b*x^2]) - (d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/b^(3/2)
```

Maple [A]
time = 0.06, size = 55, normalized size = 1.02

method	result	size
default	$d\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{cx}{a\sqrt{b\,x^2+a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+c*x/a/(b*x
^2+a)^(1/2)
```

Maxima [A]
time = 0.33, size = 46, normalized size = 0.85

$$\frac{cx}{\sqrt{bx^2+a}\,a} - \frac{dx}{\sqrt{bx^2+a}\,b} + \frac{d\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*x/(sqrt(b*x^2 + a)*a) - d*x/(sqrt(b*x^2 + a)*b) + d*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

Fricas [A]
time = 0.51, size = 167, normalized size = 3.09

$$\left[\frac{2\,(b^2c-abd)\sqrt{bx^2+a}\;x+(abdx^2+a^2d)\sqrt{b}\;\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\;\sqrt{b}\;x-a\right)}{2\,(ab^3x^2+a^2b^2)},\frac{(b^2c-abd)\sqrt{bx^2+a}\;x-(abdx^2+a^2d)\sqrt{-b}\;\arctan\left(\frac{\sqrt{-b}\;x}{\sqrt{bx^2+a}}\right)}{ab^3x^2+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - a*b*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), ((b^2*
c - a*b*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A]
time = 2.36, size = 60, normalized size = 1.11

$$d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\;x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\;b\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
c*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.83, size = 50, normalized size = 0.93

$$-\frac{d\log\left(\left|-\sqrt{b}\;x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{(bc-ad)x}{\sqrt{bx^2+a}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + (b*c - a*d)*x/(sqrt(b*x
^2 + a)*a*b)
```

Mupad [B]

time = 5.12, size = 53, normalized size = 0.98

$$\frac{d\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}+\frac{c\,x}{a\,\sqrt{b\,x^2+a}}-\frac{d\,x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^(3/2),x)
```

```
[Out] (d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (c*x)/(a*(a + b*x^2)^(1/2)
) - (d*x)/(b*(a + b*x^2)^(1/2))
```

3.85 $\int \frac{1}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

```
[Out] x/a/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {197}

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 15, normalized size = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
default	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
trager	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/a/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

Fricas [A]
time = 0.55, size = 23, normalized size = 1.44

$$\frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A]
time = 0.31, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.68, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.88

$$\frac{x}{a\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(3/2),x)
```

```
[Out] x/(a*(a + b*x^2)^(1/2))
```

3.86 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)} dx$

Optimal. Leaf size=79

$$\frac{bx}{a(bc-ad)\sqrt{a+bx^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{3/2}}$$

```
[Out] -d*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/(-a*d+b*c)^(3/2)/c^(
1/2)+b*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 385, 214}

$$\frac{bx}{a\sqrt{a+bx^2}\,(bc-ad)}-\frac{d\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{bc-ad}$$
$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{bc-ad}$$
$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.23, size = 96, normalized size = 1.22

$$\frac{bx}{(abc-a^2d)\sqrt{a+bx^2}} - \frac{d\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}\,(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/((a*b*c - a^2*d)*Sqrt[a + b*x^2]) - (d*ArcTan[(-(d*x*Sqrt[a + b*x^2])
 + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(-(b*c) + a
*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(67) = 134.
time = 0.06, size = 723, normalized size = 9.15

method	result
default	$\frac{d}{(ad-bc)\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}} - \frac{2b\sqrt{-cd}\left(2b\left(x-\frac{\sqrt{}}{}\right.\right.}{(ad-bc)\left(\frac{4b(ad-bc)}{d}+\frac{4b^2c}{d}\right)\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-c*d)^(1/2)*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-(-
c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-
b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(
1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))-1/2/(-c*d)^(1/2)*
(1/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x+(-c*d)^(1/2)/d)-2*b
*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)
/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b
*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(67) = 134.
time = 0.66, size = 441, normalized size = 5.58

$$\left[\frac{4\,(b^2c^2-abcd)\sqrt{bx^2+a}\,x-(abdx^2+a^2d)\sqrt{bc^2-acd}\,\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ad)x^3+acx)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+(ab^3c^3-2\,a^2b^2c^2d+a^3bcd^2)x^2)},\ \frac{2\,(b^2c^2-abcd)\sqrt{bx^2+a}\,x+(abdx^2+a^2d)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\,((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)}{2\,(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+(ab^3c^3-2\,a^2b^2c^2d+a^3bcd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(b*
c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*
b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*
sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(a^2*b^2*c^3 - 2*a^3*b*c^2*d
 + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2), 1/2*(2*(b^
2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(-b*c^2 + a*c*
```

```
d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a
)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^3 - 2*a^3*
b*c^2*d + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{\frac{3}{2}}\,(c+dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)), x)
```

Giac [A]
time = 0.65, size = 107, normalized size = 1.35

$$-\frac{\sqrt{b}\,d\arctan\left(-\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}\,(bc-ad)}+\frac{bx}{(abc-a^2d)\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*d*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sq
rt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*c - a*d)) + b*x/((a*b*
c - a^2*d)*sqrt(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=143

$$\frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)}-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}$$

```
[Out] -1/2*d*(-a*d+4*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(
3/2)/(-a*d+b*c)^(5/2)+1/2*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(1/2)-
1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$,
Rules used = {425, 541, 12, 385, 214}

$$-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}+\frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}\,(bc-ad)^2}-\frac{dx}{2c\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]) - (d*x)/(2*c*(b*c
 - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(4*b*c - a*d)*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} + \frac{\int \frac{2bc-ad-2bdx^2}{(a+bx^2)^{3/2}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{\int \frac{ad(4bc-ad}{\sqrt{a+bx^2}\,(c}}{2ac(bc-a}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{(d(4bc-ad))}{2c}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{(d(4bc-ad))S}{}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{d(4bc-ad)\,\mathrm{ta}}{2c^{3/}}$$

Mathematica [A]
time = 0.66, size = 151, normalized size = 1.06

$$\frac{\frac{\sqrt{c}\,x(a^2d^2+abd^2x^2+2b^2c(c+dx^2))}{a(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} + \frac{d(4bc-ad)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] ((Sqrt[c]*x*(a^2*d^2 + a*b*d^2*x^2 + 2*b^2*c*(c + d*x^2)))/(a*(b*c - a*d)^2
*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*ArcTan[(-(d*x*Sqrt[a + b*x
^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c) + a*d)^(
5/2))/(2*c^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1905 vs. $2(123) = 246$.
time = 0.07, size = 1906, normalized size = 13.33

method	result	size
default	Expression too large to display	1906

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/(-c*d)^(1/2)*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-
(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^
(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*
d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))-1/4/d/c*(-1/(a*
d-b*c)*d/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3*b*(-c*d)^(1/2)/(a*d-b*c)*(1/(a*d-b*c)*d
/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/
d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((
2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*(
(x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2))/(x-(-c*d)^(1/2)/d)))-4*b/(a*d-b*c)*d*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*
d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-1/4/c/(-c*d)^(1/2)*(1/(a*d-b
*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(
1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2
)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)
*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1
/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2))/(x+(-c*d)^(1/2)/d)))-1/4/d/c*(-1/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)/
((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(1/2)+3*b*(-c*d)^(1/2)/(a*d-b*c)*(1/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b
```

```
*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a*d
-b*c)*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/
d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/
2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-4
*b/(a*d-b*c)*d*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d
+4*b^2*c/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(123) = 246$.
time = 0.77, size = 864, normalized size = 6.04

$$\left[-\frac{\left(4\,a^2bc^2d - a^3cd^2 + \left(4\,ab^2cd^2 - a^2bd^3\right)x^4 + \left(4\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)x^2\right)\sqrt{bc^2 - acd}\,\log\left(\frac{\left(8\,b^2c^2 - 8\,abcd + a^2d^2\right)x^4 + a^2c^2 + 2\left(4\,abc^2 - 3\,a^2cd\right)x^2 + 4\left(\left(2\,bc - ad\right)x^3 + acx\right)\sqrt{bc^2 - acd}\,\sqrt{bx^2 + a}}{d^2x^4 + 2\,cdx^2 + c^2}\right) - 4\left(\left(2\,b^3c^3d - ab^2c^2d^2 - a^2bcd^3\right)x^3 + \left(2\,b^3c^4 - 2\,ab^2c^3d + a^2bc^2d^2 - a^3cd^3\right)x\right)\sqrt{bx^2 + a}}{8\left(a^2b^3c^6 - 3\,a^3b^2c^5d + 3\,a^4bc^4d^2 - a^5c^3d^3 + \left(ab^4c^5d - 3\,a^2b^3c^4d^2 + 3\,a^3b^2c^3d^3 - a^4bc^2d^4\right)x^4 + \left(ab^4c^6 - 2\,a^2b^3c^5d + 2\,a^4bc^3d^3 - a^5c^2d^4\right)x^2\right)}, \frac{\left(4\,a^2bc^2d - a^3cd^2 + \left(4\,ab^2cd^2 - a^2bd^3\right)x^4 + \left(4\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)x^2\right)\sqrt{-bc^2 + acd}\,\arctan\left(\frac{\sqrt{-bc^2 + acd}\left(\left(2\,bc - ad\right)x^2 + ac\right)\sqrt{bx^2 + a}}{2\left(\left(b^2c^2 - abcd\right)x^3 + \left(abc^2 - a^2cd\right)x\right)}\right) + 2\left(\left(2\,b^3c^3d - ab^2c^2d^2 - a^2bcd^3\right)x^3 + \left(2\,b^3c^4 - 2\,ab^2c^3d + a^2bc^2d^2 - a^3cd^3\right)x\right)\sqrt{bx^2 + a}}{4\left(a^2b^3c^6 - 3\,a^3b^2c^5d + 3\,a^4bc^4d^2 - a^5c^3d^3 + \left(ab^4c^5d - 3\,a^2b^3c^4d^2 + 3\,a^3b^2c^3d^3 - a^4bc^2d^4\right)x^4 + \left(ab^4c^6 - 2\,a^2b^3c^5d + 2\,a^4bc^3d^3 - a^5c^2d^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4
*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4
+ 2*c*d*x^2 + c^2)) - 4*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
(2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a))
/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c^
5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*c
^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2), 1/4*((4*a^2*b*c
^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*
d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*
b*c^2 - a^2*c*d)*x)) + 2*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
 (2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a)
)/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c
^5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*
c^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(123) = 246.
time = 1.17, size = 318, normalized size = 2.22

$$\frac{b^2x}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{bx^2+a}}+\frac{\left(4\,b^{\frac{3}{2}}cd-a\sqrt{b}\,d^2\right)\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{-b^2c^2+abcd}}+\frac{2\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cd-\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a\sqrt{b}\,d^2+a^2\sqrt{b}\,d^2}{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4d+4\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2bc-2\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(b*x^2 + a)) + 1/2*(4*b^(3/2
)*c*d - a*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*
c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqr
t(-b^2*c^2 + a*b*c*d)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d - (
sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d^2 + a^2*sqrt(b)*d^2)/((b^2*c^3 -
 2*a*b*c^2*d + a^2*c*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2), x)
```

3.88 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2}+\frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)}+\frac{d(4bc-ad)(2bc+3ad)x\sqrt{a+bx^2}}{8ac^2(bc-ad)^3\,(c+dx^2)}-$$

```
[Out] -3/8*d*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*
x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(7/2)-1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2/(b*
x^2+a)^(1/2)+1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)/(b*x^2+a)^(1/2)
+1/8*d*(-a*d+4*b*c)*(3*a*d+2*b*c)*x*(b*x^2+a)^(1/2)/a/c^2/(-a*d+b*c)^3/(d*x
^2+c)
```

Rubi [A]
time = 0.17, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {425, 541, 12, 385, 214}

$$-\frac{3d(a^2d^2-4abcd+8b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}+\frac{dx\sqrt{a+bx^2}\,(4bc-ad)(3ad+2bc)}{8ac^2\,(c+dx^2)\,(bc-ad)^3}+\frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)^2}-\frac{dx}{4c\sqrt{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (b*(4*b*c + a*d)
*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*(2
*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*d
*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqr
t[a + b*x^2])])/(8*c^(5/2)*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{\int \frac{4bc-3ad-4bdx^2}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} -$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} +$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} +$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} +$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 13.59, size = 1392, normalized size = 6.19

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (x*(-108045*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (324135*d*x^2*Sqrt[((
b*c - a*d)*x^2)/(c*(a + b*x^2))])/c - (324135*d^2*x^4*Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))])/c^2 - (103320*d^3*x^6*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))])/c^3 + 42735*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) + (128205*d*x
^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (139545*d^2*x^4*(((b*c -
a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + (46200*d^3*x^6*(((b*c - a*d)*x^2)/(
c*(a + b*x^2)))^(3/2))/c^3 - 3864*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)
 - (4032*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c - (4032*d^2*x^4
*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^2 - (1344*d^3*x^6*(((b*c - a*
d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^3 + 108045*ArcTanh[Sqrt[((b*c - a*d)*x^2)
/(c*(a + b*x^2))]] + (324135*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b
*x^2))]])/c + (324135*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]])/c^2 + (103320*d^3*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]]
)/c^3 + (8505*(b*c - a*d)^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^
2))]])/(c^2*(a + b*x^2)^2) + (17955*d*(b*c - a*d)^2*x^6*ArcTanh[Sqrt[((b*c
- a*d)*x^2)/(c*(a + b*x^2))]])/(c^3*(a + b*x^2)^2) + (21735*d^2*(b*c - a*d)
^2*x^8*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)^2
) + (7560*d^3*(b*c - a*d)^2*x^10*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x
^2))]])/(c^5*(a + b*x^2)^2) - (78750*(b*c - a*d)*x^2*ArcTanh[Sqrt[((b*c - a
*d)*x^2)/(c*(a + b*x^2))]])/(c*(a + b*x^2)) + (236250*d*(-(b*c) + a*d)*x^4*
ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)) + (2475
90*d^2*(-(b*c) + a*d)*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])
/(c^3*(a + b*x^2)) + (80640*d^3*(-(b*c) + a*d)*x^8*ArcTanh[Sqrt[((b*c - a*d
)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)) + 64*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(9/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d
)*x^2)/(c*(a + b*x^2))] + (192*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9
/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a
 + b*x^2))])/c + (192*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hyp
ergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^
2))])/c^2 + (64*d^3*x^6*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hypergeom
etricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/
c^3))/(2520*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*(a + b*x^2)^(3/2)*(
c + d*x^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4034 vs. 2(201) = 402.
time = 0.09, size = 4035, normalized size = 17.93

method	result	size
default	Expression too large to display	4035

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/8/d/c/(-c*d)^(1/2)*(-1/2/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-5/2*b*(-c*d)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3*b*(-c*d)^(1/2)/(a*d-b*c)*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))-4*b/(a*d-b*c)*d*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-3/2*b/(a*d-b*c)*d*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))))+3/16/(-c*d)^(1/2)/c^2*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))-3/16/d/c^2*(-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3*b*(-c*d)^(1/2)/(a*d-b*c)*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d)))-4*b/(a*d-b*c)*d*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(

```
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-3/16/(-c*d)^(1/2)/c^2*(1/(a*d-b*c)*d/
((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(1/2)+2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d
)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x
+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2
*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))/(x+(-c*d)^(1/2)/d))-3/16/d/c^2*(-1/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)/((x
+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2)+3*b*(-c*d)^(1/2)/(a*d-b*c)*(1/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a*d-b*
c)*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/
((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/
d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))-4*b/
(a*d-b*c)*d*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*
b^2*c/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2))+1/8/d/c/(-c*d)^(1/2)*(-1/2/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)^2
/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(1/2)+5/2*b*(-c*d)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)/((x+
(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
)+3*b*(-c*d)^(1/2)/(a*d-b*c)*(1/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c
*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a*d-b*c
)*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/(
(x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d
)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))-4*b/(
a*d-b*c)*d*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(201) = 402$.
time = 1.17, size = 1482, normalized size = 6.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x
)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*((8
*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a^3*b*c*d^5)*x^5 +
(16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a^3*b*c^2*d^4 + 3*a
^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c^4*d^2 - 17*a^3*b*
c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9 - 4*a^3*b^3*c^8*d
 + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a*b^5*c^7*d^2 - 4*a
^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 + a^5*b*c^3*d^6)*x^6
 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3 - 2*a^4*b^2*c^5*d
^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*a^2*b^4*c^8*d - 2*
a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2*a^6*c^4*d^5)*x^2)
, 1/16*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c
 - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^
2*c*d)*x)) + 2*((8*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a
^3*b*c*d^5)*x^5 + (16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a
^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c
^4*d^2 - 17*a^3*b*c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9
 - 4*a^3*b^3*c^8*d + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a
*b^5*c^7*d^2 - 4*a^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 +
a^5*b*c^3*d^6)*x^6 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3
 - 2*a^4*b^2*c^5*d^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*
a^2*b^4*c^8*d - 2*a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2
*a^6*c^4*d^5)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. 2(201) = 402.

time = 1.66, size = 643, normalized size = 2.86

$$\frac{b^3x}{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{bx^2+a}}+\frac{3\left(8\,b^{\frac{5}{2}}c^2d-4\,ab^{\frac{3}{2}}cd^2+a^2\sqrt{b}\,d^3\right)\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{8\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\sqrt{-b^2c^2+abcd}}+\frac{16\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6b^{\frac{5}{2}}c^2d^2-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6ab^{\frac{3}{2}}cd^3+3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6a^2\sqrt{b}\,d^4+80\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{7}{2}}c^3d-104\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4ab^{\frac{5}{2}}c^2d^2+54\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{3}{2}}cd^3-9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^3\sqrt{b}\,d^4+64\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{5}{2}}c^2d^2-52\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{3}{2}}cd^3+9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^4\sqrt{b}\,d^4+10\,a^4b^{\frac{3}{2}}cd^3-3\,a^5\sqrt{b}\,d^4}{4\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(b*x^2 +
 a)) + 3/8*(8*b^(5/2)*c^2*d - 4*a*b^(3/2)*c*d^2 + a^2*sqrt(b)*d^3)*arctan(1
/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*
d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(-b^2*c^
2 + a*b*c*d)) + 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d^2 - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*a^2*sqrt(b)*d^4 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3
*d - 104*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^2 + 54*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
a^3*sqrt(b)*d^4 + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d^2 -
52*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^3 + 9*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a^4*sqrt(b)*d^4 + 10*a^4*b^(3/2)*c*d^3 - 3*a^5*sqrt(b)*d^4)/(
(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*((sqrt(b)*x - sqr
t(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3), x)
```

3.89 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4}-\frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}\,(c+d$$

```
[Out] 1/3*(-a*d+b*c)*x*(d*x^2+c)^3/a/b/(b*x^2+a)^(3/2)+1/8*d^2*(35*a^2*d^2-80*a*b
*c*d+48*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)+1/3*(-a*d+b*c)*
(7*a*d+2*b*c)*x*(d*x^2+c)^2/a^2/b^2/(b*x^2+a)^(1/2)-1/24*d*(105*a^3*d^3-170
*a^2*b*c*d^2+40*a*b^2*c^2*d+16*b^3*c^3)*x*(b*x^2+a)^(1/2)/a^2/b^4-1/12*d*(-
35*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*x*(d*x^2+c)*(b*x^2+a)^(1/2)/a^2/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {424, 540, 542, 396, 223, 212}

$$\frac{x(c+dx^2)^2(bc-ad)(7ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2(35a^2d^2-80abcd+48b^2c^2)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}-\frac{dx\sqrt{a+bx^2}\,(c+dx^2)(-35a^2d^2+24abcd+8b^2c^2)}{12a^2b^3}-\frac{dx\sqrt{a+bx^2}\,(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3)}{24a^2b^4}+\frac{x(c+dx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/24*(d*(16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sq
rt[a + b*x^2])/(a^2*b^4) - (d*(8*b^2*c^2 + 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[
a + b*x^2]*(c + d*x^2))/(12*a^2*b^3) + ((b*c - a*d)*(2*b*c + 7*a*d)*x*(c +
d*x^2)^2)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*
b*(a + b*x^2)^(3/2)) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)x(c+dx^2)^3}{3ab\,(a+bx^2)^{3/2}} + \frac{\int \frac{(c+dx^2)^2(c(2bc+ad)-d(4bc-7ad)x^2)}{(a+bx^2)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)^3}{3ab\,(a+bx^2)^{3/2}} - \frac{\int \frac{(c+dx^2)\left(acd(4bc-7ad)+d(8b^2c\right.}{\sqrt{a+bx}}}{3a^2b^2}$$

$$= -\frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}\,(c+dx^2)}{12a^2b^3} + \frac{(bc-ad)(2bc+7ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}}$$

$$= -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d(8b^2c^2+24abcd-35a}{12}$$

$$= -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d(8b^2c^2+24abcd-35a}{12}$$

$$= -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d(8b^2c^2+24abcd-35a}{12}$$

Mathematica [A]
time = 0.40, size = 202, normalized size = 0.79

$$\frac{x(-105a^5d^4+16b^5c^4x^2+20a^4bd^3(12c-7dx^2)+8ab^4c^3(3c+4dx^2)+a^3b^2d^2(-144c^2+320cdx^2-21d^2x^4)+6a^2b^3d^2x^2(-32c^2+8cdx^2+d^2x^4))}{24a^2b^4\,(a+bx^2)^{3/2}} - \frac{d^2(48b^2c^2-80abcd+35a^2d^2)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(-105*a^5*d^4 + 16*b^5*c^4*x^2 + 20*a^4*b*d^3*(12*c - 7*d*x^2) + 8*a*b^4
*c^3*(3*c + 4*d*x^2) + a^3*b^2*d^2*(-144*c^2 + 320*c*d*x^2 - 21*d^2*x^4) +
6*a^2*b^3*d^2*x^2*(-32*c^2 + 8*c*d*x^2 + d^2*x^4)))/(24*a^2*b^4*(a + b*x^2)
^(3/2)) - (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(8*b^(9/2))
```

Maple [A]
time = 0.14, size = 360, normalized size = 1.41

method	result

default	$d^4\left(\frac{x^7}{4b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{7a\left(\frac{x^5}{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2b}\right)}{4b}\right)+4c\,d^3\left(\cdots\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] d^4*(1/4*x^7/b/(b*x^2+a)^(3/2)-7/4*a/b*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))+4*c*d^3*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+6*c^2*d^2*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+4*c^3*d*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))+c^4*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))

Maxima [A]
time = 0.29, size = 392, normalized size = 1.54

$$\frac{d^4x^7}{4\,(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,cd^3x^5}{(bx^2+a)^{\frac{3}{2}}b}-\frac{7\,ad^4x^5}{8\,(bx^2+a)^{\frac{3}{2}}b^2}-2\,c^2d^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)+\frac{10\,acd^3x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b}-\frac{35\,a^2d^4x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{24\,b^2}+\frac{2\,c^4x}{3\,\sqrt{bx^2+a}\,a^2}+\frac{c^4x}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{4\,c^3dx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{4\,c^3dx}{3\,\sqrt{bx^2+a}\,ab}-\frac{2\,c^2d^2x}{\sqrt{bx^2+a}\,b^2}+\frac{10\,acd^3x}{3\,\sqrt{bx^2+a}\,b^3}-\frac{35\,a^2d^4x}{24\,\sqrt{bx^2+a}\,b^4}+\frac{6\,c^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{10\,acd^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{7}{2}}}+\frac{35\,a^2d^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="maxima")

[Out] 1/4*d^4*x^7/((b*x^2 + a)^(3/2)*b) + 2*c*d^3*x^5/((b*x^2 + a)^(3/2)*b) - 7/8*a*d^4*x^5/((b*x^2 + a)^(3/2)*b^2) - 2*c^2*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 10/3*a*c*d^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 35/24*a^2*d^4*x*(3*x^2/((b*x^2 + a)

```
^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 2/3*c^4*x/(sqrt(b*x^2 + a)*a
^2) + 1/3*c^4*x/((b*x^2 + a)^(3/2)*a) - 4/3*c^3*d*x/((b*x^2 + a)^(3/2)*b) +
 4/3*c^3*d*x/(sqrt(b*x^2 + a)*a*b) - 2*c^2*d^2*x/(sqrt(b*x^2 + a)*b^2) + 10
/3*a*c*d^3*x/(sqrt(b*x^2 + a)*b^3) - 35/24*a^2*d^4*x/(sqrt(b*x^2 + a)*b^4)
+ 6*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 10*a*c*d^3*arcsinh(b*x/sqrt(a*
b))/b^(7/2) + 35/8*a^2*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Fricas [**A**]
time = 0.65, size = 684, normalized size = 2.68

$$\left[\frac{3\left(48a^4b^2c^2d^2-80a^5bcd^3+35a^6d^4+\left(48a^2b^4c^2d^2-80a^3b^3cd^3+35a^4b^2d^4\right)x^4+2\left(48a^3b^3c^2d^2-80a^4b^2cd^3+35a^5bd^4\right)x^2\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6a^2b^4d^4x^7+3\left(16a^2b^4cd^3-7a^3b^3d^4\right)x^5+4\left(4b^6c^4+8ab^5c^3d-48a^2b^4c^2d^2+80a^3b^3cd^3-35a^4b^2d^4\right)x^3+3\left(8ab^5c^4-48a^3b^3c^2d^2+80a^4b^2cd^3-35a^5bd^4\right)x\right)\sqrt{bx^2+a}}{48\left(a^2b^7x^4+2a^3b^6x^2+a^4b^5\right)}, -\frac{3\left(48a^4b^2c^2d^2-80a^5bcd^3+35a^6d^4+\left(48a^2b^4c^2d^2-80a^3b^3cd^3+35a^4b^2d^4\right)x^4+2\left(48a^3b^3c^2d^2-80a^4b^2cd^3+35a^5bd^4\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(6a^2b^4d^4x^7+3\left(16a^2b^4cd^3-7a^3b^3d^4\right)x^5+4\left(4b^6c^4+8ab^5c^3d-48a^2b^4c^2d^2+80a^3b^3cd^3-35a^4b^2d^4\right)x^3+3\left(8ab^5c^4-48a^3b^3c^2d^2+80a^4b^2cd^3-35a^5bd^4\right)x\right)\sqrt{bx^2+a}}{24\left(a^2b^7x^4+2a^3b^6x^2+a^4b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*a^2*b^4*c^
2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^2*d^2 - 80
*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d
^4)*x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*
d^3 - 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^
2*c*d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 +
a^4*b^5), -1/24*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*
a^2*b^4*c^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^
2*d^2 - 80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*
x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*d^3
- 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^2*c*
d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(5/2), x)
```

Giac [**A**]
time = 0.72, size = 237, normalized size = 0.93

$$\frac{\left(\left(3\left(\frac{2d^4x^2}{b}+\frac{16a^2b^6cd^3-7a^3b^5d^4}{a^2b^7}\right)x^2+\frac{4\left(4b^8c^4+8ab^7c^3d-48a^2b^6c^2d^2+80a^3b^5cd^3-35a^4b^4d^4\right)}{a^2b^7}\right)x^2+\frac{3\left(8ab^7c^4-48a^3b^5c^2d^2+80a^4b^4cd^3-35a^5b^3d^4\right)}{a^2b^7}\right)x}{24\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{\left(48b^2c^2d^2-80abcd^3+35a^2d^4\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*d^4*x^2/b + (16*a^2*b^6*c*d^3 - 7*a^3*b^5*d^4)/(a^2*b^7))*x^2 +
 4*(4*b^8*c^4 + 8*a*b^7*c^3*d - 48*a^2*b^6*c^2*d^2 + 80*a^3*b^5*c*d^3 - 35*
a^4*b^4*d^4)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c^4 - 48*a^3*b^5*c^2*d^2 + 80*a^4*
b^4*c*d^3 - 35*a^5*b^3*d^4)/(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^2*c^
2*d^2 - 80*a*b*c*d^3 + 35*a^2*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^2+c\right)^4}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^4/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^4/(a + b*x^2)^(5/2), x)
```

3.90 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{d(4b^2c^2+8abcd-15a^2d^2)\,x\sqrt{a+bx^2}}{6a^2b^3}+\frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)^2}{3ab\,(a+bx^2)^{3/2}}+\frac{d^2(6bc-\cdots}{\cdots}$$

```
[Out] 1/3*(-a*d+b*c)*x*(d*x^2+c)^2/a/b/(b*x^2+a)^(3/2)+1/2*d^2*(-5*a*d+6*b*c)*arc
tanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/3*(-a*d+b*c)*(5*a*d+2*b*c)*x*(d*x
^2+c)/a^2/b^2/(b*x^2+a)^(1/2)-1/6*d*(-15*a^2*d^2+8*a*b*c*d+4*b^2*c^2)*x*(b*
x^2+a)^(1/2)/a^2/b^3
```

Rubi [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 540, 396, 223, 212}

$$\frac{x(c+dx^2)\,(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}-\frac{dx\sqrt{a+bx^2}\,(-15a^2d^2+8abcd+4b^2c^2)}{6a^2b^3}+\frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{x(c+dx^2)^2\,(bc-ad)}{3ab\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/6*(d*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(a^2*b^3) +
 ((b*c - a*d)*(2*b*c + 5*a*d)*x*(c + d*x^2))/(3*a^2*b^2*Sqrt[a + b*x^2]) +
((b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*(6*b*c - 5*a
*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)x(c+dx^2)^2}{3ab\,(a+bx^2)^{3/2}} + \frac{\int \frac{(c+dx^2)\left(c(2bc+ad)-d(2bc-5ad)x^2\right)}{(a+bx^2)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)^2}{3ab\,(a+bx^2)^{3/2}} - \frac{\int \frac{acd(2bc-5ad)+d(4b^2c^2+8abcd-15a}{\sqrt{a+bx^2}}}{3a^2b^2}$$

$$= -\frac{d(4b^2c^2+8abcd-15a^2d^2)\,x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad}{3ab\,(a}$$

$$= -\frac{d(4b^2c^2+8abcd-15a^2d^2)\,x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad}{3ab\,(a}$$

$$= -\frac{d(4b^2c^2+8abcd-15a^2d^2)\,x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad}{3ab\,(a}$$

Mathematica [A]
time = 0.28, size = 143, normalized size = 0.83

$$\frac{x(15a^4d^3+4b^4c^3x^2+3a^2b^2d^2x^2(-8c+dx^2)+6ab^3c^2(c+dx^2)+2a^3bd^2(-9c+10dx^2))}{6a^2b^3\,(a+bx^2)^{3/2}} + \frac{d^2(-6bc+5ad)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(15*a^4*d^3 + 4*b^4*c^3*x^2 + 3*a^2*b^2*d^2*x^2*(-8*c + d*x^2) + 6*a*b^3
*c^2*(c + d*x^2) + 2*a^3*b*d^2*(-9*c + 10*d*x^2)))/(6*a^2*b^3*(a + b*x^2)^(
3/2)) + (d^2*(-6*b*c + 5*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/
2))
```

Maple [A]
time = 0.10, size = 246, normalized size = 1.43

method	result
default	$d^{3}\left(\frac{x^{5}}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2b}\right)+3c\,d^{2}\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\cdots\right.$
risch	$\frac{d^{3}x\sqrt{b\,x^{2}+a}}{2b^{3}}+\frac{3d^{2}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)c}{b^{\frac{5}{2}}}-\frac{5d^{3}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)a}{2b^{\frac{7}{2}}}+\frac{a^{2}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}b\cdots}}{12b^{4}\sqrt{-a\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/
b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+3*c*d^2*(-1/3*
x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))))+3*c^2*d*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)
^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))+c^3*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(
b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 254, normalized size = 1.48

$$\frac{d^{3}x^{5}}{2\left(bx^{2}+a\right)^{\frac{3}{2}}b}-cd^{2}x\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)+\frac{5\,ad^{3}x\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)}{6\,b}+\frac{2\,c^{3}x}{3\sqrt{bx^{2}+a}\,a^{2}}+\frac{c^{3}x}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{c^{2}dx}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{c^{2}dx}{\sqrt{bx^{2}+a}\,ab}-\frac{cd^{2}x}{\sqrt{bx^{2}+a}\,b^{2}}+\frac{5\,ad^{3}x}{6\sqrt{bx^{2}+a}\,b^{3}}+\frac{3\,cd^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{5\,ad^{3}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^5/((b*x^2 + a)^(3/2)*b) - c*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
2*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*a*d^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2))/b + 2/3*c^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^3*x
/((b*x^2 + a)^(3/2)*a) - c^2*d*x/((b*x^2 + a)^(3/2)*b) + c^2*d*x/(sqrt(b*x^
2 + a)*a*b) - c*d^2*x/(sqrt(b*x^2 + a)*b^2) + 5/6*a*d^3*x/(sqrt(b*x^2 + a)*
b^3) + 3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/2*a*d^3*arcsinh(b*x/sqrt(
a*b))/b^(7/2)
```

Fricas [**A**]
time = 0.58, size = 486, normalized size = 2.83

$$\left[-\frac{3\,{\left(6\,a^{4} b c d^{2} - 5\,a^{5} d^{3} + {\left(6\,a^{2} b^{3} c d^{2} - 5\,a^{3} b^{2} d^{3}\right)} x^{4} + 2\,{\left(6\,a^{3} b^{2} c d^{2} - 5\,a^{4} b d^{3}\right)} x^{2}\right)} \sqrt{b} \log\left(-2\,b x^{2} + 2\,\sqrt{b x^{2} + a} \sqrt{b} x - a\right) - 2\,{\left(3\,a^{2} b^{3} d^{3} x^{5} + 2\,{\left(2\,b^{5} c^{3} + 3\,a b^{4} c^{2} d - 12\,a^{2} b^{3} c d^{2} + 10\,a^{3} b^{2} d^{3}\right)} x^{3} + 3\,{\left(2\,a b^{4} c^{3} - 6\,a^{3} b^{2} c d^{2} + 5\,a^{4} b d^{3}\right)} x\right)} \sqrt{b x^{2} + a}}{12\,{\left(a^{2} b^{6} x^{4} + 2\,a^{3} b^{5} x^{2} + a^{4} b^{4}\right)}}, -\frac{3\,{\left(6\,a^{4} b c d^{2} - 5\,a^{5} d^{3} + {\left(6\,a^{2} b^{3} c d^{2} - 5\,a^{3} b^{2} d^{3}\right)} x^{4} + 2\,{\left(6\,a^{3} b^{2} c d^{2} - 5\,a^{4} b d^{3}\right)} x^{2}\right)} \sqrt{-b} \arctan\left(\frac{\sqrt{-b} x}{\sqrt{b x^{2} + a}}\right) - {\left(3\,a^{2} b^{3} d^{3} x^{5} + 2\,{\left(2\,b^{5} c^{3} + 3\,a b^{4} c^{2} d - 12\,a^{2} b^{3} c d^{2} + 10\,a^{3} b^{2} d^{3}\right)} x^{3} + 3\,{\left(2\,a b^{4} c^{3} - 6\,a^{3} b^{2} c d^{2} + 5\,a^{4} b d^{3}\right)} x\right)} \sqrt{b x^{2} + a}}{6\,{\left(a^{2} b^{6} x^{4} + 2\,a^{3} b^{5} x^{2} + a^{4} b^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) - 2*(3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2
*d - 12*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*
d^2 + 5*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b
^4), -1/6*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)
*x^4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2*d - 12*a^2
*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*d^2 + 5*a^4
*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d x^{2}\right)^{3}}{\left(a + b x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(5/2), x)
```

Giac [**A**]
time = 0.65, size = 158, normalized size = 0.92

$$\frac{{\left({\left(\frac{3\,d^{3} x^{2}}{b} + \frac{2\,{\left(2\,b^{6} c^{3} + 3\,a b^{5} c^{2} d - 12\,a^{2} b^{4} c d^{2} + 10\,a^{3} b^{3} d^{3}\right)}}{a^{2} b^{5}}\right)} x^{2} + \frac{3\,{\left(2\,a b^{5} c^{3} - 6\,a^{3} b^{3} c d^{2} + 5\,a^{4} b^{2} d^{3}\right)}}{a^{2} b^{5}}\right)} x}{6\,{\left(b x^{2} + a\right)}^{\frac{3}{2}}} - \frac{{\left(6\,b c d^{2} - 5\,a d^{3}\right)} \log\left({\left| -\sqrt{b} x + \sqrt{b x^{2} + a} \right|}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*d^3*x^2/b + 2*(2*b^6*c^3 + 3*a*b^5*c^2*d - 12*a^2*b^4*c*d^2 + 10*a^
3*b^3*d^3)/(a^2*b^5))*x^2 + 3*(2*a*b^5*c^3 - 6*a^3*b^3*c*d^2 + 5*a^4*b^2*d^
3)/(a^2*b^5))*x/(b*x^2 + a)^(3/2) - 1/2*(6*b*c*d^2 - 5*a*d^3)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^3}{{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(5/2), x)
```

3.91 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$\frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)}{3ab\,(a+bx^2)^{3/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] 1/3*(-a*d+b*c)*x*(d*x^2+c)/a/b/(b*x^2+a)^(3/2)+d^2*arctanh(x*b^(1/2)/(b*x^2
+a)^(1/2))/b^(5/2)+1/3*(-a*d+b*c)*(3*a*d+2*b*c)*x/a^2/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 393, 223, 212}

$$\frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}+\frac{x(c+dx^2)\,(bc-ad)}{3ab\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)*(2*b*c + 3*a*d)*x)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*
x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/b^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx &= \frac{(bc-ad)x(c+dx^2)}{3ab\,(a+bx^2)^{3/2}} + \frac{\int \frac{c(2bc+ad)+3ad^2x^2}{(a+bx^2)^{3/2}}\,dx}{3ab} \\
&= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)}{3ab\,(a+bx^2)^{3/2}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2} \\
&= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)}{3ab\,(a+bx^2)^{3/2}} + \frac{d^2\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^2} \\
&= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)}{3ab\,(a+bx^2)^{3/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 103, normalized size = 0.98

$$\frac{-3a^3d^2x+2b^3c^2x^3-4a^2bd^2x^3+ab^2cx(3c+2dx^2)}{3a^2b^2\,(a+bx^2)^{3/2}} - \frac{d^2\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^3*d^2*x + 2*b^3*c^2*x^3 - 4*a^2*b*d^2*x^3 + a*b^2*c*x*(3*c + 2*d*x^2)
)/(3*a^2*b^2*(a + b*x^2)^(3/2)) - (d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/b^(5/2)
```

Maple [A]
time = 0.06, size = 156, normalized size = 1.49

method	result
default	$d^{2}\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+2cd\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{}{3a^{2}\sqrt{}}\right.}{2b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))))+2*c*d*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))+c^2*(1/3*x/a/(b*x^2+a)^(3/2)+2/3
*x/a^2/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 147, normalized size = 1.40

$$-\frac{1}{3}\,d^{2}x\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)+\frac{2\,c^{2}x}{3\,\sqrt{bx^{2}+a}\,a^{2}}+\frac{c^{2}x}{3\,(bx^{2}+a)^{\frac{3}{2}}a}-\frac{2\,cdx}{3\,(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,cdx}{3\,\sqrt{bx^{2}+a}\,ab}-\frac{d^{2}x}{3\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{d^{2}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2/
3*c^2*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^2*x/((b*x^2 + a)^(3/2)*a) - 2/3*c*d*x
/((b*x^2 + a)^(3/2)*b) + 2/3*c*d*x/(sqrt(b*x^2 + a)*a*b) - 1/3*d^2*x/(sqrt(
b*x^2 + a)*b^2) + d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.49, size = 318, normalized size = 3.03

$$\left[\frac{3\,(a^{2}b^{2}d^{2}x^{4}+2\,a^{3}bd^{2}x^{2}+a^{4}d^{2})\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(2\,(b^{4}c^{2}+ab^{3}cd-2\,a^{2}b^{2}d^{2})x^{3}+3\,(ab^{3}c^{2}-a^{3}bd^{2})x)\sqrt{bx^{2}+a}}{6\,(a^{2}b^{5}x^{4}+2\,a^{3}b^{4}x^{2}+a^{4}b^{3})},\ -\frac{3\,(a^{2}b^{2}d^{2}x^{4}+2\,a^{3}bd^{2}x^{2}+a^{4}d^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(2\,(b^{4}c^{2}+ab^{3}cd-2\,a^{2}b^{2}d^{2})x^{3}+3\,(ab^{3}c^{2}-a^{3}bd^{2})x)\sqrt{bx^{2}+a}}{3\,(a^{2}b^{5}x^{4}+2\,a^{3}b^{4}x^{2}+a^{4}b^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a
^3*b^4*x^2 + a^4*b^3), -1/3*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2
)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*(b^4*c^2 + a*b^3*c*d - 2
*a^2*b^2*d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*
x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(5/2), x)
```

Giac [A]
time = 0.58, size = 103, normalized size = 0.98

$$\frac{x\left(\frac{2\,(b^4c^2+ab^3cd-2\,a^2b^2d^2)x^2}{a^2b^3}+\frac{3\,(ab^3c^2-a^3bd^2)}{a^2b^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{d^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*x^2/(a^2*b^3) + 3*(a*b^3*c^2
 - a^3*b*d^2)/(a^2*b^3))/(b*x^2 + a)^(3/2) - d^2*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^2}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(5/2), x)
```

3.92 $\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*x*(d*x^2+c)/a/(b*x^2+a)^(3/2)+2/3*c*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {386, 197}

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}}\,dx&=\frac{x(c+dx^2)}{3a\left(a+bx^2\right)^{3/2}}+\frac{(2c)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\&=\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a\left(a+bx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 0.79

$$\frac{x(3ac+2bcx^2+adx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*c + 2*b*c*x^2 + a*d*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(39) = 78.
time = 0.06, size = 90, normalized size = 1.91

method	result	size
gosper	$\frac{x(a d\,x^2+2c\,x^2b+3ac)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
trager	$\frac{x(a d\,x^2+2c\,x^2b+3ac)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
default	$d\left(-\frac{x}{2b(b\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)+c\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x
^2+a)^(1/2)))+c*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 1.45

$$\frac{2\,cx}{3\,\sqrt{bx^2+a}\,a^2}+\frac{cx}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{dx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{dx}{3\,\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*x/((b*x^2 + a)^(3/2)*a) - 1/3*d*x/((b
*x^2 + a)^(3/2)*b) + 1/3*d*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [A]
time = 0.52, size = 54, normalized size = 1.15

$$\frac{((2\,bc+ad)x^3+3\,acx)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(41) = 82$.
time = 4.50, size = 144, normalized size = 3.06

$$c\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{dx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(5/2),x)
```

```
[Out] c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + d*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sq
rt(1 + b*x**2/a))
```

Giac [A]
time = 0.58, size = 40, normalized size = 0.85

$$\frac{x\left(\frac{3\,c}{a}+\frac{(2\,b^2c+abd)x^2}{a^2b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*c/a + (2*b^2*c + a*b*d)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

Mupad [B]
time = 4.78, size = 33, normalized size = 0.70

$$\frac{3\,a\,c\,x+a\,d\,x^3+2\,b\,c\,x^3}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*a*c*x + a*d*x^3 + 2*b*c*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

3.93 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {198, 197}

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a}\\ &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 0.74

$$\frac{3ax+2bx^3}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
```

```
[Out] (3*a*x + 2*b*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.05, size = 32, normalized size = 0.82

method	result	size
gosper	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
trager	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
default	$\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{bx^2+a}\,a^2}+\frac{x}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Fricas [A]
time = 0.62, size = 47, normalized size = 1.21

$$\frac{(2\,bx^3+3\,ax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

time = 0.44, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

Giac [A]
time = 0.63, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

Mupad [B]
time = 4.75, size = 28, normalized size = 0.72

$$\frac{2\,x\,(b\,x^2+a)+a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

3.94 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)} dx$

Optimal. Leaf size=122

$$\frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{5/2}}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+d^2*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)
/(b*x^2+a)^(1/2))/(-a*d+b*c)^(5/2)/c^(1/2)+1/3*b*(-5*a*d+2*b*c)*x/a^2/(-a*d
+b*c)^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {425, 541, 12, 385, 214}

$$\frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}\,(bc-ad)^2}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(bc-ad)^{5/2}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (b*(2*b*c - 5*a*d)*x)/(3*a^2*(b
*c - a*d)^2*Sqrt[a + b*x^2]) + (d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sq
rt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}} - \frac{\int \frac{-2bc+3ad-2bdx^2}{(a+bx^2)^{3/2}(c+dx^2)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\int \frac{3a^2d^2}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{3a^2(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}}{(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\right.}{(bc-}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{a+}}\right.}{\sqrt{c}(bc-ad)^{5/}}$$

Mathematica [A]
time = 0.39, size = 130, normalized size = 1.07

$$\frac{bx(-6a^2d+2b^2cx^2+ab(3c-5dx^2))}{3a^2(bc-ad)^2(a+bx^2)^{3/2}} - \frac{d^2\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x*(-6*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 5*d*x^2)))/(3*a^2*(b*c - a*d)^2*(
a + b*x^2)^(3/2)) - (d^2*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^
2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(-(b*c) + a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. $2(104) = 208$.
time = 0.06, size = 1378, normalized size = 11.30

method	result	size
default	Expression too large to display	1378

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-c*d)^(1/2)*(1/3/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-b*(-c*d)^(1/2)/(a*d-b*c)*(2/3*(2*b*
(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*
d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+16
/3*b/(4*b*(a*d-b*c)/d+4*b^2*c/d)^2*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)
/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2))+1/(a*d-b*c)*d*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2
*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(
-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)
-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))))-1/2/(-c*
d)^(1/2)*(1/3/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/(a*d-b*c)*(2/3*(2*b*(x+(-c*d
)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+(-c*d)^(1/2)
/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+16/3*b/(4*
b*(a*d-b*c)/d+4*b^2*c/d)^2*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/((x+
(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
))+1/(a*d-b*c)*d*(1/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x+(-
c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-
b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(
1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x
+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(104) = 208$.
time = 0.84, size = 764, normalized size = 6.26

$$\left[\frac{3\,(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2)\sqrt{bc^2-acd}\,\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ad)x^3+acx)\sqrt{bc^2-acd}\,\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)+4\,((2\,b^4c^3-7\,ab^3c^2d+5\,a^2b^2cd^2)x^3+3\,(ab^3c^3-3\,a^2b^2c^2d+2\,a^3bcd^2)x)\sqrt{bx^2+a}}{12\,(a^4b^3c^4-3\,a^5b^2c^3d+3\,a^6bc^2d^2-a^7cd^3+(a^2b^5c^4-3\,a^3b^4c^3d+3\,a^4b^3c^2d^2-a^5b^2cd^3)x^4+2\,(a^3b^4c^4-3\,a^4b^3c^3d+3\,a^5b^2c^2d^2-a^6bcd^3)x^2)},\ -\frac{3\,(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2)\sqrt{-bc^2+acd}\,\arctan\left(\frac{\sqrt{-bc^2+acd}\,((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)-2\,((2\,b^4c^3-7\,ab^3c^2d+5\,a^2b^2cd^2)x^3+3\,(ab^3c^3-3\,a^2b^2c^2d+2\,a^3bcd^2)x)\sqrt{bx^2+a}}{6\,(a^4b^3c^4-3\,a^5b^2c^3d+3\,a^6bc^2d^2-a^7cd^3+(a^2b^5c^4-3\,a^3b^4c^3d+3\,a^4b^3c^2d^2-a^5b^2cd^3)x^4+2\,(a^3b^4c^4-3\,a^4b^3c^3d+3\,a^5b^2c^2d^2-a^6bcd^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b*c^2 - a*c*d)*
log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2
*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 +
a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^4*c^3 - 7*a*b^3*c^2*d + 5*a^2*b^
2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x)*sqrt(b*x^
2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c*d^3 + (a^2
*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*x^4 + 2*(a^
3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x^2), -1/6*(
3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(-b*c^2 + a*c*d)*arctan
(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c
^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^4*c^3 - 7*a*b^3*c^2*d
 + 5*a^2*b^2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x
)*sqrt(b*x^2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c
*d^3 + (a^2*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*
x^4 + 2*(a^3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x
^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{2}\right)^{\frac{5}{2}} \left(c + dx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(104) = 208$.
time = 0.68, size = 320, normalized size = 2.62

$$-\frac{\sqrt{b}\, d^2 \arctan\left(\frac{\left(\sqrt{b}\, x-\sqrt{bx^2+a}\right)^2 d+2\, bc-ad}{2\, \sqrt{-b^2c^2+abcd}}\right)}{\left(b^2c^2-2\, abcd+a^2d^2\right)\sqrt{-b^2c^2+abcd}}+\frac{\left(\frac{\left(2\, b^6c^3-9\, ab^5c^2d+12\, a^2b^4cd^2-5\, a^3b^3d^3\right)x^2}{a^2b^5c^4-4\, a^3b^4c^3d+6\, a^4b^3c^2d^2-4\, a^5b^2cd^3+a^6bd^4}+\frac{3\, \left(ab^5c^3-4\, a^2b^4c^2d+5\, a^3b^3cd^2-2\, a^4b^2d^3\right)}{a^2b^5c^4-4\, a^3b^4c^3d+6\, a^4b^3c^2d^2-4\, a^5b^2cd^3+a^6bd^4}\right)x}{3\, \left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c^2 + a
*b*c*d)) + 1/3*((2*b^6*c^3 - 9*a*b^5*c^2*d + 12*a^2*b^4*c*d^2 - 5*a^3*b^3*d
^3)*x^2/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4) + 3*(a*b^5*c^3 - 4*a^2*b^4*c^2*d + 5*a^3*b^3*c*d^2 - 2*a^4*b
^2*d^3)/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4))*x/(b*x^2 + a)^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{5/2}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)), x)
```

3.95 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=202

$$\frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}}+\frac{b(4b^2c^2-16abcd-3a^2d^2)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)}+\frac{d^2(6bc-ad)}{2c}$$

```
[Out] 1/6*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(3/2)-1/2*d*x/c/(-a*d+b*c)
/(b*x^2+a)^(3/2)/(d*x^2+c)+1/2*d^2*(-a*d+6*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/
c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c)^(7/2)+1/6*b*(-3*a^2*d^2-16*a*b*
c*d+4*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {425, 541, 12, 385, 214}

$$\frac{bx(-3a^2d^2-16abcd+4b^2c^2)}{6a^2c\sqrt{a+bx^2}(bc-ad)^3}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}-\frac{dx}{2c(a+bx^2)^{3/2}(c+dx^2)(bc-ad)}+\frac{bx(3ad+2bc)}{6ac(a+bx^2)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (b*(4*b^2*c
^2 - 16*a*b*c*d - 3*a^2*d^2)*x)/(6*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]) - (
d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c - a*d)*A
rcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*
d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)} + \frac{\int \frac{2bc-ad-4bdx^2}{(a+bx^2)^{5/2}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)} - \frac{\int \frac{-4b^2c^2+12}{}}{}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}} + \frac{b(4b^2c^2-16abcd-3a^2d^2)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{d}{2c(bc-ad)(a+}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}} + \frac{b(4b^2c^2-16abcd-3a^2d^2)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{d}{2c(bc-ad)(a+}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}} + \frac{b(4b^2c^2-16abcd-3a^2d^2)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{d}{2c(bc-ad)(a+}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}} + \frac{b(4b^2c^2-16abcd-3a^2d^2)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{d}{2c(bc-ad)(a+}$$

Mathematica [A]

time = 1.30, size = 219, normalized size = 1.08

$$\frac{x(3a^4d^3+6a^3bd^3x^2-4b^4c^2x^2(c+dx^2)+3a^2b^2d(6c^2+6cdx^2+d^2x^4)+2ab^3c(-3c^2+5cdx^2+8d^2x^4))}{6a^2c(-bc+ad)^3\,(a+bx^2)^{3/2}\,(c+dx^2)}+\frac{d^2(6bc-ad)\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\,(c+dx^2)}{\sqrt{c}\,\sqrt{-bc+ad}}\right)}{2c^{3/2}(-bc+ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 4*b^4*c^2*x^2*(c + d*x^2) + 3*a^2*b^2*d*(
6*c^2 + 6*c*d*x^2 + d^2*x^4) + 2*a*b^3*c*(-3*c^2 + 5*c*d*x^2 + 8*d^2*x^4)))
/(6*a^2*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c - a
*d)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b
*c) + a*d])])/(2*c^(3/2)*(-(b*c) + a*d)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3448 vs. $2(178) = 356$.
time = 0.08, size = 3449, normalized size = 17.07

method	result	size
default	Expression too large to display	3449

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/(-c*d)^(1/2)*(1/3/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2
)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-b*(-c*d)^(1/2)/(a*d-b*c)*(2/3*(2*
b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-
c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+
16/3*b/(4*b*(a*d-b*c)/d+4*b^2*c/d)^2*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/
2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2))+1/(a*d-b*c)*d*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*
(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x
-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))))-1/4/d/
c*(-1/(a*d-b*c)*d/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/
2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-5*b*(-c*d)^(1/2)/(a*d-b*c)*(1/3/
(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(3/2)-b*(-c*d)^(1/2)/(a*d-b*c)*(2/3*(2*b*(x-(-c*d)^(1/2)/d)+2*b
*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+16/3*b/(4*b*(a*d-b*c)/d+
4*b^2*c/d)^2*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d
)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/(a*d-b*c)
```

```
*d*(1/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2)-2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x-(-c*d)^(1/2)/d)+
2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b
*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b
*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*
d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))))-4*b/(a*d-b*c)*d*(2/3*(2*b*(x-(
-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x-(-c*d)^(
1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+16/3*b
/(4*b*(a*d-b*c)/d+4*b^2*c/d)^2*(2*b*(x-(-c*d)^(1/2)/d)+2*b*(-c*d)^(1/2)/d)/
((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(1/2)))-1/4/c/(-c*d)^(1/2)*(1/3/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c
*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/(a*d-b*c)*
(2/3*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d
)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(3/2)+16/3*b/(4*b*(a*d-b*c)/d+4*b^2*c/d)^2*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-
c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2))+1/(a*d-b*c)*d*(1/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2
*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a
*d-b*c)*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*
c/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(1/2)-1/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(
1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b
*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d)))
)-1/4/d/c*(-1/(a*d-b*c)*d/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+5*b*(-c*d)^(1/2)/(a*d-b*
c)*(1/3/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(3/2)+b*(-c*d)^(1/2)/(a*d-b*c)*(2/3*(2*b*(x+(-c*d)^(1/2
)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+(-c*d)^(1/2)/d)^2*
b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+16/3*b/(4*b*(a*d
-b*c)/d+4*b^2*c/d)^2*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/((x+(-c*d)
^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/(
a*d-b*c)*d*(1/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+2*b*(-c*d)^(1/2)/(a*d-b*c)*(2*b*(x+(-c*d)^(
1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+(-c*d)^(1/2)/d)
^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/(a*d-b*c)*d
/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))))-4*b/(a*d-b*c)*d*(2/3*(
2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(1/2)/d)/(4*b*(a*d-b*c)/d+4*b^2*c/d)/((x+
(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2
)+16/3*b/(4*b*(a*d-b*c)/d+4*b^2*c/d)^2*(2*b*(x+(-c*d)^(1/2)/d)-2*b*(-c*d)^(
1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(178) = 356.
time = 1.43, size = 1440, normalized size = 7.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6
 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c
^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2
- 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((
2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2
*c*d*x^2 + c^2)) + 4*((4*b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3
+ 3*a^3*b^2*c*d^4)*x^5 + 2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 +
 6*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d
+ 6*a^3*b^2*c^3*d^2 - a^4*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b
^4*c^7 - 4*a^5*b^3*c^6*d + 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^
4 + (a^2*b^6*c^6*d - 4*a^3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*
d^4 + a^6*b^2*c^2*d^5)*x^6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5
*d^2 + 8*a^5*b^3*c^4*d^3 - 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^
3*b^5*c^7 - 7*a^4*b^4*c^6*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7
*b*c^3*d^4 + a^8*c^2*d^5)*x^2), -1/12*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*
a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt
(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((4*
b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 + 3*a^3*b^2*c*d^4)*x^5 +
2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3 + 3*a^
4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d + 6*a^3*b^2*c^3*d^2 - a^4
*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b^4*c^7 - 4*a^5*b^3*c^6*d
+ 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^4 + (a^2*b^6*c^6*d - 4*a^
3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*d^4 + a^6*b^2*c^2*d^5)*x^
6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5*d^2 + 8*a^5*b^3*c^4*d^3
```

```
- 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^3*b^5*c^7 - 7*a^4*b^4*c^6
*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7*b*c^3*d^4 + a^8*c^2*d^5)
*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{\frac{5}{2}}\,(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(178) = 356.
time = 1.15, size = 620, normalized size = 3.07

$$\frac{\left(\frac{2\,(b^8c^4-7\,ab^7c^3d+15\,a^2b^6c^2d^2-13\,a^3b^5cd^3+4\,a^4b^4d^4)x^2}{a^2b^7c^6-6\,a^3b^6c^5d+15\,a^4b^5c^4d^2-20\,a^5b^4c^3d^3+15\,a^6b^3c^2d^4-6\,a^7b^2cd^5+a^8bd^6}+\frac{3\,(ab^7c^4-6\,a^2b^6c^3d+12\,a^3b^5c^2d^2-10\,a^4b^4cd^3+3\,a^5b^3d^4)}{a^2b^7c^6-6\,a^3b^6c^5d+15\,a^4b^5c^4d^2-20\,a^5b^4c^3d^3+15\,a^6b^3c^2d^4-6\,a^7b^2cd^5+a^8bd^6}\right)x}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{\left(6\,b^{\frac{3}{2}}cd^2-a\sqrt{b}\,d^3\right)\arctan\left(-\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{-b^2c^2+abcd}}-\frac{2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cd^2-\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a\sqrt{b}\,d^3+a^2\sqrt{b}\,d^3}{(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^8*c^4 - 7*a*b^7*c^3*d + 15*a^2*b^6*c^2*d^2 - 13*a^3*b^5*c*d^3 + 4
*a^4*b^4*d^4)*x^2/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*
a^5*b^4*c^3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6) + 3*(a*
b^7*c^4 - 6*a^2*b^6*c^3*d + 12*a^3*b^5*c^2*d^2 - 10*a^4*b^4*c*d^3 + 3*a^5*b
^3*d^4)/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*a^5*b^4*c^
3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6))*x/(b*x^2 + a)^(3
/2) + 1/2*(6*b^(3/2)*c*d^2 - a*sqrt(b)*d^3)*arctan(-1/2*((sqrt(b)*x - sqrt(
b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^3*c^4 - 3*a*b^
2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(-b^2*c^2 + a*b*c*d)) - (2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*
a*sqrt(b)*d^3 + a^2*sqrt(b)*d^3)/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^
2 - a^3*c*d^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{5/2}\,{\left(d\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2), x)
```

3.96 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=313

$$-\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}+\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}+\frac{b(8b^2c^2-40abcd-3a^2d^2}{12a^2c(bc-ad)^3\sqrt{a+bx^2}\,(c}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^2+1/12*b*(3*a*d+4*b*c)*x/a/
c/(-a*d+b*c)^2/(b*x^2+a)^(3/2)/(d*x^2+c)+1/8*d^2*(3*a^2*d^2-16*a*b*c*d+48*b
^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b
*c)^(9/2)+1/12*b*(-3*a^2*d^2-40*a*b*c*d+8*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(d*
x^2+c)/(b*x^2+a)^(1/2)+1/24*d*(9*a^3*d^3-42*a^2*b*c*d^2-88*a*b^2*c^2*d+16*b
^3*c^3)*x*(b*x^2+a)^(1/2)/a^2/c^2/(-a*d+b*c)^4/(d*x^2+c)
```

Rubi [A]
time = 0.29, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {425, 541, 12, 385, 214}

$$\frac{bx(-3a^2d^2-40abcd+8b^2c^2)}{12a^2c\sqrt{a+bx^2}\,(c+dx^2)(bc-ad)^3}+\frac{d^2(3a^2d^2-16abcd+48b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}+\frac{dx\sqrt{a+bx^2}\,(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3)}{24a^2c^2(c+dx^2)(bc-ad)^4}-\frac{dx}{4c(a+bx^2)^{3/2}(c+dx^2)^2(bc-ad)}+\frac{bx(3ad+4bc)}{12ac(a+bx^2)^{3/2}(c+dx^2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + (b*(4*b*c + 3*
a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)) + (b*(8*b^2*c^
2 - 40*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]*(c +
 d*x^2)) + (d*(16*b^3*c^3 - 88*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*
Sqrt[a + b*x^2])/(24*a^2*c^2*(b*c - a*d)^4*(c + d*x^2)) + (d^2*(48*b^2*c^2
- 16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x
^2])])/(8*c^(5/2)*(b*c - a*d)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{\int \frac{4bc-3ad-6bdx^2}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2}$$

$$= -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2}$$

$$= -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2}$$

$$= -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2}$$

$$= -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2}$$

$$= -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2}$$

Mathematica [A]
time = 3.78, size = 367, normalized size = 1.17

$$\frac{\frac{\sqrt{c}\,x\left(16b^5c^3x^2(c+dx^2)^2+8ab^4c^2(3c-11dx^2)(c+dx^2)^2+3a^5d^4(5c+3dx^2)+3a^3b^2d^3x^2(-32c^2-23cdx^2+3d^2x^4)+6a^4bd^3(-8c^2-2cdx^2+3d^2x^4)-6a^2b^3cd(16c^3+32c^2dx^2+24cd^2x^4+7d^3x^6)\right)}{a^2(bc-ad)^4(a+bx^2)^{3/2}(c+dx^2)^2}-\frac{9d^2(-4bc+ad)^2\tan^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{(-bc+ad)^{9/2}}+\frac{24abcd^3\tanh^{-1}\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{bc-ad}}\right)}{(bc-ad)^{9/2}}}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] ((Sqrt[c]*x*(16*b^5*c^3*x^2*(c + d*x^2)^2 + 8*a*b^4*c^2*(3*c - 11*d*x^2)*(c
 + d*x^2)^2 + 3*a^5*d^4*(5*c + 3*d*x^2) + 3*a^3*b^2*d^3*x^2*(-32*c^2 - 23*c
*d*x^2 + 3*d^2*x^4) + 6*a^4*b*d^3*(-8*c^2 - 2*c*d*x^2 + 3*d^2*x^4) - 6*a^2*
b^3*c*d*(16*c^3 + 32*c^2*d*x^2 + 24*c*d^2*x^4 + 7*d^3*x^6)))/(a^2*(b*c - a*
d)^4*(a + b*x^2)^(3/2)*(c + d*x^2)^2) - (9*d^2*(-4*b*c + a*d)^2*ArcTan[(-(d
*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(
-(b*c) + a*d)^(9/2) + (24*a*b*c*d^3*ArcTanh[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[
b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/(b*c - a*d)^(9/2))/(24*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7120 vs. $2(285) = 570$.
time = 0.09, size = 7121, normalized size = 22.75

method	result	size
default	Expression too large to display	7121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(285) = 570$.
time = 12.31, size = 2250, normalized size = 7.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(48*a^4*b^2*c^4*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b
^4*c^2*d^4 - 16*a^3*b^3*c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3
+ 32*a^3*b^3*c^2*d^4 - 13*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^
4*d^2 + 176*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^
6)*x^4 + 2*(48*a^3*b^3*c^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*
a^6*c*d^5)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*
x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((
16*b^6*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^
5 - 9*a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^
4*d^3 + 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16
*b^6*c^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a
^4*b^2*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*
a^2*b^4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4
- 5*a^6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a
^6*b^3*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*
b^7*c^8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 +
```

```
 5*a^6*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^
8*d^2 + 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c
^3*d^7)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^
4*c^7*d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^
3*d^7)*x^4 + 2*(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*
b^2*c^6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2), -1/48*(3*(48*a^4*b^2*c^4
*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b^4*c^2*d^4 - 16*a^3*b^3*
c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3 + 32*a^3*b^3*c^2*d^4 - 1
3*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^4*d^2 + 176*a^3*b^3*c^3*
d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^6)*x^4 + 2*(48*a^3*b^3*c
^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*a^6*c*d^5)*x^2)*sqrt(-b*
c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt
(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^6
*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^5 - 9*
a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^4*d^3
+ 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16*b^6*c
^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a^4*b^2
*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*a^2*b^
4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4 - 5*a^
6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a^6*b^3
*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*b^7*c^
8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 + 5*a^6
*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^8*d^2
+ 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c^3*d^7
)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^4*c^7*
d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^3*d^7)
*x^4 + 2*(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*b^2*c^
6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(285) = 570$.
time = 2.13, size = 1010, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/3*((2*b^10*c^5 - 19*a*b^9*c^4*d + 56*a^2*b^8*c^3*d^2 - 74*a^3*b^7*c^2*d^3
 + 46*a^4*b^6*c*d^4 - 11*a^5*b^5*d^5)*x^2/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d +
28*a^4*b^7*c^6*d^2 - 56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c
^3*d^5 + 28*a^8*b^3*c^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8) + 3*(a*b^9*c^5
- 8*a^2*b^8*c^4*d + 22*a^3*b^7*c^3*d^2 - 28*a^4*b^6*c^2*d^3 + 17*a^5*b^5*c*
d^4 - 4*a^6*b^4*d^5)/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d + 28*a^4*b^7*c^6*d^2 -
56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c^3*d^5 + 28*a^8*b^3*c
^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^(5/
2)*c^2*d^2 - 16*a*b^(3/2)*c*d^3 + 3*a^2*sqrt(b)*d^4)*arctan(1/2*((sqrt(b)*x
 - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^6
- 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-
b^2*c^2 + a*b*c*d)) - 1/4*(24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d
^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^4 + 3*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a^2*sqrt(b)*d^5 + 112*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7
/2)*c^3*d^2 - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^3 + 66*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^4 - 9*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^3*sqrt(b)*d^5 + 88*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c
^2*d^3 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^4 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^5 + 14*a^4*b^(3/2)*c*d^4 - 3*a^5*sqrt(
b)*d^5)/((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a
^4*c^2*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x)
```

```
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3), x)
```

3.97 $\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{dx(a+bx^2)^4}{9c(bc-ad)(c+dx^2)^{9/2}}+\frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)(c+dx^2)^{7/2}}+\frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)(c+dx^2)^{5/2}}+\frac{8a^2(9bc-8ad)x(a+\ldots}{315c^4(bc-ad)(c+d\ldots}$$

```
[Out] -1/9*d*x*(b*x^2+a)^4/c/(-a*d+b*c)/(d*x^2+c)^(9/2)+1/63*(-8*a*d+9*b*c)*x*(b*
x^2+a)^3/c^2/(-a*d+b*c)/(d*x^2+c)^(7/2)+2/105*a*(-8*a*d+9*b*c)*x*(b*x^2+a)^
2/c^3/(-a*d+b*c)/(d*x^2+c)^(5/2)+8/315*a^2*(-8*a*d+9*b*c)*x*(b*x^2+a)/c^4/(
-a*d+b*c)/(d*x^2+c)^(3/2)+16/315*a^3*(-8*a*d+9*b*c)*x/c^5/(-a*d+b*c)/(d*x^2
+c)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 386, 197}

$$\frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}\,(bc-ad)}+\frac{8a^2x(a+bx^2)(9bc-8ad)}{315c^4(c+dx^2)^{3/2}(bc-ad)}+\frac{2ax(a+bx^2)^2(9bc-8ad)}{105c^3(c+dx^2)^{5/2}(bc-ad)}+\frac{x(a+bx^2)^3(9bc-8ad)}{63c^2(c+dx^2)^{7/2}(bc-ad)}-\frac{dx(a+bx^2)^4}{9c(c+dx^2)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] -1/9*(d*x*(a + b*x^2)^4)/(c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*c - 8*a*
d)*x*(a + b*x^2)^3)/(63*c^2*(b*c - a*d)*(c + d*x^2)^(7/2)) + (2*a*(9*b*c -
8*a*d)*x*(a + b*x^2)^2)/(105*c^3*(b*c - a*d)*(c + d*x^2)^(5/2)) + (8*a^2*(9
*b*c - 8*a*d)*x*(a + b*x^2))/(315*c^4*(b*c - a*d)*(c + d*x^2)^(3/2)) + (16*
a^3*(9*b*c - 8*a*d)*x)/(315*c^5*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = -\frac{dx(a+bx^2)^4}{9c(bc-ad)(c+dx^2)^{9/2}} + \frac{(9bc-8ad)\int \frac{(a+bx^2)^3}{(c+dx^2)^{9/2}}\,dx}{9c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^4}{9c(bc-ad)(c+dx^2)^{9/2}} + \frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)(c+dx^2)^{7/2}} + \frac{(2a(9bc-8ad))\int \frac{(a+bx^2)^2}{(c+dx^2)^{7/2}}\,d}{21c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^4}{9c(bc-ad)(c+dx^2)^{9/2}} + \frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)(c+dx^2)^{7/2}} + \frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)(c+dx^2)^{5/2}}$$

$$= -\frac{dx(a+bx^2)^4}{9c(bc-ad)(c+dx^2)^{9/2}} + \frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)(c+dx^2)^{7/2}} + \frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)(c+dx^2)^{5/2}}$$

$$= -\frac{dx(a+bx^2)^4}{9c(bc-ad)(c+dx^2)^{9/2}} + \frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)(c+dx^2)^{7/2}} + \frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)(c+dx^2)^{5/2}}$$

Mathematica [**A**]
time = 0.35, size = 163, normalized size = 0.73

$$\frac{5b^3c^3x^7(9c+2dx^2)+3ab^2c^2x^5(63c^2+36cdx^2+8d^2x^4)+3a^2bcx^3(105c^3+126c^2dx^2+72cd^2x^4+16d^3x^6)+a^3(315c^4x+840c^3dx^3+1008c^2d^2x^5+576cd^3x^7+128d^4x^9)}{315c^5(c+dx^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] (5*b^3*c^3*x^7*(9*c + 2*d*x^2) + 3*a*b^2*c^2*x^5*(63*c^2 + 36*c*d*x^2 + 8*d
^2*x^4) + 3*a^2*b*c*x^3*(105*c^3 + 126*c^2*d*x^2 + 72*c*d^2*x^4 + 16*d^3*x^
6) + a^3*(315*c^4*x + 840*c^3*d*x^3 + 1008*c^2*d^2*x^5 + 576*c*d^3*x^7 + 12
8*d^4*x^9))/(315*c^5*(c + d*x^2)^(9/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(204) = 408.
time = 0.09, size = 536, normalized size = 2.39

method	result
gosper	$\frac{x\left(128a^{3}d^{4}x^{8}+48a^{2}bc\,d^{3}x^{8}+24a\,b^{2}c^{2}d^{2}x^{8}+10b^{3}c^{3}d\,x^{8}+576a^{3}c\,d^{3}x^{6}+216a^{2}b\,c^{2}d^{2}x^{6}+108a\,b^{2}c^{3}d\,x^{6}+45b^{3}c^{4}x^{6}+1008a^{3}c^{2}d^{2}x^{4}+378\right.}{315\left(d\,x^{2}+c\right)^{\frac{9}{2}}c^{5}}$
trager	$\frac{x\left(128a^{3}d^{4}x^{8}+48a^{2}bc\,d^{3}x^{8}+24a\,b^{2}c^{2}d^{2}x^{8}+10b^{3}c^{3}d\,x^{8}+576a^{3}c\,d^{3}x^{6}+216a^{2}b\,c^{2}d^{2}x^{6}+108a\,b^{2}c^{3}d\,x^{6}+45b^{3}c^{4}x^{6}+1008a^{3}c^{2}d^{2}x^{4}+378\right.}{315\left(d\,x^{2}+c\right)^{\frac{9}{2}}c^{5}}$

default $b^3\left(-\frac{x^5}{4d(dx^2+c)^{\frac{9}{2}}}+\frac{5c\left(-\frac{x^3}{6d\left(dx^2+c\right)^{\frac{9}{2}}}+\frac{c\left(-\frac{x}{8d\left(dx^2+c\right)^{\frac{9}{2}}}+\frac{c\left(\frac{x}{9c\left(dx^2+c\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63c\left(dx^2+c\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35c\left(dx^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(dx^2+c\right)^{\frac{3}{2}}}\right.}{}\right)}{9c}}{c}\right)}{8d}\right)}{2d}\right)}{4d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/4*x^5/d/(d*x^2+c)^(9/2)+5/4*c/d*(-1/6*x^3/d/(d*x^2+c)^(9/2)+1/2*c/d
*(-1/8*x/d/(d*x^2+c)^(9/2)+1/8*c/d*(1/9*x/c/(d*x^2+c)^(9/2)+8/9/c*(1/7*x/c/
(d*x^2+c)^(7/2)+6/7/c*(1/5*x/c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(d*x^2+c)^(3/
2)+2/3*x/c^2/(d*x^2+c)^(1/2)))))))+3*a*b^2*(-1/6*x^3/d/(d*x^2+c)^(9/2)+1/2
*c/d*(-1/8*x/d/(d*x^2+c)^(9/2)+1/8*c/d*(1/9*x/c/(d*x^2+c)^(9/2)+8/9/c*(1/7*
x/c/(d*x^2+c)^(7/2)+6/7/c*(1/5*x/c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(d*x^2+c)
^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))))))+3*a^2*b*(-1/8*x/d/(d*x^2+c)^(9/2)+1/
8*c/d*(1/9*x/c/(d*x^2+c)^(9/2)+8/9/c*(1/7*x/c/(d*x^2+c)^(7/2)+6/7/c*(1/5*x/
c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))
))))+a^3*(1/9*x/c/(d*x^2+c)^(9/2)+8/9/c*(1/7*x/c/(d*x^2+c)^(7/2)+6/7/c*(1/5
*x/c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/
2)))))
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(204) = 408$.
time = 0.35, size = 465, normalized size = 2.08

$$-\frac{b^3x^5}{4\,(dx^2+c)^{\frac{9}{2}}d}-\frac{5\,b^3cx^3}{24\,(dx^2+c)^{\frac{9}{2}}d^2}-\frac{ab^2x^3}{2\,(dx^2+c)^{\frac{9}{2}}d}+\frac{128\,a^3x}{315\,\sqrt{dx^2+c}\,c^5}+\frac{64\,a^3x}{315\,(dx^2+c)^{\frac{3}{2}}c^4}+\frac{16\,a^3x}{105\,(dx^2+c)^{\frac{5}{2}}c^3}+\frac{8\,a^3x}{63\,(dx^2+c)^{\frac{7}{2}}c^2}+\frac{a^3x}{9\,(dx^2+c)^{\frac{9}{2}}c}+\frac{b^3x}{84\,(dx^2+c)^{\frac{5}{2}}d^3}+\frac{2\,b^3x}{63\,\sqrt{dx^2+c}\,c^2d^3}+\frac{b^3x}{63\,(dx^2+c)^{\frac{3}{2}}cd^3}+\frac{5\,b^3cx}{504\,(dx^2+c)^{\frac{7}{2}}d^3}-\frac{5\,b^3c^2x}{72\,(dx^2+c)^{\frac{9}{2}}d^3}+\frac{ab^2x}{42\,(dx^2+c)^{\frac{7}{2}}d^2}+\frac{8\,ab^2x}{105\,\sqrt{dx^2+c}\,c^3d^2}+\frac{4\,ab^2x}{105\,(dx^2+c)^{\frac{3}{2}}c^2d^2}+\frac{ab^2x}{35\,(dx^2+c)^{\frac{5}{2}}cd^2}-\frac{ab^2cx}{6\,(dx^2+c)^{\frac{9}{2}}d^2}-\frac{a^2bx}{3\,(dx^2+c)^{\frac{9}{2}}d}+\frac{16\,a^2bx}{105\,\sqrt{dx^2+c}\,c^4d}+\frac{8\,a^2bx}{105\,(dx^2+c)^{\frac{3}{2}}c^3d}+\frac{2\,a^2bx}{35\,(dx^2+c)^{\frac{5}{2}}c^2d}+\frac{a^2bx}{21\,(dx^2+c)^{\frac{7}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^3*x^5/((d*x^2 + c)^(9/2)*d) - 5/24*b^3*c*x^3/((d*x^2 + c)^(9/2)*d^2)
 - 1/2*a*b^2*x^3/((d*x^2 + c)^(9/2)*d) + 128/315*a^3*x/(sqrt(d*x^2 + c)*c^5
) + 64/315*a^3*x/((d*x^2 + c)^(3/2)*c^4) + 16/105*a^3*x/((d*x^2 + c)^(5/2)*
c^3) + 8/63*a^3*x/((d*x^2 + c)^(7/2)*c^2) + 1/9*a^3*x/((d*x^2 + c)^(9/2)*c)
 + 1/84*b^3*x/((d*x^2 + c)^(5/2)*d^3) + 2/63*b^3*x/(sqrt(d*x^2 + c)*c^2*d^3
) + 1/63*b^3*x/((d*x^2 + c)^(3/2)*c*d^3) + 5/504*b^3*c*x/((d*x^2 + c)^(7/2)
*d^3) - 5/72*b^3*c^2*x/((d*x^2 + c)^(9/2)*d^3) + 1/42*a*b^2*x/((d*x^2 + c)^
(7/2)*d^2) + 8/105*a*b^2*x/(sqrt(d*x^2 + c)*c^3*d^2) + 4/105*a*b^2*x/((d*x^
2 + c)^(3/2)*c^2*d^2) + 1/35*a*b^2*x/((d*x^2 + c)^(5/2)*c*d^2) - 1/6*a*b^2*
c*x/((d*x^2 + c)^(9/2)*d^2) - 1/3*a^2*b*x/((d*x^2 + c)^(9/2)*d) + 16/105*a^
2*b*x/(sqrt(d*x^2 + c)*c^4*d) + 8/105*a^2*b*x/((d*x^2 + c)^(3/2)*c^3*d) + 2
/35*a^2*b*x/((d*x^2 + c)^(5/2)*c^2*d) + 1/21*a^2*b*x/((d*x^2 + c)^(7/2)*c*d
)
```

<u>Fricas</u> [**A**]
time = 4.67, size = 229, normalized size = 1.02

$$\frac{(2\,(5\,b^3c^3d+12\,ab^2c^2d^2+24\,a^2bcd^3+64\,a^3d^4)x^9+315\,a^3c^4x+9\,(5\,b^3c^4+12\,ab^2c^3d+24\,a^2bc^2d^2+64\,a^3cd^3)x^7+63\,(3\,ab^2c^4+6\,a^2bc^3d+16\,a^3c^2d^2)x^5+105\,(3\,a^2bc^4+8\,a^3c^3d)x^3)\sqrt{dx^2+c}}{315\,(c^5d^5x^{10}+5\,c^6d^4x^8+10\,c^7d^3x^6+10\,c^8d^2x^4+5\,c^9dx^2+c^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(2*(5*b^3*c^3*d + 12*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 64*a^3*d^4)*x^9
 + 315*a^3*c^4*x + 9*(5*b^3*c^4 + 12*a*b^2*c^3*d + 24*a^2*b*c^2*d^2 + 64*a^
3*c*d^3)*x^7 + 63*(3*a*b^2*c^4 + 6*a^2*b*c^3*d + 16*a^3*c^2*d^2)*x^5 + 105*
(3*a^2*b*c^4 + 8*a^3*c^3*d)*x^3)*sqrt(d*x^2 + c)/(c^5*d^5*x^10 + 5*c^6*d^4*
x^8 + 10*c^7*d^3*x^6 + 10*c^8*d^2*x^4 + 5*c^9*d*x^2 + c^10)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**(11/2),x)
```

```
[Out] Integral((a + b*x**2)**3/(c + d*x**2)**(11/2), x)
```

Giac [A]
time = 0.90, size = 218, normalized size = 0.97

$$\frac{\left(\left(\left(x^2\left(\frac{2\,(5\,b^3c^3d^5+12\,ab^2c^2d^6+24\,a^2bcd^7+64\,a^3d^8)x^2}{c^5d^4}+\frac{9\,(5\,b^3c^4d^4+12\,ab^2c^3d^5+24\,a^2bc^2d^6+64\,a^3cd^7)}{c^5d^4}\right)+\frac{63\,(3\,ab^2c^4d^4+6\,a^2bc^3d^5+16\,a^3c^2d^6)}{c^5d^4}\right)x^2+\frac{105\,(3\,a^2bc^4d^4+8\,a^3c^3d^5)}{c^5d^4}\right)x^2+\frac{315\,a^3}{c}\right)x}{315\,(dx^2+c)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((x^2*(2*(5*b^3*c^3*d^5 + 12*a*b^2*c^2*d^6 + 24*a^2*b*c*d^7 + 64*a^3
*d^8)*x^2/(c^5*d^4) + 9*(5*b^3*c^4*d^4 + 12*a*b^2*c^3*d^5 + 24*a^2*b*c^2*d^
6 + 64*a^3*c*d^7)/(c^5*d^4)) + 63*(3*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^5 + 16*a
^3*c^2*d^6)/(c^5*d^4))*x^2 + 105*(3*a^2*b*c^4*d^4 + 8*a^3*c^3*d^5)/(c^5*d^4
))*x^2 + 315*a^3/c)*x/(d*x^2 + c)^(9/2)
```

Mupad [B]
time = 5.15, size = 326, normalized size = 1.46

$$\frac{x\left(\frac{a^3}{9\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{9\,d}-\frac{a\,b^2}{3\,c}\right)}{d}+\frac{a^2\,b}{3\,c}\right)}{d}\right)}{{\left(d\,x^2+c\right)}^{9/2}}-\frac{x\left(\frac{b^3}{5\,d^3}-\frac{16\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-4\,b^3\,c^3}{105\,c^3\,d^3}\right)}{{\left(d\,x^2+c\right)}^{5/2}}+\frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{8\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{63\,c^2\,d^3}\right)}{{\left(d\,x^2+c\right)}^{7/2}}+\frac{x\,(64\,a^3\,d^3+24\,a^2\,b\,c\,d^2+12\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}{315\,c^4\,d^3\,{\left(d\,x^2+c\right)}^{3/2}}+\frac{x\,(128\,a^3\,d^3+48\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+10\,b^3\,c^3)}{315\,c^5\,d^3\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/(c + d*x^2)^(11/2),x)
```

```
[Out] (x*(a^3/(9*c) - (c*((c*(b^3/(9*d) - (a*b^2)/(3*c)))/d + (a^2*b)/(3*c)))/d))
/(c + d*x^2)^(9/2) - (x*(b^3/(5*d^3) - (16*a^3*d^3 - 4*b^3*c^3 + 3*a*b^2*c^
```

```
2*d + 6*a^2*b*c*d^2)/(105*c^3*d^3)))/(c + d*x^2)^(5/2) + (x*((c*(b^3/(7*d^2
) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (8*a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2)/(63*c^2*d^3)))/(c + d*x^2)^(7/2) + (x*(64*a^3*d^3 + 5*b^
3*c^3 + 12*a*b^2*c^2*d + 24*a^2*b*c*d^2))/(315*c^4*d^3*(c + d*x^2)^(3/2)) +
 (x*(128*a^3*d^3 + 10*b^3*c^3 + 24*a*b^2*c^2*d + 48*a^2*b*c*d^2))/(315*c^5*
d^3*(c + d*x^2)^(1/2))
```

3.98 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{dx(a+bx^2)^3}{7c(bc-ad)(c+dx^2)^{7/2}}+\frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)(c+dx^2)^{5/2}}+\frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)(c+dx^2)^{3/2}}+\frac{8a^2(7bc-6ad)x}{105c^4(bc-ad)\sqrt{c+dx^2}}$$

```
[Out] -1/7*d*x*(b*x^2+a)^3/c/(-a*d+b*c)/(d*x^2+c)^(7/2)+1/35*(-6*a*d+7*b*c)*x*(b*
x^2+a)^2/c^2/(-a*d+b*c)/(d*x^2+c)^(5/2)+4/105*a*(-6*a*d+7*b*c)*x*(b*x^2+a)/
c^3/(-a*d+b*c)/(d*x^2+c)^(3/2)+8/105*a^2*(-6*a*d+7*b*c)*x/c^4/(-a*d+b*c)/(d
*x^2+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 386, 197}

$$\frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}\,(bc-ad)}+\frac{4ax(a+bx^2)(7bc-6ad)}{105c^3(c+dx^2)^{3/2}(bc-ad)}+\frac{x(a+bx^2)^2(7bc-6ad)}{35c^2(c+dx^2)^{5/2}(bc-ad)}-\frac{dx(a+bx^2)^3}{7c(c+dx^2)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] -1/7*(d*x*(a + b*x^2)^3)/(c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*c - 6*a*
d)*x*(a + b*x^2)^2)/(35*c^2*(b*c - a*d)*(c + d*x^2)^(5/2)) + (4*a*(7*b*c -
6*a*d)*x*(a + b*x^2))/(105*c^3*(b*c - a*d)*(c + d*x^2)^(3/2)) + (8*a^2*(7*b
*c - 6*a*d)*x)/(105*c^4*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = -\frac{dx(a+bx^2)^3}{7c(bc-ad)\,(c+dx^2)^{7/2}} + \frac{(7bc-6ad)\int \frac{(a+bx^2)^2}{(c+dx^2)^{7/2}}\,dx}{7c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^3}{7c(bc-ad)\,(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\,(c+dx^2)^{5/2}} + \frac{(4a(7bc-6ad))\int \frac{a+bx^2}{(c+dx^2)^{5/2}}}{35c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^3}{7c(bc-ad)\,(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\,(c+dx^2)^{5/2}} + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)\,(c+dx^2)^{3/2}}$$

$$= -\frac{dx(a+bx^2)^3}{7c(bc-ad)\,(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\,(c+dx^2)^{5/2}} + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)\,(c+dx^2)^{3/2}}$$

Mathematica [A]
time = 0.20, size = 107, normalized size = 0.61

$$\frac{3b^2c^2x^5(7c+2dx^2)+2abcx^3(35c^2+28cdx^2+8d^2x^4)+3a^2(35c^3x+70c^2dx^3+56cd^2x^5+16d^3x^7)}{105c^4\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
[Out] (3*b^2*c^2*x^5*(7*c + 2*d*x^2) + 2*a*b*c*x^3*(35*c^2 + 28*c*d*x^2 + 8*d^2*x
^4) + 3*a^2*(35*c^3*x + 70*c^2*d*x^3 + 56*c*d^2*x^5 + 16*d^3*x^7))/(105*c^4
*(c + d*x^2)^(7/2))
```

Maple [A]
time = 0.07, size = 301, normalized size = 1.73

method	result
gosper	$\frac{x(48a^2d^3x^6+16abc\,d^2x^6+6b^2c^2d\,x^6+168a^2c\,d^2x^4+56ab\,c^2d\,x^4+21b^2c^3x^4+210a^2c^2d\,x^2+70ab\,c^3x^2+105a^2c^3)}{105(d\,x^2+c)^{\frac{7}{2}}c^4}$
trager	$\frac{x(48a^2d^3x^6+16abc\,d^2x^6+6b^2c^2d\,x^6+168a^2c\,d^2x^4+56ab\,c^2d\,x^4+21b^2c^3x^4+210a^2c^2d\,x^2+70ab\,c^3x^2+105a^2c^3)}{105(d\,x^2+c)^{\frac{7}{2}}c^4}$

default	$b^{2}\left(-\frac{x^{3}}{4d(dx^{2}+c)^{\frac{7}{2}}}+\frac{3c\left(-\frac{x}{6d\left(dx^{2}+c\right)^{\frac{7}{2}}}+\frac{c\left(\frac{x}{7c\left(dx^{2}+c\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(dx^{2}+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^{2}\sqrt{dx^{2}+c}}\right)}{7c}}{c}\right)}{6d}\right)}{4d}\right)+2a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/4*x^3/d/(d*x^2+c)^(7/2)+3/4*c/d*(-1/6*x/d/(d*x^2+c)^(7/2)+1/6*c/d*(
1/7*x/c/(d*x^2+c)^(7/2)+6/7/c*(1/5*x/c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(d*x^
2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))))))+2*a*b*(-1/6*x/d/(d*x^2+c)^(7/2)+1
/6*c/d*(1/7*x/c/(d*x^2+c)^(7/2)+6/7/c*(1/5*x/c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x
/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)))))+a^2*(1/7*x/c/(d*x^2+c)^(7/
2)+6/7/c*(1/5*x/c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/
(d*x^2+c)^(1/2))))
```

Maxima [A]
time = 0.29, size = 249, normalized size = 1.43

$$-\frac{b^{2}x^{3}}{4\,(dx^{2}+c)^{\frac{7}{2}}d}+\frac{16\,a^{2}x}{35\,\sqrt{dx^{2}+c}\,c^{4}}+\frac{8\,a^{2}x}{35\,(dx^{2}+c)^{\frac{3}{2}}c^{3}}+\frac{6\,a^{2}x}{35\,(dx^{2}+c)^{\frac{5}{2}}c^{2}}+\frac{a^{2}x}{7\,(dx^{2}+c)^{\frac{7}{2}}c}+\frac{3\,b^{2}x}{140\,(dx^{2}+c)^{\frac{5}{2}}d^{2}}+\frac{2\,b^{2}x}{35\,\sqrt{dx^{2}+c}\,c^{2}d^{2}}+\frac{b^{2}x}{35\,(dx^{2}+c)^{\frac{3}{2}}cd^{2}}-\frac{3\,b^{2}cx}{28\,(dx^{2}+c)^{\frac{7}{2}}d^{2}}-\frac{2\,abx}{7\,(dx^{2}+c)^{\frac{7}{2}}d}+\frac{16\,abx}{105\,\sqrt{dx^{2}+c}\,c^{3}d}+\frac{8\,abx}{105\,(dx^{2}+c)^{\frac{3}{2}}c^{2}d}+\frac{2\,abx}{35\,(dx^{2}+c)^{\frac{5}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*x^3/((d*x^2 + c)^(7/2)*d) + 16/35*a^2*x/(sqrt(d*x^2 + c)*c^4) + 8/
35*a^2*x/((d*x^2 + c)^(3/2)*c^3) + 6/35*a^2*x/((d*x^2 + c)^(5/2)*c^2) + 1/7
*a^2*x/((d*x^2 + c)^(7/2)*c) + 3/140*b^2*x/((d*x^2 + c)^(5/2)*d^2) + 2/35*b
^2*x/(sqrt(d*x^2 + c)*c^2*d^2) + 1/35*b^2*x/((d*x^2 + c)^(3/2)*c*d^2) - 3/2
8*b^2*c*x/((d*x^2 + c)^(7/2)*d^2) - 2/7*a*b*x/((d*x^2 + c)^(7/2)*d) + 16/10
5*a*b*x/(sqrt(d*x^2 + c)*c^3*d) + 8/105*a*b*x/((d*x^2 + c)^(3/2)*c^2*d) + 2
/35*a*b*x/((d*x^2 + c)^(5/2)*c*d)
```

Fricas [A]
time = 2.37, size = 151, normalized size = 0.87

$$\frac{\left(2\left(3\,b^2c^2d+8\,abcd^2+24\,a^2d^3\right)x^7+105\,a^2c^3x+7\left(3\,b^2c^3+8\,abc^2d+24\,a^2cd^2\right)x^5+70\left(abc^3+3\,a^2c^2d\right)x^3\right)\sqrt{dx^2+c}}{105\left(c^4d^4x^8+4\,c^5d^3x^6+6\,c^6d^2x^4+4\,c^7dx^2+c^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*b^2*c^2*d + 8*a*b*c*d^2 + 24*a^2*d^3)*x^7 + 105*a^2*c^3*x + 7*(
3*b^2*c^3 + 8*a*b*c^2*d + 24*a^2*c*d^2)*x^5 + 70*(a*b*c^3 + 3*a^2*c^2*d)*x^
3)*sqrt(d*x^2 + c)/(c^4*d^4*x^8 + 4*c^5*d^3*x^6 + 6*c^6*d^2*x^4 + 4*c^7*d*x
^2 + c^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(9/2), x)
```

Giac [A]
time = 0.88, size = 138, normalized size = 0.79

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,b^2c^2d^4+8\,abcd^5+24\,a^2d^6\right)x^2}{c^4d^3}+\frac{7\left(3\,b^2c^3d^3+8\,abc^2d^4+24\,a^2cd^5\right)}{c^4d^3}\right)+\frac{70\left(abc^3d^3+3\,a^2c^2d^4\right)}{c^4d^3}\right)x^2+\frac{105\,a^2}{c}\right)x}{105\left(dx^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*b^2*c^2*d^4 + 8*a*b*c*d^5 + 24*a^2*d^6)*x^2/(c^4*d^3) + 7
*(3*b^2*c^3*d^3 + 8*a*b*c^2*d^4 + 24*a^2*c*d^5)/(c^4*d^3)) + 70*(a*b*c^3*d^
3 + 3*a^2*c^2*d^4)/(c^4*d^3))*x^2 + 105*a^2/c)*x/(d*x^2 + c)^(7/2)
```

Mupad [B]
time = 4.99, size = 176, normalized size = 1.01

$$\frac{x\left(\frac{a^2}{7c}+\frac{c\left(\frac{b^2}{7d}-\frac{2ab}{7c}\right)}{d}\right)}{(dx^2+c)^{7/2}}-\frac{x\left(\frac{b^2}{5d^2}-\frac{6a^2d^2+2abcd-b^2c^2}{35c^2d^2}\right)}{(dx^2+c)^{5/2}}+\frac{x(24a^2d^2+8abcd+3b^2c^2)}{105c^3d^2(dx^2+c)^{3/2}}+\frac{x(48a^2d^2+16abcd+6b^2c^2)}{105c^4d^2\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(9/2),x)
```

```
[Out] (x*(a^2/(7*c) + (c*(b^2/(7*d) - (2*a*b)/(7*c)))/d))/(c + d*x^2)^(7/2) - (x*
(b^2/(5*d^2) - (6*a^2*d^2 - b^2*c^2 + 2*a*b*c*d)/(35*c^2*d^2)))/(c + d*x^2)
^(5/2) + (x*(24*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/(105*c^3*d^2*(c + d*x^2)^
(3/2)) + (x*(48*a^2*d^2 + 6*b^2*c^2 + 16*a*b*c*d))/(105*c^4*d^2*(c + d*x^2)
^(1/2))
```

3.99 $\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\left(c+dx^2\right)^{3/2}}+\frac{2(bc+4ad)x}{15c^3d\sqrt{c+dx^2}}$$

```
[Out] -1/5*(-a*d+b*c)*x/c/d/(d*x^2+c)^(5/2)+1/15*(4*a*d+b*c)*x/c^2/d/(d*x^2+c)^(3
/2)+2/15*(4*a*d+b*c)*x/c^3/d/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {393, 198, 197}

$$\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\left(c+dx^2\right)^{3/2}}-\frac{x(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] -1/5*((b*c - a*d)*x)/(c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*x)/(15*c^2*d*
(c + d*x^2)^(3/2)) + (2*(b*c + 4*a*d)*x)/(15*c^3*d*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = -\frac{(bc-ad)x}{5cd\,(c+dx^2)^{5/2}} + \frac{(bc+4ad)\int \frac{1}{(c+dx^2)^{5/2}}\,dx}{5cd}$$
$$= -\frac{(bc-ad)x}{5cd\,(c+dx^2)^{5/2}} + \frac{(bc+4ad)x}{15c^2d\,(c+dx^2)^{3/2}} + \frac{(2(bc+4ad))\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{15c^2d}$$
$$= -\frac{(bc-ad)x}{5cd\,(c+dx^2)^{5/2}} + \frac{(bc+4ad)x}{15c^2d\,(c+dx^2)^{3/2}} + \frac{2(bc+4ad)x}{15c^3d\sqrt{c+dx^2}}$$

Mathematica [A]
time = 0.12, size = 60, normalized size = 0.66

$$\frac{15ac^2x+5bc^2x^3+20acdx^3+2bcdx^5+8ad^2x^5}{15c^3\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] (15*a*c^2*x + 5*b*c^2*x^3 + 20*a*c*d*x^3 + 2*b*c*d*x^5 + 8*a*d^2*x^5)/(15*c
^3*(c + d*x^2)^(5/2))
```

Maple [A]
time = 0.06, size = 132, normalized size = 1.45

method	result
gosper	$\frac{x\left(8a\,d^2x^4+2bcd\,x^4+20acd\,x^2+5b\,c^2x^2+15c^2a\right)}{15\left(d\,x^2+c\right)^{\frac{5}{2}}c^3}$
trager	$\frac{x\left(8a\,d^2x^4+2bcd\,x^4+20acd\,x^2+5b\,c^2x^2+15c^2a\right)}{15\left(d\,x^2+c\right)^{\frac{5}{2}}c^3}$
default	$b\left(-\frac{x}{4d\left(d\,x^2+c\right)^{\frac{5}{2}}}+\frac{c\left(\frac{x}{5c\left(d\,x^2+c\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(d\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{d\,x^2+c}}}{c}\right)}{4d}\right)+a\left(\frac{x}{5c\left(d\,x^2+c\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(d\,x^2+c\right)^{\frac{3}{2}}}+\frac{}{15c^2\sqrt{(}}}{c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/4*x/d/(d*x^2+c)^(5/2)+1/4*c/d*(1/5*x/c/(d*x^2+c)^(5/2)+4/5/c*(1/3*x/c
/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))))+a*(1/5*x/c/(d*x^2+c)^(5/2)+4/
5/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)))
```

Maxima [A]
time = 0.30, size = 103, normalized size = 1.13

$$\frac{8\,ax}{15\,\sqrt{dx^2+c}\,c^3}+\frac{4\,ax}{15\,(dx^2+c)^{\frac{3}{2}}c^2}+\frac{ax}{5\,(dx^2+c)^{\frac{5}{2}}c}-\frac{bx}{5\,(dx^2+c)^{\frac{5}{2}}d}+\frac{2\,bx}{15\,\sqrt{dx^2+c}\,c^2d}+\frac{bx}{15\,(dx^2+c)^{\frac{3}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*a*x/(sqrt(d*x^2 + c)*c^3) + 4/15*a*x/((d*x^2 + c)^(3/2)*c^2) + 1/5*a*x
/((d*x^2 + c)^(5/2)*c) - 1/5*b*x/((d*x^2 + c)^(5/2)*d) + 2/15*b*x/(sqrt(d*x
^2 + c)*c^2*d) + 1/15*b*x/((d*x^2 + c)^(3/2)*c*d)
```

Fricas [A]
time = 1.18, size = 87, normalized size = 0.96

$$\frac{(2\,(bcd+4\,ad^2)x^5+15\,ac^2x+5\,(bc^2+4\,acd)x^3)\sqrt{dx^2+c}}{15\,(c^3d^3x^6+3\,c^4d^2x^4+3\,c^5dx^2+c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(b*c*d + 4*a*d^2)*x^5 + 15*a*c^2*x + 5*(b*c^2 + 4*a*c*d)*x^3)*sqrt(
d*x^2 + c)/(c^3*d^3*x^6 + 3*c^4*d^2*x^4 + 3*c^5*d*x^2 + c^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(83) = 166.
time = 11.43, size = 566, normalized size = 6.22

$$a\left(\frac{15c^5x}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{35c^4dx^3}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{28c^3d^2x^5}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{8c^2d^3x^7}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}\right)+b\left(\frac{5cx^3}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}+\frac{2dx^5}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] a*(15*c**5*x/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1
+ d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3
*x**6*sqrt(1 + d*x**2/c)) + 35*c**4*d*x**3/(15*c**(17/2)*sqrt(1 + d*x**2/c)
 + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 +
 d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c)) + 28*c**3*d**2*x**5
/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c)
+ 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1
 + d*x**2/c)) + 8*c**2*d**3*x**7/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(
15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c)
 + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c))) + b*(5*c*x**3/(15*c**(9/2)*s
qrt(1 + d*x**2/c) + 30*c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**
2*x**4*sqrt(1 + d*x**2/c)) + 2*d*x**5/(15*c**(9/2)*sqrt(1 + d*x**2/c) + 30*
```

```
c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**2*x**4*sqrt(1 + d*x**2/
c)))
```

Giac [A]
time = 1.00, size = 72, normalized size = 0.79

$$\frac{\left(x^2\left(\frac{2\,(bcd^3+4\,ad^4)x^2}{c^3d^2}+\frac{5\,(bc^2d^2+4\,acd^3)}{c^3d^2}\right)+\frac{15\,a}{c}\right)x}{15\,(dx^2+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*(2*(b*c*d^3 + 4*a*d^4)*x^2/(c^3*d^2) + 5*(b*c^2*d^2 + 4*a*c*d^3)/
(c^3*d^2)) + 15*a/c)*x/(d*x^2 + c)^(5/2)
```

Mupad [B]
time = 4.85, size = 87, normalized size = 0.96

$$\frac{8\,a\,d\,x\,(d\,x^2+c)^2-3\,b\,c^3\,x+2\,b\,c\,x\,(d\,x^2+c)^2+b\,c^2\,x\,(d\,x^2+c)+3\,a\,c^2\,d\,x+4\,a\,c\,d\,x\,(d\,x^2+c)}{15\,c^3\,d\,(d\,x^2+c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(c + d*x^2)^(7/2),x)
```

```
[Out] (8*a*d*x*(c + d*x^2)^2 - 3*b*c^3*x + 2*b*c*x*(c + d*x^2)^2 + b*c^2*x*(c + d
*x^2) + 3*a*c^2*d*x + 4*a*c*d*x*(c + d*x^2))/(15*c^3*d*(c + d*x^2)^(5/2))
```

3.100 $\int \frac{1}{(c+dx^2)^{5/2}} \, dx$

Optimal. Leaf size=39

$$\frac{x}{3c\,(c+dx^2)^{3/2}}+\frac{2x}{3c^2\sqrt{c+dx^2}}$$

```
[Out] 1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {198, 197}

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(-5/2),x]
```

```
[Out] x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+dx^2)^{5/2}}\,dx &= \frac{x}{3c\,(c+dx^2)^{3/2}}+\frac{2\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{3c}\\ &= \frac{x}{3c\,(c+dx^2)^{3/2}}+\frac{2x}{3c^2\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 29, normalized size = 0.74

$$\frac{3cx+2dx^3}{3c^2\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(-5/2),x]
[Out] (3*c*x + 2*d*x^3)/(3*c^2*(c + d*x^2)^(3/2))
```

Maple [A]
time = 0.06, size = 32, normalized size = 0.82

method	result	size
gosper	$\frac{x\left(2 d\,x^{2}+3 c\right)}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}} c^{2}}$	26
trager	$\frac{x\left(2 d\,x^{2}+3 c\right)}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}} c^{2}}$	26
default	$\frac{x}{3 c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{2 x}{3 c^{2} \sqrt{d\,x^{2}+c}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{dx^2+c}\,c^2}+\frac{x}{3\,(dx^2+c)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] 2/3*x/(sqrt(d*x^2 + c)*c^2) + 1/3*x/((d*x^2 + c)^(3/2)*c)
```

Fricas [A]
time = 1.10, size = 47, normalized size = 1.21

$$\frac{(2\,dx^3+3\,cx)\sqrt{dx^2+c}}{3\,(c^2d^2x^4+2\,c^3dx^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*d*x^3 + 3*c*x)*sqrt(d*x^2 + c)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

time = 0.42, size = 95, normalized size = 2.44

$$\frac{3cx}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}+\frac{2dx^3}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**2+c)**(5/2),x)
```

```
[Out] 3*c*x/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*x**2/c)
) + 2*d*x**3/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*
x**2/c))
```

Giac [A]
time = 0.70, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*d*x^2/c^2 + 3/c)/(d*x^2 + c)^(3/2)
```

Mupad [B]
time = 4.79, size = 28, normalized size = 0.72

$$\frac{2\,x\,(d\,x^2+c)+c\,x}{3\,c^2\,(d\,x^2+c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^2)^(5/2),x)
```

```
[Out] (2*x*(c + d*x^2) + c*x)/(3*c^2*(c + d*x^2)^(3/2))
```

3.101 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}$$

```
[Out] b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(3/2)/a^(1/
2)-d*x/c/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {390, 385, 211}

$$\frac{b\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad}\\ &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}\\ &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.26, size = 99, normalized size = 1.25

$$\frac{dx}{c(-bc+ad)\sqrt{c+dx^2}}-\frac{b\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*x)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (b*ArcTan[(a*Sqrt[d] + b*x*(Sqrt
[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^
(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 732 vs. 2(67) = 134.
time = 0.13, size = 733, normalized size = 9.28

method	result

default	$-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\,\left(2d\left(x-\cdots\right.\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\sqrt{-\cdots}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*
(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2))))-1/2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)
*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)
/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2)
)^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(67) = 134.
time = 3.94, size = 442, normalized size = 5.59

$$\left[-\frac{4\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3)x^2)},\ -\frac{2\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{2\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(-
a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*c^3*
d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1/2*(2
*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b*c - a
^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4 - 2*a^2*
b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 0.81, size = 107, normalized size = 1.35

$$\frac{b\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,(bc-ad)}-\frac{dx}{(bc^2-acd)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2 -
 a*c*d)*sqrt(d*x^2 + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.102 $\int \frac{1}{(a+bx^2)^2 \sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=100

$$\frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/2*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)
/(-a*d+b*c)^(3/2)+1/2*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 385, 211}

$$\frac{(bc-2ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)} + \frac{(bc-2ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.33, size = 122, normalized size = 1.22

$$-\frac{bx\sqrt{c+dx^2}}{2a(-bc+ad)(a+bx^2)} + \frac{(-bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*x*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-(b*c) + 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b
*c - a*d])])/(2*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(84) = 168$.
time = 0.06, size = 822, normalized size = 8.22

method	result

default	$-\dfrac{\dfrac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\dfrac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right.}{b}}{}\right.}{}}{4ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/a*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b)^(1/2)/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-1/4/a/(-
a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4
/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)-1/4/b/a*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(84) = 168.
time = 1.71, size = 459, normalized size = 4.59

$$\left[\frac{4\,(ab^2c-a^2bd)\sqrt{dx^2+c}\,x-(abc-2\,a^2d+(b^2c-2\,abd)x^2)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{8\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^2)},\ \frac{2\,(ab^2c-a^2bd)\sqrt{dx^2+c}\,x+\sqrt{abc-a^2d}\,(abc-2\,a^2d+(b^2c-2\,abd)x^2)\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{4\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3
*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a^2*d)*
(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^
3*b^2*c*d + a^4*b*d^2)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(84) = 168.
time = 0.94, size = 225, normalized size = 2.25

$$-\frac{1}{2}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad-bc^2\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(abcd-a^2d^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sqrt
(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d - b*
c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.103 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=149

$$\frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\,(a+bx^2)}+\frac{bx(c+dx^2)^{3/2}}{4a(bc-ad)\,(a+bx^2)^2}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*b*x*(d*x^2+c)^(3/2)/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*c*(-4*a*d+3*b*c)*arcta
n(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(3/2)+1/8*
(-4*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/a^2/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {390, 386, 385, 211}

$$\frac{c(3bc-4ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{c+dx^2}\,(3bc-4ad)}{8a^2\,(a+bx^2)\,(bc-ad)}+\frac{bx(c+dx^2)^{3/2}}{4a\,(a+bx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*a^2*(b*c - a*d)*(a + b*x^2)) + (b*x*
(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*(b*c - a*d)^
(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^3}\,dx = \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{(3bc-4ad)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{(c(3bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8a^2(bc-ad)}$$

$$= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{8a^2(bc-ad)}$$

$$= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{c(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.97, size = 151, normalized size = 1.01

$$\frac{x\sqrt{c+dx^2}\left(-5abc+4a^2d-3b^2cx^2+2abdx^2\right)}{8a^2(-bc+ad)\left(a+bx^2\right)^2} - \frac{c(3bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] (x*Sqrt[c + d*x^2]*(-5*a*b*c + 4*a^2*d - 3*b^2*c*x^2 + 2*a*b*d*x^2))/(8*a^2
*(-(b*c) + a*d)*(a + b*x^2)^2) - (c*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*S
qrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5/2)*(b
*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4154 vs. $2(129) = 258$.
time = 0.08, size = 4155, normalized size = 27.89

method	result	size
default	Expression too large to display	4155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16/b/a^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a
*d-b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-
a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2
))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)
*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-1/8/b/a
/(-a*b)^(1/2)*(1/2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))^2*(d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/2*d*(-a*b
)^(1/2)/(a*d-b*c)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1
/2)/(a*d-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1
/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)
^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^
(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-1/2
*d/(a*d-b*c)*b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-
1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
```

```
*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+3/16/(-a*b)^(1/2)/a^2*((d*(x-1/b*(-a*
b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1
/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*
(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2))))-3/16/b/a^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1
/2)/(a*d-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1
/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)
^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^
(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-3/1
6/(-a*b)^(1/2)/a^2*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+
d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/8/b/a/(-a*b)^(1/2)*(1/2/(a*d-b*
c)*b/(x+1/b*(-a*b)^(1/2))^2*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2*d*(-a*b)^(1/2)/(a*d-b*c)*(1/(a*d-
b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*((d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d
^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(129) = 258$.

time = 2.10, size = 698, normalized size = 4.68

$$\left[-\frac{\left(3\,a^{2}bc^{2}-4\,a^{3}cd+\left(3\,b^{3}c^{2}-4\,ab^{2}cd\right)x^{4}+2\,\left(3\,ab^{2}c^{2}-4\,a^{2}bcd\right)x^{2}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\,\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\,\left(\left(bc-2\,ad\right)x^{3}-acx\right)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\,\left(\left(3\,ab^{3}c^{2}-5\,a^{2}b^{2}cd+2\,a^{3}bd^{2}\right)x^{3}+\left(5\,a^{2}b^{2}c^{2}-9\,a^{3}bcd+4\,a^{4}d^{2}\right)x\right)\sqrt{dx^{2}+c}}{32\,\left(a^{5}b^{2}c^{2}-2\,a^{6}bcd+a^{7}d^{2}+\left(a^{3}b^{4}c^{2}-2\,a^{4}b^{3}cd+a^{5}b^{2}d^{2}\right)x^{4}+2\,\left(a^{4}b^{3}c^{2}-2\,a^{5}b^{2}cd+a^{6}bd^{2}\right)x^{2}\right)},\ \frac{\left(3\,a^{2}bc^{2}-4\,a^{3}cd+\left(3\,b^{3}c^{2}-4\,ab^{2}cd\right)x^{4}+2\,\left(3\,ab^{2}c^{2}-4\,a^{2}bcd\right)x^{2}\right)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{\sqrt{abc-a^{2}d}\,\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}}{2\,\left(\left(abcd-a^{2}d^{2}\right)x^{3}+\left(abc^{2}-a^{2}cd\right)x\right)}\right)+2\,\left(\left(3\,ab^{3}c^{2}-5\,a^{2}b^{2}cd+2\,a^{3}bd^{2}\right)x^{3}+\left(5\,a^{2}b^{2}c^{2}-9\,a^{3}bcd+4\,a^{4}d^{2}\right)x\right)\sqrt{dx^{2}+c}}{16\,\left(a^{5}b^{2}c^{2}-2\,a^{6}bcd+a^{7}d^{2}+\left(a^{3}b^{4}c^{2}-2\,a^{4}b^{3}cd+a^{5}b^{2}d^{2}\right)x^{4}+2\,\left(a^{4}b^{3}c^{2}-2\,a^{5}b^{2}cd+a^{6}bd^{2}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b
^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - 4*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^
2 - 9*a^3*b*c*d + 4*a^4*d^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d
 + a^7*d^2 + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c
^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x^2), 1/16*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b
^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c -
 a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3*a*b^3*c^2
- 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^2 - 9*a^3*b*c*d + 4*a^4*d
^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (a^3*b^4*c^2
 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*
b*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(129) = 258.
time = 2.77, size = 487, normalized size = 3.27

$$-\frac{\left(3\,bc^{2}\sqrt{d}-4\,acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{8\,\left(a^{2}bc-a^{3}d\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{3\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b^{3}c^{2}\sqrt{d}-4\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}ab^{2}cd^{\frac{3}{2}}-9\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b^{3}c^{3}\sqrt{d}+30\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ab^{2}c^{2}d^{\frac{3}{2}}-40\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{2}bcd^{\frac{5}{2}}+16\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{3}d^{\frac{7}{2}}+9\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{3}c^{4}\sqrt{d}-28\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{2}c^{3}d^{\frac{3}{2}}+16\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}bc^{2}d^{\frac{5}{2}}-3\,b^{3}c^{5}\sqrt{d}+2\,ab^{2}c^{4}d^{\frac{3}{2}}}{4\,\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)^{2}\left(a^{2}b^{2}c-a^{3}bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*b*c^2*sqrt(d) - 4*a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a
```

```
*b*c*d - a^2*d^2)) - 1/4*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^3*c^2*sqrt(d)
 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^2*c*d^(3/2) - 9*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b^3*c^3*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c^
2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b*c*d^(5/2) + 16*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a^3*d^(7/2) + 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^
3*c^4*sqrt(d) - 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^3*d^(3/2) + 16*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c^2*d^(5/2) - 3*b^3*c^5*sqrt(d) + 2*a*
b^2*c^4*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^2*(a^2*b
^2*c - a^3*b*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^3,x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^3, x)
```

3.104 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx$

Optimal. Leaf size=199

$$\frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\,(a+bx^2)}+\frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)\,(a+bx^2)^2}+\frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3}+\frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{\sqrt{bc}\cdots}{\sqrt{a}\,\sqrt{\cdots}}\right.}{16a^{7/2}(bc-ad)^{3/2}}$$

```
[Out] 1/24*(-6*a*d+5*b*c)*x*(d*x^2+c)^(3/2)/a^2/(-a*d+b*c)/(b*x^2+a)^2+1/6*b*x*(d
*x^2+c)^(5/2)/a/(-a*d+b*c)/(b*x^2+a)^3+1/16*c^2*(-6*a*d+5*b*c)*arctan(x*(-a
*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(3/2)+1/16*c*(-6*
a*d+5*b*c)*x*(d*x^2+c)^(1/2)/a^3/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.07, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {390, 386, 385, 211}

$$\frac{c^2(5bc-6ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}+\frac{cx\sqrt{c+dx^2}\,(5bc-6ad)}{16a^3\,(a+bx^2)\,(bc-ad)}+\frac{x(c+dx^2)^{3/2}\,(5bc-6ad)}{24a^2\,(a+bx^2)^2\,(bc-ad)}+\frac{bx(c+dx^2)^{5/2}}{6a\,(a+bx^2)^3\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (c*(5*b*c - 6*a*d)*x*Sqrt[c + d*x^2])/(16*a^3*(b*c - a*d)*(a + b*x^2)) + ((
5*b*c - 6*a*d)*x*(c + d*x^2)^(3/2))/(24*a^2*(b*c - a*d)*(a + b*x^2)^2) + (b
*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b*x^2)^3) + (c^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(16*a^(7/2)*(b*c -
 a*d)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{(5bc-6ad)\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^3}\,dx}{6a(bc-ad)}$$

$$= \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)\,(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{(c(5bc-6ad))\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx}{8a^2(bc-ad)}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\,(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)\,(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{(c^2(5b}{}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\,(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)\,(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{(c^2(5b}{}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\,(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)\,(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{c^2(5bc}{}$$

Mathematica [A]
time = 15.22, size = 179, normalized size = 0.90

$$\frac{-\frac{\sqrt{a}\,x\sqrt{c+dx^2}\,(15b^3c^2x^4+8ab^2cx^2(5c-dx^2)-6a^3d(5c+2dx^2)+a^2b(33c^2-22cdx^2-4d^2x^4))}{(-bc+ad)(a+bx^2)^3} + \frac{3c^2(5bc-6ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}}{48a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

[Out] (-((Sqrt[a]*x*Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 8*a*b^2*c*x^2*(5*c - d*x^2)
 - 6*a^3*d*(5*c + 2*d*x^2) + a^2*b*(33*c^2 - 22*c*d*x^2 - 4*d^2*x^4)))/((-(
b*c) + a*d)*(a + b*x^2)^3)) + (3*c^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(48*a^(7/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 12957 vs. $2(175) = 350$.
time = 0.09, size = 12958, normalized size = 65.12

method	result	size
default	Expression too large to display	12958

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(175) = 350$.
time = 1.98, size = 972, normalized size = 4.88

$$\left[-\frac{3\,(5\,a^3bc^3-6\,a^4c^2d+(5\,b^4c^3-6\,ab^3c^2d)x^6+3\,(5\,ab^3c^3-6\,a^2b^2c^2d)x^4+3\,(5\,a^2b^2c^3-6\,a^3bc^2d)x^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,((15\,ab^4c^3-23\,a^2b^3c^2d+4\,a^3b^2cd^2+4\,a^4bd^3)x^5+2\,(20\,a^2b^3c^3-31\,a^3b^2c^2d+5\,a^4bcd^2+6\,a^5d^3)x^3+3\,(11\,a^3b^2c^3-21\,a^4bc^2d+10\,a^5cd^2)x)\sqrt{dx^2+c}}{192\,(a^7b^2c^2-2\,a^8bcd+a^9d^2+(a^4b^5c^2-2\,a^5b^4cd+a^6b^3d^2)x^6+3\,(a^5b^4c^2-2\,a^6b^3cd+a^7b^2d^2)x^4+3\,(a^6b^3c^2-2\,a^7b^2cd+a^8bd^2)x^2)},\ \frac{3\,(5\,a^3bc^3-6\,a^4c^2d+(5\,b^4c^3-6\,ab^3c^2d)x^6+3\,(5\,ab^3c^3-6\,a^2b^2c^2d)x^4+3\,(5\,a^2b^2c^3-6\,a^3bc^2d)x^2)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2\,((15\,ab^4c^3-23\,a^2b^3c^2d+4\,a^3b^2cd^2+4\,a^4bd^3)x^5+2\,(20\,a^2b^3c^3-31\,a^3b^2c^2d+5\,a^4bcd^2+6\,a^5d^3)x^3+3\,(11\,a^3b^2c^3-21\,a^4bc^2d+10\,a^5cd^2)x)\sqrt{dx^2+c}}{96\,(a^7b^2c^2-2\,a^8bcd+a^9d^2+(a^4b^5c^2-2\,a^5b^4cd+a^6b^3d^2)x^6+3\,(a^5b^4c^2-2\,a^6b^3cd+a^7b^2d^2)x^4+3\,(a^6b^3c^2-2\,a^7b^2cd+a^8bd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="fricas")

[Out] [-1/192*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^3*c^2*d)*x^6 + 3
*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6*a^3*b*c^2*d)*x^
2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^
2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b
*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*a*b^4*c^
3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^5 + 2*(20*a^2*b^3*c
^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5*d^3)*x^3 + 3*(11*a^3*b^2*c^3
- 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^2 + c))/(a^7*b^2*c^2 - 2*a^8*b
*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x^6 + 3*(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3*(a^6*b^3*c^2 - 2*a^7*b^2*c*d

```
+ a^8*b*d^2)*x^2), 1/96*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^
3*c^2*d)*x^6 + 3*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6
*a^3*b*c^2*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c
 - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^5 + 2*(20*a^2*b^3*c^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5
*d^3)*x^3 + 3*(11*a^3*b^2*c^3 - 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^
2 + c))/(a^7*b^2*c^2 - 2*a^8*b*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d
 + a^6*b^3*d^2)*x^6 + 3*(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3
*(a^6*b^3*c^2 - 2*a^7*b^2*c*d + a^8*b*d^2)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(175) = 350.
time = 1.62, size = 919, normalized size = 4.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] -1/16*(5*b*c^3*sqrt(d) - 6*a*c^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x
^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c - a^4*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/24*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^5*c^3*s
qrt(d) - 18*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b^4*c^2*d^(3/2) - 75*(sqrt(d
)*x - sqrt(d*x^2 + c))^8*b^5*c^4*sqrt(d) + 240*(sqrt(d)*x - sqrt(d*x^2 + c)
)^8*a*b^4*c^3*d^(3/2) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*b^3*c^2*d^(
5/2) - 96*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^3*b^2*c*d^(7/2) + 96*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*a^4*b*d^(9/2) + 150*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b
^5*c^5*sqrt(d) - 620*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^4*c^4*d^(3/2) + 96
8*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*b^3*c^3*d^(5/2) - 720*(sqrt(d)*x - sq
rt(d*x^2 + c))^6*a^3*b^2*c^2*d^(7/2) + 64*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a
^4*b*c*d^(9/2) + 128*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^5*d^(11/2) - 150*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*b^5*c^6*sqrt(d) + 600*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a*b^4*c^5*d^(3/2) - 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b^3*c^4
*d^(5/2) + 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b^2*c^3*d^(7/2) + 96*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a^4*b*c^2*d^(9/2) + 75*(sqrt(d)*x - sqrt(d*x^2
```

```
 + c))^2*b^5*c^7*sqrt(d) - 210*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^4*c^6*d^
(3/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^3*c^5*d^(5/2) + 48*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*a^3*b^2*c^4*d^(7/2) - 15*b^5*c^8*sqrt(d) + 8*a*b^4
*c^7*d^(3/2) + 4*a^2*b^3*c^6*d^(5/2))/((a^3*b^3*c - a^4*b^2*d)*((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{\left(b\,x^2+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^4,x)
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^4, x)
```

3.105 $\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=20

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

[Out] d*x/b/c/(d*x^2+c)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {21, 197}

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]

[Out] (d*x)/(b*c*Sqrt[c + d*x^2])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx &= \frac{d\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx}{b}\\ &= \frac{dx}{bc\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.07, size = 19, normalized size = 0.95

method	result	size
gosper	$\frac{dx}{bc\sqrt{d\,x^2+c}}$	19
default	$\frac{dx}{bc\sqrt{d\,x^2+c}}$	19
trager	$\frac{dx}{bc\sqrt{d\,x^2+c}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/b/c/(d*x^2+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + b*c/d)*sqrt(d*x^2 + c)), x)
```

Fricas [A]
time = 1.08, size = 27, normalized size = 1.35

$$\frac{\sqrt{dx^2+c}\,dx}{bcdx^2+bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(d*x^2 + c)*d*x/(b*c*d*x^2 + b*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{d\int \frac{1}{c\sqrt{c+dx^2}+dx^2\sqrt{c+dx^2}}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x**2)/(d*x**2+c)**(1/2),x)
```

```
[Out] d*Integral(1/(c*sqrt(c + d*x**2) + d*x**2*sqrt(c + d*x**2)), x)/b
```

Giac [A]
time = 0.94, size = 18, normalized size = 0.90

$$\frac{dx}{\sqrt{dx^2+c}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] d*x/(sqrt(d*x^2 + c)*b*c)
```

Mupad [B]
time = 4.77, size = 18, normalized size = 0.90

$$\frac{d\,x}{b\,c\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^2)^(1/2)*(b*x^2 + (b*c)/d)),x)
```

```
[Out] (d*x)/(b*c*(c + d*x^2)^(1/2))
```

3.106 $\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Maple [A]
time = 0.12, size = 28, normalized size = 1.12

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^2+1}\,x}{x^2-1}\right)}{2}$	28
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-3\,\mathrm{RootOf}\left(\_Z^2+2\right)x^2+4\sqrt{-x^2+1}\,x+\mathrm{RootOf}\left(\_Z^2+2\right)}{x^2+1}\right)}{4}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 1.01, size = 23, normalized size = 0.92

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^2+1}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(20) = 40.
time = 0.77, size = 51, normalized size = 2.04

$$\frac{1}{4}\sqrt{2}\left(\pi\mathrm{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}\,x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^
2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

Mupad [B]
time = 0.37, size = 79, normalized size = 3.16

$$\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)\,1\mathrm{i}}{4}-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 1)),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i)
/4 - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i)
)*1i)/4
```

3.107 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}$$

```
[Out] arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {385, 211}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 1.43

$$-\frac{\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
- a*d])]/(Sqrt[a]*Sqrt[b*c - a*d]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(39) = 78$.
time = 0.07, size = 306, normalized size = 6.24

method	result
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.
time = 0.94, size = 241, normalized size = 4.92

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,(abc-a^2d)}, \frac{\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{2\,\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-
a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^2*d
), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a^2*d)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.60, size = 70, normalized size = 1.43

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(
a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{x\,\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)}{\sqrt{-a\,(a\,d-b\,c)}} & \text{if } 0<b\,c-a\,d \\ \frac{\ln\left(\frac{\sqrt{a\,(d\,x^2+c)}+x\,\sqrt{a\,d-b\,c}}{\sqrt{a\,(d\,x^2+c)}-x\,\sqrt{a\,d-b\,c}}\right)}{2\,\sqrt{a\,(a\,d-b\,c)}} & \text{if } b\,c-a\,d<0 \\ \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx & \text{if } b\,c-a\,d\notin\mathbb{R}\vee a\,d=b\,c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*d + b*c, atan((x*(- a*d + b*c)^(1/2))/(a^(1/2)*(c + d*x^2
)^(1/2)))/(-a*(a*d - b*c))^(1/2), - a*d + b*c < 0, log(((a*(c + d*x^2))^(1/
2) + x*(a*d - b*c)^(1/2))/((a*(c + d*x^2))^(1/2) - x*(a*d - b*c)^(1/2)))/(2
*(a*(a*d - b*c))^(1/2)), ~in(- a*d + b*c, 'real') | a*d == b*c, int(1/((a +
 b*x^2)*(c + d*x^2)^(1/2)), x))
```

3.108 $\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=15

$$-\frac{2x}{\sqrt{1+x^2}}+\sinh^{-1}(x)$$

[Out] arcsinh(x)-2*x/(x^2+1)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {393, 221}

$$\sinh^{-1}(x)-\frac{2x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(1 + x^2)^(3/2),x]

[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx &= -\frac{2x}{\sqrt{1+x^2}}+\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= -\frac{2x}{\sqrt{1+x^2}}+\sinh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.67

$$-\frac{2x}{\sqrt{1+x^2}}+\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcTanh[x/Sqrt[1 + x^2]]
```

Maple [A]
time = 0.10, size = 14, normalized size = 0.93

method	result	size
default	$\operatorname{arcsinh}(x)-\frac{2x}{\sqrt{x^2+1}}$	14
risch	$\operatorname{arcsinh}(x)-\frac{2x}{\sqrt{x^2+1}}$	14
trager	$-\frac{2x}{\sqrt{x^2+1}}+\ln\left(\sqrt{x^2+1}+x\right)$	22
meijerg	$-\frac{x}{\sqrt{x^2+1}}+\frac{-\frac{\sqrt{\pi}\,x}{\sqrt{x^2+1}}+\sqrt{\pi}\,\operatorname{arcsinh}(x)}{\sqrt{\pi}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)-2*x/(x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 13, normalized size = 0.87

$$-\frac{2\,x}{\sqrt{x^2+1}}+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*x/sqrt(x^2 + 1) + arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(13) = 26.
time = 0.87, size = 44, normalized size = 2.93

$$-\frac{2\,x^2+(x^2+1)\log\left(-x+\sqrt{x^2+1}\right)+2\,\sqrt{x^2+1}\,x+2}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^2 + 1)*log(-x + sqrt(x^2 + 1)) + 2*sqrt(x^2 + 1)*x + 2)/(x^2 +
 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 2.34, size = 31, normalized size = 2.07

$$\frac{x^2 \operatorname{asinh}(x)}{x^2+1} - \frac{2x}{\sqrt{x^2+1}} + \frac{\operatorname{asinh}(x)}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)**(3/2),x)
```

```
[Out] x**2*asinh(x)/(x**2 + 1) - 2*x/sqrt(x**2 + 1) + asinh(x)/(x**2 + 1)
```

Giac [A]
time = 0.58, size = 25, normalized size = 1.67

$$-\frac{2\,x}{\sqrt{x^2+1}} - \log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2*x/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

Mupad [B]
time = 0.04, size = 27, normalized size = 1.80

$$\frac{\operatorname{asinh}(x) + x^2 \operatorname{asinh}(x) - 2\,x\,\sqrt{x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 + 1)^(3/2),x)
```

```
[Out] (asinh(x) + x^2*asinh(x) - 2*x*(x^2 + 1)^(1/2))/(x^2 + 1)
```

3.109 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx$

Optimal. Leaf size=648

$$\frac{18144a^3x(a-bx^2)^{2/3}}{1235}-\frac{23544a^2x(a-bx^2)^{5/3}}{6175}-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

```
[Out] 18144/1235*a^3*x*(-b*x^2+a)^(2/3)-23544/6175*a^2*x*(-b*x^2+a)^(5/3)-378/475
*a*x*(-b*x^2+a)^(5/3)*(b*x^2+3*a)-3/25*x*(-b*x^2+a)^(5/3)*(b*x^2+3*a)^2-725
76/1235*a^4*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+24192/1235*3^(3/4)*a^
(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3
^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a
^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-36288/1235*3^(1/4)*a^(13/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+
a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {427, 542, 396, 201, 241, 310, 225, 1893}

$$\frac{24192\sqrt{2}\,3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36288\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{72576a^4x}{1235\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18144a^3x(a-bx^2)^{2/3}}{1235}-\frac{23544a^2x(a-bx^2)^{5/3}}{6175}-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (18144*a^3*x*(a - b*x^2)^(2/3))/1235 - (23544*a^2*x*(a - b*x^2)^(5/3))/6175
 - (378*a*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/475 - (3*x*(a - b*x^2)^(5/3)*(
3*a + b*x^2)^2)/25 - (72576*a^4*x)/(1235*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36288*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))]
```

```
, -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24192*Sqrt[2]*3^(3/4)*
a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx = -\frac{3}{25}x\left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 - \frac{3\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)\left(-78a^2b-42a\right.}{25b}$$

$$= -\frac{378}{475}ax\left(a-bx^2\right)^{5/3} \left(3a+bx^2\right) - \frac{3}{25}x\left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 + \frac{9\int (a-}{}$$

$$= -\frac{23544a^2x(a-bx^2)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3} \left(3a+bx^2\right) - \frac{3}{25}x\left(a-bx^2\right)^{5/}$$

$$= \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3} (3a+$$

$$= \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3} (3a+$$

$$= \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3} (3a+$$

$$= \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3} (3a+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 12.21, size = 99, normalized size = 0.15

$$-\frac{3\left(-15255a^4x+3390a^3bx^3+8992a^2b^2x^5+2626ab^3x^7+247b^4x^9-40320a^4x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{6175\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (-3*(-15255*a^4*x + 3390*a^3*b*x^3 + 8992*a^2*b^2*x^5 + 2626*a*b^3*x^7 + 24
7*b^4*x^9 - 40320*a^4*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3
/2, (b*x^2)/a]))/(6175*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{2}{3}} \left(b\,x^2+3a\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3)
, x)
```

Sympy [A]
time = 2.22, size = 136, normalized size = 0.21

$$27a^{\frac{11}{3}}x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) + 9a^{\frac{8}{3}}bx^{3}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) + \frac{9a^{\frac{5}{3}}b^{2}x^{5}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right)}{5} + \frac{a^{\frac{2}{3}}b^{3}x^{7}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**3,x)
```

```
[Out] 27*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a
**(8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a
**(5/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
+ a**(2/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)
/7
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a-b\,x^2\right)^{2/3}\,\left(b\,x^2+3\,a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x)
[Out] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^3, x)
```

3.110 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx$

Optimal. Leaf size=617

$$\frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}$$

```
[Out] 7776/1729*a^2*x*(-b*x^2+a)^(2/3)-252/247*a*x*(-b*x^2+a)^(5/3)-3/19*x*(-b*x^
2+a)^(5/3)*(b*x^2+3*a)-31104/1729*a^3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/
2)))+10368/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b
*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-
b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-15552/1729
*3^(1/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a
^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*
((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(
1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 396, 201, 241, 310, 225, 1893}

$$\frac{10368\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{15552\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (7776*a^2*x*(a - b*x^2)^(2/3))/1729 - (252*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/19 - (31104*a^3*x)/(1729*((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))) - (15552*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (103
```

```
68*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx &= -\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3\int\left(a-bx^2\right)^{2/3}\left(-60a^2b-28ab^2x^2\right)\,dx}{19b}\\ &= -\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)+\frac{1}{247}\left(2592a^2\right)\int (a\\ &= \frac{7776a^2x(a-bx^2)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\\ &= \frac{7776a^2x(a-bx^2)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\\ &= \frac{7776a^2x(a-bx^2)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\\ &= \frac{7776a^2x(a-bx^2)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 9.52, size = 176, normalized size = 0.29

$$\frac{x(a-bx^2)^{2/3}\left(21a(45a^2+10abx^2+b^2x^4)\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{7}{2};\frac{bx^2}{a}\right)+8bx^2(18a^2+9abx^2+b^2x^4)\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{9}{2};\frac{bx^2}{a}\right)+4b(3ax+bx^3)^2\Gamma\left(\frac{1}{3}\right)\,{}_3F_2\left(\frac{1}{3},\frac{3}{2},2;1,\frac{9}{2};\frac{bx^2}{a}\right)\right)}{105a\left(1-\frac{bx^2}{a}\right)^{2/3}\Gamma\left(-\frac{2}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-2/3]*Hype
rgeometric2F1[-2/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^
2*x^4)*Gamma[1/3]*Hypergeometric2F1[1/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*x
+ b*x^3)^2*Gamma[1/3]*HypergeometricPFQ[{1/3, 3/2, 2}, {1, 9/2}, (b*x^2)/a]
))/(105*a*(1 - (b*x^2)/a)^(2/3)*Gamma[-2/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{2}{3}} \left(b\,x^2+3a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A]
time = 1.78, size = 99, normalized size = 0.16

$$9a^{\frac{8}{3}}x{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) + 2a^{\frac{5}{3}}bx^{3}{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) + \frac{a^{\frac{2}{3}}b^{2}x^{5}{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a**
(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + a**(2
/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a-b\,x^2\right)}^{2/3}\,{\left(b\,x^2+3\,a\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x)
```

```
[Out] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^2, x)
```

3.111 $\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\,dx$

Optimal. Leaf size=588

$$\frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a\right.}$$

```
[Out] 18/13*a*x*(-b*x^2+a)^(2/3)-3/13*x*(-b*x^2+a)^(5/3)-72/13*a^2*x/(-(-b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))+24/13*3^(3/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+
(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^
(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)-36/13*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2
+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-
I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/
3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)
```

Rubi [A]

time = 0.31, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {396, 201, 241, 310, 225, 1893}

$$\frac{24\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (18*a*x*(a - b*x^2)^(2/3))/13 - (3*x*(a - b*x^2)^(5/3))/13 - (72*a^2*x)/(13
*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3
]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) + (24*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
```

```
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1
3*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\,dx &= -\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{1}{13}(42a)\int \left(a-bx^2\right)^{2/3}\,dx\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{1}{13}\left(24a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}-\frac{\left(36a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x}}\right.}{13bx}\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{\left(36a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)}{\sqrt{-a+}}\right.}{13bx}\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.84, size = 62, normalized size = 0.11

$$\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(-a+bx^2+\frac{14a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (3*x*(a - b*x^2)^(2/3)*(-a + b*x^2 + (14*a*Hypergeometric2F1[-2/3, 1/2, 3/2
, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/13
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{2}+a\right)^{\frac{2}{3}} \left(b\,x^{2}+3a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="fricas")
[Out] integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A]
time = 1.29, size = 63, normalized size = 0.11

$$3a^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) + \frac{a^{\frac{2}{3}}bx^{3}\,{}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a),x)
[Out] 3*a**(5/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + a**(2
/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a-b\,x^2\right)}^{2/3}\,\left(b\,x^2+3\,a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(2/3)*(3*a + b*x^2),x)
[Out] int((a - b*x^2)^(2/3)*(3*a + b*x^2), x)
```

3.112 $\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx$

Optimal. Leaf size=740

$$\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}+\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}}+\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right.}{\sqrt{3}\,\sqrt{b}}$$

```
[Out] 3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+2^(1/3)*a^(1/6)*arctanh(x*b^(1/
2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))/b^(1/2)-1/3*2^(1/3)*a^(1/6)*
arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)+1/3*2^(1/3)*a^(1/6)*arctan(a^(1/6)*(a^(1
/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*3^(1/2)/b^(1/2)+1/3*2^(1/3
)*a^(1/6)*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*3^(1/2)/b^(1/2)-3^(3/4)*a^(1/3)
*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)
))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)
+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3/2*3^(1/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x
^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+
1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 740, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {405, 241, 310, 225, 1893, 402}

$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}}+\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}}+\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}+\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\,\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}-\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (3*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (2^(1/3)*a^(1/6)*ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTa
n[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sq
rt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b])
 + (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*
```

```
x^2)^(1/3))])/Sqrt[b] + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a
 - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (Sqrt[2]*3^(3/4)*a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 405

```
Int[((a_) + (b_.)*(x_)^2)^(2/3)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[1/(a + b*x^2)^(1/3), x], x] - Dist[(b*c - a*d)/d, Int[1/((a + b*x^2)
```

```
^(1/3)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& EqQ[b*c + 3*a*d, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = (4a)\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx - \int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} + \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} - \cdots$$

$$= \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} + \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} - \cdots$$

$$= \frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}} + \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} + \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\cdots}{\cdots}\right)}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 7.19, size = 162, normalized size = 0.22

$$\frac{9ax(a-bx^2)^{2/3}F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9aF_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-2bx^2\left(F_1\left(\frac{3}{2};-\frac{2}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (9*a*x*(a - b*x^2)^(2/3)*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2
)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
2)/a] - 2*b*x^2*(AppellF1[3/2, -2/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2
*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{b\,x^{2}+3a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a - bx^{2}\right)^{\frac{2}{3}}}{3a + bx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a),x)
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^2\right)}^{2/3}}{b\,x^2+3\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2),x)
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2), x)
```

3.113 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx$

Optimal. Leaf size=584

$$\frac{x(a-bx^2)^{2/3}}{6a\left(3a+bx^2\right)}-\frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}}{4\,3^{3/4}a^{2/3}bx\sqrt{-\cdots}}$$

```
[Out] 1/6*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)-1/6*x/a/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-
3^(1/2)))+1/18*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1
/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a
^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*
x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/12*(a^(1/3
)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2
+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+
a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {423, 21, 241, 310, 225, 1893}

$$\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{x(a-bx^2)^{2/3}}{6a\left(3a+bx^2\right)}-\frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) - x/(6*a*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqr
t[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(4*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
```

```
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(3*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} - \frac{\int \frac{-a-\frac{bx^2}{3}}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{6a}$$

$$= \frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{18a}$$

$$= \frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} - \frac{\sqrt{-bx^2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{12abx}$$

$$= \frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} + \frac{\sqrt{-bx^2}\,\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{12abx} - \frac{\left(\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right.}\right.}{}$$

$$= \frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} - \frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 86, normalized size = 0.15

$$\frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} + \frac{x\sqrt[3]{\frac{a-bx^2}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{18a\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) + (x*((a - b*x^2)/a)^(1/3)*Hyperg
eometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(18*a*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{\left(b\,x^{2}+3a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(2/3)/(b^2*x^4 + 6*a*b*x^2 + 9*a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a - bx^{2}\right)^{\frac{2}{3}}}{\left(3a + bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**2,x)
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^2\right)}^{2/3}}{{\left(b\,x^2+3\,a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x)
```

```
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^2, x)
```

3.114 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3} dx$

Optimal. Leaf size=818

$$\frac{x(a-bx^2)^{2/3}}{12a(3a+bx^2)^2}+\frac{x(a-bx^2)^{2/3}}{36a^2(3a+bx^2)}-\frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\tan^{-1}\left(\sqrt{\ }\right.}{}$$

[Out] 1/12*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)^2+1/36*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+3*a)-1/36*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/144*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(11/6)/b^(1/2)-1/432*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(11/6)/b^(1/2)+1/432*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/432*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/108*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/72*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.46, antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {423, 541, 544, 241, 310, 225, 1893, 402}

$$\frac{(a-bx^2)^{2/3}x}{36a^2(bx^2+3a)}-\frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{12a(bx^2+3a)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\,2^{2/3}a^{11/6}\sqrt{b}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{24\,3^{3/4}a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{18\sqrt{2}\sqrt[4]{3}\,a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

Antiderivative was successfully verified.

[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]

[Out] (x*(a - b*x^2)^(2/3))/(12*a*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(36*a^2*(3*a + b*x^2)) - x/(36*a^2*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]

```
) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(
216*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(
1/3)*(a - b*x^2)^(1/3)))]/(72*2^(2/3)*a^(11/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]
]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(24*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*
x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(18*Sqrt[2]*3^(1/4)*a^
(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} - \frac{\int \frac{-3a+\frac{5bx^2}{3}}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx}{12a}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\,(3a+bx^2)} + \frac{\int \frac{16a^2b+\frac{8}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{288a^3b}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\,(3a+bx^2)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{108a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{36a}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{72\;2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{b}\,x}\right.}{72\;2^{2/3}\sqrt{3}\,a^{11/6}}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{72\;2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{b}\,x}\right.}{72\;2^{2/3}\sqrt{3}\,a^{11/6}}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\,(3a+bx^2)} - \frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\right.}{72\;2^{2/3}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.29, size = 252, normalized size = 0.31

$$\frac{\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+\frac{27x\left(6-\frac{5bx^2}{a}-\frac{b^2x^4}{a^2}+\frac{18(3a+bx^2)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)}{(3a+bx^2)^2}}{972\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

```
[Out] ((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b
*x^2)/a])/a^3 + (27*x*(6 - (5*b*x^2)/a - (b^2*x^4)/a^2 + (18*(3*a + b*x^2)*
AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
```

```
2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a]))))/(3*a + b*x^2)^2)/(972*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{\left(b\,x^{2}+3a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a - bx^{2}\right)^{\frac{2}{3}}}{\left(3a + bx^{2}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**3,x)
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^2\right)}^{2/3}}{{\left(b\,x^2+3\,a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x)
```

```
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^3, x)
```

3.115 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4} \, dx$

Optimal. Leaf size=849

$$\frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)^3}+\frac{x(a-bx^2)^{2/3}}{54a^2(3a+bx^2)^2}+\frac{x(a-bx^2)^{2/3}}{144a^3(3a+bx^2)}-\frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{7\tan^{-1}\left(\cdots\right.}{1296\ 2^{2/3}\sqrt{\cdots}}$$

```
[Out] 1/18*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)^3+1/54*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+
3*a)^2+1/144*x*(-b*x^2+a)^(2/3)/a^3/(b*x^2+3*a)-1/144*x/a^3/(-(-b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))+7/2592*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*
(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-7/7776*arctanh(x*b^(1/2)/a^(1/2
))*2^(1/3)/a^(17/6)/b^(1/2)+7/7776*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+
a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+7/7776*arctan
(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+1/432*(a^(1/3)
-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+
a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/288*(a^(1/3)-(-b*x^2+a)^(1/3))*El
lipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*
(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(
2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(
1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {423, 541, 544, 241, 310, 225, 1893, 402}

$$\frac{(a-bx^2)^{2/3}x}{144a^3(bx^2+3a)}-\frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{54a^2(bx^2+3a)^2}+\frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)^3}+\frac{7\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{7\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3888\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\ 2^{2/3}a^{17/6}\sqrt{b}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{96\ 3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{72\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(18*a*(3*a + b*x^2)^3) + (x*(a - b*x^2)^(2/3))/(54*a^
2*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(144*a^3*(3*a + b*x^2)) - x/(144
*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(Sqrt[3]*Sqrt
```

```
[a])/(Sqrt[b]*x)])/(1296*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + (7*ArcTan[(Sqr
t[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(1296*2^(
2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3888*2^(
2/3)*a^(17/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)
*(a - b*x^2)^(1/3)))])/(1296*2^(2/3)*a^(17/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]]
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(96*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x
^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*a^(
8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} - \frac{\int \frac{-5a+\frac{11bx^2}{3}}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx}{18a}$$

$$= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{\int \frac{64a^2b-\frac{80}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx}{864a^3b}$$

$$= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)} - \frac{\int \frac{-368a^3b^2-48a^2b^3x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{20736a^5b^2}$$

$$= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{432a^3} + \frac{7\int \frac{}{\sqrt[3]{a-}}}{}$$

$$= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{1296\;2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{7\,\mathrm{ta}}{}$$

$$= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{1296\;2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{7\,\mathrm{ta}}{}$$

$$= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)} - \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a} - \sqrt[3]{}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.15, size = 265, normalized size = 0.31

$$\frac{x\left(\frac{9a(a-bx^2)(75a^2+26abx^2+3b^2x^4)}{(3a+bx^2)^3} + bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right) + \frac{621a^3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}\right)}{3888a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*((9*a*(a - b*x^2)*(75*a^2 + 26*a*b*x^2 + 3*b^2*x^4))/(3*a + b*x^2)^3 + b
*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^
```

```
2)/a] + (621*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3
*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*
b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/
2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))))/(3888*a^4*(a - b*x^2)^(1/3)
)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{\left(b\,x^{2}+3a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a - bx^{2}\right)^{\frac{2}{3}}}{\left(3a + bx^{2}\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**4,x)
```

```
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^2\right)}^{2/3}}{{\left(b\,x^2+3\,a\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x)
```

```
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^4, x)
```

3.116 $\int (a-bx^2)^{5/3} (3a+bx^2)^3 \, dx$

Optimal. Leaf size=668

$$\frac{2809728a^4x(a-bx^2)^{2/3}}{267995}+\frac{1404864a^3x(a-bx^2)^{5/3}}{191425}-\frac{33264a^2x(a-bx^2)^{8/3}}{14725}-\frac{432}{775}ax(a-bx^2)^{8/3}(3a+bx^2)-$$

```
[Out] 2809728/267995*a^4*x*(-b*x^2+a)^(2/3)+1404864/191425*a^3*x*(-b*x^2+a)^(5/3)
-33264/14725*a^2*x*(-b*x^2+a)^(8/3)-432/775*a*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a
)-3/31*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a)^2-11238912/267995*a^5*x/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))+3746304/267995*3^(3/4)*a^(16/3)*(a^(1/3)-(-b*x^2
+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+
a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)-5619456/267995*3^(1/4)*a^(16/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*
EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)
^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2
^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {427, 542, 396, 201, 241, 310, 225, 1893}

$$\frac{3746304\sqrt{2}\,3^{3/4}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{5619456\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2809728a^4x(a-bx^2)^{2/3}}{267995}+\frac{1404864a^3x(a-bx^2)^{5/3}}{191425}-\frac{33264a^2x(a-bx^2)^{8/3}}{14725}-\frac{432}{775}ax(a-bx^2)^{8/3}(3a+bx^2)-\frac{3}{31}x(a-bx^2)^{8/3}(3a+bx^2)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (2809728*a^4*x*(a - b*x^2)^(2/3))/267995 + (1404864*a^3*x*(a - b*x^2)^(5/3)
)/191425 - (33264*a^2*x*(a - b*x^2)^(8/3))/14725 - (432*a*x*(a - b*x^2)^(8/
3)*(3*a + b*x^2))/775 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2)^2)/31 - (11238
912*a^5*x)/(267995*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5619456*
3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2
/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x
```

```
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) + (3746304*Sqrt[2]*3^(3/4)*a^(16/3)*(a^(1/3
) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx = -\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2 - \frac{3\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\left(-96a^2b-48a\right.}{31b}$$

$$= -\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) - \frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2 + \frac{9\int (a-}{}$$

$$= -\frac{33264a^2x(a-bx^2)^{8/3}}{14725} - \frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) - \frac{3}{31}x\left(a-bx^2\right)^{8}$$

$$= \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/3}}{14725} - \frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a\right.$$

$$= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995} + \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/}}{14725}$$

$$= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995} + \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/}}{14725}$$

$$= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995} + \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/}}{14725}$$

$$= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995} + \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/}}{14725}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 13.80, size = 110, normalized size = 0.16

$$\frac{3\left(5815935a^5x-5312355a^4bx^3-1675114a^3b^2x^5+749658a^2b^3x^7+378651ab^4x^9+43225b^5x^{11}+6243840a^5x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{1339975\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (3*(5815935*a^5*x - 5312355*a^4*b*x^3 - 1675114*a^3*b^2*x^5 + 749658*a^2*b^
3*x^7 + 378651*a*b^4*x^9 + 43225*b^5*x^11 + 6243840*a^5*x*(1 - (b*x^2)/a)^(
1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(1339975*(a - b*x^2)^(1/
3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{2}+a\right)^{\frac{5}{3}} \left(b\,x^{2}+3a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
[Out] integral(-(b^4*x^8 + 8*a*b^3*x^6 + 18*a^2*b^2*x^4 - 27*a^4)*(-b*x^2 + a)^(2
/3), x)
```

Sympy [A]
time = 2.90, size = 139, normalized size = 0.21

$$27a^{\frac{14}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{18a^{\frac{8}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}-\frac{8a^{\frac{5}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}-\frac{a^{\frac{2}{3}}b^4x^9\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**3,x)
[Out] 27*a**(14/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 18*
a**(8/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
 - 8*a**(5/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)
```

```
/a)/7 - a**(2/3)*b**4*x**9*hyper((-2/3, 9/2), (11/2,), b*x**2*exp_polar(2*I
*pi)/a)/9
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a-b\,x^2\right)^{5/3}\,\left(b\,x^2+3\,a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x)
[Out] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^3, x)
```

3.117 $\int (a-bx^2)^{5/3} (3a+bx^2)^2 \, dx$

Optimal. Leaf size=637

$$\frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax(a-bx^2)^{8/3}-\frac{3}{25}x(a-bx^2)^{8/3}(3a+bx^2)-\frac{\quad}{8645\left(\left(1\right.\right.}$$

```
[Out] 28512/8645*a^3*x*(-b*x^2+a)^(2/3)+14256/6175*a^2*x*(-b*x^2+a)^(5/3)-306/475
*a*x*(-b*x^2+a)^(8/3)-3/25*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a)-114048/8645*a^4*x
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+38016/8645*3^(3/4)*a^(13/3)*(a^(1/
3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)-57024/8645*3^(1/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(
1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*
x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)
+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 396, 201, 241, 310, 225, 1893}

$$\frac{38016\sqrt{2}\,3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{57024\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax(a-bx^2)^{8/3}-\frac{3}{25}x(a-bx^2)^{8/3}(3a+bx^2)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (28512*a^3*x*(a - b*x^2)^(2/3))/8645 + (14256*a^2*x*(a - b*x^2)^(5/3))/6175
 - (306*a*x*(a - b*x^2)^(8/3))/475 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2))/
25 - (114048*a^4*x)/(8645*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5
7024*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
```

```
]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (38016*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx=-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{3\int \left(a-bx^2\right)^{5/3}\left(-78a^2b-34ab^2x^2\right)\,dx}{25b}$$

$$=-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)+\frac{1}{475}\left(4752a^2\right)\int \left(a-\right.$$

$$=\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

$$=\frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x$$

$$=\frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x$$

$$=\frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x$$

$$=\frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.63, size = 173, normalized size = 0.27

$$\frac{x(a-bx^2)^{2/3}\left(21a(45a^2+10abx^2+b^2x^4)\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};\frac{7}{2};\frac{bx^2}{a}\right)+8bx^2(18a^2+9abx^2+b^2x^4)\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(-\frac{2}{3},\frac{3}{2};\frac{9}{2};\frac{bx^2}{a}\right)+4b(3ax+bx^3)^2\Gamma\left(-\frac{2}{3}\right)\,{}_3F_2\left(-\frac{2}{3},\frac{3}{2},2;1,\frac{9}{2};\frac{bx^2}{a}\right)\right)}{105\left(1-\frac{bx^2}{a}\right)^{2/3}\Gamma\left(-\frac{5}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-5/3]*Hype
rgeometric2F1[-5/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^
2*x^4)*Gamma[-2/3]*Hypergeometric2F1[-2/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*
x + b*x^3)^2*Gamma[-2/3]*HypergeometricPFQ[{-2/3, 3/2, 2}, {1, 9/2}, (b*x^2
)/a]))/(105*(1 - (b*x^2)/a)^(2/3)*Gamma[-5/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{5}{3}}\left(b\,x^2+3a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 5*a*b^2*x^4 + 3*a^2*b*x^2 - 9*a^3)*(-b*x^2 + a)^(2/3),
 x)
```

Sympy [A]
time = 2.59, size = 131, normalized size = 0.21

$$9a^{\frac{11}{3}}x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) - a^{\frac{8}{3}}bx^{3}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) - a^{\frac{5}{3}}b^{2}x^{5}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right) - \frac{a^{\frac{2}{3}}b^{3}x^{7}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(
8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(5/
3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(2
/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/7
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a-b\,x^2\right)^{5/3}\,{\left(b\,x^2+3\,a\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x)
```

```
[Out] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^2, x)
```

3.118 $\int (a-bx^2)^{5/3} (3a+bx^2)\, dx$

Optimal. Leaf size=608

$$3600\sqrt[4]{}$$

$$\frac{1800a^2x(a-bx^2)^{2/3}}{1729}+\frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-$$

```
[Out] 1800/1729*a^2*x*(-b*x^2+a)^(2/3)+180/247*a*x*(-b*x^2+a)^(5/3)-3/19*x*(-b*x^
2+a)^(8/3)-7200/1729*a^3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+2400/172
9*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+
a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))
*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-3600/1729*3^(1/4)*a^(10/
3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/
2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/
3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/
3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {396, 201, 241, 310, 225, 1893}

$$\frac{2400\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{3600\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{1800a^2x(a-bx^2)^{2/3}}{1729}+\frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

```
[Out] (1800*a^2*x*(a - b*x^2)^(2/3))/1729 + (180*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(8/3))/19 - (7200*a^3*x)/(1729*((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))) - (3600*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3
)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (2400*Sqrt[2]*3^(3/4
```

```
)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int (a-bx^2)^{5/3}(3a+bx^2)\,dx = -\frac{3}{19}x(a-bx^2)^{8/3}+\frac{1}{19}(60a)\int (a-bx^2)^{5/3}\,dx$$

$$= \frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}+\frac{1}{247}(600a^2)\int (a-bx^2)^{2/3}\,dx$$

$$= \frac{1800a^2x(a-bx^2)^{2/3}}{1729}+\frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}+\frac{(2400a^3)}{\ldots}$$

$$= \frac{1800a^2x(a-bx^2)^{2/3}}{1729}+\frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}-\frac{(3600a^3\ldots}{\ldots}$$

$$= \frac{1800a^2x(a-bx^2)^{2/3}}{1729}+\frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}+\frac{(3600a^3\ldots}{\ldots}$$

$$= \frac{1800a^2x(a-bx^2)^{2/3}}{1729}+\frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}-\frac{\ldots}{1729((1\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.12, size = 68, normalized size = 0.11

$$\frac{3}{19}x(a-bx^2)^{2/3}\left(-(a-bx^2)^2+\frac{20a^2\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

```
[Out] (3*x*(a - b*x^2)^(2/3)*(-(a - b*x^2)^2 + (20*a^2*Hypergeometric2F1[-5/3, 1/
2, 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/19
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{2}+a\right)^{\frac{5}{3}} \left(b\,x^{2}+3a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A]
time = 2.04, size = 100, normalized size = 0.16

$$3a^{\frac{8}{3}}x\,{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\right| \frac{bx^{2}e^{2i\pi}}{a}\right) - \frac{2a^{\frac{5}{3}}bx^{3}\,{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\right| \frac{bx^{2}e^{2i\pi}}{a}\right)}{3} - \frac{a^{\frac{2}{3}}b^{2}x^{5}\,{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\right| \frac{bx^{2}e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a),x)
```

```
[Out] 3*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 2*a**
(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3 - a**
(2/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a-b\,x^2\right)}^{5/3}\,\left(b\,x^2+3\,a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/3)*(3*a + b*x^2),x)
```

```
[Out] int((a - b*x^2)^(5/3)*(3*a + b*x^2), x)
```

3.119 $\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx$

Optimal. Leaf size=765

$$-\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{4\sqrt[3]{2}\,a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}\,a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{}}{}\right.}{\sqrt{}}$$

```
[Out] -3/7*x*(-b*x^2+a)^(2/3)+96/7*a*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+4*
2^(1/3)*a^(7/6)*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)
))/b^(1/2)-4/3*2^(1/3)*a^(7/6)*arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)+4/3*2^(1/
3)*a^(7/6)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1
/2))*3^(1/2)/b^(1/2)+4/3*2^(1/3)*a^(7/6)*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*
3^(1/2)/b^(1/2)-32/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((
-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2
/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3
)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+48/7*3
^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1
/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a
^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-
b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 544, 241, 310, 225, 1893, 402}

$$-\frac{32\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{48\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{4\sqrt[3]{2}\,a^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}\,a^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}\,a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}-\frac{4\sqrt[3]{2}\,a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 + (96*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqr
t[3]*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/
3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (4*2^(1/3)*a^(7/6)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTanh[(Sq
```

```
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] + (48*3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
 + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) - (32*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{3\int \frac{\frac{16a^2b}{3}-\frac{32}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{7b}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} - \frac{1}{7}(32a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx + \left(16a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{4\sqrt[3]{2}\,a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} + \frac{4\sqrt[3]{2}\,a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{\ldots}\right.}{\sqrt{b}\ldots}\right.}{\sqrt{3}\,\sqrt{b}}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{4\sqrt[3]{2}\,a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} + \frac{4\sqrt[3]{2}\,a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{\ldots}\right.}{\sqrt{b}\ldots}\right.}{\sqrt{3}\,\sqrt{b}}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{4\sqrt[3]{2}\,a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.73, size = 231, normalized size = 0.30

$$\frac{x\left(-32bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+27\left(-a+bx^2+\frac{48a^3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}\right)\right)}{63\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (x*(-32*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + 27*(-a + b*x^2 + (48*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2
)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2
)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3
*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))))/(
63*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{5}{3}}}{b\,x^2+3a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-bx^2)^{\frac{5}{3}}}{3a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a),x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-b\,x^2)^{5/3}}{b\,x^2+3\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/3)/(3*a + b*x^2),x)
[Out] int((a - b*x^2)^(5/3)/(3*a + b*x^2), x)
```

3.120 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx$

Optimal. Leaf size=775

$$\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}-\frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\sqrt{b}}-\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\right.}{}\right.}{\sqrt{3}\,\sqrt{}}$$

```
[Out] 2/3*x*(-b*x^2+a)^(2/3)/(b*x^2+3*a)-11/3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))-2^(1/3)*a^(1/6)*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a
)^(1/3)))/b^(1/2)+1/3*2^(1/3)*a^(1/6)*arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)-1/
3*2^(1/3)*a^(1/6)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)
/x/b^(1/2))*3^(1/2)/b^(1/2)-1/3*2^(1/3)*a^(1/6)*arctan(3^(1/2)*a^(1/2)/x/b^
(1/2))*3^(1/2)/b^(1/2)+11/9*3^(3/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*Elli
pticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^
2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*
(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
-11/6*3^(1/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/
3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/
2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(
1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 775, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {424, 544, 241, 310, 225, 1893, 402}

$$\frac{11\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{11\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2\,3^{3/4}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{\sqrt{3}\sqrt{b}}-\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\sqrt{b}}+\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}-\frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}+\frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (2*x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)) - (11*x)/(3*((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b
]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*
a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan
```

```
h[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] - (
11*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3
/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (11*Sqrt[2]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(3*3^(1/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \frac{2x(a-bx^2)^{2/3}}{3\,(3a+bx^2)} + \frac{\int \frac{-2a^2b+\frac{22}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{6ab}$$

$$= \frac{2x(a-bx^2)^{2/3}}{3\,(3a+bx^2)} + \frac{11}{9}\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx - (4a)\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$

$$= \frac{2x(a-bx^2)^{2/3}}{3\,(3a+bx^2)} - \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} - \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{\cdots}\right.}{\sqrt{b}\,x}\right.}{\sqrt{3}\,\sqrt{b}}$$

$$= \frac{2x(a-bx^2)^{2/3}}{3\,(3a+bx^2)} - \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} - \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{\cdots}\right.}{\sqrt{b}\,x}\right.}{\sqrt{3}\,\sqrt{b}}$$

$$= \frac{2x(a-bx^2)^{2/3}}{3\,(3a+bx^2)} - \frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{\sqrt[3]{2}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{3}\,\sqrt{b}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.11, size = 235, normalized size = 0.30

$$\frac{x\left(\frac{11bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a}+\frac{27\left(2a-2bx^2-\frac{9a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)}{3a+bx^2}\right)}{81\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*((11*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a])/a + (27*(2*a - 2*b*x^2 - (9*a^2*AppellF1[1/2, 1/3, 1, 3/2,
(b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3
*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/
a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2
)))/(81*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{5}{3}}}{\left(b\,x^{2}+3a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a - bx^{2}\right)^{\frac{5}{3}}}{\left(3a + bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-b\,x^2)^{5/3}}{(b\,x^2+3\,a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x)
[Out] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^2, x)
```

3.121 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx$

Optimal. Leaf size=815

$$\frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2}-\frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)}+\frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{18\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}}{\ldots}\right)}{18\,2^{2/}\ldots}$$

```
[Out] 1/3*x*(-b*x^2+a)^(2/3)/(b*x^2+3*a)^2-1/18*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)+
1/18*x/a/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/36*arctanh(x*b^(1/2)/a^(
1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(5/6)/b^(1/2)-1/108*arct
anh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/108*arctan(a^(1/6)*(a^(1/3
)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(5/6)*3^(1/2)/b^(1
/2)+1/108*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(5/6)*3^(1/2)/b^(1/2)
-1/54*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^
(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^
(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/36*(a^(1/3)-(-b*x^2
+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)
+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 815, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {424, 12, 482, 544, 241, 310, 225, 1893, 402}

$$-\frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)}+\frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{3(bx^2+3a)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{18\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{18\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{54\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{12\,3^{3/4}a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{9\sqrt{2}\sqrt[4]{3}\,a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)^2) - (x*(a - b*x^2)^(2/3))/(18*a*(3*
a + b*x^2)) + x/(18*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(18*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcT
```

```
an[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(18
*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(54*2^(2/3
)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b
*x^2)^(1/3)))]/(18*2^(2/3)*a^(5/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) -
 (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)
^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1
 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(12*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
 ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 225</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

<u>Rule 241</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

<u>Rule 310</u>

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

<u>Rule 402</u>

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
```

```
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx &= \frac{x(a-bx^2)^{2/3}}{3\,(3a+bx^2)^2} + \frac{\int \frac{16ab^2x^2}{3\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx}{12ab} \\
&= \frac{x(a-bx^2)^{2/3}}{3\,(3a+bx^2)^2} + \frac{1}{9}(4b)\int \frac{x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx \\
&= \frac{x(a-bx^2)^{2/3}}{3\,(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)} + \frac{\int \frac{a-\frac{bx^2}{3}}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{18a} \\
&= \frac{x(a-bx^2)^{2/3}}{3\,(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)} + \frac{1}{9}\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{54a} \\
&= \frac{x(a-bx^2)^{2/3}}{3\,(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{18\;2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{\cdots}\right.}{\sqrt{b}\,x}\cdots\right.}{18\;2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}\cdots} \\
&= \frac{x(a-bx^2)^{2/3}}{3\,(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{18\;2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{\cdots}\right.}{\sqrt{b}\,x}\cdots\right.}{18\;2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}\cdots} \\
&= \frac{x(a-bx^2)^{2/3}}{3\,(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)} + \frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{18\;2^{2/3}\sqrt{3}\,\cdots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 252, normalized size = 0.31

$$
\frac{-\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^2}+\frac{27x\left(3a-4bx^2+\frac{b^2x^4}{a}+\frac{9a\left(3a+bx^2\right)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)}{(3a+bx^2)^2}}{486\sqrt[3]{a-bx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (-((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a])/a^2) + (27*x*(3*a - 4*b*x^2 + (b^2*x^4)/a + (9*a*(3*a + b*x^2)*
```

```
AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a]))))/(3*a + b*x^2)^2)/(486*(a - b*x^2)^(1/3))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{5}{3}}}{\left(b\,x^{2}+3a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a - bx^{2}\right)^{\frac{5}{3}}}{\left(3a + bx^{2}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^2\right)^{5/3}}{\left(b\,x^2+3\,a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x)
```

```
[Out] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^3, x)
```

3.122 $\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=659

$$-\frac{1552608a^3x(a-bx^2)^{2/3}}{43225}-\frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175}-\frac{18}{19}ax(a-bx^2)^{2/3}(3a+bx^2)^2-\frac{3}{25}x(a-bx^2)^{2/3}$$

```
[Out] -1552608/43225*a^3*x*(-b*x^2+a)^(2/3)-36288/6175*a^2*x*(-b*x^2+a)^(2/3)*(b*
x^2+3*a)-18/19*a*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)^2-3/25*x*(-b*x^2+a)^(2/3)*(
b*x^2+3*a)^3-3794688/8645*a^4*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+126
4896/8645*3^(3/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a
)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*
3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+
a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1897344/8645*3^(
1/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/
3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b
*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 542, 396, 241, 310, 225, 1893}

$$\frac{1264896\sqrt{2}\,3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{1897344\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{1552608a^3x(a-bx^2)^{2/3}}{43225}-\frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175}-\frac{18}{19}ax(a-bx^2)^{2/3}(3a+bx^2)^2-\frac{3}{25}x(a-bx^2)^{2/3}(3a+bx^2)^3$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (-1552608*a^3*x*(a - b*x^2)^(2/3))/43225 - (36288*a^2*x*(a - b*x^2)^(2/3)*(
3*a + b*x^2))/6175 - (18*a*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (3*x*(
a - b*x^2)^(2/3)*(3*a + b*x^2)^3)/25 - (3794688*a^4*x)/(8645*((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))) - (1897344*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3
)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
```

```
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (
1264896*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(86
45*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{25}x(a-bx^2)^{2/3}(3a+bx^2)^3 - \frac{3\int \frac{(3a+bx^2)^2(-78a^2b-50ab^2x^2)}{\sqrt[3]{a-bx^2}}dx}{25b}$$

$$= -\frac{18}{19}ax(a-bx^2)^{2/3}(3a+bx^2)^2 - \frac{3}{25}x(a-bx^2)^{2/3}(3a+bx^2)^3 + \frac{9\int \frac{(3a+bx^2)(1632a^3b^2+}{\sqrt[3]{a-bx}}}{475b^2}$$

$$= -\frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175} - \frac{18}{19}ax(a-bx^2)^{2/3}(3a+bx^2)^2 - \frac{3}{25}x(a-bx^2)^{2/3}$$

$$= -\frac{1552608a^3x(a-bx^2)^{2/3}}{43225} - \frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175} - \frac{18}{19}ax(a-bx^2)^{2/3}(3a$$

$$= -\frac{1552608a^3x(a-bx^2)^{2/3}}{43225} - \frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175} - \frac{18}{19}ax(a-bx^2)^{2/3}(3a$$

$$= -\frac{1552608a^3x(a-bx^2)^{2/3}}{43225} - \frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175} - \frac{18}{19}ax(a-bx^2)^{2/3}(3a$$

$$= -\frac{1552608a^3x(a-bx^2)^{2/3}}{43225} - \frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175} - \frac{18}{19}ax(a-bx^2)^{2/3}(3a$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 15.06, size = 98, normalized size = 0.15

$$\frac{3x\left(-941085a^4+727830a^3bx^2+184044a^2b^2x^4+27482ab^3x^6+1729b^4x^8+2108160a^4\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{43225\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*x*(-941085*a^4 + 727830*a^3*b*x^2 + 184044*a^2*b^2*x^4 + 27482*a*b^3*x^6
 + 1729*b^4*x^8 + 2108160*a^4*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3,
1/2, 3/2, (b*x^2)/a]))/(43225*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+3a\right)^{4}}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^4
)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A]
time = 3.09, size = 165, normalized size = 0.25

$$81a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+36a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{54a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{12a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}+\frac{b^4x^9\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{9}{2}\\\frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**4/(-b*x**2+a)**(1/3),x)
[Out] 81*a**(11/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 36*a
**(8/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 54*a
**(5/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5 +
 12*a**(2/3)*b**3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a
```

```
)/7 + b**4*x**9*hyper((1/3, 9/2), (11/2,), b*x**2*exp_polar(2*I*pi)/a)/(9*a
**(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+3\,a\right)}^4}{{\left(a-b\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^4/(a - b*x^2)^(1/3),x)
[Out] int((3*a + b*x^2)^4/(a - b*x^2)^(1/3), x)
```

3.123 $\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=628

$$-\frac{15768a^2x(a-bx^2)^{2/3}}{1729}-\frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2)-\frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2)^2-\frac{215136}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

```
[Out] -15768/1729*a^2*x*(-b*x^2+a)^(2/3)-324/247*a*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)
-3/19*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)^2-215136/1729*a^3*x/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))+71712/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+
(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^
(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)-107568/1729*3^(1/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-
(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
)),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/
(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 542, 396, 241, 310, 225, 1893}

$$\frac{71712\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{107568\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{15768a^2x(a-bx^2)^{2/3}}{1729}-\frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2)-\frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2)^2$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (-15768*a^2*x*(a - b*x^2)^(2/3))/1729 - (324*a*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2))/247 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (215136*a^3*x)/(1
729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (107568*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
```

```
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))^2)]) + (71712*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
/; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2)^2 - \frac{3\int \frac{(3a+bx^2)(-60a^2b-36ab^2x^2)}{\sqrt[3]{a-bx^2}}\,dx}{19b}$$

$$= -\frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2) - \frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2)^2 + \frac{9\int \frac{888a^3b^2+584a^2b^3x^2}{\sqrt[3]{a-bx^2}}}{247b^2}$$

$$= -\frac{15768a^2x(a-bx^2)^{2/3}}{1729} - \frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2) - \frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2$$

$$= -\frac{15768a^2x(a-bx^2)^{2/3}}{1729} - \frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2) - \frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2$$

$$= -\frac{15768a^2x(a-bx^2)^{2/3}}{1729} - \frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2) - \frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2$$

$$= -\frac{15768a^2x(a-bx^2)^{2/3}}{1729} - \frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2) - \frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 15.05, size = 88, normalized size = 0.14

$$\frac{3\left(-8343a^3x+7041a^2bx^3+1211ab^2x^5+91b^3x^7+23904a^3x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{1729\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*(-8343*a^3*x + 7041*a^2*b*x^3 + 1211*a*b^2*x^5 + 91*b^3*x^7 + 23904*a^3*
x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(1729
*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+3a)^3}{(-b\,x^2+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
)/(b*x^2 - a), x)
```

Sympy [A]
time = 2.46, size = 129, normalized size = 0.21

$$27a^{\frac{8}{3}}x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)+9a^{\frac{5}{3}}bx^{3}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)+\frac{9a^{\frac{2}{3}}b^{2}x^{5}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)}{5}+\frac{b^{3}x^{7}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)}{7\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(8/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a**
(5/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a**(
2/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5 + b*
*3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+3\,a\right)}^3}{{\left(a-b\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^3/(a - b*x^2)^(1/3),x)
```

```
[Out] int((3*a + b*x^2)^3/(a - b*x^2)^(1/3), x)
```

3.124 $\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=597

$$-\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{}$$

```
[Out] -198/91*a*x*(-b*x^2+a)^(2/3)-3/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)-3240/91*a^
2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1080/91*3^(3/4)*a^(7/3)*(a^(1/3
)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)
*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))
^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)-1620/91*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))
*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a
)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*
2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {427, 396, 241, 310, 225, 1893}

$$\frac{1080\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

```
[Out] (-198*a*x*(a - b*x^2)^(2/3))/91 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13
- (3240*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (1620*3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
```

```
(a - b*x^2)^(1/3))^2)]) + (1080*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3\int \frac{-42a^2b-22ab^2x^2}{\sqrt[3]{a-bx^2}}\,dx}{13b}$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)+\frac{1}{91}\left(1080a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{\left(1620a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-a}}\right.}{91bx}$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)+\frac{\left(1620a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{(1+\sqrt{}}{\sqrt{-}}\right.}{91bx}$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 13.13, size = 158, normalized size = 0.26

$$\frac{x\sqrt[3]{1-\frac{bx^2}{a}}\left(63a(45a^2+10abx^2+b^2x^4)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{7}{2};\frac{bx^2}{a}\right)+8bx^2(18a^2+9abx^2+b^2x^4)\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{9}{2};\frac{bx^2}{a}\right)+4b(3ax+bx^3)^2\,{}_3F_2\left(\frac{4}{3},\frac{3}{2},2;1,\frac{9}{2};\frac{bx^2}{a}\right)\right)}{315a\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/3)*(63*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Hypergeometr
ic2F1[1/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^2*x^4)*Hy
pergeometric2F1[4/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*x + b*x^3)^2*Hypergeom
etricPFQ[{4/3, 3/2, 2}, {1, 9/2}, (b*x^2)/a]))/(315*a*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+3 a\right)^{2}}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral(-(b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A]
time = 1.91, size = 94, normalized size = 0.16

$$9a^{\frac{5}{3}}x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)+2a^{\frac{2}{3}}bx^{3}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)+\frac{b^{2}x^{5}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(1/3),x)
```

```
[Out] 9*a**(5/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a**(
2/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + b**2*x*
*5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+3\,a\right)}^2}{{\left(a-b\,x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^2/(a - b*x^2)^(1/3),x)
```

```
[Out] int((3*a + b*x^2)^2/(a - b*x^2)^(1/3), x)
```

3.125 $\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=568

$$-\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\cdots}{\left(\left(1\cdots\right.\right.}}}{7bx\sqrt{\cdots}}$$

```
[Out] -3/7*x*(-b*x^2+a)^(2/3)-72/7*a*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+24
/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+
a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))
*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-36/7*3^(1/4)*a^(4/3)*(a^
(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b
*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {396, 241, 310, 225, 1893}

$$\frac{24\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 - (72*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(4/3)*(a
^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
```

```
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{1}{7}(24a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{\left(36a\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{\left(36a\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}-\frac{\left(36\right.}{}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 62, normalized size = 0.11

$$\frac{3x\left(-a+bx^2+8a\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{7\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*x*(-a + b*x^2 + 8*a*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/
2, (b*x^2)/a]))/(7*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,x^2+3a}{\left(-b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A]
time = 1.15, size = 60, normalized size = 0.11

$$3a^{\frac{2}{3}}x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{bx^3\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(1/3),x)
```

```
[Out] 3*a**(2/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + b*x**3
*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2+3\,a}{{\left(a-b\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)/(a - b*x^2)^(1/3),x)
[Out] int((3*a + b*x^2)/(a - b*x^2)^(1/3), x)
```

3.126 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{\dots}\right.}\right.}{2\,2^{2/3}a^{5/6}\sqrt{b}}$$

```
[Out] 1/4*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a
^(5/6)/b^(1/2)-1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/12
*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/
3)/a^(5/6)*3^(1/2)/b^(1/2)+1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a
^(5/6)*3^(1/2)/b^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {402}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\Big(}{6\,2^{2/3}a^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 5.94, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\,(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)
/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{3}}\left(b\,x^2+3a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a-bx^2}\cdot(3a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
```

```
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

3.127 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$

Optimal. Leaf size=787

$$\frac{x(a-bx^2)^{2/3}}{24a^2(3a+bx^2)}-\frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{\ldots}\right.}{\sqrt{b}\,x}\right.}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}\ldots}$$

```
[Out] 1/24*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+3*a)-1/24*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))+1/16*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^
(1/3)))*2^(1/3)/a^(11/6)/b^(1/2)-1/48*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^
(11/6)/b^(1/2)+1/48*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/
2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/48*arctan(3^(1/2)*a^(1/2)/
x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/72*(a^(1/3)-(-b*x^2+a)^(1/3))
*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a
)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b
/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*
(1-3^(1/2)))^2)^(1/2)-1/48*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a
)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*
3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/
3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {425, 544, 241, 310, 225, 1893, 402}

$$\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{12\sqrt{2}\,\sqrt[4]{3}\,a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{16\,3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\,\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\,2^{2/3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{x(a-bx^2)^{2/3}}{24a^2(3a+bx^2)}-\frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(24*a^2*(3*a + b*x^2)) - x/(24*a^2*((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/
3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(
a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcT
```

```
anh[(Sqrt[b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x
)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt
[b]) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4
)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(12*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1
/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \frac{x(a-bx^2)^{2/3}}{24a^2\,(3a+bx^2)} - \frac{\int \frac{-7ab-\frac{b^2x^2}{3}}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{24a^2b}$$

$$= \frac{x(a-bx^2)^{2/3}}{24a^2\,(3a+bx^2)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{72a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{4a}$$

$$= \frac{x(a-bx^2)^{2/3}}{24a^2\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right.}{\sqrt{b}\,x}\right.}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}$$

$$= \frac{x(a-bx^2)^{2/3}}{24a^2\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right.}{\sqrt{b}\,x}\right.}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}$$

$$= \frac{x(a-bx^2)^{2/3}}{24a^2\,(3a+bx^2)} - \frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 234, normalized size = 0.30

$$\frac{x\left(\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+\frac{27\left(\frac{a-bx^2}{a^2}+\frac{63F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)}{3a+bx^2}\right)}{648\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3
*(b*x^2)/a])/a^3 + (27*((a - b*x^2)/a^2 + (63*AppellF1[1/2, 1/3, 1, 3/2, (b
*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(
b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]
 + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2))
)/(648*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}} \left(b\,x^{2}+3a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a - bx^{2}}\,\left(3a + bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,{\left(b\,x^2+3\,a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x)
```

```
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2), x)
```

3.128 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$

Optimal. Leaf size=818

$$\frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2}+\frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)}-\frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{144\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}+\frac{5\tan^{-1}\cdots}{\cdots}$$

```
[Out] 1/48*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+3*a)^2+5/288*x*(-b*x^2+a)^(2/3)/a^3/(b*x
^2+3*a)-5/288*x/a^3/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+5/288*arctanh(x
*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/
2)-5/864*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+5/864*arctan(a
^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/
6)*3^(1/2)/b^(1/2)+5/864*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)
*3^(1/2)/b^(1/2)+5/864*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1
/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1
/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1
/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-5/57
6*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2
)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 544, 241, 310, 225, 1893, 402}

$$\frac{5(a-bx^2)^{2/3}x}{288a^3(bx^2+3a)}-\frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{48a^2(bx^2+3a)^2}+\frac{5\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{144\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{144\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{432\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\,2^{2/3}a^{17/6}\sqrt{b}}-\frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{192\,3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}+\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{144\sqrt{2}\sqrt[4]{3}\,a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(48*a^2*(3*a + b*x^2)^2) + (5*x*(a - b*x^2)^(2/3))/(2
88*a^3*(3*a + b*x^2)) - (5*x)/(288*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) + (5*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a
^(17/6)*Sqrt[b]) + (5*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2
```

```
)^(1/3)))/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (5*ArcTanh
[(Sqrt[b]*x)/Sqrt[a]])/(432*2^(2/3)*a^(17/6)*Sqrt[b]) + (5*ArcTanh[(Sqrt[b]
*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(144*2^(2/3)*a^(17/6)
*Sqrt[b]) - (5*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(19
2*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (5*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(144*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} - \frac{\int \frac{-15ab+\frac{5b^2x^2}{3}}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx}{48a^2b}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} + \frac{\int \frac{100a^2b^2+\frac{20}{3}ab^3x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{1152a^4b^2}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} + \frac{5\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{864a^3} + \frac{5\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a}}{72a^2}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{144\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}}{}\right.}{14}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{144\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}}{}\right.}{14}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} - \frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 255, normalized size = 0.31

$$\frac{x\left(\frac{27a(a-bx^2)(21a+5bx^2)}{(3a+bx^2)^2}+5bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{6075a^3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}\right)}{7776a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*((27*a*(a - b*x^2)*(21*a + 5*b*x^2))/(3*a + b*x^2)^2 + 5*b*x^2*(1 - (b*x
^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + (6075*
a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(
9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-Appel
lF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/
2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(7776*a^4*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}} \left(b\,x^{2}+3a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a - bx^{2}}\,\left(3a + bx^{2}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**3,x)
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,{\left(b\,x^2+3\,a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x)
```

```
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3), x)
```

3.129 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx$

Optimal. Leaf size=623

10

$$\frac{2538}{91}ax\left(a-bx^2\right)^{2/3}+\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)+\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}+\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+-$$

```
[Out] 2538/91*a*x*(-b*x^2+a)^(2/3)+81/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)+6*x*(b*x^
2+3*a)^2/(-b*x^2+a)^(1/3)+20088/91*a^2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2)))-6696/91*3^(3/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^
2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I
-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x
^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+10044/91*3^(1
/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)
*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x
^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 623, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {424, 542, 396, 241, 310, 225, 1893}

$$-\frac{6696\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{10044\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2538}{91}ax\left(a-bx^2\right)^{2/3}+\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}+\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (2538*a*x*(a - b*x^2)^(2/3))/91 + (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13
 + (6*x*(3*a + b*x^2)^2)/(a - b*x^2)^(1/3) + (20088*a^2*x)/(91*((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))) + (10044*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
```

```
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (669
6*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} - \frac{3\int \frac{(3a+bx^2)(6a^2b+18ab^2x^2)}{\sqrt[3]{a-bx^2}}dx}{2ab}$$

$$= \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} + \frac{9\int \frac{-132a^3b^2-188a^2b^3x^2}{\sqrt[3]{a-bx^2}}dx}{26ab^2}$$

$$= \frac{2538}{91}ax(a-bx^2)^{2/3} + \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} - \frac{1}{91}(6696a^2)\int$$

$$= \frac{2538}{91}ax(a-bx^2)^{2/3} + \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} + \frac{\left(10044a^2\sqrt{-bx^2}\right.}{}$$

$$= \frac{2538}{91}ax(a-bx^2)^{2/3} + \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} - \frac{\left(10044a^2\sqrt{-bx^2}\right.}{}$$

$$= \frac{2538}{91}ax(a-bx^2)^{2/3} + \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} + \frac{200}{91\left(\left(1-\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 15.05, size = 76, normalized size = 0.12

$$-\frac{3x\left(-3051a^2+132abx^2+7b^2x^4+2232a^2\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{91\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (-3*x*(-3051*a^2 + 132*a*b*x^2 + 7*b^2*x^4 + 2232*a^2*(1 - (b*x^2)/a)^(1/3)
*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(91*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+3a)^3}{(-b\,x^2+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3)
/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3a + bx^2)^3}{(a - bx^2)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(4/3),x)
```

```
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+3\,a\right)^3}{\left(a-b\,x^2\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^3/(a - b*x^2)^(4/3),x)
[Out] int((3*a + b*x^2)^3/(a - b*x^2)^(4/3), x)
```

3.130 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx$

Optimal. Leaf size=592

$$\frac{45}{7}x\left(a-bx^2\right)^{2/3}+\frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}}+\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-\right.}{}$$

[Out] 45/7*x*(-b*x^2+a)^(2/3)+6*x*(b*x^2+3*a)/(-b*x^2+a)^(1/3)+324/7*a*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-108/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+162/7*3^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.31, antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {424, 396, 241, 310, 225, 1893}

$$-\frac{108\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}}+\frac{45}{7}x\left(a-bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]

[Out] (45*x*(a - b*x^2)^(2/3))/7 + (6*x*(3*a + b*x^2))/(a - b*x^2)^(1/3) + (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3

```
/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} - \frac{3\int \frac{6a^2b+10ab^2x^2}{\sqrt[3]{a-bx^2}}\,dx}{2ab}$$

$$= \frac{45}{7}x(a-bx^2)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} - \frac{1}{7}(108a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{45}{7}x(a-bx^2)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} + \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-\right.}{7bx}$$

$$= \frac{45}{7}x(a-bx^2)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} - \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{\right.}{7bx}$$

$$= \frac{45}{7}x(a-bx^2)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} + \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{162\sqrt[4]{3}\sqrt{2+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 14.40, size = 168, normalized size = 0.28

$$\frac{x\sqrt[3]{1-\frac{bx^2}{a}}\,\Gamma\left(\frac{1}{3}\right)\left(63a(45a^2+10abx^2+b^2x^4)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{2};\frac{bx^2}{a}\right)+32bx^2(18a^2+9abx^2+b^2x^4)\,{}_2F_1\left(\frac{3}{2},\frac{7}{3};\frac{9}{2};\frac{bx^2}{a}\right)+16b(3ax+bx^3)^2\,{}_3F_2\left(\frac{3}{2},2,\frac{7}{3};1,\frac{9}{2};\frac{bx^2}{a}\right)\right)}{945a^2\sqrt[3]{a-bx^2}\,\Gamma\left(\frac{4}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/3)*Gamma[1/3]*(63*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*H
ypergeometric2F1[1/2, 4/3, 7/2, (b*x^2)/a] + 32*b*x^2*(18*a^2 + 9*a*b*x^2 +
 b^2*x^4)*Hypergeometric2F1[3/2, 7/3, 9/2, (b*x^2)/a] + 16*b*(3*a*x + b*x^3
)^2*HypergeometricPFQ[{3/2, 2, 7/3}, {1, 9/2}, (b*x^2)/a]))/(945*a^2*(a - b
*x^2)^(1/3)*Gamma[4/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+3a\right)^{2}}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*
x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3a+bx^{2}\right)^{2}}{\left(a-bx^{2}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(4/3),x)
```

```
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+3\,a\right)}^2}{{\left(a-b\,x^2\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^2/(a - b*x^2)^(4/3),x)
[Out] int((3*a + b*x^2)^2/(a - b*x^2)^(4/3), x)
```

3.131 $\int \frac{3a+bx^2}{\left(a-bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=561

$$\frac{6x}{\sqrt[3]{a-bx^2}}+\frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}\cdots}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1-\sqrt{3}\cdots\right.\right.}}}$$

```
[Out] 6*x/(-b*x^2+a)^(1/3)+9*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3*3^(3/4)*
a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+
3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((
a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+9/2*3^(1/4)*a^(1/3)*(a^(1/3)-(-b*x^
2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3
)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {393, 241, 310, 225, 1893}

$$-\frac{3\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{6x}{\sqrt[3]{a-bx^2}}+\frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

```
[Out] (6*x)/(a - b*x^2)^(1/3) + (9*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
 + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) - (3*Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/
```

```
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \frac{6x}{\sqrt[3]{a-bx^2}} - 3\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} + \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx}$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} - \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx} + \frac{\left(9\sqrt{\frac{1}{2}\left(2+\right.}\right.}{}$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} + \frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 53, normalized size = 0.09

$$\frac{6x - 3x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
[Out] (6*x - 3*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a
])/(a - b*x^2)^(1/3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{b\,x^2+3a}{(-b\,x^2+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 3.17, size = 60, normalized size = 0.11

$$\frac{3x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[3]{a}} + \frac{bx^3\,{}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(4/3),x)
```

```
[Out] 3*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/3) + b*x**3
*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2+3\,a}{{\left(a-b\,x^2\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)/(a - b*x^2)^(4/3),x)
[Out] int((3*a + b*x^2)/(a - b*x^2)^(4/3), x)
```

3.132 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx$

Optimal. Leaf size=776

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-\cdots}\right.}{\sqrt{b}\,x}\right.}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}$$

```
[Out] 3/8*x/a^2/(-b*x^2+a)^(1/3)+3/8*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)+1/16*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3
)/a^(11/6)/b^(1/2)-1/48*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(11/6)/b^(1/2)
+1/48*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*
2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/48*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(
1/3)/a^(11/6)*3^(1/2)/b^(1/2)-1/8*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-
b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))
,2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b/x*2^(1/2)/(-a^
(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)+3/16*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)
*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3)/b/x/(-a^(1/3
)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)
```

Rubi [A]
time = 0.36, antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {425, 544, 241, 310, 225, 1893, 402}

$$-\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\,a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{16a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{8\,2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\,2^{2/3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(8*a^2*(a - b*x^2)^(1/3)) + (3*x)/(8*a^2*((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]
*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[
```

```
b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)
*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b]) + (3*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*a^(5/3
)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)]) - (3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(4*Sqrt[2]*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}dx = \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{3\int \frac{-\frac{ab}{3}-\frac{b^2x^2}{3}}{\sqrt[3]{a-bx^2}(3a+bx^2)}dx}{8a^2b}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}dx}{8a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}dx}{4a}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right.}{\sqrt{b}x}\right.}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right.}{\sqrt{b}x}\right.}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 7.34, size = 226, normalized size = 0.29

$$\frac{x\left(-\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+27\left(\frac{1}{a^2}-\frac{3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}\right)\right)}{72\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*(-((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1
/3*(b*x^2)/a])/a^3) + 27*(a^(-2) - (3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a,
 -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a,
 -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*
x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))))/(72*(
a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}} \left(b\,x^{2}+3 a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - bx^{2}\right)^{\frac{4}{3}} \cdot \left(3a + bx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a),x)
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\,x^2\right)^{4/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x)
```

```
[Out] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)), x)
```

3.133 $\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^2}\,dx$

Optimal. Leaf size=807

$$\frac{x}{12a^3\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\sqrt[3]{a-bx^2}\,\left(3a+bx^2\right)}+\frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{16\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}+$$

```
[Out] 1/12*x/a^3/(-b*x^2+a)^(1/3)+1/24*x/a^2/(-b*x^2+a)^(1/3)/(b*x^2+3*a)+1/12*x/
a^3/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/32*arctanh(x*b^(1/2)/a^(1/6)/
(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-1/96*arctanh(x
*b^(1/2)/a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+1/96*arctan(a^(1/6)*(a^(1/3)-2^(
1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+
1/96*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)-1/3
6*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2
)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/24*(a^(1/3)-(-b*x^2+a)^
(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b
*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2
)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(
-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 807, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 544, 241, 310, 225, 1893, 402}

$$\frac{x}{12a^3\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\sqrt[3]{a-bx^2}\,(bx^2+3a)}+\frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{16\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{16\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{48\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8\,3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] x/(12*a^3*(a - b*x^2)^(1/3)) + x/(24*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)) +
 x/(12*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(16*2^(2/3)*S
```

```
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(48*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(16*2^(2/3)*a^(17/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(8*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

<u>Rule 225</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

<u>Rule 241</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

<u>Rule 310</u>

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

<u>Rule 402</u>

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{-7ab+\frac{5b^2x^2}{3}}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx}{24a^2b}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{-\frac{8}{3}a^2b^2+\frac{16}{9}ab^3x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{64a^4b^2}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{36a^3} + \frac{\int \frac{}{\sqrt[3]{a-}}}{}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{16\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{\tan^{-}}{}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{16\,2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{\tan^{-}}{}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} + \frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.14, size = 236, normalized size = 0.29

$$\frac{x\left(-2bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(7a+2bx^2+\frac{9a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)}{3a+bx^2}\right)}{648a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(-2*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1
/3*(b*x^2)/a] + (27*a*(7*a + 2*b*x^2 + (9*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b
*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(
b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]
```

```
 + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2))
)/(648*a^4*(a - b*x^2)^(1/3))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}} \left(b\,x^{2}+3a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - bx^{2}\right)^{\frac{4}{3}} \left(3a + bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**2,x)
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{4/3}\,{\left(b\,x^2+3\,a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x)
```

```
[Out] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2), x)
```

3.134 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx$

Optimal. Leaf size=849

$$\frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)}-\frac{19x(a-bx^2)^{2/3}}{1152a^4\,(3a+bx^2)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}$$

```
[Out] 1/48*x/a^2/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2+17/192*x/a^3/(-b*x^2+a)^(1/3)/(b*
x^2+3*a)-19/1152*x*(-b*x^2+a)^(2/3)/a^4/(b*x^2+3*a)+19/1152*x/a^4/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+7/576*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(
1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(23/6)/b^(1/2)-7/1728*arctanh(x*b^(1/2)/a
^(1/2))*2^(1/3)/a^(23/6)/b^(1/2)+7/1728*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b
*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)+7/1728*a
rctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)-19/3456*(
a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(
-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)*3^(3/4)/a^(11/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+19/2304*(a^(1/3)-(-b*x^2+a)
^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-
b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/
2)+1/2*2^(1/2))*3^(1/4)/a^(11/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 544, 241, 310, 225, 1893, 402}

$$-\frac{19(a-bx^2)^{2/3}x}{1152a^4(bx^2+3a)}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(bx^2+3a)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\,(bx^2+3a)^2}+\frac{7\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\,2^{2/3}\sqrt{3}\,a^{23/6}\sqrt{b}}+\frac{7\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\,2^{2/3}\sqrt{3}\,a^{23/6}\sqrt{b}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{19\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{768\,3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}-\frac{19\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{576\sqrt{2}\sqrt[4]{3}\,a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] x/(48*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)^2) + (17*x)/(192*a^3*(a - b*x^2)^
(1/3)*(3*a + b*x^2)) - (19*x*(a - b*x^2)^(2/3))/(1152*a^4*(3*a + b*x^2)) +
(19*x)/(1152*a^4*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(
```

```
Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) + (7*
ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])
/(864*2^(2/3)*a^(23/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3)
+ 2^(1/3)*(a - b*x^2)^(1/3)))])/(288*2^(2/3)*a^(23/6)*Sqrt[b]) + (19*Sqrt[2
 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(768*3^(3/4)*a^(11/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) - (19*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(576*Sq
rt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} - \frac{\int \frac{-15ab+\frac{11b^2x^2}{3}}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx}{48a^2b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{-6a^2b^2-\frac{170}{9}a}{\sqrt[3]{a-bx^2}\,(3a}}{128a^4b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^2}{1152a^4(3a+b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^2}{1152a^4(3a+b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^2}{1152a^4(3a+b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^2}{1152a^4(3a+b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^2}{1152a^4(3a+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 256, normalized size = 0.30

$$\frac{x\left(-19bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(273a^2+140abx^2+19b^2x^4+\frac{333a^2(3a+bx^2)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)}{(3a+bx^2)^2}\right)}{31104a^5\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(-19*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + (27*a*(273*a^2 + 140*a*b*x^2 + 19*b^2*x^4 + (333*a^2*(3*a
+ b*x^2)*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*Appell
F1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)
/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2)^2))/(31104*a^5*(a - b*x^2)^(1/3))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}} \left(b\,x^{2}+3a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - bx^{2}\right)^{\frac{4}{3}} \left(3a + bx^{2}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{4/3}\,{\left(b\,x^2+3\,a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x)
```

```
[Out] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3), x)
```

3.135 $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=653

$$-\frac{3240}{91}ax\left(a-bx^2\right)^{2/3}-\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}}+\frac{3x(3a+bx^2)^3}{2\left(a-bx^2\right)^{4/3}}-\frac{36936a^2\cdots}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}$$

```
[Out] -3240/91*a*x*(-b*x^2+a)^(2/3)-81/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)-9/2*x*(b
*x^2+3*a)^2/(-b*x^2+a)^(1/3)+3/2*x*(b*x^2+3*a)^3/(-b*x^2+a)^(4/3)-36936/91*
a^2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+12312/91*3^(3/4)*a^(7/3)*(a^(
1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1
/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)-18468/91*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x
^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+
1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {424, 540, 542, 396, 241, 310, 225, 1893}

$$\frac{12312\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{18468\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x(3a+bx^2)^3}{2\left(a-bx^2\right)^{4/3}}-\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}}-\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240}{91}ax\left(a-bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3240*a*x*(a - b*x^2)^(2/3))/91 - (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/1
3 - (9*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2)^3)/(2*
(a - b*x^2)^(4/3)) - (36936*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) - (18468*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^
(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a
```

```
^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (12312*Sqrt[2]*3^(3/4)*a^(7/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
+ (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{3\int \frac{(3a+bx^2)^2(-12a^2b+20ab^2x^2)}{(a-bx^2)^{4/3}}\,dx}{8ab}$$

$$= -\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{9\int \frac{(3a+bx^2)(-48a^3b^2-48a^2b^3x^2)}{\sqrt[3]{a-bx^2}}\,dx}{16a^2b^2}$$

$$= -\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} + \frac{27\int \frac{768a^4b^3+640a^3b^4x^2}{\sqrt[3]{a-bx^2}}}{208a^2b^3}$$

$$= -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3} - \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$= -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3} - \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$= -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3} - \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

$$= -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3} - \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 15.08, size = 96, normalized size = 0.15

$$-\frac{3\left(1647a^3x - 4743a^2bx^3 + 177ab^2x^5 + 7b^3x^7 - 4104a^2x(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{91\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3*(1647*a^3*x - 4743*a^2*b*x^3 + 177*a*b^2*x^5 + 7*b^3*x^7 - 4104*a^2*x*(
a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a
]))/(91*(a - b*x^2)^(4/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+3a\right)^{4}}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^4
)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3a+bx^{2}\right)^{4}}{\left(a-bx^{2}\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**4/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**4/(a - b*x**2)**(7/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+3\,a\right)^4}{\left(a-b\,x^2\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^4/(a - b*x^2)^(7/3),x)
```

```
[Out] int((3*a + b*x^2)^4/(a - b*x^2)^(7/3), x)
```

3.136 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=596

$$-\frac{27}{14}x\left(a-bx^2\right)^{2/3}+\frac{3x(3a+bx^2)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{162\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a\right.}}{}$$

```
[Out] -27/14*x*(-b*x^2+a)^(2/3)+3/2*x*(b*x^2+3*a)^2/(-b*x^2+a)^(4/3)-324/7*a*x/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+108/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^
2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2
+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)-162/7*3^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE
((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b
/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {424, 21, 396, 241, 310, 225, 1893}

$$\frac{108\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3x(3a+bx^2)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{27}{14}x\left(a-bx^2\right)^{2/3}-\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

```
[Out] (-27*x*(a - b*x^2)^(2/3))/14 + (3*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(4/3))
- (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (162*3^(1/4)*
Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
```

```
*x^2)^(1/3))^2)]) + (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} - \frac{3\int \frac{(3a+bx^2)(-12a^2b+12ab^2x^2)}{(a-bx^2)^{4/3}}\,dx}{8ab}$$

$$= \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} + \frac{9}{2}\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{27}{14}x(a-bx^2)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} + \frac{1}{7}(108a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{27}{14}x(a-bx^2)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} - \frac{\left(162a\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a}\right.}{7bx}$$

$$= -\frac{27}{14}x(a-bx^2)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} + \frac{\left(162a\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x\right.}{7bx}$$

$$= -\frac{27}{14}x(a-bx^2)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} - \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{162\sqrt[4]{3}\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 15.06, size = 83, normalized size = 0.14

$$\frac{81a^2x+90abx^3-3b^2x^5+108ax(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{7\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

```
[Out] (81*a^2*x + 90*a*b*x^3 - 3*b^2*x^5 + 108*a*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1
/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(7*(a - b*x^2)^(4/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+3a\right)^3}{\left(-b\,x^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3a + bx^2)^3}{(a - bx^2)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(7/3),x)
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(7/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2 + 3\,a)^3}{(a - b\,x^2)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^3/(a - b*x^2)^(7/3),x)
[Out] int((3*a + b*x^2)^3/(a - b*x^2)^(7/3), x)
```

3.137 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=44

$$\frac{9x}{2\sqrt[3]{a-bx^2}}+\frac{3x(3a+bx^2)}{2\,(a-bx^2)^{4/3}}$$

```
[Out] 9/2*x/(-b*x^2+a)^(1/3)+3/2*x*(b*x^2+3*a)/(-b*x^2+a)^(4/3)
```

Rubi [A]

time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {424, 391}

$$\frac{3x(3a+bx^2)}{2\,(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
[Out] (9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^(4/3))
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx &= \frac{3x(3a+bx^2)}{2\,(a-bx^2)^{4/3}}-\frac{3\int \frac{-12a^2b+4ab^2x^2}{(a-bx^2)^{4/3}}\,dx}{8ab}\\ &= \frac{9x}{2\sqrt[3]{a-bx^2}}+\frac{3x(3a+bx^2)}{2\,(a-bx^2)^{4/3}}\end{aligned}$$

Mathematica [A]
time = 15.04, size = 24, normalized size = 0.55

$$\frac{9ax - 3bx^3}{(a - bx^2)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
[Out] (9*a*x - 3*b*x^3)/(a - b*x^2)^(4/3)
```

Maple [A]
time = 0.07, size = 24, normalized size = 0.55

method	result	size
gosper	$\frac{3x(-b\,x^2+3a)}{(-b\,x^2+a)^{\frac{4}{3}}}$	24
trager	$\frac{3x(-b\,x^2+3a)}{(-b\,x^2+a)^{\frac{4}{3}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x,method=_RETURNVERBOSE)
[Out] 3/(-b*x^2+a)^(4/3)*x*(-b*x^2+3*a)
```

Maxima [A]
time = 0.33, size = 33, normalized size = 0.75

$$\frac{3\,(bx^3 - 3\,ax)}{(bx^2 - a)(-bx^2 + a)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="maxima")
[Out] 3*(b*x^3 - 3*a*x)/((b*x^2 - a)*(-b*x^2 + a)^(1/3))
```

Fricas [A]
time = 1.23, size = 42, normalized size = 0.95

$$-\frac{3\,(bx^3 - 3\,ax)(-bx^2 + a)^{\frac{2}{3}}}{b^2x^4 - 2\,abx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -3*(b*x^3 - 3*a*x)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(7/3),x)
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(7/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3), x)
```

Mupad [B]
time = 4.78, size = 27, normalized size = 0.61

$$\frac{3\,x\,(a-b\,x^2)+6\,a\,x}{(a-b\,x^2)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)^2/(a - b*x^2)^(7/3),x)
[Out] (3*x*(a - b*x^2) + 6*a*x)/(a - b*x^2)^(4/3)
```

3.138 $\int \frac{3a+bx^2}{\left(a-bx^2\right)^{7/3}} \, dx$

Optimal. Leaf size=590

$$\frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{}$$

[Out] 3/2*x/(-b*x^2+a)^(4/3)+9/4*x/a/(-b*x^2+a)^(1/3)+9/4*x/a/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3/4*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+9/8*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.31, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {393, 205, 241, 310, 225, 1893}

$$-\frac{3\ 3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2\sqrt{2}\,a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]

[Out] (3*x)/(2*(a - b*x^2)^(4/3)) + (9*x)/(4*a*(a - b*x^2)^(1/3)) + (9*x)/(4*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (3*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^

```
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*Sqrt[2]*a^(2/3)*b*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \frac{3x}{2(a-bx^2)^{4/3}} + \frac{3}{2}\int \frac{1}{(a-bx^2)^{4/3}}\,dx$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} - \frac{3\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{4a}$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} + \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{8abx}$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} - \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right.}{8abx}$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} + \frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 74, normalized size = 0.13

$$\frac{15ax-9bx^3-3x(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{4a(a-bx^2)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

```
[Out] (15*a*x - 9*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1
[1/3, 1/2, 3/2, (b*x^2)/a])/(4*a*(a - b*x^2)^(4/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{b\,x^{2}+3a}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b
*x^2 - a^3), x)
```

Sympy [A]
time = 5.62, size = 60, normalized size = 0.10

$$\frac{3x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{4}{3}}} + \frac{bx^3\,{}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(7/3),x)
```

```
[Out] 3*x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(4/3) + b*x**3
*hyper((3/2, 7/3), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(7/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b\,x^2+3\,a}{{\left(a-b\,x^2\right)}^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a + b*x^2)/(a - b*x^2)^(7/3),x)
```

```
[Out] int((3*a + b*x^2)/(a - b*x^2)^(7/3), x)
```

3.139 $\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx$

Optimal. Leaf size=796

$$\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{32\ 2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{}$$

```
[Out] 3/32*x/a^2/(-b*x^2+a)^(4/3)+21/64*x/a^3/(-b*x^2+a)^(1/3)+21/64*x/a^3/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/64*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2
^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-1/192*arctanh(x*b^(1/2)/
a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+1/192*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b
*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+1/192*ar
ctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)-7/64*(a^(1
/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(
-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x
^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+21/128*(a^(1/3)-(-b*x^2+a)^(1/3)
)*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+
a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2
*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 544, 241, 310, 225, 1893, 402}

$$\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{32\ 2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{32\ 2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{21\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{128a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}-\frac{7\ 3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{32\sqrt{2}\,a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(32*a^2*(a - b*x^2)^(4/3)) + (21*x)/(64*a^3*(a - b*x^2)^(1/3)) + (21*
x)/(64*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(32*2^(2/3)*S
```

```
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(96*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(32*2^(2/3)*a^(17/6)*Sqrt[b]) + (21*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(128*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (7
*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(32*Sqrt[2]*a^(8/3)*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx = \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{3\int \frac{\frac{23ab}{3}+\frac{5b^2x^2}{3}}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx}{32a^2b}$$

$$= \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{9\int \frac{-\frac{68}{9}a^2b^2-\frac{28}{9}ab^3x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{256a^4b^2}$$

$$= \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} - \frac{7\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{64a^3} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a\cdots}}{16a^2}$$

$$= \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{32\ 2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\cdots}{\cdots}\right)}{32\cdots}$$

$$= \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{32\ 2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\cdots}{\cdots}\right)}{32\cdots}$$

$$= \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.70, size = 248, normalized size = 0.31

$$\frac{x\left(-7bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+27a\left(\frac{9a-7bx^2}{a-bx^2}-\frac{51a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}\right)\right)}{576a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*(-7*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1
/3*(b*x^2)/a] + 27*a*((9*a - 7*b*x^2)/(a - b*x^2) - (51*a^2*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a]))))))/(576*a^4*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}} \left(b\,x^{2}+3a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - bx^{2}\right)^{\frac{7}{3}} \cdot \left(3a + bx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a),x)
[Out] Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{7/3}\,\left(b\,x^2+3\,a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x)
```

```
[Out] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)), x)
```

3.140 $\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)^2}\,dx$

Optimal. Leaf size=827

$$\frac{5x}{384a^3\left(a-bx^2\right)^{4/3}}+\frac{79x}{768a^4\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}+\frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+$$

```
[Out] 5/384*x/a^3/(-b*x^2+a)^(4/3)+79/768*x/a^4/(-b*x^2+a)^(1/3)+1/24*x/a^2/(-b*x
^2+a)^(4/3)/(b*x^2+3*a)+79/768*x/a^4/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)+3/256*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/
3)/a^(23/6)/b^(1/2)-1/256*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(23/6)/b^(1/
2)+1/256*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2
))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)+1/256*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))
*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)-79/2304*(a^(1/3)-(-b*x^2+a)^(1/3))*Ellipt
icF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)/b/x*2^(
1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)+79/1536*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(
1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(11/3)
/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 827, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 544, 241, 310, 225, 1893, 402}

$$\frac{79x}{768a^4\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(bx^2+3a\right)}+\frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{5x}{384a^3\left(a-bx^2\right)^{4/3}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{79\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{512\,3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}-\frac{79\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{384\sqrt{2}\sqrt[4]{3}\,a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (5*x)/(384*a^3*(a - b*x^2)^(4/3)) + (79*x)/(768*a^4*(a - b*x^2)^(1/3)) + x/
(24*a^2*(a - b*x^2)^(4/3)*(3*a + b*x^2)) + (79*x)/(768*a^4*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) + (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x
```

```
)])/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (Sqrt[3]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1
/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[
b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (3*ArcT
anh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(128*2^(2
/3)*a^(23/6)*Sqrt[b]) + (79*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(512*3^(3/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (79*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(384*Sqrt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} - \frac{\int \frac{-7ab+\frac{11b^2x^2}{3}}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx}{24a^2b}$$

$$= \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} - \frac{\int \frac{-\frac{194}{3}a^2b^2-\frac{50}{9}ab^3x^2}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx}{256a^4b^2}$$

$$= \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} - \frac{3}{}$$

$$= \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} - \frac{79}{}$$

$$= \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} + \frac{\sqrt{}}{}$$

$$= \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} + \frac{\sqrt{}}{}$$

$$= \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} + \frac{}{76}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.18, size = 259, normalized size = 0.31

$$\frac{x\left(-79bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(\frac{299a^2-148abx^2-79b^2x^4}{a-bx^2}-\frac{387a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)}{3a+bx^2}\right)}{20736a^5\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(-79*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + (27*a*((299*a^2 - 148*a*b*x^2 - 79*b^2*x^4)/(a - b*x^2) -
```

```
(387*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*Appell
F1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)
/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2)))/(20736*a^5*(a - b*x^2)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}} \left(b\,x^{2}+3a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - bx^{2}\right)^{\frac{7}{3}} \left(3a + bx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{7/3}\,{\left(b\,x^2+3\,a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x)
[Out] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2), x)
```

3.141 $\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx$

Optimal. Leaf size=252

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,\sqrt[3]{-a}\,\sqrt{a}\,\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}\,\left(\sqrt[3]{-a}-\sqrt[3]{2}\,\sqrt[3]{-a+bx^2}\right)}{\sqrt[3]{-a}\,\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,\sqrt[3]{-a}\,\sqrt{a}\,\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\,\sqrt{a}\,\sqrt{b}}-\frac{\tanh^{-1}\Big(-$$

[Out] 1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/(-a)^(1/3)/a^(1/2)/b^(1/2)-1/4*arctanh((-a)^(1/3)*x*b^(1/2)/((-a)^(1/3)+2^(1/3)*(b*x^2-a)^(1/3))/a^(1/2))*2^(1/3)/(-a)^(1/3)/a^(1/2)/b^(1/2)-1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/(-a)^(1/3)*3^(1/2)/a^(1/2)/b^(1/2)-1/12*arctan(((-a)^(1/3)-2^(1/3)*(b*x^2-a)^(1/3))*3^(1/2)*a^(1/2)/(-a)^(1/3)/x/b^(1/2))*2^(1/3)/(-a)^(1/3)*3^(1/2)/a^(1/2)/b^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {402}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,\sqrt[3]{-a}\,\sqrt{a}\,\sqrt{b}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,\sqrt[3]{-a}\,\sqrt{a}\,\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{b}\,x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\,2^{2/3}\sqrt[3]{-a}\,\sqrt{a}\,\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\,\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]

[Out] -1/2*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1/3)*(-a + b*x^2)^(1/3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) - ArcTanh[((-a)^(1/3)*Sqrt[b]*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*x^2)^(1/3)))]/(2*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}{\sqrt[3]{-a}\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.18, size = 163, normalized size = 0.65

$$-\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{-a+bx^2}\,(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] (-9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((-a + b*x^2
)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + A
ppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b\,x^2-3a)\,(b\,x^2-a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-3*a)/(b*x**2-a)**(1/3),x)
```

```
[Out] -Integral(1/(3*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{1}{\left(b\,x^2-a\right)^{1/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)),x)
```

```
[Out] -int(1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)), x)
```

3.142 $\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=202

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\ 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}\,x}\right)}{2\ 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}$$

```
[Out] 1/4*arctan(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(b*x^2+a)^(1/3)))*2^(1/3)/a^(
5/6)/b^(1/2)-1/12*arctan(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)-1/12*ar
ctanh(a^(1/6)*(a^(1/3)-2^(1/3)*(b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/
a^(5/6)*3^(1/2)/b^(1/2)-1/12*arctanh(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(
5/6)*3^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {401}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}\,x}\right)}{2\ 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\ 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[b
]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sq
rt[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right.}{2\ 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.02, size = 166, normalized size = 0.82

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)}{(3a-bx^2)\sqrt[3]{a+bx^2}\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)+2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)/(3*a)])/((3*a - b*x
^2)*(a + b*x^2)^(1/3)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)
/(3*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)]
- AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b\,x^2+3a)\,(b\,x^2+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-3a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+3*a)/(b*x**2+a)**(1/3),x)
```

```
[Out] -Integral(1/(-3*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{1/3}\,\left(3\,a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)), x)
```

3.143 $\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$

Optimal. Leaf size=204

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{\ldots}\right.}{\sqrt{\ldots}}\right)}{2\,2^{2/3}c^{5/6}\ldots}$$

```
[Out] -1/4*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*(3*d*x^2+c)^(1/3))/x/d^(1/2))*2^(1/3)
/c^(5/6)/d^(1/2)-1/4*arctanh(1/x/d^(1/2)*c^(1/2))*2^(1/3)/c^(5/6)/d^(1/2)-1
/12*arctan(x*3^(1/2)*d^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)/d^(1/2)+1/4*a
rctan(x*3^(1/2)*d^(1/2)/c^(1/6)/(c^(1/3)+2^(1/3)*(3*d*x^2+c)^(1/3)))*3^(1/2
)*2^(1/3)/c^(5/6)/d^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {401}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d])
+ (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c + 3*d*
x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[Sqrt[c]/(Sqrt[d]*x)]/(
2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3*d*x
^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeqQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}} + \frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}}{2\,2^{2/3}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.15, size = 153, normalized size = 0.75

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)}{(c-dx^2)\sqrt[3]{c+3dx^2}\left(3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)+2dx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c])/((c - d*x^2)*(c
 + 3*d*x^2)^(1/3)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c]
+ 2*d*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*d*x^2)/c, (d*x^2)/c] - AppellF1[3
/2, 4/3, 1, 5/2, (-3*d*x^2)/c, (d*x^2)/c])))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(-d\,x^2+c)\,(3d\,x^2+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

```
[Out] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-c\sqrt[3]{c+3dx^2}+dx^2\sqrt[3]{c+3dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**2+c)/(3*d*x**2+c)**(1/3),x)
[Out] -Integral(1/(-c*(c + 3*d*x**2)**(1/3) + d*x**2*(c + 3*d*x**2)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-d\,x^2\right)\,{\left(3\,d\,x^2+c\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x)
[Out] int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)), x)
```

3.144 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\cdots\right.}\right.}{2\,2^{2/3}a^{5/6}\sqrt{b}\cdots}$$

```
[Out] 1/4*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a
^(5/6)/b^(1/2)-1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/12
*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/
3)/a^(5/6)*3^(1/2)/b^(1/2)+1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a
^(5/6)*3^(1/2)/b^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {402}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{2\,2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\ }}{\sqrt{\ }}\right.}{6\,2^{2/3}a^{5/6}\sqrt{\ }}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.03, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\,(3a+bx^2)\left(9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)
/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{3}}\left(b\,x^2+3a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a-bx^2}\cdot(3a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
```

```
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

3.145 $\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

```
[Out] 1/4*arctan(c^(1/6)*(c^(1/3)-2^(1/3)*(-3*d*x^2+c)^(1/3))/x/d^(1/2))*2^(1/3)/
c^(5/6)/d^(1/2)+1/4*arctan(1/x/d^(1/2)*c^(1/2))*2^(1/3)/c^(5/6)/d^(1/2)-1/1
2*arctanh(x*3^(1/2)*d^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)/d^(1/2)+1/4*ar
ctanh(x*3^(1/2)*d^(1/2)/c^(1/6)/(c^(1/3)+2^(1/3)*(-3*d*x^2+c)^(1/3)))*3^(1/
2)*2^(1/3)/c^(5/6)/d^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {402}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\text{ArcTan}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[(c^(1/6)*(
c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqr
t[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqr
t[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c
 - 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}\,x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}}{\sqrt{c}}\right.}{2\,2^{2/3}\sqrt{3}\,c^{5/6}\sqrt{}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.16, size = 156, normalized size = 0.76

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)}{\sqrt[3]{c-3dx^2}\,(c+dx^2)\left(3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)+2dx^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/c)])/((c - 3*d*x^2
)^(1/3)*(c + d*x^2)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/
c)] + 2*d*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (3*d*x^2)/c, -((d*x^2)/c)] + App
ellF1[3/2, 4/3, 1, 5/2, (3*d*x^2)/c, -((d*x^2)/c)])))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3d\,x^2+c\right)^{\frac{1}{3}}\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

```
[Out] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x**2+c)**(1/3)/(d*x**2+c),x)
```

```
[Out] Integral(1/((c - 3*d*x**2)**(1/3)*(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x^2+c)\,{(c-3\,d\,x^2)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)),x)
```

```
[Out] int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)), x)
```

3.146 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1
/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{402\}$

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.63, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 20.49, size = 938, normalized size = 8.30

method	result	size
trager	Expression too large to display	938

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/432*ln((RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5+225*RootOf(_Z^6+
108)^4*x^4-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+72*RootOf(_Z^6+108)^4*x
^3+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-189*RootOf(_Z^6+108)^4*x^2-864
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3+648*(-x^2+1)^(2/3)*x^4-648*RootOf(_Z
^6+108)^2*(-x^2+1)^(1/3)*x^2-4536*(-x^2+1)^(2/3)*x^3+27*RootOf(_Z^6+108)^4+
324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+1944*(-x^2+1)^(2/3)*x^2+1944*(-x^2+
1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+108)^4+1/72*ln((RootOf(_Z^6+108)^4*x^6-7
2*RootOf(_Z^6+108)^4*x^5+225*RootOf(_Z^6+108)^4*x^4-36*RootOf(_Z^6+108)^2*(
-x^2+1)^(1/3)*x^5+72*RootOf(_Z^6+108)^4*x^3+648*RootOf(_Z^6+108)^2*(-x^2+1)
^(1/3)*x^4-189*RootOf(_Z^6+108)^4*x^2-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)
*x^3+648*(-x^2+1)^(2/3)*x^4-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2-4536*
(-x^2+1)^(2/3)*x^3+27*RootOf(_Z^6+108)^4+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1
/3)*x+1944*(-x^2+1)^(2/3)*x^2+1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6
+108)+1/36*RootOf(_Z^6+108)*ln(-(-1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2+1)^(2/
3)*x^3-3888*(-x^2+1)^(2/3)*x^2-3888*(-x^2+1)^(2/3)*x-189*RootOf(_Z^6+108)^4
*x^2+3402*RootOf(_Z^6+108)*x^2+72*RootOf(_Z^6+108)^4*x^3-1296*RootOf(_Z^6+1
08)*x^3+RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5+225*RootOf(_Z^6+10
8)^4*x^4-18*RootOf(_Z^6+108)*x^6+1296*RootOf(_Z^6+108)*x^5-4050*RootOf(_Z^6
+108)*x^4-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+648*RootOf(_Z^6+108)^2*(
-x^2+1)^(1/3)*x^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-648*RootOf(_Z^6
+108)^2*(-x^2+1)^(1/3)*x^2+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+6*RootOf
```

```
(_Z^6+108)^5*(-x^2+1)^(1/3)*x^5-108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4+1
44*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^3+108*RootOf(_Z^6+108)^5*(-x^2+1)^(1
/3)*x^2-54*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x+27*RootOf(_Z^6+108)^4-486*Ro
otOf(_Z^6+108))/(x^2+3)^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. 2(81) = 162.
time = 1.79, size = 1943, normalized size = 17.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
```

```
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.147 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/1
2*arctanh(1/x*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {401}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\text{ArcTan}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx=-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{\ }\right.}{\ }\right.}{2\ 2^{2/3}\sqrt{\ }}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.84, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Maple [F]
time = 83.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^2+3)(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. $2(77) = 154$.
time = 1.43, size = 1685, normalized size = 15.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
```

6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
+ 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
- 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
x^2 + 27) - 864(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/

```
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^2+1\right)^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.148 $\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=96

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log\left(3+x^2\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x}+(1+x)^{2/3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(x^2+3)*2^(1/3)+3/4*ln(2^(1/3)*(1-x)^(1/3)+(1+x)^(2/3))*2^(1/3)+1/2*
arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1+x)^(2/3)/(1-x)^(1/3)*3^(1/2))*3^(1/2)*2^
(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1022}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1 - x)^(1/3
))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/3)*(1 - x)^(1/3) + (
1 + x)^(2/3)])/(2*2^(2/3))
```

Rule 1022

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log\left(3+x^2\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x}+(1+\right.}{2\ 2^{2/3}}$$

Mathematica [A]
time = 0.23, size = 155, normalized size = 1.61

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{-2^{2/3}-2^{2/3}x+\sqrt[3]{1-x^2}}\right)+2\log\left(2^{2/3}+2^{2/3}x+2\sqrt[3]{1-x^2}\right)-\log\left(-\sqrt[3]{2}-2\sqrt[3]{2}x-\sqrt[3]{2}x^2+2^{2/3}(1+x)\sqrt[3]{1-x^2}-2(1-x^2)^{2/3}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) - 2^(2/3)*x + (1 - x^
2)^(1/3))] + 2*Log[2^(2/3) + 2^(2/3)*x + 2*(1 - x^2)^(1/3)] - Log[-2^(1/3)
- 2*2^(1/3)*x - 2^(1/3)*x^2 + 2^(2/3)*(1 + x)*(1 - x^2)^(1/3) - 2*(1 - x^2)
^(2/3)])/(2*2^(2/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.69, size = 1033, normalized size = 10.76

method	result	size
trager	Expression too large to display	1033

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(4*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+6*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2+12*RootOf(RootOf
(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+18*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x-18*RootOf(RootOf
(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(-x^2+1)^(2/3)-6*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(-x^2+1)^(
1/3)*x-9*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)*x-6*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(-x^2+1)^(1/3)+2*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-9*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)+3*Root
Of(_Z^3-2)*x^2-12*(-x^2+1)^(2/3)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+4*_Z^2)+9*RootOf(_Z^3-2))/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x+3)/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x-3))+1/2*RootOf(_Z^3-2)*ln((12*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2+36*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+6*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x-18*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(-x^2+1)^(2/3)
-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(-x^
2+1)^(1/3)*x-9*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)*x-12*RootOf(RootOf(_Z^3-2)^2
+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(-x^2+1)^(1/3)+6*RootOf(RootOf(
```

```
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-9*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)
+RootOf(_Z^3-2)*x^2+36*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*
x+6*RootOf(_Z^3-2)*x-6*(-x^2+1)^(2/3)-18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)-3*RootOf(_Z^3-2))/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x+3)/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x-3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(70) = 140$.
time = 10.19, size = 285, normalized size = 2.97

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(x^4+3x^3+3x^2+9x)(-x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^6-18x^5-117x^4-36x^3+207x^2+54x-27)+12(x^5+19x^4+42x^3+6x^2-27x-9)(-x^2+1)^{\frac{1}{3}}\right)}{6(x^6+54x^5+171x^4+108x^3-81x^2-162x-27)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{6\cdot 4^{\frac{2}{3}}(x^2+3x)(-x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^4+18x^3+24x^2-18x-9)-6(x^3+7x^2+3x-3)(-x^2+1)^{\frac{1}{3}}}{x^4+6x^2+9}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(x^2+3)+6\cdot 4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}(x+1)+12(-x^2+1)^{\frac{2}{3}}}{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(x^4 + 3*x^3 +
3*x^2 + 9*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^6 - 18*x^5 - 117*x^4 - 36*x^3 +
207*x^2 + 54*x - 27) + 12*(x^5 + 19*x^4 + 42*x^3 + 6*x^2 - 27*x - 9)*(-x^2
+ 1)^(1/3))/(x^6 + 54*x^5 + 171*x^4 + 108*x^3 - 81*x^2 - 162*x - 27)) - 1/2
4*4^(2/3)*log((6*4^(2/3)*(x^2 + 3*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^4 + 18*x
^3 + 24*x^2 - 18*x - 9) - 6*(x^3 + 7*x^2 + 3*x - 3)*(-x^2 + 1)^(1/3))/(x^4
+ 6*x^2 + 9)) + 1/12*4^(2/3)*log((4^(2/3)*(x^2 + 3) + 6*4^(1/3)*(-x^2 + 1)^
(1/3)*(x + 1) + 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\,dx-\int \left(-\frac{3}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x) - Integral(-
3/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x-3}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] -int((x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.149 $\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\,\sqrt[3]{1+x}}\right)}{2^{2/3}}+\frac{\log\left(3+x^2\right)}{2\,2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\,\sqrt[3]{1+x}\right)}{2\,2^{2/3}}$$

```
[Out] 1/4*ln(x^2+3)*2^(1/3)-3/4*ln((1-x)^(2/3)+2^(1/3)*(1+x)^(1/3))*2^(1/3)-1/2*a
rctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-x)^(2/3)/(1+x)^(1/3)*3^(1/2))*3^(1/2)*2^(
1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1022}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\,\sqrt[3]{x+1}}\right)}{2^{2/3}}+\frac{\log\left(x^2+3\right)}{2\,2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\,\sqrt[3]{x+1}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 + x)^(1/3))
])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^(2/3) + 2^(1/3)*(1 +
 x)^(1/3)])/(2*2^(2/3))
```

Rule 1022

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,\left(3+x^2\right)}\,dx=\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\,\sqrt[3]{1+x}}\right)}{2^{2/3}}+\frac{\log\left(3+x^2\right)}{2\,2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\,\sqrt[3]{1+\right.}{2\,2^{2/3}}$$

Mathematica [A]
time = 0.20, size = 151, normalized size = 1.59

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{-2^{2/3}+2^{2/3}x+\sqrt[3]{1-x^2}}\right)-2\log\left(-2^{2/3}+2^{2/3}x-2\sqrt[3]{1-x^2}\right)+\log\left(\sqrt[3]{2}-2\sqrt[3]{2}x+\sqrt[3]{2}x^2+2^{2/3}(-1+x)\sqrt[3]{1-x^2}+2(1-x^2)^{2/3}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) + 2^(2/3)*x + (1 - x
^2)^(1/3))] - 2*Log[-2^(2/3) + 2^(2/3)*x - 2*(1 - x^2)^(1/3)] + Log[2^(1/3)
 - 2*2^(1/3)*x + 2^(1/3)*x^2 + 2^(2/3)*(-1 + x)*(1 - x^2)^(1/3) + 2*(1 - x^
2)^(2/3)])/(2*2^(2/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.49, size = 1042, normalized size = 10.97

method	result	size
trager	Expression too large to display	1042

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*ln(-(4*RootOf(RootOf(_Z
^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2+6*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2-12*RootOf(RootOf
(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x-18*RootOf(RootO
f(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x+18*RootOf(_Z^3+2
)^2*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+6*Ro
otOf(_Z^3+2)*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_
Z^2)*x+9*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)*x-6*RootOf(_Z^3+2)*(-x^2+1)^(1/3)*
RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-2*RootOf(RootOf(_Z^3+2)
^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2-9*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)-3*Root
Of(_Z^3+2)*x^2-12*(-x^2+1)^(2/3)-6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3
+2)+4*_Z^2)-9*RootOf(_Z^3+2))/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2
)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3)/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z
^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3))+1/2*RootOf(_Z^3+2)*ln((12*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2+2*RootOf(R
ootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2-36*RootOf(
RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x-6*RootOf(
RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x+18*RootOf(_
Z^3+2)^2*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)
+12*RootOf(_Z^3+2)*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+
2)+4*_Z^2)*x+9*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)*x-12*RootOf(_Z^3+2)*(-x^2+1)
^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-6*RootOf(RootOf(
```

```
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2-9*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)
-RootOf(_Z^3+2)*x^2+36*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*
x+6*RootOf(_Z^3+2)*x-6*(-x^2+1)^(2/3)+18*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)+3*RootOf(_Z^3+2))/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(
_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3)/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*Roo
tOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(70) = 140.
time = 9.40, size = 315, normalized size = 3.32

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}(x^4-3x^3+3x^2-9x)(-x^2+1)^{\frac{2}{3}}+12(-1)^{\frac{1}{3}}(x^5-19x^4+42x^3-6x^2-27x+9)(-x^2+1)^{\frac{1}{3}}+4^{\frac{1}{3}}(x^6+18x^5-117x^4+36x^3+207x^2-54x-27)\right)}{6(x^6-54x^5+171x^4-108x^3-81x^2+162x-27)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^2-3x)(-x^2+1)^{\frac{2}{3}}-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^4-18x^3+24x^2+18x-9)-6(x^3-7x^2+3x+3)(-x^2+1)^{\frac{1}{3}}}{x^4+6x^2+9}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}(x-1)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^2+3)-12(-x^2+1)^{\frac{2}{3}}}{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(-1)
^(2/3)*(x^4 - 3*x^3 + 3*x^2 - 9*x)*(-x^2 + 1)^(2/3) + 12*(-1)^(1/3)*(x^5 -
19*x^4 + 42*x^3 - 6*x^2 - 27*x + 9)*(-x^2 + 1)^(1/3) + 4^(1/3)*(x^6 + 18*x^
5 - 117*x^4 + 36*x^3 + 207*x^2 - 54*x - 27))/(x^6 - 54*x^5 + 171*x^4 - 108*
x^3 - 81*x^2 + 162*x - 27)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-(6*4^(2/3)*(-1)^
(1/3)*(x^2 - 3*x)*(-x^2 + 1)^(2/3) - 4^(1/3)*(-1)^(2/3)*(x^4 - 18*x^3 + 24*
x^2 + 18*x - 9) - 6*(x^3 - 7*x^2 + 3*x + 3)*(-x^2 + 1)^(1/3))/(x^4 + 6*x^2
+ 9)) + 1/12*4^(2/3)*(-1)^(1/3)*log(-(6*4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3)
*(x - 1) + 4^(2/3)*(-1)^(1/3)*(x^2 + 3) - 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((x + 3)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+3}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.150 $\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\,\sqrt{b}\,x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}$$

```
[Out] 1/12*arctan(1/3*(a^(1/3)-(b*x^2+a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^(5
/6)/d+1/12*arctan(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d-1/12*arctanh(a^(
1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {403}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\,\sqrt{b}\,x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTan[
(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) - (S
qrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*Sqrt[3]*a^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.84, size = 169, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{a+bx^2}\left(9a+bx^2\right)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(a +
 b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
-1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/9*(b
*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}\left(\frac{9ad}{b}+d\,x^2\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{1}{9a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(9*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)
```

3.151 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\,\sqrt{b}\,x}\right)}{12a^{5/6}d}$$

```
[Out] 1/12*arctanh(1/3*(a^(1/3)-(-b*x^2+a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^
(5/6)/d-1/12*arctanh(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d-1/12*arctan(a
^(1/6)*(a^(1/3)-(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{404\}$

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\,\sqrt{b}\,x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] -1/4*(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/(Sqrt[
b]*x)])/(Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(1
2*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a - b*x^2)^(1/3))^2/(3*a^(1/6)*
Sqrt[b]*x)])/(12*a^(5/6)*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.93, size = 167, normalized size = 1.09

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\sqrt[3]{a-bx^2}\,(9a-bx^2)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(a - b*
x^2)^(1/3)*(9*a - b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2
)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{3}}\left(-\frac{9ad}{b}+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{1}{-9a\sqrt[3]{a-bx^2}+bx^2\sqrt[3]{a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(-9*a*d/b+d*x**2),x)
[Out] b*Integral(1/(-9*a*(a - b*x**2)**(1/3) + b*x**2*(a - b*x**2)**(1/3)), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)),x)
[Out] int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)), x)
```

3.152 $\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}\,x}\right)}{12a^{5/6}d}$$

```
[Out] -1/12*arctanh(1/3*(a^(1/3)+(b*x^2-a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^
(5/6)/d+1/12*arctanh(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d+1/12*arctan(a
^(1/6)*(a^(1/3)+(b*x^2-a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {404}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}\,x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a + b*x^2)^(1/3))^2/(3*a^(1/6)*S
qrt[b]*x)])/(12*a^(5/6)*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d} - -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.93, size = 168, normalized size = 1.11

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\left(9a-bx^2\right)\sqrt[3]{-a+bx^2}\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(9*a -
b*x^2)*(-a + b*x^2)^(1/3)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^
2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2-a\right)^{\frac{1}{3}}\left(-\frac{9ad}{b}+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{1}{-9a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(-9*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)/
d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(b\,x^2-a\right)}^{1/3}\,\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)),x)
```

```
[Out] int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)), x)
```

3.153 $\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}\,x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}$$

```
[Out] -1/12*arctan(1/3*(a^(1/3)+(-b*x^2-a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^
(5/6)/d-1/12*arctan(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d+1/12*arctanh(a
^(1/6)*(a^(1/3)+(-b*x^2-a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {403}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}\,x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] -1/12*(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(a^(5/6)*d) - (Sqrt[b]*ArcT
an[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d)
+ (Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a - b*x^2)^(1/3)))/(Sqrt[b
]*x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{a}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)^2}{3\sqrt[6]{a}\,\sqrt{b}\,x}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\,\tanh^{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.91, size = 172, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{-a-bx^2}\,(9a+bx^2)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(-a
- b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
 -1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/9*(
b*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2-a\right)^{\frac{1}{3}}\left(\frac{9ad}{b}+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{1}{9a\sqrt[3]{-a-bx^2}+bx^2\sqrt[3]{-a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/3)/(9*a*d/b+d*x**2),x)
[Out] b*Integral(1/(9*a*(-a - b*x**2)**(1/3) + b*x**2*(-a - b*x**2)**(1/3)), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(-b\,x^2-a\right)}^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)
[Out] int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)
```

3.154 $\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}+\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)^2}{3\sqrt[6]{2}\,\sqrt{b}\,x}\right)}{12\,2^{5/6}d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{2}\,\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d}$$

```
[Out] 1/24*arctan(1/6*(2^(1/3)-(b*x^2+2)^(1/3))^2*2^(5/6)/x/b^(1/2))*b^(1/2)*2^(1
/6)/d+1/24*arctan(1/6*x*b^(1/2)*2^(1/2))*b^(1/2)*2^(1/6)/d-1/24*arctanh(2^(
1/6)*(2^(1/3)-(b*x^2+2)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)*2^(1/6)/d*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {403}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\,\sqrt{b}\,x}\right)}{12\,2^{5/6}d}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{2}\,\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[b]*ArcTan[
(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d) - (S
qrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2 + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{2}}\right)}{12\;2^{5/6}d} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}\,x}\right)}{12\;2^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\cdots\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.29, size = 148, normalized size = 0.98

$$-\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)}{d\sqrt[3]{2+bx^2}\,(18+bx^2)\left(-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (-27*b*x*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1/18*(b*x^2)])/(d*(2 + b
*x^2)^(1/3)*(18 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1/1
8*(b*x^2)] + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -1/2*(b*x^2), -1/18*(b*x^2)]
 + 3*AppellF1[3/2, 4/3, 1, 5/2, -1/2*(b*x^2), -1/18*(b*x^2)])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+2\right)^{\frac{1}{3}}\left(\frac{18d}{b}+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2+2}+18\sqrt[3]{bx^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/3)/(18*d/b+d*x**2),x)
[Out] b*Integral(1/(b*x**2*(b*x**2 + 2)**(1/3) + 18*(b*x**2 + 2)**(1/3)), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{18\,d}{b}+d\,x^2\right)\,{\left(b\,x^2+2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)),x)
[Out] int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)), x)
```

3.155 $\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}\,x}\right)}{12\,2^{5/6}d}$$

```
[Out] -1/24*arctanh(1/6*(2^(1/3)+(b*x^2-2)^(1/3))^2*2^(5/6)/x/b^(1/2))*b^(1/2)*2^
(1/6)/d+1/24*arctanh(1/6*x*b^(1/2)*2^(1/2))*b^(1/2)*2^(1/6)/d+1/24*arctan(2
^(1/6)*(2^(1/3)+(b*x^2-2)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)*2^(1/6)/d*3^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {404}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}\,x}\right)}{12\,2^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*
2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2 + b*x^2)^(1/3))^2/(3*2^(1/6)*S
qrt[b]*x)])/(12*2^(5/6)*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{3\sqrt{2}}\right)}{12\,2^{5/6}d} - \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 5.66, size = 148, normalized size = 1.01

$$\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)}{d\left(-18+bx^2\right)\sqrt[3]{-2+bx^2}\left(27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)+bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (27*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18])/(d*(-18 + b*x^2)
*(-2 + b*x^2)^(1/3)*(27*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18] +
 b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/2, (b*x^2)/18] + 3*AppellF1[3/2,
 4/3, 1, 5/2, (b*x^2)/2, (b*x^2)/18])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2-2\right)^{\frac{1}{3}}\left(-\frac{18d}{b}+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{1}{bx^2\sqrt[3]{bx^2-2}-18\sqrt[3]{bx^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2)**(1/3)/(-18*d/b+d*x**2),x)
[Out] b*Integral(1/(b*x**2*(b*x**2 - 2)**(1/3) - 18*(b*x**2 - 2)**(1/3)), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{1}{\left(\frac{18\,d}{b}-d\,x^2\right)\,{\left(b\,x^2-2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)),x)
[Out] int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)), x)
```

3.156 $\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx$

Optimal. Leaf size=123

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}\,d}+\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)^2}{3\sqrt[6]{2}\,\sqrt{3}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)}{x}\right)}{4\,2^{5/6}d}$$

```
[Out] -1/8*arctanh(2^(1/6)*(2^(1/3)-(3*x^2+2)^(1/3))/x)*2^(1/6)/d+1/24*arctan(1/1
8*(2^(1/3)-(3*x^2+2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)+1/24*arc
tan(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {403}

$$\frac{\text{ArcTan}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\,\sqrt{3}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d}+\frac{\text{ArcTan}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\,2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 + 3*x^2)^(1/
3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/6)*(2^(1
/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}\,d}+\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)^2}{3\sqrt[6]{2}\,\sqrt{3}\,x}\right)}{4\,2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)}{x}\right)}{4\,2^{5/6}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 5.29, size = 136, normalized size = 1.11

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\left(6+x^2\right)\sqrt[3]{2+3x^2}\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(6 + x^2)*(2 + 3
*x^2)^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(App
ellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2
, (-3*x^2)/2, -1/6*x^2])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 76.53, size = 548, normalized size = 4.46

method	result	size
trager	Expression too large to display	548

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(RootOf(_Z^6+54)*ln(-(16*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+5
4)+576*_Z^2)*RootOf(_Z^6+54)^6*x-768*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(
_Z^6+54)+576*_Z^2)^2*RootOf(_Z^6+54)^5*x-RootOf(_Z^6+54)^5*(3*x^2+2)^(1/3)*
x+72*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+5
4)^4*(3*x^2+2)^(1/3)*x-1152*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+
576*_Z^2)^2*RootOf(_Z^6+54)^3*(3*x^2+2)^(1/3)*x-36*RootOf(RootOf(_Z^6+54)^2
-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3*x^2+72*RootOf(RootOf(_Z^
6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+18*RootOf(_Z^6+54
)^2*(3*x^2+2)^(1/3)-432*(3*x^2+2)^(1/3)*RootOf(RootOf(_Z^6+54)^2-24*_Z*Root
Of(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)+54*(3*x^2+2)^(2/3))/(x^2+6))+24*RootO
f(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*ln((4*RootOf(_Z^6+54)^7
*x-192*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6
+54)^6*x-6*RootOf(_Z^6+54)^5*(3*x^2+2)^(1/3)*x+288*RootOf(RootOf(_Z^6+54)^2
-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^4*(3*x^2+2)^(1/3)*x-9*x^2*
RootOf(_Z^6+54)^4+18*RootOf(_Z^6+54)^4+108*RootOf(_Z^6+54)^2*(3*x^2+2)^(1/3
)-324*(3*x^2+2)^(2/3))/(x^2+6)))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2961 vs. $2(90) = 180$.
time = 51.46, size = 2961, normalized size = 24.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

```
[Out] 1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*log(1/24*(5184*sqrt(3)*(1/864)^(5
/6)*(3*d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(5/6) + 12*(32*sqrt(3)*(1/
864)^(1/6)*d*(d^(-6))^(1/6)*x^3 + 3*(1/4)^(2/3)*(d^4*x^4 + 28*d^4*x^2 + 4*d
^4)*(d^(-6))^(2/3))*(3*x^2 + 2)^(2/3) + (1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4
 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^(1/3) + 6*(27*x^4 + sqrt(3)*sqrt(1/6)*(d^
3*x^5 + 140*d^3*x^3 + 36*d^3*x)*sqrt(d^(-6)) + 36*x^2 + 12)*(3*x^2 + 2)^(1/
3))/(x^6 + 18*x^4 + 108*x^2 + 216)) + 1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(
1/6)*log(1/96*(5184*sqrt(3)*(1/864)^(5/6)*(3*d^5*x^5 + 20*d^5*x^3 + 12*d^5*
x)*(d^(-6))^(5/6) + 12*(32*sqrt(3)*(1/864)^(1/6)*d*(d^(-6))^(1/6)*x^3 + 3*(
1/4)^(2/3)*(d^4*x^4 + 28*d^4*x^2 + 4*d^4)*(d^(-6))^(2/3))*(3*x^2 + 2)^(2/3)
 + (1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^(1/
3) + 6*(27*x^4 + sqrt(3)*sqrt(1/6)*(d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*sqrt(
d^(-6)) + 36*x^2 + 12)*(3*x^2 + 2)^(1/3))/(x^6 + 18*x^4 + 108*x^2 + 216)) -
 1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*log(-1/96*(5184*sqrt(3)*(1/864)^
(5/6)*(3*d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(5/6) + 12*(32*sqrt(3)*(
1/864)^(1/6)*d*(d^(-6))^(1/6)*x^3 - 3*(1/4)^(2/3)*(d^4*x^4 + 28*d^4*x^2 + 4
*d^4)*(d^(-6))^(2/3))*(3*x^2 + 2)^(2/3) - (1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x
^4 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^(1/3) - 6*(27*x^4 - sqrt(3)*sqrt(1/6)*(
d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*sqrt(d^(-6)) + 36*x^2 + 12)*(3*x^2 + 2)^(
1/3))/(x^6 + 18*x^4 + 108*x^2 + 216)) - 1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))
^(1/6)*log(-1/24*(5184*sqrt(3)*(1/864)^(5/6)*(3*d^5*x^5 + 20*d^5*x^3 + 12*d
^5*x)*(d^(-6))^(5/6) + 12*(32*sqrt(3)*(1/864)^(1/6)*d*(d^(-6))^(1/6)*x^3 -
3*(1/4)^(2/3)*(d^4*x^4 + 28*d^4*x^2 + 4*d^4)*(d^(-6))^(2/3))*(3*x^2 + 2)^(2
/3) - (1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^
(1/3) - 6*(27*x^4 - sqrt(3)*sqrt(1/6)*(d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*sq
rt(d^(-6)) + 36*x^2 + 12)*(3*x^2 + 2)^(1/3))/(x^6 + 18*x^4 + 108*x^2 + 216)
) - 1/6*(1/864)^(1/6)*(d^(-6))^(1/6)*arctan(12*(288*(1/864)^(5/6)*(23*d^5*x
^3 + 18*d^5*x)*(3*x^2 + 2)^(2/3)*(d^(-6))^(5/6) + 6*sqrt(1/6)*sqrt((1/4)^(1
/3)*d^2*(d^(-6))^(1/3))*(12*(1/864)^(5/6)*(d^5*x^6 + 642*d^5*x^4 - 36*d^5*x
^2 - 72*d^5)*(d^(-6))^(5/6) + sqrt(1/6)*(13*d^3*x^4 + 12*d^3)*(3*x^2 + 2)^(
1/3)*sqrt(d^(-6)) + (1/864)^(1/6)*(d*x^4 + 48*d*x^2 + 12*d)*(3*x^2 + 2)^(2/
```

```
3)*(d^(-6))^(1/6)) + sqrt(1/6)*(11*d^3*x^5 + 20*d^3*x^3 - 36*d^3*x)*sqrt(d^
(-6)) + (1/864)^(1/6)*(d*x^5 - 160*d*x^3 - 36*d*x)*(3*x^2 + 2)^(1/3)*(d^(-6
))^(1/6))/(x^6 - 1314*x^4 - 756*x^2 - 216)) + 1/12*(1/864)^(1/6)*(d^(-6))^(
1/6)*arctan(-(24*sqrt(1/6)*(72*(1/864)^(5/6)*(d^5*x^12 + 33744*d^5*x^10 - 8
30844*d^5*x^8 - 1118016*d^5*x^6 - 1364688*d^5*x^4 + 139968*d^5)*(d^(-6))^(5
/6) - 2*sqrt(3)*(1/4)^(1/3)*(43*d^2*x^11 + 9370*d^2*x^9 - 250632*d^2*x^7 -
387504*d^2*x^5 - 177552*d^2*x^3 + 23328*d^2*x)*(d^(-6))^(1/3) - 24*(8*sqrt(
3)*(1/4)^(2/3)*(11*d^4*x^9 - 174*d^4*x^7 - 2700*d^4*x^5 - 648*d^4*x^3)*(d^(
-6))^(2/3) - (1/864)^(1/6)*(d*x^10 + 1080*d*x^8 - 28296*d*x^6 - 12960*d*x^4
 + 11664*d*x^2)*(d^(-6))^(1/6))*(3*x^2 + 2)^(2/3) + (3*x^2 + 2)^(1/3)*(6*sq
rt(1/6)*(151*d^3*x^10 + 4698*d^3*x^8 - 219816*d^3*x^6 - 138672*d^3*x^4 - 58
320*d^3*x^2 + 23328*d^3)*sqrt(d^(-6)) - sqrt(3)*(x^11 + 5990*x^9 - 155160*x
^7 + 34992*x^5 + 6480*x^3 - 23328*x)))*sqrt((5184*sqrt(3)*(1/864)^(5/6)*(3*
d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(5/6) + 12*(32*sqrt(3)*(1/864)^(1
/6)*d*(d^(-6))^(1/6)*x^3 + 3*(1/4)^(2/3)*(d^4*x^4 + 28*d^4*x^2 + 4*d^4)*(d^
(-6))^(2/3))*(3*x^2 + 2)^(2/3) + (1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*
d^2*x^2 - 72*d^2)*(d^(-6))^(1/3) + 6*(27*x^4 + sqrt(3)*sqrt(1/6)*(d^3*x^5 +
 140*d^3*x^3 + 36*d^3*x)*sqrt(d^(-6)) + 36*x^2 + 12)*(3*x^2 + 2)^(1/3))/(x^
6 + 18*x^4 + 108*x^2 + 216)) + 192*sqrt(1/6)*(4*d^3*x^11 - 905*d^3*x^9 + 11
016*d^3*x^7 - 1944*d^3*x^5 - 2592*d^3*x^3 - 11664*d^3*x)*sqrt(d^(-6)) + 24*
(576*(1/864)^(5/6)*(97*d^5*x^9 - 4644*d^5*x^7 + 33696*d^5*x^5 + 1296*d^5*x^
3 - 11664*d^5*x)*(d^(-6))^(5/6) - sqrt(3)*(1/4)^(1/3)*(d^2*x^10 + 366*d^2*x
^8 - 3576*d^2*x^6 + 38448*d^2*x^4 - 3888*d^2*x^2 + 7776*d^2)*(d^(-6))^(1/3)
)*(3*x^2 + 2)^(2/3) - sqrt(3)*(x^12 - 156*x^10 + 101916*x^8 - 359712*x^6 +
1346544*x^4 + 793152*x^2 + 46656) - 24*(2*sqrt(3)*(1/4)^(2/3)*(37*d^4*x^10
- 450*d^4*x^8 + 15432*d^4*x^6 - 28944*d^4*x^4 + 11664*d^4*x^2 + 7776*d^4)*(
d^(-6))^(2/3) - (1/864)^(1/6)*(d*x^11 - 4282*d*x^9 + 100008*d*x^7 - 327888*
d*x^5 - 304560*d*x^3 - 23328*d*x)*(d^(-6))^(1/6))*(3*x^2 + 2)^(1/3))/(x^12
- 32652*x^10 + 1036476*x^8 - 4865184*x^6 - 4770576*x^4 - 1259712*x^2 + 4199
04)) - 1/12*(1/864)^(1/6)*(d^(-6))^(1/6)*arctan((24*sqrt(1/6)*(72*(1/864)^(
5/6)*(d^5*x^12 + 33744*d^5*x^10 - 830844*d^5*x^8 - 1118016*d^5*x^6 - 136468
8*d^5*x^4 + 139968*d^5)*(d^(-6))^(5/6) + 2*sqrt(3)*(1/4)^(1/3)*(43*d^2*x^11
 + 9370*d^2*x^9 - 250632*d^2*x^7 - 387504*d^2*x^5 - 177552*d^2*x^3 + 23328*
d^2*x)*(d^(-6))^(1/3) + 24*(8*sqrt(3)*(1/4)^(2/3)*(11*d^4*x^9 - 174*d^4*x^7
 - 2700*d^4*x^5 - 648*d^4*x^3)*(d^(-6))^(2/3) + (1/864)^(1/6)*(d*x^10 + 108
0*d*x^8 - 28296*d*x^6 - 12960*d*x^4 + 11664*d*x...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt[3]{3x^2+2}+6\sqrt[3]{3x^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 + 2)**(1/3) + 6*(3*x**2 + 2)**(1/3)), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(3\,x^2+2\right)}^{1/3}\,\left(d\,x^2+6\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)),x)
[Out] int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)), x)
```

3.157 $\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx$

Optimal. Leaf size=123

$$-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}$$

```
[Out] -1/8*arctan(2^(1/6)*(2^(1/3)-(-3*x^2+2)^(1/3))/x)*2^(1/6)/d+1/24*arctanh(1/
18*(2^(1/3)-(-3*x^2+2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)-1/24*a
rctanh(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {404}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] -1/4*ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(2^(5/6)*d) - ArcTan
h[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (2 - 3*x^2)^(1/3))^
2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.86, size = 136, normalized size = 1.11

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\sqrt[3]{2-3x^2}\,(-6+x^2)\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(2 - 3*x^2)^(1/3)*(-6
 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/2
, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
x^2/6])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 72.68, size = 1064, normalized size = 8.65

method	result	size
trager	Expression too large to display	1064

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(ln((16*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*Root
Of(_Z^6-54)^6*x+768*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2
)^2*RootOf(_Z^6-54)^5*x-(-3*x^2+2)^(1/3)*RootOf(_Z^6-54)^5*x-72*RootOf(Root
Of(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^4*(-3*x^2+2)^
(1/3)*x-1152*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*Roo
tOf(_Z^6-54)^3*(-3*x^2+2)^(1/3)*x-36*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-5
4)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*x^2-72*RootOf(RootOf(_Z^6-54)^2+24*_Z*
RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3-18*(-3*x^2+2)^(1/3)*RootOf(_Z^6
-54)^2-432*(-3*x^2+2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+
576*_Z^2)*RootOf(_Z^6-54)+54*(-3*x^2+2)^(2/3))/(x^2-6))*RootOf(_Z^6-54)+24*
RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*ln(-(4*RootOf(_Z^6
-54)^7*x+288*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootO
f(_Z^6-54)^6*x+4608*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2
)^2*RootOf(_Z^6-54)^5*x-144*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+
576*_Z^2)*RootOf(_Z^6-54)^4*(-3*x^2+2)^(1/3)*x-6912*RootOf(RootOf(_Z^6-54)^
2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^3*(-3*x^2+2)^(1/3)*x-9*
x^2*RootOf(_Z^6-54)^4-216*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2+24*_Z*
RootOf(_Z^6-54)+576*_Z^2)*x^2-18*RootOf(_Z^6-54)^4-432*RootOf(RootOf(_Z^6-5
4)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3-2592*(-3*x^2+2)^(1/3
)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)-
```

```
324*(-3*x^2+2)^(2/3))/(x^2-6))+24*ln((16*RootOf(RootOf(_Z^6-54)^2+24*_Z*Roo
tOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6*x+768*RootOf(RootOf(_Z^6-54)^2+24*
_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^5*x-(-3*x^2+2)^(1/3)*RootOf(
_Z^6-54)^5*x-72*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*Ro
otOf(_Z^6-54)^4*(-3*x^2+2)^(1/3)*x-1152*RootOf(RootOf(_Z^6-54)^2+24*_Z*Root
Of(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^3*(-3*x^2+2)^(1/3)*x-36*RootOf(_Z^6
-54)^3*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*x^2-72*Root
Of(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3-18*(
-3*x^2+2)^(1/3)*RootOf(_Z^6-54)^2-432*(-3*x^2+2)^(1/3)*RootOf(RootOf(_Z^6-5
4)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)+54*(-3*x^2+2)^(2/3))/(
x^2-6))*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2831 vs. $2(90) = 180$.
time = 49.52, size = 2831, normalized size = 23.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*arctan(1/3*(13824*sqrt(3)*(1/864
)^(5/6)*(35*d^5*x^9 - 9720*d^5*x^7 + 8424*d^5*x^5 - 3168*d^5*x^3 + 432*d^5*
x)*(-3*x^2 + 2)^(2/3)*(d^(-6))^(5/6) + 48*sqrt(3)*(1/4)^(2/3)*(27*d^4*x^10
+ 4614*d^4*x^8 + 18296*d^4*x^6 - 20304*d^4*x^4 + 4464*d^4*x^2 - 288*d^4)*(-
3*x^2 + 2)^(1/3)*(d^(-6))^(2/3) - 24*sqrt(3)*(1/4)^(1/3)*(d^2*x^10 + 1178*d
^2*x^8 + 15784*d^2*x^6 - 6192*d^2*x^4 - 432*d^2*x^2 + 288*d^2)*(-3*x^2 + 2)
^(2/3)*(d^(-6))^(1/3) + 144*sqrt(3)*sqrt(1/6)*(3*d^3*x^11 - 2234*d^3*x^9 +
15672*d^3*x^7 - 14928*d^3*x^5 + 4080*d^3*x^3 - 288*d^3*x)*sqrt(d^(-6)) - 24
*sqrt(3)*(1/864)^(1/6)*(d*x^11 - 3182*d*x^9 + 169704*d*x^7 - 120816*d*x^5 +
 20304*d*x^3 - 864*d*x)*(-3*x^2 + 2)^(1/3)*(d^(-6))^(1/6) - 24*sqrt(1/6)*(1
92*sqrt(3)*(1/4)^(2/3)*(5*d^4*x^9 - 490*d^4*x^7 + 732*d^4*x^5 - 120*d^4*x^3
)*(-3*x^2 + 2)^(2/3)*(d^(-6))^(2/3) - 72*sqrt(3)*(1/864)^(5/6)*(d^5*x^12 +
4368*d^5*x^10 - 844860*d^5*x^8 + 753216*d^5*x^6 - 217296*d^5*x^4 + 13824*d^
5*x^2 + 1728*d^5)*(d^(-6))^(5/6) + 6*sqrt(3)*sqrt(1/6)*(49*d^3*x^10 - 10086
*d^3*x^8 + 14632*d^3*x^6 + 3024*d^3*x^4 - 2736*d^3*x^2 + 288*d^3)*(-3*x^2 +
```

2)^(1/3)*sqrt(d^(-6)) - 12*sqrt(3)*(1/864)^(1/6)*(d*x^10 - 214*d*x^8 - 290
48*d*x^6 + 18576*d*x^4 - 3888*d*x^2 + 288*d)*(-3*x^2 + 2)^(2/3)*(d^(-6))^(1
/6) + 2*sqrt(3)*(1/4)^(1/3)*(29*d^2*x^11 + 586*d^2*x^9 - 10680*d^2*x^7 + 39
888*d^2*x^5 - 19440*d^2*x^3 + 2592*d^2*x)*(d^(-6))^(1/3) - sqrt(3)*(x^11 +
1834*x^9 - 162264*x^7 + 126288*x^5 - 32688*x^3 + 2592*x)*(-3*x^2 + 2)^(1/3)
)*sqrt(-(96*(1/864)^(1/6)*(-3*x^2 + 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 - 12*(1/4
)^(2/3)*(d^4*x^4 + 12*d^4*x^2 - 12*d^4)*(-3*x^2 + 2)^(2/3)*(d^(-6))^(2/3) -
3456*(1/864)^(5/6)*(d^5*x^5 + 26*d^5*x^3)*(d^(-6))^(5/6) + 6*sqrt(1/6)*(d^
3*x^5 - 20*d^3*x^3 - 108*d^3*x)*(-3*x^2 + 2)^(1/3)*sqrt(d^(-6)) - (1/4)^(1/
3)*(d^2*x^6 - 186*d^2*x^4 - 468*d^2*x^2 + 72*d^2)*(d^(-6))^(1/3) - 6*(x^4 -
36*x^2 - 12)*(-3*x^2 + 2)^(1/3))/(x^6 - 18*x^4 + 108*x^2 - 216)) - sqrt(3)
*(x^12 + 6300*x^10 + 311964*x^8 + 34080*x^6 - 229392*x^4 + 91584*x^2 - 8640
))/(x^12 - 9972*x^10 + 1310076*x^8 - 1277280*x^6 + 413424*x^4 - 67392*x^2 +
5184)) - 1/12*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*arctan(1/3*(13824*sqrt(
3)*(1/864)^(5/6)*(35*d^5*x^9 - 9720*d^5*x^7 + 8424*d^5*x^5 - 3168*d^5*x^3 +
432*d^5*x)*(-3*x^2 + 2)^(2/3)*(d^(-6))^(5/6) - 48*sqrt(3)*(1/4)^(2/3)*(27*
d^4*x^10 + 4614*d^4*x^8 + 18296*d^4*x^6 - 20304*d^4*x^4 + 4464*d^4*x^2 - 28
8*d^4)*(-3*x^2 + 2)^(1/3)*(d^(-6))^(2/3) + 24*sqrt(3)*(1/4)^(1/3)*(d^2*x^10
+ 1178*d^2*x^8 + 15784*d^2*x^6 - 6192*d^2*x^4 - 432*d^2*x^2 + 288*d^2)*(-3
x^2 + 2)^(2/3)(d^(-6))^(1/3) + 144*sqrt(3)*sqrt(1/6)*(3*d^3*x^11 - 2234*d
^3*x^9 + 15672*d^3*x^7 - 14928*d^3*x^5 + 4080*d^3*x^3 - 288*d^3*x)*sqrt(d^(
-6)) - 24*sqrt(3)*(1/864)^(1/6)*(d*x^11 - 3182*d*x^9 + 169704*d*x^7 - 12081
6*d*x^5 + 20304*d*x^3 - 864*d*x)*(-3*x^2 + 2)^(1/3)*(d^(-6))^(1/6) + 24*sqr
t(1/6)*(192*sqrt(3)*(1/4)^(2/3)*(5*d^4*x^9 - 490*d^4*x^7 + 732*d^4*x^5 - 12
0*d^4*x^3)*(-3*x^2 + 2)^(2/3)*(d^(-6))^(2/3) + 72*sqrt(3)*(1/864)^(5/6)*(d^
5*x^12 + 4368*d^5*x^10 - 844860*d^5*x^8 + 753216*d^5*x^6 - 217296*d^5*x^4 +
13824*d^5*x^2 + 1728*d^5)*(d^(-6))^(5/6) - 6*sqrt(3)*sqrt(1/6)*(49*d^3*x^1
0 - 10086*d^3*x^8 + 14632*d^3*x^6 + 3024*d^3*x^4 - 2736*d^3*x^2 + 288*d^3)*
(-3*x^2 + 2)^(1/3)*sqrt(d^(-6)) + 12*sqrt(3)*(1/864)^(1/6)*(d*x^10 - 214*d*
x^8 - 29048*d*x^6 + 18576*d*x^4 - 3888*d*x^2 + 288*d)*(-3*x^2 + 2)^(2/3)*(d
^(-6))^(1/6) + 2*sqrt(3)*(1/4)^(1/3)*(29*d^2*x^11 + 586*d^2*x^9 - 10680*d^2
*x^7 + 39888*d^2*x^5 - 19440*d^2*x^3 + 2592*d^2*x)*(d^(-6))^(1/3) - sqrt(3)
*(x^11 + 1834*x^9 - 162264*x^7 + 126288*x^5 - 32688*x^3 + 2592*x)*(-3*x^2 +
2)^(1/3))*sqrt((96*(1/864)^(1/6)*(-3*x^2 + 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 +
12*(1/4)^(2/3)*(d^4*x^4 + 12*d^4*x^2 - 12*d^4)*(-3*x^2 + 2)^(2/3)*(d^(-6))
^(2/3) - 3456*(1/864)^(5/6)*(d^5*x^5 + 26*d^5*x^3)*(d^(-6))^(5/6) + 6*sqrt(
1/6)*(d^3*x^5 - 20*d^3*x^3 - 108*d^3*x)*(-3*x^2 + 2)^(1/3)*sqrt(d^(-6)) + (
1/4)^(1/3)*(d^2*x^6 - 186*d^2*x^4 - 468*d^2*x^2 + 72*d^2)*(d^(-6))^(1/3) +
6*(x^4 - 36*x^2 - 12)*(-3*x^2 + 2)^(1/3))/(x^6 - 18*x^4 + 108*x^2 - 216)) +
sqrt(3)*(x^12 + 6300*x^10 + 311964*x^8 + 34080*x^6 - 229392*x^4 + 91584*x^
2 - 8640))/(x^12 - 9972*x^10 + 1310076*x^8 - 1277280*x^6 + 413424*x^4 - 673
92*x^2 + 5184)) - 1/48*(1/864)^(1/6)*(d^(-6))^(1/6)*log(1/24*(96*(1/864)^(1
/6)*(-3*x^2 + 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 + 12*(1/4)^(2/3)*(d^4*x^4 + 12*
d^4*x^2 - 12*d^4)*(-3*x^2 + 2)^(2/3)*(d^(-6))^(2/3) - 3456*(1/864)^(5/6)*(d
^5*x^5 + 26*d^5*x^3)*(d^(-6))^(5/6) + 6*sqrt(1/6)*(d^3*x^5 - 20*d^3*x^3 - 1

```
08*d^3*x)*(-3*x^2 + 2)^(1/3)*sqrt(d^(-6)) + (1/4)^(1/3)*(d^2*x^6 - 186*d^2*
x^4 - 468*d^2*x^2 + 72*d^2)*(d^(-6))^(1/3) + 6*(x^4 - 36*x^2 - 12)*(-3*x^2
+ 2)^(1/3))/(x^6 - 18*x^4 + 108*x^2 - 216)) + 1/48*(1/864)^(1/6)*(d^(-6))^(
1/6)*log(-1/24*(96*(1/864)^(1/6)*(-3*x^2 + 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 -
12*(1/4)^(2/3)*(d^4*x^4 + 12*d^4*x^2 - 12*d^4)*(-3*x^2 + 2)^(2/3)*(d^(-6))^
(2/3) - 3456*(1/864)^(5/6)*(d^5*x^5 + 26*d^5*x^3)*(d^(-6))^(5/6) + 6*sqrt(1
/6)*(d^3*x^5 - 20*d^3*x^3 - 108*d^3*x)*(-3*x^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt[3]{2-3x^2}-6\sqrt[3]{2-3x^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/3)/(d*x**2-6*d),x)
[Out] Integral(1/(x**2*(2 - 3*x**2)**(1/3) - 6*(2 - 3*x**2)**(1/3)), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(2-3\,x^2\right)}^{1/3}\,\left(6\,d-d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)),x)
[Out] -int(1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)), x)
```

3.158 $\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}$$

```
[Out] 1/8*arctan(2^(1/6)*(2^(1/3)+(3*x^2-2)^(1/3))/x)*2^(1/6)/d-1/24*arctanh(1/18
*(2^(1/3)+(3*x^2-2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)+1/24*arct
anh(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {404}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) + ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (-2 + 3*x^2)^(1/3))^2
/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\cdots\right.}{3\sqrt{\cdots}}\right)}{4\ 2^{5/6}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.86, size = 136, normalized size = 1.14

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d(-6+x^2)\sqrt[3]{-2+3x^2}\,\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(-6 + x^2)*(-2 + 3*x^
2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/
2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
 x^2/6])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 98.38, size = 723, normalized size = 6.08

method	result	size
trager	Expression too large to display	723

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(ln((4*RootOf(_Z^6-54)^7*x+192*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(
_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6*x-6*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^5*
x-288*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2+24*_Z*Root
Of(_Z^6-54)+576*_Z^2)*x-9*x^2*RootOf(_Z^6-54)^4-18*RootOf(_Z^6-54)^4-108*(3
*x^2-2)^(1/3)*RootOf(_Z^6-54)^2+324*(3*x^2-2)^(2/3))/(x^2-6))*RootOf(_Z^6-5
4)+RootOf(_Z^6-54)*ln(-(4*RootOf(_Z^6-54)^7*x+288*RootOf(RootOf(_Z^6-54)^2+
24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6*x+4608*RootOf(RootOf(_Z^6
-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^5*x+144*(3*x^2-2)^
(1/3)*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*
_Z^2)*x+6912*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^3*RootOf(RootOf(_Z^6-54)^2+24*
_Z*RootOf(_Z^6-54)+576*_Z^2)^2*x-9*x^2*RootOf(_Z^6-54)^4-216*RootOf(_Z^6-54
)^3*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*x^2-18*RootOf(
_Z^6-54)^4-432*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*Roo
tOf(_Z^6-54)^3+2592*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_
Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)-324*(3*x^2-2)^(2/3))/(x^2-6))+24*ln((4*Ro
otOf(_Z^6-54)^7*x+192*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z
^2)*RootOf(_Z^6-54)^6*x-6*(3*x^2-2)^(1/3)*RootOf(_Z^6-54)^5*x-288*(3*x^2-2)
^(1/3)*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576
*_Z^2)*x-9*x^2*RootOf(_Z^6-54)^4-18*RootOf(_Z^6-54)^4-108*(3*x^2-2)^(1/3)*R
ootOf(_Z^6-54)^2+324*(3*x^2-2)^(2/3))/(x^2-6))*RootOf(RootOf(_Z^6-54)^2+24*
_Z*RootOf(_Z^6-54)+576*_Z^2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2681 vs. 2(86) = 172.
time = 46.32, size = 2681, normalized size = 22.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*arctan(1/3*(144*sqrt(3)*sqrt(1/6)
*(3*d^3*x^11 - 2234*d^3*x^9 + 15672*d^3*x^7 - 14928*d^3*x^5 + 4080*d^3*x^3
- 288*d^3*x)*sqrt(d^(-6)) + 24*sqrt(1/6)*(72*sqrt(3)*(1/864)^(5/6)*(d^5*x^1
2 + 4368*d^5*x^10 - 844860*d^5*x^8 + 753216*d^5*x^6 - 217296*d^5*x^4 + 1382
4*d^5*x^2 + 1728*d^5)*(d^(-6))^(5/6) + 2*sqrt(3)*(1/4)^(1/3)*(29*d^2*x^11 +
 586*d^2*x^9 - 10680*d^2*x^7 + 39888*d^2*x^5 - 19440*d^2*x^3 + 2592*d^2*x)*
(d^(-6))^(1/3) + 12*(16*sqrt(3)*(1/4)^(2/3)*(5*d^4*x^9 - 490*d^4*x^7 + 732*
d^4*x^5 - 120*d^4*x^3)*(d^(-6))^(2/3) + sqrt(3)*(1/864)^(1/6)*(d*x^10 - 214
*d*x^8 - 29048*d*x^6 + 18576*d*x^4 - 3888*d*x^2 + 288*d)*(d^(-6))^(1/6))*(3
*x^2 - 2)^(2/3) + (6*sqrt(3)*sqrt(1/6)*(49*d^3*x^10 - 10086*d^3*x^8 + 14632
*d^3*x^6 + 3024*d^3*x^4 - 2736*d^3*x^2 + 288*d^3)*sqrt(d^(-6)) + sqrt(3)*(x
^11 + 1834*x^9 - 162264*x^7 + 126288*x^5 - 32688*x^3 + 2592*x))*(3*x^2 - 2)
^(1/3))*sqrt(-(3456*(1/864)^(5/6)*(d^5*x^5 + 26*d^5*x^3)*(d^(-6))^(5/6) - 1
2*(8*(1/864)^(1/6)*d*(d^(-6))^(1/6)*x^3 + (1/4)^(2/3)*(d^4*x^4 + 12*d^4*x^2
 - 12*d^4)*(d^(-6))^(2/3))*(3*x^2 - 2)^(2/3) - (1/4)^(1/3)*(d^2*x^6 - 186*d
^2*x^4 - 468*d^2*x^2 + 72*d^2)*(d^(-6))^(1/3) + 6*(x^4 - 36*x^2 + sqrt(1/6)
*(d^3*x^5 - 20*d^3*x^3 - 108*d^3*x)*sqrt(d^(-6)) - 12)*(3*x^2 - 2)^(1/3))/(
x^6 - 18*x^4 + 108*x^2 - 216)) + 24*(576*sqrt(3)*(1/864)^(5/6)*(35*d^5*x^9
- 9720*d^5*x^7 + 8424*d^5*x^5 - 3168*d^5*x^3 + 432*d^5*x)*(d^(-6))^(5/6) +
sqrt(3)*(1/4)^(1/3)*(d^2*x^10 + 1178*d^2*x^8 + 15784*d^2*x^6 - 6192*d^2*x^4
 - 432*d^2*x^2 + 288*d^2)*(d^(-6))^(1/3))*(3*x^2 - 2)^(2/3) + sqrt(3)*(x^12
 + 6300*x^10 + 311964*x^8 + 34080*x^6 - 229392*x^4 + 91584*x^2 - 8640) + 24
*(2*sqrt(3)*(1/4)^(2/3)*(27*d^4*x^10 + 4614*d^4*x^8 + 18296*d^4*x^6 - 20304
*d^4*x^4 + 4464*d^4*x^2 - 288*d^4)*(d^(-6))^(2/3) + sqrt(3)*(1/864)^(1/6)*(
d*x^11 - 3182*d*x^9 + 169704*d*x^7 - 120816*d*x^5 + 20304*d*x^3 - 864*d*x)*
(d^(-6))^(1/6))*(3*x^2 - 2)^(1/3))/(x^12 - 9972*x^10 + 1310076*x^8 - 127728
```

```
0*x^6 + 413424*x^4 - 67392*x^2 + 5184)) - 1/12*sqrt(3)*(1/864)^(1/6)*(d^(-6
))^(1/6)*arctan(-1/3*(144*sqrt(3)*sqrt(1/6)*(3*d^3*x^11 - 2234*d^3*x^9 + 15
672*d^3*x^7 - 14928*d^3*x^5 + 4080*d^3*x^3 - 288*d^3*x)*sqrt(d^(-6)) + 24*s
qrt(1/6)*(72*sqrt(3)*(1/864)^(5/6)*(d^5*x^12 + 4368*d^5*x^10 - 844860*d^5*x
^8 + 753216*d^5*x^6 - 217296*d^5*x^4 + 13824*d^5*x^2 + 1728*d^5)*(d^(-6))^(
5/6) - 2*sqrt(3)*(1/4)^(1/3)*(29*d^2*x^11 + 586*d^2*x^9 - 10680*d^2*x^7 + 3
9888*d^2*x^5 - 19440*d^2*x^3 + 2592*d^2*x)*(d^(-6))^(1/3) - 12*(16*sqrt(3)*
(1/4)^(2/3)*(5*d^4*x^9 - 490*d^4*x^7 + 732*d^4*x^5 - 120*d^4*x^3)*(d^(-6))^
(2/3) - sqrt(3)*(1/864)^(1/6)*(d*x^10 - 214*d*x^8 - 29048*d*x^6 + 18576*d*x
^4 - 3888*d*x^2 + 288*d)*(d^(-6))^(1/6))*(3*x^2 - 2)^(2/3) + (6*sqrt(3)*sqr
t(1/6)*(49*d^3*x^10 - 10086*d^3*x^8 + 14632*d^3*x^6 + 3024*d^3*x^4 - 2736*d
^3*x^2 + 288*d^3)*sqrt(d^(-6)) - sqrt(3)*(x^11 + 1834*x^9 - 162264*x^7 + 12
6288*x^5 - 32688*x^3 + 2592*x))*(3*x^2 - 2)^(1/3))*sqrt((3456*(1/864)^(5/6)
*(d^5*x^5 + 26*d^5*x^3)*(d^(-6))^(5/6) - 12*(8*(1/864)^(1/6)*d*(d^(-6))^(1/
6)*x^3 - (1/4)^(2/3)*(d^4*x^4 + 12*d^4*x^2 - 12*d^4)*(d^(-6))^(2/3))*(3*x^2
 - 2)^(2/3) + (1/4)^(1/3)*(d^2*x^6 - 186*d^2*x^4 - 468*d^2*x^2 + 72*d^2)*(d
^(-6))^(1/3) - 6*(x^4 - 36*x^2 - sqrt(1/6)*(d^3*x^5 - 20*d^3*x^3 - 108*d^3*
x)*sqrt(d^(-6)) - 12)*(3*x^2 - 2)^(1/3))/(x^6 - 18*x^4 + 108*x^2 - 216)) +
24*(576*sqrt(3)*(1/864)^(5/6)*(35*d^5*x^9 - 9720*d^5*x^7 + 8424*d^5*x^5 - 3
168*d^5*x^3 + 432*d^5*x)*(d^(-6))^(5/6) - sqrt(3)*(1/4)^(1/3)*(d^2*x^10 + 1
178*d^2*x^8 + 15784*d^2*x^6 - 6192*d^2*x^4 - 432*d^2*x^2 + 288*d^2)*(d^(-6)
)^(1/3))*(3*x^2 - 2)^(2/3) - sqrt(3)*(x^12 + 6300*x^10 + 311964*x^8 + 34080
*x^6 - 229392*x^4 + 91584*x^2 - 8640) - 24*(2*sqrt(3)*(1/4)^(2/3)*(27*d^4*x
^10 + 4614*d^4*x^8 + 18296*d^4*x^6 - 20304*d^4*x^4 + 4464*d^4*x^2 - 288*d^4
)*(d^(-6))^(2/3) - sqrt(3)*(1/864)^(1/6)*(d*x^11 - 3182*d*x^9 + 169704*d*x^
7 - 120816*d*x^5 + 20304*d*x^3 - 864*d*x)*(d^(-6))^(1/6))*(3*x^2 - 2)^(1/3)
)/(x^12 - 9972*x^10 + 1310076*x^8 - 1277280*x^6 + 413424*x^4 - 67392*x^2 +
5184)) + 1/48*(1/864)^(1/6)*(d^(-6))^(1/6)*log(-1/24*(3456*(1/864)^(5/6)*(d
^5*x^5 + 26*d^5*x^3)*(d^(-6))^(5/6) - 12*(8*(1/864)^(1/6)*d*(d^(-6))^(1/6)*
x^3 + (1/4)^(2/3)*(d^4*x^4 + 12*d^4*x^2 - 12*d^4)*(d^(-6))^(2/3))*(3*x^2 -
2)^(2/3) - (1/4)^(1/3)*(d^2*x^6 - 186*d^2*x^4 - 468*d^2*x^2 + 72*d^2)*(d^(-
6))^(1/3) + 6*(x^4 - 36*x^2 + sqrt(1/6)*(d^3*x^5 - 20*d^3*x^3 - 108*d^3*x)*
sqrt(d^(-6)) - 12)*(3*x^2 - 2)^(1/3))/(x^6 - 18*x^4 + 108*x^2 - 216)) - 1/4
8*(1/864)^(1/6)*(d^(-6))^(1/6)*log(1/24*(3456*(1/864)^(5/6)*(d^5*x^5 + 26*d
^5*x^3)*(d^(-6))^(5/6) - 12*(8*(1/864)^(1/6)*d*(d^(-6))^(1/6)*x^3 - (1/4)^(
2/3)*(d^4*x^4 + 12*d^4*x^2 - 12*d^4)*(d^(-6))^(2/3))*(3*x^2 - 2)^(2/3) + (1
/4)^(1/3)*(d^2*x^6 - 186*d^2*x^4 - 468*d^2*x^2 + 72*d^2)*(d^(-6))^(1/3) - 6
*(x^4 - 36*x^2 - sqrt(1/6)*(d^3*x^5 - 20*d^3*x^3 - 108*d^3*x)*sqrt(d^(-6))
- 12)*(3*x^2 - 2)^(1/3))/(x^6 - 18*x^4 + 108*x^2 - 216)) - 1/24*(1/864)^(1/
6)*(d^(-6))^(1/6)*log(1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x
)*(d^(-6))^(2/3) + 2*(10*x^3 + 3*sqrt(1/6)*(d^3...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2 \sqrt[3]{3x^2-2} - 6\sqrt[3]{3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(1/3)/(d*x**2-6*d),x)
[Out] Integral(1/(x**2*(3*x**2 - 2)**(1/3) - 6*(3*x**2 - 2)**(1/3)), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(3\,x^2-2\right)}^{1/3}\,\left(6\,d-d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)),x)
[Out] -int(1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)), x)
```

3.159 $\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

```
[Out] 1/8*arctanh(2^(1/6)*(2^(1/3)+(-3*x^2-2)^(1/3))/x)*2^(1/6)/d-1/24*arctan(1/1
8*(2^(1/3)+(-3*x^2-2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)-1/24*ar
ctan(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {403}

$$-\frac{\mathrm{ArcTan}\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}-\frac{\mathrm{ArcTan}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] -1/4*ArcTan[x/Sqrt[6]]/(2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2 - 3*x^2)
^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/6)*(
2^(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx=-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}\,d}-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}\,x}\right)}{4\ 2^{5/6}\sqrt{3}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\right.}{}\right.}{4\ 2^{5/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 5.39, size = 136, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\sqrt[3]{-2-3x^2}\,(6+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(-2 - 3*x^2)^(1/
3)*(6 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(Ap
pellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/
2, (-3*x^2)/2, -1/6*x^2])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 90.71, size = 726, normalized size = 6.10

method	result	size
trager	Expression too large to display	726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(RootOf(_Z^6+54)*ln(-(4*RootOf(_Z^6+54)^7*x+288*RootOf(RootOf(_Z^6+54
)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^6*x+4608*RootOf(RootOf(
_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*RootOf(_Z^6+54)^5*x-144*(-3*x^
2-2)^(1/3)*RootOf(RootOf(_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(
_Z^6+54)^4*x-6912*(-3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6+54)^2+24*_Z*RootOf(_Z
^6+54)+576*_Z^2)^2*RootOf(_Z^6+54)^3*x-9*x^2*RootOf(_Z^6+54)^4-216*RootOf(R
ootOf(_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3*x^2+18*R
ootOf(_Z^6+54)^4+432*RootOf(RootOf(_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^
2)*RootOf(_Z^6+54)^3+2592*(-3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6+54)^2+24*_Z*R
ootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)+324*(-3*x^2-2)^(2/3))/(x^2+6))-ln(
-(4*RootOf(_Z^6+54)^7*x+192*RootOf(RootOf(_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+
576*_Z^2)*RootOf(_Z^6+54)^6*x+6*RootOf(_Z^6+54)^5*(-3*x^2-2)^(1/3)*x+288*(-
3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^2)*Roo
tOf(_Z^6+54)^4*x+9*x^2*RootOf(_Z^6+54)^4-18*RootOf(_Z^6+54)^4+108*(-3*x^2-2
)^(1/3)*RootOf(_Z^6+54)^2+324*(-3*x^2-2)^(2/3))/(x^2+6))*RootOf(_Z^6+54)-24
*ln(-(4*RootOf(_Z^6+54)^7*x+192*RootOf(RootOf(_Z^6+54)^2+24*_Z*RootOf(_Z^6+
54)+576*_Z^2)*RootOf(_Z^6+54)^6*x+6*RootOf(_Z^6+54)^5*(-3*x^2-2)^(1/3)*x+28
8*(-3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^2)
*RootOf(_Z^6+54)^4*x+9*x^2*RootOf(_Z^6+54)^4-18*RootOf(_Z^6+54)^4+108*(-3*x
^2-2)^(1/3)*RootOf(_Z^6+54)^2+324*(-3*x^2-2)^(2/3))/(x^2+6))*RootOf(RootOf(
_Z^6+54)^2+24*_Z*RootOf(_Z^6+54)+576*_Z^2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3109 vs. 2(86) = 172.
time = 49.09, size = 3109, normalized size = 26.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

```
[Out] -1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*log(1/24*(384*sqrt(3)*(1/864)^(1
/6)*(-3*x^2 - 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 + 5184*sqrt(3)*(1/864)^(5/6)*(3
*d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(5/6) + 36*(1/4)^(2/3)*(d^4*x^4
+ 28*d^4*x^2 + 4*d^4)*(-3*x^2 - 2)^(2/3)*(d^(-6))^(2/3) - 6*sqrt(3)*sqrt(1/
6)*(d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*(-3*x^2 - 2)^(1/3)*sqrt(d^(-6)) + (1/
4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^(1/3) - 1
8*(9*x^4 + 12*x^2 + 4)*(-3*x^2 - 2)^(1/3))/(x^6 + 18*x^4 + 108*x^2 + 216))
- 1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*log(1/96*(384*sqrt(3)*(1/864)^(
1/6)*(-3*x^2 - 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 + 5184*sqrt(3)*(1/864)^(5/6)*(
3*d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(5/6) + 36*(1/4)^(2/3)*(d^4*x^4
 + 28*d^4*x^2 + 4*d^4)*(-3*x^2 - 2)^(2/3)*(d^(-6))^(2/3) - 6*sqrt(3)*sqrt(1
/6)*(d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*(-3*x^2 - 2)^(1/3)*sqrt(d^(-6)) + (1
/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^(1/3) -
18*(9*x^4 + 12*x^2 + 4)*(-3*x^2 - 2)^(1/3))/(x^6 + 18*x^4 + 108*x^2 + 216))
 + 1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*log(-1/96*(384*sqrt(3)*(1/864)
^(1/6)*(-3*x^2 - 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 + 5184*sqrt(3)*(1/864)^(5/6)
*(3*d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(5/6) - 36*(1/4)^(2/3)*(d^4*x
^4 + 28*d^4*x^2 + 4*d^4)*(-3*x^2 - 2)^(2/3)*(d^(-6))^(2/3) - 6*sqrt(3)*sqrt
(1/6)*(d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*(-3*x^2 - 2)^(1/3)*sqrt(d^(-6)) -
(1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^(1/3)
+ 18*(9*x^4 + 12*x^2 + 4)*(-3*x^2 - 2)^(1/3))/(x^6 + 18*x^4 + 108*x^2 + 216
)) + 1/96*sqrt(3)*(1/864)^(1/6)*(d^(-6))^(1/6)*log(-1/24*(384*sqrt(3)*(1/86
4)^(1/6)*(-3*x^2 - 2)^(2/3)*d*(d^(-6))^(1/6)*x^3 + 5184*sqrt(3)*(1/864)^(5/
6)*(3*d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(5/6) - 36*(1/4)^(2/3)*(d^4
*x^4 + 28*d^4*x^2 + 4*d^4)*(-3*x^2 - 2)^(2/3)*(d^(-6))^(2/3) - 6*sqrt(3)*sq
rt(1/6)*(d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*(-3*x^2 - 2)^(1/3)*sqrt(d^(-6))
```

```
- (1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*d^2*x^2 - 72*d^2)*(d^(-6))^(1/3
) + 18*(9*x^4 + 12*x^2 + 4)*(-3*x^2 - 2)^(1/3))/(x^6 + 18*x^4 + 108*x^2 + 2
16)) + 1/12*(1/864)^(1/6)*(d^(-6))^(1/6)*arctan((13824*(1/864)^(5/6)*(97*d^
5*x^9 - 4644*d^5*x^7 + 33696*d^5*x^5 + 1296*d^5*x^3 - 11664*d^5*x)*(-3*x^2
- 2)^(2/3)*(d^(-6))^(5/6) + 48*sqrt(3)*(1/4)^(2/3)*(37*d^4*x^10 - 450*d^4*x
^8 + 15432*d^4*x^6 - 28944*d^4*x^4 + 11664*d^4*x^2 + 7776*d^4)*(-3*x^2 - 2)
^(1/3)*(d^(-6))^(2/3) - 24*sqrt(3)*(1/4)^(1/3)*(d^2*x^10 + 366*d^2*x^8 - 35
76*d^2*x^6 + 38448*d^2*x^4 - 3888*d^2*x^2 + 7776*d^2)*(-3*x^2 - 2)^(2/3)*(d
^(-6))^(1/3) - 24*sqrt(1/6)*(192*sqrt(3)*(1/4)^(2/3)*(11*d^4*x^9 - 174*d^4*
x^7 - 2700*d^4*x^5 - 648*d^4*x^3)*(-3*x^2 - 2)^(2/3)*(d^(-6))^(2/3) - 72*(1
/864)^(5/6)*(d^5*x^12 + 33744*d^5*x^10 - 830844*d^5*x^8 - 1118016*d^5*x^6 -
 1364688*d^5*x^4 + 139968*d^5)*(d^(-6))^(5/6) + 6*sqrt(1/6)*(151*d^3*x^10 +
 4698*d^3*x^8 - 219816*d^3*x^6 - 138672*d^3*x^4 - 58320*d^3*x^2 + 23328*d^3
)*(-3*x^2 - 2)^(1/3)*sqrt(d^(-6)) + 2*sqrt(3)*(1/4)^(1/3)*(43*d^2*x^11 + 93
70*d^2*x^9 - 250632*d^2*x^7 - 387504*d^2*x^5 - 177552*d^2*x^3 + 23328*d^2*x
)*(d^(-6))^(1/3) - 24*(1/864)^(1/6)*(d*x^10 + 1080*d*x^8 - 28296*d*x^6 - 12
960*d*x^4 + 11664*d*x^2)*(-3*x^2 - 2)^(2/3)*(d^(-6))^(1/6) - sqrt(3)*(x^11
+ 5990*x^9 - 155160*x^7 + 34992*x^5 + 6480*x^3 - 23328*x)*(-3*x^2 - 2)^(1/3
))*sqrt((384*sqrt(3)*(1/864)^(1/6)*(-3*x^2 - 2)^(2/3)*d*(d^(-6))^(1/6)*x^3
+ 5184*sqrt(3)*(1/864)^(5/6)*(3*d^5*x^5 + 20*d^5*x^3 + 12*d^5*x)*(d^(-6))^(
5/6) + 36*(1/4)^(2/3)*(d^4*x^4 + 28*d^4*x^2 + 4*d^4)*(-3*x^2 - 2)^(2/3)*(d^
(-6))^(2/3) - 6*sqrt(3)*sqrt(1/6)*(d^3*x^5 + 140*d^3*x^3 + 36*d^3*x)*(-3*x^
2 - 2)^(1/3)*sqrt(d^(-6)) + (1/4)^(1/3)*(d^2*x^6 + 1098*d^2*x^4 + 396*d^2*x
^2 - 72*d^2)*(d^(-6))^(1/3) - 18*(9*x^4 + 12*x^2 + 4)*(-3*x^2 - 2)^(1/3))/(
x^6 + 18*x^4 + 108*x^2 + 216)) + 192*sqrt(1/6)*(4*d^3*x^11 - 905*d^3*x^9 +
11016*d^3*x^7 - 1944*d^3*x^5 - 2592*d^3*x^3 - 11664*d^3*x)*sqrt(d^(-6)) - 2
4*(1/864)^(1/6)*(d*x^11 - 4282*d*x^9 + 100008*d*x^7 - 327888*d*x^5 - 304560
*d*x^3 - 23328*d*x)*(-3*x^2 - 2)^(1/3)*(d^(-6))^(1/6) - sqrt(3)*(x^12 - 156
*x^10 + 101916*x^8 - 359712*x^6 + 1346544*x^4 + 793152*x^2 + 46656))/(x^12
- 32652*x^10 + 1036476*x^8 - 4865184*x^6 - 4770576*x^4 - 1259712*x^2 + 4199
04)) + 1/12*(1/864)^(1/6)*(d^(-6))^(1/6)*arctan((13824*(1/864)^(5/6)*(97*d^
5*x^9 - 4644*d^5*x^7 + 33696*d^5*x^5 + 1296*d^5*x^3 - 11664*d^5*x)*(-3*x^2
- 2)^(2/3)*(d^(-6))^(5/6) - 48*sqrt(3)*(1/4)^(2/3)*(37*d^4*x^10 - 450*d^4*x
^8 + 15432*d^4*x^6 - 28944*d^4*x^4 + 11664*d^4*x^2 + 7776*d^4)*(-3*x^2 - 2)
^(1/3)*(d^(-6))^(2/3) + 24*sqrt(3)*(1/4)^(1/3)*(d^2*x^10 + 366*d^2*x^8 - 35
76*d^2*x^6 + 38448*d^2*x^4 - 3888*d^2*x^2 + 7776*d^2)*(-3*x^2 - 2)^(2/3)*(d
^(-6))^(1/3) + 24*sqrt(1/6)*(192*sqrt(3)*(1/4)^(2/3)*(11*d^4*x^9 - 174*d^4*
x^7 - 2700*d^4*x^5 - 648*d^4*x^3)*(-3*x^2 - 2)^(2/3)*(d^(-6))^(2/3) + 72*(1
/864)^(5/6)*(d^5*x^12 + 33744*d^5*x^10 - 830844*d^5*x^8 - 1118016*d^5*x^6 -
 1364688*d^5*x^4 + 139968*d^5)*(d^(-6))^(5/6) - 6*sqrt(1/6)*(151*d^3*x^10 +
 4698*d^3*x^8 - 219816*d^3*x^6 - 138672*d^3*x^4...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt[3]{-3x^2-2}+6\sqrt[3]{-3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(1/3)/(d*x**2+6*d),x)
[Out] Integral(1/(x**2*(-3*x**2 - 2)**(1/3) + 6*(-3*x**2 - 2)**(1/3)), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(-3\,x^2-2\right)^{1/3}\,\left(d\,x^2+6\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)),x)
[Out] int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)), x)
```

3.160 $\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx$

Optimal. Leaf size=70

$$\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)+\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

```
[Out] 1/12*arctan(1/3*x)+1/12*arctan(1/3*(1-(x^2+1)^(1/3))^2/x)-1/12*arctanh((1-(
x^2+1)^(1/3))*3^(1/2)/x)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {403}

$$\frac{1}{12}\text{ArcTan}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)+\frac{1}{12}\text{ArcTan}\left(\frac{x}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTanh[(Sqrt[3
]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx=\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)+\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.73, size = 124, normalized size = 1.77

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)}{\sqrt[3]{1+x^2}\,(9+x^2)\left(-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2])/((1 + x^2)^(1/3)*(9 + x^
2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2] + 2*x^2*(AppellF1[3/2, 1
/3, 2, 5/2, -x^2, -1/9*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -x^2, -1/9*x^2])
))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.44, size = 616, normalized size = 8.80

method	result
trager	$-144\,\mathrm{RootOf}\left(20736\_Z^4-144\_Z^2+1\right)^3\ln\left(\frac{497664\,\mathrm{RootOf}\left(20736\_Z^4-144\_Z^2+1\right)^5(x^2+1)^{\frac{1}{3}}x-995328\,\mathrm{Ro}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/3)/(x^2+9),x,method=_RETURNVERBOSE)
```

```
[Out] -144*RootOf(20736*_Z^4-144*_Z^2+1)^3*ln((497664*RootOf(20736*_Z^4-144*_Z^2+
1)^5*(x^2+1)^(1/3)*x-995328*RootOf(20736*_Z^4-144*_Z^2+1)^5*x-6912*RootOf(2
0736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(1/3)*x+20736*RootOf(20736*_Z^4-144*_Z^2+1)
^3*x-144*RootOf(20736*_Z^4-144*_Z^2+1)^2*x^2+864*RootOf(20736*_Z^4-144*_Z^2
+1)^2*(x^2+1)^(1/3)+432*RootOf(20736*_Z^4-144*_Z^2+1)^2-96*RootOf(20736*_Z^
4-144*_Z^2+1)*x-6*(x^2+1)^(2/3)+x^2-3)/(x^2+9))+RootOf(20736*_Z^4-144*_Z^2+
1)*ln((497664*RootOf(20736*_Z^4-144*_Z^2+1)^5*(x^2+1)^(1/3)*x-995328*RootOf
(20736*_Z^4-144*_Z^2+1)^5*x-6912*RootOf(20736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(1
/3)*x+20736*RootOf(20736*_Z^4-144*_Z^2+1)^3*x-144*RootOf(20736*_Z^4-144*_Z^
2+1)^2*x^2+864*RootOf(20736*_Z^4-144*_Z^2+1)^2*(x^2+1)^(1/3)+432*RootOf(207
36*_Z^4-144*_Z^2+1)^2-96*RootOf(20736*_Z^4-144*_Z^2+1)*x-6*(x^2+1)^(2/3)+x^
2-3)/(x^2+9))+RootOf(20736*_Z^4-144*_Z^2+1)*ln(-(82944*RootOf(20736*_Z^4-14
4*_Z^2+1)^5*(x^2+1)^(1/3)*x-165888*RootOf(20736*_Z^4-144*_Z^2+1)^5*x-1728*R
ootOf(20736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(1/3)*x+2304*RootOf(20736*_Z^4-144*_
Z^2+1)^3*x-24*RootOf(20736*_Z^4-144*_Z^2+1)^2*x^2+144*RootOf(20736*_Z^4-144
*_Z^2+1)^2*(x^2+1)^(1/3)+8*(x^2+1)^(1/3)*RootOf(20736*_Z^4-144*_Z^2+1)*x+72
*RootOf(20736*_Z^4-144*_Z^2+1)^2+(x^2+1)^(2/3)-(x^2+1)^(1/3))/(x^2+9))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(49) = 98$.
time = 4.00, size = 1395, normalized size = 19.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="fricas")
```

```
[Out] 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 + 32*sqrt(3)*x^3
+ 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*
x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3) - 243)
/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 89
1*x^2 + 18*(3*x^4 - 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sq
rt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 +
 81*x) + 81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/36*
arctan((384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5 - 1259712*x^3 + 36
*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 + sqrt(3)*(x^10 + 549*x^8 -
8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x^2 + 1)^(2/3) + sq
rt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 6816879*x^4 + 6022998*x
^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 3773304*x^6 - 6908733*x
^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*x^2 + 12*sqrt(3)*(
11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3) + 6*sqrt(3)*(43*x^
11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 + 177147*x) + 6*(453
*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^2 + sqrt(3)*(x^11
+ 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147*x) + 531441)*(x^2
 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 - 32*sqrt
(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sqrt(3)*(x^5 + 10*x^3 + 9*x)
+ 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3
) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 6423*x^9 + 225018*x^7
 - 1106622*x^5 - 1541835*x^3 + 3*sqrt(3)*(37*x^10 - 675*x^8 + 34722*x^6 - 9
7686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3) - 8503056*x)/(x^1
2 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^4 - 9565938*x^2 +
4782969)) + 1/36*arctan(-(384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5
- 1259712*x^3 + 36*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 - sqrt(3)*
```

```
(x^10 + 549*x^8 - 8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x
^2 + 1)^(2/3) - sqrt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 68168
79*x^4 + 6022998*x^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 37733
04*x^6 - 6908733*x^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*
x^2 - 12*sqrt(3)*(11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3)
- 6*sqrt(3)*(43*x^11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 +
177147*x) + 6*(453*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^
2 - sqrt(3)*(x^11 + 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147
*x) + 531441)*(x^2 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 1
8*(3*x^4 + 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^
5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) +
81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 642
3*x^9 + 225018*x^7 - 1106622*x^5 - 1541835*x^3 - 3*sqrt(3)*(37*x^10 - 675*x
^8 + 34722*x^6 - 97686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3)
 - 8503056*x)/(x^12 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^
4 - 9565938*x^2 + 4782969)) - 1/36*arctan(6*(11*x^5 + 30*x^3 + 6*(23*x^3 +
27*x)*(x^2 + 1)^(2/3) + (x^5 - 240*x^3 - 81*x)*(x^2 + 1)^(1/3) - 81*x)/(x^6
 - 1971*x^4 - 1701*x^2 - 729))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^2+1}\,(x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
[Out] Integral(1/((x**2 + 1)**(1/3)*(x**2 + 9)), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="giac")
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(x^2+1)}^{1/3}\,(x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)),x)
[Out] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)), x)
```

3.161 $\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx$

Optimal. Leaf size=104

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+bx^2}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,\sqrt{b}}$$

```
[Out] 1/12*arctan(1/3*(1-(b*x^2+1)^(1/3))^2/x/b^(1/2))/b^(1/2)+1/12*arctan(1/3*x*
b^(1/2))/b^(1/2)-1/12*arctanh((1-(b*x^2+1)^(1/3))*3^(1/2)/x/b^(1/2))*3^(1/2
)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {403}

$$\frac{\text{ArcTan}\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3))^2/(3*Sq
rt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(4*Sqrt[3]*Sqrt[b])
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+bx^2}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+bx^2}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\,\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 5.66, size = 137, normalized size = 1.32

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)}{\sqrt[3]{1+bx^2}\,(9+bx^2)\left(-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)+2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)])/((1 + b*x^2)^(1/
3)*(9 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)] + 2*
b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^2), -1/9*(b*x^2)] + 3*AppellF1[3/2,
 4/3, 1, 5/2, -(b*x^2), -1/9*(b*x^2)])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+1)^{\frac{1}{3}}\,(b\,x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

```
[Out] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{bx^2+1}\,(bx^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+1)**(1/3)/(b*x**2+9),x)
```

```
[Out] Integral(1/((b*x**2 + 1)**(1/3)*(b*x**2 + 9)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+1\right)}^{1/3}\,\left(b\,x^2+9\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)),x)
```

```
[Out] int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)), x)
```

3.162 $\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)$$

```
[Out] 1/12*arctanh(1/3*x)-1/12*arctanh(1/3*(1-(-x^2+1)^(1/3))^2/x)+1/12*arctan((1
-(-x^2+1)^(1/3))*3^(1/2)/x)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{404\}$

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x/3]/12 - A
rcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 9.75, size = 125, normalized size = 1.69

$$\frac{\sqrt[3]{\frac{-1+x}{-3+x}}\sqrt[3]{\frac{1+x}{-3+x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{4}{-3+x},-\frac{2}{-3+x}\right)-\sqrt[3]{\frac{-1+x}{3+x}}\sqrt[3]{\frac{1+x}{3+x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2}{3+x},\frac{4}{3+x}\right)}{4\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] (((-1 + x)/(-3 + x))^(1/3)*((1 + x)/(-3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/3,
 5/3, -4/(-3 + x), -2/(-3 + x)] - ((-1 + x)/(3 + x))^(1/3)*((1 + x)/(3 + x)
)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, 2/(3 + x), 4/(3 + x)])/(4*(1 - x^2)^(1
/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.61, size = 365, normalized size = 4.93

method	result
trager	$-\frac{\ln\left(\frac{576\left(-x^2+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(144\_Z^2+12\_Z+1\right)^2 x+24\,\mathrm{RootOf}\left(144\_Z^2+12\_Z+1\right)\left(-x^2+1\right)^{\frac{1}{3}}x-1152\,\mathrm{RootOf}\left(144\_Z^2+12\_Z+1\right)^2 x}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(-x^2+9),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln((576*(-x^2+1)^(1/3)*RootOf(144*_Z^2+12*_Z+1)^2*x+24*RootOf(144*_Z^
2+12*_Z+1)*(-x^2+1)^(1/3)*x-1152*RootOf(144*_Z^2+12*_Z+1)^2*x+12*RootOf(144
*_Z^2+12*_Z+1)*x^2+6*(-x^2+1)^(2/3)+72*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1
/3)-144*RootOf(144*_Z^2+12*_Z+1)*x+x^2+36*RootOf(144*_Z^2+12*_Z+1)-4*x+3)/(
3+x)/(x-3))-1/12*ln((48*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x+6*(-x^2+1
)^(2/3)+2*(-x^2+1)^(1/3)*x+96*RootOf(144*_Z^2+12*_Z+1)*x-x^2+6*(-x^2+1)^(1/
3)+4*x-3)/(3+x)/(x-3))-ln((48*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x+6*(
-x^2+1)^(2/3)+2*(-x^2+1)^(1/3)*x+96*RootOf(144*_Z^2+12*_Z+1)*x-x^2+6*(-x^2+
1)^(1/3)+4*x-3)/(3+x)/(x-3))*RootOf(144*_Z^2+12*_Z+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(53) = 106$.
time = 1.49, size = 269, normalized size = 3.64

$$-\frac{1}{36}\sqrt{3}\arctan\left(\frac{36\sqrt{3}(x^4-32x^3-42x^2+9)(-x^2+1)^{\frac{2}{3}}+12\sqrt{3}(x^5+27x^4-210x^3-54x^2+81x+27)(-x^2+1)^{\frac{1}{3}}+\sqrt{3}(x^6-108x^5-567x^4+1080x^3+459x^2-972x-405)}{3(x^6+108x^5-1647x^4-1080x^3+891x^2+972x+243)}\right)-\frac{1}{72}\log\left(\frac{x^3+33x^2+18(-x^2+1)^{\frac{2}{3}}(x+1)-6(x^2+6x-3)(-x^2+1)^{\frac{1}{3}}-9x-9}{x^3+9x^2+27x+27}\right)+\frac{1}{36}\log\left(-\frac{x^3-33x^2+18(-x^2+1)^{\frac{2}{3}}(x-1)+6(x^2-6x-3)(-x^2+1)^{\frac{1}{3}}-9x+9}{x^3+9x^2+27x+27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*arctan(1/3*(36*sqrt(3)*(x^4 - 32*x^3 - 42*x^2 + 9)*(-x^2 + 1)
^(2/3) + 12*sqrt(3)*(x^5 + 27*x^4 - 210*x^3 - 54*x^2 + 81*x + 27)*(-x^2 + 1
)^(1/3) + sqrt(3)*(x^6 - 108*x^5 - 567*x^4 + 1080*x^3 + 459*x^2 - 972*x - 4
05))/(x^6 + 108*x^5 - 1647*x^4 - 1080*x^3 + 891*x^2 + 972*x + 243)) - 1/72*
log((x^3 + 33*x^2 + 18*(-x^2 + 1)^(2/3)*(x + 1) - 6*(x^2 + 6*x - 3)*(-x^2 +
 1)^(1/3) - 9*x - 9)/(x^3 + 9*x^2 + 27*x + 27)) + 1/36*log(-(x^3 - 33*x^2 +
 18*(-x^2 + 1)^(2/3)*(x - 1) + 6*(x^2 - 6*x - 3)*(-x^2 + 1)^(1/3) - 9*x + 9
)/(x^3 + 9*x^2 + 27*x + 27))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{1-x^2}-9\sqrt[3]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(-x**2+9),x)
```

```
[Out] -Integral(1/(x**2*(1 - x**2)**(1/3) - 9*(1 - x**2)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(1-x^2\right)^{1/3}\,\left(x^2-9\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^2)^(1/3)*(x^2 - 9)),x)
```

```
[Out] -int(1/((1 - x^2)^(1/3)*(x^2 - 9)), x)
```

3.163 $\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{x\sqrt{-1+c^2x^2}}{2d\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{-1+c^2x^2}\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*x*(c^2*x^2-1)^(1/2)/d/(-c^2*d*x^2+d)^(3/2)+1/2*arctanh(c*x)*(c^2*x^2-1)
^(1/2)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {23, 205, 214}

$$\frac{x\sqrt{c^2x^2-1}}{2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{c^2x^2-1}\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[-1 + c^2*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1
 + c^2*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{-1+c^2x^2}\int \frac{1}{(d-c^2dx^2)^2}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{x\sqrt{-1+c^2x^2}}{2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+c^2x^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{d-c^2dx^2}}$$
$$= \frac{x\sqrt{-1+c^2x^2}}{2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+c^2x^2}\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.72

$$\frac{-cx+(-1+c^2x^2)\tanh^{-1}(cx)}{2cd^2\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d -
 c^2*d*x^2])
```

Maple [A]
time = 0.08, size = 94, normalized size = 1.19

method	result	size
default	$\frac{\sqrt{-(c^2x^2-1)d}\,(\ln(cx-1)c^2x^2-\ln(cx+1)c^2x^2+2cx-\ln(cx-1)+\ln(cx+1))}{4\sqrt{c^2x^2-1}\,d^3(cx-1)c(cx+1)}$	94
risch	$-\frac{x}{2d^2\sqrt{c^2x^2-1}\sqrt{-(c^2x^2-1)d}} - \frac{\sqrt{c^2x^2-1}\ln(cx-1)}{4d^2\sqrt{-(c^2x^2-1)d}\,c} + \frac{\sqrt{c^2x^2-1}\ln(-cx-1)}{4d^2\sqrt{-(c^2x^2-1)d}\,c}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*c
^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^3/(c*x-1)/c/(c*x+1)
```

Maxima [A]
time = 0.30, size = 70, normalized size = 0.89

$$-\frac{x}{2\left(c^2\sqrt{-d}\,d^2x^2-\sqrt{-d}\,d^2\right)} - \frac{\sqrt{-d}\,\log(cx+1)}{4\,cd^3} + \frac{\sqrt{-d}\,\log(cx-1)}{4\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x/(c^2*sqrt(-d)*d^2*x^2 - sqrt(-d)*d^2) - 1/4*sqrt(-d)*log(c*x + 1)/(c
*d^3) + 1/4*sqrt(-d)*log(c*x - 1)/(c*d^3)
```

Fricas [A]
time = 1.08, size = 315, normalized size = 3.99

$$\left[\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,cx-(c^4x^4-2\,c^2x^2+1)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\,(c^5d^3x^4-2\,c^3d^3x^2+cd^3)}, \frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,cx-(c^4x^4-2\,c^2x^2+1)\sqrt{d}\,\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)}{4\,(c^5d^3x^4-2\,c^3d^3x^2+cd^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3), 1/4*(2*sqrt(-c^2
*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arcta
n(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c
^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx-1)\,(cx+1)}}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(sqrt((c*x - 1)*(c*x + 1))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 - 1)/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2-1}}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2),x)
[Out] int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2), x)
```

3.164 $\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{dx\sqrt{-1+c^2x^2}}{2\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{-1+c^2x^2}\tanh^{-1}(cx)}{2c\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*d*x*(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2)+1/2*arctanh(c*x)*(c^2*x^2-1)
^(1/2)/c/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 205, 214}

$$\frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(c^2x^2-1\right)^{3/2}}+\frac{\left(d-c^2dx^2\right)^{3/2}\tanh^{-1}(cx)}{2cd^2\left(c^2x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (x*(d - c^2*d*x^2)^(3/2))/(2*d^2*(1 - c^2*x^2)*(-1 + c^2*x^2)^(3/2)) + ((d
- c^2*d*x^2)^(3/2)*ArcTanh[c*x])/(2*c*d^2*(-1 + c^2*x^2)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \frac{(d-c^2dx^2)^{3/2}\int \frac{1}{(d-c^2dx^2)^2}\,dx}{(-1+c^2x^2)^{3/2}}$$
$$= \frac{x(d-c^2dx^2)^{3/2}}{2d^2(1-c^2x^2)(-1+c^2x^2)^{3/2}} + \frac{(d-c^2dx^2)^{3/2}\int \frac{1}{d-c^2dx^2}\,dx}{2d(-1+c^2x^2)^{3/2}}$$
$$= \frac{x(d-c^2dx^2)^{3/2}}{2d^2(1-c^2x^2)(-1+c^2x^2)^{3/2}} + \frac{(d-c^2dx^2)^{3/2}\tanh^{-1}(cx)}{2cd^2(-1+c^2x^2)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.73

$$\frac{-cx+(-1+c^2x^2)\tanh^{-1}(cx)}{2c\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2
*d*x^2])
```

Maple [A]
time = 0.07, size = 94, normalized size = 1.27

method	result	size
default	$\frac{\sqrt{-(c^2x^2-1)d}\,(\ln(cx-1)c^2x^2-\ln(cx+1)c^2x^2+2cx-\ln(cx-1)+\ln(cx+1))}{4\sqrt{c^2x^2-1}\,d(cx-1)c(cx+1)}$	94
risch	$-\frac{x}{2\sqrt{c^2x^2-1}\,\sqrt{-(c^2x^2-1)d}}-\frac{\sqrt{c^2x^2-1}\,\ln(cx-1)}{4\sqrt{-(c^2x^2-1)d}\,c}+\frac{\sqrt{c^2x^2-1}\,\ln(-cx-1)}{4\sqrt{-(c^2x^2-1)d}\,c}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*c
^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d/(c*x-1)/c/(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

Fricas [A]
time = 1.36, size = 303, normalized size = 4.09

$$\left[\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,cx-(c^4x^4-2\,c^2x^2+1)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\,(c^5dx^4-2\,c^3dx^2+cd)}, \frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,cx-(c^4x^4-2\,c^2x^2+1)\sqrt{d}\,\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)}{4\,(c^5dx^4-2\,c^3dx^2+cd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d*x^4 - 2*c^3*d*x^2 + c*d), 1/4*(2*sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arctan(2*sq
rt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c^5*d*x
^4 - 2*c^3*d*x^2 + c*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx-1)\,(cx+1)\right)^{\frac{3}{2}}\,\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2-1)**(3/2)/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(1/(((c*x - 1)*(c*x + 1))**(3/2)*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d-c^2\,d\,x^2}\,{\left(c^2\,x^2-1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)),x)
[Out] int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)), x)
```

3.165 $\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{x\sqrt{-1+c^2x^2}}{2\left(d-c^2dx^2\right)^{3/2}}-\frac{\sqrt{-1+c^2x^2}\,\tanh^{-1}(cx)}{2cd\sqrt{d-c^2dx^2}}$$

```
[Out] -1/2*x*(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2)-1/2*arctanh(c*x)*(c^2*x^2-1)^
(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {23, 205, 214}

$$\frac{x\sqrt{d-c^2dx^2}}{2d^2\left(1-c^2x^2\right)\sqrt{c^2x^2-1}}+\frac{\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{2cd^2\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[d - c^2*d*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[-1 + c^2*x^2]) + (Sqrt[d
- c^2*d*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{1}{(d-c^2dx^2)^2}\,dx}{\sqrt{-1+c^2x^2}}$$
$$= \frac{x\sqrt{d-c^2dx^2}}{2d^2(1-c^2x^2)\sqrt{-1+c^2x^2}} + \frac{\sqrt{d-c^2dx^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{-1+c^2x^2}}$$
$$= \frac{x\sqrt{d-c^2dx^2}}{2d^2(1-c^2x^2)\sqrt{-1+c^2x^2}} + \frac{\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{2cd^2\sqrt{-1+c^2x^2}}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.75

$$\frac{cx+(1-c^2x^2)\tanh^{-1}(cx)}{2cd\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (c*x + (1 - c^2*x^2)*ArcTanh[c*x])/(2*c*d*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d
*x^2])
```

Maple [A]
time = 0.08, size = 94, normalized size = 1.24

method	result	size
default	$-\frac{\sqrt{-\left(c^2x^2-1\right)d}\,\left(\ln(cx-1)c^2x^2-\ln(cx+1)c^2x^2+2cx-\ln(cx-1)+\ln(cx+1)\right)}{4\sqrt{c^2x^2-1}\,d^2(cx-1)c(cx+1)}$	94
risch	$\frac{x}{2d\sqrt{c^2x^2-1}\,\sqrt{-\left(c^2x^2-1\right)d}}+\frac{\sqrt{c^2x^2-1}\,\ln(cx-1)}{4d\sqrt{-\left(c^2x^2-1\right)d}\,c}-\frac{\sqrt{c^2x^2-1}\,\ln(-cx-1)}{4d\sqrt{-\left(c^2x^2-1\right)d}\,c}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*
c^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^2/(c*x-1)/c/(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

Fricas [A]
time = 1.19, size = 314, normalized size = 4.13

$$\left[-\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,cx+(c^4x^4-2\,c^2x^2+1)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\,(c^5d^2x^4-2\,c^3d^2x^2+cd^2)},-\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,cx-(c^4x^4-2\,c^2x^2+1)\sqrt{d}\,\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)}{4\,(c^5d^2x^4-2\,c^3d^2x^2+cd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] [-1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x + (c^4*x^4 - 2*c^2*x^2
+ 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*
x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^
4 + 3*c^2*x^2 - 1)))/(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2), -1/4*(2*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arc
tan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/
(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx-1)\,(cx+1)}\,\left(-d\,(cx-1)\,(cx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral(1/(sqrt((c*x - 1)*(c*x + 1))*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d-c^2\,d\,x^2\right)}^{3/2}\,\sqrt{c^2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)),x)
[Out] int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)), x)
```

3.166 $\int (a+bx^2)^{3/2} \sqrt{c+dx^2} \, dx$

Optimal. Leaf size=328

$$\frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}}-\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d}+\frac{\sqrt{c}\,(2b^2c^2}{}$$

```
[Out] 1/5*b*x*(d*x^2+c)^(3/2)*(b*x^2+a)^(1/2)/d+1/15*(7*a*c-2*b*c^2/d+3*a^2*d/b)*
x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-1/15*c^(3/2)*(-9*a*d+b*c)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)+1/15*(-3*a^2*d^2-7*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*c^(1/2)*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)-2/15*(-3*a*d+b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A]

time = 0.19, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$\frac{\sqrt{c}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{b}+7ac-\frac{2bc^2}{d}\right)}{15\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d}-\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

```
[Out] ((7*a*c - (2*b*c^2)/d + (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*Sqrt[c + d*x^2]
) - (2*(b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d) + (b*x*Sqrt[
a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) + (Sqrt[c]*(2*b^2*c^2 - 7*a*b*c*d - 3*a
^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(15*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
 - (c^(3/2)*(b*c - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(15*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx = \frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d} + \frac{\int \frac{\sqrt{c+dx^2}\left(-a(bc-5ad)-2b(bc-3ad)x^2\right)}{\sqrt{a+bx^2}}dx}{5d}$$

$$= -\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d} + \frac{\int \frac{-abc(bc-}{\sqrt{}}}{}$$

$$= -\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d} - \frac{(ac(bc-}{}$$

$$= \frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}} - \frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{}}{}$$

$$= \frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}} - \frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.38, size = 243, normalized size = 0.74

$$\frac{\sqrt{\frac{b}{a}}\,dx\left(a+bx^2\right)\left(c+dx^2\right)\left(6ad+b\left(c+3dx^2\right)\right)-ic\left(-2b^2c^2+7abcd+3a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-2ic\left(b^2c^2-4abcd+3a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{15\sqrt{\frac{b}{a}}\,d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*a*d + b*(c + 3*d*x^2)) - I*c*(-2*
b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(b^2*c^2 - 4*a*b*c*d
 + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Maple [A]
time = 0.12, size = 543, normalized size = 1.66

method	result

risch	$\frac{x(3bd\,x^2+6ad+bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15d}+\frac{\left(-\frac{(3a^2d^2+7abcd-2b^2c^2)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x}}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{b\,x^3\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{5}+\frac{\left(2abd+b^2c-\frac{b(4ad+4bc)}{5}\right)x\sqrt{bd\,x^4+ad\,x^2+c}}{3bd}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+9\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+4\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+6\sqrt{-\frac{b}{a}}\,a^2d^3x^3+10\sqrt{-\frac{b}{a}}\,abc\,d^2x^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*b^2*d^3*x^7+9*(-b/a)^(
1/2)*a*b*d^3*x^5+4*(-b/a)^(1/2)*b^2*c*d^2*x^5+6*(-b/a)^(1/2)*a^2*d^3*x^3+10
*(-b/a)^(1/2)*a*b*c*d^2*x^3+(-b/a)^(1/2)*b^2*c^2*d*x^3+6*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-8
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*a*b*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-2*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*b^2*c^3+6*(-b/a)^(1/2)*a^2*c*d^2*x+(-b/a)^(1/2)*a*b*c^2*d*x)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2\right)^{\frac{3}{2}} \sqrt{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2), x)
```

3.167 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=249

$$\frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}-\frac{\sqrt{c}\,(bc+ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}\sqrt{c+dx^2}}+\frac{2c^{3/\cdots}}{\cdots}$$

```
[Out] 1/3*(a*d+b*c)*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+2/3*c^(3/2)*(1/(1+d*x^2/c
))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d
*x^2+c)^(1/2)-1/3*(a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)
^(1/2)/b/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*x*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {428, 545, 429, 506, 422}

$$\frac{2c^{3/2}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}\,(ad+bc)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{x\sqrt{a+bx^2}\,(ad+bc)}{3b\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) + (x*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/3 - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
```

```
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{2}{3}\int \frac{ac+\frac{1}{2}(bc+ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$= \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{1}{3}(2ac)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx+\frac{1}{3}(bc+ad)\int$$

$$= \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{2c^{3/2}\sqrt{a+bx^2}\,F\Big(\tan^{-1}\Big(}{3\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$= \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}-\frac{\sqrt{c}(bc+ad)\sqrt{a+bx^2}\,E\Big(}{3b\sqrt{d}\sqrt{\frac{c(a+b}{a(c+d}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.14, size = 198, normalized size = 0.80

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)\,(c+dx^2)-ic(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3\sqrt{\frac{b}{a}}\,d\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*
(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 328, normalized size = 1.32

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3}+\frac{\left(\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}-\frac{(ad+bc)c\sqrt{1+}}{}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3}+\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\right.}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b}}\right.}{\sqrt{b}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b*d^2*x^5+(-b/a)^(1/2)*a*
d^2*x^3+(-b/a)^(1/2)*b*c*d*x^3+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*a*c*d+EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*b*c^2+a*c*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*b*c^2+(-b/a)^(1/2)*a*c*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx^2}\,\sqrt{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.168 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] d*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = c\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$
$$= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} - \frac{(cd)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b}$$
$$= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\right.}{a\sqrt{d}\,\sqrt{\frac{c(a+b}{a(c+d}}}$$

Mathematica [A]
time = 0.92, size = 86, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{a+bx^2}\,\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A]
time = 0.07, size = 101, normalized size = 0.50

method	result
default	$\frac{\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}\,c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac}}-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1 \cdots}}{\cdots}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/
a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.169 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a+bx^2}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

```
[Out] (1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(1/2)/b^(1/2)/(b*x^2+a)^(1/
2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {422}

$$\frac{\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a+bx^2}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(
Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a+bx^2}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.13, size = 133, normalized size = 1.58

$$\frac{x(c+dx^2)+\frac{ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}}}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(c + d*x^2) + (I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/Sqrt[b/a])/(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 181, normalized size = 2.15

method	result
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\left(\sqrt{-\frac{b}{a}}\,d\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{E}}{(bd\,x^4+ad\,x^2+c\,x^2b+ac)a\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{(bd\,x^2+bc)x}{ba\sqrt{\left(x^2+\frac{a}{b}\right)(bd\,x^2+bc)}}+\frac{\left(\frac{d}{b}-\frac{ad-bc}{ab}-\frac{c}{a}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b}}\right.}{\sqrt{b\,x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-b/a)^(1/2)*d*x^3+((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c-((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c+(-b/a
)^(1/2)*c*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(3/2), x)
```

3.170 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}+\frac{(2bc-ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\,(bc-ad)\sqrt{a+bx^2}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{c^{3/2}\sqrt{d}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\right.}{3a^2(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\,\sqrt{c+dx}}$$

```
[Out] -1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-a*
d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*x*(d*x^2+c)^(1/2
)/a/(b*x^2+a)^(3/2)+1/3*(-a*d+2*b*c)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2
)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c
)^(1/2)/a^(3/2)/(-a*d+b*c)/b^(1/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {423, 539, 429, 422}

$$\frac{\sqrt{c+dx^2}\,(2bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\,\sqrt{a+bx^2}\,(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{c^{3/2}\sqrt{d}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + ((2*b*c - a*d)*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a^(3/2)*Sqrt[
b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (c^
(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^2*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-2c-dx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{3a} \\
&= \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} - \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)} + \frac{(2bc-ad)\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx}{3a(bc-ad)} \\
&= \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} + \frac{(2bc-ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\,(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}} - \frac{c^{3/2}\sqrt{d}\,\sqrt{a+b}}{3a^2(bc-ac}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.91, size = 243, normalized size = 1.03

$$\frac{\sqrt{\frac{b}{a}}\,x(c+dx^2)\left(2a^2d-2b^2cx^2+ab(-3c+dx^2)\right)+ic(-2bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-2ic(-bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3a^2\sqrt{\frac{b}{a}}\,(-bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(2*a^2*d - 2*b^2*c*x^2 + a*b*(-3*c + d*x^2)) + I*c
*(-2*b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*(a + b*x^2
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(-(b*c) + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d
*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(277) = 554$.
time = 0.09, size = 617, normalized size = 2.60

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{x\sqrt{bd x^4+ad x^2+c x^2 b+ac}}{3b^2 a\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd x^2+bc\right)x(ad-2bc)}{3b a^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{\left(\frac{d}{3ab}-\frac{a}{3}\right.}{}\right.}{}$
default	$\frac{\sqrt{-\frac{b}{a}}\, ab d^2 x^5-2\sqrt{-\frac{b}{a}}\, b^2 cd x^5+2\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)abcd x^2-2\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,\mathrm{E}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-b/a)^(1/2)*a*b*d^2*x^5-2*(-b/a)^(1/2)*b^2*c*d*x^5+2*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x
^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*b^2*c^2*x^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^2+2*(-b/a)^(1
/2)*a^2*d^2*x^3-2*(-b/a)^(1/2)*a*b*c*d*x^3-2*(-b/a)^(1/2)*b^2*c^2*x^3+2*((b
*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*a^2*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*a*b*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2+2*(-b/a)^(1/2)
*a^2*c*d*x-3*(-b/a)^(1/2)*a*b*c^2*x)/(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(a*d-b*c)
/a^2/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(5/2), x)
```

3.171 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$

Optimal. Leaf size=309

$$\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}+\frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{\left(8b^2c^2-13abcd+3a^2d^2\right)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}\,(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

```
[Out] -2/15*c^(3/2)*(-3*a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a
)^(1/2)/a^3/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/
5*x*(d*x^2+c)^(1/2)/a/(b*x^2+a)^(5/2)+1/15*(-3*a*d+4*b*c)*x*(d*x^2+c)^(1/2)
/a^2/(-a*d+b*c)/(b*x^2+a)^(3/2)+1/15*(3*a^2*d^2-13*a*b*c*d+8*b^2*c^2)*(1/(1
+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^
(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(5/2)/(-a*d+b*c)^2/b^(1/2)/(b*x^
2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {423, 541, 539, 429, 422}

$$-\frac{2c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,(2bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}\,(bc-ad)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{c+dx^2}\,(4bc-3ad)}{15a^2\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}\sqrt{a+bx^2}\,(bc-ad)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(5*a*(a + b*x^2)^(5/2)) + ((4*b*c - 3*a*d)*x*Sqrt[c + d
*x^2])/(15*a^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 13*a*b*c*d +
3*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)
/(b*c)])/(15*a^(5/2)*Sqrt[b]*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x
^2))/(c*(a + b*x^2))]) - (2*c^(3/2)*Sqrt[d]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*
d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{7/2}}\,dx = \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{-4c-3dx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx}{5a}$$

$$= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{c(8bc-9ad)+d(4bc-3ad)x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{15a^2(bc-ad)}$$

$$= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{(2cd(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{15a^2(bc-ad)^2}$$

$$= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{(8b^2c^2-13abcd+3a^2d^2)\sqrt{c+dx^2}\,E\Big(}{15a^{5/2}\sqrt{b}\,(bc-ad)^2\sqrt{a+bx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.12, size = 285, normalized size = 0.92

$$\frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+a(-bc+ad)(-4bc+3ad)\left(a+bx^2\right)+\left(8b^2c^2-13abcd+3a^2d^2\right)\left(a+bx^2\right)^2\right)+ic\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\left(8b^2c^2-13abcd+3a^2d^2\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+\left(-8b^2c^2+17abcd-9a^2d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{15a^3\sqrt{\frac{b}{a}}(bc-ad)^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-4*b*c +
3*a*d)*(a + b*x^2) + (8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^2) +
I*c*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((8*b^2*c^2 - 13*
a*b*c*d + 3*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8*b
^2*c^2 + 17*a*b*c*d - 9*a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. 2(343) = 686.
time = 0.09, size = 1411, normalized size = 4.57

method	result
elliptic	$\frac{\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{5b^3a\left(x^2+\frac{a}{b}\right)^3}+\frac{(3ad-4bc)x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{15a^2(ad-bc)b^2\left(x^2+\frac{a}{b}\right)^2}+\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*a^2*b^2*c*d^2*x^4-17*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^2*d*x^4-3*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2
*c*d^2*x^4+9*(-b/a)^(1/2)*a^4*d^3*x^3+8*(-b/a)^(1/2)*b^4*c^3*x^5+16*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
a*b^3*c^3*x^2-16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*x^2-17*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d+13*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b
*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*b^4*c^3*x^4-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^3*x^4+9*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2+8*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*b^2*c^3-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3-26*(-b/a)^(1/2)*a^3*b
*c^2*d*x+13*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b^3*c^2*d*x^4+18*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^2*x^2-34*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*
b^2*c^2*d*x^2-6*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^2*x^2+26*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d*x^2+3*(-b/a)
^(1/2)*a^2*b^2*d^3*x^7+8*(-b/a)^(1/2)*b^4*c^2*d*x^7+9*(-b/a)^(1/2)*a^3*b*d^
3*x^5+20*(-b/a)^(1/2)*a*b^3*c^3*x^3+9*(-b/a)^(1/2)*a^4*c*d^2*x+15*(-b/a)^(1
/2)*a^2*b^2*c^3*x-13*(-b/a)^(1/2)*a*b^3*c*d^2*x^7-30*(-b/a)^(1/2)*a^2*b^2*c
*d^2*x^5+7*(-b/a)^(1/2)*a*b^3*c^2*d*x^5-17*(-b/a)^(1/2)*a^3*b*c*d^2*x^3-18*
(-b/a)^(1/2)*a^2*b^2*c^2*d*x^3)/(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(a*d-b*c)^2/a^
3/(b*x^2+a)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(7/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(7/2), x)
```

3.172 $\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} \, dx$

Optimal. Leaf size=410

$$-\frac{2(bc+ad)\left(b^2c^2-6abcd+a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}}+\frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{2(4bc-a$$

```
[Out] -2/35*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/d/(d*x^2+
c)^(1/2)-1/35*c^(3/2)*(a^2*d^2-18*a*b*c*d+b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1
+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(
1/2))*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)+2/35*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*c^(1/2)*(b*x^2+a)^(1/2)/b^2/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)+2/35*(-a*d+4*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b+1/7*d*x*(
b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b+1/35*(9*a*c+b*c^2/d-2*a^2*d/b)*x*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$\frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{35b^2d\sqrt{c+dx^2}}+\frac{1}{35}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{2a^2d}{b}+9ac+\frac{bc^2}{d}\right)+\frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b}+\frac{2x(a+bx^2)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{35b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(35*b^2*
d*Sqrt[c + d*x^2]) + ((9*a*c + (b*c^2)/d - (2*a^2*d)/b)*x*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/35 + (2*(4*b*c - a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/
(35*b) + (d*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + (2*Sqrt[c]*(b*c +
a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b^2*c^2 - 18*a*b*c*d + a^2*d^2)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
5*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}dx = \frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b} + \frac{\int \frac{(a+bx^2)^{3/2}(c(7bc-ad)+2d(4bc-ad)x^2)}{\sqrt{c+dx^2}}dx}{7b}$$

$$= \frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b} + \frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b} + \frac{\int \sqrt{c}\ldots}{\ldots}$$

$$= \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2(4bc-ad)x(a+bx^2)^3\ldots}{35b}$$

$$= \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2(4bc-ad)x(a+bx^2)^3\ldots}{35b}$$

$$= -\frac{2(bc+ad)\left(b^2c^2-6abcd+a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}} + \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\ldots\right.$$

$$= -\frac{2(bc+ad)\left(b^2c^2-6abcd+a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}} + \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\ldots\right.$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 4.39, size = 302, normalized size = 0.74

$$\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)\left(a^2d^2+abd(17c+8dx^2)+b^2(c^2+8cdx^2+5d^2x^4)\right)+2ic\left(b^3c^3-5ab^2c^2d-5a^2bcd^2+a^3d^3\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\left(2b^3c^3-11ab^2c^2d+8a^2bcd^2+a^3d^3\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{35b\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(17*c + 8*d*x^2) +
b^2*(c^2 + 8*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(b^3*c^3 - 5*a*b^2*c^2*d - 5*a
^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b^3*c^3 - 11*a*b^2*c^2*d + 8*a^2
*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

<u>**Maple**</u> [A]
time = 0.10, size = 780, normalized size = 1.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(5*(-b/a)^(1/2)*b^3*d^4*x^9+13*(-b/a)^
(1/2)*a*b^2*d^4*x^7+13*(-b/a)^(1/2)*b^3*c*d^3*x^7+9*(-b/a)^(1/2)*a^2*b*d^4*
x^5+38*(-b/a)^(1/2)*a*b^2*c*d^3*x^5+9*(-b/a)^(1/2)*b^3*c^2*d^2*x^5+(-b/a)^(
1/2)*a^3*d^4*x^3+26*(-b/a)^(1/2)*a^2*b*c*d^3*x^3+26*(-b/a)^(1/2)*a*b^2*c^2*
d^2*x^3+(-b/a)^(1/2)*b^3*c^3*d*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+8*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*d^2-
11*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a*b^2*c^3*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+10*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c
^2*d^2+10*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b^2*c^3*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4+(-b/a)^(1/2)*a^3*c*d^3*x+17*(
-b/a)^(1/2)*a^2*b*c^2*d^2*x+(-b/a)^(1/2)*a*b^2*c^3*d*x)/b/d^2/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^{\frac{3}{2}} \left(c+dx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2),x)
[Out] Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x)
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2), x)
```

3.173 $\int \sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,dx$

Optimal. Leaf size=336

$$\frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}}+\frac{2(3bc-ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15b}+\frac{dx(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{5b}-\frac{\sqrt{c}\,(3}{}$$

```
[Out] 1/15*(-2*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/(d*x^2+c)^(1/2)
+1/15*c^(3/2)*(-a*d+9*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/b/
d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(-2*a^2*d^2+7*
a*b*c*d+3*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b^
2/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/5*d*x*(b*x^2+a)
^(3/2)*(d*x^2+c)^(1/2)/b+2/15*(-a*d+3*b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/b
```

Rubi [A]
time = 0.18, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(-2a^2d^2+7abcd+3b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,(-2a^2d^2+7abcd+3b^2c^2)}{15b^2\sqrt{c+dx^2}}+\frac{c^{3/2}\sqrt{a+bx^2}\,(9bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{5b}+\frac{2x\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(3bc-ad)}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*Sqrt[c + d*
x^2]) + (2*(3*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b) + (d*x*(
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) - (Sqrt[c]*(3*b^2*c^2 + 7*a*b*c*d -
 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(15*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx = \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b} + \frac{\int \frac{\sqrt{a+bx^2}\,(c(5bc-ad)+2d(3bc-ad)x^2)}{\sqrt{c+dx^2}}\,dx}{5b}$$

$$= \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} + \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b} + \frac{\int \frac{acd(9bc-ad}{\sqrt{a}}}{}$$

$$= \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} + \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b} + \frac{(ac(9bc-}{}$$

$$= \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}} + \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} +$$

$$= \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}} + \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.46, size = 246, normalized size = 0.73

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)(6bc+ad+3bdx^2)+ic(-3b^2c^2-7abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(-3b^2c^2+2abcd+a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{15b\sqrt{\frac{b}{a}}\,d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*b*c + a*d + 3*b*d*x^2) + I*c*(-3*
b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-3*b^2*c^2 + 2*a*b*c*d
+ a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 545, normalized size = 1.62

method	result

risch	$\frac{x(3bd\,x^2+ad+6bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15b} - \frac{\left(-\frac{(2a^2d^2-7abcd-3b^2c^2)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x}}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{d\,x^3\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{5}+\frac{\left(a\,d^2+2bcd-\frac{d(4ad+4bc)}{5}\right)x\sqrt{bd\,x^4+ad\,x^2+c}}{3bd}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+9\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+\sqrt{-\frac{b}{a}}\,a^2d^3x^3+10\sqrt{-\frac{b}{a}}\,abc\,d^2x^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*b^2*d^3*x^7+4*(-b/a)^(
1/2)*a*b*d^3*x^5+9*(-b/a)^(1/2)*b^2*c*d^2*x^5+(-b/a)^(1/2)*a^2*d^3*x^3+10*(
-b/a)^(1/2)*a*b*c*d^2*x^3+6*(-b/a)^(1/2)*b^2*c^2*d*x^3+((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+2*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a*b*c^2*d-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+3*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*b^2*c^3+(-b/a)^(1/2)*a^2*c*d^2*x+6*(-b/a)^(1/2)*a*b*c^2*d*x)/d/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)/b/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2), x)
```

3.174 $\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=273

$$\frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}}+\frac{dx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3b}-\frac{2\sqrt{c}\,\sqrt{d}\,(2bc-ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\right.}{3b^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] 2/3*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b^2/(d*x^2+c)^(1/2)+1/3*c^(3/2)*(-a*d+
3*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/a/b/d^(1/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)+1/3*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b
```

Rubi [A]
time = 0.11, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {427, 545, 429, 506, 422}

$$-\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,(2bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}\,(3bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2dx\sqrt{a+bx^2}\,(2bc-ad)}{3b^2\sqrt{c+dx^2}}+\frac{dx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) + (d*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (2*Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - a*
d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(3*a*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3b} + \frac{\int \frac{c(3bc-ad)+2d(2bc-ad)x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{3b}$$

$$= \frac{dx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3b} + \frac{(2d(2bc-ad))\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{3b} + \frac{(c(3bc-ad)) \int}{}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}} + \frac{dx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3b} + \frac{c^{3/2}(3bc-ad)\sqrt{a+bx^2}\,F\Big(\tan}{3ab\sqrt{d}\,\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}} + \frac{dx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3b} - \frac{2\sqrt{c}\,\sqrt{d}\,(2bc-ad)\sqrt{a+bx^2}\,}{3b^2\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.16, size = 199, normalized size = 0.73

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)\,(c+dx^2)+2ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3b\sqrt{\frac{b}{a}}\,\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(-2*b*c + a*d)*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

Maple [A]
time = 0.08, size = 330, normalized size = 1.21

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{dx\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3b}+\frac{\left(c^2-\frac{acd}{3b}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c}}\right.}{}$

default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{}$
risch	$\frac{dx\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3b}-\frac{\left(-\frac{\left(2a\,d^2-4bcd\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{Ellip}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-b/a)^(1/2)*b*d^2*x^5+(-b/a)^(1/2)*a*
d^2*x^3+(-b/a)^(1/2)*b*c*d*x^3+a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*a*c*d+4*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*b*c^2+(-b/a)^(1/2)*a*c*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/b/(-b/a)^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/sqrt(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(1/2), x)
```

3.175 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}}+\frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}}+\frac{\sqrt{c}\,\sqrt{d}\,(bc-2ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] -d*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/a/b^2/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/b/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+(-2*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1
/2)*(b*x^2+a)^(1/2)/a/b^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(
-a*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {424, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,(bc-2ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{d}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{a+bx^2}\,(bc-2ad)}{ab^2\sqrt{c+dx^2}}+\frac{x\sqrt{c+dx^2}\,(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(a*b^2*Sqrt[c + d*x^2])) + ((b*c - a*
d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*(b*c - 2*a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sq
rt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(a*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd-d(bc-2ad)x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{ab}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{b} - \frac{(d(bc-2ad))\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{\cdot}}}{ab}$$

$$= -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}} + \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{c^{3/2}\sqrt{d}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{ab\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+d}}$$

$$= -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}} + \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{\sqrt{c}\,\sqrt{d}\,(bc-2ad)\sqrt{a+bx^2}\,E\Big(\mathrm{ta}}{ab^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\surd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.91, size = 191, normalized size = 0.72

$$\frac{-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)+(bc-ad)\left(\sqrt{\frac{b}{a}}\,x(c+dx^2)-ic\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{a^2\left(\frac{b}{a}\right)^{3/2}\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2)
 - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)]))/(a^2*(b/a)^(3/2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 332, normalized size = 1.24

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3-ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{Elliptic}\right.}{}$

elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(-\frac{\left(bd\,x^2+bc\right)(ad-bc)x}{b^2a\sqrt{\left(x^2+\frac{a}{b}\right)(bd\,x^2+bc)}}+\frac{\left(-\frac{d(ad-2bc)}{b^2}+\frac{(ad-bc)^2}{b^2a}+\frac{c(ad-bc)}{ba}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b/a)^(1/2)*a*d^2*x^3+(-b/a)^(1/2)*b*c*d*x^3-a*c*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+2
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*a*c*d-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*b*c^2-(-b/a)^(1/2)*a*c*d*x+(-b/a)^(1/2)*b*c^2*x)*(d*x^
2+c)^(1/2)*(b*x^2+a)^(1/2)/b/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(3/2), x)
```

3.176 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=229

$$\frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}}+\frac{2(bc+ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a\,(c+dx^2)}{c\,(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.\right.}{3a^2b\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}\,\sqrt{c+}}$$

```
[Out] -1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/b/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*(-a*d+b*c)*x*(d*x^2+c)^(1
/2)/a/b/(b*x^2+a)^(3/2)+2/3*(a*d+b*c)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/
2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+
c)^(1/2)/a^(3/2)/b^(3/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {424, 539, 429, 422}

$$\frac{2\sqrt{c+dx^2}\,(ad+bc)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\,\sqrt{\frac{a\,(c+dx^2)}{c\,(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2b\sqrt{c+dx^2}\,\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}+\frac{x\sqrt{c+dx^2}\,(bc-ad)}{3ab\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b*c + a*d)*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3
*a^(3/2)*b^(3/2)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (
c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*a^2*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx &= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}}+\frac{\int \frac{c(2bc+ad)+d(bc+2ad)x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{3ab}\\
&= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}}-\frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ab}+\frac{(2(bc+ad))\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx}{3ab}\\
&= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}}+\frac{2(bc+ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\,\sqrt{c}}{3a}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.73, size = 232, normalized size = 1.01

$$\frac{\sqrt{\frac{b}{a}}\,x(c+dx^2)\left(a^2d+2b^2cx^2+ab(3c+2dx^2)\right)+2ic(bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(2bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3a^3\left(\frac{b}{a}\right)^{3/2}(a+bx^2)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(a^2*d + 2*b^2*c*x^2 + a*b*(3*c + 2*d*x^2)) + (2*I
)*c*(b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b*c + a*d)*(a + b*x^2)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)])/(3*a^3*(b/a)^(3/2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(269) = 538$.
time = 0.09, size = 607, normalized size = 2.65

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(-\frac{(ad-bc)x\sqrt{bdx^4+adx^2+cx^2b+ac}}{3ab^3\left(x^2+\frac{a}{b}\right)^2}+\frac{2\left(bdx^2+bc\right)x(ad+bc)}{3b^2a^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bdx^2+bc\right)}}+\frac{\left(\frac{d^2}{b^2}\right.}{}\right.}{}$
default	$\frac{2\sqrt{-\frac{b}{a}}\,abd^2x^5+2\sqrt{-\frac{b}{a}}\,b^2cdx^5+\sqrt{\frac{bx^2+a}{a}}\,\sqrt{\frac{dx^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcdx^2+2\sqrt{\frac{bx^2+a}{a}}\,\sqrt{\frac{dx^2+c}{c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(-b/a)^(1/2)*a*b*d^2*x^5+2*(-b/a)^(1/2)*b^2*c*d*x^5+((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x
^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*b^2*c^2*x^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^2+(-b/a)^(1
/2)*a^2*d^2*x^3+5*(-b/a)^(1/2)*a*b*c*d*x^3+2*(-b/a)^(1/2)*b^2*c^2*x^3+((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2+(-b/a)^(1/2)*a
^2*c*d*x+3*(-b/a)^(1/2)*a*b*c^2*x)/(d*x^2+c)^(1/2)/a^2/(-b/a)^(1/2)/(b*x^2+
a)^(3/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(5/2), x)
```

3.177 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx$

Optimal. Leaf size=315

$$\frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}}+\frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}}+\frac{(8b^2c^2-3abcd-2a^2d^2)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\right.}{15a^{5/2}b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] -1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^
(1/2)/a^3/b/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/5*
(-a*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)^(5/2)+2/15*(a*d+2*b*c)*x*(d*x^2+
c)^(1/2)/a^2/b/(b*x^2+a)^(3/2)+1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(1/(1+
b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(
1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(5/2)/b^(3/2)/(-a*d+b*c)/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {424, 541, 539, 429, 422}

$$-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,(4bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3b\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2x\sqrt{c+dx^2}\,(ad+2bc)}{15a^2b\,(a+bx^2)^{3/2}}+\frac{\sqrt{c+dx^2}\,(-2a^2d^2-3abcd+8b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}\sqrt{a+bx^2}\,(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\,(bc-ad)}{5ab\,(a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(5*a*b*(a + b*x^2)^(5/2)) + (2*(2*b*c + a*d
)*x*Sqrt[c + d*x^2])/(15*a^2*b*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 3*a*b*c*d
 - 2*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a
*d)/(b*c)])/(15*a^(5/2)*b^(3/2)*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*
x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*(4*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*b*(b*c - a*d
)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{\int \frac{c(4bc+ad)+d(3bc+2ad)x^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx}{5ab}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}} - \frac{\int \frac{-c(bc-ad)(8bc+ad)-2d(bc-ad)(2bc+ad)x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}}{15a^2b(bc-ad)}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}} - \frac{(cd(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+d}}}{15a^2b(bc-ad)}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}} + \frac{(8b^2c^2-3abcd-2a^2d^2)\sqrt{c+dx^2}}{15a^{5/2}b^{3/2}(bc-ad)\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.93, size = 285, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+2a(bc-ad)(2bc+ad)(a+bx^2)+(8b^2c^2-3abcd-2a^2d^2)(a+bx^2)^2\right)-ic(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left((-8b^2c^2+3abcd+2a^2d^2)E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(8b^2c^2-7abcd-a^2d^2)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{15a^4\left(\frac{b}{a}\right)^{3/2}(bc-ad)(a+bx^2)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 2*a*(b*c - a*d)*(2*b*c + a*
d)*(a + b*x^2) + (8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*(a + b*x^2)^2) - I*c*(
a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((-8*b^2*c^2 + 3*a*b*c
*d + 2*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*b^2*c^2
 - 7*a*b*c*d - a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(1
5*a^4*(b/a)^(3/2)*(b*c - a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(349) = 698$.
time = 0.09, size = 1410, normalized size = 4.48

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(-\frac{(ad-bc)x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{5b^4a\left(x^2+\frac{a}{b}\right)^3}+\frac{2(ad+2bc)x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+}}{15b^3a^2\left(x^2+\frac{a}{b}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a^2*b^2*c*d^2*x^4+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^2*d*x^4-2*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c*
d^2*x^4+(-b/a)^(1/2)*a^4*d^3*x^3-8*(-b/a)^(1/2)*b^4*c^3*x^5-16*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3
*c^3*x^2+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b^3*c^3*x^2+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d-3*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d-
8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*b^4*c^3*x^4+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^3*x^4+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2-8*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2
*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*a^4*c*d^2+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3+11*(-b/a)^(1/2)*a^3*b*c^2*d*x-
3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*a*b^3*c^2*d*x^4+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^2*x^2+14*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d*x
^2-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^3*b*c*d^2*x^2-6*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d*x^2+2*(-b/a)^(1/2)*a^2*b
^2*d^3*x^7-8*(-b/a)^(1/2)*b^4*c^2*d*x^7+6*(-b/a)^(1/2)*a^3*b*d^3*x^5-20*(-b
/a)^(1/2)*a*b^3*c^3*x^3+(-b/a)^(1/2)*a^4*c*d^2*x-15*(-b/a)^(1/2)*a^2*b^2*c^
3*x+3*(-b/a)^(1/2)*a*b^3*c*d^2*x^7+10*(-b/a)^(1/2)*a^2*b^2*c*d^2*x^5-17*(-b
/a)^(1/2)*a*b^3*c^2*d*x^5+17*(-b/a)^(1/2)*a^3*b*c*d^2*x^3-7*(-b/a)^(1/2)*a^
2*b^2*c^2*d*x^3)/(d*x^2+c)^(1/2)/a^3/(a*d-b*c)/(-b/a)^(1/2)/(b*x^2+a)^(5/2)
/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^2)^{\frac{3}{2}}}{(a + bx^2)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(7/2),x)
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{{\left(b\,x^2+a\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x)
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(7/2), x)
```

3.178 $\int \sqrt{2+bx^2}\,\sqrt{3+dx^2}\,dx$

Optimal. Leaf size=235

$$\frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}}+\frac{1}{3}x\sqrt{2+bx^2}\,\sqrt{3+dx^2}-\frac{\sqrt{2}\,(3b+2d)\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}}+\frac{2\sqrt{2}}{}$$

```
[Out] 1/3*(3*b+2*d)*x*(b*x^2+2)^(1/2)/b/(d*x^2+3)^(1/2)-1/3*(3*b+2*d)*(1/(3*d*x^2
+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),
1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/b/d^(1/2)/((b*x^2+2)/(d*x^2+3)
)^(1/2)/(d*x^2+3)^(1/2)+2*(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF
(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)
^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+1/3*x*(b*x^2+2)^
(1/2)*(d*x^2+3)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {428, 545, 429, 506, 422}

$$\frac{2\sqrt{2}\,\sqrt{bx^2+2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{dx^2+3}\,\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\,(3b+2d)\sqrt{bx^2+2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\,\sqrt{dx^2+3}\,\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{1}{3}x\sqrt{bx^2+2}\,\sqrt{dx^2+3}+\frac{x(3b+2d)\sqrt{bx^2+2}}{3b\sqrt{dx^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

```
[Out] ((3*b + 2*d)*x*Sqrt[2 + b*x^2])/(3*b*Sqrt[3 + d*x^2]) + (x*Sqrt[2 + b*x^2]*
Sqrt[3 + d*x^2])/3 - (Sqrt[2]*(3*b + 2*d)*Sqrt[2 + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
d*x^2)]*Sqrt[3 + d*x^2]) + (2*Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*
Sqrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
```

```
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \sqrt{2+bx^2}\,\sqrt{3+dx^2}\,dx = \frac{1}{3}x\sqrt{2+bx^2}\,\sqrt{3+dx^2} + \frac{2}{3}\int \frac{6+\frac{1}{2}(3b+2d)x^2}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx$$

$$= \frac{1}{3}x\sqrt{2+bx^2}\,\sqrt{3+dx^2} + 4\int \frac{1}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx + \frac{1}{3}(3b+2d)\int \frac{\cdots}{\sqrt{2\cdots}}$$

$$= \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}} + \frac{1}{3}x\sqrt{2+bx^2}\,\sqrt{3+dx^2} + \frac{2\sqrt{2}\,\sqrt{2+bx^2}\,F\left(\tan^{-1}\cdots\right.}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\cdots}$$

$$= \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}} + \frac{1}{3}x\sqrt{2+bx^2}\,\sqrt{3+dx^2} - \frac{\sqrt{2}\,(3b+2d)\sqrt{2+bx^2}\,E\cdots}{3b\sqrt{d}\,\sqrt{\frac{2+b\cdots}{3+d\cdots}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.96, size = 127, normalized size = 0.54

$$\frac{\sqrt{b}\,dx\sqrt{2+bx^2}\,\sqrt{3+dx^2}-i\sqrt{3}\,(3b+2d)E\left(i\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)+i\sqrt{3}\,(3b-2d)F\left(i\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{3\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] - I*Sqrt[3]*(3*b + 2*d)*Ellipt
icE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] + I*Sqrt[3]*(3*b - 2*d)*El
lipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)])/(3*Sqrt[b]*d)
```

Maple [A]
time = 0.11, size = 303, normalized size = 1.29

method	result
risch	$\frac{x\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}{3}+\frac{\left(\frac{2\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{(3b+2d)\sqrt{3d}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{x\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}{3}+\frac{2\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3d}}{3},\cdots\right.}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2}\cdots}\right.}{\sqrt{\cdots}}$
default	$\frac{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\left(b^2d\,x^5\sqrt{-d}+3b^2x^3\sqrt{-d}+2bd\,x^3\sqrt{-d}+3\sqrt{2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)*(b^2*d*x^5*(-d)^(1/2)+3*b^2*x^3*(-d)^(1
/2)+2*b*d*x^3*(-d)^(1/2)+3*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2
^(1/2)*3^(1/2)*(b/d)^(1/2))*b*(d*x^2+3)^(1/2)*(b*x^2+2)^(1/2)-2*2^(1/2)*Ell
ipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*d*(d*x^2+3
```

```
)^(1/2)*(b*x^2+2)^(1/2)+3*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^
(1/2)*3^(1/2)*(b/d)^(1/2))*b*(d*x^2+3)^(1/2)*(b*x^2+2)^(1/2)+2*2^(1/2)*Elli
pticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*d*(d*x^2+3)
^(1/2)*(b*x^2+2)^(1/2)+6*b*x*(-d)^(1/2))/(b*d*x^4+3*b*x^2+2*d*x^2+6)/(-d)^(
1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + 2}\,\sqrt{dx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+2)**(1/2)*(d*x**2+3)**(1/2),x)
[Out] Integral(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2),x)
[Out] int((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2), x)
```

3.179 $\int \sqrt{3-6x^2}\,\sqrt{2+4x^2}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{2}{3}}\,x\sqrt{1-4x^4}+\frac{2F\left(\sin^{-1}\left(\sqrt{2}\,x\right)\middle|-1\right)}{\sqrt{3}}$$

[Out] 2/3*EllipticF(x*2^(1/2),I)*3^(1/2)+1/3*x*6^(1/2)*(-4*x^4+1)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {254, 201, 227}

$$\frac{2F\left(\text{ArcSin}\left(\sqrt{2}\,x\right)\middle|-1\right)}{\sqrt{3}}+\sqrt{\frac{2}{3}}\,\sqrt{1-4x^4}\,x$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]

[Out] Sqrt[2/3]*x*Sqrt[1 - 4*x^4] + (2*EllipticF[ArcSin[Sqrt[2]*x], -1])/Sqrt[3]

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rubi steps

$$\int \sqrt{3-6x^2}\,\sqrt{2+4x^2}\,dx = \int \sqrt{6-24x^4}\,dx$$
$$= \sqrt{\frac{2}{3}}\,x\sqrt{1-4x^4} + 4\int \frac{1}{\sqrt{6-24x^4}}\,dx$$
$$= \sqrt{\frac{2}{3}}\,x\sqrt{1-4x^4} + \frac{2F\left(\sin^{-1}\left(\sqrt{2}\,x\right)\middle|-1\right)}{\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 22, normalized size = 0.58

$$\sqrt{6}\,x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

```
[Out] Sqrt[6]*x*Hypergeometric2F1[-1/2, 1/4, 5/4, 4*x^4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(29) = 58.
time = 0.14, size = 75, normalized size = 1.97

method	result
default	$-\frac{\sqrt{-6x^2+3}\,\sqrt{2}\,\sqrt{2x^2+1}\,\left(\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+3}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{2},i\right)-12x^5+3x\right)}{9(4x^4-1)}$
elliptic	$-\frac{\sqrt{-6x^2+3}\,\sqrt{4x^2+2}\,\sqrt{-24x^4+6}\,\left(\frac{x\sqrt{-24x^4+6}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{2}\right.}{\sqrt{-24x^4+6}}\right.}{6(4x^4-1)}$
risch	$-\frac{x(2x^2-1)(2x^2+1)\sqrt{(-6x^2+3)(4x^2+2)}\,\sqrt{6}}{3\sqrt{-(2x^2-1)(2x^2+1)}\,\sqrt{-6x^2+3}\,\sqrt{4x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{2},i\right)}{3\sqrt{-4x^4+1}\,\sqrt{-6x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-6*x^2+3)^(1/2)*2^(1/2)*(2*x^2+1)^(1/2)*(2^(1/2)*3^(1/2)*(-6*x^2+3)^(
1/2)*(2*x^2+1)^(1/2)*EllipticF(x*2^(1/2),I)-12*x^5+3*x)/(4*x^4-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Fricas [A]
time = 0.24, size = 21, normalized size = 0.55

$$\frac{1}{3}\sqrt{4\,x^2+2}\,\sqrt{-6\,x^2+3}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3)*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{6}\int\sqrt{1-2x^2}\,\sqrt{2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x**2+3)**(1/2)*(4*x**2+2)**(1/2),x)
[Out] sqrt(6)*Integral(sqrt(1 - 2*x**2)*sqrt(2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{4\,x^2+2}\,\sqrt{3-6\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 2)^(1/2)*(3 - 6*x^2)^(1/2),x)
[Out] int((4*x^2 + 2)^(1/2)*(3 - 6*x^2)^(1/2), x)
```

3.180 $\int \sqrt{2+4x^2}\,\sqrt{3+6x^2}\,dx$

Optimal. Leaf size=20

$$\sqrt{6}\,x+2\sqrt{\frac{2}{3}}\,x^3$$

```
[Out] 2/3*x^3*6^(1/2)+x*6^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {22}

$$2\sqrt{\frac{2}{3}}\,x^3+\sqrt{6}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*x + 2*Sqrt[2/3]*x^3
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+4x^2}\,\sqrt{3+6x^2}\,dx &= \sqrt{\frac{2}{3}}\int \left(3+6x^2\right)\,dx\\ &= \sqrt{6}\,x+2\sqrt{\frac{2}{3}}\,x^3\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 0.75

$$\sqrt{6}\left(x+\frac{2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*(x + (2*x^3)/3)
```

Maple [C] Result contains higher order function than in optimal. Order 2 vs. order 1.
time = 0.06, size = 38, normalized size = 1.90

method	result	size
gosper	$\frac{x(2x^2+3)\sqrt{4x^2+2}}{\sqrt{6x^2+3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(2*x^2+3)*(4*x^2+2)^(1/2)*(6*x^2+3)^(1/2)/(2*x^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(14) = 28.
time = 0.63, size = 38, normalized size = 1.90

$$\frac{(2\,x^3+3\,x)\sqrt{6\,x^2+3}\,\sqrt{4\,x^2+2}}{3\,(2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*x^3 + 3*x)*sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2)/(2*x^2 + 1)
```

Sympy [A]
time = 1.43, size = 17, normalized size = 0.85

$$\frac{2\sqrt{6}\,x^3}{3}+\sqrt{6}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+2)**(1/2)*(6*x**2+3)**(1/2),x)
[Out] 2*sqrt(6)*x**3/3 + sqrt(6)*x
```

Giac [A]
time = 1.13, size = 17, normalized size = 0.85

$$\frac{1}{3}\,\sqrt{3}\,\sqrt{2}\,\left(2\,x^{3}+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*sqrt(2)*(2*x^3 + 3*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{4\,x^2+2}\,\sqrt{6\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2),x)
[Out] int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2), x)
```

3.181 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}}-\frac{\sqrt{2}\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}+\frac{\sqrt{2}\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

```
[Out] x*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*E
llipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(
b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+(1/(3*d*
x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/
2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3
))^(1/2)/(d*x^2+3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{2}\sqrt{bx^2+2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\sqrt{bx^2+2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx &= 2\int \frac{1}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx\\ &= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} + \frac{\sqrt{2}\,\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}} - 3\int \frac{\sqrt{2+bx^2}}{(3+dx^2)^{3/2}}\,dx\\ &= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\,\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}} + \frac{\sqrt{2}\,\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\right.\right.}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.73, size = 37, normalized size = 0.20

$$\frac{\sqrt{2}\,E\left(\sin^{-1}\left(\frac{\sqrt{-d}\,x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]
```

Maple [A]
time = 0.07, size = 37, normalized size = 0.20

method	result
default	$\frac{\mathrm{EllipticE}\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{\sqrt{-d}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+2\right)\left(d\,x^{2}+3\right)}\left(\frac{\sqrt{3d\,x^{2}+9}\,\sqrt{2b\,x^{2}+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^{4}+3b\,x^{2}+2d\,x^{2}+6}}-\frac{\sqrt{3d\,x^{2}+9}}{\ldots}\right)}{\sqrt{b\,x^{2}+2}\,\sqrt{d\,x^{2}\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^(1/2)
/(-d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2),x)
```

```
[Out] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2), x)
```

3.182 $\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}}+\frac{(c+4d)\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

```
[Out] -EllipticE(1/2*x,2*(-d/c)^(1/2))*(d*x^2+c)^(1/2)/d/(1+d*x^2/c)^(1/2)+(c+4*d
)*EllipticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/d/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {434, 437, 435, 432, 430}

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,F\left(\text{ArcSin}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\,E\left(\text{ArcSin}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] -((Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c]
)) + ((c + 4*d)*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sq
rt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx &= -\frac{\int \frac{\sqrt{c+dx^2}}{\sqrt{4-x^2}}\,dx}{d} - \frac{(-c-4d)\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx}{d}\\
&= -\frac{\sqrt{c+dx^2}\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{4-x^2}}\,dx}{d\sqrt{1+\frac{dx^2}{c}}} - \frac{\left((-c-4d)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{4-x^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}}\\
&= -\frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}} + \frac{(c+4d)\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.63, size = 60, normalized size = 0.66

$$\frac{2\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]

[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -1/4*c/d])/(Sqrt[-(d/c)]*Sqrt[c + d*x^2])

Maple [A]
time = 0.09, size = 78, normalized size = 0.86

method	result
default	$\frac{\left(c\,\mathrm{EllipticF}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+4\,\mathrm{EllipticF}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)d-c\,\mathrm{EllipticE}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)\sqrt{\frac{d\,x^2+c}{c}}}{\sqrt{d\,x^2+c}\;d}$
elliptic	$\frac{\sqrt{-\left(d\,x^2+c\right)\left(x^2-4\right)}\left(\frac{4\sqrt{-x^2+4}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)}{\sqrt{-d\,x^4-c\,x^2+4d\,x^2+4c}}+\frac{c\sqrt{-x^2+4}\,\sqrt{1+\frac{d}{}}}{}\right.}{\sqrt{-x^2+4}\,\sqrt{d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] (c*EllipticF(1/2*x,2*(-d/c)^(1/2))+4*EllipticF(1/2*x,2*(-d/c)^(1/2))*d-c*EllipticE(1/2*x,2*(-d/c)^(1/2)))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(c + d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{4-x^2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((4 - x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.183 $\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=150

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}+\frac{4\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

```
[Out] x*(d*x^2+c)^(1/2)/d/(x^2+4)^(1/2)-(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+4)^(1/
2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/d/((d*x^2+c)/c/(x^2+4))^(1/2)+4*(1/(x^2
+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/c/((d
*x^2+c)/c/(x^2+4))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {433, 429, 506, 422}

$$\frac{4\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}-\frac{\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}+\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[x
/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))]) + (4*S
qrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt[(
c + d*x^2)/(c*(4 + x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx &= 4\int \frac{1}{\sqrt{4+x^2}\,\sqrt{c+dx^2}}\,dx+\int \frac{x^2}{\sqrt{4+x^2}\,\sqrt{c+dx^2}}\,dx\\
&= \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}+\frac{4\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\,\sqrt{\frac{c+dx^2}{c(4+x^2)}}}-\frac{4\int \frac{\sqrt{c+dx^2}}{(4+x^2)^{3/2}}\,dx}{d}\\
&= \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\,\sqrt{\frac{c+dx^2}{c(4+x^2)}}}+\frac{4\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\,\sqrt{\frac{c+dx^2}{c(4+x^2)}}}
\end{aligned}
$$

Mathematica [A]
time = 0.64, size = 60, normalized size = 0.40

$$
\frac{2\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], c/(4*d)])/(Sqrt[-(
d/c)]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 53, normalized size = 0.35

method	result
default	$\frac{2\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}}, \frac{\sqrt{\frac{c}{d}}}{2}\right)\sqrt{\frac{d\,x^{2}+c}{c}}}{\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(d\,x^{2}+c)\,(x^{2}+4)}\left(\frac{2\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{x^{2}+4}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}}, \frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^{4}+c\,x^{2}+4d\,x^{2}+4c}} - \frac{2\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{x^{2}+}}{}\right.}{\sqrt{d\,x^{2}+c}\,\sqrt{x^{2}+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^(
1/2)/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+4}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+4}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 4)^(1/2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^2 + 4)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.184 $\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\mid\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {435}

$$\frac{E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\mid\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\mid\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.28, size = 20, normalized size = 1.00

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\mid\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Maple [A]
time = 0.10, size = 23, normalized size = 1.15

method	result
default	$\frac{\sqrt{2}\,\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)+2\,\text{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)}{6}$
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-1)}\,\left(\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}-\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\left(\text{EllipticF}\left(x,\right.\right.}{3\sqrt{3x^4-5x^2+}}\right.}{\sqrt{-x^2+1}\,\sqrt{-3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*(EllipticF(x,1/2*6^(1/2))+2*EllipticE(x,1/2*6^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 1.49, size = 34, normalized size = 1.70

$$\begin{cases} \frac{\sqrt{3}\,E\left(\operatorname{asin}\left(\frac{\sqrt{6}\,x}{2}\right)\middle|\,\frac{2}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

```
[Out] int((1 - x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.185 $\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {435}

$$\frac{2E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.29, size = 21, normalized size = 1.00

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A]
time = 0.10, size = 18, normalized size = 0.86

method	result
default	$\frac{2\,\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\sqrt{3}}{3}$
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-4)}\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)}{3\sqrt{3x^4-14x^2+8}}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-x^2}}{\cdots}\right.}{\sqrt{-x^2+4}\,\sqrt{-3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*6^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 1.69, size = 36, normalized size = 1.71

$$\begin{cases} \frac{2\sqrt{3}\,E\left(\operatorname{asin}\left(\frac{\sqrt{6}\,x}{2}\right)\middle|\frac{1}{6}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 1/6)/3, (x > -sqrt(6)/3)
 & (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

```
[Out] int((4 - x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.186 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}} \, dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

[Out] 1/3*EllipticE(1/2*x*6^(1/2),2/3*6^(1/2))*3^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {435}

$$\frac{E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]

[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}} \, dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.33, size = 20, normalized size = 1.00

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Maple [A]
time = 0.10, size = 29, normalized size = 1.45

method	result
default	$-\frac{\sqrt{2}\left(5\,\text{EllipticF}\left(2x,\frac{\sqrt{6}}{4}\right)-8\,\text{EllipticE}\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{12}$
elliptic	$\frac{\sqrt{(3x^2-2)(4x^2-1)}\left(\frac{\sqrt{-4x^2+1}\,\sqrt{-6x^2+4}\,\text{EllipticF}\left(2x,\frac{\sqrt{6}}{4}\right)}{4\sqrt{12x^4-11x^2+2}}-\frac{2\sqrt{-4x^2+1}\,\sqrt{-6x^2+4}\,(\text{Ellipt}\ldots}{3\sqrt{12x^4-1\ldots}}\right.}{\sqrt{-4x^2+1}\,\sqrt{-3x^2+2}\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*2^(1/2)*(5*EllipticF(2*x,1/4*6^(1/2))-8*EllipticE(2*x,1/4*6^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 1.43, size = 34, normalized size = 1.70

$$\begin{cases}\frac{\sqrt{3}\,E\left(\operatorname{asin}\left(\frac{\sqrt{6}\,x}{2}\right)\middle|\frac{8}{3}\right)}{3} & \text{for}\ x>-\frac{\sqrt{6}}{3}\wedge x<\frac{\sqrt{6}}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 8/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{1-4\,x^2}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 4*x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

```
[Out] int((1 - 4*x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.187 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=4

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] EllipticE(x,I)
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {435}

$$E(\text{ArcSin}(x)|-1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
[Out] EllipticE[ArcSin[x], -1]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E\left(\sin^{-1}(x)\middle|-1\right)$$

Mathematica [A]
time = 0.27, size = 4, normalized size = 1.00

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
[Out] EllipticE[ArcSin[x], -1]
```

Maple [A]
time = 0.10, size = 5, normalized size = 1.25

method	result	size
default	$\mathrm{EllipticE}\left(x, i\right)$	5
elliptic	$\frac{\sqrt{-x^{4}+1}\, \left(\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \mathrm{EllipticF}\left(x, i\right)}{\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\mathrm{EllipticF}\left(x, i\right)-\mathrm{EllipticE}\left(x, i\right)\right)}{\sqrt{-x^{4}+1}}\right)}{\sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticE(x,I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(3) = 6$.
time = 0.24, size = 21, normalized size = 5.25

$$-\frac{\sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x^2 + 1)*sqrt(-x^2 + 1)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.
time = 1.31, size = 10, normalized size = 2.50

$$\begin{cases} E\left(\operatorname{asin}{\left(x \right)}\middle| -1\right) & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((elliptic_e(asin(x), -1), (x > -1) & (x < 1)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.25

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

3.188 $\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {435}

$$\frac{E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.27, size = 20, normalized size = 1.00

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.95

method	result
default	$\frac{\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{3}}{3}$
elliptic	$\frac{\sqrt{-\left(3x^2-2\right)\left(x^2+1\right)}\left(\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}-\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{x^2+}}{}\right.}{\sqrt{x^2+1}\sqrt{-3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [A]
time = 0.23, size = 21, normalized size = 1.05

$$-\frac{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)/x
```

Sympy [A]
time = 1.56, size = 36, normalized size = 1.80

$$\begin{cases} \frac{\sqrt{3}\,E\left(\operatorname{asin}\left(\frac{\sqrt{6}\,x}{2}\right)\middle|-\frac{2}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{x^2+1}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

```
[Out] int((x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.189 $\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {435}

$$\frac{2E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.30, size = 21, normalized size = 1.00

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.90

method	result
default	$\frac{2\,\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\sqrt{3}}{3}$
elliptic	$\frac{\sqrt{-\left(3x^{2}-2\right)\left(x^{2}+4\right)}\left(\frac{\sqrt{6}\,\sqrt{-6x^{2}+4}\,\sqrt{x^{2}+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)}{3\sqrt{-3x^{4}-10x^{2}+8}}-\frac{\sqrt{6}\,\sqrt{-6x^{2}+4}\,\sqrt{x^{2}-}}{}\right.}{\sqrt{x^{2}+4}\,\sqrt{-3x^{2}+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Fricas [A]
time = 0.27, size = 21, normalized size = 1.00

$$-\frac{\sqrt{x^{2}+4}\,\sqrt{-3\,x^{2}+2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)/x
```

Sympy [A]
time = 1.62, size = 37, normalized size = 1.76

$$\begin{cases} \frac{2\sqrt{3}\,E\left(\operatorname{asin}\left(\frac{\sqrt{6}\,x}{2}\right)\middle|-\frac{1}{6}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -1/6)/3, (x > -sqrt(6)/3
) & (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{x^2+4}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 4)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

```
[Out] int((x^2 + 4)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.190 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{435\}$

$$\frac{E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.32, size = 20, normalized size = 1.00

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.95

method	result
default	$\frac{\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\sqrt{3}}{3}$
elliptic	$\frac{\sqrt{-\left(3x^2-2\right)\left(4x^2+1\right)}\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{4x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)}{6\sqrt{-12x^4+5x^2+2}}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{4x}}{}\right.}{\sqrt{4x^2+1}\,\sqrt{-3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [A]
time = 0.22, size = 23, normalized size = 1.15

$$-\frac{\sqrt{4\,x^2+1}\,\sqrt{-3\,x^2+2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)/x
```

Sympy [A]
time = 1.60, size = 36, normalized size = 1.80

$$\begin{cases} \frac{\sqrt{3}\,E\left(\operatorname{asin}\left(\frac{\sqrt{6}\,x}{2}\right)\middle|-\frac{8}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -8/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{4\,x^2+1}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

```
[Out] int((4*x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.191 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=13

$$-E\left(\sin^{-1}(x)\middle|-1\right)+2F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -EllipticE(x,I)+2*EllipticF(x,I)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {434, 435, 254, 227}

$$2F(\text{ArcSin}(x)|-1)-E(\text{ArcSin}(x)|-1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
[Out] -EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx &= 2\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1+x^2}}\,dx - \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx\\ &= -E\left(\sin^{-1}(x)\middle|-1\right) + 2\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= -E\left(\sin^{-1}(x)\middle|-1\right) + 2F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 12, normalized size = 0.92

$$-iE\left(i\sinh^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (-I)*EllipticE[I*ArcSinh[x], -1]
```

Maple [A]
time = 0.08, size = 14, normalized size = 1.08

method	result	size
default	$-\operatorname{EllipticE}(x,i)+2\operatorname{EllipticF}(x,i)$	14
elliptic	$\frac{\sqrt{-x^4+1}\left(\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\operatorname{EllipticF}(x,i)}{\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^4+1}}\right)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -EllipticE(x,I)+2*EllipticF(x,I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Fricas [A]
time = 0.31, size = 20, normalized size = 1.54

$$\frac{\sqrt{x^2+1}\,\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 1)*sqrt(-x^2 + 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int\frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.192 $\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)+\frac{5F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{3\sqrt{2}}$$

```
[Out] 5/6*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)-1/3*EllipticE(x,1/2*I*6^(1/2))*2^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {434, 435, 430}

$$\frac{5F\left(\text{ArcSin}(x)\middle|-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}\,E\left(\text{ArcSin}(x)\middle|-\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -1/3*(Sqrt[2]*EllipticE[ArcSin[x], -3/2]) + (5*EllipticF[ArcSin[x], -3/2])/
(3*Sqrt[2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-x^2}}\,dx\right) + \frac{5}{3}\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+3x^2}}\,dx$$
$$= -\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right) + \frac{5F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{3\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 27, normalized size = 0.87

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -2/3])/Sqrt[3]
```

Maple [A]
time = 0.09, size = 27, normalized size = 0.87

method	result
default	$\frac{\left(5\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)-2\,\mathrm{EllipticE}\left(x,\frac{i\sqrt{6}}{2}\right)\right)\sqrt{2}}{6}$
elliptic	$\frac{\sqrt{-\left(3x^2+2\right)\left(x^2-1\right)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}+\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(x,\frac{i}{}\right.\right.}{3\sqrt{-3x^4+x^2-}}\right.}{\sqrt{-x^2+1}\,\sqrt{3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(5*EllipticF(x,1/2*I*6^(1/2))-2*EllipticE(x,1/2*I*6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [**A**]
time = 0.23, size = 23, normalized size = 0.74

$$\frac{\sqrt{3\,x^2+2}\,\sqrt{-x^2+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(3*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-x^2}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.193 $\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)+\frac{7}{3}\sqrt{2}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

```
[Out] -1/3*EllipticE(1/2*x,I*6^(1/2))*2^(1/2)+7/3*EllipticF(1/2*x,I*6^(1/2))*2^(1
/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {434, 435, 430}

$$\frac{7}{3}\sqrt{2}\,F\left(\text{ArcSin}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}\,E\left(\text{ArcSin}\left(\frac{x}{2}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -1/3*(Sqrt[2]*EllipticE[ArcSin[x/2], -6]) + (7*Sqrt[2]*EllipticF[ArcSin[x/2
], -6])/3
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{4-x^2}}\,dx\right) + \frac{14}{3}\int \frac{1}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx$$
$$= -\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right) + \frac{7}{3}\sqrt{2}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 27, normalized size = 0.77

$$-\frac{2iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A]
time = 0.09, size = 31, normalized size = 0.89

method	result
default	$\frac{\left(7\,\mathrm{EllipticF}\left(\frac{x}{2},i\sqrt{6}\right)-\mathrm{EllipticE}\left(\frac{x}{2},i\sqrt{6}\right)\right)\sqrt{2}}{3}$
elliptic	$\frac{\sqrt{-\left(3x^2+2\right)\left(x^2-4\right)}\left(\frac{2\sqrt{-x^2+4}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(\frac{x}{2},i\sqrt{6}\right)}{\sqrt{-3x^4+10x^2+8}}+\frac{\sqrt{-x^2+4}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{x}{2}\right.\right.}{3\sqrt{-3x^4+10x}}\right.}{\sqrt{-x^2+4}\,\sqrt{3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(7*EllipticF(1/2*x,I*6^(1/2))-EllipticE(1/2*x,I*6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Fricas [A]
time = 0.35, size = 23, normalized size = 0.66

$$\frac{\sqrt{3\,x^2+2}\,\sqrt{-x^2+4}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-2)\,(x+2)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(3*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{4-x^2}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

```
[Out] int((4 - x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.194 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{2}{3}\sqrt{2}\,E\left(\sin^{-1}(2x)|-\frac{3}{8}\right)+\frac{11F\left(\sin^{-1}(2x)|-\frac{3}{8}\right)}{6\sqrt{2}}$$

```
[Out] 11/12*EllipticF(2*x,1/4*I*6^(1/2))*2^(1/2)-2/3*EllipticE(2*x,1/4*I*6^(1/2))
*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {434, 435, 430}

$$\frac{11F\left(\text{ArcSin}(2x)\,|-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{2}{3}\sqrt{2}\,E\left(\text{ArcSin}(2x)\,\middle|-\frac{3}{8}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-2*Sqrt[2]*EllipticE[ArcSin[2*x], -3/8])/3 + (11*EllipticF[ArcSin[2*x], -3
/8])/(6*Sqrt[2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = -\left(\frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-4x^2}}\,dx\right) + \frac{11}{3}\int \frac{1}{\sqrt{1-4x^2}\,\sqrt{2+3x^2}}\,dx$$
$$= -\frac{2}{3}\sqrt{2}\,E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right) + \frac{11F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{6\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 27, normalized size = 0.77

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -8/3])/Sqrt[3]
```

Maple [A]
time = 0.10, size = 31, normalized size = 0.89

method	result
default	$\frac{\left(11\,\mathrm{EllipticF}\left(2x,\frac{i\sqrt{6}}{4}\right)-8\,\mathrm{EllipticE}\left(2x,\frac{i\sqrt{6}}{4}\right)\right)\sqrt{2}}{12}$
elliptic	$\frac{\sqrt{-\left(3x^2+2\right)\left(4x^2-1\right)}\left(\frac{\sqrt{-4x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(2x,\frac{i\sqrt{6}}{4}\right)}{4\sqrt{-12x^4-5x^2+2}}+\frac{2\sqrt{-4x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{Elliptic}\right.}{3\sqrt{-12x^4-}}\right.}{\sqrt{-4x^2+1}\,\sqrt{3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(11*EllipticF(2*x,1/4*I*6^(1/2))-8*EllipticE(2*x,1/4*I*6^(1/2)))*2^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [A]
time = 0.15, size = 23, normalized size = 0.66

$$\frac{\sqrt{3\,x^2+2}\,\sqrt{-4\,x^2+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(2x-1)(2x+1)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(2*x - 1)*(2*x + 1))/sqrt(3*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-4\,x^2}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 4*x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((1 - 4*x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.195 $\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}+\frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] 1/3*x*(3*x^2+2)^(1/2)/(x^2+1)^(1/2)+1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+
1)^(1/2),1/2*I*2^(1/2))*(3*x^2+2)^(1/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)-1
/3*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)*(3*x^
2+2)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{3x^2+2}\,F\left(\text{ArcTan}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\text{ArcTan}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}+\frac{\sqrt{3x^2+2}\,x}{3\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]) + (Sqrt[2 +
 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)
/(1 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx &= \int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+3x^2}}\,dx+\int \frac{x^2}{\sqrt{1+x^2}\,\sqrt{2+3x^2}}\,dx\\ &= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}}+\frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+3x^2}{1+x^2}}}-\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{(1+x^2)^{3/2}}\,dx\\ &= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}}-\frac{\sqrt{2}\,\sqrt{2+3x^2}\,E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\,\sqrt{\frac{2+3x^2}{1+x^2}}}+\frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+3x^2}{1+x^2}}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 27, normalized size = 0.21

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[3]
```

Maple [A]
time = 0.08, size = 30, normalized size = 0.23

method	result

default	$-\frac{i\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)+2\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)\sqrt{2}}{6}$
elliptic	$\frac{\sqrt{(3x^2+2)(x^2+1)}\left(-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)\right.}{3\sqrt{3x^4+5x^2+2}}\right.}{\sqrt{3x^2+2}\,\sqrt{x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*(EllipticF(I*x,1/2*6^(1/2))+2*EllipticE(I*x,1/2*6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt(3*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.196 $\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=136

$$\frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}-\frac{\sqrt{2}\,\sqrt{2+3x^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\,\sqrt{\frac{2+3x^2}{4+x^2}}}+\frac{2\sqrt{2}\,\sqrt{2+3x^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{4+x^2}\,\sqrt{\frac{2+3x^2}{4+x^2}}}$$

```
[Out] 1/3*x*(3*x^2+2)^(1/2)/(x^2+4)^(1/2)-1/3*(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+
4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(x^2+4))^(1/2)+2*(1/
(x^2+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)
/((3*x^2+2)/(x^2+4))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {433, 429, 506, 422}

$$\frac{2\sqrt{2}\,\sqrt{3x^2+2}\,F\left(\text{ArcTan}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{x^2+4}\,\sqrt{\frac{3x^2+2}{x^2+4}}}-\frac{\sqrt{2}\,\sqrt{3x^2+2}\,E\left(\text{ArcTan}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\,\sqrt{\frac{3x^2+2}{x^2+4}}}+\frac{\sqrt{3x^2+2}\,x}{3\sqrt{x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)]) + (2*Sqrt[2
]*Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x/2], -5])/(Sqrt[4 + x^2]*Sqrt[(2 + 3*x^
2)/(4 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx &= 4\int \frac{1}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx+\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx\\ &= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}+\frac{2\sqrt{2}\sqrt{2+3x^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}-\frac{4}{3}\int \frac{\sqrt{2+3x^2}}{(4+x^2)^{3/2}}\,dx\\ &= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}+\frac{2\sqrt{2}\sqrt{2+3x^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 27, normalized size = 0.20

$$-\frac{2iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A]
time = 0.08, size = 26, normalized size = 0.19

method	result

default	$-\frac{i\left(5\,\mathrm{EllipticF}\left(\frac{ix}{2},\sqrt{6}\right)+\mathrm{EllipticE}\left(\frac{ix}{2},\sqrt{6}\right)\right)\sqrt{2}}{3}$
elliptic	$\frac{\sqrt{(3x^2+2)(x^2+4)}\left(-\frac{2i\sqrt{x^2+4}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(\frac{ix}{2},\sqrt{6}\right)}{\sqrt{3x^4+14x^2+8}}+\frac{i\sqrt{x^2+4}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{ix}{2},\sqrt{6}\right.\right.}{3\sqrt{3x^4+14x^2+8}}\right.}{\sqrt{3x^2+2}\,\sqrt{x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*(5*EllipticF(1/2*I*x,6^(1/2))+EllipticE(1/2*I*x,6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+4}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4)/sqrt(3*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 4)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((x^2 + 4)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.197 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=148

$$\frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}}-\frac{2\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}+\frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}$$

```
[Out] 4/3*x*(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2)+1/4*(1/(4*x^2+1))^(1/2)*EllipticF(2*x
/(4*x^2+1)^(1/2),1/4*10^(1/2))*(3*x^2+2)^(1/2)*2^(1/2)/((3*x^2+2)/(4*x^2+1)
)^(1/2)-2/3*(1/(4*x^2+1))^(1/2)*EllipticE(2*x/(4*x^2+1)^(1/2),1/4*10^(1/2))
*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(4*x^2+1))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{3x^2+2}\,F\left(\text{ArcTan}(2x)\,|\,\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}-\frac{2\sqrt{2}\sqrt{3x^2+2}\,E\left(\text{ArcTan}(2x)\,|\,\frac{5}{8}\right)}{3\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}+\frac{4\sqrt{3x^2+2}\,x}{3\sqrt{4x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (4*x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (2*Sqrt[2]*Sqrt[2 + 3*x^2]*Elli
pticE[ArcTan[2*x], 5/8])/(3*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
+ (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[2*x], 5/8])/(2*Sqrt[2]*Sqrt[(2 + 3*x^2)
/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx &= 4\int \frac{x^2}{\sqrt{2+3x^2}\,\sqrt{1+4x^2}}\,dx + \int \frac{1}{\sqrt{2+3x^2}\,\sqrt{1+4x^2}}\,dx \\
&= \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} + \frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}} - \frac{4}{3}\int \frac{\sqrt{2+3x^2}}{(1+4x^2)^{3/2}}\,dx \\
&= \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{2\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}} + \frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 27, normalized size = 0.18

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3])/Sqrt[3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 20, normalized size = 0.14

method	result

default	$-\frac{i\,\text{EllipticE}\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\sqrt{3}}{3}$
elliptic	$\frac{\sqrt{(3x^2+2)(4x^2+1)}\left(-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{4x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)}{6\sqrt{12x^4+11x^2+2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{4x^2+}}{}\right.}{\sqrt{3x^2+2}\,\sqrt{4x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*I*EllipticE(1/2*I*x*6^(1/2),2/3*6^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(4*x**2 + 1)/sqrt(3*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

```
[Out] int((4*x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.198 $\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-2x^2}\,E\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+2x^2}}$$

```
[Out] 1/2*EllipticE(x*2^(1/2),1/2*2^(1/2))*(-2*x^2+1)^(1/2)*2^(1/2)/(2*x^2-1)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {438, 435}

$$\frac{\sqrt{1-2x^2}\,E\left(\text{ArcSin}\left(\sqrt{2}\,x\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt[-1 + 2*x^
2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx &= \frac{\sqrt{1-2x^2}\int \frac{\sqrt{1-x^2}}{\sqrt{1-2x^2}}\,dx}{\sqrt{-1+2x^2}}\\ &= \frac{\sqrt{1-2x^2}\,E\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 35, normalized size = 0.88

$$\frac{\sqrt{1-2x^2}\,E\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{2}\right)}{\sqrt{-2+4x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]

[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/Sqrt[-2 + 4*x^2]

Maple [A]
time = 0.10, size = 32, normalized size = 0.80

method	result
default	$\frac{\left(\mathrm{EllipticF}\left(x,\sqrt{2}\right)+\mathrm{EllipticE}\left(x,\sqrt{2}\right)\right)\sqrt{-2x^2+1}}{2\sqrt{2x^2-1}}$
elliptic	$\frac{\sqrt{-\left(2x^2-1\right)\left(x^2-1\right)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,\mathrm{EllipticF}\left(x,\sqrt{2}\right)}{\sqrt{-2x^4+3x^2-1}}-\frac{\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,\left(\mathrm{EllipticF}\right.}{2\sqrt{-2x^4+3x}}\right.}{\sqrt{-x^2+1}\,\sqrt{2x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(EllipticF(x,2^(1/2))+EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/(2*x^2-1)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)

Fricas [A]
time = 0.15, size = 23, normalized size = 0.58

$$\frac{\sqrt{2\,x^2-1}\,\sqrt{-x^2+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(2*x^2 - 1)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(2*x^2 - 1)^(1/2), x)
```

3.199 $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=423

$$-\frac{8(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}}+\frac{b\left(24b^2c^2-71abcd+71a^2d^2\right)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{105d^3}-$$

```
[Out] -8/105*(-2*a*d+b*c)*(11*a^2*d^2-11*a*b*c*d+6*b^2*c^2)*x*(b*x^2+a)^(1/2)/d^3
/(d*x^2+c)^(1/2)+8/105*(-2*a*d+b*c)*(11*a^2*d^2-11*a*b*c*d+6*b^2*c^2)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*
x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105*(-7*a*d+3*b*c)*(15*a^2*d^2-11*a*b*c*d+8
*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*
(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-6/35*b*(-2*a*d+b*c)*x*(b*x^2+a
)^(3/2)*(d*x^2+c)^(1/2)/d^2+1/7*b*x*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/d+1/105
*b*(71*a^2*d^2-71*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3
```

Rubi [A]
time = 0.29, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\sqrt{a+bx^2}\,(3bc-7ad)\left(15a^2d^2-11abcd+8b^2c^2\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{8\sqrt{c}\sqrt{a+bx^2}\,(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-71abcd+24b^2c^2\right)}{105d^3}-\frac{8x\sqrt{a+bx^2}\,(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)}{105d^3\sqrt{c+dx^2}}-\frac{6bx(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(bc-2ad)}{35d^2}+\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-8*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*x*Sqrt[a + b*x^2])/
(105*d^3*Sqrt[c + d*x^2]) + (b*(24*b^2*c^2 - 71*a*b*c*d + 71*a^2*d^2)*x*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (6*b*(b*c - 2*a*d)*x*(a + b*x^2)^
(3/2)*Sqrt[c + d*x^2])/(35*d^2) + (b*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(
7*d) + (8*Sqrt[c]*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^
(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(3*
b*c - 7*a*d)*(8*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{(a+bx^2)^{3/2}(-a(bc-7ad)-6b(bc-2ad)x^2)}{\sqrt{c+dx^2}}\,dx}{7d}$$

$$= -\frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2} + \frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} + \frac{\int \sqrt{a+bx^2}\,(a(\ldots}{\ldots}$$

$$= \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+\ldots}}{35d^2}$$

$$= \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+\ldots}}{35d^2}$$

$$= -\frac{8(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} + \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\ldots}{105d^3}$$

$$= -\frac{8(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} + \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\ldots}{105d^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.97, size = 321, normalized size = 0.76

$$\frac{b\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)\left(122a^2d^2+abd(-89c+66dx^2)+3b^2(8c^2-6cdx^2+5d^2x^4)\right)-8ibc(-6b^3c^3+23ab^2c^2d-33a^2bcd^2+22a^3d^3)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(48b^4c^4-208ab^3c^3d+353a^2b^2c^2d^2-298a^3bcd^3+105a^4d^4)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{105\sqrt{\frac{b}{a}}\,d^4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(122*a^2*d^2 + a*b*d*(-89*c + 66*d
*x^2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - (8*I)*b*c*(-6*b^3*c^3 + 23
*a*b^2*c^2*d - 33*a^2*b*c*d^2 + 22*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(48*b^4*c^4 - 2
08*a*b^3*c^3*d + 353*a^2*b^2*c^2*d^2 - 298*a^3*b*c*d^3 + 105*a^4*d^4)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)])/(105*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.10, size = 852, normalized size = 2.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*(-b/a)^(1/2)*b^4*d^4*x^9+81*(-b/a
)^(1/2)*a*b^3*d^4*x^7-3*(-b/a)^(1/2)*b^4*c*d^3*x^7+188*(-b/a)^(1/2)*a^2*b^2
*d^4*x^5-26*(-b/a)^(1/2)*a*b^3*c*d^3*x^5+6*(-b/a)^(1/2)*b^4*c^2*d^2*x^5+122
*(-b/a)^(1/2)*a^3*b*d^4*x^3+99*(-b/a)^(1/2)*a^2*b^2*c*d^3*x^3-83*(-b/a)^(1/
2)*a*b^3*c^2*d^2*x^3+24*(-b/a)^(1/2)*b^4*c^3*d*x^3+105*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*d^4-298*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a^3*b*c*d^3+353*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^2-208*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*d+48*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*b^4*c^4+176*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^3-264*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^2+184*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b
^3*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*b^4*c^4+122*(-b/a)^(1/2)*a^3*b*c*d^3*x-89*(-b/a)^(1/2)*a
^2*b^2*c^2*d^2*x+24*(-b/a)^(1/2)*a*b^3*c^3*d*x)/d^4/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{7}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(7/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{7/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(7/2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(7/2)/(c + d*x^2)^(1/2), x)
```

3.200 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=344

$$\frac{(8b^2c^2-23abcd+23a^2d^2)\,x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}}-\frac{4b(bc-2ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15d^2}+\frac{bx(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{5d}-\frac{\sqrt{c}}{\cdots}$$

```
[Out] 1/15*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/d^2/(d*x^2+c)^(1/2
)-1/15*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1
/2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
+1/15*(15*a^2*d^2-11*a*b*c*d+4*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(
1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/
2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+
1/5*b*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d-4/15*b*(-2*a*d+b*c)*x*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]

time = 0.18, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(15a^2d^2-11abcd+4b^2c^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(23a^2d^2-23abcd+8b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,(23a^2d^2-23abcd+8b^2c^2)}{15d^2\sqrt{c+dx^2}}-\frac{4bx\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-2ad)}{15d^2}+\frac{bx(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*d^2*Sqrt[c +
d*x^2]) - (4*b*(b*c - 2*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) +
(b*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 23*a*
b*c*d + 23*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) + (Sqrt[c]*(4*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{\int \frac{\sqrt{a+bx^2}\,(-a(bc-5ad)-4b(bc-2ad)x^2)}{\sqrt{c+dx^2}}\,dx}{5d}$$

$$= -\frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{\int \frac{a(4b^2c^2-11abcd+15a^2}{\sqrt{a+}}}{}$$

$$= -\frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{(a(4b^2c^2-11abcd}{}$$

$$= \frac{(8b^2c^2-23abcd+23a^2d^2)\,x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} - \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+}{}$$

$$= \frac{(8b^2c^2-23abcd+23a^2d^2)\,x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} - \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+}{}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 2.91, size = 260, normalized size = 0.76

$$\frac{b\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)(-4bc+11ad+3bdx^2) - ibc(8b^2c^2-23abcd+23a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right) - i(-8b^3c^3+27ab^2c^2d-34a^2bcd^2+15a^3d^3)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{15\sqrt{\frac{b}{a}}\,d^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 11*a*d + 3*b*d*x^2) - I*
b*c*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-8*b^3*c^3 + 27*
a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])
```

<u>**Maple**</u> [A]
time = 0.10, size = 615, normalized size = 1.79

method	result

elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{b^2x^3\sqrt{bdx^4+adx^2+cx^2b+ac}}{5d}+\frac{\left(3ab^2-\frac{b^2(4ad+4bc)}{5d}\right)x\sqrt{bdx^4+adx^2+cx\cdots}}{3bd}\cdots\right.}{\cdots}$
risch	$\frac{bx(3bdx^2+11ad-4bc)\sqrt{bx^2+a}\sqrt{dx^2+c}}{15d^2}+\frac{\left(-\frac{(23a^2bd^2-23ab^2cd+8b^3c^2)c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\cdots\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}b^3d^3x^7+14\sqrt{-\frac{b}{a}}ab^2d^3x^5-\sqrt{-\frac{b}{a}}b^3cd^2x^5+11\sqrt{-\frac{b}{a}}a^2bd^3x^3+10\sqrt{-\frac{b}{a}}ab^2\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*b^3*d^3*x^7+14*(-b/a)^
(1/2)*a*b^2*d^3*x^5-(-b/a)^(1/2)*b^3*c*d^2*x^5+11*(-b/a)^(1/2)*a^2*b*d^3*x^
3+10*(-b/a)^(1/2)*a*b^2*c*d^2*x^3-4*(-b/a)^(1/2)*b^3*c^2*d*x^3+15*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^
3*d^3-34*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^2*b*c*d^2+27*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3+23*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*b*c*d^2-23*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3+11*(-b/a)^(1/2)*a^2*b*
c*d^2*x-4*(-b/a)^(1/2)*a*b^2*c^2*d*x)/d^3/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b
/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^(1/2), x)
```

3.201 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=260

$$-\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}}+\frac{bx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d}+\frac{2\sqrt{c}\,(bc-2ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] -2/3*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+2/3*(-2*a*d+b*c)*(1/(
1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d
*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*(-3*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)+1/3*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {427, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}\,\sqrt{a+bx^2}\,(bc-2ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d}-\frac{2x\sqrt{a+bx^2}\,(bc-2ad)}{3d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-2*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*d*Sqrt[c + d*x^2]) + (b*x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*d) + (2*Sqrt[c]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 3*a*d)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(
3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{\int \frac{-a(bc-3ad)-2b(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d}$$

$$= \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{(a(bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d} - \frac{(2b(bc-2ad))\int}{}$$

$$= -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{\sqrt{c}\,(bc-3ad)\sqrt{a+bx^2}\,F\Big(\text{ta}}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$= -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{2\sqrt{c}\,(bc-2ad)\sqrt{a+bx^2}\,E\Big(\text{t}}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.27, size = 216, normalized size = 0.83

$$\frac{b\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2) - 2ibc(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right) - i(2b^2c^2-5abcd+3a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3\sqrt{\frac{b}{a}}\,d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - (2*I)*b*c*(-(b*c) + 2*a*d)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - I*(2*b^2*c^2 - 5*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^
2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 399, normalized size = 1.53

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{bx\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3d} + \frac{\left(a^2-\frac{abc}{3d}\right)\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\Big(x\sqrt{}}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+c}}\right.}{}$

risch	$\frac{bx\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3d}+\frac{\left(-\frac{\left(4abd-2b^2c\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{Ellip}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,d}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(\sqrt{-\frac{b}{a}}\,b^2d^2x^5+\sqrt{-\frac{b}{a}}\,ab\,d^2x^3+\sqrt{-\frac{b}{a}}\,b^2cd\,x^3+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b^2*d^2*x^5+(-b/a)^(1/2)*
a*b*d^2*x^3+(-b/a)^(1/2)*b^2*c*d*x^3+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*d^2-5*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*b^2*c^2+4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(-b/a)^(1/2)*a*b*c*d*x)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
[Out] Integral((a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^(1/2), x)
```

3.202 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*
x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx &= a\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} + \frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} - c\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} + \frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\right.\right.}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
\end{aligned}$$

Mathematica [A]
time = 0.86, size = 86, normalized size = 0.44

$$\frac{\sqrt{a+bx^2}\,\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{\frac{a+bx^2}{a}}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]

[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])

Maple [A]
time = 0.07, size = 158, normalized size = 0.81

method	result
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\left(a \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\,\mathrm{El}\cdots\right.}{(bd x^4+ad x^2+c x^2 b+ac)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{a\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+c x^2 b+ac}}-\frac{bc\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1\cdots}}{\cdots}\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.203 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] (1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {429}

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A]
time = 0.69, size = 86, normalized size = 0.99

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}F\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
(a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.07, size = 100, normalized size = 1.15

method	result	size
default	$\frac{\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+c\,x^2b+ac)}$	100
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)
^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [A]
time = 0.10, size = 41, normalized size = 0.47

$$-\frac{\sqrt{ac}\,\sqrt{-\frac{b}{a}}\ \text{ellipticF}\left(x\sqrt{-\frac{b}{a}},\ \frac{ad}{bc}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*c)*sqrt(-b/a)*ellipticF(x*sqrt(-b/a), a*d/(b*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.204 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=273

$$-\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}}+\frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\,\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{}}{a(bc}$$

```
[Out] -d*x*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/(d*x^2+c)^(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2
+a)^(1/2)/a/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+b*x*
(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {425, 21, 433, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}\,(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{a\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((d*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*x*Sqrt[c + d*
x^2])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c
- a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{\int \frac{ad+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{}}}{a(bc-ad}$$

$$= -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}} + \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,F(\mathrm{ta}}{a(bc-ad)\sqrt{\frac{c(a+}{a(c+}}}$$

$$= -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}} + \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,E(\mathrm{ta}}{a(bc-ad)\sqrt{\frac{c(a+}{a(c+}}}$$

Mathematica [A]
time = 2.07, size = 112, normalized size = 0.41

$$\frac{-bx(c+dx^2)+\frac{ad\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)\middle|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}}}{a(-bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(b*x*(c + d*x^2)) + (a*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
E[ArcSin[Sqrt[-(d/c)]*x], (b*c)/(a*d)])/Sqrt[-(d/c)])/(a*(-(b*c) + a*d)*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.07, size = 248, normalized size = 0.91

method	result

default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bd\, x^{3}+a\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) d-\sqrt{\frac{d\, x^{2}+c}{c}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) \sqrt{\frac{b\, x^{2}\cdots}{a}}\right.}{\sqrt{-\frac{b}{a}}\, a(ad-bc)(bd\, x^{4}+ad\, x^{2}\cdots}$
elliptic	$\frac{\sqrt{(b\, x^{2}+a)\,(d\, x^{2}+c)}\left(-\frac{(bd\, x^{2}+bc)\, x}{a(ad-bc)\sqrt{\left(x^{2}+\frac{a}{b}\right)(bd\, x^{2}+bc)}}+\frac{\left(\frac{1}{a}+\frac{bc}{a(ad-bc)}\right)\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \mathrm{EllipticF}\cdots}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+\cdots}}\right.}{\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b/a)^(1/2)*b*d*x^3+a*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*b*c+((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*b*c-(-b/a)^(1/2)*b*c*x)
*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/a/(a*d-b*c)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.205 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=255

$$\frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{2\sqrt{b}\,(bc-2ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}(bc-ad)^2\sqrt{a+bx^2}\,\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\,\sqrt{d}\,(bc-3ad)\sqrt{a+b}}{3a^2(bc-ad)^2\sqrt{}}$$

```
[Out] -1/3*(-3*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/
2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(
1/2)/a^2/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*b
*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*(-2*a*d+b*c)*(1/(1+b*x^
2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2)
,(1-a*d/b/c)^(1/2))*b^(1/2)*(d*x^2+c)^(1/2)/a^(3/2)/(-a*d+b*c)^2/(b*x^2+a)^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {425, 539, 429, 422}

$$\frac{2\sqrt{b}\,\sqrt{c+dx^2}\,(bc-2ad)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{a+bx^2}\,(bc-ad)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,(bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}\,(bc-ad)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{bx\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*Sqrt[b]*(b*c
 - 2*a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/
(b*c)])/(3*a^(3/2)*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(b*c - a*d)^2*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)(a+bx^2)^{3/2}} - \frac{\int \frac{-2bc+3ad-bdx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)(a+bx^2)^{3/2}} - \frac{(d(bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)^2} + \frac{(2b(}{}$$

$$= \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{2\sqrt{b}\,(bc-2ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1\right.}{3a^{3/2}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.27, size = 261, normalized size = 1.02

$$\frac{b\sqrt{\frac{b}{a}}\,x(c+dx^2)\left(-5a^2d+2b^2cx^2+ab(3c-4dx^2)\right)-2ibc(-bc+2ad)(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-i(2b^2c^2-5abcd+3a^2d^2)(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3a^2\sqrt{\frac{b}{a}}(bc-ad)^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(-5*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 4*d*x^2)) -
 (2*I)*b*c*(-(b*c) + 2*a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*b^2*c^2 - 5*a*b*
c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(b*c - a*d)^2*(a
 + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(295) = 590$.
time = 0.09, size = 752, normalized size = 2.95

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(-\frac{x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3ba(ad-bc)\left(x^2+\frac{a}{b}\right)^2}-\frac{2\left(bd\,x^2+bc\right)x(2ad-bc)}{3a^2(ad-bc)^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(-\frac{\cdots}{3a(\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-4\sqrt{-\frac{b}{a}}\,a\,b^2d^2x^5+2\sqrt{-\frac{b}{a}}\,b^3cd\,x^5+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2b\,d^2x^2-5\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+\cdots}{c}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-4*(-b/a)^(1/2)*a*b^2*d^2*x^5+2*(-b/a)^(1/2)*b^3*c*d*x^5+3*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*
b*d^2*x^2-5*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b^2*c*d*x^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^2*x^2+4*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c*d*x^
2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*b^3*c^2*x^2-5*(-b/a)^(1/2)*a^2*b*d^2*x^3-(-b/a)^(1/2)*a*b^2*c*d*
x^3+2*(-b/a)^(1/2)*b^3*c^2*x^3+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*d^2-5*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d+2*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a*b^2*c^2+4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*a^2*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2-5*(-b/a)^(1/2)*a^2*b*c*d*x
+3*(-b/a)^(1/2)*a*b^2*c^2*x)/(d*x^2+c)^(1/2)/(a*d-b*c)^2/a^2/(-b/a)^(1/2)/(
b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

3.206 $\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=334

$$\frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}}+\frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}}+\frac{\sqrt{b}\left(8b^2c^2-23abcd+23a^2d^2\right)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\right.\right.}{15a^{5/2}(bc-ad)^3\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

```
[Out] -1/15*(15*a^2*d^2-11*a*b*c*d+4*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(
1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/
2)*d^(1/2)*(b*x^2+a)^(1/2)/a^3/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)+1/5*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(5/2)+4/15*
b*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)/a^2/(-a*d+b*c)^2/(b*x^2+a)^(3/2)+1/15*(23*
a^2*d^2-23*a*b*c*d+8*b^2*c^2)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*Ellip
ticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*b^(1/2)*(d*x^2+
c)^(1/2)/a^(5/2)/(-a*d+b*c)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Rubi [A]
time = 0.19, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {425, 541, 539, 429, 422}

$$\frac{4bx\sqrt{c+dx^2}\,(bc-2ad)}{15a^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}+\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{a+bx^2}\,(bc-ad)^3\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}\,(bc-ad)^3\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*b*(b*c - 2*a
*d)*x*Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (Sqrt[b]*
(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqr
t[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*(b*c - a*d)^3*Sqrt[a + b*x^
2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(4*b^2*c^2 - 1
1*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\,(a+bx^2)^{5/2}} - \frac{\int \frac{-4bc+5ad-3bdx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx}{5a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\,(a+bx^2)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\,(a+bx^2)^{3/2}} + \frac{\int \frac{8b^2c^2-19abcd+15a^2d^2 \cdots}{(a+bx^2)^{3/2}\sqrt{c \cdots}}}{15a^2(bc - \cdots}$$

$$= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\,(a+bx^2)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\,(a+bx^2)^{3/2}} - \frac{(d(4b^2c^2-11abcd \cdots}{\cdots}$$

$$= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\,(a+bx^2)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\,(a+bx^2)^{3/2}} + \frac{\sqrt{b}\,(8b^2c^2-23abc \cdots}{15a^{5/\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.55, size = 301, normalized size = 0.90

$$\frac{b\sqrt{\frac{b}{a}}\,x(c+dx^2)\left(3a^2(bc-ad)^2+4a(bc-2ad)(bc-ad)\,(a+bx^2)+(8b^2c^2-23abcd+23a^2d^2)\,(a+bx^2)^2\right)+i(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(bc(8b^2c^2-23abcd+23a^2d^2)\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)+(-8b^3c^3+27ab^2c^2d-34a^2bcd^2+15a^3d^3)\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{15a^3\sqrt{\frac{b}{a}}\,(bc-ad)^3\,(a+bx^2)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 4*a*(b*c - 2*a*d)*(b*c -
a*d)*(a + b*x^2) + (8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*(a + b*x^2)^2) + I
*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*c*(8*b^2*c^2 - 23
*a*b*c*d + 23*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8
*b^3*c^3 + 27*a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^3*(a + b*x^2)^
(5/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1606 vs. $2(368) = 736$.
time = 0.08, size = 1607, normalized size = 4.81

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(-\frac{x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{5b^2a(ad-bc)\left(x^2+\frac{a}{b}\right)^3}-\frac{4(2ad-bc)x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{15b\,a^2(ad-bc)^2\left(x^2+\frac{a}{b}\right)^2}-\cdots\right.}{\cdots}$

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(15*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^3*b^2*d^3*x^4+30*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*b*d^3*x^2-16*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^4*c^3
*x^2+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*a*b^4*c^3*x^2-34*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*b*c*d^2+27*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b^2*c^2*d+
23*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a^4*b*c*d^2-23*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b^2*c^2*d-8*(-b/a)^(1/2)*b^5*c^3*x^5+54
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*a^2*b^3*c^2*d*x^2+15*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^5*d^3-23*(-b/a)^(1/2)*a^2*b^3*d^3*x^7
-8*(-b/a)^(1/2)*b^5*c^2*d*x^7-54*(-b/a)^(1/2)*a^3*b^2*d^3*x^5-34*(-b/a)^(1/
2)*a^4*b*d^3*x^3-20*(-b/a)^(1/2)*a*b^4*c^3*x^3-15*(-b/a)^(1/2)*a^2*b^3*c^3*
x-34*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^2*b^3*c*d^2*x^4+27*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^4*c^2*d*x^4+23*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^3*c
*d^2*x^4-23*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b^4*c^2*d*x^4-68*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b^2*c*d^2*x^2+46*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^
3*b^2*c*d^2*x^2-46*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^3*c^2*d*x^2+23*(-b/a)^(1/2)*a*b^4*c*d^2*x^7
+35*(-b/a)^(1/2)*a^2*b^3*c*d^2*x^5+3*(-b/a)^(1/2)*a*b^4*c^2*d*x^5-13*(-b/a)
^(1/2)*a^3*b^2*c*d^2*x^3+43*(-b/a)^(1/2)*a^2*b^3*c^2*d*x^3-34*(-b/a)^(1/2)*
a^4*b*c*d^2*x+41*(-b/a)^(1/2)*a^3*b^2*c^2*d*x-8*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^5*c^3*x^4+8*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b^5*c^3*x^4-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a^2*b^3*c^3+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^3*c^3)/(d*x^2+c)^(1/2)/(a*
d-b*c)^3/a^3/(-b/a)^(1/2)/(b*x^2+a)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{7/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)), x)
```

3.207 $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=445

$$\frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}}-\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}}-\frac{b(24b^2c^2-43abcd+15a\ldots}{1\ldots}$$

```
[Out] -(-a*d+b*c)*x*(b*x^2+a)^(5/2)/c/d/(d*x^2+c)^(1/2)+1/15*(-15*a^3*d^3+103*a^2
*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*x*(b*x^2+a)^(1/2)/c/d^3/(d*x^2+c)^(1/2
)-1/15*(-15*a^3*d^3+103*a^2*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(7/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)+1/15*b*(45*a^2*d^2-61*a*b*c*d+24*b^2*c^2)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(d*x^2+c)^(1/2)+1/5*b*(-5*a*d+6*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^
(1/2)/c/d^2-1/15*b*(15*a^2*d^2-43*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/c/d^3
```

Rubi [A]
time = 0.26, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {424, 542, 545, 429, 506, 422}

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\,(45a^2d^2-61abcd+24b^2c^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\,(15a^2d^2-43abcd+24b^2c^2)}{15cd^3}-\frac{\sqrt{a+bx^2}\,(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}\,d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3)}{15cd^3\sqrt{c+dx^2}}+\frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(6bc-5ad)}{5cd^2}-\frac{x(a+bx^2)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*Sqrt[a + b
*x^2])/(15*c*d^3*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(5/2))/(c*d*
Sqrt[c + d*x^2]) - (b*(24*b^2*c^2 - 43*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/(15*c*d^3) + (b*(6*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*Sq
rt[c + d*x^2])/(5*c*d^2) - ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2
 - 15*a^3*d^3)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(15*Sqrt[c]*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) + (b*Sqrt[c]*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(7/2)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} + \frac{\int \frac{(a+bx^2)^{3/2}(abc+b(6bc-5ad)x^2)}{\sqrt{c+dx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} + \frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2} + \frac{\int \sqrt{a+bx^2}\,(-2abc}{}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15cd^3} + \frac{b(6}{}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15cd^3} + \frac{b(6}{}$$

$$= \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} - \frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b}{}$$

$$= \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} - \frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.49, size = 318, normalized size = 0.71

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)\left(-45a^2bcd^2+15a^3d^3+ab^2cd(61c+16dx^2)-3b^3c(8c^2+2cdx^2-d^2x^4)\right)+ibc(-48b^3c^3+128ab^2c^2d-103a^2bcd^2+15a^3d^3)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)+4ibc(12b^3c^3-38ab^2c^2d+41a^2bcd^2-15a^3d^3)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{15\sqrt{\frac{b}{a}}\,cd^4\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-45*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*c*d*(61*c
+ 16*d*x^2) - 3*b^3*c*(8*c^2 + 2*c*d*x^2 - d^2*x^4)) + I*b*c*(-48*b^3*c^3 +
 128*a*b^2*c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*I)*b*c*(1
2*b^3*c^3 - 38*a*b^2*c^2*d + 41*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*
Sqrt[b/a]*c*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 755, normalized size = 1.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*b^4*c*d^3*x^7+19*(-b/a
)^(1/2)*a*b^3*c*d^3*x^5-6*(-b/a)^(1/2)*b^4*c^2*d^2*x^5+15*(-b/a)^(1/2)*a^3*
b*d^4*x^3-29*(-b/a)^(1/2)*a^2*b^2*c*d^3*x^3+55*(-b/a)^(1/2)*a*b^3*c^2*d^2*x
^3-24*(-b/a)^(1/2)*b^4*c^3*d*x^3+60*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^3-164*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c
^2*d^2+152*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a*b^3*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4-15*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^3+103*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a^2*b^2*c^2*d^2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*d+48*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4+15*(-b/a)^(1
/2)*a^4*d^4*x-45*(-b/a)^(1/2)*a^3*b*c*d^3*x+61*(-b/a)^(1/2)*a^2*b^2*c^2*d^2
*x-24*(-b/a)^(1/2)*a*b^3*c^3*d*x)/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^
(1/2)/c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^{\frac{7}{2}}}{(c + dx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(7/2)/(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{7/2}}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(7/2)/(c + d*x^2)^(3/2), x)
```

3.208 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{(8b^2c^2-13abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}}-\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}}+\frac{b(4bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cd^2}+\frac{(8b^2c$$

```
[Out] -(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^(1/2)-1/3*(3*a^2*d^2-13*a*b*c*d
+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/c/d^2/(d*x^2+c)^(1/2)+1/3*(3*a^2*d^2-13*a*b*c
*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(5/2)/c^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*b*(-3*a*d+2*b*c)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*
x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*b*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/c/d^2
```

Rubi [A]

time = 0.18, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {424, 542, 545, 429, 506, 422}

$$\frac{\sqrt{a+bx^2}\,(3a^2d^2-13abcd+8b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}\,d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\,(3a^2d^2-13abcd+8b^2c^2)}{3cd^2\sqrt{c+dx^2}}-\frac{2b\sqrt{c}\,\sqrt{a+bx^2}\,(2bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(4bc-3ad)}{3cd^2}-\frac{x(a+bx^2)^{3/2}\,(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -1/3*((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(c*d^2*Sqrt[c
 + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) + (b*(
4*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d^2) + ((8*b^2*c^2 -
 13*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{\int \frac{\sqrt{a+bx^2}\,(abc+b(4bc-3ad)x^2)}{\sqrt{c+dx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cd^2} + \frac{\int \frac{-2abc(2bc-3ad)-b(8b^2c^2}{\sqrt{a+bx^2}\,\sqrt{}}}{3cd^2}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cd^2} - \frac{(2ab(2bc-3ad))\int -}{3}$$

$$= -\frac{(8b^2c^2-13abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a}}{3cd}$$

$$= -\frac{(8b^2c^2-13abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a}}{3cd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.68, size = 256, normalized size = 0.74

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)\left(-6abcd+3a^2d^2+b^2c(4c+dx^2)\right)+ibc(8b^2c^2-13abcd+3a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ibc(8b^2c^2-17abcd+9a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3\sqrt{\frac{b}{a}}\,cd^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(4*c + d*x^2)) +
 I*b*c*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(8*b^2*c^2
- 17*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])
```

Maple [A]
time = 0.11, size = 539, normalized size = 1.56

method	result

elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{(bdx^2+ad)(a^2d^2-2abcd+b^2c^2)x}{d^3c\sqrt{(x^2+\frac{c}{d})(bdx^2+ad)}}+\frac{b^2x\sqrt{bdx^4+adx^2+cx^2b+ac}}{3d^2}+\frac{\left(\frac{b(3a^2d^2-3\cdots}{d}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}b^3cd^2x^5+3\sqrt{-\frac{b}{a}}a^2bd^3x^3-5\sqrt{-\frac{b}{a}}ab^2cd^2x^3+4\sqrt{-\frac{b}{a}}b^3c^2dx^3+9\sqrt{\frac{bx^2+a}{a}}\cdots\right.}{\cdots}$
risch	$\frac{b^2x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d^2}+\frac{\left(b\left(-\frac{(7abd^2-5b^2cd)c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\cdots\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+cx^2b+\cdots}}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b^3*c*d^2*x^5+3*(-b/a)^(1
/2)*a^2*b*d^3*x^3-5*(-b/a)^(1/2)*a*b^2*c*d^2*x^3+4*(-b/a)^(1/2)*b^3*c^2*d*x
^3+9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^2*b*c*d^2-17*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3-3*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2
*b*c*d^2+13*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b^2*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3+3*(-b/a)^(1/2)*a^3*d^3*x-6*
(-b/a)^(1/2)*a^2*b*c*d^2*x+4*(-b/a)^(1/2)*a*b^2*c^2*d*x)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)/d^3/c/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^(3/2), x)
```

3.209 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}+\frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}-\frac{(2bc-ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{b\sqrt{c}\,\sqrt{}}{}$$

```
[Out] -(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)^(1/2)+(-a*d+2*b*c)*x*(b*x^2+a)^
(1/2)/c/d/(d*x^2+c)^(1/2)-(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2
+a)^(1/2)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+b
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)
/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {424, 545, 429, 506, 422}

$$\frac{b\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\,(2bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,d^{3/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\,(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\,(2bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -(((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*c - a*d)*x
*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{\int \frac{abc+b(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(ab)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{d} + \frac{(b(2bc-ad))\int \frac{}{\sqrt{a+bx^2}}}{cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{b\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right.}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx}}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\right.\right.}{\sqrt{c}\,d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.09, size = 196, normalized size = 0.76

$$\frac{ibc(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)+(-bc+ad)\left(\sqrt{\frac{b}{a}}\,dx(a+bx^2)-2ibc\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}\,cd^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^
2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Maple [A]
time = 0.08, size = 345, normalized size = 1.34

method	result
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(\sqrt{-\frac{b}{a}}\,ab\,d^2x^3-\sqrt{-\frac{b}{a}}\,b^2cd\,x^3+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd\right.}{}$

elliptic	$$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{(bd\,x^2+ad)(ad-bc)x}{d^2c\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{\left(\frac{b(2ad-bc)}{d^2}+\frac{(ad-bc)^2}{d^2c}-\frac{a(ad-bc)}{dc}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d}{}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*a*b*d^2*x^3-(-b/a)^(1/2)*b^2*
c*d*x^3+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(
-b/a)^(1/2)*a^2*d^2*x-(-b/a)^(1/2)*a*b*c*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
/d^2/c/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] (1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {422}

$$\frac{\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.06, size = 136, normalized size = 1.62

$$\frac{\frac{x(a+bx^2)}{c}+\frac{ia\sqrt{\frac{b}{a}}\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{d}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((x*(a + b*x^2))/c + (I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/d)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 188, normalized size = 2.24

method	result
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(\sqrt{-\frac{b}{a}}\,bd\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{b\,x^2+a}{a}}\,bc-\sqrt{\frac{d\,x^2+c}{c}}\,\text{EllipticE}\right.}{(bd\,x^4+ad\,x^2+c\,x^2b+ac)dc\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\left(\frac{(bd\,x^2+ad)\,x}{dc\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{\left(\frac{b}{d}+\frac{ad-bc}{cd}-\frac{a}{c}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2}}\right.}{\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b*d*x^3+((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*b*c-((d*x^2+c)
/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*b*c
+(-b/a)^(1/2)*a*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d/c/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^(3/2), x)
```

3.211 $\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/d^(1/2
)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*d^(1/2)*(b*x^2+a)^(1/2)/(-a*d+b*c)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {425, 21, 433, 429, 506, 422}

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}} + \frac{\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad} + \frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{}}}{c(bc-a}$$

$$= \frac{b\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,(bc-ad)\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}\,\sqrt{c+dx^2}} - \frac{d\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,(bc-ad)\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}\,\sqrt{c+dx^2}} + \frac{b\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\right.\right.}{a\sqrt{d}\,(bc-ad)\sqrt{\frac{c\,(a+b}{a\,(c+d}}}$$

Mathematica [A]
time = 2.10, size = 112, normalized size = 0.58

$$\frac{-dx(a+bx^2)+\frac{bc\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}}{c(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(d*x*(a + b*x^2)) + (b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
E[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(c*(b*c - a*d)*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.07, size = 144, normalized size = 0.74

method	result

default	$\frac{\left(\sqrt{-\frac{b}{a}}\, b d\,x^{3}-\sqrt{\frac{d\,x^{2}+c}{c}}\, \mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{b\,x^{2}+a}{a}}\, bc+\sqrt{-\frac{b}{a}}\, adx\right)\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}}{c\sqrt{-\frac{b}{a}}\,(ad-bc)(bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac)}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)\,(d\,x^{2}+c)}\left(\frac{(bd\,x^{2}+ad)x}{c(ad-bc)\sqrt{\left(x^{2}+\frac{c}{d}\right)(bd\,x^{2}+ad)}}+\frac{\left(\frac{1}{c}-\frac{ad}{c(ad-bc)}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+c}}\right.}{\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b/a)^(1/2)*b*d*x^3-((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*((b*x^2+a)/a)^(1/2)*b*c+(-b/a)^(1/2)*a*d*x)*(d*x^2+c)^(1/2)*(b*x^2
+a)^(1/2)/c/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^{2}}\,(c+dx^{2})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.212 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{d}\,(bc+ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\,(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}{a(bc-ad)^2\sqrt{\;}}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+(a*d+b*c)*(1/(1+d*x^2/c))^
(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*
c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)^2/c^(1/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/
2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)
*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*
x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {425, 539, 429, 422}

$$-\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}\,(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}\,(ad+bc)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}\,(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(b*c + a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[c]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (2*b*Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}} - \frac{\int \frac{ad-bdx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx}{a(bc-ad)}$$

$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}} - \frac{(2bd)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{(bc-ad)^2} + \frac{(d(b\ldots}{\ldots}$$

$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}} + \frac{\sqrt{d}\,(bc+ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\ldots\right.\right.}{a\sqrt{c}\,(bc-ad)^2\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}\,\sqrt{\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.51, size = 224, normalized size = 0.93

$$\frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}\,x\left(a^2d^2+abd^2x^2+b^2c\left(c+dx^2\right)\right)+ibc(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)+ibc(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{bc(bc-ad)^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)) + I*b*c
*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*(b*c - a
*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.09, size = 354, normalized size = 1.46

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, ab\, d^{2} x^{3}+\sqrt{-\frac{b}{a}}\, b^{2} c d\, x^{3}-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, \mathrm{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) a b c d-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, \mathrm{Ellipt}\right.}{}$
elliptic	$\frac{\sqrt{\left(b\, x^{2}+a\right) \left(d\, x^{2}+c\right)}\, \left(-\frac{2 b d \left(-\frac{(a d+b c) x^{3}}{2 a c \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}-\frac{\left(a^{2} d^{2}+b^{2} c^{2}\right) x}{2 a c \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) b d}\right)}{\sqrt{\left(x^{4}+\frac{(a d+b c) x^{2}}{b d}+\frac{a c}{b d}\right) b d}}+\frac{\left(\frac{1}{a c}-\frac{a^{2} d^{2}+b^{2} c^{2}}{a c \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}\right) \sqrt{1+}}{\sqrt{-\frac{b}{a}}\, \sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b/a)^(1/2)*a*b*d^2*x^3+(-b/a)^(1/2)*b^2*c*d*x^3-((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^
2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*b*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(-b/a)^(1/2)*a^2*d^2*x+(-b/a)^(1/2)*b^2
*c^2*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/(-b/a)^(1/2)/a/(a*d-b*c)^2/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x)
```

3.213 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}+\frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{d}\left(2b^2c^2-7abcd-3a^2d^2\right)\sqrt{a+}}{3a^2\sqrt{c}\,(bc-ad)^3\sqrt{\frac{c(}{a(}}}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)+2/3*b*(-3*a*d+b*c)*x/a
^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*(-3*a^2*d^2-7*a*b*c*d+2
*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-a*d+b
*c)^3/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(-9*a*d
+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-a
*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {425, 541, 539, 429, 422}

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-9ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2bx(bc-3ad)}{3a^2\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (2*b*(b*c - 3*a
*d)*x)/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(2*
b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*(b*c - a*d)^3*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*Sqrt[d]*(b*c - 9*a*d)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
*a^2*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} - \frac{\int \frac{-2bc+3ad-3bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} +$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} -$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.73, size = 337, normalized size = 1.04

$$\frac{\sqrt{\frac{b}{a}}\,x\left(3a^4d^3+6a^3bd^3x^2-2b^4c^2x^2\left(c+dx^2\right)+a^2b^2d\left(8c^2+8cdx^2+3d^2x^4\right)+ab^3c\left(-3c^2+4cdx^2+7d^2x^4\right)\right)+ibc\left(-2b^2c^2+7abcd+3a^2d^2\right)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)+2ibc\left(b^2c^2-4abcd+3a^2d^2\right)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{3a^2\sqrt{\frac{b}{a}}\,c(-bc+ad)^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 2*b^4*c^2*x^2*(c + d*x^2) + a^2
*b^2*d*(8*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + a*b^3*c*(-3*c^2 + 4*c*d*x^2 + 7*d^
2*x^4)) + I*b*c*(-2*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] + (2*I)*b*c*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3
*a^2*Sqrt[b/a]*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(357) = 714$.
time = 0.10, size = 964, normalized size = 2.98

method	result
elliptic	$\frac{\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3a(ad-bc)^2\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd\,x^2+bc\right)bx(7ad-2bc)}{3a^2(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{}{c(ad-bc)^3}\right.}{}$

default	$-\frac{-3\sqrt{-\frac{b}{a}}\,a^2b^2d^3x^5-7\sqrt{-\frac{b}{a}}\,a\,b^3c\,d^2x^5+2\sqrt{-\frac{b}{a}}\,b^4c^2d\,x^5+6\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-3*(-b/a)^(1/2)*a^2*b^2*d^3*x^5-7*(-b/a)^(1/2)*a*b^3*c*d^2*x^5+2*(-b/
a)^(1/2)*b^4*c^2*d*x^5+6*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c*d^2*x^2-8*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^2*d*x^2+
2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*b^4*c^3*x^2+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c*d^2*x^2+7*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^2*d*x^2-2*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*b^4*c^3*x^2-6*(-b/a)^(1/2)*a^3*b*d^3*x^3-8*(-b/a)^(1/2)*a^2*b^2*c*d^
2*x^3-4*(-b/a)^(1/2)*a*b^3*c^2*d*x^3+2*(-b/a)^(1/2)*b^4*c^3*x^3+6*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^
3*b*c*d^2-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3+3*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^2+7*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*a^2*b^2*c^2*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3-3*(-b/a)^(1/2)*a^4*d^3*x-8*(-b/a)^(
1/2)*a^2*b^2*c^2*d*x+3*(-b/a)^(1/2)*a*b^3*c^3*x)/(d*x^2+c)^(1/2)/(a*d-b*c)^
3/a^2/(-b/a)^(1/2)/c/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{5/2}\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x)
```

3.214 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] (1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {429}

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 0.99

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}F\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
(a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.00, size = 100, normalized size = 1.15

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+c\,x^2b+ac)}$	100
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)
^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [A]
time = 0.17, size = 41, normalized size = 0.47

$$-\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}\,\mathrm{ellipticF}\left(x\sqrt{-\frac{b}{a}},\ \frac{ad}{bc}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a*c)*sqrt(-b/a)*ellipticF(x*sqrt(-b/a), a*d/(b*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.215 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[Out] EllipticF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {432, 430}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*
x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{c+dx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Mathematica [A]
time = 0.73, size = 87, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,F\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], -((
a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 103, normalized size = 1.18

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{\frac{b}{a}}\,(-bd\,x^4+ad\,x^2-c\,x^2b+ac)}$	103
elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-c\,x^2b+ac}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a
)^(1/2)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*
x^2+a*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [**A**]
time = 0.16, size = 39, normalized size = 0.45

$$\frac{\sqrt{ac}\,\sqrt{\frac{b}{a}}\,\mathrm{ellipticF}\left(x\sqrt{\frac{b}{a}},\,-\frac{ad}{bc}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*c)*sqrt(b/a)*ellipticF(x*sqrt(b/a), -a*d/(b*c))/(b*c)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.216 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[Out] EllipticF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {432, 430}

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}$$

$$= \frac{\left(\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Mathematica [A]
time = 0.75, size = 89, normalized size = 1.02

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
-((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Maple [A]
time = 0.08, size = 103, normalized size = 1.18

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}}{\sqrt{\frac{d}{c}}\,(-bd\,x^4-ad\,x^2+c\,x^2b+ac)}$	103
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(-d\,x^2+c)}\,\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+c\,x^2b+ac}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c
)^(1/2)*(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*
x^2+a*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Fricas [**A**]
time = 0.13, size = 39, normalized size = 0.45

$$\frac{\sqrt{ac}\sqrt{\frac{d}{c}}\,\text{ellipticF}\left(x\sqrt{\frac{d}{c}},-\frac{bc}{ad}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*c)*sqrt(d/c)*ellipticF(x*sqrt(d/c), -b*c/(a*d))/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

3.217 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

```
[Out] EllipticF(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(1-b*x^2/a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {432, 430}

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*
x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}$$
$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a-bx^2}\sqrt{c-dx^2}}$$
$$= \frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Mathematica [A]
time = 0.75, size = 88, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,F\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], (a*
d)/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Maple [A]
time = 0.08, size = 104, normalized size = 1.18

method	result	size
default	$\frac{\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+c}}{\sqrt{\frac{d}{c}}\,(bd\,x^4-ad\,x^2-c\,x^2b+ac)}$	104
elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(-d\,x^2+c)}\,\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-c\,x^2b+ac}}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c
)^(1/2)*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(b*d*x^4-a*d*x^2-b*c*
x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Fricas [A]
time = 0.14, size = 38, normalized size = 0.43

$$\frac{\sqrt{ac}\,\sqrt{\frac{d}{c}}\,\text{ellipticF}\left(x\sqrt{\frac{d}{c}},\,\frac{bc}{ad}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*c)*sqrt(d/c)*ellipticF(x*sqrt(d/c), b*c/(a*d))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-bx^2}\,\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
[Out] int(1/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

3.218 $\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*10^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {430}

$$\frac{F\left(\text{ArcSin}(x)\,\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+5x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.22, size = 12, normalized size = 1.00

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Maple [A]
time = 0.10, size = 14, normalized size = 1.17

method	result	size
default	$\frac{\operatorname{EllipticF}\left(x, \frac{i\sqrt{10}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-\left(x^{2}-1\right)\left(5x^{2}+2\right)}\,\sqrt{10x^{2}+4}\,\operatorname{EllipticF}\left(x, \frac{i\sqrt{10}}{2}\right)}{2\sqrt{5x^{2}+2}\,\sqrt{-5x^{4}+3x^{2}+2}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*I*10^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 0.12, size = 8, normalized size = 0.67

$$\frac{1}{2}\sqrt{2}\,\operatorname{ellipticF}\left(x, -\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*ellipticF(x, -5/2)
```

Sympy [A]
time = 1.62, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\sqrt{2}\,F\left(\operatorname{asin}{\left(x \right)}\middle| -\frac{5}{2}\right)}{2} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -5/2)/2, (x > -1) & (x < 1)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(5*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.219 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{F(\sin^{-1}(x)|-2)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,I*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{430\}$

$$\frac{F(\text{ArcSin}(x)|-2)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{F(\sin^{-1}(x)|-2)}{\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 58, normalized size = 5.80

$$-\frac{i\sqrt{1-x^2}\sqrt{1+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)}{2\sqrt{1+x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

[Out] ((-1/2*I)*Sqrt[1 - x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/2])/Sqrt[1 + x^2 - 2*x^4]

Maple [A]
time = 0.11, size = 14, normalized size = 1.40

method	result	size
default	$\frac{\operatorname{EllipticF}\left(x,i\sqrt{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-\left(x^2-1\right)\left(2x^2+1\right)}\,\operatorname{EllipticF}\left(x,i\sqrt{2}\right)}{\sqrt{-4x^4+2x^2+2}}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*EllipticF(x,I*2^(1/2))*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)

Fricas [A]
time = 0.14, size = 8, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\,\operatorname{ellipticF}(x,-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*ellipticF(x, -2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{1-x^2}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(4*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.220 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {430}

$$\frac{F\left(\text{ArcSin}(x)\,\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.22, size = 12, normalized size = 1.00

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Maple [A]
time = 0.08, size = 14, normalized size = 1.17

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-\left(3x^{2}+2\right)\left(x^{2}-1\right)}\,\sqrt{6x^{2}+4}\,\mathrm{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^{2}+2}\,\sqrt{-3x^{4}+x^{2}+2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 0.11, size = 8, normalized size = 0.67

$$\frac{1}{2}\sqrt{2}\,\mathrm{ellipticF}\left(x, -\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*ellipticF(x, -3/2)
```

Sympy [A]
time = 1.61, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\sqrt{2}\,F\left(\operatorname{asin}\left(x\right)\middle| -\frac{3}{2}\right)}{2} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -3/2)/2, (x > -1) & (x < 1)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.221 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {254, 227}

$$\frac{F(\text{ArcSin}(x)|-1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx &= \int \frac{1}{\sqrt{2-2x^4}}\,dx\\ &= \frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 10.02, size = 10, normalized size = 1.00

$$\frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A]
time = 0.09, size = 10, normalized size = 1.00

method	result	size
default	$\frac{\text{EllipticF}(x,i)\sqrt{2}}{2}$	10
elliptic	$\frac{\sqrt{-x^4+1}\ \text{EllipticF}(x,i)}{\sqrt{-2x^4+2}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 0.12, size = 8, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\,\text{ellipticF}(x,-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*ellipticF(x, -1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(8) = 16.
time = 10.95, size = 73, normalized size = 7.30

$$-\frac{\sqrt{2}\,G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}+\frac{\sqrt{2}\,G_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
(0,)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3/2)) + sqrt(2)*meijerg(((-1/4, 0,
 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*pi**(
3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.222 $\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {430}

$$\frac{F\left(\text{ArcSin}(x)\,\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 18, normalized size = 1.50

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[2]], -2]
```

Maple [A]
time = 0.09, size = 14, normalized size = 1.17

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x, \frac{i\sqrt{2}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-\left(x^{2}-1\right)\left(x^{2}+2\right)}\,\sqrt{2x^{2}+4}\,\mathrm{EllipticF}\left(x, \frac{i\sqrt{2}}{2}\right)}{2\sqrt{x^{2}+2}\,\sqrt{-x^{4}-x^{2}+2}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 0.19, size = 8, normalized size = 0.67

$$\frac{1}{2}\sqrt{2}\,\mathrm{ellipticF}\left(x, -\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*ellipticF(x, -1/2)
```

Sympy [A]
time = 1.22, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\sqrt{2}\,F\left(\operatorname{asin}{\left(x \right)}\middle| - \frac{1}{2}\right)}{2} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -1/2)/2, (x > -1) & (x < 1)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.223 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{430\}$

$$\frac{F\left(\text{ArcSin}(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.20, size = 12, normalized size = 1.00

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]

Maple [A]
time = 0.10, size = 13, normalized size = 1.08

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x, \frac{\sqrt{2}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{\left(x^{2}-1\right)\left(x^{2}-2\right)}\,\sqrt{-2x^{2}+4}\,\mathrm{EllipticF}\left(x, \frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^{2}+2}\,\sqrt{x^{4}-3x^{2}+2}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 0.24, size = 8, normalized size = 0.67

$$\frac{1}{2}\sqrt{2}\,\mathrm{ellipticF}\left(x, \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*ellipticF(x, 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x-1\right)\left(x+1\right)}\,\sqrt{2-x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2 - x^2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.224 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {22, 212}

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] ArcTanh[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx &= \frac{\int \frac{1}{1-x^2}\,dx}{\sqrt{2}}\\ &= \frac{\tanh^{-1}(x)}{\sqrt{2}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(8) = 16.
time = 0.01, size = 26, normalized size = 3.25

$$-\frac{\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] -((Log[1 - x]/2 - Log[1 + x]/2)/Sqrt[2])
```

Maple [A]
time = 0.08, size = 8, normalized size = 1.00

method	result	size
meijerg	$\frac{\operatorname{arctanh}(x)\sqrt{2}}{2}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(x)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(7) = 14.
time = 1.25, size = 68, normalized size = 8.50

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{x^6+5x^4-2\sqrt{2}\left(x^3+x\right)\sqrt{-x^2+1}\sqrt{-2x^2+2}-5x^2-1}{x^6-3x^4+3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(x^6 + 5*x^4 - 2*sqrt(2)*(x^3 + x)*sqrt(-x^2 + 1)*sqrt(-2*
x^2 + 2) - 5*x^2 - 1)/(x^6 - 3*x^4 + 3*x^2 - 1))
```

Sympy [A]
time = 0.92, size = 22, normalized size = 2.75

$$-\sqrt{2}\left(\begin{cases}-\frac{\operatorname{acoth}(x)}{2} & \text{for}\ x^2>1\\ -\frac{\operatorname{atanh}(x)}{2} & \text{for}\ x^2<1\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(2)*Piecewise((-acoth(x)/2, x**2 > 1), (-atanh(x)/2, x**2 < 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.225 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}}$$

[Out] 1/2*EllipticF(x,1/2*6^(1/2))*2^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{430\}$

$$\frac{F\left(\text{ArcSin}(x)\,\middle|\,\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]

[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.21, size = 12, normalized size = 1.00

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]

[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]

Maple [A]
time = 0.10, size = 13, normalized size = 1.08

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x, \frac{\sqrt{6}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{\left(3x^{2}-2\right)\left(x^{2}-1\right)}\,\sqrt{-6x^{2}+4}\,\mathrm{EllipticF}\left(x, \frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^{2}+2}\,\sqrt{3x^{4}-5x^{2}+2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*EllipticF(x,1/2*6^(1/2))*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)

Fricas [A]
time = 0.15, size = 8, normalized size = 0.67

$$\frac{1}{2}\sqrt{2}\,\mathrm{ellipticF}\left(x, \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*ellipticF(x, 3/2)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.
time = 1.34, size = 34, normalized size = 2.83

$$\begin{cases} \frac{\sqrt{3}\,F\left(\operatorname{asin}\left(\frac{\sqrt{6}\,x}{2}\right)\middle|\,\frac{2}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.226 $\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{F(\sin^{-1}(x)|\,2)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{430\}$

$$\frac{F(\text{ArcSin}(x)|2)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{1-x^2}}\,dx = \frac{F(\sin^{-1}(x)|\,2)}{\sqrt{2}}$$

Mathematica [A]
time = 0.06, size = 10, normalized size = 1.00

$$\frac{F(\sin^{-1}(x)|\,2)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Maple [A]
time = 0.10, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x,\sqrt{2}\right)\sqrt{2}}{2}$	11
elliptic	$\frac{\sqrt{\left(2x^{2}-1\right)\left(x^{2}-1\right)}\,\mathrm{EllipticF}\left(x,\sqrt{2}\right)}{\sqrt{4x^{4}-6x^{2}+2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [A]
time = 0.18, size = 8, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\,\mathrm{ellipticF}(x,2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*ellipticF(x, 2)
```

Sympy [A]
time = 1.86, size = 39, normalized size = 3.90

$$\frac{\sqrt{2}\left(\begin{cases}\frac{\sqrt{2}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2} & \text{for}\ x>-\frac{\sqrt{2}}{2}\wedge x<\frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sqrt(2
)/2) & (x < sqrt(2)/2)))/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-4\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 4*x^2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.227 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {430}

$$\frac{F\left(\text{ArcSin}(x)\,\middle|\,\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.22, size = 12, normalized size = 1.00

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Maple [A]
time = 0.10, size = 13, normalized size = 1.08

method	result	size
default	$\frac{\operatorname{EllipticF}\left(x, \frac{\sqrt{10}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{\left(5x^{2}-2\right)\left(x^{2}-1\right)}\,\sqrt{-10x^{2}+4}\,\operatorname{EllipticF}\left(x, \frac{\sqrt{10}}{2}\right)}{2\sqrt{-5x^{2}+2}\,\sqrt{5x^{4}-7x^{2}+2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [A]
time = 0.21, size = 8, normalized size = 0.67

$$\frac{1}{2}\sqrt{2}\operatorname{ellipticF}\left(x, \frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*ellipticF(x, 5/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.
time = 1.34, size = 34, normalized size = 2.83

$$\begin{cases} \frac{\sqrt{5}\,F\left(\operatorname{asin}\left(\frac{\sqrt{10}\,x}{2}\right)\middle|\,\frac{2}{5}\right)}{5} & \text{for}\: x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), 2/5)/5, (x > -sqrt(10)/5)
 & (x < sqrt(10)/5)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-5\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 5*x^2)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.228 $\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{2+5x^2}\,F\left(\tan^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+5x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*6^(1/2))*(5*x^2+2)^(1
/2)*2^(1/2)/((5*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{429\}$

$$\frac{\sqrt{5x^2+2}\,F\left(\text{ArcTan}(x)\,|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\frac{5x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 5*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+5x^2}}\,dx = \frac{\sqrt{2+5x^2}\,F\left(\tan^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 19, normalized size = 0.37

$$-\frac{iF\left(i\sinh^{-1}(x)|\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 5/2])/Sqrt[2]
```

Maple [A]
time = 0.08, size = 17, normalized size = 0.33

method	result	size
default	$-\frac{i\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{10}}{2}\right)\sqrt{2}}{2}$	17
elliptic	$-\frac{i\sqrt{\left(x^{2}+1\right)\left(5x^{2}+2\right)}\,\sqrt{10x^{2}+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{10}}{2}\right)}{2\sqrt{5x^{2}+2}\,\sqrt{5x^{4}+7x^{2}+2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*EllipticF(I*x,1/2*10^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [A]
time = 0.48, size = 10, normalized size = 0.20

$$-\frac{1}{2}i\sqrt{2}\,\mathrm{ellipticF}\left(i\,x,\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(2)*ellipticF(I*x, 5/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^{2}+1}\,\sqrt{5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(5*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.229 $\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{1+2x^2}\,F(\tan^{-1}(x)|-1)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{1+2x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),I)*(2*x^2+1)^(1/2)*2^(1/2)/
((2*x^2+1)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {429}

$$\frac{\sqrt{2x^2+1}\,F(\text{ArcTan}(x)|-1)}{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\frac{2x^2+1}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(1 +
 2*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1+2x^2}\,F(\tan^{-1}(x)|-1)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 17, normalized size = 0.35

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|\,2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], 2])/Sqrt[2]
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.31

method	result	size
default	$-\frac{i\,\mathrm{EllipticF}\left(ix,\sqrt{2}\right)\sqrt{2}}{2}$	15
elliptic	$-\frac{i\sqrt{\left(x^{2}+1\right)\left(2x^{2}+1\right)}\,\mathrm{EllipticF}\left(ix,\sqrt{2}\right)}{\sqrt{4x^{4}+6x^{2}+2}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*EllipticF(I*x,2^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [A]
time = 0.34, size = 10, normalized size = 0.20

$$-\frac{1}{2}i\sqrt{2}\,\mathrm{ellipticF}(i\,x,2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*I*sqrt(2)*ellipticF(I*x, 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{x^{2}+1}\sqrt{2x^{2}+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(x**2 + 1)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.230 $\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*(3*x^2+2)^(1
/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {429}

$$\frac{\sqrt{3x^2+2}\,F\left(\text{ArcTan}(x)\,|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 3*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+3x^2}}\,dx = \frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 19, normalized size = 0.37

$$-\frac{iF\left(i\sinh^{-1}(x)|\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 3/2])/Sqrt[2]
```

Maple [A]
time = 0.07, size = 17, normalized size = 0.33

method	result	size
default	$-\frac{i \operatorname{EllipticF}\left(i x, \frac{\sqrt{6}}{2}\right) \sqrt{2}}{2}$	17
elliptic	$-\frac{i \sqrt{\left(3 x^{2}+2\right)\left(x^{2}+1\right)}\, \sqrt{6 x^{2}+4}\, \operatorname{EllipticF}\left(i x, \frac{\sqrt{6}}{2}\right)}{2 \sqrt{3 x^{2}+2}\, \sqrt{3 x^{4}+5 x^{2}+2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*EllipticF(I*x,1/2*6^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [A]
time = 0.24, size = 10, normalized size = 0.20

$$-\frac{1}{2} i \sqrt{2}\, \operatorname{ellipticF}\left(i x, \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(2)*ellipticF(I*x, 3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2}+1}\, \sqrt{3 x^{2}+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.231 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {22, 209}

$$\frac{\text{ArcTan}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx &= \sqrt{2}\int \frac{1}{2+2x^2}\,dx \\ &= \frac{\tan^{-1}(x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

Maple [A]
time = 0.06, size = 8, normalized size = 1.00

method	result	size
meijerg	$\frac{\arctan(x)\sqrt{2}}{2}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(7) = 14$.
time = 1.20, size = 34, normalized size = 4.25

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2x^2+2}\sqrt{x^2+1}\,x}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*sqrt(2*x^2 + 2)*sqrt(x^2 + 1)*x/(x^4 - 1))
```

Sympy [A]
time = 0.95, size = 8, normalized size = 1.00

$$\frac{\sqrt{2}\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*atan(x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.232 $\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*(x^2+2)^(1/2)*
2^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {429}

$$\frac{\sqrt{x^2+2}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 +
x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+x^2}}\,dx = \frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 19, normalized size = 0.40

$$-\frac{iF\left(i\sinh^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 1/2])/Sqrt[2]
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 15, normalized size = 0.32

method	result	size
default	$-i\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2}, \sqrt{2}\right)$	15
elliptic	$-\frac{i\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\,\sqrt{2}\,\sqrt{2x^2+4}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2}, \sqrt{2}\right)}{2\sqrt{x^2+2}\,\sqrt{x^4+3x^2+2}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.35, size = 10, normalized size = 0.21

$$-i\,\mathrm{ellipticF}\left(\frac{1}{2}i\,\sqrt{2}\,x, 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*ellipticF(1/2*I*sqrt(2)*x, 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.233 $\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {430}

$$F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]

[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 19, normalized size = 1.90

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]

[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]

Maple [A]
time = 0.09, size = 14, normalized size = 1.40

method	result	size
default	$\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)$	14
elliptic	$\frac{\sqrt{-\left(x^{2}-2\right)\left(x^{2}+1\right)}\,\sqrt{2}\,\sqrt{-2x^{2}+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{2\sqrt{-x^{2}+2}\,\sqrt{-x^{4}+x^{2}+2}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

Fricas [A]
time = 0.18, size = 8, normalized size = 0.80

$$\mathrm{ellipticF}\left(\frac{1}{2}\,\sqrt{2}\,x, -2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] ellipticF(1/2*sqrt(2)*x, -2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-x^{2}}\,\sqrt{x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - x**2)*sqrt(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - x^2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.234 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {254, 227}

$$\frac{F(\text{ArcSin}(x)|-1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx &= \int \frac{1}{\sqrt{2-2x^4}}\,dx\\ &= \frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 10.02, size = 10, normalized size = 1.00

$$\frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A]
time = 0.08, size = 10, normalized size = 1.00

method	result	size
default	$\frac{\mathrm{EllipticF}(x,i)\sqrt{2}}{2}$	10
elliptic	$\frac{\sqrt{-x^4+1}\ \mathrm{EllipticF}(x,i)}{\sqrt{-2x^4+2}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [A]
time = 0.36, size = 8, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\,\mathrm{ellipticF}(x,-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*ellipticF(x, -1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(8) = 16.
time = 11.00, size = 76, normalized size = 7.60

$$\frac{\sqrt{2}\,iG_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}-\frac{\sqrt{2}\,iG_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
 (0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4, 0, 1/4, 1/2, 3/
4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16*pi
**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.235 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{430\}$

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.21, size = 20, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A]
time = 0.08, size = 19, normalized size = 0.95

method	result	size
default	$\frac{\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-\left(3x^{2}-2\right)\left(x^{2}+1\right)}\,\sqrt{6}\,\sqrt{-6x^{2}+4}\,\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^{2}+2}\,\sqrt{-3x^{4}-x^{2}+2}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [A]
time = 0.27, size = 16, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\,\text{ellipticF}\left(\frac{1}{2}\sqrt{3}\sqrt{2}\,x,-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*ellipticF(1/2*sqrt(3)*sqrt(2)*x, -2/3)
```

Sympy [A]
time = 1.49, size = 36, normalized size = 1.80

$$\begin{cases} \frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle| -\frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.236 $\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)$$

```
[Out] 1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{430\}$

$$\frac{1}{2}F\left(\text{ArcSin}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{1+x^2}}\,dx = \frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)$$

Mathematica [A]
time = 0.05, size = 16, normalized size = 1.00

$$\frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Maple [A]
time = 0.10, size = 15, normalized size = 0.94

method	result	size
default	$\frac{\operatorname{EllipticF}\left(x\sqrt{2}, \frac{i\sqrt{2}}{2}\right)}{2}$	15
elliptic	$\frac{\sqrt{-\left(2x^{2}-1\right)\left(x^{2}+1\right)}\,\sqrt{2}\,\operatorname{EllipticF}\left(x\sqrt{2}, \frac{i\sqrt{2}}{2}\right)}{2\sqrt{-4x^{4}-2x^{2}+2}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [A]
time = 0.27, size = 9, normalized size = 0.56

$$\frac{1}{2}\,\operatorname{ellipticF}\left(\sqrt{2}\,x, -\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*ellipticF(sqrt(2)*x, -1/2)
```

Sympy [A]
time = 2.09, size = 41, normalized size = 2.56

$$\frac{\sqrt{2}\left(\begin{cases}\frac{\sqrt{2}\,F\left(\operatorname{asin}\left(\sqrt{2}\,x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for}\ x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -sqrt(
2)/2) & (x < sqrt(2)/2)))/2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-4\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.237 $\int \frac{1}{\sqrt{2-5x^2}\,\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}\,x\right)\middle|-\frac{2}{5}\right)}{\sqrt{5}}$$

```
[Out] 1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{430\}$

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{5}{2}}\,x\right)\middle|-\frac{2}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2}\,\sqrt{1+x^2}}\,dx = \frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}\,x\right)\middle|-\frac{2}{5}\right)}{\sqrt{5}}$$

Mathematica [A]
time = 0.21, size = 20, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}\,x\right)\middle|-\frac{2}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.95

method	result	size
default	$\frac{\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{i\sqrt{10}}{5}\right)\sqrt{5}}{5}$	19
elliptic	$\frac{\sqrt{-\left(5x^{2}-2\right)\left(x^{2}+1\right)}\,\sqrt{10}\,\sqrt{-10x^{2}+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{i\sqrt{10}}{5}\right)}{10\sqrt{-5x^{2}+2}\,\sqrt{-5x^{4}-3x^{2}+2}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [A]
time = 0.15, size = 16, normalized size = 0.80

$$\frac{1}{5}\sqrt{5}\,\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{5}\,\sqrt{2}\,x,-\frac{2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*ellipticF(1/2*sqrt(5)*sqrt(2)*x, -2/5)
```

Sympy [A]
time = 1.50, size = 36, normalized size = 1.80

$$\begin{cases}\frac{\sqrt{5}\,F\left(\operatorname{asin}\left(\frac{\sqrt{10}\,x}{2}\right)\middle|-\frac{2}{5}\right)}{5} & \text{for}\ x>-\frac{\sqrt{10}}{5}\wedge x<\frac{\sqrt{10}}{5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), -2/5)/5, (x > -sqrt(10)/5
) & (x < sqrt(10)/5)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-5\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.238 $\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F\left(\text{ArcSin}(x)\,\middle|-\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{2+5x^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+5x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 32, normalized size = 1.00

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A]
time = 0.09, size = 37, normalized size = 1.16

method	result	size
default	$-\frac{i\,\mathrm{EllipticF}\left(\frac{ix\sqrt{10}}{2},\frac{i\sqrt{10}}{5}\right)\sqrt{-x^2+1}\,\sqrt{5}}{5\sqrt{x^2-1}}$	37
elliptic	$-\frac{i\sqrt{(x^2-1)(5x^2+2)}\,\sqrt{10}\,\sqrt{10x^2+4}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{10}}{2},\frac{i\sqrt{10}}{5}\right)}{10\sqrt{x^2-1}\,\sqrt{5x^2+2}\,\sqrt{5x^4-3x^2-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*I*10^(1/2))*(-x^2+1)^(1/2)*5^(1/2)/(x
^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

[Out] 0

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\,\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(5*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.239 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-x^2}\,F(\sin^{-1}(x)|-2)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,I*2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F(\text{ArcSin}(x)|-2)}{\sqrt{2}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{2+4x^2}}\,dx &= \frac{\sqrt{1-x^2}\,\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+4x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}\,F(\sin^{-1}(x)|-2)}{\sqrt{2}\,\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 1.00

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)|-2\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A]
time = 0.10, size = 34, normalized size = 1.13

method	result	size
default	$-\frac{i\,\mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{2}}{2}\right)\sqrt{-x^2+1}}{2\sqrt{x^2-1}}$	34
elliptic	$-\frac{i\sqrt{(x^2-1)(2x^2+1)}\,\sqrt{2}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{2}}{2}\right)}{2\sqrt{x^2-1}\,\sqrt{4x^4-2x^2-2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*EllipticF(I*x*2^(1/2),1/2*I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{x^2-1}\,\sqrt{4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.240 $\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F\left(\text{ArcSin}(x)\,\middle|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{2+3x^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+3x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 32, normalized size = 1.00

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A]
time = 0.10, size = 37, normalized size = 1.16

method	result	size
default	$-\frac{i\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{-x^2+1}\,\sqrt{3}}{3\sqrt{x^2-1}}$	37
elliptic	$-\frac{i\sqrt{(3x^2+2)(x^2-1)}\,\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^2+2}\,\sqrt{x^2-1}\,\sqrt{3x^4-x^2-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*I*6^(1/2))*(-x^2+1)^(1/2)*3^(1/2)/(x^2
-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\,\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.241 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)$$

```
[Out] 1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {259, 228}

$$\frac{1}{2}F\left(\text{ArcSin}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]/2
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx &= \frac{\sqrt{-2+2x^4}\int \frac{1}{\sqrt{-2+2x^4}}\,dx}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\\ &= \frac{1}{2}F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 46, normalized size = 1.84

$$\frac{x\sqrt{1-x^4}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)}{\sqrt{-1+x^2}\sqrt{2+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] (x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[-1 + x^2]*Sqr
t[2 + 2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 30, normalized size = 1.20

method	result	size
default	$-\frac{i\,\mathrm{EllipticF}(ix,i)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	30
elliptic	$-\frac{i\sqrt{x^4-1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{\sqrt{x^2-1}\,\sqrt{2x^4-2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*EllipticF(I*x,I)*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

[Out] 0

Sympy [C] Result contains complex when optimal does not.
time = 10.97, size = 75, normalized size = 3.00

$$\frac{\sqrt{2}\, i G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2}, 1, 1 & \frac{3}{4}, \frac{3}{4}, \frac{5}{4} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} & 0 \end{matrix} \middle| {\frac{e^{2i\pi}}{x^{4}}} \right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}\, i G_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} & 1 \\ 0, \frac{1}{2}, 0 & -\frac{1}{4}, \frac{1}{4}, \frac{1}{4} \end{matrix} \middle| {\frac{1}{x^{4}}} \right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
 (0,)), exp_polar(2*I*pi)/x**4)/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4,
0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*pi*
*(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.242 $\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F\left(\text{ArcSin}(x)\,|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{2+x^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2+x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [**A**]
time = 0.19, size = 32, normalized size = 1.00

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [**A**]
time = 0.09, size = 34, normalized size = 1.06

method	result	size
default	$-\frac{i\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{2}\right)\sqrt{-x^2+1}}{\sqrt{x^2-1}}$	34
elliptic	$-\frac{i\sqrt{(x^2-1)(x^2+2)}\,\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{x^2-1}\,\sqrt{x^2+2}\,\sqrt{x^4+x^2-2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -I*EllipticF(1/2*I*x*2^(1/2),I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\,(x+1)}\,\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.243 $\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=12

$$-F\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

[Out] -(x^2)^(1/2)/x*EllipticF(1/2*(-2*x^2+4)^(1/2),2^(1/2))

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{431\}$

$$-F\left(\text{ArcCos}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]

[Out] -EllipticF[ArcCos[x/Sqrt[2]], 2]

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = -F\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(12) = 24.
time = 10.04, size = 47, normalized size = 3.92

$$\frac{\sqrt{1-x^2}\sqrt{1-\frac{x^2}{2}}\,F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{-2+3x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - x^2/2]*EllipticF[ArcSin[x], 1/2])/Sqrt[-2 + 3*x^2 -
 x^4]
```

Maple [A]
time = 0.10, size = 28, normalized size = 2.33

method	result	size
default	$\frac{\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{2}\right)\sqrt{-x^{2}+1}}{\sqrt{x^{2}-1}}$	28
elliptic	$\frac{\sqrt{-\left(x^{2}-1\right)\left(x^{2}-2\right)}\,\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{-x^{2}+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{2}\right)}{2\sqrt{-x^{2}+2}\,\sqrt{x^{2}-1}\,\sqrt{-x^{4}+3x^{2}-2}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(1/2*x*2^(1/2),2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\,\sqrt{2-x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+2)**(1/2)/(x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.244 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{-1+x^2}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

```
[Out] -1/2*arctanh(x)*(x^2-1)^(1/2)*2^(1/2)/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {23, 213}

$$-\frac{\sqrt{x^2-1}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] -((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{-1+x^2}\int \frac{1}{-1+x^2}\,dx}{\sqrt{2-2x^2}}\\
&= -\frac{\sqrt{-1+x^2}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.38

$$\frac{(-1+x^2)\,(\log(1-x)-\log(1+x))}{2\sqrt{2}\,\sqrt{-\left(-1+x^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] ((-1 + x^2)*(Log[1 - x] - Log[1 + x]))/(2*Sqrt[2]*Sqrt[-(-1 + x^2)^2])
```

Maple [A]
time = 0.11, size = 24, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{2}\,\operatorname{arctanh}(x)}{2\sqrt{x^2-1}}$	24
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}(x^2-1)}\,\operatorname{arctanh}(x)}{2\sqrt{\operatorname{signum}(x^2-1)}}$	26
risch	$\frac{\sqrt{\frac{-2x^2+2}{x^2-1}}\,\sqrt{x^2-1}\,\left(\frac{\sqrt{-2}\,\ln(x+1)}{4}-\frac{\sqrt{-2}\,\ln(x-1)}{4}\right)}{\sqrt{-2x^2+2}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)*arctanh(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [A]
time = 0.99, size = 34, normalized size = 1.17

$$\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x^2-1}\,\sqrt{-2\,x^2+2}\,x}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)*x/(x^4 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{1-x^2}\sqrt{x^2-1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(x**2 - 1)), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{x^2-1}\,\sqrt{2-2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.245 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F\left(\text{ArcSin}(x)\,\middle|\,\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 40, normalized size = 1.25

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right)}{\sqrt{3}\,\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/(Sqrt[3]*Sqrt[-1 + x^2]
)
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	29
elliptic	$\frac{\sqrt{-\left(3x^2-2\right)\left(x^2-1\right)}\,\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{x^2-1}\,\sqrt{-3x^4+5x^2-2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x,1/2*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [C] Result contains complex when optimal does not.
time = 1.55, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{\sqrt{3}\, iF\left(\operatorname{asin}\left(\frac{\sqrt{6}\, x}{2}\right)\middle|\,\frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(3)*I*elliptic_f(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3
) & (x < sqrt(6)/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.246 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-x^2}\,F(\sin^{-1}(x)|\,2)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F(\text{ArcSin}(x)|2)}{\sqrt{2}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}\,F(\sin^{-1}(x)|\,2)}{\sqrt{2}\,\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 1.20

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{2}\right)}{2\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2]*x], 1/2])/(2*Sqrt[-1 + x^2])
```

Maple [A]
time = 0.12, size = 27, normalized size = 0.90

method	result	size
default	$\frac{\text{EllipticF}\left(x,\sqrt{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	27
elliptic	$\frac{\sqrt{-\left(2x^2-1\right)\left(x^2-1\right)}\,\sqrt{-x^2+1}\,\text{EllipticF}\left(x,\sqrt{2}\right)}{\sqrt{x^2-1}\,\sqrt{-4x^4+6x^2-2}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [A]
time = 2.05, size = 42, normalized size = 1.40

$$\frac{\sqrt{2}\left(\begin{cases} -\frac{\sqrt{2}\,iF\left(\operatorname{asin}\left(\sqrt{2}\,x\right)\middle|\frac{1}{2}\right)}{2} & \text{for}\ x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2} \end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sqr
t(2)/2) & (x < sqrt(2)/2)))/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-4\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.247 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F\left(\text{ArcSin}(x)\,\middle|\,\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 40, normalized size = 1.25

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}\,x\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], 2/5])/(Sqrt[5]*Sqrt[-1 + x^2]
)
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\operatorname{EllipticF}\left(x,\frac{\sqrt{10}}{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	29
elliptic	$\frac{\sqrt{-\left(5x^2-2\right)\left(x^2-1\right)}\,\sqrt{-x^2+1}\,\sqrt{-10x^2+4}\,\operatorname{EllipticF}\left(x,\frac{\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{x^2-1}\,\sqrt{-5x^4+7x^2-2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [C] Result contains complex when optimal does not.
time = 1.56, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{\sqrt{5}\, iF\left(\operatorname{asin}\left(\frac{\sqrt{10}\, x}{2}\right)\middle|\frac{2}{5}\right)}{5} & \text{for}\: x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(5)*I*elliptic_f(asin(sqrt(10)*x/2), 2/5)/5, (x > -sqrt(10)
/5) & (x < sqrt(10)/5)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-5\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.248 $\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{2+5x^2}\,F\left(\tan^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-1-x^2}\,\sqrt{\frac{2+5x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*6^(1/2)
)*(5*x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((5*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {429}

$$\frac{\sqrt{5x^2+2}\,F\left(\text{ArcTan}(x)\,|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-x^2-1}\,\sqrt{\frac{5x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 5*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2+5x^2}}\,dx = \frac{\sqrt{2+5x^2}\,F\left(\tan^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-1-x^2}\,\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 39, normalized size = 0.74

$$-\frac{i\sqrt{1+x^2}\,F\left(i\sinh^{-1}(x)|\frac{5}{2}\right)}{\sqrt{2}\,\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 5/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.68

method	result	size
default	$\frac{i\,\mathrm{EllipticF}\left(\frac{ix\sqrt{10}}{2},\frac{\sqrt{10}}{5}\right)\sqrt{5}\,\sqrt{-x^{2}-1}}{5\sqrt{x^{2}+1}}$	36
elliptic	$-\frac{i\sqrt{-\left(x^{2}+1\right)\left(5x^{2}+2\right)}\,\sqrt{10}\,\sqrt{10x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{10}}{2},\frac{\sqrt{10}}{5}\right)}{10\sqrt{-x^{2}-1}\,\sqrt{5x^{2}+2}\,\sqrt{-5x^{4}-7x^{2}-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*10^(1/2))/(x^2+1)^(1/2)*5^(1/2)*(-x^2-
1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [A]
time = 0.31, size = 10, normalized size = 0.19

$$\frac{1}{2}i\sqrt{-2}\,\mathrm{ellipticF}\left(i\,x,\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(-2)*ellipticF(I*x, 5/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-x^{2}-1}\,\sqrt{5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(5*x**2 + 2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((- x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.249 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{1+2x^2}\,F(\tan^{-1}(x)|-1)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),I)*(2*x^2+1)^
(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((2*x^2+1)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{429\}$

$$\frac{\sqrt{2x^2+1}\,F(\text{ArcTan}(x)|-1)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(1
+ 2*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1+2x^2}\,F(\tan^{-1}(x)|-1)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 37, normalized size = 0.73

$$-\frac{i\sqrt{1+x^2}\,F\left(i\sinh^{-1}(x)\middle|2\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A]
time = 0.10, size = 33, normalized size = 0.65

method	result	size
default	$\frac{i \operatorname{EllipticF}\left(i x \sqrt{2}, \frac{\sqrt{2}}{2}\right) \sqrt{-x^{2}-1}}{2 \sqrt{x^{2}+1}}$	33
elliptic	$-\frac{i \sqrt{-\left(x^{2}+1\right)\left(2 x^{2}+1\right)}\, \sqrt{2}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(i x \sqrt{2}, \frac{\sqrt{2}}{2}\right)}{2 \sqrt{-x^{2}-1}\, \sqrt{-4 x^{4}-6 x^{2}-2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*EllipticF(I*x*2^(1/2),1/2*2^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [A]
time = 0.31, size = 10, normalized size = 0.20

$$\frac{1}{2} i \sqrt{-2}\, \operatorname{ellipticF}(i x, 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*sqrt(-2)*ellipticF(I*x, 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2} \int \frac{1}{\sqrt{-x^{2}-1}\, \sqrt{2 x^{2}+1}}\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((- x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.250 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2)
)*(3*x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {429}

$$\frac{\sqrt{3x^2+2}\,F\left(\text{ArcTan}(x)\,|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 3*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 39, normalized size = 0.74

$$-\frac{i\sqrt{1+x^2}\,F\left(i\sinh^{-1}(x)|\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 3/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.68

method	result	size
default	$\frac{i \,\mathrm{EllipticF}\left(\frac{i x \sqrt{6}}{2}, \frac{\sqrt{6}}{3}\right) \sqrt{3}\, \sqrt{-x^{2}-1}}{3 \sqrt{x^{2}+1}}$	36
elliptic	$-\frac{i \sqrt{-\left(3 x^{2}+2\right)\left(x^{2}+1\right)}\, \sqrt{6}\, \sqrt{6 x^{2}+4}\, \sqrt{x^{2}+1}\, \mathrm{EllipticF}\left(\frac{i x \sqrt{6}}{2}, \frac{\sqrt{6}}{3}\right)}{6 \sqrt{-x^{2}-1}\, \sqrt{3 x^{2}+2}\, \sqrt{-3 x^{4}-5 x^{2}-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))/(x^2+1)^(1/2)*3^(1/2)*(-x^2-1)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [A]
time = 0.20, size = 10, normalized size = 0.19

$$\frac{1}{2} i \sqrt{-2}\, \mathrm{ellipticF}\left(i\, x, \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(-2)*ellipticF(I*x, 3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{2}-1}\, \sqrt{3 x^{2}+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(3*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((- x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.251 $\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{\sqrt{2}\,\sqrt{-1-x^2}}$$

```
[Out] 1/2*arctan(x)*(x^2+1)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {23, 209}

$$\frac{\sqrt{x^2+1}\,\text{ArcTan}(x)}{\sqrt{2}\,\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2+2x^2}}\,dx &= \frac{\sqrt{2+2x^2}\,\int \frac{1}{2+2x^2}\,dx}{\sqrt{-1-x^2}}\\ &= \frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{\sqrt{2}\,\sqrt{-1-x^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$\frac{(1+x^2)\tan^{-1}(x)}{\sqrt{2}\sqrt{-(1+x^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
[Out] ((1 + x^2)*ArcTan[x])/(Sqrt[2]*Sqrt[-(1 + x^2)^2])
```

Maple [A]
time = 0.08, size = 24, normalized size = 0.86

method	result	size
meijerg	$-\frac{i\sqrt{2}\,\arctan(x)}{2}$	9
default	$-\frac{\sqrt{-x^2-1}\,\sqrt{2}\,\arctan(x)}{2\sqrt{x^2+1}}$	24
risch	$\frac{\sqrt{\frac{(-x^2-1)(2x^2+2)}{(x^2+1)^2}}\,(x^2+1)\left(-\frac{i\sqrt{-2}\,\ln(x+i)}{4}+\frac{i\sqrt{-2}\,\ln(x-i)}{4}\right)}{\sqrt{-x^2-1}\,\sqrt{2x^2+2}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-x^2-1)^(1/2)*2^(1/2)/(x^2+1)^(1/2)*arctan(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(23) = 46.
time = 1.07, size = 104, normalized size = 3.71

$$\frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}\,x+\sqrt{2}\,(x^4-1)\right)}{x^4+2x^2+1}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}\,x-\sqrt{2}\,(x^4-1)\right)}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x + sqrt(2)*(x^4 - 1))/
(x^4 + 2*x^2 + 1)) - 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x
- sqrt(2)*(x^4 - 1))/(x^4 + 2*x^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-x^2-1}\,\sqrt{x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 1)), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{1}{\sqrt{-x^2-1}\,\sqrt{2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.252 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
(x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {429}

$$\frac{\sqrt{x^2+2}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(2 +
 x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 53, normalized size = 1.08

$$-\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-\left(\left(1+x^2\right)\left(2+x^2\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x], 1/2])/(Sqrt[2]*Sq
rt[-((1 + x^2)*(2 + x^2))])
```

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 33, normalized size = 0.67

method	result	size
default	$\frac{i\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-x^{2}-1}}{2\sqrt{x^{2}+1}}$	33
elliptic	$-\frac{i\sqrt{-\left(x^{2}+1\right)\left(x^{2}+2\right)}\,\sqrt{x^{2}+1}\,\sqrt{2x^{2}+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^{2}-1}\,\sqrt{x^{2}+2}\,\sqrt{-x^{4}-3x^{2}-2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.18, size = 16, normalized size = 0.33

$$\frac{1}{2}i\,\sqrt{2}\,\sqrt{-2}\,\mathrm{ellipticF}\left(\frac{1}{2}i\,\sqrt{2}\,x,2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*sqrt(2)*sqrt(-2)*ellipticF(1/2*I*sqrt(2)*x, 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{2}-1}\,\sqrt{x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)),x)
```

```
[Out] int(1/((- x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.253 $\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-1-x^2}}$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))*(x^2+1)^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {432, 430}

$$\frac{\sqrt{x^2+1}\,F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[x/Sqrt[2]], -2])/Sqrt[-1 - x^2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2-x^2}}\,dx &= \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-x^2}\,\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}\\ &= \frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-1-x^2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 39, normalized size = 1.26

$$-\frac{i\sqrt{1+x^2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A]
time = 0.09, size = 34, normalized size = 1.10

method	result	size
default	$\frac{i\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-2x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-x^2-1}\,\sqrt{-x^2+2}\,\sqrt{x^4-x^2-2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,1/2*I*2^(1/2))/(x^2+1)^(1/2)*2^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [A]
time = 0.26, size = 16, normalized size = 0.52

$$-\frac{1}{2}\,\sqrt{2}\,\sqrt{-2}\,\mathrm{ellipticF}\left(\frac{1}{2}\,\sqrt{2}\,x,-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(-2)*ellipticF(1/2*sqrt(2)*x, -2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-x^2}\,\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(-x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(2 - x**2)*sqrt(-x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.254 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-\frac{1}{x^4}}\,x^2F(\csc^{-1}(x)|-1)}{\sqrt{2-2x^2}\sqrt{-1-x^2}}$$

```
[Out] -x^2*EllipticF(1/x,I)*(1-1/x^4)^(1/2)/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {259, 228}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{2\sqrt{-x^2-1}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]],
 1/2])/(2*Sqrt[-1 - x^2]*Sqrt[1 - x^2])
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{-2+2x^4}\int \frac{1}{\sqrt{-2+2x^4}}\,dx}{\sqrt{2-2x^2}\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{2\sqrt{-1-x^2}\sqrt{1-x^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 48, normalized size = 1.14

$$\frac{x\sqrt{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)}{\sqrt{2-2x^2}\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[2 - 2*x^2]*Sq
rt[-1 - x^2])
```

Maple [A]
time = 0.09, size = 30, normalized size = 0.71

method	result	size
default	$\frac{i\,\mathrm{EllipticF}(ix,i)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	30
elliptic	$-\frac{i\sqrt{x^4-1}\,\sqrt{x^2+1}\,\mathrm{EllipticF}(ix,i)}{\sqrt{-x^2-1}\,\sqrt{2x^4-2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*EllipticF(I*x,I)*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [A]
time = 0.18, size = 8, normalized size = 0.19

$$-\frac{1}{2}\sqrt{-2}\,\text{ellipticF}(x,-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*ellipticF(x, -1)
```

Sympy [A]
time = 11.16, size = 73, normalized size = 1.74

$$\frac{\sqrt{2}\,G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}-\frac{\sqrt{2}\,G_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
[Out] sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4), (
0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*meijerg(((-1/4, 0, 1/4, 1/2, 3/4),
(1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3
/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.255 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}\,\sqrt{-1-x^2}}$$

```
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*(x^2+1)^(1/2)*3^(1/2)/(-x^2-1)^(
1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {432, 430}

$$\frac{\sqrt{x^2+1}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}\,\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^2
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}$$

Mathematica [A]
time = 0.20, size = 40, normalized size = 1.00

$$\frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^2
])
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.85

method	result	size
default	$\frac{i\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)\sqrt{-x^2-1}\,\sqrt{2}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(3x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{-x^2-1}\,\sqrt{3x^4+x^2-2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*I*6^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [**A**]
time = 0.22, size = 22, normalized size = 0.55

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{-2}\,\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{3}\sqrt{2}\,x,-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*sqrt(-2)*ellipticF(1/2*sqrt(3)*sqrt(2)*x, -2/3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2-3x^2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x**2)*sqrt(-x**2 - 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-x^2-1}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

```
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.256 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

```
[Out] 1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))*(x^2+1)^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {432, 430}

$$\frac{\sqrt{x^2+1}\,F\left(\text{ArcSin}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx &= \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}\\ &= \frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 1.00

$$\frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

Maple [A]
time = 0.10, size = 34, normalized size = 0.94

method	result	size
default	$\frac{i\,\mathrm{EllipticF}\left(ix,i\sqrt{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(2x^2-1)(x^2+1)}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(ix,i\sqrt{2}\right)}{\sqrt{-x^2-1}\,\sqrt{4x^4+2x^2-2}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*EllipticF(I*x,I*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [A]
time = 0.24, size = 15, normalized size = 0.42

$$-\frac{1}{4}\,\sqrt{2}\,\sqrt{-2}\,\mathrm{ellipticF}\left(\sqrt{2}\,x,-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(-2)*ellipticF(sqrt(2)*x, -1/2)
```

Sympy [A]
time = 1.97, size = 44, normalized size = 1.22

$$\frac{\sqrt{2}\left(\begin{cases}-\frac{\sqrt{2}\,iF\left(\operatorname{asin}\left(\sqrt{2}\,x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for}\ x>-\frac{\sqrt{2}}{2}\wedge x<\frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
[Out] sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -sq
rt(2)/2) & (x < sqrt(2)/2)))/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-4\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.257 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}\,x\right)|-\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-1-x^2}}$$

```
[Out] 1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*(x^2+1)^(1/2)*5^(1/2)/(-x^2-1)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {432, 430}

$$\frac{\sqrt{x^2+1}\,F\left(\text{ArcSin}\left(\sqrt{\frac{5}{2}}\,x\right)|-\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^2
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}} = \frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}\,x\right)|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}$$

Mathematica [A]
time = 0.21, size = 40, normalized size = 1.00

$$\frac{\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}\,x\right)|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^2
])
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.85

method	result	size
default	$\frac{i\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{10}}{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(5x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-10x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{-x^2-1}\,\sqrt{5x^4+3x^2-2}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,1/2*I*10^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [A]
time = 0.26, size = 22, normalized size = 0.55

$$-\frac{1}{10}\sqrt{5}\sqrt{2}\sqrt{-2}\,\text{ellipticF}\left(\frac{1}{2}\sqrt{5}\sqrt{2}\,x,-\frac{2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/10*sqrt(5)*sqrt(2)*sqrt(-2)*ellipticF(1/2*sqrt(5)*sqrt(2)*x, -2/5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2-5x^2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 5*x**2)*sqrt(-x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-x^2-1}\,\sqrt{2-5\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.258 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}$$

$$= \frac{\left(\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Mathematica [A]
time = 0.88, size = 87, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c)
/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [A]
time = 0.07, size = 104, normalized size = 1.20

method	result

default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}\,a\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{(-bd\,x^4-ad\,x^2+c\,x^2b+ac)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(-d\,x^2+c)}\left(\frac{a\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+c\,x^2b+ac}}-\frac{a\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)/(
d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c - d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.259 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}$$

$$= \frac{\left(\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Mathematica [A]
time = 0.89, size = 90, normalized size = 1.00

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.
time = 0.08, size = 168, normalized size = 1.87

method	result

default	$\frac{\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\left(a\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bc\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\right.\right.}{(-bd\,x^4-ad\,x^2+c\,x^2b+ac)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-\left(b\,x^2+a\right)\left(-d\,x^2+c\right)}\left(-\frac{a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2-c\,x^2b-ac}}-\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\right.}{\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*
(a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1/2
),(-a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))/(-b*d*x
^4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(c - d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-b\,x^2-a}}{\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- a - b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
```

```
[Out] int((- a - b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.260 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}$$

$$= \frac{\left(\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Mathematica [A]
time = 0.66, size = 88, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c)
/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.
time = 0.08, size = 167, normalized size = 1.90

method	result

default	$\frac{\left(-a\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bc\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bc\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)\sqrt{b\,x^2+a}}{\left(-bd\,x^4-ad\,x^2+c\,x^2b+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-\left(b\,x^2+a\right)\left(-d\,x^2+c\right)}\left(\frac{a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2-c\,x^2b-ac}}+\frac{bc\sqrt{1+\frac{b\,x^2}{a}}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/
2),(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))*(b*x^2
+a)^(1/2)*(d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)/(-b*d*x^
4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.261 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}$$
$$= \frac{\left(\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$
$$= \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Mathematica [A]
time = 0.69, size = 91, normalized size = 1.00

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [A]
time = 0.08, size = 108, normalized size = 1.19

method	result

default	$\frac{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(bd\,x^{4}+ad\,x^{2}-c\,x^{2}b-ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+c\,x^{2}b+ac}}+\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{}}{}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(d/c)^(1/2)*(-b*x^2-a)^(1/2)*(d*x^2-c)^(1/2
)*a*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*c/
a/d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a-bx^{2}}}{\sqrt{-c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-b\,x^2-a}}{\sqrt{d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
```

```
[Out] int((- a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.262 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\,\sqrt{a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}$$
$$= \frac{\left(\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$
$$= \frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Mathematica [A]
time = 0.89, size = 88, normalized size = 1.00

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/(
a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [A]
time = 0.08, size = 105, normalized size = 1.19

method	result

default	$\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(bd\,x^{4}-ad\,x^{2}-c\,x^{2}b+ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-c\,x^{2}b+ac}}-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{}}{}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/
2)*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(
d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a-bx^{2}}}{\sqrt{c-dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(c - d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-b\,x^2}}{\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
```

```
[Out] int((a - b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.263 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{c}\,\sqrt{-a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(b*x^2-a)^(1/2)*(1-d*x
^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{bx^2-a}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}$$
$$= \frac{\left(\sqrt{-a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$
$$= \frac{\sqrt{c}\,\sqrt{-a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\mid\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c-dx^2}}$$

Mathematica [**A**]
time = 0.88, size = 89, normalized size = 1.00

$$\frac{\sqrt{-a+bx^2}\,\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)\mid\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{\frac{a-bx^2}{a}}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(74) = 148$.
time = 0.09, size = 161, normalized size = 1.81

method	result

default	$\frac{\sqrt{b\,x^2-a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\left(a\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\,\mathrm{E}\right.}{(bd\,x^4-ad\,x^2-c\,x^2b+ac)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-(-b\,x^2+a)\,(-d\,x^2+c)}\left(-\frac{a\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2+c\,x^2b-ac}}+\frac{bc\sqrt{1-\frac{b\,x^2}{a}}}{}\right.}{\sqrt{b\,x^2-a}\,\sqrt{-d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*
(a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2),(
a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4-a*d*
x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c - d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2-a}}{\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 - a)^(1/2)/(c - d*x^2)^(1/2),x)
```

```
[Out] int((b*x^2 - a)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.264 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{c}\,\sqrt{a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}$$

$$= \frac{\left(\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Mathematica [A]
time = 0.73, size = 89, normalized size = 1.00

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/(
a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(74) = 148$.
time = 0.09, size = 162, normalized size = 1.82

method	result

default	$\frac{\left(-a \,\mathrm{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) d+bc \,\mathrm{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)-bc \,\mathrm{EllipticE}\left(x \sqrt{\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)\right) \sqrt{-b\,x^{2}+a}\, \sqrt{d\,x^{2}-}}{\left(bd\,x^{4}-ad\,x^{2}-c\,x^{2}b+ac\right) \sqrt{\frac{b}{a}}\, d}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a \sqrt{1-\frac{b\,x^{2}}{a}}\, \sqrt{1-\frac{d\,x^{2}}{c}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\, \sqrt{-bd\,x^{4}+ad\,x^{2}+c\,x^{2}b-ac}}-\frac{bc \sqrt{1-\frac{b\,x^{2}}{a}}}{}\right.}{\sqrt{-b\,x^{2}+a}\, \sqrt{d\,x^{2}-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2),
(a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2+a)^(
1/2)*(d*x^2-c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)/(b*d*x^4-a*d
*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-b\,x^2}}{\sqrt{d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
```

```
[Out] int((a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.265 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{c}\,\sqrt{-a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(b*x^2-a)^(1/2)*(1-d*x
^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {438, 437, 435}

$$\frac{\sqrt{c}\,\sqrt{bx^2-a}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}$$

$$= \frac{\left(\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Mathematica [A]
time = 0.67, size = 90, normalized size = 1.00

$$\frac{\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [A]
time = 0.08, size = 107, normalized size = 1.19

method	result

default	$\frac{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\operatorname{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}+ad\,x^{2}+c\,x^{2}b-ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-c\,x^{2}b+ac}}+\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}}{}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(d/c)^(1/2)*(b*x^2-a)^(1/2)*(d*x^2-c)^(1/2
)*a*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*c/
a/d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a+bx^{2}}}{\sqrt{-c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2-a}}{\sqrt{d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 - a)^(1/2)/(d*x^2 - c)^(1/2),x)
```

```
[Out] int((b*x^2 - a)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.266 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*
x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = a\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - c\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx$$

$$= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\right.\right.}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [A]
time = 0.02, size = 86, normalized size = 0.44

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]

[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])

Maple [A]
time = 0.00, size = 158, normalized size = 0.81

method	result
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\left(a\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\,\mathrm{El}\right.}{\left(bd\,x^4+ad\,x^2+c\,x^2b+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}\left(\frac{a\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}-\frac{bc\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1}}{}\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.267 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] x*(-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-
b*x^2-a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/(
1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx &= -\left(a\int \frac{1}{\sqrt{-a-bx^2}\,\sqrt{c+dx^2}}\,dx\right)-b\int \frac{x^2}{\sqrt{-a-bx^2}\,\sqrt{c+dx^2}}\,dx\\ &= \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}-c\int \frac{\sqrt{-a-bx^2}}{(c+dx^2)^{3/2}}\,dx\\ &= \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\mathrm{t}\right.}{\sqrt{d}\,\sqrt{\frac{c(a+}{a(c+}}}\end{aligned}$$

Mathematica [A]
time = 0.87, size = 89, normalized size = 0.44

$$\frac{\sqrt{-a-bx^2}\,\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{\frac{a+bx^2}{a}}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]

[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])

Maple [A]
time = 0.08, size = 104, normalized size = 0.51

method	result
default	$\frac{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}+c}\,a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac\right)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-c\,x^{2}b-ac}}+\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}}{}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}+}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a - bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{-b\,x^2 - a}}{\sqrt{d\,x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- a - b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((- a - b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.268 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*(b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b
*x^2+a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1/(
1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx &= a\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{-c-dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{-c-dx^2}}\,dx\\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} + \frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + c\int \frac{\sqrt{a+bx^2}}{(-c-dx^2)^{3/2}}\,dx\\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\right.}{\sqrt{d}\,\sqrt{-c-dx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.86, size = 89, normalized size = 0.44

$$\frac{\sqrt{a+bx^2}\,\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{\frac{a+bx^2}{a}}\,\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A]
time = 0.08, size = 108, normalized size = 0.53

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}\;a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\;\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}-ad\,x^{2}-c\,x^{2}b-ac\right)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-c\,x^{2}b-ac}}-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{\ldots}}{\ldots}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}-\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-
d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{-d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.269 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=212

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}-\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*(-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(
-b*x^2-a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/
2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{-a-bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = -\left(a\int \frac{1}{\sqrt{-a-bx^2}\,\sqrt{-c-dx^2}}\,dx\right) - b\int \frac{x^2}{\sqrt{-a-bx^2}\,\sqrt{-c-dx^2}}\,dx$$

$$= \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}} + \frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + c\int \frac{\sqrt{-a-bx^2}}{(-c-dx^2)^{3/2}}\,dx$$

$$= \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\,\sqrt{-a-bx^2}\,F\left(\tan^{-1}\right.}{\sqrt{d}\,\sqrt{-c-d}}$$

Mathematica [A]
time = 0.74, size = 92, normalized size = 0.43

$$\frac{\sqrt{-a-bx^2}\,\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{\frac{a+bx^2}{a}}\,\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A]
time = 0.08, size = 165, normalized size = 0.78

method	result
default	$\frac{\left(-a\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bc\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bc\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{-b\,x^{2}-a}\,\sqrt{-\cdots}}{\left(bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac}}+\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1\cdots}}{\cdots}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2-a
)^(1/2)*(-d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{-b\,x^2-a}}{\sqrt{-d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
```

```
[Out] int((- a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.270 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,d\sqrt{a-bx^2}\,\sqrt{c+dx^2}}$$

```
[Out] -EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^
(1/2)*(d*x^2+c)^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {434, 438, 437, 435, 432, 430}

$$\frac{\sqrt{a}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{\frac{dx^2}{c}+1}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,d\sqrt{a-bx^2}\,\sqrt{c+dx^2}}-\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\,\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sq
rt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = -\frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx}{d} + \frac{(bc+ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx}{d}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,d}{d\sqrt{c+dx^2}}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int -}{d\sqrt{a-bx^2}\sqrt{c+d}}$$

$$= -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1}}{\sqrt{b}\,d\sqrt{a-}}$$

Mathematica [**A**]
time = 0.87, size = 89, normalized size = 0.47

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((b
*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [**A**]
time = 0.08, size = 161, normalized size = 0.85

method	result
default	$\frac{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\left(a\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bc\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)-\right.}{(-bd\,x^4+ad\,x^2-c\,x^2b+ac)\sqrt{\frac{b}{a}}\,d}$

elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(d\,x^2+c)}\left(\frac{a\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-c\,x^2b+ac}}+\frac{bc\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1}}{}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(
a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2),(
-a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))/(-b*d*x^4+a
*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-b\,x^2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.271 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}-\frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{\sqrt{b}\,d\sqrt{-a+bx^2}\sqrt{c+dx^2}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(
1/2)*(d*x^2+c)^(1/2)/d/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)-(a*d+b*c)*Elliptic
F(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/d/b^(1/2)/(b*x^2-a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {434, 438, 437, 435, 432, 430}

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt
[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
- (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqr
t[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx}{d} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{c+dx^2}}\,dx}{d}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}}{d\sqrt{c+dx^2}}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int}{d\sqrt{-a+bx^2}\sqrt{c}}$$

$$= \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}}{\sqrt{b}\,d\sqrt{-a}}$$

Mathematica [A]
time = 0.86, size = 90, normalized size = 0.47

$$\frac{\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.08, size = 107, normalized size = 0.56

method	result
default	$\frac{\sqrt{b\,x^2-a}\,\sqrt{d\,x^2+c}\,a\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{(-bd\,x^4+ad\,x^2-c\,x^2b+ac)\sqrt{-\frac{d}{c}}}$

elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+c\,x^{2}b-ac}}+\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\cdots}{\cdots}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(
-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2-a}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 - a)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((b*x^2 - a)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.272 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}+\frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{\sqrt{b}\,d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(
1/2)*(-d*x^2-c)^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {434, 438, 437, 435, 432, 430}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,d\sqrt{a-bx^2}\sqrt{-c-dx^2}}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqr
t[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
+ (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt
[-c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}dx = \frac{b\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}dx}{d} + \frac{(bc+ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{-c-dx^2}}dx}{d}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}}{d\sqrt{-c-dx^2}}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int}{d\sqrt{a-bx^2}\sqrt{-c}}$$

$$= \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}}{\sqrt{b}\,d\sqrt{a}}$$

Mathematica [A]
time = 0.66, size = 92, normalized size = 0.47

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((b
*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.56

method	result
default	$\frac{\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2-c}\,a\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{(bd\,x^4-ad\,x^2+c\,x^2b-ac)\sqrt{-\frac{d}{c}}}$

elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+c\,x^{2}b-ac}}-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}}{\ldots}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/
(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-bx^{2}}}{\sqrt{-c-dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-b\,x^2}}{\sqrt{-d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.273 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}-\frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\right.}{\sqrt{b}\,d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$

```
[Out] -EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^
(1/2)*(-d*x^2-c)^(1/2)/d/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)-(a*d+b*c)*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/d/b^(1/2)/(b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {434, 438, 437, 435, 432, 430}

$$-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,d\sqrt{bx^2-a}\sqrt{-c-dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c
])) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]
*Sqrt[-c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{b\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx}{d} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{-c-dx^2}}\,dx}{d}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{1+}}}{d\sqrt{-c-dx^2}}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right.}{d\sqrt{-a+bx^2}\sqrt{}}$$

$$= -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}\,(bc+ad)\sqrt{1-\frac{bx^2}{a}}}{\sqrt{b}\,d\sqrt{}}$$

Mathematica [A]
time = 0.61, size = 93, normalized size = 0.47

$$\frac{\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A]
time = 0.08, size = 166, normalized size = 0.84

method	result
default	$\frac{\left(-a\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bc\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bc\,\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)\sqrt{b\,x^2-a}\,\sqrt{-}}{(-bd\,x^4+ad\,x^2-c\,x^2b+ac)\sqrt{\frac{b}{a}}\,d}$

elliptic	$\dfrac{\sqrt{(-b\,x^2+a)\,(d\,x^2+c)}\left(-\dfrac{a\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-c\,x^2b+ac}}-\dfrac{bc\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1}}{}\right.}{\sqrt{b\,x^2-a}\,\sqrt{-d\,x^2-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2)
,(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))*(b*x^2-a)
^(1/2)*(-d*x^2-c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(-b*d*x^4+
a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2-a}}{\sqrt{-d\,x^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 - a)^(1/2)/(- c - d*x^2)^(1/2),x)
[Out] int((b*x^2 - a)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.274 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,\sqrt{a-bx^2}\,\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*
x^2+c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,\sqrt{a-bx^2}\,\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\,\sqrt{1+\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,\sqrt{a-bx^2}\,\sqrt{1+\frac{dx^2}{c}}}$$

Mathematica [A]
time = 0.88, size = 87, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{a-bx^2}\,\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d)
/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A]
time = 0.08, size = 104, normalized size = 1.20

method	result

default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,c\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{(-bd\,x^4+ad\,x^2-c\,x^2b+ac)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(d\,x^2+c)}\left(\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-c\,x^2b+ac}}-\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+}}{}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(
b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(c + d*x**2)/sqrt(a - b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
[Out] int((c + d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.275 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d
*x^2-c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

Mathematica [A]
time = 0.88, size = 90, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.
time = 0.08, size = 168, normalized size = 1.87

method	result

default	$\frac{\sqrt{-d\,x^{2}-c}\,\sqrt{-b\,x^{2}+a}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\left(ad\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+c\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{b}{a}}\right.\right.}{\left(-bd\,x^{4}+ad\,x^{2}-c\,x^{2}b+ac\right)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+c\,x^{2}b-ac}}-\frac{da\sqrt{1+\frac{d\,x}{c}}}{}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*
(a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2),
(-b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))/(-b*d*x
^4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c-dx^{2}}}{\sqrt{a-bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-d\,x^2-c}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- c - d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

```
[Out] int((- c - d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.276 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*
x^2+c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\,\sqrt{1+\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\,\sqrt{-a+bx^2}\,\sqrt{1+\frac{dx^2}{c}}}$$

Mathematica [**A**]
time = 0.75, size = 88, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-a+bx^2}\,\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d)
/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.
time = 0.08, size = 167, normalized size = 1.90

method	result

default	$\frac{\left(-ad\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-c\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+ad\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)\sqrt{d\,x^2+c}}{(-bd\,x^4+ad\,x^2-c\,x^2b+ac)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-(-b\,x^2+a)\,(d\,x^2+c)}\left(\frac{c\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2+c\,x^2b-ac}}+\frac{da\sqrt{1+\frac{d\,x^2}{c}}}{}\right.}{\sqrt{b\,x^2-a}\,\sqrt{d\,x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2)
,(-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))*(d*x^2
+c)^(1/2)*(b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.277 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d
*x^2-c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

Mathematica [A]
time = 0.63, size = 91, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A]
time = 0.08, size = 108, normalized size = 1.19

method	result

default	$\frac{\sqrt{-d\,x^2-c}\,\sqrt{b\,x^2-a}\,c\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{(bd\,x^4-ad\,x^2+c\,x^2b-ac)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(d\,x^2+c)}\left(-\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-c\,x^2b+ac}}+\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1}}{}\right.}{\sqrt{b\,x^2-a}\,\sqrt{-d\,x^2-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)*(-d*x^2-c)^(1/2)*(b*x^2-a)^(1/2
)*c*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/
b/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-d\,x^2-c}}{\sqrt{b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
[Out] int((- c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.278 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d*
x^2+c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Mathematica [A]
time = 0.90, size = 88, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/(
b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(73) = 146$.
time = 0.07, size = 160, normalized size = 1.82

method	result

default	$\frac{\left(-ad\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+c\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+ad\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2-}}{(bd\,x^4-ad\,x^2-c\,x^2b+ac)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(-d\,x^2+c)}\left(\frac{c\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-c\,x^2b+ac}}-\frac{da\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{}}{}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(b
*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2+c)^(
1/2)*(-b*x^2+a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)/(b*d*x^4-a*
d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-d\,x^2}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

```
[Out] int((c - d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.279 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*x
^2-c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$
$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Mathematica [A]
time = 0.87, size = 89, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A]
time = 0.08, size = 106, normalized size = 1.19

method	result

default	$\frac{\sqrt{d\,x^{2}-c}\,\sqrt{-b\,x^{2}+a}\,c\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\operatorname{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(bd\,x^{4}-ad\,x^{2}-c\,x^{2}b+ac\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+c\,x^{2}b-ac}}+\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}}{\ldots}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(b
/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c+dx^{2}}}{\sqrt{a-bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-c + d*x**2)/sqrt(a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2-c}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2 - c)^(1/2)/(a - b*x^2)^(1/2),x)
[Out] int((d*x^2 - c)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.280 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d*
x^2+c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Mathematica [A]
time = 0.81, size = 89, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/(
b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A]
time = 0.08, size = 106, normalized size = 1.19

method	result

default	$\frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2-a}\,c\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{(-bd\,x^4+ad\,x^2+c\,x^2b-ac)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-(-b\,x^2+a)\,(-d\,x^2+c)}\left(\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2+c\,x^2b-ac}}-\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{}}{}\right.}{\sqrt{b\,x^2-a}\,\sqrt{-d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x^2+c)^(1/2)*(b*x^2-a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(
b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-d\,x^2}}{\sqrt{b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
```

```
[Out] int((c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.281 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\,\sqrt{-a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*x
^2-c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {438, 437, 435}

$$\frac{\sqrt{a}\,\sqrt{1-\frac{bx^2}{a}}\,\sqrt{dx^2-c}\,E\left(\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\,\sqrt{bx^2-a}\,\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$
$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Mathematica [A]
time = 0.71, size = 90, normalized size = 1.00

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(75) = 150$.
time = 0.08, size = 163, normalized size = 1.81

method	result

default	$\frac{\sqrt{d\,x^2-c}\,\sqrt{b\,x^2-a}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\left(ad\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-c\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b-ad\,\mathrm{Elli}\right.}{(bd\,x^4-ad\,x^2-c\,x^2b+ac)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(-d\,x^2+c)}\left(-\frac{c\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-c\,x^2b+ac}}+\frac{da\sqrt{1-\frac{d\,x^2}{c}}}{}\right.}{\sqrt{b\,x^2-a}\,\sqrt{d\,x^2-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2-c)^(1/2)*(b*x^2-a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*(
a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),(b*c
/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4-a*d*x
^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2-c}}{\sqrt{b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2 - c)^(1/2)/(b*x^2 - a)^(1/2),x)
```

```
[Out] int((d*x^2 - c)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.282 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] d*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx &= c\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx \\
&= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} - \frac{(cd)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b} \\
&= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan\right.}{a\sqrt{d}\,\sqrt{\frac{c(a+}{a(c+}}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 0.42

$$
\frac{\sqrt{\frac{a+bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{a+bx^2}\,\sqrt{\frac{c+dx^2}{c}}}
$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]

[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])

Maple [A]
time = 0.00, size = 101, normalized size = 0.50

method	result
default	$\frac{\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}\,c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)}{\left(b d\,x^{4}+a d\,x^{2}+c\,x^{2} b+a c\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}+c\,x^{2} b+a c}}-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+}}{}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/
a)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.283 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$-\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -d*x*(b*x^2+a)^(1/2)/b/(-d*x^2-c)^(1/2)-c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(b*x^2+a)^(1/2)/a/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(-d*x^2-
c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {433, 429, 506, 422}

$$-\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] -((d*x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
 d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx &= -\left(c\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{-c-dx^2}}\,dx\right)-d\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{-c-dx^2}}\,dx\\
&= -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(cd)\int \frac{\sqrt{a+bx^2}}{(-c-dx^2)^{3/2}}\,dx}{b}\\
&= -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\,F}{a\sqrt{d}\,\sqrt{-c-}}
\end{aligned}$$

Mathematica [A]
time = 0.87, size = 89, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\,\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{a+bx^2}\,\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A]
time = 0.08, size = 161, normalized size = 0.75

method	result
default	$\frac{\left(-ad\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+c\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+ad\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{-d\,x^2-c}\,\sqrt{b}}{(bd\,x^4+ad\,x^2+c\,x^2b+ac)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\left(-\frac{c\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2-c\,x^2b-ac}}+\frac{da\sqrt{1+\frac{d\,x^2}{c}}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2),
(b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2-c
)^(1/2)*(b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{-d\,x^2-c}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- c - d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((- c - d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.284 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=214

$$-\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\right.}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] -d*x*(-b*x^2-a)^(1/2)/b/(d*x^2+c)^(1/2)-c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(-b*x^2-a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1/2)/b/(c*(b*x
^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {433, 429, 506, 422}

$$-\frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] -((d*x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[-a -
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[-a - b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = c\int \frac{1}{\sqrt{-a-bx^2}\,\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{-a-bx^2}\,\sqrt{c+dx^2}}\,dx$$

$$= -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} + \frac{(cd)\int \frac{\sqrt{-a-bx^2}}{(c+dx^2)^{3/2}}}{b}$$

$$= -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}} + \frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-b}}{a\sqrt{d}}$$

Mathematica [A]
time = 0.86, size = 89, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\,\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{-a-bx^2}\,\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]

[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])

Maple [A]
time = 0.08, size = 162, normalized size = 0.76

method	result
default	$\frac{\sqrt{d x^2+c}\,\sqrt{-b x^2-a}\,\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{b x^2+a}{a}}\left(ad\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-c\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b-a\right.}{(bd x^4+ad x^2+c x^2b+ac)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-(b x^2+a)(d x^2+c)}\left(\frac{c\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{b x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd x^4-ad x^2-c x^2b-ac}}-\frac{da\sqrt{1+\frac{d x^2}{c}}}{}\right.}{\sqrt{-b x^2-a}\,\sqrt{d x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(a*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{-b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.285 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=222

$$\frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\right.}{a\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] d*x*(-b*x^2-a)^(1/2)/b/(-d*x^2-c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(-b*x^2-a)^(1/2)/a/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+
d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1/2)/b/(-d*x^
2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {433, 429, 506, 422}

$$\frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (d*x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*Sqrt[-a - b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx &= -\left(c\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\right)-d\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\\ &= \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}+\frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{(cd)\int \frac{\sqrt{-a-bx^2}}{(-c-dx^2)^{3/2}}}{b}\\ &= \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{-a-bx^2}}{a\sqrt{d}\,\sqrt{-}}\end{aligned}$$

Mathematica [A]
time = 0.81, size = 92, normalized size = 0.41

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A]
time = 0.08, size = 111, normalized size = 0.50

method	result
default	$\frac{\sqrt{-d\,x^{2}-c}\,\sqrt{-b\,x^{2}-a}\,c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(-bd\,x^{4}-ad\,x^{2}-c\,x^{2}b-ac\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac}}+\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1}}{}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)*(-d*x^2-c)^(1/2)*(-b*x^2-a)^(
1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{-d\,x^2-c}}{\sqrt{-b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
```

```
[Out] int((- c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.286 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\,\sqrt{c-dx^2}}+\frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{c-dx^2}}$$

```
[Out] -EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1
/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/
c)^(1/2)/b/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {434, 438, 437, 435, 432, 430}

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{c-dx^2}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sq
rt[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx}{b} + \frac{(bc+ad)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{b}$$

$$= -\frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}}{b\sqrt{c-dx^2}}$$

$$= -\frac{\left(d\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int}{b\sqrt{a+bx^2}\sqrt{c-}}$$

$$= -\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1}}{b\sqrt{d}\sqrt{a+}}$$

Mathematica [**A**]
time = 0.86, size = 89, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [**A**]
time = 0.07, size = 161, normalized size = 0.85

method	result
default	$\frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\left(ad\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+c\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b-\right.}{(-bd\,x^4-ad\,x^2+c\,x^2b+ac)\sqrt{\frac{d}{c}}\,b}$

elliptic	$\frac{\sqrt{(b\,x^2+a)\,(-d\,x^2+c)}\,\left(\frac{c\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+c\,x^2b+ac}}+\frac{da\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(
a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(-b
*c/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))/(-b*d*x^4-a
*d*x^2+b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-d\,x^2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
[Out] int((c - d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.287 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\,\sqrt{-c+dx^2}}-\frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/
2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)-(a*d+b*c)*Elliptic
F(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)
^(1/2)/b/d^(1/2)/(b*x^2+a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {434, 438, 437, 435, 432, 430}

$$\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\,\sqrt{dx^2-c}}-\frac{\sqrt{c}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt
[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
- (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[-c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx}{b} - \frac{(bc+ad)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{-c+dx^2}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}}}{b\sqrt{-c+dx^2}}$$

$$= \frac{\left(d\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\,\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1-\frac{dx^2}{c}}\right)\int}{b\sqrt{a+bx^2}\,\sqrt{-c}}$$

$$= \frac{\sqrt{c}\,\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\,\sqrt{-c+dx^2}} - \frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{}}{b\sqrt{d}\,\sqrt{a}}$$

Mathematica [A]
time = 0.85, size = 90, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\,\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{a+bx^2}\,\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A]
time = 0.08, size = 107, normalized size = 0.56

method	result
default	$\frac{\sqrt{d\,x^2-c}\,\sqrt{b\,x^2+a}\,c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{\left(-bd\,x^4-ad\,x^2+c\,x^2b+ac\right)\sqrt{-\frac{b}{a}}}$

elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-c\,x^{2}b-ac}}+\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}}{}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2-c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)/(
-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2-c}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2 - c)^(1/2)/(a + b*x^2)^(1/2),x)
[Out] int((d*x^2 - c)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.288 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\,\sqrt{c-dx^2}}+\frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1
/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/
c)^(1/2)/b/d^(1/2)/(-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {434, 438, 437, 435, 432, 430}

$$\frac{\sqrt{c}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{c-dx^2}}+\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
+ (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqr
t[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{d\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx}{b} + \frac{(bc+ad)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{c-dx^2}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{1-\frac{dx}{c}}}}{b\sqrt{c-dx^2}}$$

$$= \frac{\left(d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int}{b\sqrt{-a-bx^2}\sqrt{c}}$$

$$= \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{}}{b\sqrt{d}\sqrt{-}}$$

Mathematica [A]
time = 0.68, size = 92, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A]
time = 0.09, size = 109, normalized size = 0.56

method	result
default	$\frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2-a}\,c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{(bd\,x^4+ad\,x^2-c\,x^2b-ac)\sqrt{-\frac{b}{a}}}$

elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-c\,x^{2}b-ac}}-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}}{}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/
(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-dx^{2}}}{\sqrt{-a-bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-d\,x^2}}{\sqrt{-b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
[Out] int((c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.289 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\,\sqrt{-c+dx^2}}-\frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\right.}{b\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{-c+dx^2}}$$

```
[Out] -EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(
1/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)-(a*d+b*c)*Ellipt
icF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/
c)^(1/2)/b/d^(1/2)/(-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {434, 438, 437, 435, 432, 430}

$$-\frac{\sqrt{c}\,\sqrt{\frac{bx^2}{a}+1}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{dx^2-c}}-\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{-a-bx^2}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\,\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2
])) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]
*Sqrt[-c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{d\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx}{b} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c+dx^2}}\,dx}{b}$$

$$= -\frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{1-\right.}}}{b\sqrt{-c+dx^2}}$$

$$= -\frac{\left(d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right.}{b\sqrt{-a-bx^2}\,\sqrt{}}$$

$$= -\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}\,(bc+ad)\sqrt{1+\frac{bx}{a}}}{b\sqrt{d}\,\sqrt{}}$$

Mathematica [A]
time = 0.66, size = 93, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A]
time = 0.08, size = 166, normalized size = 0.84

method	result
default	$\frac{\left(-ad\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-c\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+ad\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)\sqrt{d\,x^2-c}\,\sqrt{-}}{(-bd\,x^4-ad\,x^2+c\,x^2b+ac)\sqrt{\frac{d}{c}}\,b}$

elliptic	$\frac{\sqrt{(b\,x^2+a)(-d\,x^2+c)}\left(-\frac{c\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+c\,x^2b+ac}}-\frac{da\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{}}{}\right.}{\sqrt{-b\,x^2-a}\,\sqrt{d\,x^2-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),(
-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))*(d*x^2-c)
^(1/2)*(-b*x^2-a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(-b*d*x^4-
a*d*x^2+b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2-c}}{\sqrt{-b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2 - c)^(1/2)/(- a - b*x^2)^(1/2),x)
[Out] int((d*x^2 - c)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.290 $\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\,\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}}$$

```
[Out] 1/2*(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*
d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2
+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{429\}$

$$\frac{\sqrt{bx^2+2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\,\sqrt{d}\,\sqrt{dx^2+3}\,\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] (Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(
Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx=\frac{\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\,\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}}$$

Mathematica [A]
time = 0.57, size = 37, normalized size = 0.47

$$\frac{F\left(\sin^{-1}\left(\frac{\sqrt{-b}\,x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{\sqrt{3}\,\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] EllipticF[ArcSin[(Sqrt[-b]*x)/Sqrt[2]], (2*d)/(3*b)]/(Sqrt[3]*Sqrt[-b])
```

Maple [A]
time = 0.08, size = 38, normalized size = 0.49

method	result	size
default	$\frac{\sqrt{2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)}{2\sqrt{-d}}$	38
elliptic	$\frac{\sqrt{(b\,x^2+2)\,(d\,x^2+3)}\,\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{2\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1
/2))/(-d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Fricas [A]
time = 0.45, size = 34, normalized size = 0.44

$$-\frac{\sqrt{6}\,\sqrt{2}\,\sqrt{-b}\,\mathrm{ellipticF}\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{-b}\,x,\,\frac{2\,d}{3\,b}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*sqrt(2)*sqrt(-b)*ellipticF(1/2*sqrt(2)*sqrt(-b)*x, 2/3*d/b)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+2}\,\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)),x)
```

```
[Out] int(1/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)), x)
```

3.291 $\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

```
[Out] EllipticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {432, 430}

$$\frac{\sqrt{\frac{dx^2}{c}+1}\,F\left(\text{ArcSin}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{\sqrt{4-x^2}\sqrt{1+\frac{dx^2}{c}}}dx}{\sqrt{c+dx^2}} = \frac{\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Mathematica [A]
time = 0.50, size = 40, normalized size = 1.03

$$\frac{\sqrt{\frac{c+dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
[Out] (Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d\,x^2+c}{c}}\;\mathrm{EllipticF}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)}{\sqrt{d\,x^2+c}}$	38
elliptic	$\frac{\sqrt{-\left(d\,x^2+c\right)\left(x^2-4\right)}\;\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)}{\sqrt{d\,x^2+c}\;\sqrt{-d\,x^4-c\,x^2+4d\,x^2+4c}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(1/2*x,2*(-d/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Fricas [A]
time = 0.50, size = 14, normalized size = 0.36

$$\frac{\text{ellipticF}\left(\frac{1}{2}x, -\frac{4d}{c}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] ellipticF(1/2*x, -4*d/c)/sqrt(c)
```

Sympy [A]
time = 1.16, size = 20, normalized size = 0.51

$$\begin{cases} \frac{F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle| -\frac{4d}{c}\right)}{\sqrt{c}} & \text{for}\: x > -2 \wedge x < 2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Piecewise((elliptic_f(asin(x/2), -4*d/c)/sqrt(c), (x > -2) & (x < 2)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{4-x^2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.292 $\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

```
[Out] (1/(x^2+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2
)/c/((d*x^2+c)/c/(x^2+4))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {429}

$$\frac{\sqrt{c+dx^2}\,F\left(\text{ArcTan}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt
[(c + d*x^2)/(c*(4 + x^2))])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.48, size = 47, normalized size = 0.77

$$-\frac{i\sqrt{\frac{c+dx^2}{c}}\,F\left(i\sinh^{-1}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[(c + d*x^2)/c]*EllipticF[I*ArcSinh[x/2], (4*d)/c])/Sqrt[c + d*x^
2]
```

Maple [A]
time = 0.08, size = 53, normalized size = 0.87

method	result	size
default	$\frac{\sqrt{\frac{d\,x^{2}+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)}{2\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}}$	53
elliptic	$\frac{\sqrt{\left(d\,x^{2}+c\right)\left(x^{2}+4\right)}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)}{2\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^{4}+c\,x^{2}+4d\,x^{2}+4c}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-d/c)^(1/2),1/2*(c/d)^
(1/2))/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.37, size = 15, normalized size = 0.25

$$-\frac{i\,\mathrm{ellipticF}\left(\frac{1}{2}i\,x,\frac{4\,d}{c}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*ellipticF(1/2*I*x, 4*d/c)/sqrt(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+dx^2}\,\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2+4}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.293 $\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=6

$$-F\left(\cos^{-1}(x)\middle|\,2\right)$$

```
[Out] -(x^2)^(1/2)/x*EllipticF((-x^2+1)^(1/2),2^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {431}

$$-F(\text{ArcCos}(x)|2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] -EllipticF[ArcCos[x], 2]
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -F\left(\cos^{-1}(x)\middle|\,2\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(6) = 12.
time = 0.23, size = 27, normalized size = 4.50

$$\frac{\sqrt{1-2x^2}\,F\left(\sin^{-1}(x)\middle|\,2\right)}{\sqrt{-1+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticF[ArcSin[x], 2])/Sqrt[-1 + 2*x^2]
```

Maple [A]
time = 0.10, size = 25, normalized size = 4.17

method	result	size
default	$\frac{\text{EllipticF}\left(x,\sqrt{2}\right)\sqrt{-2x^{2}+1}}{\sqrt{2x^{2}-1}}$	25
elliptic	$\frac{\sqrt{-\left(2x^{2}-1\right)\left(x^{2}-1\right)}\,\sqrt{-2x^{2}+1}\,\text{EllipticF}\left(x,\sqrt{2}\right)}{\sqrt{2x^{2}-1}\,\sqrt{-2x^{4}+3x^{2}-1}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)/(2*x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x-1\right)\left(x+1\right)}\,\sqrt{2x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.17

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)), x)
```

3.294 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{E(\sin^{-1}(cx)|-1)}{c}+\frac{2F(\sin^{-1}(cx)|-1)}{c}$$

```
[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

Rubi [A]

time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {434, 435, 254, 227}

$$\frac{2F(\text{ArcSin}(cx)|-1)}{c}-\frac{E(\text{ArcSin}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

`)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]`

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx \\ &= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx \\ &= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}\,x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 28, normalized size = 1.22

method	result
default	$\frac{(2\,\mathrm{EllipticF}(x\,\mathrm{csgn}(c)c,i)-\mathrm{EllipticE}(x\,\mathrm{csgn}(c)c,i))\mathrm{csgn}(c)}{c}$
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{c^2},\dots\right)\dots\right)}{\sqrt{c^2}\sqrt{-c^4x^4+1}}\dots\right)}{\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Fricas [A]
time = 0.22, size = 30, normalized size = 1.30

$$\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{1-c^2\,x^2}}{\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2),x)
[Out] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2), x)
```

3.295 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}}-\frac{\sqrt{2}\,\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}}+\frac{\sqrt{2}\,\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}}$$

```
[Out] x*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*E
llipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(
b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+(1/(3*d*
x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/
2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3
))^(1/2)/(d*x^2+3)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{2}\,\sqrt{bx^2+2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{dx^2+3}\,\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\,\sqrt{bx^2+2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{dx^2+3}\,\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx &= 2\int \frac{1}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx\\ &= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} + \frac{\sqrt{2}\,\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}} - 3\int \frac{\sqrt{2+bx^2}}{(3+dx^2)^{3/2}}\,dx\\ &= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\,\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}} + \frac{\sqrt{2}\,\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\right.\right.}{\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.20

$$\frac{\sqrt{2}\,E\left(\sin^{-1}\left(\frac{\sqrt{-d}\,x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]
```

Maple [A]
time = 0.00, size = 37, normalized size = 0.20

method	result
default	$\frac{\text{EllipticE}\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3}, \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{\sqrt{-d}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)\,(d\,x^2+3)}\left(\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,\text{EllipticF}\left(\frac{x\sqrt{-3d}}{3}, \frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}} - \frac{\sqrt{3d\,x^2+9}}{\ldots}\right)}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2 \ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^(1/2)
/(-d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2),x)
```

```
[Out] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2), x)
```

3.296 $\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] -1/3*(x^2)^(1/2)/x*EllipticE(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {436}

$$-\frac{E\left(\text{ArcCos}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(EllipticE[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])
```

Rule 436

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.32, size = 35, normalized size = 1.84

$$\frac{\sqrt{-1+3x^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3-9x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (Sqrt[-1 + 3*x^2]*EllipticE[ArcSin[Sqrt[3/2]*x], 2])/Sqrt[3 - 9*x^2]
```

Maple [A]
time = 0.11, size = 37, normalized size = 1.95

method	result
default	$-\frac{\mathrm{EllipticE}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2}\right)\sqrt{-3x^2+1}\,\sqrt{3}}{3\sqrt{3x^2-1}}$
elliptic	$\frac{\sqrt{-\left(3x^2-2\right)\left(3x^2-1\right)}\left(-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)}{6\sqrt{-9x^4+9x^2-2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{\cdots}}{\cdots}\right.}{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*EllipticE(1/2*x*2^(1/2)*3^(1/2),2^(1/2))*(-3*x^2+1)^(1/2)*3^(1/2)/(3*x
^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [A]
time = 0.14, size = 23, normalized size = 1.21

$$-\frac{\sqrt{3\,x^2-1}\,\sqrt{-3\,x^2+2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2 - 1}}{\sqrt{2 - 3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{3\,x^2 - 1}}{\sqrt{2 - 3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
```

```
[Out] int((3*x^2 - 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.297 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] 1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2),((-b-(-4*a*c+b
^2)^(1/2))/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2)/c^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.017$, Rules used = {435}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,\sqrt{c}}$$

Mathematica [A]
time = 2.30, size = 95, normalized size = 1.00

$$\frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(80) = 160$.
time = 0.33, size = 809, normalized size = 8.52

method	result
elliptic	$\frac{\sqrt{\frac{-2c\,x^2+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}}\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{-\frac{\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)\left(-2c\,x^2+\sqrt{-4ac+b^2}+b\right.}{ac}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((-2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1
/2))*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)
^(1/2)/(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2*2^(1/2)/(c/(-b+(-4*a*c+b^2)^(1/
2)))^(1/2)*(1-2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)*(1-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2
)^(1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*Ell
ipticF(x*2^(1/2)*(c/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a
*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))
+2*c/(-b+(-4*a*c+b^2)^(1/2))*2^(1/2)/(c/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2
*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)*(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1
/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))-4*c^2/
(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(-2*c/(b+(-4*a*c+b
^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2))-b/a)*(EllipticF(x*2^(1/2)*(c/(-b+(-4*
a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*
c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(x*2^(1/2)*(c/(-b+
(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-
4*a*c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt
(b^2 - 4*a*c)) + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)
))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))),
x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}}{\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2),x)
```

```
[Out] int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2), x)
```

3.298 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] 1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2),((b+(-4*a*c+b^
2)^(1/2))/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2)/c^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.017$, Rules used = {435}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqr
t[2]*Sqrt[c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Mathematica [A]
time = 2.38, size = 95, normalized size = 1.01

$$\frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]))])/
(Sqrt[2]*Sqrt[c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(76) = 152$.
time = 0.26, size = 1388, normalized size = 14.77

method	result	size
elliptic	Expression too large to display	1388

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((-2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1/
2))*(-(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(
1/2)/(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^
2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)
^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a
*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a
```

```
*x^2)^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
)+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF(
1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^
2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(-2*c/(b+(-4*a*c+b^2)
^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2-4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-2
*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4
*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a
*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-(-4
*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(-2*c/(b+(-4*a*c+b^2)^(1
/2))-2*c/(b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^(1/2)/a)*(EllipticF(1/2*x*(-2*
((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(-2*c/(b+(-4*a*c+b^2)^(1/2))-2*
c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*
c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(1
/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2
)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(-2*c/(b+(-4*a*c+b^2)^
(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b
^2-4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqr
t(b^2 - 4*a*c)) + 1), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**
2)))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))
, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}}{\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2),x)
```

```
[Out] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2), x)
```

3.299 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=478

$$\frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}-\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\middle|-\frac{2\sqrt{\cdots}}{b-\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] x*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)-1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)*EllipticE(x*2^(1/
2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^
(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+
2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1
/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*
(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x^2/(b
-(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {433, 429, 506, 422}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,F\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\middle|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,E\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\middle|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{x\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])] - (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
```

```
*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*
Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{(2c)\int \frac{x^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{b-\sqrt{b^2-4ac}} + \int \frac{}{\sqrt{1+\frac{2}{b-\sqrt{b}}}}$$

$$= \frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,F\Big(\text{ta}}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}}$$

$$= \frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} - \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,E\Big(\text{ta}}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}}$$

Mathematica [A]
time = 2.39, size = 102, normalized size = 0.21

$$\frac{\sqrt{-b-\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b
- Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(S
qrt[2]*Sqrt[c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(457) = 914$.
time = 0.22, size = 1388, normalized size = 2.90

method	result	size
elliptic	Expression too large to display	1388

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1/
2))*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(
1/2)/(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b
^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2
)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*
a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
))+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF
(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b
^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)
^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))+2
*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4
*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a
*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1
+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-(-4
*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(2*c/(b+(-4*a*c+b^2)^(1/
2))+2*c/(b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^(1/2)/a)*(EllipticF(1/2*x*(-2*(
(-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/
(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)
/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(1/2
*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^
(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/
2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+
4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(
b^2 - 4*a*c)) + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)
))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}}{\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2),x)
```

```
[Out] int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2), x)
```

3.300 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=215

$$-\frac{\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\,bF\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.\right.}{\sqrt{c}\sqrt{b-\sqrt{b^2-}}}$$

```
[Out] b*EllipticF(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((-b+(-4*a*c+b^2
)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2)-1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((-b
+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))*
2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {434, 435, 430}

$$\frac{\sqrt{2}\,bF\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] -(((b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]))])/(Sq
rt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) + (Sqrt[2]*b*EllipticF[ArcSin[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/
(b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{(2b)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{b-\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4a\right.}{}$$

$$= -\frac{\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\middle|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} +$$

Mathematica [A]
time = 2.25, size = 102, normalized size = 0.47

$$\frac{\sqrt{-b-\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)\middle|\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

[Out] (Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c])

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(173) = 346$.
time = 0.19, size = 872, normalized size = 4.06

method	result
elliptic	$\dfrac{\sqrt{\frac{2cx^2+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}}\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{-\frac{\left(2cx^2+\sqrt{-4ac+b^2}-b\right)\left(2cx^2+\sqrt{-4ac+b^2}+b\right)}{ac}}\left(\ \cdots \right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1/2))*(-(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)/(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF(x*(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))/c/(-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2)))^(1/2))-4*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2))-b/a)*(EllipticF(x*(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2))-

```
2*c/(b-(-4*a*c+b^2)^(1/2)))/c/(-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2)
)*(b+(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(x*(-2*c/(b+(-4*a*c+b^2)^(1/2)))^
(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))/c/(
-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt
(b^2 - 4*a*c)) + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**
2)))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))),
x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}}{\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2),x)
```

```
[Out] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2), x)
```

3.301 $\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2^{-2-m}\sqrt{x^2}\,(2-4x^2)^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};(1-2x^2)^2\right)}{(1+m)x}$$

```
[Out] -2^(-2-m)*(-4*x^2+2)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],(-2*x^2+1
)^2)*(x^2)^(1/2)/(1+m)/x
```

Rubi [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {440}

$$xF_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

```
[Out] x*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = xF_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.39, size = 122, normalized size = 1.97

$$\frac{3x(1-2x^2)^m F_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)}{\sqrt{1-x^2}\,\left(3F_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)+x^2\left(-4mF_1\left(\frac{3}{2};1-m,\frac{1}{2};\frac{5}{2};2x^2,x^2\right)+F_1\left(\frac{3}{2};-m,\frac{3}{2};\frac{5}{2};2x^2,x^2\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

```
[Out] (3*x*(1 - 2*x^2)^m*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2])/(Sqrt[1 - x^2]*
(3*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2] + x^2*(-4*m*AppellF1[3/2, 1 - m,
 1/2, 5/2, 2*x^2, x^2] + AppellF1[3/2, -m, 3/2, 5/2, 2*x^2, x^2])))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-2x^{2}+1\right)^{m}}{\sqrt{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

```
[Out] int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*(-2*x^2 + 1)^m/(x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(1-2x^{2}\right)^{m}}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)**m/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((1 - 2*x**2)**m/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-2\,x^2)^m}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 2*x^2)^m/(1 - x^2)^(1/2),x)
[Out] int((1 - 2*x^2)^m/(1 - x^2)^(1/2), x)
```

3.302 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)|-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\,\sqrt{-1+x^2}}$$

```
[Out] EllipticF(x,I*3^(1/2)+2*I)*(-x^2+1)^(1/2)/(x^2-1)^(1/2)/(2-3^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {432, 430}

$$\frac{\sqrt{1-x^2}\,F\left(\text{ArcSin}(x)\,\middle|\,-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\,\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -7 - 4*Sqrt[3]])/(Sqrt[7 - 4*Sqrt[3]]*S
qrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}$$

Mathematica [**A**]
time = 0.89, size = 48, normalized size = 1.04

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|\frac{1}{-7+4\sqrt{3}}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], (-7 + 4*Sqrt[3])^(-1)])/(Sqrt[7 - 4*Sqr
t[3]]*Sqrt[-1 + x^2])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(37) = 74.
time = 0.22, size = 117, normalized size = 2.54

method	result
default	$-\frac{i\,\mathrm{EllipticF}\left(\frac{ix}{-2+\sqrt{3}},2i-i\sqrt{3}\right)\sqrt{-x^2+1}\sqrt{-\left(-x^2+4\sqrt{3}-7\right)\left(-4\sqrt{3}+7\right)}\left(-2+\sqrt{3}\right)\sqrt{x^2-1}}{\left(4\sqrt{3}-7\right)\left(-x^4+4x^2\sqrt{3}-6x^2-4\sqrt{3}+7\right)}$
elliptic	$-\frac{i\sqrt{-\left(x^2-1\right)\left(-x^2+4\sqrt{3}-7\right)}\sqrt{-4\sqrt{3}+7}\sqrt{1-\frac{x^2}{4\sqrt{3}-7}}\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix}{\sqrt{-4\sqrt{3}}}\right.}{\sqrt{x^2-1}\sqrt{7+x^2-4\sqrt{3}}\sqrt{6x^2-7+x^4-4x^2\sqrt{3}+4\sqrt{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*EllipticF(I*x/(-2+3^(1/2)),2*I-I*3^(1/2))*(-x^2+1)^(1/2)*(-(-x^2+4*3^(1/
2)-7)*(-4*3^(1/2)+7))^(1/2)/(4*3^(1/2)-7)*(-2+3^(1/2))*(x^2-1)^(1/2)*(7+x^2
-4*3^(1/2))^(1/2)/(-x^4+4*x^2*3^(1/2)-6*x^2-4*3^(1/2)+7)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{x^2-4\sqrt{3}+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(7+x**2-4*3**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 - 4*sqrt(3) + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,\sqrt{x^2-4\,\sqrt{3}+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 - 4*3^(1/2) + 7)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 - 4*3^(1/2) + 7)^(1/2)), x)
```

3.303 $\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}\,x^2}\,\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{6}\sqrt{3+\sqrt{3}}\,F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

```
[Out] -1/6*(x^2*(9-3*3^(1/2)))^(1/2)/x/(9-3*3^(1/2))^(1/2)*EllipticF(1/3*(9-x^2*(
9-3*3^(1/2)))^(1/2),1/2*(2+2*3^(1/2))^(1/2))*(3+3^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = $\{431\}$

$$-\frac{1}{6}\sqrt{3+\sqrt{3}}\,F\left(\text{ArcCos}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]),x]
```

```
[Out] -1/6*(Sqrt[3 + Sqrt[3]]*EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqr
t[3])/2])
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}\,x^2}\,\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx=-\frac{1}{6}\sqrt{3+\sqrt{3}}\,F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

Mathematica [A]

time = 2.06, size = 81, normalized size = 1.72

$$\frac{\sqrt{-3+3\sqrt{3}-2\sqrt{3}\,x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1+\sqrt{3}}\,x}{\sqrt[4]{3}}\right)\middle|2-\sqrt{3}\right)}{3^{3/4}\sqrt{6-6\sqrt{3}+4\sqrt{3}\,x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^
2]),x]
```

```
[Out] (Sqrt[-3 + 3*Sqrt[3] - 2*Sqrt[3]*x^2]*EllipticF[ArcSin[(Sqrt[1 + Sqrt[3]]*x
)/3^(1/4)], 2 - Sqrt[3]])/(3^(3/4)*Sqrt[6 - 6*Sqrt[3] + 4*Sqrt[3]*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(64) = 128.
time = 0.43, size = 207, normalized size = 4.40

method	result
default	$\dfrac{\sqrt{x^2\sqrt{3}-3x^2+3}\,\sqrt{3-3\sqrt{3}+2x^2\sqrt{3}}\,\sqrt{2}\,\sqrt{-\left(4x^2\sqrt{3}-6x^2-3\sqrt{3}+3\right)\left(\sqrt{3}-1\right)}\,\sqrt{\cdots}}{18\left(\sqrt{3}-1\right)^2\left(2x^4\sqrt{3}-2x^4-6\cdots\right)}$
elliptic	$\dfrac{\sqrt{\left(x^2\sqrt{3}-3x^2+3\right)\left(3-3\sqrt{3}+2x^2\sqrt{3}\right)}\,\sqrt{6}\,\sqrt{9-\frac{6\left(2\sqrt{3}-3\right)x^2}{\sqrt{3}-1}}\,\sqrt{9-\frac{6\sqrt{3}\,x^2}{\sqrt{3}-1}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{\cdots}\right)}{18\sqrt{x^2\sqrt{3}-3x^2+3}\,\sqrt{3-3\sqrt{3}+2x^2\sqrt{3}}\,\sqrt{\frac{2\sqrt{3}-3}{\sqrt{3}-1}}\,\sqrt{18x^2\sqrt{3}-18x^2+6\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/18*(x^2*3^(1/2)-3*x^2+3)^(1/2)*(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2)*2^(1/2)/
(3^(1/2)-1)^2*(-(4*x^2*3^(1/2)-6*x^2-3*3^(1/2)+3)*(3^(1/2)-1))^(1/2)*(-(3-3
*3^(1/2)+2*x^2*3^(1/2))*(3^(1/2)-1))^(1/2)*EllipticF(1/3*x*2^(1/2)*3^(1/2)/
(3^(1/2)-1)*((2*3^(1/2)-3)*(3^(1/2)-1))^(1/2),1/(3^(1/2)-1)*((3^(1/2)-1)*(1
+3^(1/2)))^(1/2))*(-3+3^(1/2))/(2*x^4*3^(1/2)-2*x^4-6*x^2*3^(1/2)+6*x^2+3*3
^(1/2)-3)/((2*3^(1/2)-3)*(3^(1/2)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3
)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+\sqrt{3}\,x^2+3}\,\sqrt{2\sqrt{3}\,x^2-3\sqrt{3}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x**2*(-3+3**(1/2)))**(1/2)/(3-3*3**(1/2)+2*3**(1/2)*x**2)**(
1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*x**2 + sqrt(3)*x**2 + 3)*sqrt(2*sqrt(3)*x**2 - 3*sqrt(3
) + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3
)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\left(\sqrt{3}-3\right)\,x^2+3}\,\sqrt{2\,\sqrt{3}\,x^2-3\,\sqrt{3}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2*(3^(1/2) - 3) + 3)^(1/2)*(2*3^(1/2)*x^2 - 3*3^(1/2) + 3)^(1/2))
,x)
```

```
[Out] int(1/((x^2*(3^(1/2) - 3) + 3)^(1/2)*(2*3^(1/2)*x^2 - 3*3^(1/2) + 3)^(1/2))
, x)
```

3.304 $\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\,2^{3/4}+2\sqrt[4]{2}\,\sqrt{2+3x^2}}{2\sqrt{3}\,x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{2+3x^2}}{2\sqrt{3}\,x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

```
[Out] -1/12*arctan(1/6*(2*2^(3/4)+2*2^(1/4)*(3*x^2+2)^(1/2))/x/(3*x^2+2)^(1/4)*3^
(1/2))*2^(1/4)*3^(1/2)-1/12*arctanh(1/6*(2*2^(3/4)-2*2^(1/4)*(3*x^2+2)^(1/2
))/x/(3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{406\}$

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[4]{2}\,\sqrt{3x^2+2}+2\,2^{3/4}}{2\sqrt{3}\,x\sqrt[4]{3x^2+2}}\right)}{2\,2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{3x^2+2}}{2\sqrt{3}\,x\sqrt[4]{3x^2+2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] -1/2*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2
)^(1/4))]/(2^(3/4)*Sqrt[3]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2
])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx=-\frac{\tan^{-1}\left(\frac{2\,2^{3/4}+2\sqrt[4]{2}\,\sqrt{2+3x^2}}{2\sqrt{3}\,x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{2+3x^2}}{2\sqrt{3}\,x\sqrt[4]{2+3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Mathematica [A]
time = 0.26, size = 119, normalized size = 0.92

$$\frac{\tan^{-1}\left(\frac{3\sqrt{2}\,x^2-4\sqrt{2+3x^2}}{2\,2^{3/4}\sqrt{3}\,x\sqrt[4]{2+3x^2}}\right)+\tanh^{-1}\left(\frac{2\,2^{3/4}\sqrt{3}\,x\sqrt[4]{2+3x^2}}{3\sqrt{2}\,x^2+4\sqrt{2+3x^2}}\right)}{4\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] (ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 + 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 + 3*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 + 3*x^2)^(1/4))/(3*Sqrt[2]*x^2
+ 4*Sqrt[2 + 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.55, size = 186, normalized size = 1.44

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)\ln\left(\frac{6\left(3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)-\left(3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)^2\mathrm{RootOf}\left(\right.}{3x^2+4}\right.}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln((6*(3*x^2+2)^(3/4)*RootOf(_Z^2+RootO
f(_Z^4+72)^2)-(3*x^2+2)^(1/4)*RootOf(_Z^4+72)^2*RootOf(_Z^2+RootOf(_Z^4+72)
^2)+18*(3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2+4))+1/24*RootOf(_Z^4
+72)*ln((6*(3*x^2+2)^(3/4)*RootOf(_Z^4+72)+(3*x^2+2)^(1/4)*RootOf(_Z^4+72)^
3+18*(3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+72)^2*x)/(3*x^2+4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(88) = 176.
time = 2.55, size = 553, normalized size = 4.29

$$\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x^3+54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{3}{4}}x+12\sqrt{2}\,(3x^2+4)\sqrt{3x^2+2}+72x^2-\left(18^{\frac{3}{4}}\sqrt{2}\,(3x^3-4x)\sqrt{3x^2+2}+72(3x^2+2)^{\frac{1}{4}}x^2+6\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^3+4x)+48\sqrt{2}\,(3x^2+2)^{\frac{3}{4}}\right)\sqrt{\frac{3\sqrt{2}\,x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x+4\sqrt{3x^2+2}}{3x^2+4}}}{6(9x^4-24x^2-16)}\right)-\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x^3-54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{3}{4}}x-12\sqrt{2}\,(3x^2+4)\sqrt{3x^2+2}-72x^2-\left(18^{\frac{3}{4}}\sqrt{2}\,(3x^3-4x)\sqrt{3x^2+2}-72(3x^2+2)^{\frac{1}{4}}x^2+6\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^3+4x)-48\sqrt{2}\,(3x^2+2)^{\frac{3}{4}}\right)\sqrt{\frac{3\sqrt{2}\,x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x+4\sqrt{3x^2+2}}{3x^2+4}}}{6(9x^4-24x^2-16)}\right)+\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{36\left(3\sqrt{2}\,x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x+4\sqrt{3x^2+2}\right)}{3x^2+4}\right)-\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{36\left(3\sqrt{2}\,x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x+4\sqrt{3x^2+2}\right)}{3x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="fricas")

[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x^3 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 + 4)*sqrt(3*x^2 + 2) + 72*x^2 - (18^(3/4)*sqrt(2)*(3*x^3 - 4*x)*sqrt(3*x^2 + 2) + 72*(3*x^2 + 2)^(1/4)*x^2 + 6*18^(1/4)*sqrt(2)*(3*x^3 + 4*x) + 48*sqrt(2)*(3*x^2 + 2)^(3/4))*sqrt((3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4)))/(9*x^4 - 24*x^2 - 16)) - 1/72*18^(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x^3 - 54*x^4 + 24*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 + 4)*sqrt(3*x^2 + 2) - 72*x^2 - (18^(3/4)*sqrt(2)*(3*x^3 - 4*x)*sqrt(3*x^2 + 2) - 72*(3*x^2 + 2)^(1/4)*x^2 + 6*18^(1/4)*sqrt(2)*(3*x^3 + 4*x) - 48*sqrt(2)*(3*x^2 + 2)^(3/4))*sqrt((3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4)))/(9*x^4 - 24*x^2 - 16)) + 1/288*18^(3/4)*sqrt(2)*log(36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4)) - 1/288*18^(3/4)*sqrt(2)*log(36*(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{3x^2+2}\cdot(3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2+2)**(1/4)/(3*x**2+4),x)

[Out] Integral(1/((3*x**2 + 2)**(1/4)*(3*x**2 + 4)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="giac")

[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(3\,x^2+2\right)}^{1/4}\,\left(3\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)),x)

[Out] int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)), x)

3.305 $\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

```
[Out] 1/12*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^(
1/2))*2^(1/4)*3^(1/2)+1/12*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)
/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {406}

$$\frac{\text{ArcTan}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Mathematica [A]
time = 0.25, size = 119, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{3\sqrt{2}\,x^2-4\sqrt{2-3x^2}}{2\ 2^{3/4}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)+\tanh^{-1}\left(\frac{2\ 2^{3/4}\sqrt{3}\,x\sqrt[4]{2-3x^2}}{3\sqrt{2}\,x^2+4\sqrt{2-3x^2}}\right)}{4\ 2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))/(3*Sqrt[2]*x^2
+ 4*Sqrt[2 - 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.52, size = 186, normalized size = 1.55

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)\ln\left(\frac{6\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)+\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)^2\mathrm{Roo}}{3x^2-4}\right.}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln((6*(-3*x^2+2)^(3/4)*RootOf(_Z^2+Root
Of(_Z^4+72)^2)+(-3*x^2+2)^(1/4)*RootOf(_Z^4+72)^2*RootOf(_Z^2+RootOf(_Z^4+7
2)^2)+18*(-3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))+1/24*RootOf(_
Z^4+72)*ln((6*(-3*x^2+2)^(3/4)*RootOf(_Z^4+72)-(-3*x^2+2)^(1/4)*RootOf(_Z^4
+72)^3+18*(-3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(89) = 178.
time = 2.42, size = 553, normalized size = 4.61

$$\frac{1}{72}\cdot 18^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3+54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x+12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}-72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}-72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)-48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)-\frac{1}{72}\cdot 18^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3-54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x-12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}+72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}+72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)+48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)+\frac{1}{288}\cdot 18^{\frac{1}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)-\frac{1}{288}\cdot 18^{\frac{1}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) - 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) - 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) - 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) -
 1/72*18^(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 - 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) + 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) + 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) + 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) +
 1/288*18^(3/4)*sqrt(2)*log(-36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)) - 1/288*18^(3/4)*sqrt(2)*l
og(-36*(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 4*sqrt(-3
*x^2 + 2))/(3*x^2 - 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(2-3\,x^2\right)}^{1/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
[Out] -int(1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.306 $\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*(2*2^(3/4)+2*2^(1/4)*(b*x^2+2)^(1/2))/x/(b*x^2+2)^(1/4)/b^(
1/2))*2^(1/4)/b^(1/2)-1/4*arctanh(1/2*(2*2^(3/4)-2*2^(1/4)*(b*x^2+2)^(1/2))
/x/(b*x^2+2)^(1/4)/b^(1/2))*2^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {406}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2
)^(1/4))]/(2^(3/4)*Sqrt[b]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2
])/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx=-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.22, size = 119, normalized size = 0.92

$$\frac{\tan^{-1}\left(\frac{2^{3/4}bx^2-4\sqrt[4]{2}\sqrt{2+bx^2}}{4\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)+\tanh^{-1}\left(\frac{2\ 2^{3/4}\sqrt{b}\,x\sqrt[4]{2+bx^2}}{\sqrt{2}\,bx^2+4\sqrt{2+bx^2}}\right)}{4\ 2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] (ArcTan[(2^(3/4)*b*x^2 - 4*2^(1/4)*Sqrt[2 + b*x^2])/(4*Sqrt[b]*x*(2 + b*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 + b*x^2)^(1/4))/(Sqrt[2]*b*x^2
+ 4*Sqrt[2 + b*x^2])])/(4*2^(3/4)*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+2)^{\frac{1}{4}}\,(b\,x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

```
[Out] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(86) = 172.
time = 7.25, size = 755, normalized size = 5.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(b*x^
2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 + b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(b*x^
```

```
2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x + 4*b*x^2 + 4*sqrt(1/2)*(b^2*x^2 + 4*b)*s
qrt(b*x^2 + 2)*sqrt(b^(-2)) - 2*sqrt(1/2)*(4*(b*x^2 + 2)^(1/4)*b*x^2 + 2*sq
rt(2)*(1/2)^(3/4)*(b^3*x^3 + 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(b*x^2
+ 2)^(3/4)*b*sqrt(b^(-2)) + sqrt(2)*(1/2)^(1/4)*(b^2*x^3 - 4*b*x)*sqrt(b*x^
2 + 2)*(b^(-2))^(1/4))*sqrt((2*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b
^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(b*x^2 + 2))/(b*x^2
 + 4)))/(b^2*x^4 - 8*b*x^2 - 16)) - 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*
arctan((2*sqrt(2)*(1/2)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 - b^
2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x - 4*b*
x^2 - 4*sqrt(1/2)*(b^2*x^2 + 4*b)*sqrt(b*x^2 + 2)*sqrt(b^(-2)) + 2*sqrt(1/2
)*(4*(b*x^2 + 2)^(1/4)*b*x^2 - 2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3 + 4*b^2*x)*(b
^(-2))^(3/4) + 16*sqrt(1/2)*(b*x^2 + 2)^(3/4)*b*sqrt(b^(-2)) - sqrt(2)*(1/2
)^(1/4)*(b^2*x^3 - 4*b*x)*sqrt(b*x^2 + 2)*(b^(-2))^(1/4))*sqrt(-(2*sqrt(2)*
(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1/2)*b^2*sqrt(b^(
-2))*x^2 - 2*sqrt(b*x^2 + 2))/(b*x^2 + 4)))/(b^2*x^4 - 8*b*x^2 - 16)) + 1/1
6*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(1/2*(2*sqrt(2)*(1/2)^(3/4)*(b*x^2
+ 2)^(1/4)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(b
*x^2 + 2))/(b*x^2 + 4)) - 1/16*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(-1/2*
(2*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1/2)*b
^2*sqrt(b^(-2))*x^2 - 2*sqrt(b*x^2 + 2))/(b*x^2 + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{bx^2 + 2}\,(bx^2 + 4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/4)/(b*x**2+4),x)
[Out] Integral(1/((b*x**2 + 2)**(1/4)*(b*x**2 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+2\right)}^{1/4}\,\left(b\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)),x)
[Out] int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)), x)
```

3.307 $\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

```
[Out] 1/4*arctan(1/2*(2-2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x/(-b*x^2+2)^(1/4)/b^(1
/2))*2^(1/4)/b^(1/2)+1/4*arctanh(1/2*(2+2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x
/(-b*x^2+2)^(1/4)/b^(1/2))*2^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {406}

$$\frac{\text{ArcTan}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[
b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.22, size = 123, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{2^{3/4}bx^2-4\sqrt[4]{2}\sqrt{2-bx^2}}{4\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)+\tanh^{-1}\left(\frac{2\ 2^{3/4}\sqrt{b}\,x\sqrt[4]{2-bx^2}}{\sqrt{2}\,bx^2+4\sqrt{2-bx^2}}\right)}{4\ 2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] (ArcTan[(2^(3/4)*b*x^2 - 4*2^(1/4)*Sqrt[2 - b*x^2])/(4*Sqrt[b]*x*(2 - b*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))/(Sqrt[2]*b*x^2
+ 4*Sqrt[2 - b*x^2])])/(4*2^(3/4)*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+2\right)^{\frac{1}{4}}\left(-b\,x^{2}+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

```
[Out] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(93) = 186$.
time = 6.06, size = 776, normalized size = 6.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(-b*x
^2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 + b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(-b*
```

```
x^2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x - 4*b*x^2 + 4*sqrt(1/2)*(b^2*x^2 - 4*b)
*sqrt(-b*x^2 + 2)*sqrt(b^(-2)) - 2*sqrt(1/2)*(4*(-b*x^2 + 2)^(1/4)*b*x^2 +
2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3 - 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(-b
*x^2 + 2)^(3/4)*b*sqrt(b^(-2)) - sqrt(2)*(1/2)^(1/4)*(b^2*x^3 + 4*b*x)*sqrt
(-b*x^2 + 2)*(b^(-2))^(1/4))*sqrt(-(2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4
)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(-b*x^2 + 2
))/(b*x^2 - 4)))/(b^2*x^4 + 8*b*x^2 - 16)) + 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2
))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(1/
4)*x^3 - b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(3/4)*b^2*(b^(-2))^(3
/4)*x + 4*b*x^2 - 4*sqrt(1/2)*(b^2*x^2 - 4*b)*sqrt(-b*x^2 + 2)*sqrt(b^(-2))
 + 2*sqrt(1/2)*(4*(-b*x^2 + 2)^(1/4)*b*x^2 - 2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3
 - 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(-b*x^2 + 2)^(3/4)*b*sqrt(b^(-2))
 + sqrt(2)*(1/2)^(1/4)*(b^2*x^3 + 4*b*x)*sqrt(-b*x^2 + 2)*(b^(-2))^(1/4))*s
qrt((2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1
/2)*b^2*sqrt(b^(-2))*x^2 - 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4)))/(b^2*x^4 + 8*b
*x^2 - 16)) + 1/16*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(-1/2*(2*sqrt(2)*(
1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(
-2))*x^2 + 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4)) - 1/16*sqrt(2)*(1/2)^(1/4)*(b^(
-2))^(1/4)*log(1/2*(2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(
3/4)*x - sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 - 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{bx^2\sqrt[4]{-bx^2+2} - 4\sqrt[4]{-bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2)**(1/4)/(-b*x**2+4),x)
[Out] -Integral(1/(b*x**2*(-b*x**2 + 2)**(1/4) - 4*(-b*x**2 + 2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{(2-b\,x^2)^{1/4}\,(b\,x^2-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)),x)
[Out] -int(1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)), x)
```

3.308 $\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

```
[Out] -1/6*arctan(1/3*a^(3/4)*(1+(3*x^2+a)^(1/2)/a^(1/2))/x/(3*x^2+a)^(1/4)*3^(1/
2))/a^(3/4)*3^(1/2)-1/6*arctanh(1/3*a^(3/4)*(1-(3*x^2+a)^(1/2)/a^(1/2))/x/(
3*x^2+a)^(1/4)*3^(1/2))/a^(3/4)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {406}

$$-\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] -1/2*ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^
(1/4))]/(Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))
/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Mathematica [A]
time = 0.21, size = 121, normalized size = 1.01

$$\frac{-\tan^{-1}\left(\frac{-3x^2+2\sqrt{a}\,\sqrt{a+3x^2}}{2\sqrt{3}\,\sqrt[4]{a}\,x\sqrt[4]{a+3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{3}\,\sqrt[4]{a}\,x\sqrt[4]{a+3x^2}}{3x^2+2\sqrt{a}\,\sqrt{a+3x^2}}\right)}{4\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
[Out] (-ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a + 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a + 3*x
^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a + 3*x^2)^(1/4))/(3*x^2 + 2*Sq
rt[a]*Sqrt[a + 3*x^2])])/(4*Sqrt[3]*a^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(3x^2+a)^{\frac{1}{4}}\,(3x^2+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
[Out] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(89) = 178.
time = 4.69, size = 286, normalized size = 2.38

$$\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{3x^2+a}\,a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{-a\sqrt{-\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}(3x^2+a)^{\frac{1}{4}}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+(3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+(3x^2+a)^{\frac{3}{4}}}{3x^2+2a}\right)+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-(3x^2+a)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-(3x^2+a)^{\frac{3}{4}}}{3x^2+2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/36)^(1/4)*(-1/a^3)^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(-1/a^3
)^(3/4) + (1/36)^(1/4)*sqrt(3*x^2 + a)*a*(-1/a^3)^(1/4))*sqrt(-a*sqrt(-1/a^
3)) - (1/36)^(1/4)*(3*x^2 + a)^(1/4)*a*(-1/a^3)^(1/4))/x) - 1/4*(1/36)^(1/4
)*(-1/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4)
+ (3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) +
(3*x^2 + a)^(3/4))/(3*x^2 + 2*a)) + 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log(-(1
8*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^2 + a)^(1/4)*a^2
*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) - (3*x^2 + a)^(3/4))/(3*x
^2 + 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+3x^2}\cdot(2a+3x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+a)**(1/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(1/((a + 3*x**2)**(1/4)*(2*a + 3*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(3\,x^2+2\,a)\,{(3\,x^2+a)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)),x)
```

```
[Out] int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)), x)
```

3.309 $\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

```
[Out] 1/6*arctan(1/3*a^(3/4)*(1-(-3*x^2+a)^(1/2)/a^(1/2))/x/(-3*x^2+a)^(1/4)*3^(1
/2))/a^(3/4)*3^(1/2)+1/6*arctanh(1/3*a^(3/4)*(1+(-3*x^2+a)^(1/2)/a^(1/2))/x
/(-3*x^2+a)^(1/4)*3^(1/2))/a^(3/4)*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{406\}$

$$\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}\,a^{3/4}}$$

Mathematica [A]
time = 0.20, size = 121, normalized size = 1.01

$$\frac{-\tan^{-1}\left(\frac{-3x^2+2\sqrt{a}\,\sqrt{a-3x^2}}{2\sqrt{3}\,\sqrt[4]{a}\,x\sqrt[4]{a-3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{3}\,\sqrt[4]{a}\,x\sqrt[4]{a-3x^2}}{3x^2+2\sqrt{a}\,\sqrt{a-3x^2}}\right)}{4\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] (-ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a - 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a - 3*x
^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a - 3*x^2)^(1/4))/(3*x^2 + 2*Sq
rt[a]*Sqrt[a - 3*x^2])])/(4*Sqrt[3]*a^(3/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+a\right)^{\frac{1}{4}}\left(-3x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

```
[Out] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(89) = 178.
time = 5.02, size = 286, normalized size = 2.38

$$\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{-3x^2+a}\,a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{a\sqrt{-\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-3x^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(-3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-\left(-3x^2+a\right)^{\frac{3}{4}}}{3x^2-2a}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2+a}\,a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(-3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+\left(-3x^2+a\right)^{\frac{3}{4}}}{3x^2-2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="fricas")

[Out] (1/36)^(1/4)*(-1/a^3)^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(-1/a^3)^(3/4) - (1/36)^(1/4)*sqrt(-3*x^2 + a)*a*(-1/a^3)^(1/4))*sqrt(a*sqrt(-1/a^3)) - (1/36)^(1/4)*(-3*x^2 + a)^(1/4)*a*(-1/a^3)^(1/4))/x) + 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) + (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) - (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a)) - 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2a\sqrt[4]{a-3x^2}+3x^2\sqrt[4]{a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2+a)**(1/4)/(-3*x**2+2*a),x)

[Out] -Integral(1/(-2*a*(a - 3*x**2)**(1/4) + 3*x**2*(a - 3*x**2)**(1/4)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="giac")

[Out] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,a-3\,x^2\right)\,{\left(a-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)),x)

[Out] int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)), x)

3.310 $\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -1/2*arctan(a^(3/4)*(1+(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)^(1/4)/b^(1/2))/
a^(3/4)/b^(1/2)-1/2*arctanh(a^(3/4)*(1-(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)
^(1/4)/b^(1/2))/a^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {406}

$$-\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^
(1/4))]/(a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))
/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.22, size = 119, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{bx^2-2\sqrt{a}\sqrt{a+bx^2}}{2\sqrt[4]{a}\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt[4]{a}\sqrt{b}\,x\sqrt[4]{a+bx^2}}{bx^2+2\sqrt{a}\sqrt{a+bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
[Out] (ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a + b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2
)^(1/4))] + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2)^(1/4))/(b*x^2 + 2*Sqrt
[a]*Sqrt[a + b*x^2])])/(4*a^(3/4)*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,(b\,x^2+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
[Out] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(88) = 176.
time = 30.38, size = 337, normalized size = 2.81

$$\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^2+a}\,a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{-ab\sqrt{-\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}(bx^2+a)^{\frac{1}{4}}a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(bx^2+a)^{\frac{3}{4}}}{bx^2+2\,a}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-(bx^2+a)^{\frac{3}{4}}}{bx^2+2\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
-1/(a^3*b^2))^(3/4) + (1/4)^(1/4)*sqrt(b*x^2 + a)*a*(-1/(a^3*b^2))^(1/4))*s
qrt(-a*b*sqrt(-1/(a^3*b^2))) - (1/4)^(1/4)*(b*x^2 + a)^(1/4)*a*(-1/(a^3*b^2
))^(1/4))/x) - 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt
(b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (b*x^2 + a)^(1/4)*a^2*b*sqrt(-
1/(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (b*x^2 + a)^(3/4))/
(b*x^2 + 2*a)) + 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*s
qrt(b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/4)*a^2*b*sqr
t(-1/(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (b*x^2 + a)^(3/4
))/(b*x^2 + 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^2}\cdot(2a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4)/(b*x**2+2*a),x)
```

```
[Out] Integral(1/((a + b*x**2)**(1/4)*(2*a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{1/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

3.311 $\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] 1/2*arctan(a^(3/4)*(1-(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2+a)^(1/4)/b^(1/2))
/a^(3/4)/b^(1/2)+1/2*arctanh(a^(3/4)*(1+(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2
+a)^(1/4)/b^(1/2))/a^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{406\}$

$$\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(S
qrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.23, size = 123, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{bx^2-2\sqrt{a}\sqrt{a-bx^2}}{2\sqrt[4]{a}\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt[4]{a}\sqrt{b}\,x\sqrt[4]{a-bx^2}}{bx^2+2\sqrt{a}\sqrt{a-bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
[Out] (ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a - b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2
)^(1/4))] + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2)^(1/4))/(b*x^2 + 2*Sqrt
[a]*Sqrt[a - b*x^2])])/(4*a^(3/4)*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(-b\,x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
[Out] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(92) = 184.
time = 28.58, size = 343, normalized size = 2.77

$$\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{-bx^2+a}\,a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{ab\sqrt{-\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-bx^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(-bx^2+a\right)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^2-2\,a}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(-bx^2+a\right)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^2-2\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
-1/(a^3*b^2))^(3/4) - (1/4)^(1/4)*sqrt(-b*x^2 + a)*a*(-1/(a^3*b^2))^(1/4))*
sqrt(a*b*sqrt(-1/(a^3*b^2))) - (1/4)^(1/4)*(-b*x^2 + a)^(1/4)*a*(-1/(a^3*b^
2))^(1/4))/x) + 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sq
rt(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^2 + a)^(1/4)*a^2*b*sq
rt(-1/(a^3*b^2)) + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (-b*x^2 + a)^(3
/4))/(b*x^2 - 2*a)) - 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/
4)*sqrt(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (-b*x^2 + a)^(1/4)*a^2
*b*sqrt(-1/(a^3*b^2)) + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (-b*x^2 +
a)^(3/4))/(b*x^2 - 2*a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2a\sqrt[4]{a-bx^2}+bx^2\sqrt[4]{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - b*x**2)**(1/4) + b*x**2*(a - b*x**2)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-b\,x^2\right)}^{1/4}\,\left(2\,a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x)
```

```
[Out] int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)), x)
```

3.312 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(1
/2)/(3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 0.92

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1+3x^2}}{x}\right)-\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (ArcTan[(Sqrt[2/3]*(-1 + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x
^2)^(1/4)])/(2*Sqrt[6])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.00, size = 138, normalized size = 2.26

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{3}{4}}-3\sqrt{3x^2-1}\,x-\mathrm{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{1}{4}}+3x}{3x^2-2}\right)}{12}-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*RootOf(_Z^2+6)*ln(-(RootOf(_Z^2+6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x
-RootOf(_Z^2+6)*(3*x^2-1)^(1/4)+3*x)/(3*x^2-2))-1/12*RootOf(_Z^2-6)*ln((Roo
tOf(_Z^2-6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1)^(1
/4)+3*x)/(3*x^2-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(43) = 86.
time = 3.39, size = 104, normalized size = 1.70

$$\frac{1}{12}\sqrt{6}\,\arctan\left(\frac{\sqrt{6}\,(3\,x^2-1)^{\frac{1}{4}}}{3\,x}\right)+\frac{1}{24}\sqrt{6}\,\log\left(-\frac{9\,x^4-6\,\sqrt{6}\,(3\,x^2-1)^{\frac{1}{4}}x^3+12\,\sqrt{3\,x^2-1}\,x^2-4\,\sqrt{6}\,(3\,x^2-1)^{\frac{3}{4}}x+12\,x^2-4}{9\,x^4-12\,x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(9
*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6)
*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(3\,x^2-1)^{1/4}\,(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.313 $\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/(-3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(
1/2)/(-3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]*
x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 407

```
Int[1/((((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 0.92

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1-3x^2}}{x}\right)+\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*(-ArcTan[(Sqrt[2/3]*(-1 - 3*x^2)^(1/4))/x] + ArcTanh[(Sqrt[3/2]*x)/(-1
 - 3*x^2)^(1/4)])/Sqrt[6]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.96, size = 137, normalized size = 2.25

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\left(-3x^2-1\right)^{\frac{3}{4}}+3\sqrt{-3x^2-1}\,x-\mathrm{RootOf}\left(\_Z^2-6\right)\left(-3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2+2}\right)}{12}-\frac{\mathrm{RootOf}\left(\_Z^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*RootOf(_Z^2-6)*ln((RootOf(_Z^2-6)*(-3*x^2-1)^(3/4)+3*(-3*x^2-1)^(1/2)
*x-RootOf(_Z^2-6)*(-3*x^2-1)^(1/4)-3*x)/(3*x^2+2))-1/12*RootOf(_Z^2+6)*ln((
RootOf(_Z^2+6)*(-3*x^2-1)^(3/4)+3*(-3*x^2-1)^(1/2)*x+RootOf(_Z^2+6)*(-3*x^2
-1)^(1/4)+3*x)/(3*x^2+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.49, size = 243, normalized size = 3.98

$$-\frac{1}{24}\sqrt{6}\log\left(\frac{\sqrt{6}\sqrt{-3x^2-1}\,x-\sqrt{6}\,x+2\left(-3x^2-1\right)^{\frac{3}{4}}-2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)+\frac{1}{24}\sqrt{6}\log\left(-\frac{\sqrt{6}\sqrt{-3x^2-1}\,x-\sqrt{6}\,x-2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)+\frac{1}{24}i\sqrt{6}\log\left(\frac{i\sqrt{6}\sqrt{-3x^2-1}\,x+i\sqrt{6}\,x+2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)-\frac{1}{24}i\sqrt{6}\log\left(\frac{-i\sqrt{6}\sqrt{-3x^2-1}\,x-i\sqrt{6}\,x+2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*log(1/3*(sqrt(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x + 2*(-3*x^2 -
 1)^(3/4) - 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)) + 1/24*sqrt(6)*log(-1/3*(sqr
t(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x - 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)
^(1/4))/(3*x^2 + 2)) + 1/24*I*sqrt(6)*log(1/3*(I*sqrt(6)*sqrt(-3*x^2 - 1)*x
 + I*sqrt(6)*x + 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
- 1/24*I*sqrt(6)*log(1/3*(-I*sqrt(6)*sqrt(-3*x^2 - 1)*x - I*sqrt(6)*x + 2*(
-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2\sqrt[4]{-3x^2-1}+2\sqrt[4]{-3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)/(-3*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(3*x**2*(-3*x**2 - 1)**(1/4) + 2*(-3*x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(-3\,x^2-1\right)}^{1/4}\,\left(3\,x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)),x)
```

```
[Out] -int(1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)), x)
```

3.314 $\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\,\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\,\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)-1/4*arct
anh(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\,\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*Sqrt[b]) - A
rcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\,\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\,\sqrt{b}}$$

Mathematica [A]
time = 0.12, size = 67, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{-1+bx^2}}{\sqrt{b}\,x}\right)-\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] (ArcTan[(Sqrt[2]*(-1 + b*x^2)^(1/4))/(Sqrt[b]*x)] - ArcTanh[(Sqrt[b]*x)/(Sq
rt[2]*(-1 + b*x^2)^(1/4))])/(2*Sqrt[2]*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}-2\right)\left(b\,x^{2}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(55) = 110$.
time = 7.38, size = 274, normalized size = 3.56

$$\left[\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,(bx^{2}-1)^{\frac{1}{4}}}{\sqrt{b}\,x}\right)+\sqrt{2}\,\sqrt{b}\,\log\left(-\frac{b^{2}x^{4}-2\,\sqrt{2}\,(bx^{2}-1)^{\frac{1}{4}}b^{\frac{3}{2}}x^{3}+4\,\sqrt{bx^{2}-1}\,bx^{2}+4\,bx^{2}-4\,\sqrt{2}\,(bx^{2}-1)^{\frac{3}{4}}\sqrt{b}\,x-4}{b^{2}x^{4}-4\,bx^{2}+4}\right)}{8\,b},\ \frac{2\,\sqrt{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\,(bx^{2}-1)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\sqrt{2}\,\sqrt{-b}\,\log\left(-\frac{b^{2}x^{4}+2\,\sqrt{2}\,(bx^{2}-1)^{\frac{1}{4}}\sqrt{-b}\,bx^{3}-4\,\sqrt{bx^{2}-1}\,bx^{2}+4\,bx^{2}-4\,\sqrt{2}\,(bx^{2}-1)^{\frac{3}{4}}\sqrt{-b}\,x-4}{b^{2}x^{4}-4\,bx^{2}+4}\right)}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sqr
t(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*sq
rt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x - 4)/
(b^2*x^4 - 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*x^2
- 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 + 2*sqrt(2)*(b*
x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(
2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^2-2)\sqrt[4]{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2)/(b*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((b*x**2 - 2)*(b*x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2-1\right)}^{1/4}\,\left(b\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)),x)
```

```
[Out] int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

3.315 $\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)-1/4*arc
tanh(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*Sqrt[b]) - A
rcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.87

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1-bx^2}}{\sqrt{b}\,x}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] -1/2*(-ArcTan[(Sqrt[2]*(-1 - b*x^2)^(1/4))/(Sqrt[b]*x)] + ArcTanh[(Sqrt[b]*
x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))])/(Sqrt[2]*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}-2\right)\left(-b\,x^{2}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(57) = 114.
time = 6.82, size = 273, normalized size = 3.46

$$\left[\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,\left(-bx^{2}-1\right)^{\frac{1}{4}}}{\sqrt{b}\,x}\right)+\sqrt{2}\,\sqrt{b}\,\log\left(-\frac{b^{2}x^{4}+4\,\sqrt{-bx^{2}-1}\,bx^{2}-4\,bx^{2}-2\,\sqrt{2}\,\left(\left(-bx^{2}-1\right)^{\frac{1}{4}}bx^{3}+2\left(-bx^{2}-1\right)^{\frac{3}{4}}x\right)\sqrt{b}-4}{b^{2}x^{4}+4\,bx^{2}+4}\right)}{8\,b},\ \frac{2\,\sqrt{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\,\left(-bx^{2}-1\right)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\sqrt{2}\,\sqrt{-b}\,\log\left(-\frac{b^{2}x^{4}-4\,\sqrt{-bx^{2}-1}\,bx^{2}-4\,bx^{2}+2\,\sqrt{2}\,\left(\left(-bx^{2}-1\right)^{\frac{1}{4}}bx^{3}-2\left(-bx^{2}-1\right)^{\frac{3}{4}}x\right)\sqrt{-b}-4}{b^{2}x^{4}+4\,bx^{2}+4}\right)}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sq
rt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(2
)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*x^4
 + 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1
/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 - 1)*b
*x^2 - 4*b*x^2 + 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)^(3/4)
*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{bx^2\sqrt[4]{-bx^2-1}+2\sqrt[4]{-bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2)/(-b*x**2-1)**(1/4),x)
[Out] -Integral(1/(b*x**2*(-b*x**2 - 1)**(1/4) + 2*(-b*x**2 - 1)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(-b\,x^2-1\right)}^{1/4}\,\left(b\,x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)),x)
[Out] -int(1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)), x)
```

3.316 $\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)-1/12*ar
ctanh(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{407\}$

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(Sqrt[6]*a^(3/4)) -
 ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}\,a^{3/4}}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.91

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a+3x^2}}{x}\right)-\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a+3x^2}}{x}\right)}{2\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] (ArcTan[(Sqrt[2/3]*a^(1/4)*(-a + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[2/3]*a^(1
/4)*(-a + 3*x^2)^(1/4))/x])/(2*Sqrt[6]*a^(3/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-2a\right)\left(3x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(59) = 118$.
time = 4.84, size = 276, normalized size = 3.25

$$-\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\frac{1}{a^3}^{\frac{3}{4}}+\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{3x^2-a}\,a\frac{1}{a^3}^{\frac{1}{4}}\right)\sqrt{a\sqrt{\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(3x^2-a\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x+\left(3x^2-a\right)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+\left(3x^2-a\right)^{\frac{3}{4}}}{3x^2-2a}\right)+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x-\left(3x^2-a\right)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x-\left(3x^2-a\right)^{\frac{3}{4}}}{3x^2-2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/36)^(1/4)*(a^(-3))^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(a^(-3
))^(3/4) + (1/36)^(1/4)*sqrt(3*x^2 - a)*a*(a^(-3))^(1/4))*sqrt(a*sqrt(a^(-3
))) - (1/36)^(1/4)*(3*x^2 - a)^(1/4)*a*(a^(-3))^(1/4))/x) - 1/4*(1/36)^(1/4
)*(a^(-3))^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x
+ (3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x +
(3*x^2 - a)^(3/4))/(3*x^2 - 2*a)) + 1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*log(-(1
8*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^2 - a)^(1/4)*a^2
*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x - (3*x^2 - a)^(3/4))/(3*x
^2 - 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2*a)/(3*x**2-a)**(1/4),x)
```

```
[Out] Integral(1/((-2*a + 3*x**2)*(-a + 3*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(2\,a-3\,x^2\right)\,{\left(3\,x^2-a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)),x)
```

```
[Out] -int(1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)), x)
```

3.317 $\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)-1/12*a
rctanh(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(Sqrt[6]*a^(3/4)) -
 ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Mathematica [A]
time = 0.15, size = 77, normalized size = 0.91

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a-3x^2}}{x}\right)+\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a-3x^2}}{x}\right)}{2\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*(-ArcTan[(Sqrt[2/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x] + ArcTanh[(Sqrt[2/3
]*a^(1/4)*(-a - 3*x^2)^(1/4))/x])/(Sqrt[6]*a^(3/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2-2a\right)\left(-3x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(59) = 118.
time = 6.42, size = 278, normalized size = 3.27

$$-\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\frac{1}{a^3}^{\frac{3}{4}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{-3x^2-a}\,a\frac{1}{a^3}^{\frac{1}{4}}\right)\sqrt{-a\sqrt{\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-3x^2-a\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x+\left(-3x^2-a\right)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x-\left(-3x^2-a\right)^{\frac{3}{4}}}{3x^2+2a}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3x^2-a}\,a^2\frac{1}{a^3}^{\frac{3}{4}}x-\left(-3x^2-a\right)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+\left(-3x^2-a\right)^{\frac{3}{4}}}{3x^2+2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/36)^(1/4)*(a^(-3))^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(a^(-3
))^(3/4) - (1/36)^(1/4)*sqrt(-3*x^2 - a)*a*(a^(-3))^(1/4))*sqrt(-a*sqrt(a^(
-3))) - (1/36)^(1/4)*(-3*x^2 - a)^(1/4)*a*(a^(-3))^(1/4))/x) + 1/4*(1/36)^(
1/4)*(a^(-3))^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/
4)*x + (-3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4
)*x - (-3*x^2 - a)^(3/4))/(3*x^2 + 2*a)) - 1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*
log((18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (-3*x^2 - a)^(
1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3*x^2 - a)^(3
/4))/(3*x^2 + 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{2a\sqrt[4]{-a-3x^2}+3x^2\sqrt[4]{-a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2*a)/(-3*x**2-a)**(1/4),x)
[Out] -Integral(1/(2*a*(-a - 3*x**2)**(1/4) + 3*x**2*(-a - 3*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(3\,x^2+2\,a\right)\,{\left(-3\,x^2-a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)),x)
[Out] -int(1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)), x)
```

3.318 $\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*2^(1/2)/
b^(1/2)-1/4*arctanh(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*
2^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(3
/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2
*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.14, size = 88, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}{\sqrt{b}\,x}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] (ArcTan[(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)] - ArcTanh[(Sqrt[2
]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)])/(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}-2a\right)\left(b\,x^{2}-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(71) = 142.
time = 26.00, size = 338, normalized size = 3.35

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^2-a}\,a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{ab\sqrt{\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}(bx^2-a)^{\frac{1}{4}}a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(bx^2-a)^{\frac{3}{4}}}{bx^2-2\,a}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-(bx^2-a)^{\frac{3}{4}}}{bx^2-2\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
1/(a^3*b^2))^(3/4) + (1/4)^(1/4)*sqrt(b*x^2 - a)*a*(1/(a^3*b^2))^(1/4))*sqr
```

```
t(a*b*sqrt(1/(a^3*b^2))) - (1/4)^(1/4)*(b*x^2 - a)^(1/4)*a*(1/(a^3*b^2))^(1
/4))/x) - 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(b*x^2
 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b
^2)) + (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4))/(b*x^2 -
2*a)) + 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(b*x^2
- a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^
2)) + (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (b*x^2 - a)^(3/4))/(b*x^2 - 2
*a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2*a)/(b*x**2-a)**(1/4),x)
```

```
[Out] Integral(1/((-2*a + b*x**2)*(-a + b*x**2)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(b\,x^2-a\right)}^{1/4}\,\left(2\,a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)),x)
```

```
[Out] -int(1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)), x)
```

3.319 $\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*2^(1/2)
/b^(1/2)-1/4*arctanh(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(3/4
)*2^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(3
/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2
*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.13, size = 90, normalized size = 0.87

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}\,x}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}\,x}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] -1/2*(-ArcTan[(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)] + ArcTanh[(
Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)*Sqrt[b])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2-2a\right)\left(-b\,x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(73) = 146.
time = 25.81, size = 350, normalized size = 3.40

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{-bx^2-a}\,a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)\sqrt{-ab\sqrt{\frac{1}{a^3b^2}}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-bx^2-a\right)^{\frac{1}{4}}a\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+\left(-bx^2-a\right)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-\left(-bx^2-a\right)^{\frac{3}{4}}}{bx^2+2\,a}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}\,a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-\left(-bx^2-a\right)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+\left(-bx^2-a\right)^{\frac{3}{4}}}{bx^2+2\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a^3*b*(
1/(a^3*b^2))^(3/4) - (1/4)^(1/4)*sqrt(-b*x^2 - a)*a*(1/(a^3*b^2))^(1/4))*sq
```

```
rt(-a*b*sqrt(1/(a^3*b^2))) - (1/4)^(1/4)*(-b*x^2 - a)^(1/4)*a*(1/(a^3*b^2))
^(1/4))/x) + 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(-
b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1/
(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (-b*x^2 - a)^(3/4))/(b
*x^2 + 2*a)) - 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(
-b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1
/(a^3*b^2)) - (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2 - a)^(3/4))/(
b*x^2 + 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{2a\sqrt[4]{-a-bx^2}+bx^2\sqrt[4]{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2*a)/(-b*x**2-a)**(1/4),x)
[Out] -Integral(1/(2*a*(-a - b*x**2)**(1/4) + b*x**2*(-a - b*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(-b\,x^2-a\right)}^{1/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)),x)
[Out] -int(1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

3.320 $\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*arctanh(1/2*x/(x^2-1)^(
1/4)*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {407}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(-1 +
 x^2)^(1/4))]/(2*Sqrt[2])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

Mathematica [A]

time = 0.08, size = 48, normalized size = 0.91

$$\frac{-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1+x^2}}{x}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (-ArcTan[(Sqrt[2]*(-1 + x^2)^(1/4))/x] + ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4
))])/(2*Sqrt[2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.11, size = 121, normalized size = 2.28

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{3}{4}}-\sqrt{x^2-1}\,x+\mathrm{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{1}{4}}-x}{x^2-2}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+2)/(x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*RootOf(_Z^2-2)*ln(-(RootOf(_Z^2-2)*(x^2-1)^(3/4)-(x^2-1)^(1/2)*x+RootO
f(_Z^2-2)*(x^2-1)^(1/4)-x)/(x^2-2))+1/4*RootOf(_Z^2+2)*ln((RootOf(_Z^2+2)*(
x^2-1)^(3/4)+(x^2-1)^(1/2)*x-RootOf(_Z^2+2)*(x^2-1)^(1/4)-x)/(x^2-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(39) = 78.
time = 3.42, size = 91, normalized size = 1.72

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{x^4+2\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^2-1}\,x^2+4\sqrt{2}\left(x^2-1\right)^{\frac{3}{4}}x+4x^2-4}{x^4-4x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) + 1/8*sqrt(2)*log(-(x^4 + 2*
sqrt(2)*(x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^2 - 1)*x^2 + 4*sqrt(2)*(x^2 - 1)^(3/
4)*x + 4*x^2 - 4)/(x^4 - 4*x^2 + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+2)/(x**2-1)**(1/4),x)
[Out] -Integral(1/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(x^2-1\right)}^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
[Out] -int(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.321 $\int \frac{(a+bx^2)^{7/4}}{c+dx^2} dx$

Optimal. Leaf size=362

$$\frac{6abx}{5d\sqrt[4]{a+bx^2}}-\frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}}+\frac{2bx(a+bx^2)^{3/4}}{5d}-\frac{6a^{3/2}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}}+\frac{2\sqrt{a}\sqrt{b}(b$$

```
[Out] 6/5*a*b*x/d/(b*x^2+a)^(1/4)-2*b*(-a*d+b*c)*x/d^2/(b*x^2+a)^(1/4)+2/5*b*x*(b
*x^2+a)^(3/4)/d-6/5*a^(3/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan
(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/d/(b*x^2+a)^(1/4)+2*(-a*d+b*c)*(1+b*x
^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1
/2)*b^(1/2)/d^2/(b*x^2+a)^(1/4)+a^(1/4)*(a*d-b*c)^(3/2)*EllipticPi((b*x^2+a
)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^(5/2
)/x-a^(1/4)*(a*d-b*c)^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1
/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^(5/2)/x
```

Rubi [A]
time = 0.19, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {411, 201, 235, 233, 202, 408, 504, 1232}

$$-\frac{6a^{3/2}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x}+\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(bc-ad)E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}}-\frac{2bx(bc-ad)}{d^2\sqrt[4]{a+bx^2}}+\frac{6abx}{5d\sqrt[4]{a+bx^2}}+\frac{2bx(a+bx^2)^{3/4}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

```
[Out] (6*a*b*x)/(5*d*(a + b*x^2)^(1/4)) - (2*b*(b*c - a*d)*x)/(d^2*(a + b*x^2)^(1
/4)) + (2*b*x*(a + b*x^2)^(3/4))/(5*d) - (6*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)
^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*d*(a + b*x^2)^(1/4))
 + (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*(-(b*c) + a*d)
^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]
), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x) - (a^(1/4)*(-(b*c) +
 a*d)^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
```

```
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \frac{b\int (a+bx^2)^{3/4}\,dx}{d} - \frac{(bc-ad)\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx}{d}$$

$$= \frac{2bx(a+bx^2)^{3/4}}{5d} + \frac{(3ab)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5d} - \frac{(b(bc-ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{d^2} + \frac{(bc-ad)^2 \cdots}{\cdots}$$

$$= \frac{2bx(a+bx^2)^{3/4}}{5d} + \frac{\left(2(bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+\cdots}\right.}{d^2x}$$

$$= \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} + \frac{2bx(a+bx^2)^{3/4}}{5d} - \frac{\left((bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} + \frac{2bx(a+bx^2)^{3/4}}{5d} - \frac{6a^{3/2}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\cdots\right.\right.}{5d\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.33, size = 346, normalized size = 0.96

$$\frac{x\left(\frac{b(-5bc+8ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(-3ac(5a^2d+2abdx^2+2b^2x^2(c+dx^2))F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2(a+bx^2)(c+dx^2)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{15d\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

```
[Out] (x*((b*(-5*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(5*a^2*d + 2*a*b*d*x^2 + 2*b^
2*x^2*(c + d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/
a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/
c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
```

```
+ b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(15*d*(a
+ b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{7}{4}}}{d\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/4)/(d*x^2+c),x)
[Out] int((b*x^2+a)^(7/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{\frac{7}{4}}}{c+dx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/4)/(d*x**2+c),x)
[Out] Integral((a + b*x**2)**(7/4)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{7/4}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(7/4)/(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(7/4)/(c + d*x^2), x)
```

3.322 $\int \frac{(a+bx^2)^{5/4}}{c+dx^2} \, dx$

Optimal. Leaf size=302

$$\frac{2bx\sqrt[4]{a+bx^2}}{3d}+\frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{a}\,\sqrt{b}\,(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\right.}{d^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/3*b*x*(b*x^2+a)^(1/4)/d+2/3*a^(3/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/d/(b*x^2+a)^(3/4)-2*(-a*d+b
*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*
arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(
1/2))*a^(1/2)*b^(1/2)/d^2/(b*x^2+a)^(3/4)+a^(1/4)*(-a*d+b*c)*EllipticPi((b*
x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d
^2/x+a^(1/4)*(-a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/
(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^2/x
```

Rubi [A]
time = 0.14, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {411, 201, 239, 237, 410, 109, 418, 1232}

$$\frac{2a^{3/2}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(bc-ad)F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d^2\left(a+bx^2\right)^{3/4}}+\frac{2bx\sqrt[4]{a+bx^2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (2*b*x*(a + b*x^2)^(1/4))/(3*d) + (2*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*
EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*d*(a + b*x^2)^(3/4)) - (2*S
qrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(3/4)) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b
*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b
*x^2)^(1/4)/a^(1/4)], -1])/(d^2*x) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/(d^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \frac{b\int \sqrt[4]{a+bx^2}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx}{d}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{(ab)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{3d} - \frac{(b(bc-ad))\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{}{(a+bx^2}}{d^2}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{\left((bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2d^2x} + -$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3d\,(a+bx^2)^{3/4}} - \frac{2\sqrt{a}\,\sqrt{b}\,(bc-ad)}{}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3d\,(a+bx^2)^{3/4}} - \frac{2\sqrt{a}\,\sqrt{b}\,(bc-ad)}{}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3d\,(a+bx^2)^{3/4}} - \frac{2\sqrt{a}\,\sqrt{b}\,(bc-ad)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.82, size = 348, normalized size = 1.15

$$\frac{x\left(\frac{b(-3bc+4ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(-3ac(3a^2d+2abdx^2+2b^2x^2(c+dx^2))F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2(a+bx^2)(c+dx^2)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{9d\,(a+bx^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (x*((b*(-3*b*c + 4*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(3*a^2*d + 2*a*b*d*x^2 + 2*b^
2*x^2*(c + d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/
a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2
```

```
)/c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(9*d
*(a + b*x^2)^(3/4))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}{d\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/4)/(d*x^2+c),x)
[Out] int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{\frac{5}{4}}}{c+dx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/4)/(d*x**2+c),x)
[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{5/4}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/4)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(5/4)/(c + d*x^2), x)
```

3.323 $\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx$

Optimal. Leaf size=244

$$\frac{2bx}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\,\sqrt{-bc+ad}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\mathrm{s}\right.}{d^{3/2}x}$$

```
[Out] 2*b*x/d/(b*x^2+a)^(1/4)-2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x
*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/d/(b*x^2+a)^(1/4)+a^(1/4)*Ellip
ticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(a*d-b*c)
^(1/2)*(-b*x^2/a)^(1/2)/d^(3/2)/x-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4
),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/d^(3/
2)/x
```

Rubi [A]
time = 0.11, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {411, 235, 233, 202, 408, 504, 1232}

$$\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\sqrt{ad-bc}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\sqrt{ad-bc}\,\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}-\frac{2\sqrt{a}\,\sqrt{b}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}}+\frac{2bx}{d\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

```
[Out] (2*b*x)/(d*(a + b*x^2)^(1/4)) - (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*El
lipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(1/4)) + (a^(1/4)
*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x) - (a^(1
/4)*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt
[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx}{d}$$

$$= -\frac{\left(2(bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{dx} + \frac{\left(b\sqrt[4]{1 \right.}{}$$

$$= \frac{2bx}{d\sqrt[4]{a+bx^2}} + \frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}\,x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \right.}{d^{3/2}x}$$

$$= \frac{2bx}{d\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\,\sqrt{-bc+ad}\,\sqrt{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.79, size = 161, normalized size = 0.66

$$\frac{6acx(a+bx^2)^{3/4}F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(6acF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(-4adF_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(3/4)*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)
/c)] + 3*b*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(3/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{4}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/(d*x**2+c),x)
[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{3/4}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(3/4)/(c + d*x^2), x)
```

3.324 $\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$

Optimal. Leaf size=199

$$\frac{2\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right.}{dx}$$

```
[Out] 2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)*b^(1/2)/d/(b*x^2+a)^(3/4)-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^
(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d/x-a^(1/4)*Elli
pticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a
)^(1/2)/d/x
```

Rubi [A]
time = 0.10, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {411, 239, 237, 410, 109, 418, 1232}

$$-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}+\frac{2\sqrt{a}\,\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(d*(a + b*x^2)^(3/4)) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(d*x) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{d}$$

$$= -\frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}\,(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2dx} + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)}{d\,(a+b}$$

$$= \frac{2\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\,(a+bx^2)^{3/4}} + \frac{\left(2(bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{}}\right.}{d}$$

$$= \frac{2\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\,(a+bx^2)^{3/4}} - \frac{\sqrt{-\frac{bx^2}{a}}\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{-bc+ad}}\right)}\right.}{dx}$$

$$= \frac{2\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{d\,(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-}\right.}{dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 7.41, size = 160, normalized size = 0.80

$$\frac{6acx\sqrt[4]{a+bx^2}\,F_1\left(\frac{1}{2};-\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(6acF_1\left(\frac{1}{2};-\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(-4adF_1\left(\frac{3}{2};-\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(1/4)*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)
/c)] + b*c*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a + bx^2}}{c + dx^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{d\,x^2+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(1/4)/(c + d*x^2), x)
```

3.325 $\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx$

Optimal. Leaf size=167

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\sqrt{-bc+ad}\,x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right.\right.}{\sqrt{d}\sqrt{-bc+ad}\,x}$$

```
[Out] a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2)
,I)*(-b*x^2/a)^(1/2)/x/d^(1/2)/(a*d-b*c)^(1/2)-a^(1/4)*EllipticPi((b*x^2+a)
^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/x/d^(1/2
)/(a*d-b*c)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {408, 504, 1232}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\,x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\,x\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
 - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \frac{\left(2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{x}$$

$$= -\frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}\,x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{\sqrt{d}\,x} + \cdots$$

$$= \frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\,\sqrt{-bc+ad}\,x} - \frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\Pi \cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.99, size = 160, normalized size = 0.96

$$-\frac{6acxF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt[4]{a+bx^2}\,(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((a + b*x
^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a + bx^2} \, (c + dx^2)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)^{1/4}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)), x)
```

3.326 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)x}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)x}$$

```
[Out] a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2)
,I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)/x+a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4
),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)/x
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {410, 109, 418, 1232}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x) + (a^(1/4)*Sqr
t[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
```

```
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}\,(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2x}$$

$$= -\frac{\left(2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\,(-bc+ad-dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{x}$$

$$= \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} + \frac{\sqrt{-\frac{bx}{a}}}{}$$

$$= \frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)x} + \frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}}{}$$

Mathematica [A]
time = 7.60, size = 120, normalized size = 0.79

$$\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\left(\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)+\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{(bc-ad)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*(EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1] + EllipticPi[(Sqrt[a]*Sqrt[d])
/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]))/((b*c - a*d)*
x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{4}} \left(d\,x^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{2}\right)^{\frac{3}{4}} \left(c + dx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/4}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)), x)
```

3.327 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)} \, dx$

Optimal. Leaf size=233

$$\frac{2\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(bc-ad)\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(-bc+ad)^{3/2}x}-$$

```
[Out] 2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))*b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/4)/a^(1/2)+a^(1/4)*EllipticPi((b*x^2+a)
^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(1/2)
/(a*d-b*c)^(3/2)/x-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/
2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(3/2)/x
```

Rubi [A]
time = 0.11, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {412, 203, 202, 408, 504, 1232}

$$\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}+\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

```
[Out] (2*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[a]*(b*c - a*d)*(a + b*x^2)^(1/4)) + (a^(1/4)*Sqrt[d]*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x) - (a^(1/4)*Sqrt[d]*Sqrt[-((
b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x
^2)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \frac{b\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{bc-ad} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx}{bc-ad}$$

$$= -\frac{\left(2d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} + \frac{\left(b\sqrt[4]{1\right.}{a(}$$

$$= \frac{2\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\left(\sqrt{d}\,\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \right.}{\left(\sqrt{-\right.}$$

$$= \frac{2\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\,\sqrt{d}\,\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+a}}\right.}{(-bc+a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.53, size = 327, normalized size = 1.40

$$\frac{x\left(\frac{bdx^2\sqrt[4]{1+\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} + \frac{6\left(3ac(ad-b(c+2dx^2))F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2(c+dx^2)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{3a(-bc+ad)\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

```
[Out] (x*((b*d*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/c + (6*(3*a*c*(a*d - b*(c + 2*d*x^2))*AppellF1[1/2, 1/4, 1,
 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*AppellF1[3/2,
1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -
((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*
x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*
x^2)/c)])))))/(3*a*(-(b*c) + a*d)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{4}}\,(d\,x^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{2}\right)^{\frac{5}{4}} \left(c + dx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{5/4}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)), x)
```

3.328 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$

Optimal. Leaf size=254

$$\frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,(bc-ad)\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\,d\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\mathrm{si}\right.}{(bc-ad)^2x}$$

```
[Out] 2/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/4)+2/3*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(
x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(s
in(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(3/
4)/a^(1/2)-a^(1/4)*d*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a
*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)^2/x-a^(1/4)*d*EllipticPi((b*x^
2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*
d+b*c)^2/x
```

Rubi [A]
time = 0.11, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {412, 205, 239, 237, 410, 109, 418, 1232}

$$-\frac{\sqrt[4]{a}\,d\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}-\frac{\sqrt[4]{a}\,d\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}+\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\left(a+bx^2\right)^{3/4}(bc-ad)}+\frac{2bx}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(1 + (b*x^2)/a)^(3
/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*(b*c - a*d)*(a
+ b*x^2)^(3/4)) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)^2*x) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx = \frac{b\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx}{bc-ad} - \frac{d\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{bc-ad}$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{3a(bc-ad)} - \frac{\left(d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{\quad}{\sqrt{-\frac{b}{\quad}}}\right.}{2(bc-\quad}$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{\left(2d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\,(-bc+ad-dx^4)}\,d\right.}{(bc-ad)x}$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,(bc-ad)\left(a+bx^2\right)^{3/4}} - \frac{(}{\quad}$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,(bc-ad)\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{\quad}}{\quad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.77, size = 331, normalized size = 1.30

$$\frac{x\left(-\frac{bdx^2\left(1+\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(3ac(-3bc+3ad-2bdx^2)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2(c+dx^2)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{9a(-bc+ad)\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

```
[Out] (x*(-((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a
), -((d*x^2)/c)])/c) + (6*(3*a*c*(-3*b*c + 3*a*d - 2*b*d*x^2)*AppellF1[1/2,
 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*Appell
F1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4,
 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 3
/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/4, 2,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^
2)/a), -((d*x^2)/c)])))))/(9*a*(-(b*c) + a*d)*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{4}} \left(d\,x^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{2}\right)^{\frac{7}{4}} \left(c + dx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c),x)
[Out] Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{7/4}\,\left(d\,x^2+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)), x)
```

3.329 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx$

Optimal. Leaf size=274

$$\frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}}+\frac{2\sqrt{b}\,(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}(bc-ad)^2\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\,d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\Pi\Big(-\frac{\sqrt{}}{\sqrt{-}}}{(-}$$

```
[Out] 2/5*b*x/a/(-a*d+b*c)/(b*x^2+a)^(5/4)+2/5*(-8*a*d+3*b*c)*(1+b*x^2/a)^(1/4)*(
cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2
)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(
-a*d+b*c)^2/(b*x^2+a)^(1/4)+a^(1/4)*d^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1
/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(5/2)/x-
a^(1/4)*d^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c
)^(1/2),I)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(5/2)/x
```

Rubi [A]
time = 0.26, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 541, 544, 235, 233, 202, 408, 504, 1232}

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,(3bc-8ad)E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^2}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}-\frac{\sqrt[4]{a}\,d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}+\frac{2bx}{5a\,(a+bx^2)^{5/4}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + (2*Sqrt[b]*(3*b*c - 8*a*d)*(1
 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)
*(b*c - a*d)^2*(a + b*x^2)^(1/4)) + (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x) - (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} - \frac{2\int \frac{\frac{1}{2}(-3bc+5ad)-\frac{3}{2}bdx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx}{5a(bc-ad)}$$

$$= \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{4\int \frac{\frac{1}{4}(-3b^2c^2+8abcd+5a^2d^2)}{\sqrt[4]{a+bx^2}\,(}}{5a^2(bc-}$$

$$= \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,d}{(bc-ad)^2}$$

$$= \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{\left(2d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}}{}$$

$$= \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} - \frac{\left(d^{3/2}\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}\,x^2\right)\sqrt{}}\right.}{(bc-ad)^2x}$$

$$= \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} + \frac{2\sqrt{b}\,(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right|}{5a^{3/2}(bc-ad)^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.50, size = 419, normalized size = 1.53

$$\frac{x\left(\frac{bd(-3bc+8ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}-\frac{6\left(3ac\left(5a^3d^2+3b^3cx^2(c+2dx^2)-a^2bd(10c+13dx^2)+ab^2(5c^2-16d^2x^4)\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2(c+dx^2)(9a^2d-3b^2cx^2-4ab(c-2dx^2))\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(a+bx^2)(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{15a^2(bc-ad)^2\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

```
[Out] (x*((b*d*(-3*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5
/2, -((b*x^2)/a), -((d*x^2)/c)])/c - (6*(3*a*c*(5*a^3*d^2 + 3*b^3*c*x^2*(c
+ 2*d*x^2) - a^2*b*d*(10*c + 13*d*x^2) + a*b^2*(5*c^2 - 16*d^2*x^4))*Appell
F1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(9*a^2
*d - 3*b^2*c*x^2 - 4*a*b*(c - 2*d*x^2))*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x
^2)/c)])))))/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{9}{4}} \left(d\,x^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{9}{4}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{9/4}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)), x)
```

3.330 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx$

Optimal. Leaf size=304

$$\frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}}+\frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt{b}\,(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right.}{21a^{3/2}(bc-ad)^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/7*b*x/a/(-a*d+b*c)/(b*x^2+a)^(7/4)+2/21*b*(-12*a*d+5*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^2+a)^(3/4)+2/21*(-12*a*d+5*b*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(
x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(s
in(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(-a*d+b*c)^2/(b*
x^2+a)^(3/4)+a^(1/4)*d^2*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2
)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)^3/x+a^(1/4)*d^2*EllipticPi
((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2
)/(-a*d+b*c)^3/x
```

Rubi [A]
time = 0.24, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 541, 544, 239, 237, 410, 109, 418, 1232}

$$\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{2bx(5bc-12ad)}{21a^2\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{\sqrt[4]{a}\,d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^3}+\frac{\sqrt[4]{a}\,d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^3}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + (2*b*(5*b*c - 12*a*d)*x)/(21*
a^2*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(5*b*c - 12*a*d)*(1 + (b*
x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*(b*c
 - a*d)^2*(a + b*x^2)^(3/4)) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/((b*c - a*d)^3*x) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a
]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*
c - a*d)^3*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} - \frac{2\int \frac{\frac{1}{2}(-5bc+7ad)-\frac{5}{2}bdx^2}{(a+bx^2)^{7/4}(c+dx^2)}\,dx}{7a(bc-ad)}$$

$$= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} + \frac{4\int \frac{\frac{1}{4}(5b^2c^2-12abcd+\ldots}{(a+b\ldots}}{21a^2\ldots}$$

$$= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} + \frac{d^2\int \frac{1}{(a+bx^2)^{3/4}(c+d\ldots}}{(bc-ad)^2}$$

$$= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} + \frac{\left(d^2\sqrt{-\frac{bx^2}{a}}\right)\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} + \frac{2\sqrt{b}\,(5bc-12ad\ldots}{21a\ldots}$$

$$= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} + \frac{2\sqrt{b}\,(5bc-12ad\ldots}{21a\ldots}$$

$$= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} + \frac{2\sqrt{b}\,(5bc-12ad\ldots}{21a\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.53, size = 431, normalized size = 1.42

$$-\frac{x\left(\frac{bd(-5bc+12ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(3ac\left(21a^3d^2+5b^3cx^2(3c+2dx^2)-3a^2bd(14c+3dx^2)+ab^2(21c^2-20cdx^2-24d^2x^4)\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2(c+dx^2)(15a^2d-5b^2cx^2+ab(-8c+12dx^2))\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(a+bx^2)(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{63a^2(bc-ad)^2(a+bx^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

```
[Out] -1/63*(x*((b*d*(-5*b*c + 12*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/
4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(3*a*c*(21*a^3*d^2 + 5*b^3*c
*x^2*(3*c + 2*d*x^2) - 3*a^2*b*d*(14*c + 3*d*x^2) + a*b^2*(21*c^2 - 20*c*d*
x^2 - 24*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 b*x^2*(c + d*x^2)*(15*a^2*d - 5*b^2*c*x^2 + a*b*(-8*c + 12*d*x^2))*(4*a*d*
AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2
, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(-6*
a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*App
ellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7
/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(a^2*(b*c - a*d)^2*(a + b*x^2)
^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{11}{4}}\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{\frac{11}{4}}\,(c+dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(11/4)*(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{11/4}\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)), x)
```

3.331 $\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=340

$$\frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}}-\frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)}-\frac{\sqrt{a}\,\sqrt{b}\,(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\,\sqrt{-bc}}{}$$

```
[Out] 1/2*b*(-a*d+5*b*c)*x/c/d^2/(b*x^2+a)^(1/4)-1/2*(-a*d+b*c)*x*(b*x^2+a)^(3/4)
/c/d/(d*x^2+c)-1/2*(-a*d+5*b*c)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*ar
ctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/d^2/(b*x^2+a)^(1/4)+1/4
*a^(1/4)*(2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/
(a*d-b*c)^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/c/d^(5/2)/x-1/4*a^(1/4)
*(2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)
^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/c/d^(5/2)/x
```

Rubi [A]
time = 0.20, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {424, 544, 235, 233, 202, 408, 504, 1232}

$$\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\sqrt{ad-bc}\,(2ad+5bc)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x}-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,\sqrt{ad-bc}\,(2ad+5bc)\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x}-\frac{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{\frac{bx^2}{a}+1}\,(5bc-ad)E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}}+\frac{bx(5bc-ad)}{2cd^2\sqrt[4]{a+bx^2}}-\frac{x(a+bx^2)^{3/4}(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

```
[Out] (b*(5*b*c - a*d)*x)/(2*c*d^2*(a + b*x^2)^(1/4)) - ((b*c - a*d)*x*(a + b*x^2
)^(3/4))/(2*c*d*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(5*b*c - a*d)*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d^2*(a + b*x^2)^
(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/(4*c*d^(5/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)
*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin
[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(5/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} + \frac{\int \frac{a(bc+ad)+\frac{1}{2}b(5bc-ad)x^2}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx}{2cd}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} + \frac{(b(5bc-ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4cd^2} - \frac{((bc-ad)(5bc+2ad))\int \frac{\ }{\sqrt[4]{\ }}}{4cd^2}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} - \frac{\left((bc-ad)(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad}\right.}{2cd^2x}$$

$$= \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} + \frac{\left((bc-ad)(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} - \frac{\sqrt{a}\,\sqrt{b}\,(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\right.\right.}{2cd^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.24, size = 340, normalized size = 1.00

$$\frac{x\left(-b(-5bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{6c\left(-6ac(2a^2d-b^2cx^2+abdx^2)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+(-bc+ad)x^2(a+bx^2)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{12c^2d\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(-(b*(-5*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2,
 -((b*x^2)/a), -((d*x^2)/c)]) + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d*x
^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a*d)
*x^2*(a + b*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/
c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d
*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*
(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF
1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))))/(12*c^2*d*(a + b*x^2)^
(1/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{7}{4}}}{\left(d\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{\frac{7}{4}}}{\left(c+dx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(7/4)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{7/4}}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(7/4)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(7/4)/(c + d*x^2)^2, x)
```

3.332 $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=279

$$-\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)}+\frac{\sqrt{a}\,\sqrt{b}\,(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\,(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a}\,(3bc+2ad)\sqrt{-\frac{bx^2}{a}}}{}$$

```
[Out] -1/2*(-a*d+b*c)*x*(b*x^2+a)^(1/4)/c/d/(d*x^2+c)+1/2*(a*d+3*b*c)*(1+b*x^2/a)
^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b
^(1/2)/c/d^2/(b*x^2+a)^(3/4)-1/4*a^(1/4)*(2*a*d+3*b*c)*EllipticPi((b*x^2+a)
^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^2/x
-1/4*a^(1/4)*(2*a*d+3*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/
2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^2/x
```

Rubi [A]

time = 0.18, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {424, 544, 239, 237, 410, 109, 418, 1232}

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(2ad+3bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(2ad+3bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}+\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\,(a+bx^2)^{3/4}}-\frac{x\sqrt[4]{a+bx^2}\,(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x*(a + b*x^2)^(1/4))/(c*d*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]
*(3*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(2*c*d^2*(a + b*x^2)^(3/4)) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*d^2*x) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(
1/4)/a^(1/4)], -1])/(4*c*d^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)} + \frac{\int \frac{a(bc+ad)+\frac{1}{2}b(3bc+ad)x^2}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{2cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)} + \frac{(b(3bc+ad))\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{4cd^2} - \frac{((bc-ad)(3bc+2ad))\int \frac{}{(a+\ldots}}{4cd^2}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)} - \frac{\left((bc-ad)(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}\,(a+bx)^{3/4}\ldots}\right.}{8cd^2x}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)} + \frac{\sqrt{a}\,\sqrt{b}\,(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\,(a+bx^2)^{3/4}} + \ldots$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)} + \frac{\sqrt{a}\,\sqrt{b}\,(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\,(a+bx^2)^{3/4}} - \ldots$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)} + \frac{\sqrt{a}\,\sqrt{b}\,(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\,(a+bx^2)^{3/4}} - \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.24, size = 341, normalized size = 1.22

$$\frac{x\left(b(3bc+ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{6c\left(-6ac(2a^2d-b^2cx^2+abdx^2)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+(-bc+ad)x^2(a+bx^2)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{12c^2d\,(a+bx^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(b*(3*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -(
(b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d*x^2)*
AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a*d)*x^2
*(a + b*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
+ 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x
```

```
^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF
1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*d*(a + b*x^2)^
(3/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{4}}}{(d\,x^{2}+c)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
[Out] int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^{2})^{\frac{5}{4}}}{(c+dx^{2})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{5/4}}{{\left(d\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/4)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(5/4)/(c + d*x^2)^2, x)
```

3.333 $\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=309

$$-\frac{bx}{2cd\sqrt[4]{a+bx^2}}+\frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}+\frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\,(bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\Big(-}{4cd}$$

```
[Out] -1/2*b*x/c/d/(b*x^2+a)^(1/4)+1/2*x*(b*x^2+a)^(3/4)/c/(d*x^2+c)+1/2*(1+b*x^2
/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2
)*b^(1/2)/c/d/(b*x^2+a)^(1/4)+1/4*a^(1/4)*(2*a*d+b*c)*EllipticPi((b*x^2+a)^
(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^(3/2
)/x/(a*d-b*c)^(1/2)-1/4*a^(1/4)*(2*a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1
/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^(3/2)/x/(a*d-b*
c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {423, 544, 235, 233, 202, 408, 504, 1232}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}+\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}}-\frac{bx}{2cd\sqrt[4]{a+bx^2}}+\frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*(b*x)/(c*d*(a + b*x^2)^(1/4)) + (x*(a + b*x^2)^(3/4))/(2*c*(c + d*x^2)
) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(2*c*d*(a + b*x^2)^(1/4)) + (a^(1/4)*(b*c + 2*a*d)*Sqrt[-((b*x
^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x
^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x) - (a^(1/4)*(b*c
 + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d
], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*
x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/4}}{\left(c+dx^2\right)^2}\,dx = \frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)} - \frac{\int \frac{-a+\frac{bx^2}{2}}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx}{2c}$$

$$= \frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)} - \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4cd} + \frac{(bc+2ad)\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx}{4cd}$$

$$= \frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)} + \frac{\left((bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right.}{2cdx}$$

$$= -\frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)} - \frac{\left((bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{c\right.}}\right.}{4cd^{3/2}x}$$

$$= -\frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)} + \frac{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{2cd\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.15, size = 232, normalized size = 0.75

$$\frac{x\left(-\frac{bx^2\sqrt[4]{1+\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}+\frac{6\left(\frac{a+bx^2}{c}-\frac{6a^2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}\right)}{c+dx^2}\right)}{12\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(-((b*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/c^2) + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)
/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -(
(d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
)/(c + d*x^2)))/(12*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{\left(d\,x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
[Out] int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2}\right)^{\frac{3}{4}}}{\left(c + dx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{3/4}}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(3/4)/(c + d*x^2)^2, x)
```

3.334 $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=278

$$\frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\,(bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-bc+ad}};\mathrm{si}\right.}{4cd(bc-ad)x}$$

```
[Out] 1/2*x*(b*x^2+a)^(1/4)/c/(d*x^2+c)+1/2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/d/(b*x^2+a)^(3/4)
-1/4*a^(1/4)*(-2*a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/
2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d/(-a*d+b*c)/x-1/4*a^(1/4)*(-2*a*d
+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)
*(-b*x^2/a)^(1/2)/c/d/(-a*d+b*c)/x
```

Rubi [A]
time = 0.14, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {423, 544, 239, 237, 410, 109, 418, 1232}

$$-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,(bc-2ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,(bc-2ad)\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}+\frac{\sqrt{a}\,\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^(1/4))/(2*c*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^
(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d*(a + b*x^2)^(3/4)
) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c
- a*d)*x) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*S
qrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*
(b*c - a*d)*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)} - \frac{\int \frac{-a-\frac{bx^2}{2}}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{2c}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)} + \frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{4cd} - \frac{\left(\frac{bc}{2}-ad\right)\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{2cd}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)} - \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}\,(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{4cdx} + \frac{\left(b\left(\right.\right.}{}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)} + \frac{\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd\,(a+bx^2)^{3/4}} + \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)}{}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)} + \frac{\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd\,(a+bx^2)^{3/4}} - \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)}{}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)} + \frac{\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2cd\,(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}\,(bc-2ad)\sqrt{-\frac{bx^2}{a}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.16, size = 232, normalized size = 0.83

$$\frac{x\left(\frac{bx^2\left(1+\frac{bx^2}{a}\right)^{3/4} F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2} + \frac{6\left(\frac{a+bx^2}{c} - \frac{6a^2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}\right)}{c+dx^2}\right)}{12\,(a+bx^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*((b*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])/c^2 + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 3/4, 1, 3/2, -
```

```
((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a)
, -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*
x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
/(c + d*x^2)))/(12*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(d\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx^{2}}}{\left(c+dx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(1/4)/(c + d*x^2)^2, x)
```

3.335 $\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx$

Optimal. Leaf size=336

$$\frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}}-\frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)}-\frac{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{a}\,(3bc-2ad)}{}$$

```
[Out] 1/2*b*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)-1/2*d*x*(b*x^2+a)^(3/4)/c/(-a*d+b*c)/(
d*x^2+c)-1/2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/4)-1/4*a^(1/4)*(-2
*a*d+3*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(
1/2),I)*(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(3/2)/x/d^(1/2)+1/4*a^(1/4)*(-2*a*d+3*
b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*
(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(3/2)/x/d^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {425, 544, 235, 233, 202, 408, 504, 1232}

$$-\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,(3bc-2ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}\,x(ad-bc)^{3/2}}+\frac{\sqrt[4]{a}\,\sqrt{-\frac{bx^2}{a}}\,(3bc-2ad)\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}\,x(ad-bc)^{3/2}}-\frac{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2c\sqrt[4]{a+bx^2}\,(bc-ad)}+\frac{bx}{2c\sqrt[4]{a+bx^2}\,(bc-ad)}-\frac{dx(a+bx^2)^{3/4}}{2c\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (d*x*(a + b*x^2)^(3/4))/(2*c*(b
*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (a^
(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqr
t[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*Sqrt[d]*(-(b
*c) + a*d)^(3/2)*x) + (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticP
i[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = -\frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} + \frac{\int \frac{2bc-ad+\frac{1}{2}bdx^2}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} + \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4c(bc-ad)} + \frac{(3bc-2ad)\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)} \cdots}{4c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} + \frac{\left((3bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad+\cdots}\right.}{2c(bc-ad)x \cdots}$$

$$= \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} - \frac{\left((3bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Sub}\cdots}{\cdots}$$

$$= \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} - \frac{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan\cdots\right.}{2c(bc-ad)\sqrt[4]{a+\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 392, normalized size = 1.17

$$\frac{-6acxF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(-6c(-2bc+2ad+bdx^2)+bdx^2\sqrt[4]{1+\frac{bx^2}{a}}\,(c+dx^2)\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-dx^3\left(6c(a+bx^2)-bx^2\sqrt[4]{1+\frac{bx^2}{a}}\,(c+dx^2)\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{12c^2(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)]*(-6*c*(-2*
b*c + 2*a*d + b*d*x^2) + b*d*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*AppellF1
[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]) - d*x^3*(6*c*(a + b*x^2) -
b*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2
)/a), -((d*x^2)/c)])*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))/(12*
c^2*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1,
 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)]))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{4}} \left(d\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^{2}}\,\left(c+dx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{1/4}\,\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)^2), x)
```

3.336 $\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=292

$$-\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}-\frac{\sqrt{a}\,\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\left(a+bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\,(5bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\Bigg(-\frac{\sqrt{}}{\sqrt{-}}}{4c(bc}$$

```
[Out] -1/2*d*x*(b*x^2+a)^(1/4)/c/(-a*d+b*c)/(d*x^2+c)-1/2*(1+b*x^2/a)^(3/4)*(cos(
1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*
EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/(-a
*d+b*c)/(b*x^2+a)^(3/4)+1/4*a^(1/4)*(-2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/
4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c
)^2/x+1/4*a^(1/4)*(-2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)
*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^2/x
```

Rubi [A]
time = 0.16, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {425, 544, 239, 237, 410, 109, 418, 1232}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^2}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^2}-\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2c\left(a+bx^2\right)^{3/4}(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x*(a + b*x^2)^(1/4))/(c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]
*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b
*c - a*d)*(a + b*x^2)^(3/4)) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*
EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4
)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b
*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^
2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} + \frac{\int \frac{2bc-ad-\frac{1}{2}bdx^2}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} - \frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{4c(bc-ad)} + \frac{(5bc-2ad)\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,d}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} + \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}\,(a+bx)^3}\right.}{8c(bc-ad)x}$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)(a+bx^2)^{3/4}} -$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)(a+bx^2)^{3/4}} +$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)(a+bx^2)^{3/4}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.24, size = 336, normalized size = 1.15

$$\frac{x\left(\frac{bdx^2\left(1+\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{-bc+ad}+\frac{c\left(36ac(-2bc+2ad+bdx^2)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-6dx^2(a+bx^2)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(bc-ad)(c+dx^2)\left(-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{12c^2(a+bx^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/(-(b*c) + a*d) + (c*(36*a*c*(-2*b*c + 2*a*d + b*d*x^2)*Appe
llF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 6*d*x^2*(a + b*x^2)*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF
```

```
1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))/((b*c - a*d)*(c + d*x^2)
*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*
d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3
/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(12*c^2*(a + b*x^2)^(3/4)
)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(d\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{2}\right)^{\frac{3}{4}}\left(c+dx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{3/4}\,{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)^2), x)
```

3.337 $\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=314

$$-\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,\left(c+dx^2\right)}+\frac{\sqrt{b}\,(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,c(bc-ad)^2\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{a}\,\sqrt{d}\,(7bc-2ad)}{}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)/(d*x^2+c)+1/2*(a*d+4*b*c)*(1+b*x^2/a)
^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2
)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/c
/(-a*d+b*c)^2/(b*x^2+a)^(1/4)/a^(1/2)-1/4*a^(1/4)*(-2*a*d+7*b*c)*EllipticPi
((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^
2/a)^(1/2)/c/(a*d-b*c)^(5/2)/x+1/4*a^(1/4)*(-2*a*d+7*b*c)*EllipticPi((b*x^2
+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(1/
2)/c/(a*d-b*c)^(5/2)/x
```

Rubi [A]
time = 0.28, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {425, 541, 544, 235, 233, 202, 408, 504, 1232}

$$-\frac{\sqrt[4]{a}\,\sqrt{d}\,\sqrt{-\frac{bx^2}{a}}\,(7bc-2ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{5/2}}+\frac{\sqrt[4]{a}\,\sqrt{d}\,\sqrt{-\frac{bx^2}{a}}\,(7bc-2ad)\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{5/2}}+\frac{\sqrt{b}\,\sqrt[4]{\frac{bx^2}{a}+1}\,(ad+4bc)E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,c\sqrt[4]{a+bx^2}\,(bc-ad)^2}-\frac{dx}{2c\sqrt[4]{a+bx^2}\,\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c
+ a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
2*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(1/4)) - (a^(1/4)*Sqrt[d]*(7*b*c - 2*
a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]),
 ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(5/2)*x) + (a^
(1/4)*Sqrt[d]*(7*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d
])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c)
 + a*d)^(5/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} + \frac{\int \frac{2bc-ad-\frac{3}{2}bdx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} - \frac{\int \frac{\frac{1}{2}(2b^2c^2+4abcd\cdots}{\sqrt[4]{a+\cdots}}}{ac(\cdots}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} - \frac{(d(7bc-2ad)\cdots}{4c\cdots}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} - \frac{\left(d(7bc-2ad\cdots\right.}{\cdots}$$

$$= -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} + \frac{\left(\sqrt{d}\,(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \cdots\right.}{4\cdots}$$

$$= -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} + \frac{\sqrt{b}\,(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\cdots\right.\right.}{2\sqrt{a}\,c(bc-ad)^2\sqrt[4]{a+bx^2}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.38, size = 380, normalized size = 1.21

$$\frac{x\left(-bd(4bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{c\left(36ac(2a^2d^2+abd(-4c+dx^2)+2b^2c(c+2dx^2))F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-6x^2(a^2d^2+abd^2x^2+4b^2c(c+dx^2))\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{12ac^2(bc-ad)^2\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*(-(b*d*(4*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)]) + (c*(36*a*c*(2*a^2*d^2 + a*b*d*(-4*c + d*x^
2) + 2*b^2*c*(c + 2*d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] - 6*x^2*(a^2*d^2 + a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4*a*d*Appell
F1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1
, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4
, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/
2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a
), -((d*x^2)/c)])))))/(12*a*c^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{4}} \left(d\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{2}\right)^{\frac{5}{4}} \left(c + dx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{5/4}\,{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)^2), x)
```

3.338 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx$

Optimal. Leaf size=345

$$\frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}(c+dx^2)}+\frac{\sqrt{b}\,(4bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\cdots\right.\right.}{6\sqrt{a}\,c(bc-ad)^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] 1/6*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(3/4)-1/2*d*x/c/(-a*d+b*c)
/(b*x^2+a)^(3/4)/(d*x^2+c)+1/6*(3*a*d+4*b*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arc
tan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*Ellipti
cF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/c/(-a*d+b*c)^2/(b*x^
2+a)^(3/4)/a^(1/2)-1/4*a^(1/4)*d*(-2*a*d+9*b*c)*EllipticPi((b*x^2+a)^(1/4)/
a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^3
/x-1/4*a^(1/4)*d*(-2*a*d+9*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*
d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^3/x
```

Rubi [A]

time = 0.26, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 541, 544, 239, 237, 410, 109, 418, 1232}

$$-\frac{\sqrt[4]{a}\,d\sqrt{-\frac{bx^2}{a}}\,(9bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^3}-\frac{\sqrt[4]{a}\,d\sqrt{-\frac{bx^2}{a}}\,(9bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^3}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\,c\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{bx(3ad+4bc)}{6ac\left(a+bx^2\right)^{3/4}(bc-ad)^2}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}(c+dx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (d*x)/(2*c*
(b*c - a*d)*(a + b*x^2)^(3/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c + 3*a*d)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*c*
(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x) - (a^(1/4)*d*(9*b*c - 2*a*d)*S
qrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(
a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-\frac{5}{2}bdx^2}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} - \frac{\int \frac{1}{2}\left(-2b^2c \ldots\right.}{\ldots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} - \frac{(d(9bc - \ldots}{\ldots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} - \frac{\left(d(9bc - \ldots\right.}{\ldots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\sqrt{b}\,(4bc \ldots}{\ldots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\sqrt{b}\,(4bc \ldots}{\ldots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\sqrt{b}\,(4bc \ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.38, size = 387, normalized size = 1.12

$$\frac{x\left(bd(4bc+3ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{c\left(36ac(6a^2d^2+3abd(-4c+dx^2)+2b^2c(3c+2dx^2))F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-6x^2(3a^2d^2+3abd^2x^2+4b^2c(c+dx^2))\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(c+dx^2)\left(6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right)}{36ac^2(bc-ad)^2\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*(b*d*(4*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)] + (c*(36*a*c*(6*a^2*d^2 + 3*a*b*d*(-4*c + d*x
^2) + 2*b^2*c*(3*c + 2*d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((
d*x^2)/c)] - 6*x^2*(3*a^2*d^2 + 3*a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4*a*d
*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/
2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1
[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3
/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2,
-((b*x^2)/a), -((d*x^2)/c)])))))/(36*a*c^2*(b*c - a*d)^2*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{7}{4}}\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{2}\right)^{\frac{7}{4}} \left(c + dx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{7/4}\,{\left(d\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)^2), x)
```

3.339 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx$

Optimal. Leaf size=371

$$\frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}+\frac{\sqrt{b}\left(12b^2c^2-52abcd-5a^2d^2\right)\sqrt[4]{1+\frac{bx^2}{a}}\,E}{10a^{3/2}c(bc-ad)^3\sqrt[4]{a+}}$$

```
[Out] 1/10*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(5/4)-1/2*d*x/c/(-a*d+b*c
)/(b*x^2+a)^(5/4)/(d*x^2+c)+1/10*(-5*a^2*d^2-52*a*b*c*d+12*b^2*c^2)*(1+b*x^
2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^
(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/
2)/a^(3/2)/c/(-a*d+b*c)^3/(b*x^2+a)^(1/4)-1/4*a^(1/4)*d^(3/2)*(-2*a*d+11*b*
c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(
-b*x^2/a)^(1/2)/c/(a*d-b*c)^(7/2)/x+1/4*a^(1/4)*d^(3/2)*(-2*a*d+11*b*c)*Ell
ipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/
a)^(1/2)/c/(a*d-b*c)^(7/2)/x
```

Rubi [A]
time = 0.39, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 541, 544, 235, 233, 202, 408, 504, 1232}

$$\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c\sqrt[4]{a+bx^2}(bc-ad)^3}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{7/2}}+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}+\frac{bx(5ad+4bc)}{10ac\left(a+bx^2\right)^{5/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(10*a*c*(b*c - a*d)^2*(a + b*x^2)^(5/4)) - (d*x)/(2*c
*(b*c - a*d)*(a + b*x^2)^(5/4)*(c + d*x^2)) + (Sqrt[b]*(12*b^2*c^2 - 52*a*b
*c*d - 5*a^2*d^2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(10*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(1/4)) - (a^(1/4)*d^(3/2)
*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(
7/2)*x) + (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(
Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]
)/(4*c*(-(b*c) + a*d)^(7/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)(a+bx^2)^{5/4}(c+dx^2)} + \frac{\int \frac{2bc-ad-\frac{7}{2}bdx^2}{(a+bx^2)^{9/4}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{5/4}(c+dx^2)} - \frac{\int \frac{1}{2}(-6b^2c^2}{}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} + \frac{b(12b^2c^2-52abcd-5a^2d^2)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}} - \frac{}{2c(bc-ad)(a}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} + \frac{b(12b^2c^2-52abcd-5a^2d^2)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}} - \frac{}{2c(bc-ad)(a}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} + \frac{b(12b^2c^2-52abcd-5a^2d^2)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}} - \frac{}{2c(bc-ad)(a}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{5/4}(c+dx^2)} - \frac{\Big(d^{3/2}(11}{}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{5/4}(c+dx^2)} + \frac{\sqrt{b}\,(12b^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.76, size = 536, normalized size = 1.44

$$\frac{bd(-12b^2c^2+52abcd+5a^2d^2)\,x^3\sqrt[4]{1+\frac{bx^2}{a}}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{6c\left(-6acx\left(10a^4d^3+15a^3bd^2(-2c+dx^2)-6b^4c^2x^2(c+2dx^2)+a^2b^2d(30c^2+26cdx^2+5d^2x^4)+2ab^3c(-5c^2+5cdx^2+26d^2x^4)\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^3\left(5a^4d^3+10a^3bd^3x^2-12b^4c^2x^2(c+dx^2)+a^2b^2d(56c^2+56cdx^2+5d^2x^4)+4ab^3c(-4c^2+9cdx^2+13d^2x^4)\right)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(a+bx^2)(c+dx^2)\left(6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}}{60a^2c^2(bc-ad)^3\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*d*(-12*b^2*c^2 + 52*a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(1/4)*Appel
lF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*x*(10*a^4*
d^3 + 15*a^3*b*d^2*(-2*c + d*x^2) - 6*b^4*c^2*x^2*(c + 2*d*x^2) + a^2*b^2*d
*(30*c^2 + 26*c*d*x^2 + 5*d^2*x^4) + 2*a*b^3*c*(-5*c^2 + 5*c*d*x^2 + 26*d^2
*x^4))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^3*(5*a^4*
d^3 + 10*a^3*b*d^3*x^2 - 12*b^4*c^2*x^2*(c + d*x^2) + a^2*b^2*d*(56*c^2 + 5
6*c*d*x^2 + 5*d^2*x^4) + 4*a*b^3*c*(-4*c^2 + 9*c*d*x^2 + 13*d^2*x^4))*(4*a*
d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2
, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(6*a
*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*Appe
llF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4,
 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(60*a^2*c^2*(b*c - a*d)^3*(a + b*x
^2)^(1/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{9}{4}}\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{\frac{9}{4}}(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{9/4}\,{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)^2), x)
```

3.340 $\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=419

$$\frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}}+\frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}+\frac{\sqrt{b}\,(20b^2}{}$$

```
[Out] 1/14*b*(7*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(7/4)+1/42*b*(-21*a^2*d^2
-76*a*b*c*d+20*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^2+a)^(3/4)-1/2*d*x/c/(-a*
d+b*c)/(b*x^2+a)^(7/4)/(d*x^2+c)+1/42*(-21*a^2*d^2-76*a*b*c*d+20*b^2*c^2)*(
1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcta
n(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))
*b^(1/2)/a^(3/2)/c/(-a*d+b*c)^3/(b*x^2+a)^(3/4)+1/4*a^(1/4)*d^2*(-2*a*d+13*
b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)
*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^4/x+1/4*a^(1/4)*d^2*(-2*a*d+13*b*c)*Elliptic
Pi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1
/2)/c/(-a*d+b*c)^4/x
```

Rubi [A]
time = 0.33, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 541, 544, 239, 237, 410, 109, 418, 1232}

$$\frac{bx(-21a^2d^2-76abcd+20b^2c^2)}{42a^2c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(-21a^2d^2-76abcd+20b^2c^2)\,F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{42a^{3/2}c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt[4]{a}\,d^2\sqrt{-\frac{bx^2}{a}}\,(13bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^4}+\frac{\sqrt[4]{a}\,d^2\sqrt{-\frac{bx^2}{a}}\,(13bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{ArcSin}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^4}-\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}+\frac{bx(7ad+4bc)}{14ac\left(a+bx^2\right)^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 7*a*d)*x)/(14*a*c*(b*c - a*d)^2*(a + b*x^2)^(7/4)) + (b*(20*b^2
*c^2 - 76*a*b*c*d - 21*a^2*d^2)*x)/(42*a^2*c*(b*c - a*d)^3*(a + b*x^2)^(3/4
)) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(7/4)*(c + d*x^2)) + (Sqrt[b]*(20*b
^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(42*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(3/4)) +
(a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt
[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c
 - a*d)^4*x) + (a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1
])/(4*c*(b*c - a*d)^4*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
```

```
x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)(a+bx^2)^{7/4}(c+dx^2)} + \frac{\int \frac{2bc-ad-\frac{9}{2}bdx^2}{(a+bx^2)^{11/4}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{7/4}(c+dx^2)} - \frac{\int \frac{1}{2}(-10b$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{}{2c(bc-ad)}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{}{2c(bc-ad)}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{}{2c(bc-ad)}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{}{2c(bc-ad)}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{}{2c(bc-ad)}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{}{2c(bc-ad)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.67, size = 550, normalized size = 1.31

$$\frac{\frac{bd(-20b^2c^2+76abcd+21a^2d^2)x^3\left(1+\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(-bc+ad)^3} + \frac{6c\left(-6acx\left(42a^4d^3+63a^3bd^2(-2c+dx^2)-10b^4c^2x^2(3c+2dx^2)+a^2b^2d(126c^2-38cdx^2+21d^2x^4)+2ab^3c(-21c^2+41cdx^2+38d^2x^4)\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^3\left(21a^4d^3+42a^3bd^3x^2-20b^4c^2x^2(c+dx^2)+4ab^3c(-8c^2+11cdx^2+19d^2x^4)+a^2b^2d(88c^2+88cdx^2+21d^2x^4)\right)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}{(bc-ad)^3(a+bx^2)(c+dx^2)\left(6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}}{252a^2c^2(a+bx^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] ((b*d*(-20*b^2*c^2 + 76*a*b*c*d + 21*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(3/4)*App
ellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(-(b*c) + a*d)^3 + (6*
c*(-6*a*c*x*(42*a^4*d^3 + 63*a^3*b*d^2*(-2*c + d*x^2) - 10*b^4*c^2*x^2*(3*c
 + 2*d*x^2) + a^2*b^2*d*(126*c^2 - 38*c*d*x^2 + 21*d^2*x^4) + 2*a*b^3*c*(-2
1*c^2 + 41*c*d*x^2 + 38*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] + x^3*(21*a^4*d^3 + 42*a^3*b*d^3*x^2 - 20*b^4*c^2*x^2*(c + d*
x^2) + 4*a*b^3*c*(-8*c^2 + 11*c*d*x^2 + 19*d^2*x^4) + a^2*b^2*d*(88*c^2 + 8
8*c*d*x^2 + 21*d^2*x^4))*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -(
(d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])
))/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)*(6*a*c*AppellF1[1/2, 3/4, 1, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d
*x^2)/c)]))))/(252*a^2*c^2*(a + b*x^2)^(3/4))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{11}{4}} \left(d\,x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{11}{4}}\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(11/4)*(c + d*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^{11/4}\,{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)^2), x)
```

3.341 $\int (a+bx^2)^p (c+dx^2)^q \,dx$

Optimal. Leaf size=79

$$x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((1+b*x
^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx$$
$$= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int \left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right.$$
$$= x\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.
time = 0.20, size = 172, normalized size = 2.18

$$\frac{3acx(a+bx^2)^p \left(c+dx^2\right)^q F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3acF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(bcpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a
), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)
/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int((a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.342 $\int (a+bx^2)^p (c+dx^2)^3 \, dx$

Optimal. Leaf size=296

$$\frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))\,x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)}-\frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p}\,(c+dx^2)}{b^2(5+2p)(7+2p)}$$

```
[Out] d*(15*a^2*d^2-8*a*b*c*d*(6+p)+b^2*c^2*(4*p^2+28*p+57))*x*(b*x^2+a)^(1+p)/b^
3/(8*p^3+60*p^2+142*p+105)-d*(5*a*d-b*c*(11+2*p))*x*(b*x^2+a)^(1+p)*(d*x^2+
c)/b^2/(4*p^2+24*p+35)+d*x*(b*x^2+a)^(1+p)*(d*x^2+c)^2/b/(7+2*p)-(15*a^3*d^
3-9*a^2*b*c*d^2*(7+2*p)+3*a*b^2*c^2*d*(4*p^2+24*p+35)-b^3*c^3*(8*p^3+60*p^2
+142*p+105))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b^3/(8*p^3+6
0*p^2+142*p+105)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.20, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {427, 542, 396, 252, 251}

$$\frac{dx(a+bx^2)^{p+1}(15a^2d^2-8abcd(p+6)+b^2c^2(4p^2+28p+57))}{b^3(2p+3)(2p+5)(2p+7)}-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(15a^3d^3-9a^2bcd^2(2p+7)+3ab^2c^2d(4p^2+24p+35)-b^3c^3(8p^3+60p^2+142p+105))\;{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b^3(2p+3)(2p+5)(2p+7)}-\frac{dx(c+dx^2)(a+bx^2)^{p+1}(5ad-bc(2p+11))}{b^2(2p+5)(2p+7)}+\frac{dx(c+dx^2)^2(a+bx^2)^{p+1}}{b(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

```
[Out] (d*(15*a^2*d^2 - 8*a*b*c*d*(6 + p) + b^2*c^2*(57 + 28*p + 4*p^2))*x*(a + b*
x^2)^(1 + p))/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)) - (d*(5*a*d - b*c*(11 + 2
*p))*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b^2*(5 + 2*p)*(7 + 2*p)) + (d*x*(a
 + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b*(7 + 2*p)) - ((15*a^3*d^3 - 9*a^2*b*c*d
^2*(7 + 2*p) + 3*a*b^2*c^2*d*(35 + 24*p + 4*p^2) - b^3*c^3*(105 + 142*p + 6
0*p^2 + 8*p^3))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx = \frac{dx(a+bx^2)^{1+p} (c+dx^2)^2}{b(7+2p)} + \frac{\int (a+bx^2)^p (c+dx^2) (-c(ad-bc(7+2p)) - \dots}{b(7+2p)}$$

$$= -\frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p} (c+dx^2)}{b^2(5+2p)(7+2p)} + \frac{dx(a+bx^2)^{1+p} (c+dx^2)^2}{b(7+2p)} \dots$$

$$= \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))\, x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-b \dots}{\dots}$$

$$= \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))\, x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-b \dots}{\dots}$$

$$= \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))\, x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-b \dots}{\dots}$$

Mathematica [A]

time = 5.13, size = 136, normalized size = 0.46

$$\frac{1}{35}x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(35c^3\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+dx^2\left(35c^2\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+dx^2\left(21c\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+5dx^2\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^p*(35*c^3*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d*
x^2*(35*c^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + d*x^2*(21*c*Hyp
ergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)] + 5*d*x^2*Hypergeometric2F1[7/2,
 -p, 9/2, -((b*x^2)/a)]))))/(35*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x^2 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 20.04, size = 121, normalized size = 0.41

$$a^pc^3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+a^pc^2dx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+\frac{3a^pcd^2x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+\frac{a^pd^3x^7\,{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**3,x)
```

```
[Out] a**p*c**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*c**2*
d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*a**p*c*d**2*x
**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*d**3*x**7*h
yper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^p*(c + d*x^2)^3, x)
```

3.343 $\int (a+bx^2)^p (c+dx^2)^2 \, dx$

Optimal. Leaf size=176

$$-\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)}+\frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)}+\frac{(3a^2d^2-2abcd(5+2p)+b^2c^2(15+16p+4}{b^2(3+}$$

```
[Out] -d*(3*a*d-b*c*(7+2*p))*x*(b*x^2+a)^(1+p)/b^2/(4*p^2+16*p+15)+d*x*(b*x^2+a)^
(1+p)*(d*x^2+c)/b/(5+2*p)+(3*a^2*d^2-2*a*b*c*d*(5+2*p)+b^2*c^2*(4*p^2+16*p+
15))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b^2/(4*p^2+16*p+15)/
((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {427, 396, 252, 251}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3a^2d^2-2abcd(2p+5)+b^2c^2(4p^2+16p+15))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b^2(2p+3)(2p+5)}-\frac{dx(a+bx^2)^{p+1}(3ad-bc(2p+7))}{b^2(2p+3)(2p+5)}+\frac{dx(c+dx^2)(a+bx^2)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

```
[Out] -((d*(3*a*d - b*c*(7 + 2*p))*x*(a + b*x^2)^(1 + p))/(b^2*(3 + 2*p)*(5 + 2*p
))) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p)) + ((3*a^2*d^2 - 2
*a*b*c*d*(5 + 2*p) + b^2*c^2*(15 + 16*p + 4*p^2))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b^2*(3 + 2*p)*(5 + 2*p)*(1 + (b*x^2)
/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int (a+bx^2)^p (c+dx^2)^2\,dx = \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)} + \frac{\int (a+bx^2)^p(-c(ad-bc(5+2p))-d(3ad-bc(}{b(5+2p)}$$

$$= -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)} + \frac{(3a^2d^2 -}{}$$

$$= -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)} + \frac{\left((3a^2d^2\right.}{}$$

$$= -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)} + \frac{(3a^2d^2 -}{}$$

Mathematica **[A]**
time = 5.10, size = 106, normalized size = 0.60

$$\frac{1}{15}x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(15c^2\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+dx^2\left(10c\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+3dx^2\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*(15*c^2*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d*
x^2*(10*c*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 3*d*x^2*Hypergeom
etric2F1[5/2, -p, 7/2, -((b*x^2)/a)])))/(15*(1 + (b*x^2)/a)^p)
```

Maple **[F]**
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x^2 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.93, size = 88, normalized size = 0.50

$$a^p c^2 x {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{2a^p c d x^3 {}_2F_1\left(\left.\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + \frac{a^p d^2 x^5 {}_2F_1\left(\left.\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**2,x)
```

```
[Out] a**p*c**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + 2*a**p*c*d
*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*d**2*x**5
*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^p*(c + d*x^2)^2, x)
```

3.344 $\int \left(a+bx^2\right)^p \left(c+dx^2\right) dx$

Optimal. Leaf size=93

$$\frac{dx(a+bx^2)^{1+p}}{b(3+2p)} - \frac{(ad-bc(3+2p))x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(3+2p)}$$

```
[Out] d*x*(b*x^2+a)^(1+p)/b/(3+2*p)-(a*d-b*c*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/
2, -p],[3/2],-b*x^2/a)/b/(3+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$x\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c-\frac{ad}{2bp+3b}\right) {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) + \frac{dx\left(a+bx^2\right)^{p+1}}{b(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2),x]
```

```
[Out] (d*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((c - (a*d)/(3*b + 2*b*p))*x*(a +
 b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)\,dx = \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} - \left(-c+\frac{ad}{3b+2bp}\right)\int \left(a+bx^2\right)^p\,dx$$
$$= \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} - \left(\left(-c+\frac{ad}{3b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right.$$
$$= \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} + \left(c-\frac{ad}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\right.$$

Mathematica [A]
time = 0.07, size = 90, normalized size = 0.97

$$\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(d(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p+(-ad+bc(3+2p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{b(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2),x]
[Out] (x*(a + b*x^2)^p*(d*(a + b*x^2)*(1 + (b*x^2)/a)^p + (-(a*d) + b*c*(3 + 2*p)
)*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(b*(3 + 2*p)*(1 + (b*x^2)
/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c),x)
[Out] int((b*x^2+a)^p*(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="fricas")
[Out] integral((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.68, size = 53, normalized size = 0.57

$$a^{p}cx\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2}, -p \\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right) + \frac{a^{p}dx^{3}\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c),x)
[Out] a**p*c*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*x**3*h
yper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^2+a\right)}^p\,\left(d\,x^2+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(c + d*x^2),x)
[Out] int((a + b*x^2)^p*(c + d*x^2), x)
```

3.345 $\int (a+bx^2)^p \, dx$

Optimal. Leaf size=44

$$x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

```
[Out] x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {252, 251}

$$x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p dx\\ &= x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p,x]
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p,x)
[Out] int((b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.07, size = 22, normalized size = 0.50

$$a^p x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p,x)
```

```
[Out] a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

Mupad [B]
time = 5.57, size = 41, normalized size = 0.93

$$\frac{x\,{\left(b\,x^2+a\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p,x)
```

```
[Out] (x*(a + b*x^2)^p*hypergeom([1/2, -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

3.346 $\int \frac{(a+bx^2)^p}{c+dx^2}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,-p,1,3/2,-b*x^2/a,-d*x^2/c)/c/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c*
(1 + (b*x^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{c+dx^2}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{c+dx^2}\,dx\\ &= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.22, size = 162, normalized size = 2.84

$$-\frac{3acx(a+bx^2)^p F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(-3acF_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(-bcpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)*(-3*a*c*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/
c)] + 2*x^2*(-(b*c*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c
)]) + a*d*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d*x^2 + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^p/(c + d*x^2), x)
```

3.347 $\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,-p,2,3/2,-b*x^2/a,-d*x^2/c)/c^2/((1+b*x^2/a)^p)
```

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
2*(1 + (b*x^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(c+dx^2)^2}\,dx\\ &= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.25, size = 162, normalized size = 2.84

$$-\frac{3acx(a+bx^2)^p F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^2\left(-3acF_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(bcpF_1\left(\frac{3}{2};1-p,2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2adF_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)^2*(-3*a*c*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] - 2*a*d*AppellF1[3/2, -p, 3, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(d*x**2+c)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^p/(c + d*x^2)^2, x)
```

3.348 $\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,-p,3,3/2,-b*x^2/a,-d*x^2/c)/c^3/((1+b*x^2/a)^p)
```

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
3*(1 + (b*x^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(c+dx^2)^3}\,dx\\ &= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.33, size = 162, normalized size = 2.84

$$-\frac{3acx(a+bx^2)^p F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^3\left(-3acF_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(bcpF_1\left(\frac{3}{2};1-p,3;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-3adF_1\left(\frac{3}{2};-p,4;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)^3*(-3*a*c*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 3, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] - 3*a*d*AppellF1[3/2, -p, 4, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^p/(c + d*x^2)^3, x)
```

3.349 $\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}} \, dx$

Optimal. Leaf size=53

$$\frac{x(a+bx^2)^{-\frac{bc}{2bc-2ad}} (c+dx^2)^{\frac{ad}{2bc-2ad}}}{ac}$$

```
[Out] x*(d*x^2+c)^(a*d/(-2*a*d+2*b*c))/a/c/((b*x^2+a)^(b*c/(-2*a*d+2*b*c)))
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{389\}$

$$\frac{x(a+bx^2)^{-\frac{bc}{2bc-2ad}} (c+dx^2)^{\frac{ad}{2bc-2ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/(2*b*c
 - 2*a*d)),x]
```

```
[Out] (x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(2*b*c - 2*
a*d)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}} \, dx = \frac{x(a+bx^2)^{-\frac{bc}{2bc-2ad}} (c+dx^2)^{\frac{ad}{2bc-2ad}}}{ac}$$

Mathematica [A]
time = 0.41, size = 52, normalized size = 0.98

$$\frac{x(a+bx^2)^{\frac{bc}{-2bc+2ad}} (c+dx^2)^{\frac{ad}{2bc-2ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/
(2*b*c - 2*a*d)),x]
```

```
[Out] (x*(a + b*x^2)^((b*c)/(-2*b*c + 2*a*d))*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d))
)/(a*c)
```

Maple [A]
time = 0.13, size = 71, normalized size = 1.34

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)^{1-\frac{2 a d-3 b c}{2 (a d-b c)}} \left(d\,x^{2}+c\right)^{1-\frac{3 a d-2 b c}{2 (a d-b c)}} x}{a c}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1-1/2*(2*a*d-3*b*c)/(a*d-b*c))*(d*x^2+c)^(1-1/2*(3*a*d-2*b*c)/(a
*d-b*c))/a/c*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

Fricas [A]
time = 0.96, size = 91, normalized size = 1.72

$$\frac{bdx^5 + (bc + ad)x^3 + acx}{(bx^2 + a)^{\frac{3\,bc-2\,ad}{2\,(bc-ad)}}(dx^2 + c)^{\frac{2\,bc-3\,ad}{2\,(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^5 + (b*c + a*d)*x^3 + a*c*x)/((b*x^2 + a)^(1/2*(3*b*c - 2*a*d)/(b*c
- a*d))*(d*x^2 + c)^(1/2*(2*b*c - 3*a*d)/(b*c - a*d))*a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^{-\frac{bc}{-2ad+2bc}-1}\left(c+dx^2\right)^{\frac{ad}{-2ad+2bc}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(-1-b*c/(-2*a*d+2*b*c))*(d*x**2+c)**(-1+a*d/(-2*a*d+2
*b*c)),x)
```

```
[Out] Integral((a + b*x**2)**(-b*c/(-2*a*d + 2*b*c) - 1)*(c + d*x**2)**(a*d/(-2*a
*d + 2*b*c) - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

Mupad [B]
time = 5.74, size = 131, normalized size = 2.47

$$\frac{x\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}+\frac{x^3\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}\left(a\,d+b\,c\right)}{a\,c}+\frac{b\,d\,x^5\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}}{a\,c}}{\left(d\,x^2+c\right)^{\frac{a\,d}{2\,a\,d-2\,b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1)/(c + d*x^2)^((a*d)/(2*a*d - 2*b
*c) + 1),x)
```

```
[Out] (x*(a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1) + (x^3*(a + b*x^2)^((b*c)/(2*a*d
 - 2*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^5*(a + b*x^2)^((b*c)/(2*a*d - 2*
b*c) - 1))/(a*c))/(c + d*x^2)^((a*d)/(2*a*d - 2*b*c) + 1)
```

Chapter 4

Appendix

Local contents

4.1 Download section 1564
4.2 Listing of Grading functions 1564

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```